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“Photons beyond qubits”, Olomouc, 16/04/2014

# Multimode state generation and detection & hybrid quantum-classical entanglement

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*Florence, Italy*



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# Photons beyond qubits

How to go beyond?

New forms of qubit encodings

Continuous variables  
Spatial, temporal, spectral modes

More modes (qudits)

Hybrid discrete/CV entangled qubits



# Our tools

Single-photon-level light manipulation

Measuring light at the quantum level

Single-photon addition and subtraction

Balanced homodyne detection

Sequences and superpositions of quantum operators

Multi-mode state analysis and m-m operator superpositions

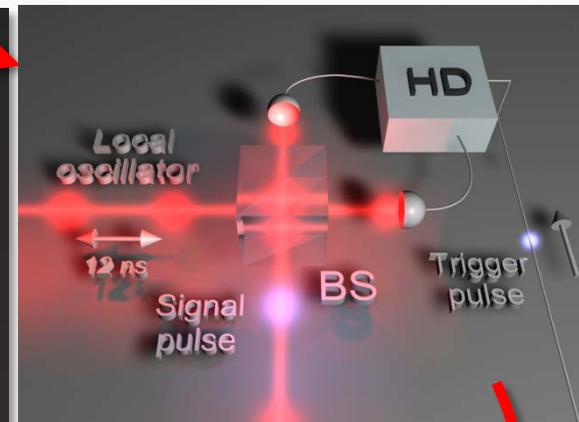
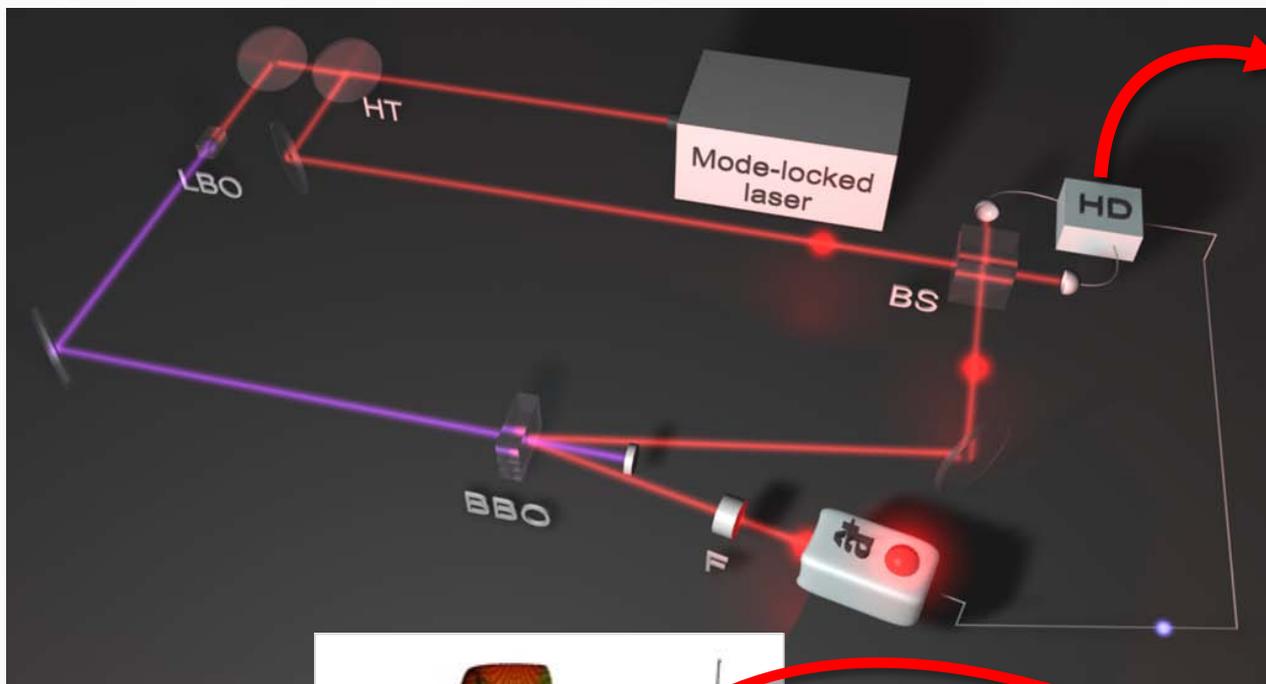
Direct probing of  
fundamental quantum rules

Quantum processes:  
noiseless amplification, ...

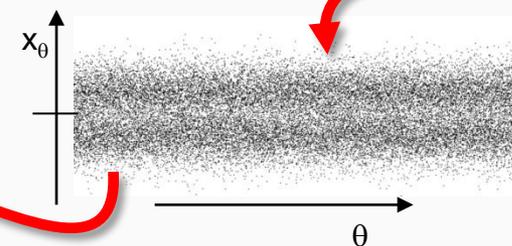
Accessing states in arbitrary spectro-temporal modes  
Generating hybrid quantum-classical entangled states



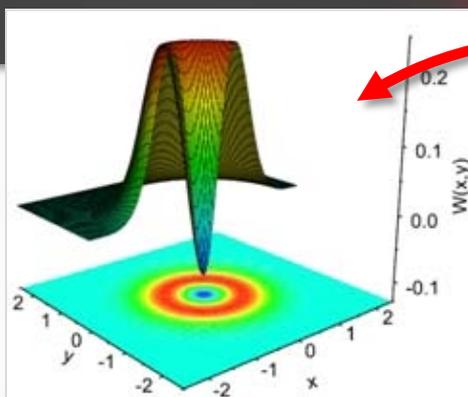
# Single photons in the lab



Acquire homodyne data



Collect quadrature distributions



Reconstruct the  
Wigner function and  
the density matrix



# The shape of a single photon

## Manipulating the color and shape of single photons

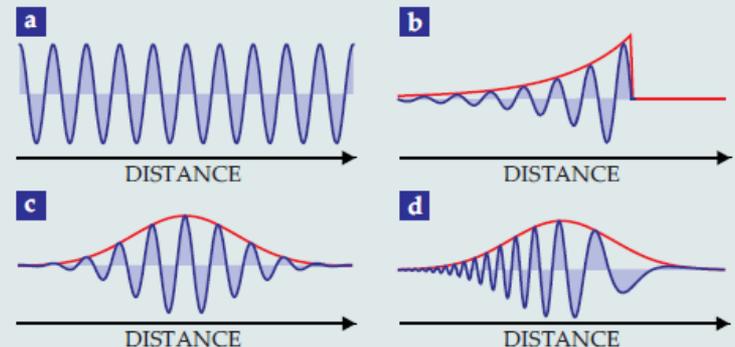
In a future quantum internet, individual photons might well be the agents that carry information between different kinds of devices. But physicists must first learn to tailor some of their essential features.

Michael G. Raymer  
and Kartik Srinivasan

physics  
today

November 2012

**Figure 1. Single photons** exist in a variety of shapes. These four examples show various photons at an instant in time; the red lines in panels b–d indicate wavepacket envelopes. (a) A monochromatic photon produced by an ideal laser. (b) A decaying exponential packet spontaneously emitted from an excited atom. (c) A Gaussian packet created by nonlinear optical processes. (d) A so-called chirped-frequency packet resulting from dispersive propagation in optical fiber.



Optimized detection

Efficient coupling to atomic quantum memories

Multimode encoding and detection in higher-dimensional Hilbert spaces

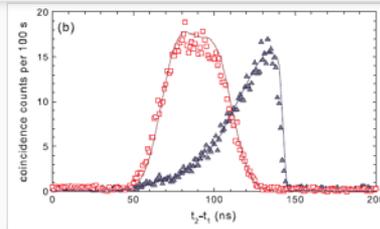


# The shape of “long” photons

PRL 101, 103601 (2008) PHYSICAL REVIEW LETTERS week ending 5 SEPTEMBER 2008

## Electro-Optic Modulation of Single Photons

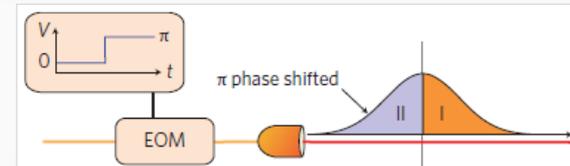
Pavel Kolchin,\* Chinmay Belthangady, Shengwang Du, G. Y. Yin, and S. E. Harris



nature photonics LETTERS  
PUBLISHED ONLINE: 13 JULY 2009 | DOI: 10.1038/NPHOTON.2009.115

## Phase shaping of single-photon wave packets

H. P. Specht, J. Bochmann, M. Mücke, B. Weber, E. Figueroa, D. L. Moehring\* and G. Rempe



Amplitude and phase modulation has been recently achieved for narrowband, long (100ns - 1 $\mu$ s), photons  
Both modulation and detection can be performed with standard electronics

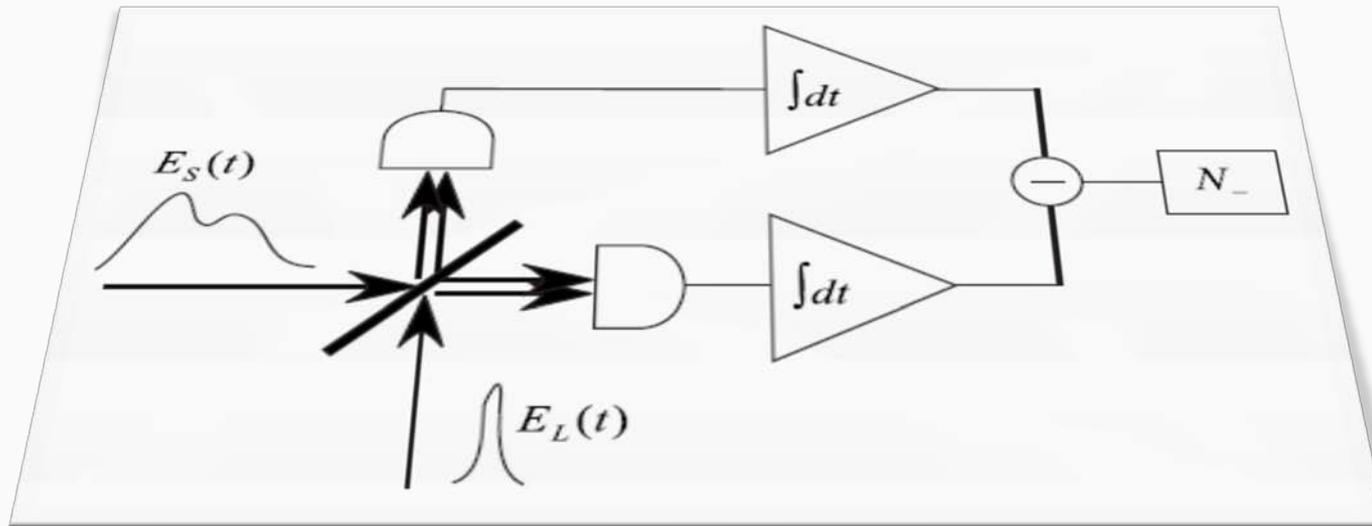
## In our case:

Manipulate and characterize the spectrotemporal mode of broadband, ultrashort (<100 fs), quantum states

Ultrafast + Quantum  
Optics

# Modal selectivity of homodyne detection

Homodyne detection only extracts information from the signal field in the mode defined by the local oscillator

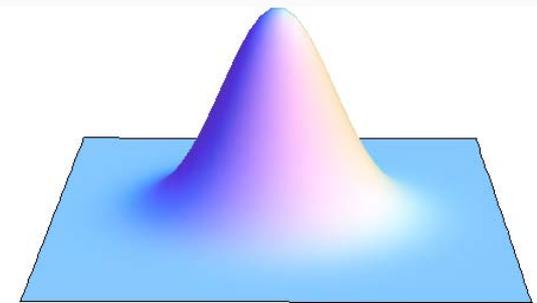
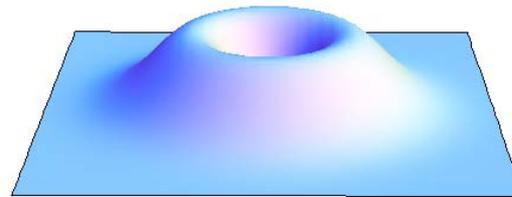
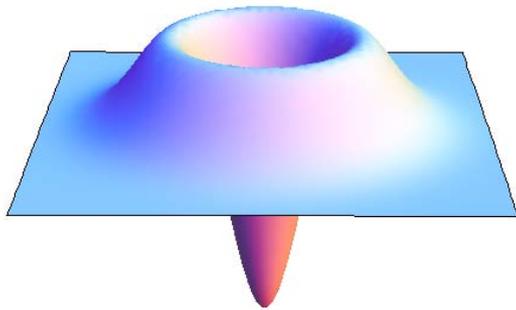
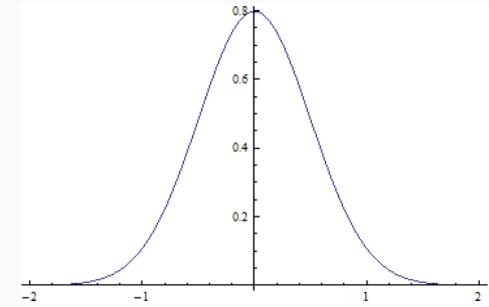
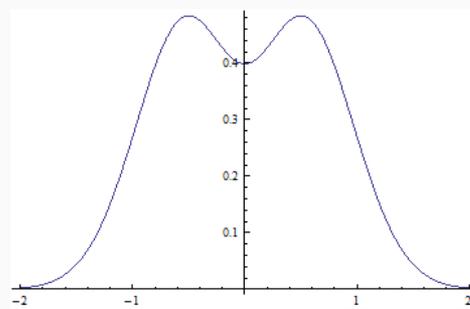
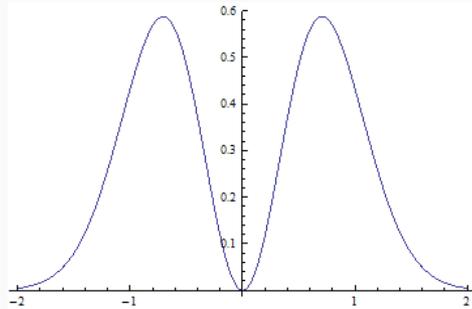
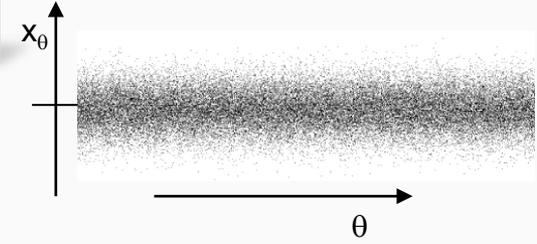
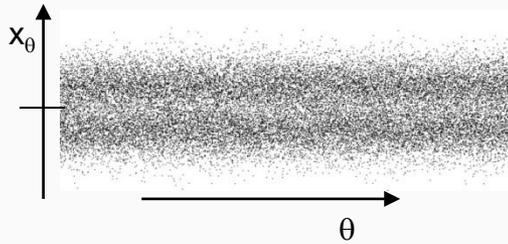


The (spatial, temporal, spectral, polarization) mode of the local oscillator has to be perfectly matched to that of the state to be measured, otherwise detection efficiency rapidly drops to zero



# Wigner function of a single photon

$$\eta |1\rangle \langle 1| + (1 - \eta) |0\rangle \langle 0|$$



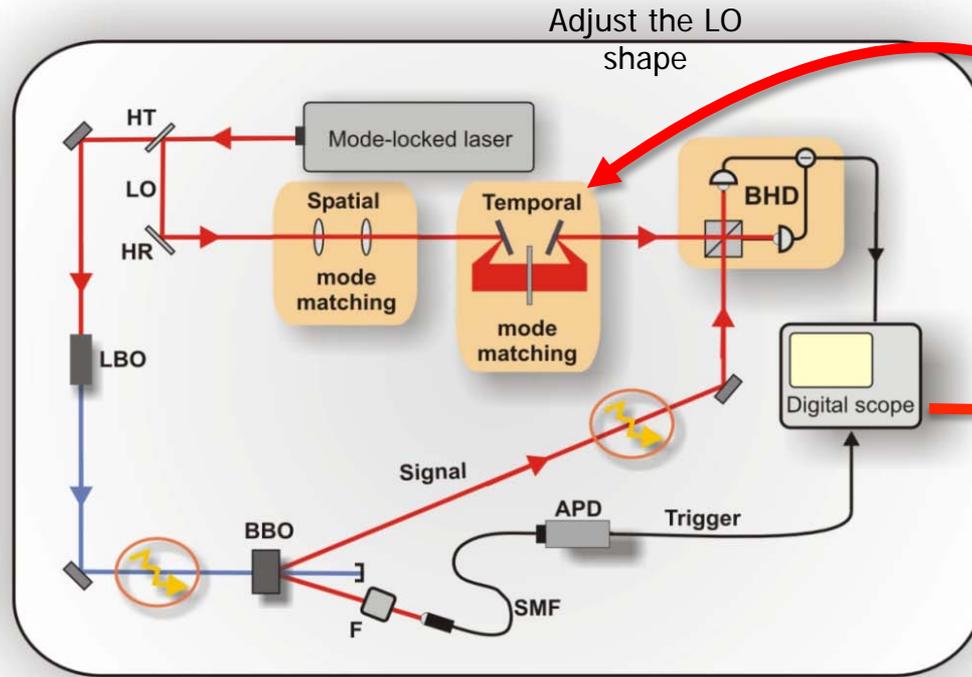
Single photon

decreasing detection  
efficiency

Vacuum

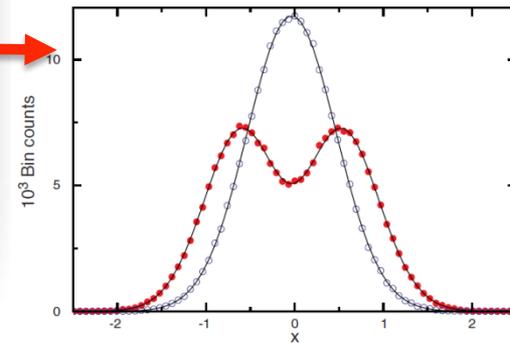


# Searching for the photon shape



Shaping the LO to maximize the homodyne detection efficiency

Calculate the efficiency  $\eta$



Single-photon quadrature distribution

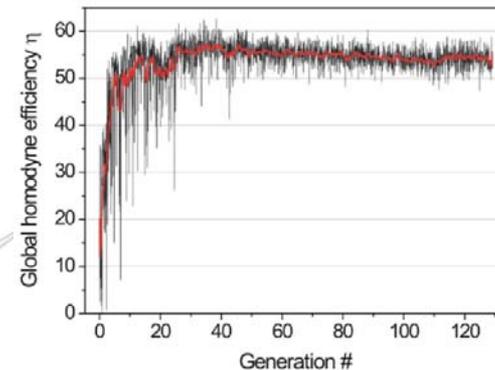
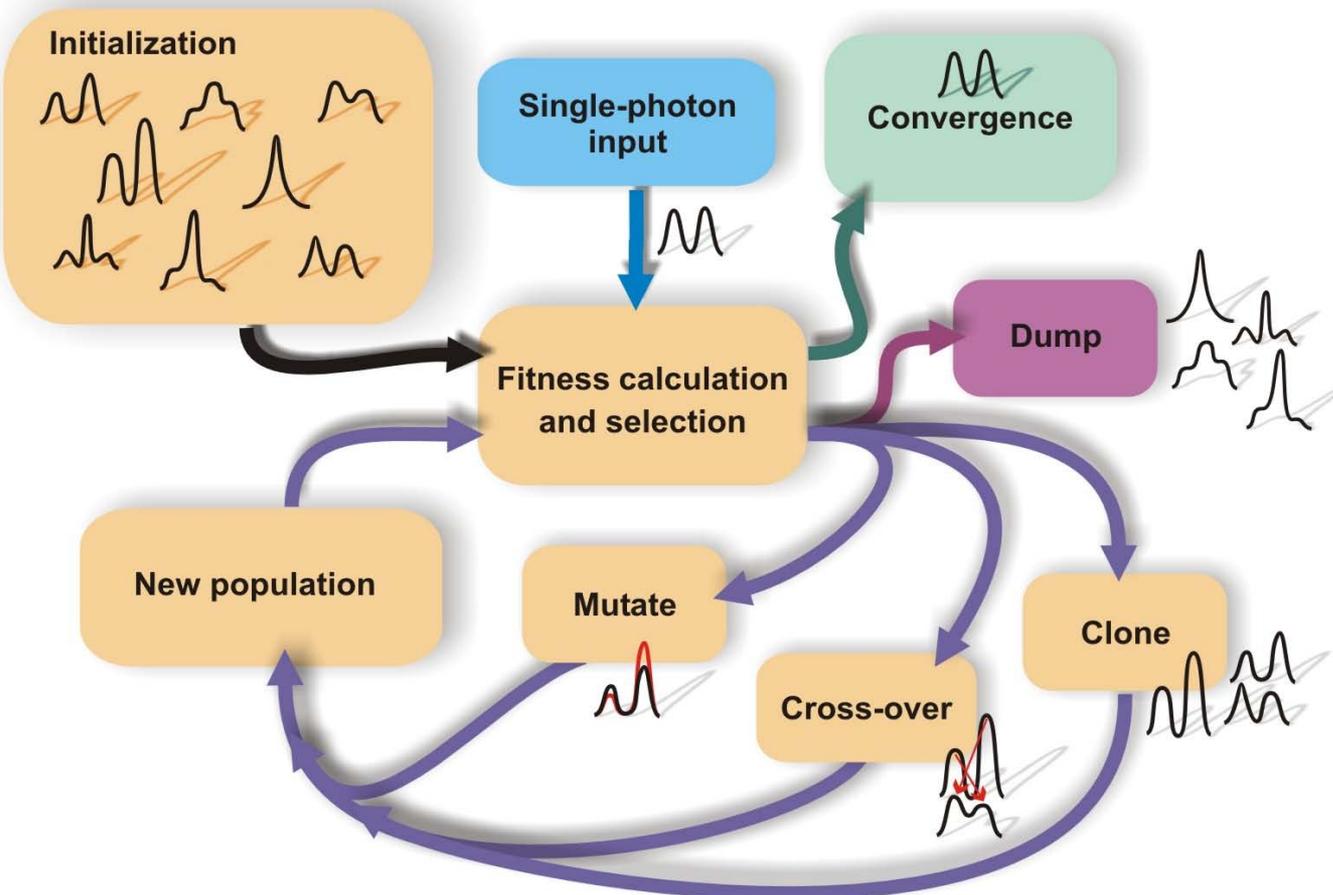
$$\eta |1\rangle \langle 1| + (1 - \eta) |0\rangle \langle 0|$$

$\eta$  quantifies the amount of pure single photon in the detected mixed state



# Genetic search of the photon shape

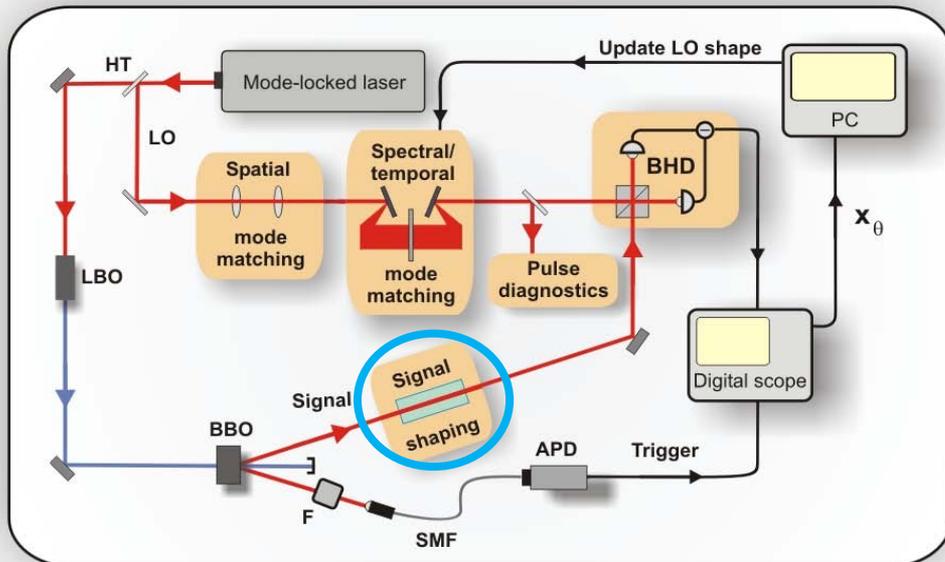
Using an evolutionary algorithm to find the best LO pulse shape



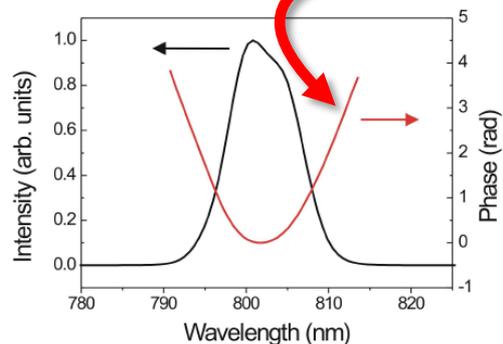
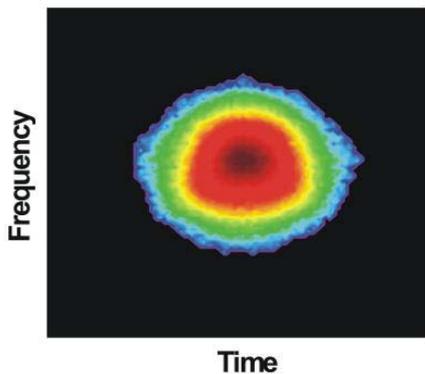
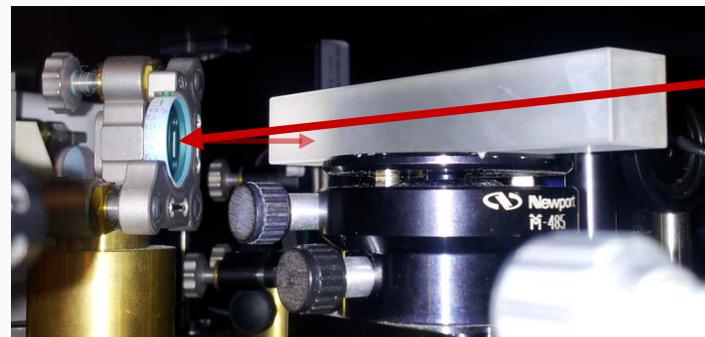
No preliminary information required!



# Shaping the photon



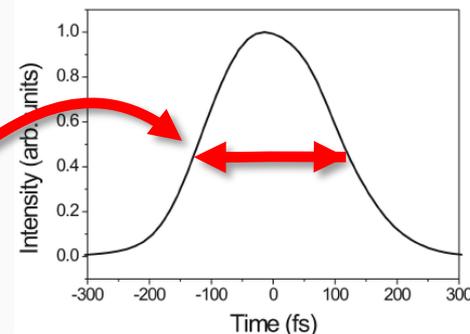
Linear dispersion by propagation through a 10-cm-long block of BK7 glass



Quadratic spectral phase  
(positively chirped  
single-photon pulse)

$$\Delta\lambda_{\text{FWHM}} \sim 9.5 \text{ nm}$$

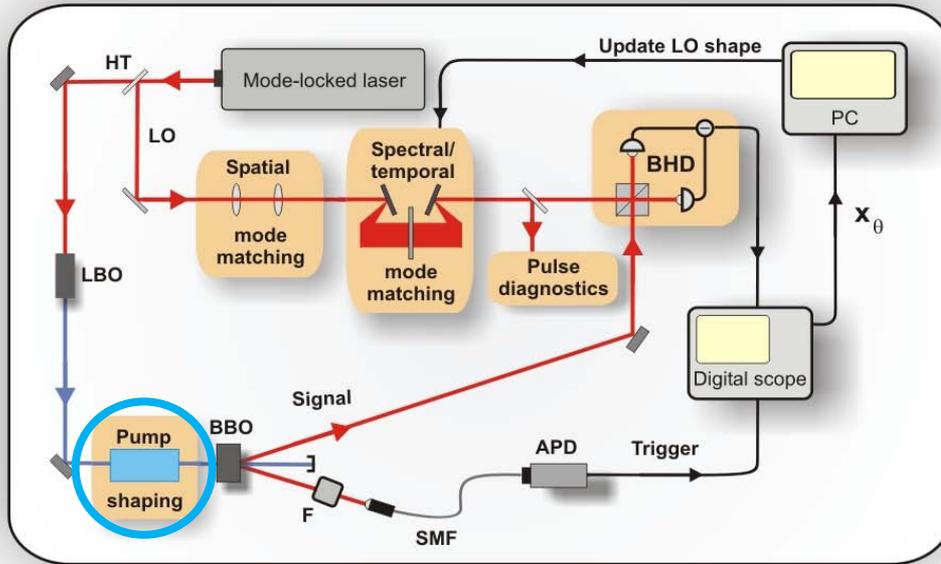
$$\Delta\tau_{\text{FWHM}} > 200 \text{ fs}$$



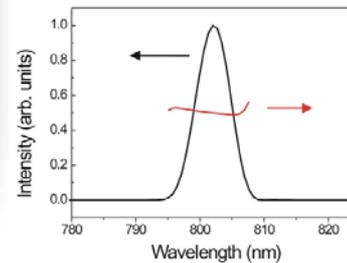
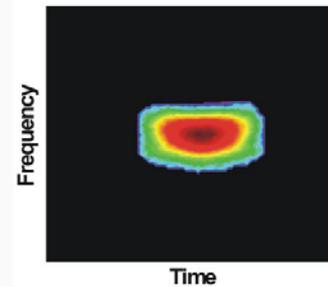
Detection efficiency would drop from  $\sim 60\%$  to  $\sim 40\%$  without shaping the LO!



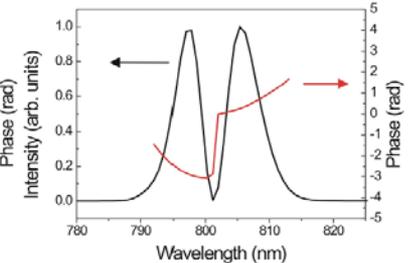
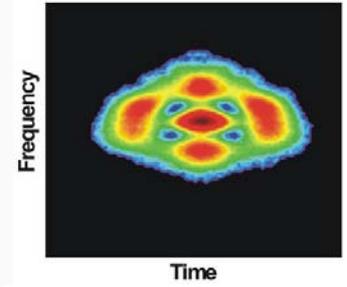
# More shaping by pump modulation



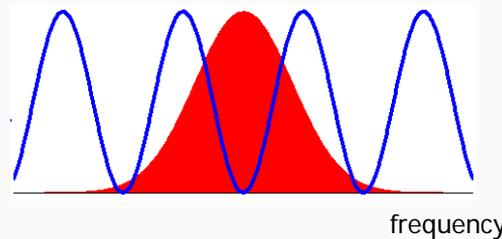
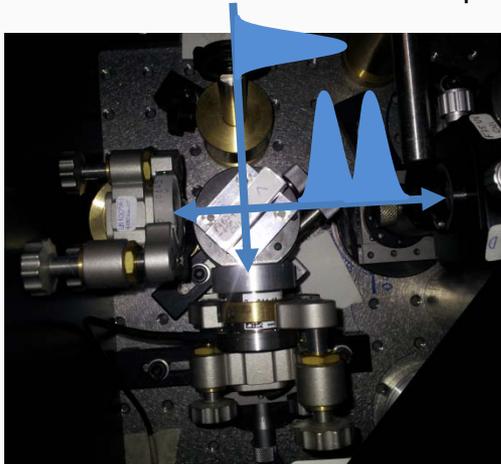
Spectral narrowing



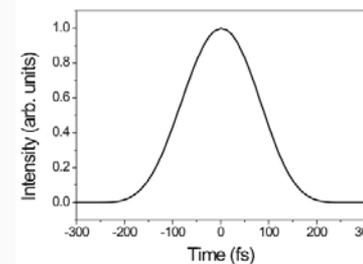
Double spectrotemporal peaks



Michelson interferometer on the pump

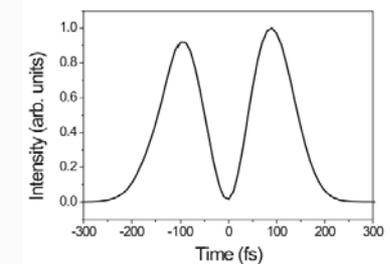


Sinusoidal modulation of the pump spectrum



$$\Delta\lambda_{FWHM} \sim 6 \text{ nm}$$

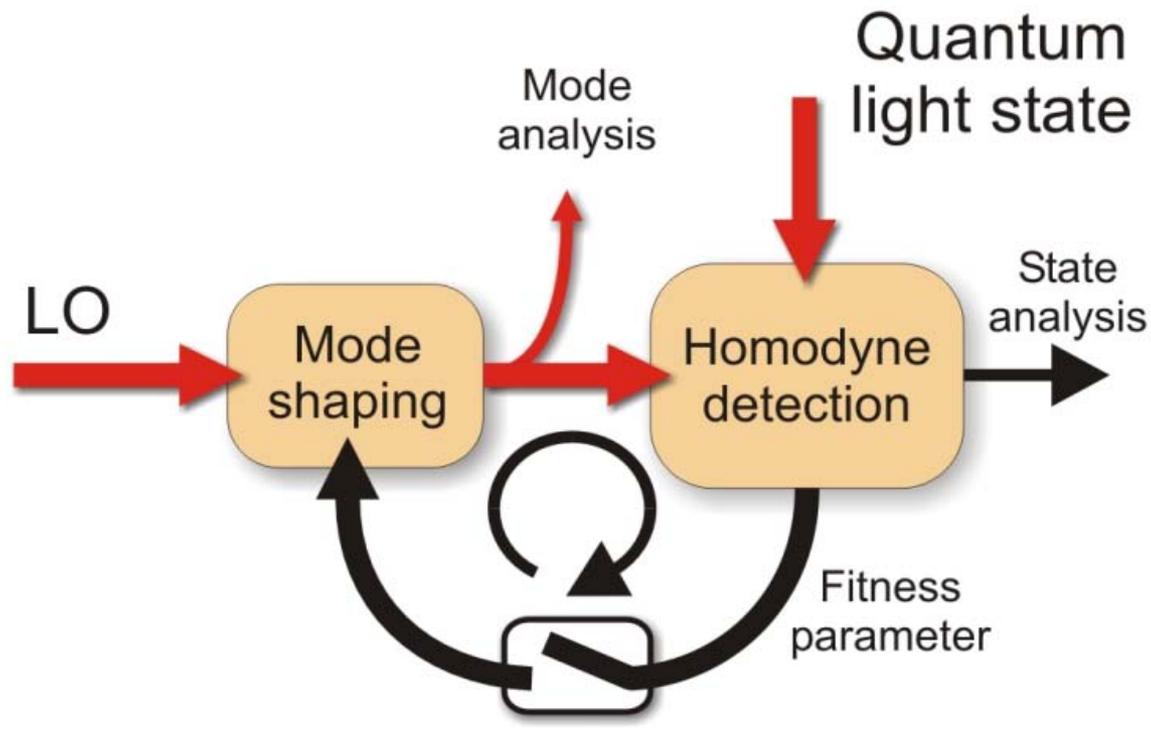
$$\Delta\tau_{FWHM} \sim 180 \text{ fs}$$





# Exploiting the modal selectivity

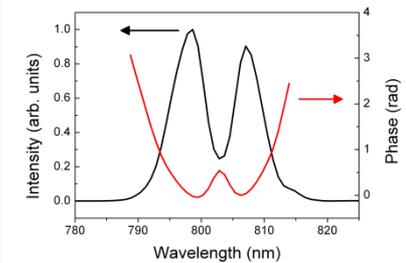
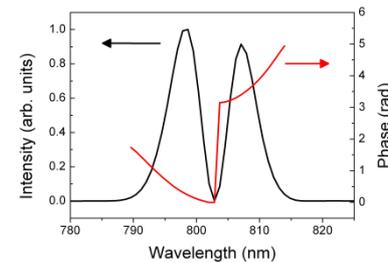
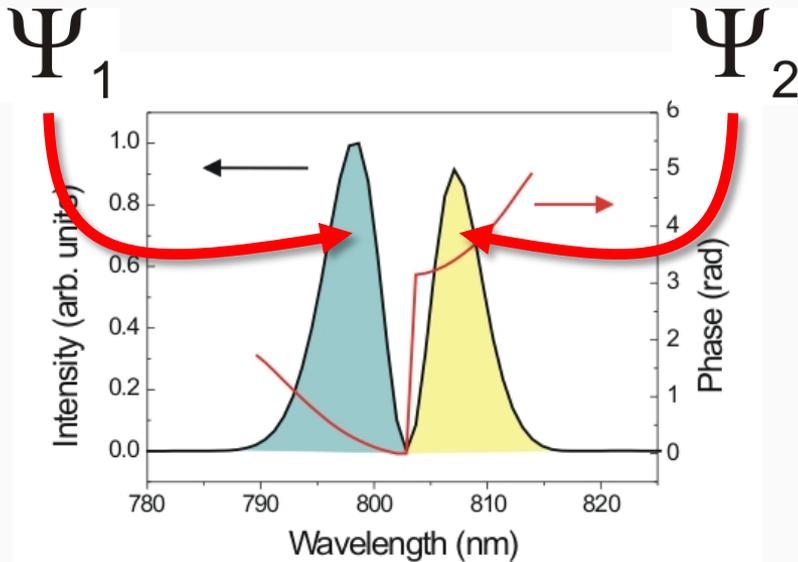
From the measurement of the mode  
to mode-selective analysis of quantum states



Instead of measuring the unknown mode, we can use a shaped LO  
to analyze the state in given modes

# A single-photon spectral qubit

Probing coherence by HD with shaped LO modes

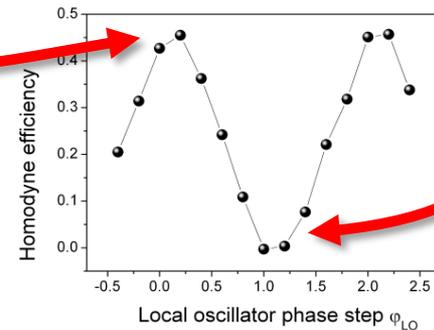
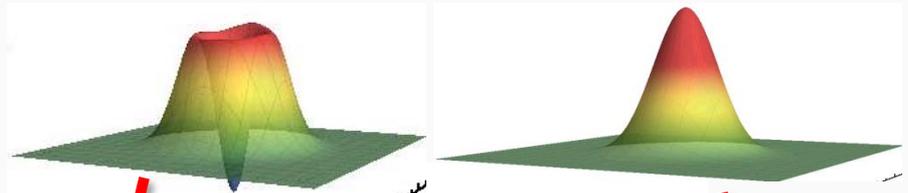


$$\frac{(|1\rangle_{\Psi_1} |0\rangle_{\Psi_2} + |0\rangle_{\Psi_1} |1\rangle_{\Psi_2})}{\sqrt{2}}$$

$$\frac{(|1\rangle_{\Psi_1} |0\rangle_{\Psi_2} - |0\rangle_{\Psi_1} |1\rangle_{\Psi_2})}{\sqrt{2}}$$

The single photon is in a coherent superposition of two distinct spectral modes

$$\frac{(|1\rangle_{\Psi_1} |0\rangle_{\Psi_2} + |0\rangle_{\Psi_1} |1\rangle_{\Psi_2})}{\sqrt{2}}$$

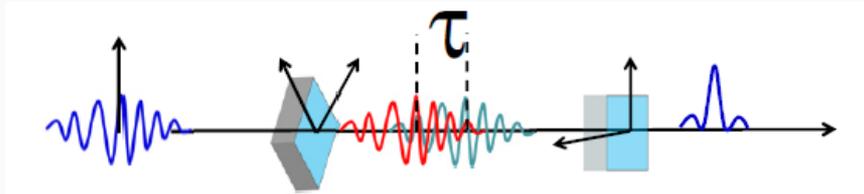


C. Polycarpou, K. Cassemiro, G. Venturi, A. Zavatta, & MB,  
*Phys. Rev. Lett.* **109**, 053602 (2012)



# Exploring single-photon spectral qudits

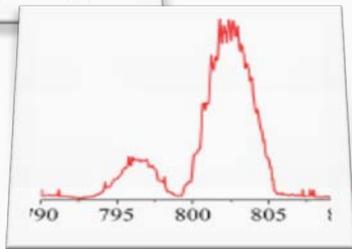
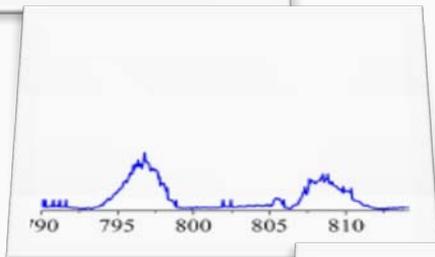
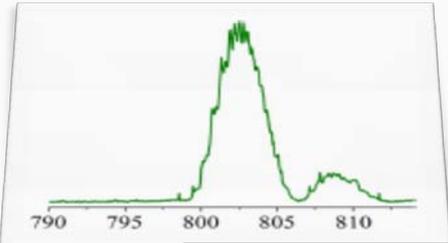
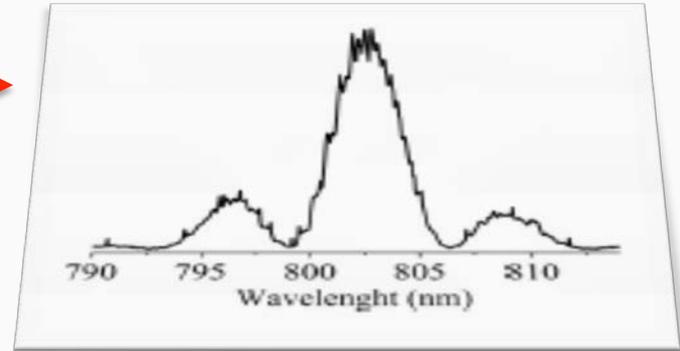
Generating more frequency bins by properly shaping the pump



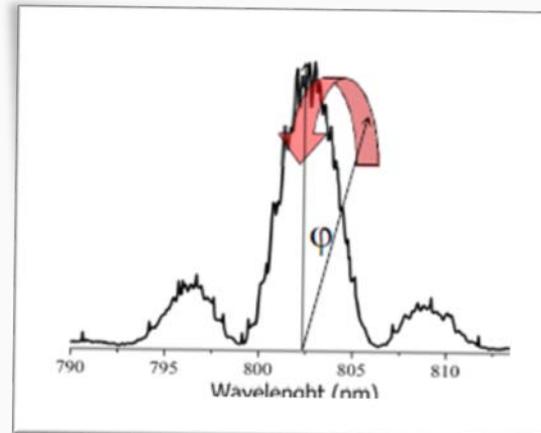
Use a birefringent crystal for stable pump shaping



A 3-peaked single photon



A shaped LO can probe combinations of the three modes with different relative weights and phases

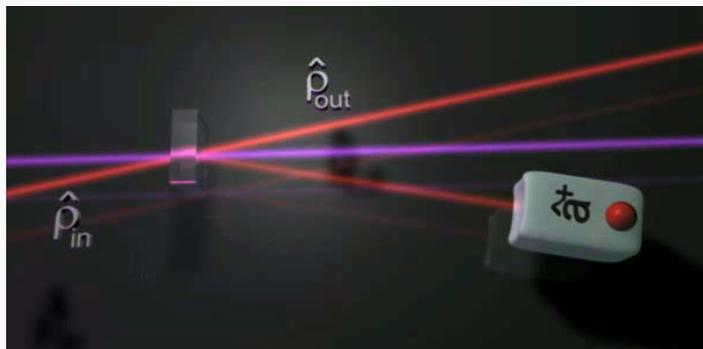


work in progress...



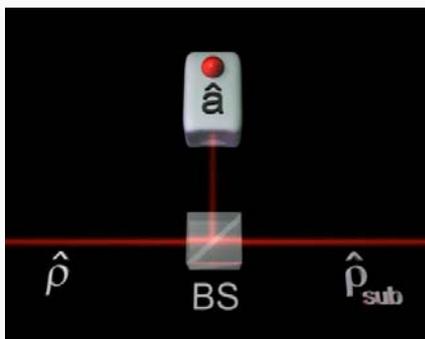
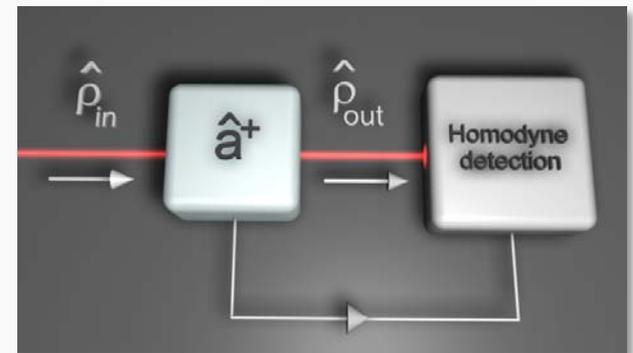
# Beyond the single photon

Introducing our (single mode) quantum toolbox



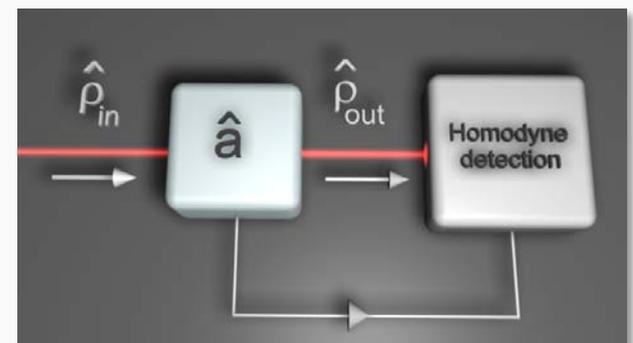
$$\hat{a}^\dagger$$

- small PDC gain
- small photon numbers



$$\hat{a}$$

- small BS reflectivity
- small photon numbers

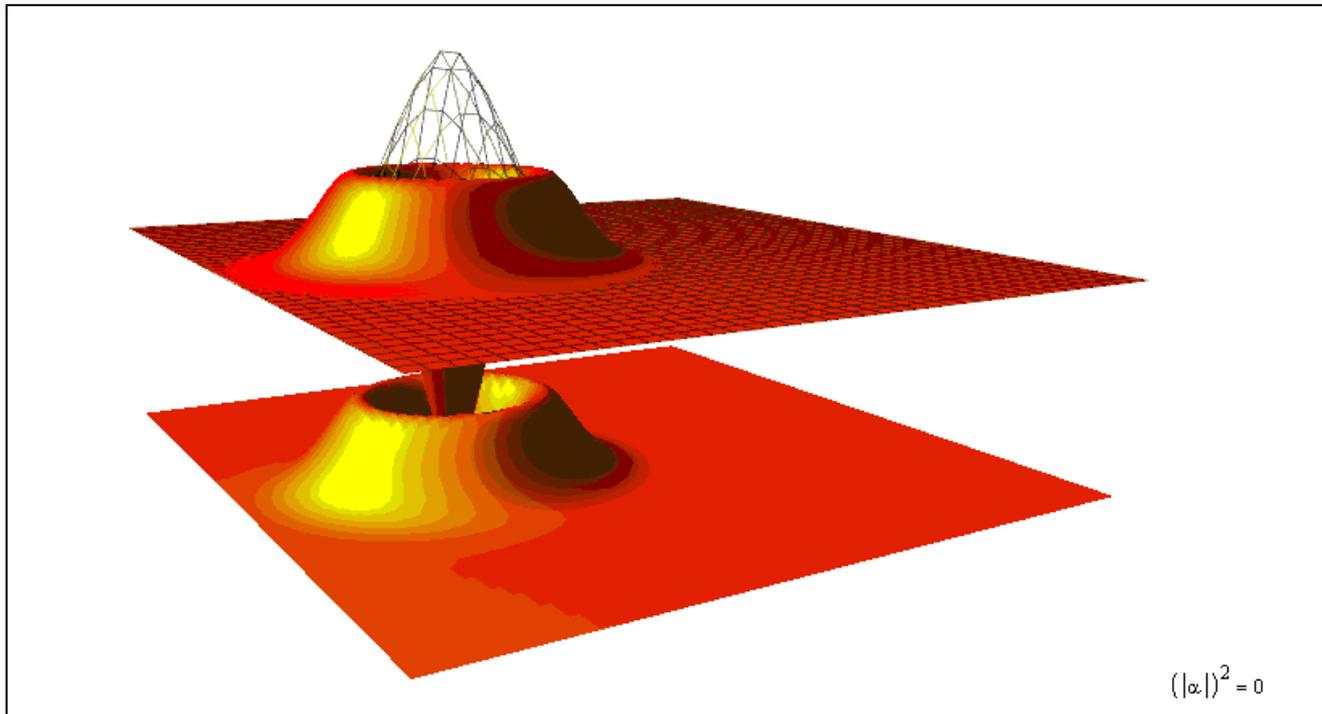


Non-deterministic (heralded) generation schemes



# Single-photon-added coherent states

$$\hat{a}^\dagger |\alpha\rangle$$



The SPACS resembles a coherent state for large input amplitudes  $|\alpha|$



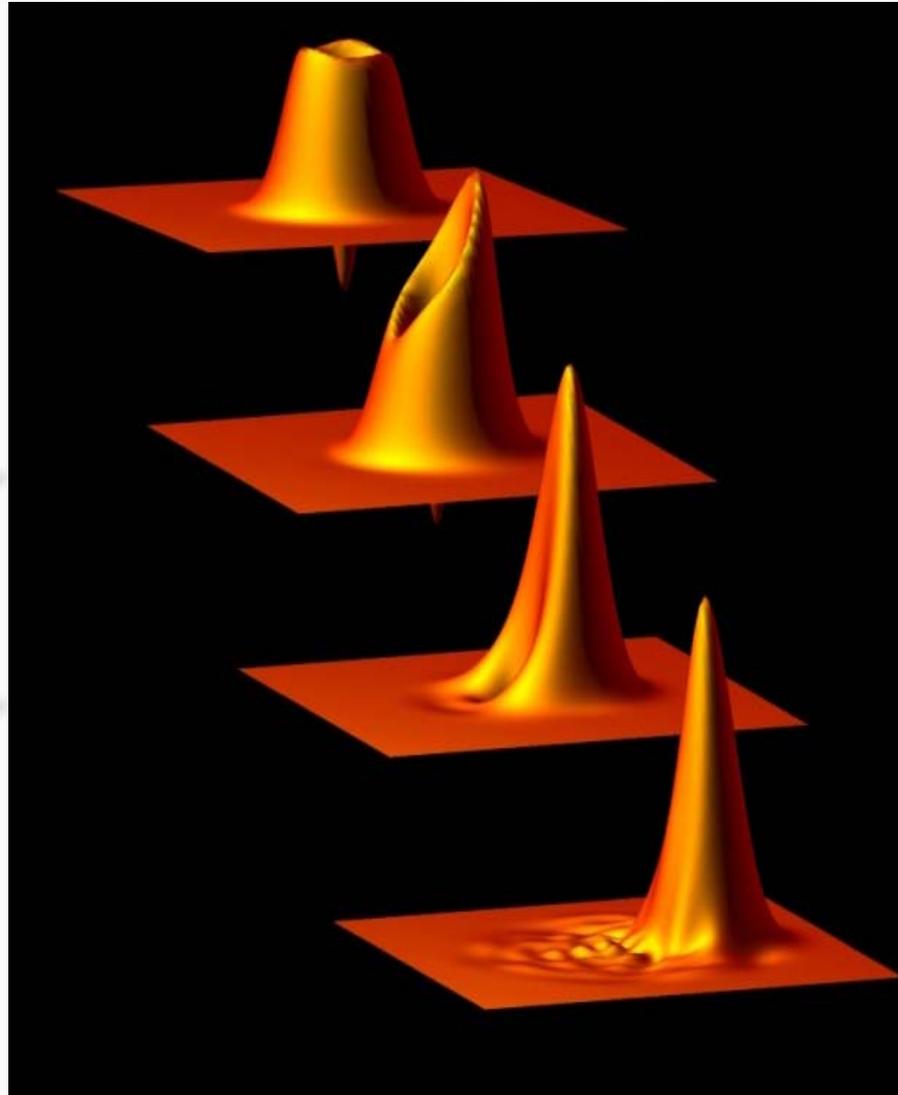
# Quantum-to-classical transition

$$|\alpha| = 0$$

$$|\alpha| = 0.387$$

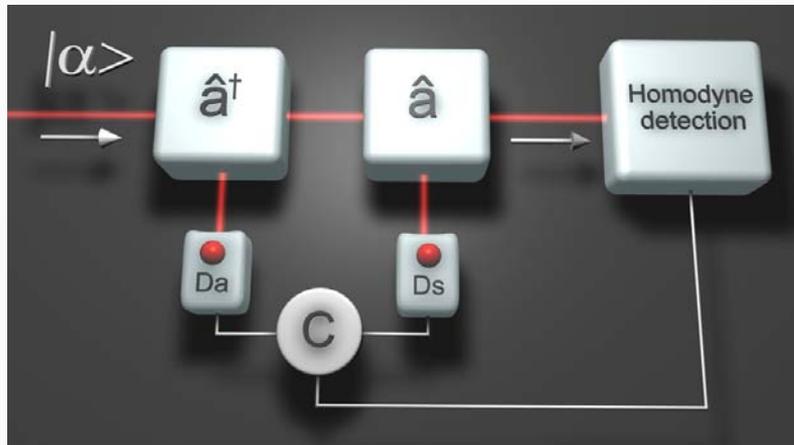
$$|\alpha| = 0.723$$

$$|\alpha| = 3.74$$





# Sequences of quantum operators



## Noiseless amplification

ARTICLES

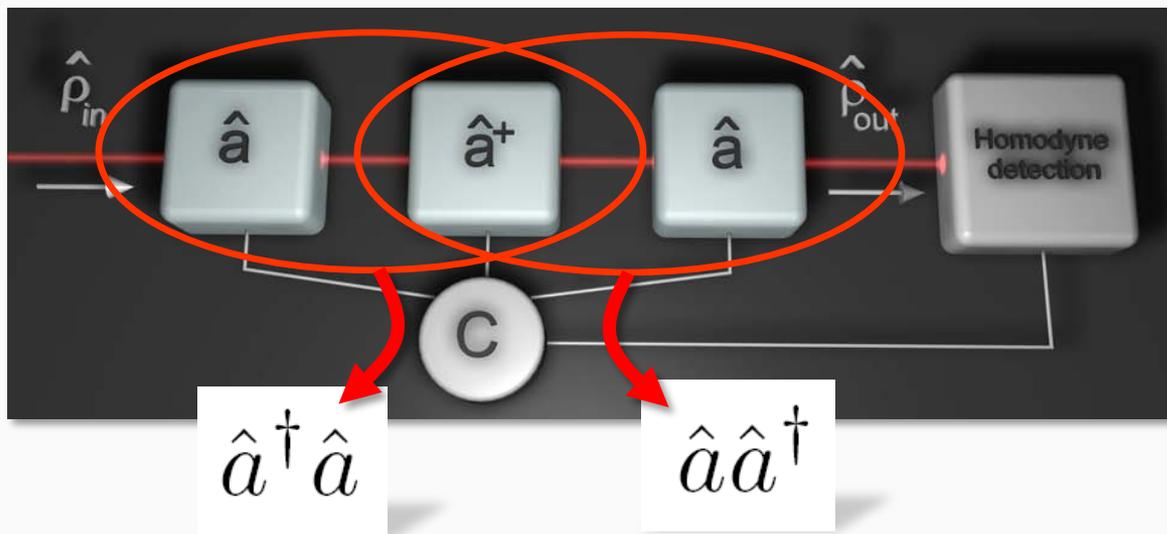
PUBLISHED ONLINE: 21 NOVEMBER 2010 | DOI: 10.1038/NPHOTON.2010.260

nature  
photonics

### A high-fidelity noiseless amplifier for quantum light states

A. Zavatta<sup>1,2</sup>, J. Fiurášek<sup>3</sup> and M. Bellini<sup>1,2\*</sup>

## Direct test of quantum non-commutativity



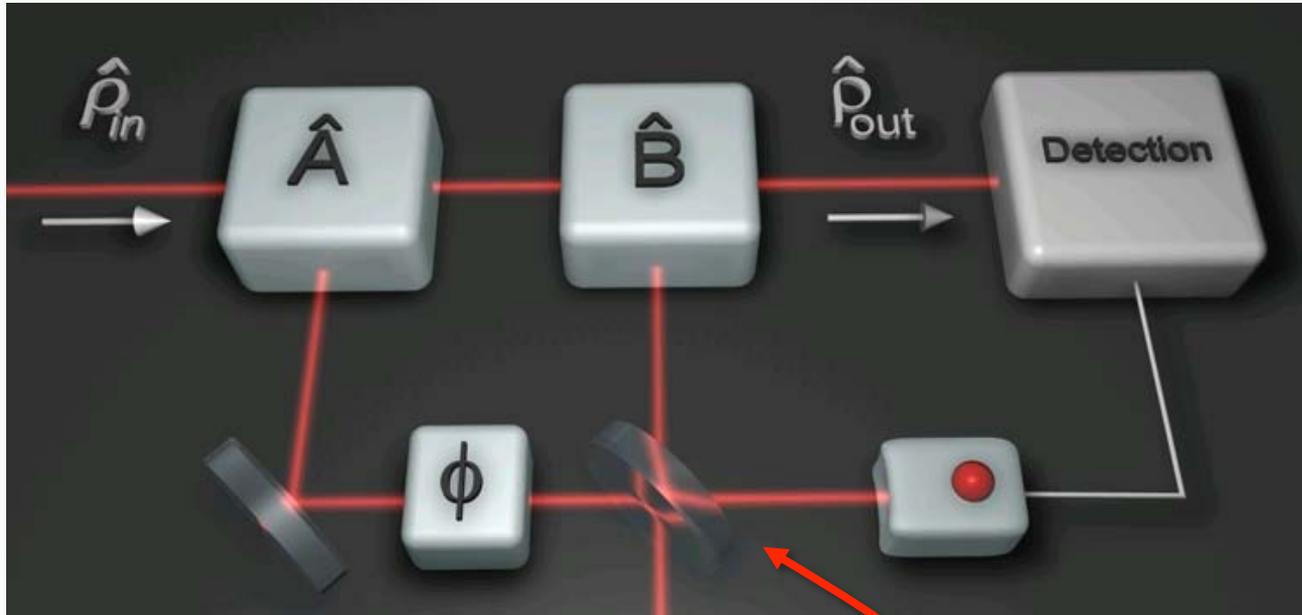
Science

$$[\hat{a}, \hat{a}^\dagger] \neq 0$$

V. Parigi, A. Zavatta, M.S. Kim, & MB  
*Science* **317**, 1890 (2007)



# Superpositions of quantum operators



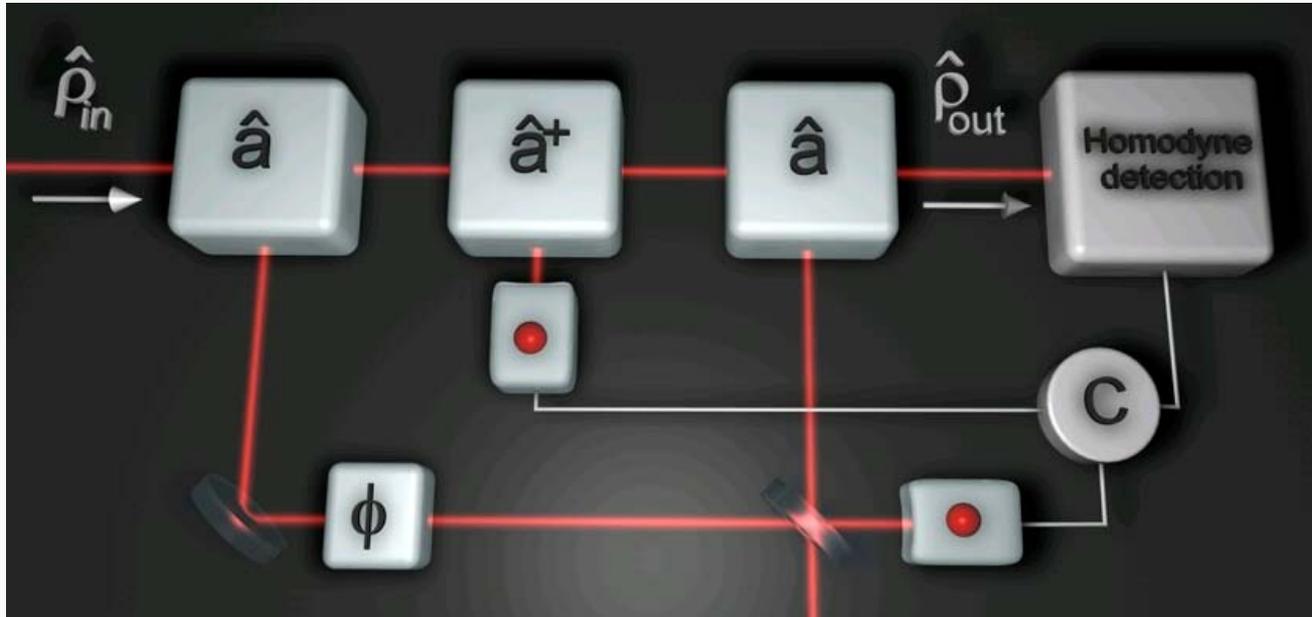
Erases the information about  
the origin of a "click"

$$|\alpha| \hat{A} + e^{i\phi} |\beta| \hat{B}$$

Arbitrary superpositions of operators  
can be implemented



# Experimental test of commutation rules



$$[\hat{a}, \hat{a}^\dagger] = \hat{a}\hat{a}^\dagger - \hat{a}^\dagger\hat{a} = \mathbf{1}$$

M. S. Kim, H. Jeong, A. Zavatta, V. Parigi, & MB, *PRL* **101**, 260401 (2008)

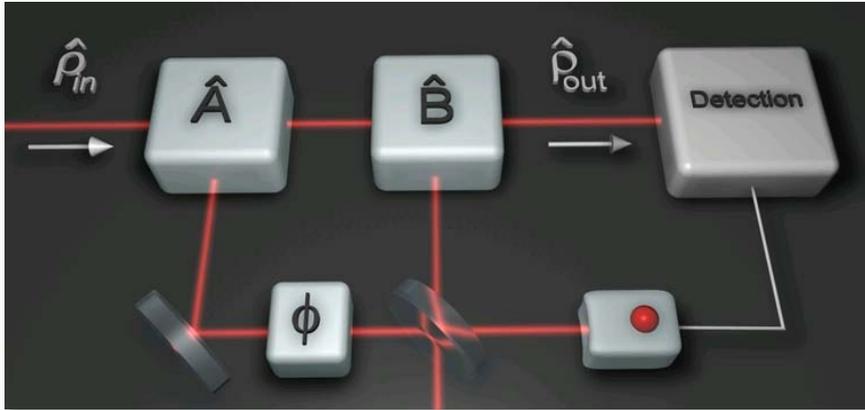
A. Zavatta, V. Parigi, M. S. Kim, H. Jeong, & MB, *PRL* **103**, 140406 (2009)

Theory

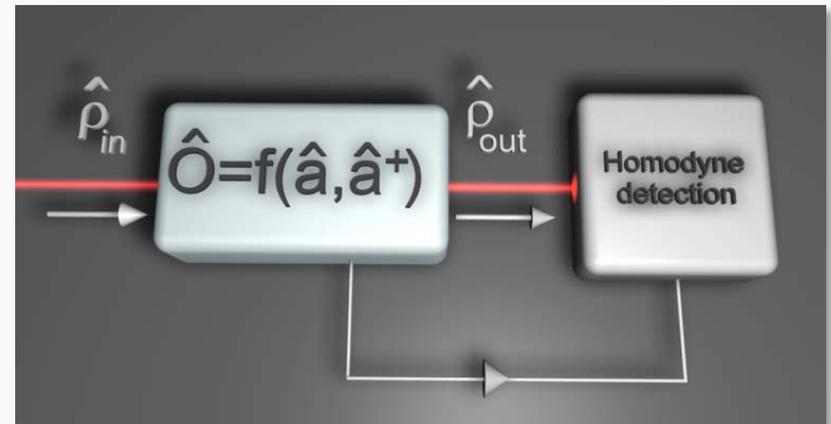
Experiment

# Going multi-mode

So far, all operations have been performed in a ~~single~~, ~~well-defined~~, mode



Single-mode superposition of quantum operators



Generic combination of sequences and superpositions of basic operators

How to extend our quantum tools to work in a multi-mode regime?

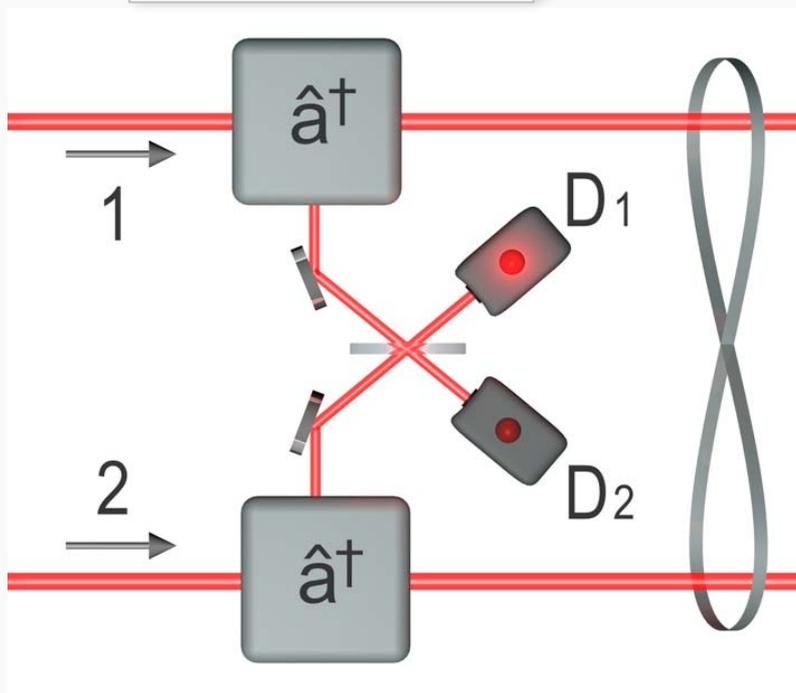


# Superposition of operators on two modes

$$r\hat{a}_1^\dagger + t\hat{a}_2^\dagger$$

Delocalized photon addition on modes 1 and 2

$|0\rangle_1$



$$\frac{1}{\sqrt{2}}(|1\rangle_1 |0\rangle_2 + |0\rangle_1 |1\rangle_2)$$

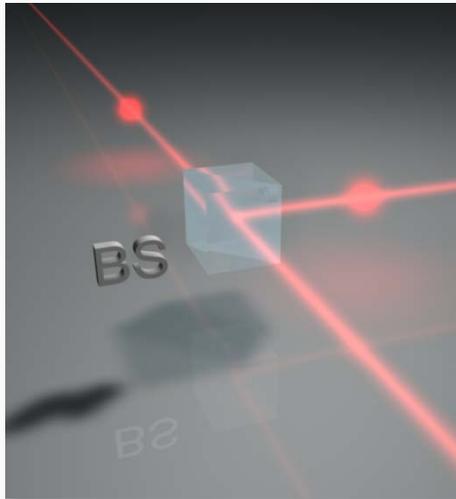
## Single-photon path-entangled state

Two distinct spatial modes get entangled by sharing a single photon

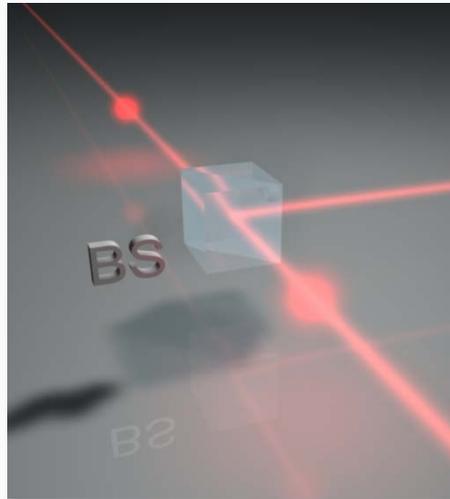


# Single-photon entanglement

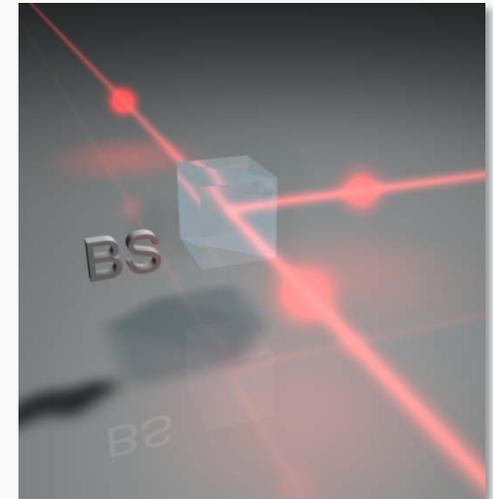
Indistinguishable alternatives



+



=



Classical particle



Quantum particle

“Here” and “there”  
at the same time

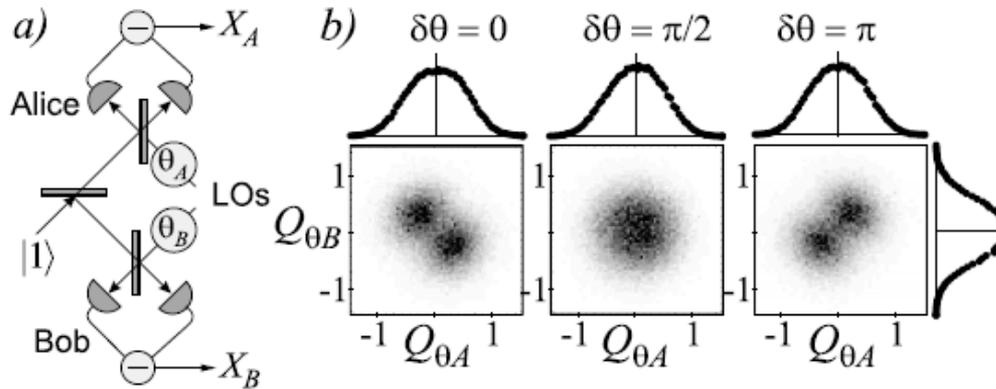
$$\frac{1}{\sqrt{2}} (|1\rangle_A |0\rangle_B + |0\rangle_A |1\rangle_B)$$

Single-photon path-entangled state

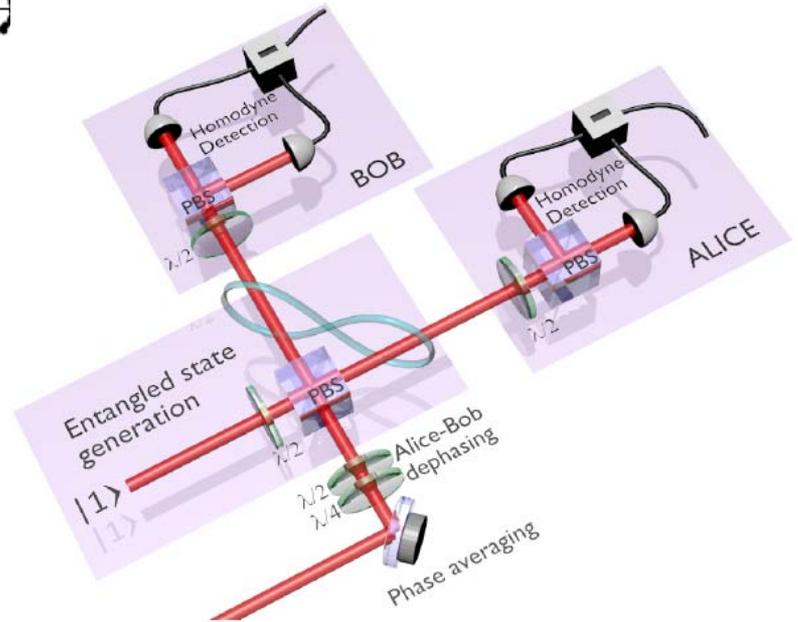
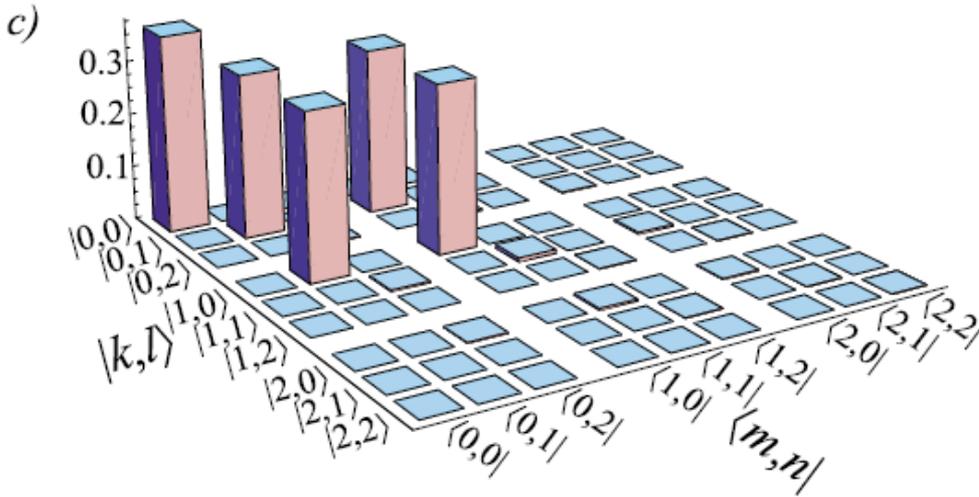


# Experimental single-photon entanglement

Babichev S. A., J. Appel, and A. I. Lvovsky, 2004, Phys. Rev. Lett. **92**, 193601.



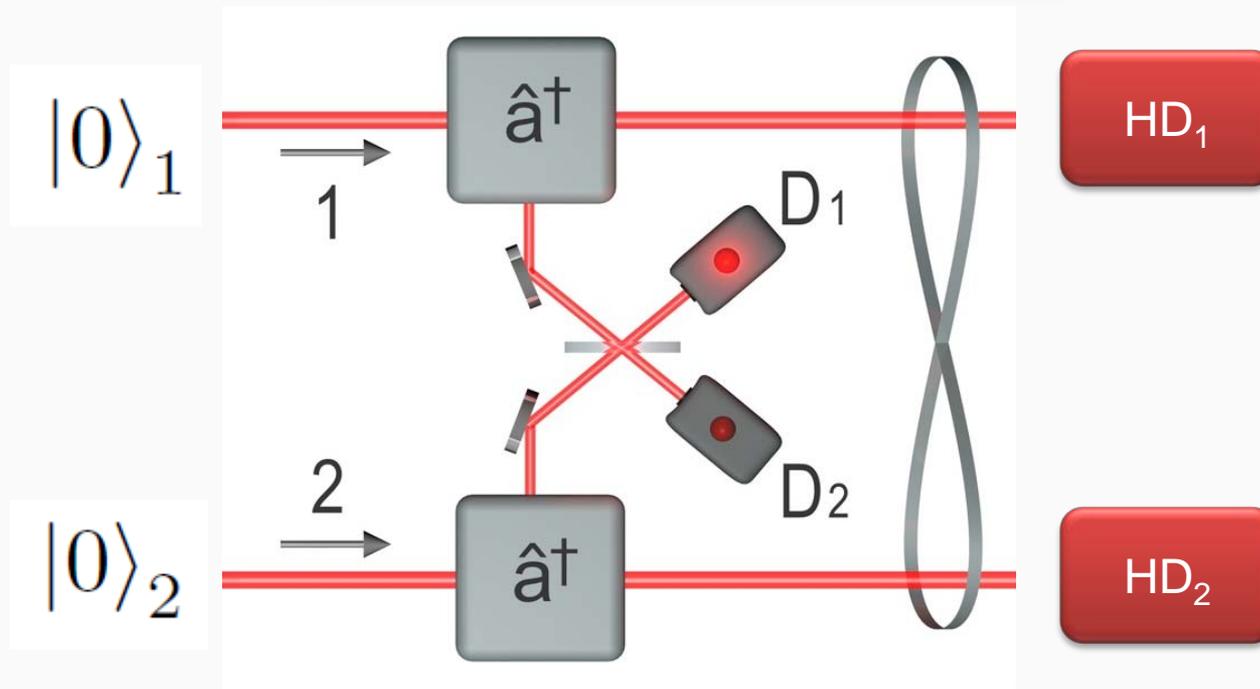
$$\frac{1}{\sqrt{2}} (|1\rangle_A |0\rangle_B + |0\rangle_A |1\rangle_B)$$



O. Morin, J.-D. Bancal, M. Ho, P. Sekatski, V. D'Auria, N. Gisin, J. Laurat and N. Sangouard. 2013 Phys. Rev. Lett. 110, 130401

# Superposition of operators on two modes

$$\frac{1}{\sqrt{2}}(|1\rangle_1 |0\rangle_2 + |0\rangle_1 |1\rangle_2)$$



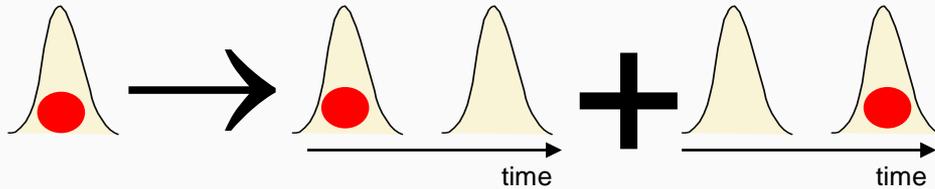
It requires two PDC sources and two homodyne detectors

Not a very clever solution...



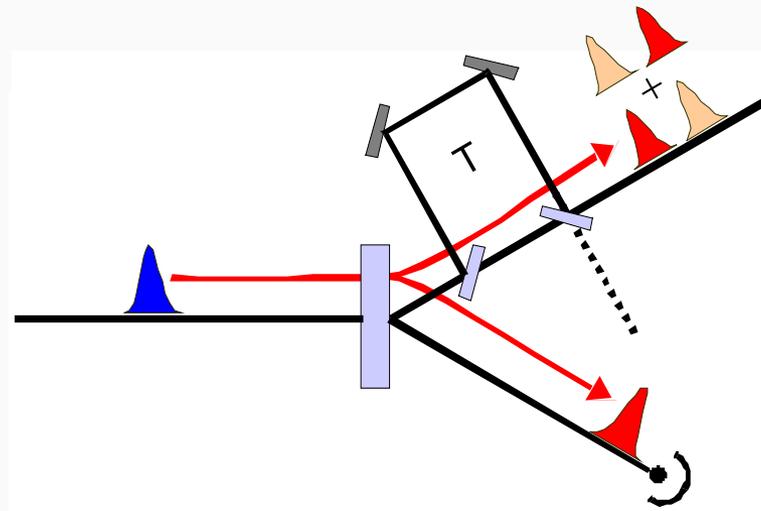
# Time-delocalized single photons

Spatial modes  $\rightarrow$  Temporal modes



$$|\Psi\rangle = \frac{1}{\sqrt{2}} (|1\rangle_t |0\rangle_{t+T} + |0\rangle_t |1\rangle_{t+T})$$

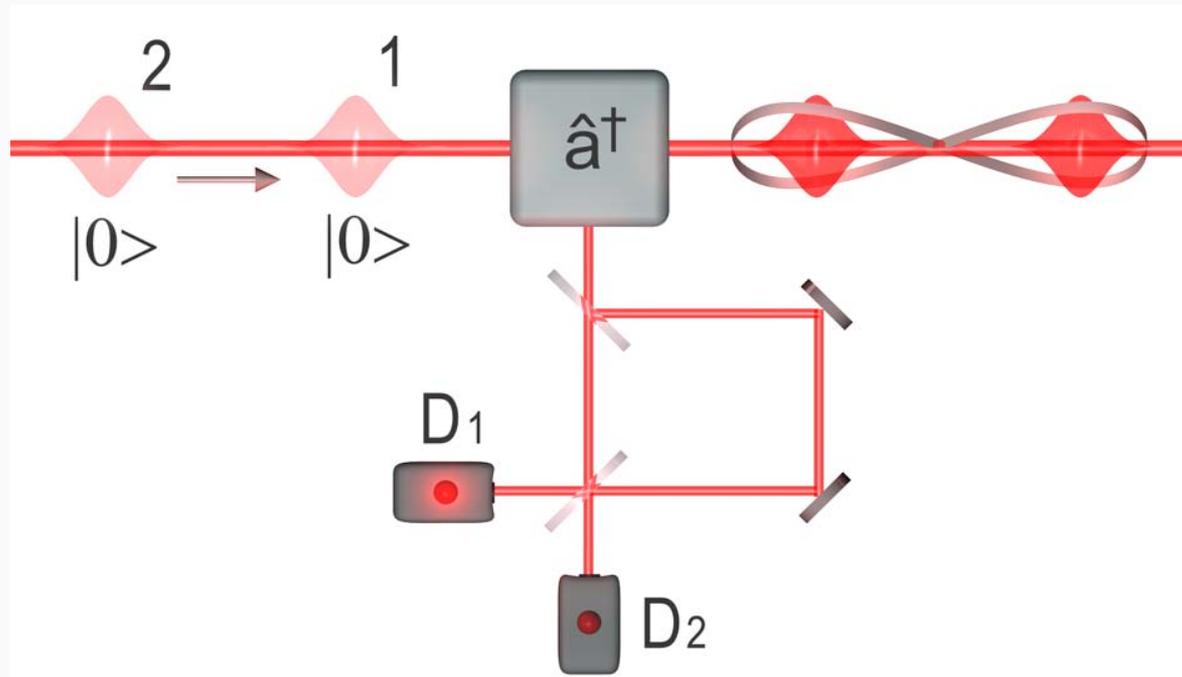
“Sooner” and “later”  
at the same time  
(?!?)



High losses ...

# A single-photon time-bin qubit

Coherent superposition of two photon-addition processes onto two different traveling wavepacket temporal modes



$$(\hat{a}_t^\dagger + \hat{a}_{t+T}^\dagger) |0\rangle_t |0\rangle_{t+T} = |1\rangle_t |0\rangle_{t+T} + |0\rangle_t |1\rangle_{t+T}$$

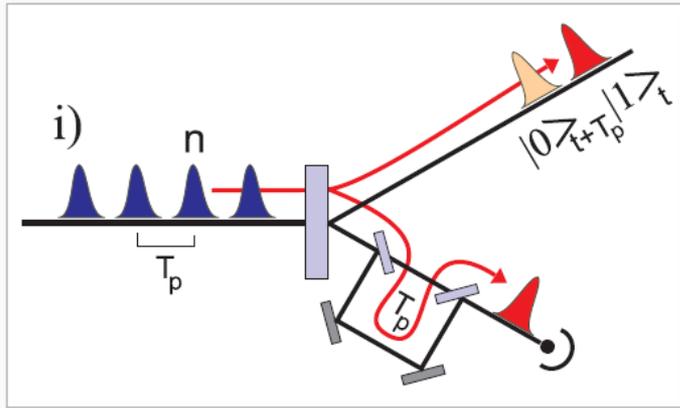
A. Zavatta, M. D'Angelo, V. Parigi and MB, *Phys. Rev. Lett.*, **96**, 020502 (2006)

M. D'Angelo, A. Zavatta, V. Parigi and MB, *Phys. Rev. A*, **74**, 052114 (2006)

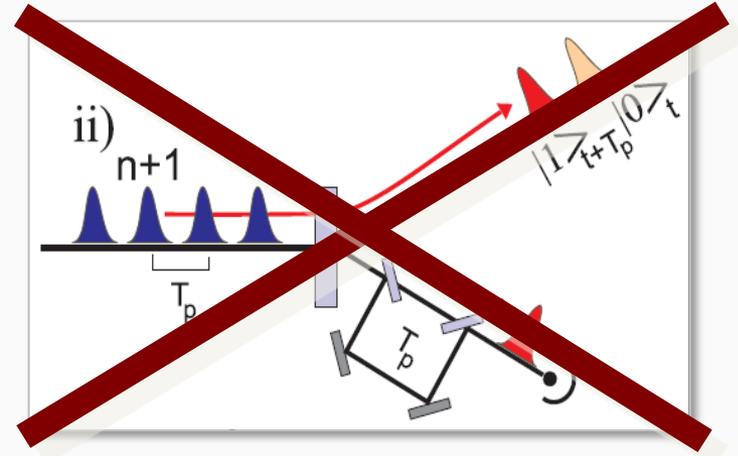
S. Takeda, T. Mizuta, M. Fuwa, J. Yoshikawa, H. Yonezawa, and A. Furusawa, *Phys. Rev. A* **87**, 043803 (2013)



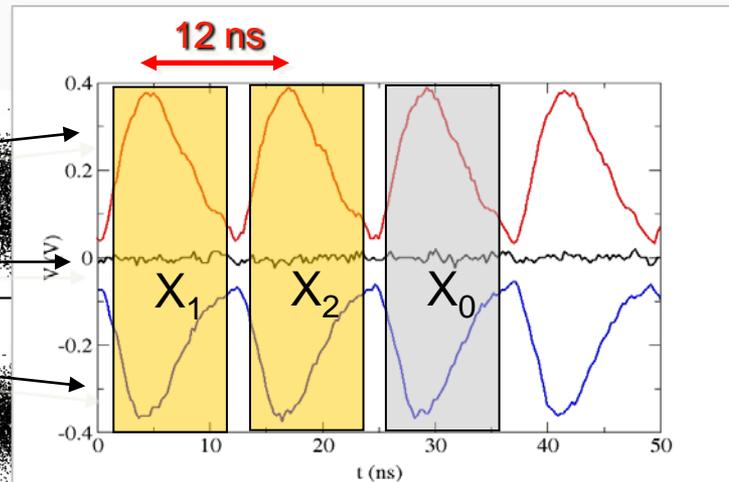
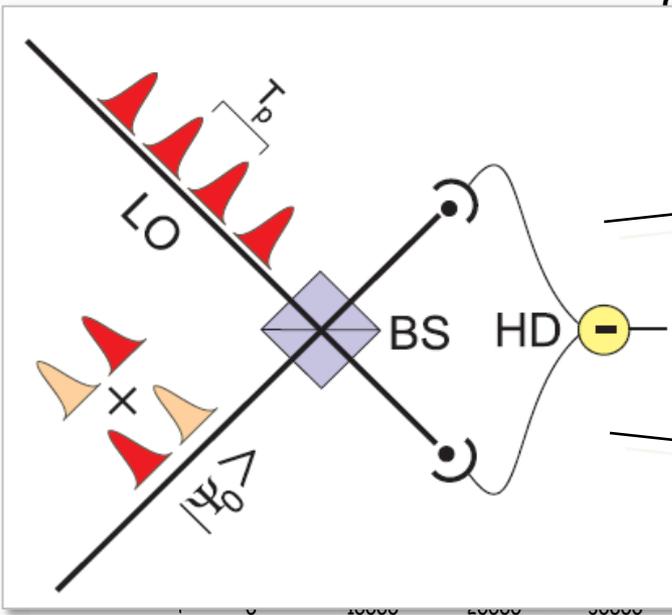
# Conditional delocalization & multimode HD



$$\hat{a}_t^\dagger$$



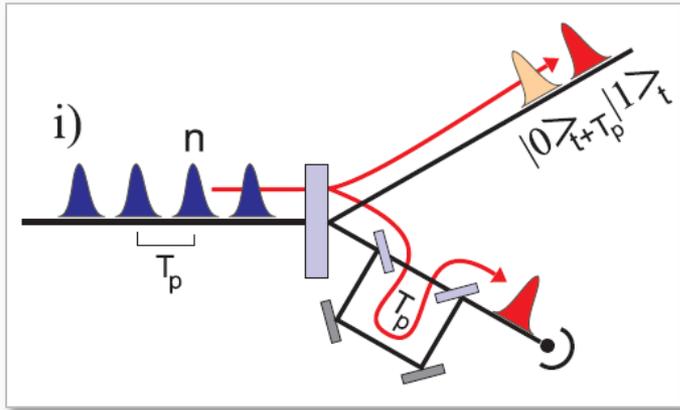
Only one possible path



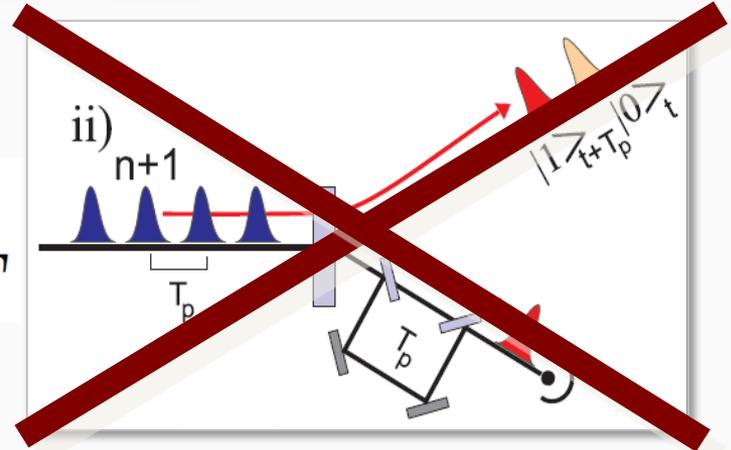
All localized  
or three consecutive  
time-bins are  
analyzed for each  
trigger event



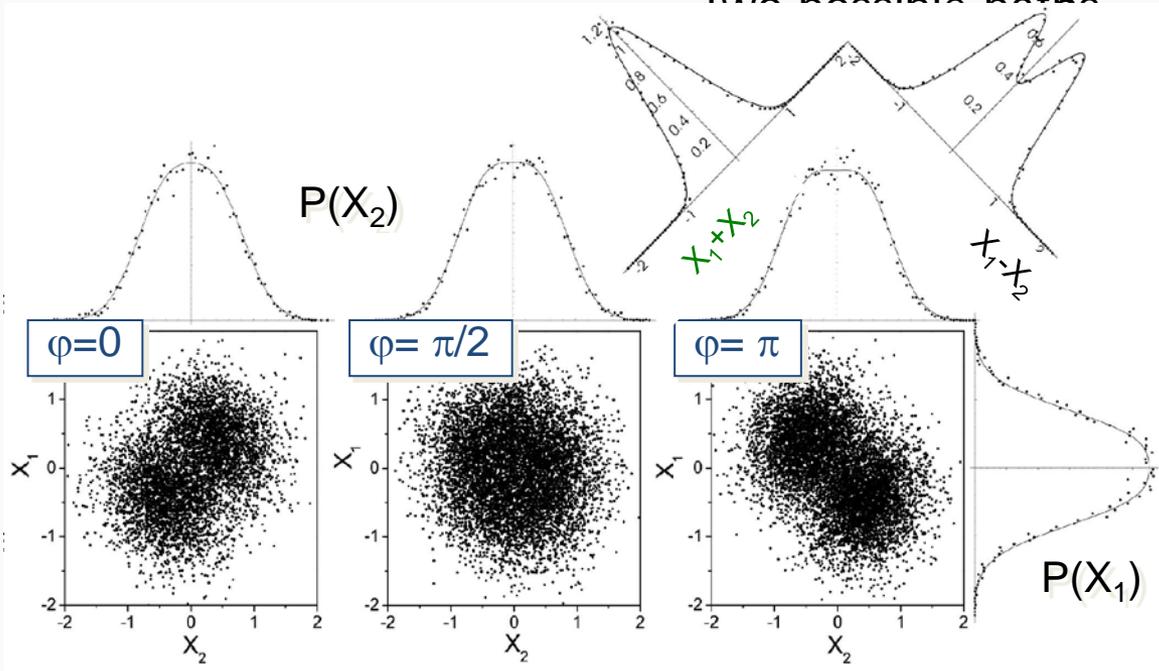
# Conditional time delocalization



$$\hat{a}_t^\dagger + \hat{a}_{t+T}^\dagger$$



Two possible paths



$$+ |0\rangle_t |1\rangle_{t+T}$$

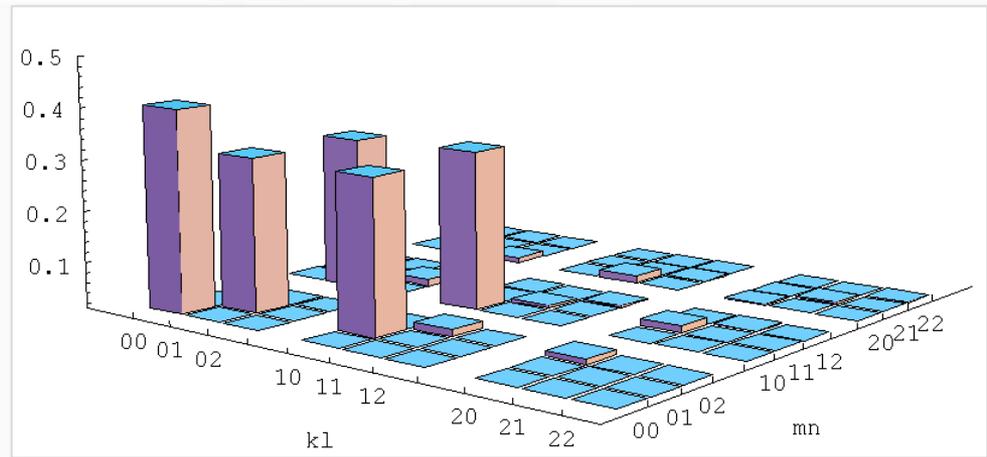
Coherence manifests in the correlations between the quadrature of the two modes

statistical mixture of vacuum and single photon



# Entanglement and nonlocality

Reconstructed two-mode  
density matrix



Entanglement verified by means of Peres' criterion  
(negativity of partial transpose)

A. Peres, *Phys. Rev. Lett.* **77**, 1413 (1996)  
G. Vidal and R. F. Werner, *Phys. Rev. A* **65**, 032314 (2002)

Within some assumptions, Bell's-type inequalities are violated  
(use a "tomographic" approach via the Wigner function)

No dichotomization of CV

K. Banaszek and K. Wodkiewicz, *Phys. Rev. A* **58**, 4345 (1998)  
K. Banaszek and K. Wodkiewicz, *Phys. Rev. Lett.* **82**, 2009 (1999)

A. Zavatta, M. D'Angelo, V. Parigi and MB, *Phys. Rev. Lett.*, **96**, 020502 (2006)  
M. D'Angelo, A. Zavatta, V. Parigi and MB, *Phys. Rev. A*, **74**, 052114 (2006)

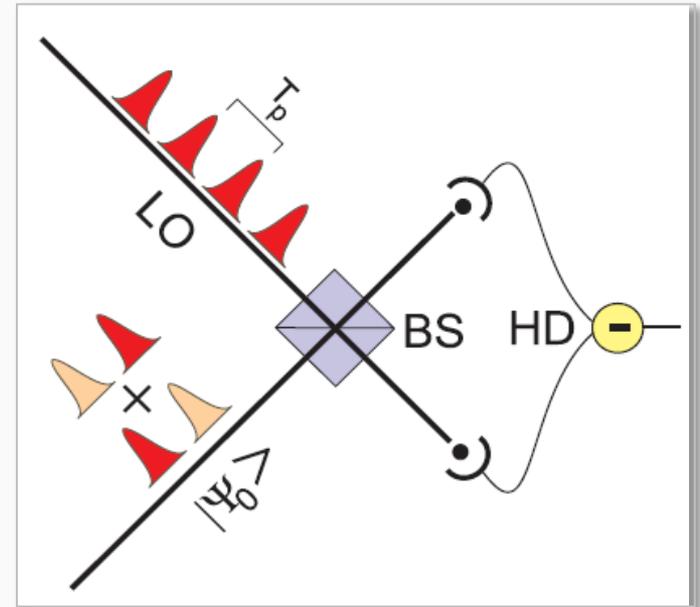


# Going hybrid

$$\frac{1}{\sqrt{2}} (|1\rangle_1 |0\rangle_2 + |0\rangle_1 |1\rangle_2)$$

**Discrete variable entangled state**

+

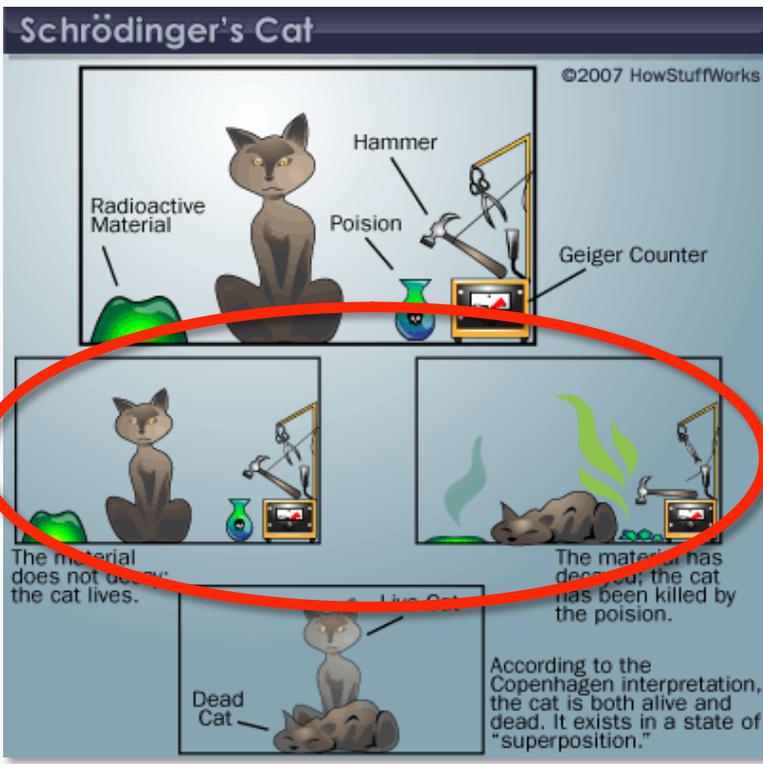


**Multi-mode homodyne detection  
(continuous variables)**

DV entanglement analyzed with CV detection



# Hybrid quantum-classical entanglement



Schrödinger's cat paradox

Entanglement of quantum and classical objects

$$|\Psi\rangle = \frac{1}{\sqrt{2}} (|\uparrow\rangle |\text{Smiley}\rangle + |\downarrow\rangle |\text{Sad}\rangle)$$



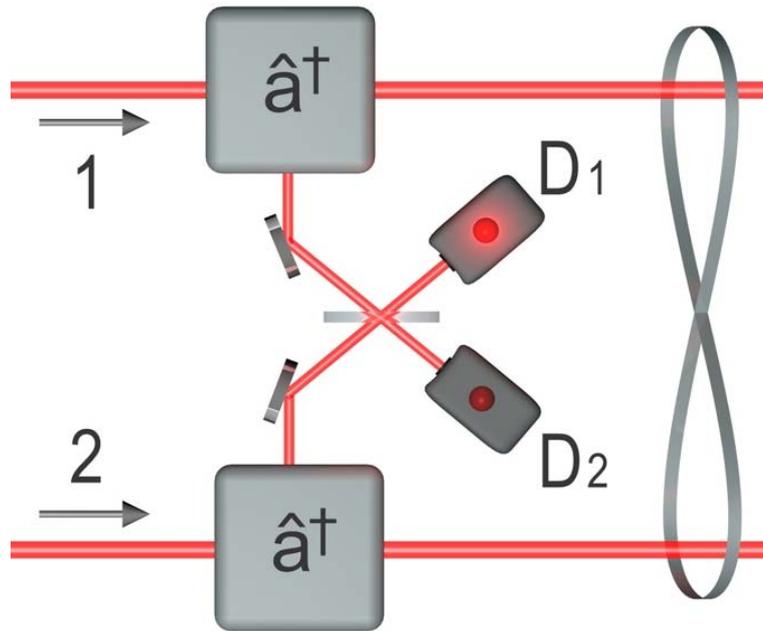
Most quantum state  
Particle-like  
Discrete variables

Most classical state  
Wave-like  
Continuous variables

Fundamental importance  
Useful resource for optical quantum information processing

Hard to produce (cross-Kerr nonlinearity,...)

# Creating a hybrid entangled state



Using the appropriate beam-splitter ratio:

$$t = \sqrt{1 - |r|^2} = \frac{1}{\sqrt{1 + |\alpha|^2}}$$

$$\frac{1}{\sqrt{2}} \left( |1\rangle|\alpha\rangle + |0\rangle \frac{\hat{a}^\dagger |\alpha\rangle}{\sqrt{|\alpha|^2 + 1}} \right)$$

If  $\alpha$  is not too small  $\frac{\hat{a}^\dagger |\alpha\rangle}{\sqrt{1 + |\alpha|^2}} \approx |\alpha'\rangle$

$$\mathcal{F} \approx 0.98 \text{ for } \alpha = 2 \text{ and } g\alpha = 2.414$$

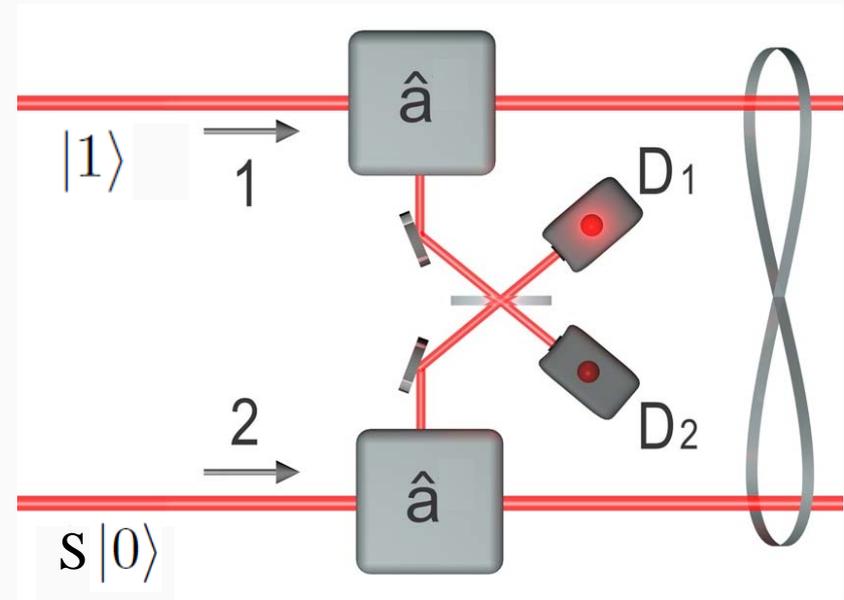
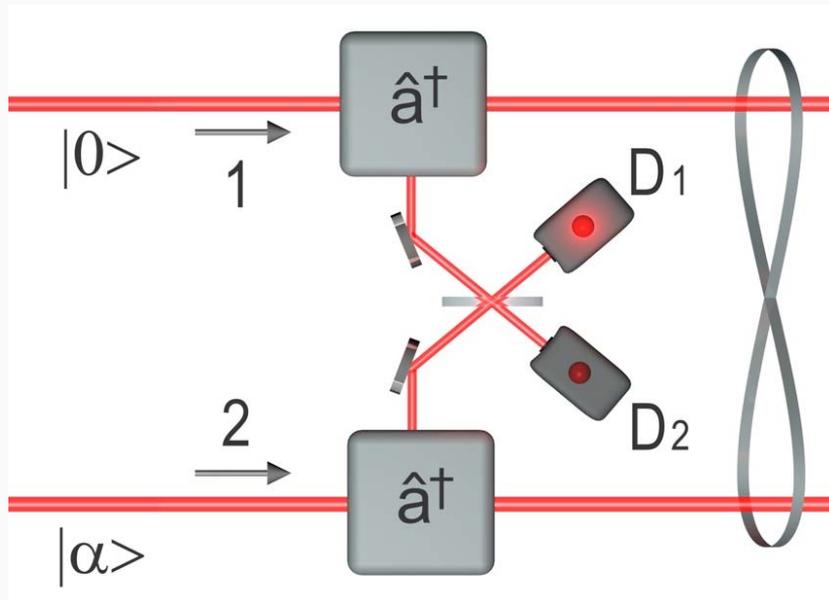
$$\mathcal{F} \approx 0.998 \text{ for } \alpha = 4 \text{ and } g\alpha = 4.236$$

$$\approx \frac{1}{\sqrt{2}} \left( |1\rangle|\alpha\rangle + |0\rangle|\alpha'\rangle \right)$$

Hybrid single-photon (discrete) and  
coherent state (continuous variable)  
entangled qubits



# Different approaches to hybrid entanglement



$$\approx \frac{1}{\sqrt{2}} (|1\rangle|\alpha\rangle + |0\rangle|\alpha'\rangle)$$



see Julien's and Ulrik's schemes for hybrid entanglement generation

Need some nonclassical resources somewhere

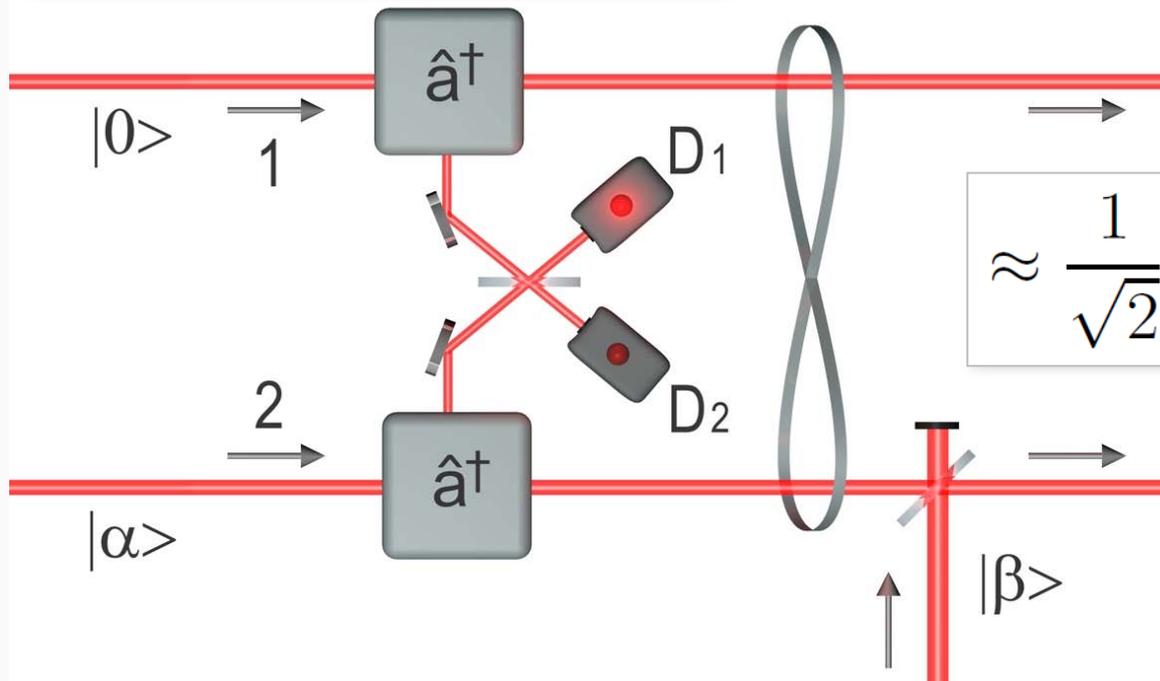


# Symmetric hybrid entangled state

$$\approx \frac{1}{\sqrt{2}} (|1\rangle|\alpha\rangle + |0\rangle|\alpha'\rangle)$$

The right amount of phase-space displacement in the second mode can produce a symmetric hybrid state

$$D\left(-\frac{\alpha + \alpha'}{2}\right)$$



$$\approx \frac{1}{\sqrt{2}} (|0\rangle|\alpha_f\rangle + |1\rangle|-\alpha_f\rangle)$$

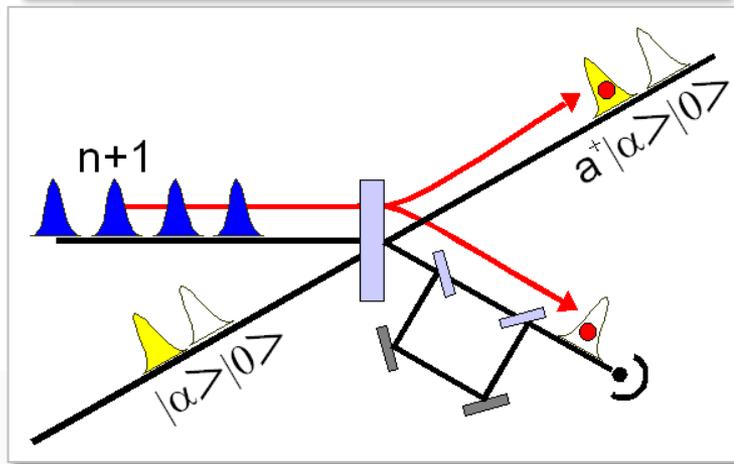
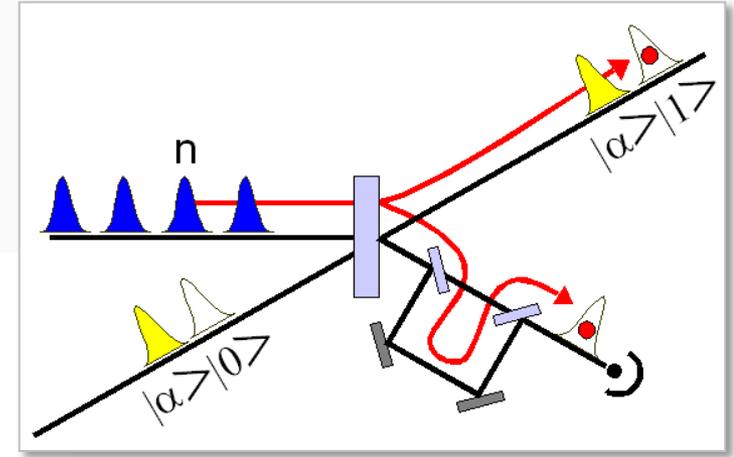
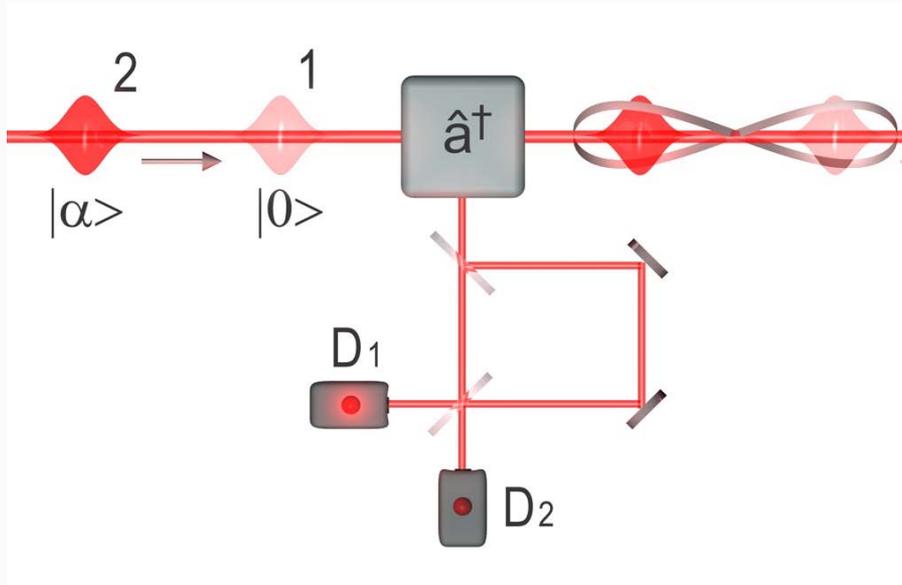
with a final (small) coherent state amplitude of

$$\alpha_f = (\alpha' - \alpha)/2$$

In principle the coherent state amplitude can be made larger (and with higher fidelity) by tele-amplifying mode 2



# Creating a time-bin hybrid entangled state

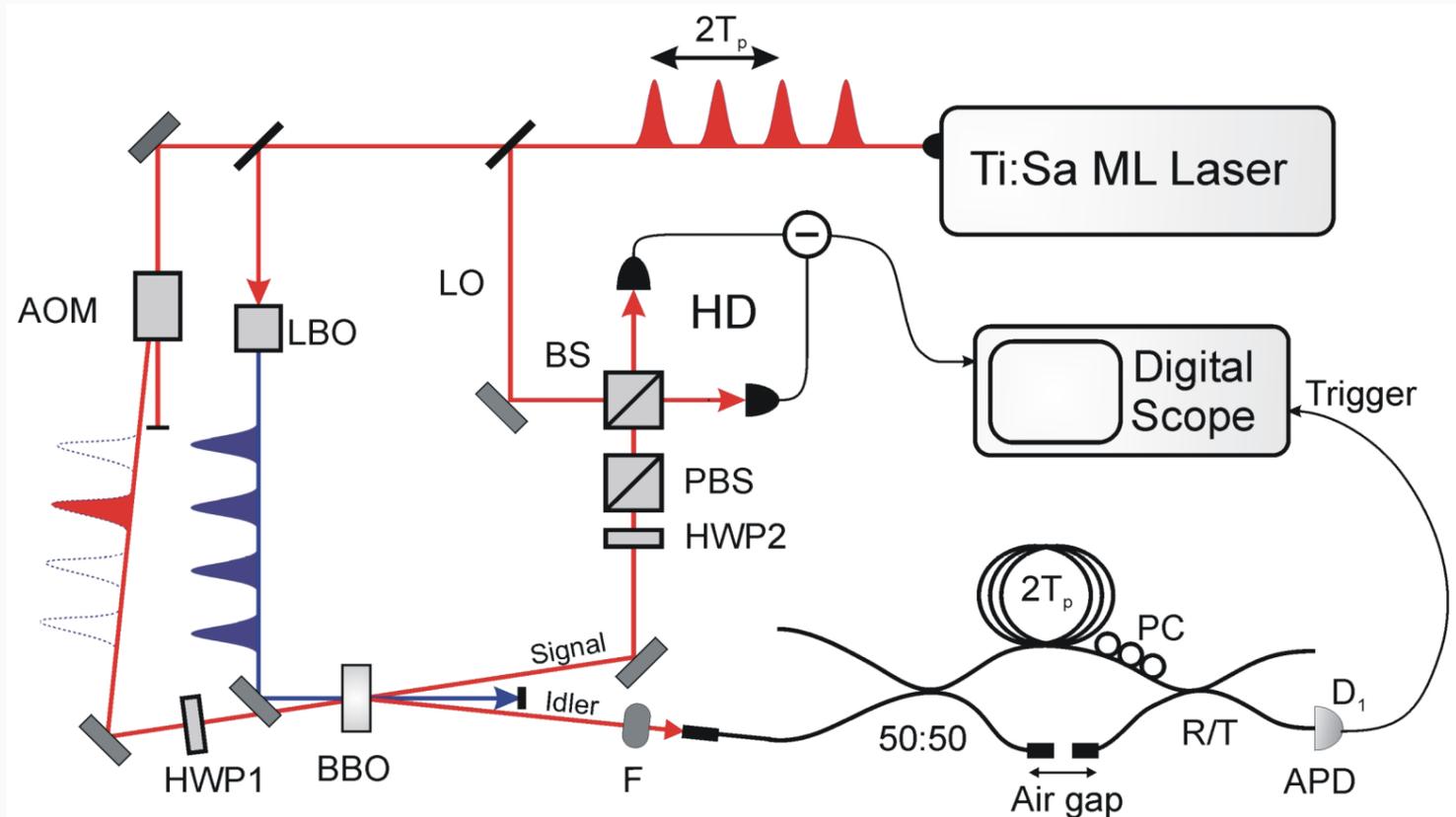


Temporal-mode version of the hybrid entangled state

$$(\hat{a}_t^\dagger + \hat{a}_{t+T}^\dagger) |0\rangle_t |\alpha\rangle_{t+T} = |1\rangle_t |\alpha\rangle_{t+T} + |0\rangle_t \hat{a}_{t+T}^\dagger |\alpha\rangle_{t+T}$$



# Experimental setup



Entangling quantum and classical states of light  
arXiv 1309:6192



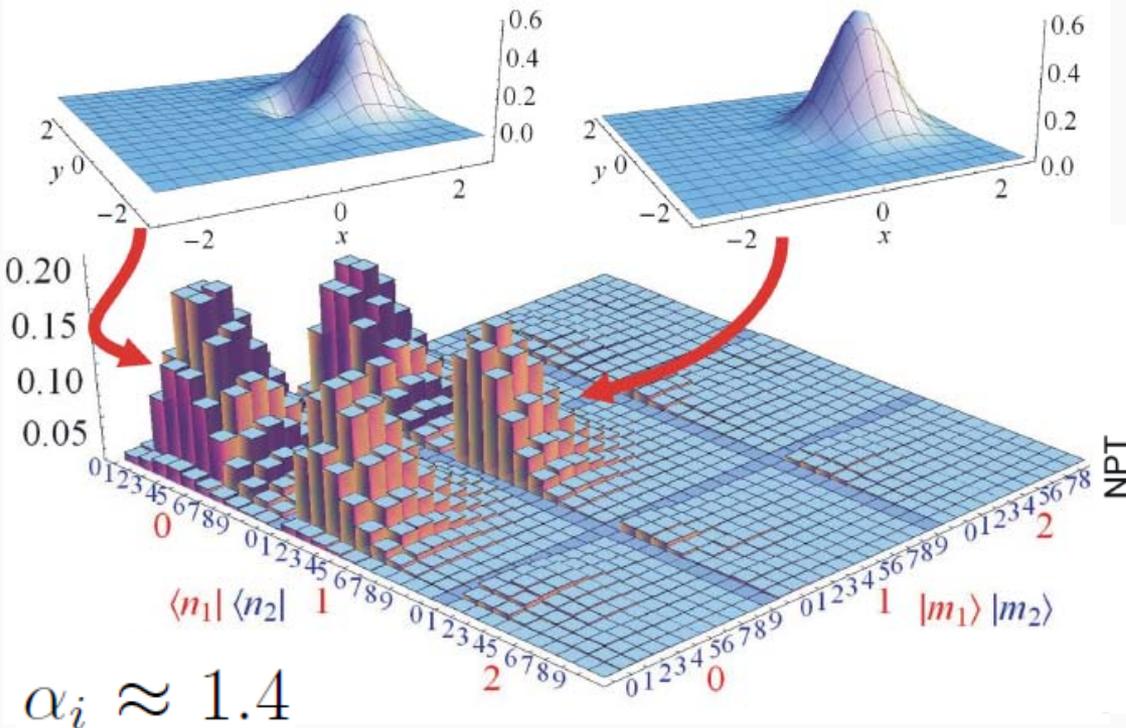
# Experimental hybrid entangled state

$$\approx \frac{1}{\sqrt{2}} (|1\rangle|\alpha\rangle + |0\rangle|\alpha'\rangle)$$

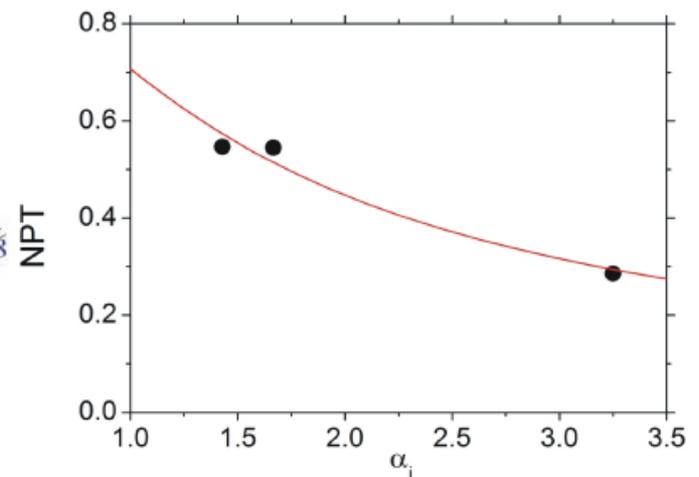
Off-diagonal terms prove the coherent superposition character of the state

$$NPT=0.55$$

Entangled hybrid state



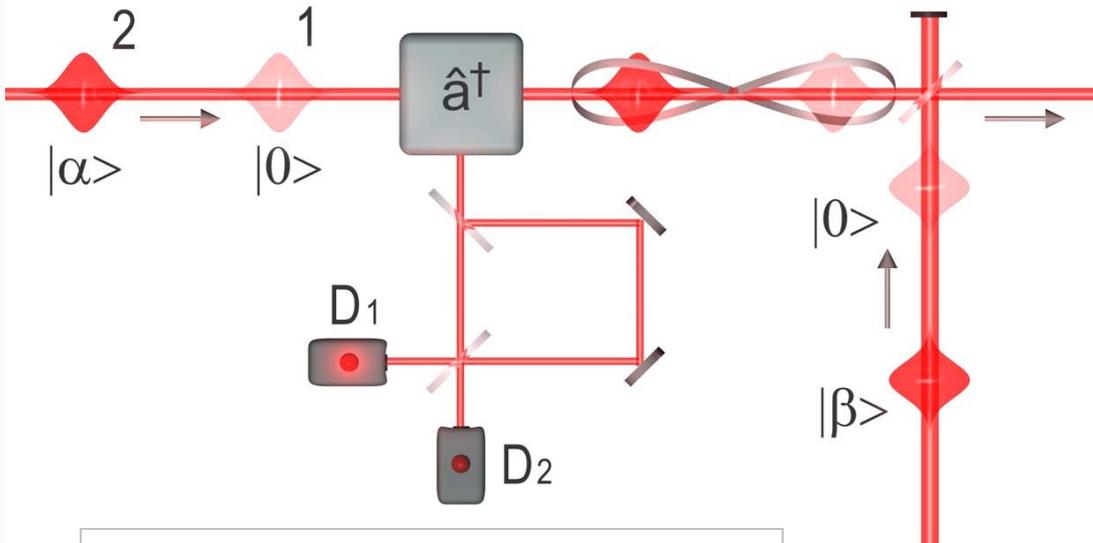
Reconstructed two-mode density matrix  
(corrected for detection efficiency)



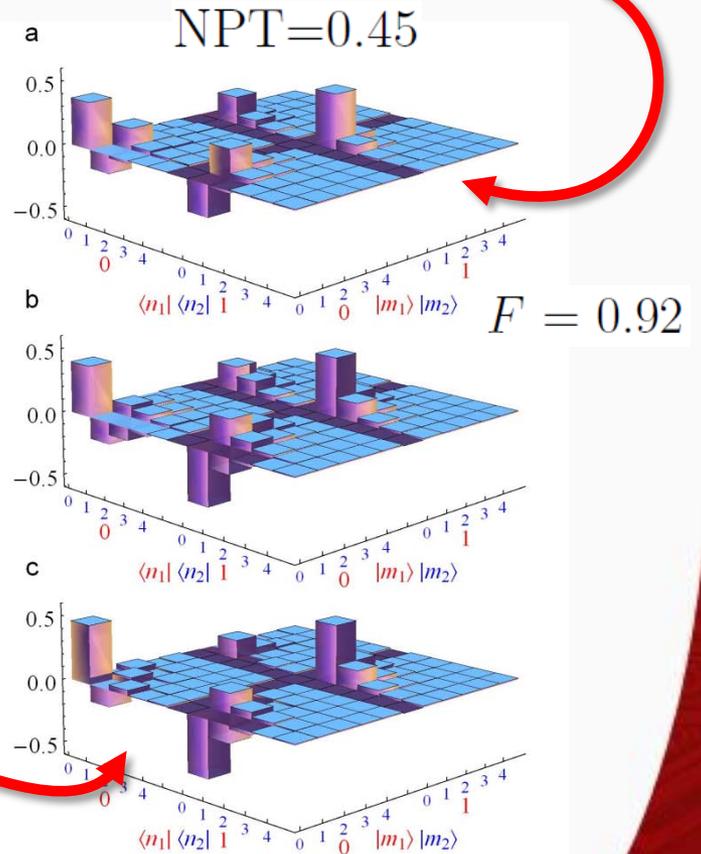
Measured negativity of the partial  
transpose



# Symmetric time-bin hybrid entangled state



Reconstructed two-mode density matrix  
(corrected for detection efficiency)



$$\approx \frac{1}{\sqrt{2}} (|1\rangle|\alpha\rangle + |0\rangle|\alpha'\rangle)$$

Displace the second mode

$$\approx \frac{1}{\sqrt{2}} (|0\rangle|\alpha_f\rangle + |1\rangle|-\alpha_f\rangle)$$

$$\alpha_f \approx 0.31$$

Entangling quantum and classical states of light

arXiv 1309:6192



# Conclusions

New quantum state engineering tools

New multi-mode techniques



Accessing unknown and multiple modes

Experimental Schmidt mode retrieval

Higher-dimensionality encoding of quantum information

Generating (small-scale) optical hybrid entanglement

Explore quantum-classical entanglement

Interface qubits encoded in discrete and continuous variables

Deterministic linear-optical gate operations and teleportation

Extend to larger amplitudes (tele-amplification)



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# Credits

- ❖ **Alessandro Zavatta**
- ❖ **Samuele Grandi**
- ❖ **Luca Costanzo**
- ❖ **Antonio Sales Coelho**
- ❖ **Katiuscia Cassemiro**
- ❖ **Constantina Policarpou**
  
- ❖ **Myungshik Kim**
- ❖ **Jaromir Fiurasek**
- ❖ **Werner Vogel**
- ❖ **Hyunseok Jeong**
- ❖ **Timothy C. Ralph**



# Thank you!

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