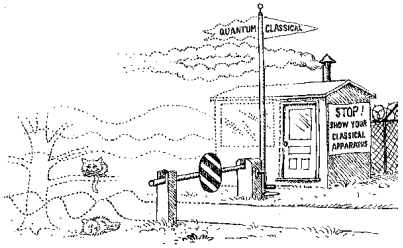
A closer look at quantum states: From ”classical entanglement” to single mode single photon sources

Christoph Marquardt[1,2,3,5], Stefan Berg-Johansen[1,2], Vanessa Chille[1,2], Marion Semmler[1,2], Christian Gabriel[1,2], Andrea Aiello[1,2], Falk Töppel[1,2], Elisabeth Giacobino[1,6], Michael Förtsch[1,2,3], Gerhard Schunk[1,2,3], Josef U. Fürst[1,2], Dmitry Strekalov[1,2], Thomas Gerrits[4], Martin J. Stevens[4], Florian Sedlmeir[1,2], Harald G. L. Schwefel[1,2,3], Sae Woo Nam[4], Gerd Leuchs[1,2]



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[2] Institute for Optics, Information and Photonics,University Erlangen-Nürnberg, Germany

[3]SAOT, School in Advanced Optical Technology, Erlangen, Germany

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Bundesministerium für Bildung und Forschung

SEVENTH FRAMEWORK PROGRAMME

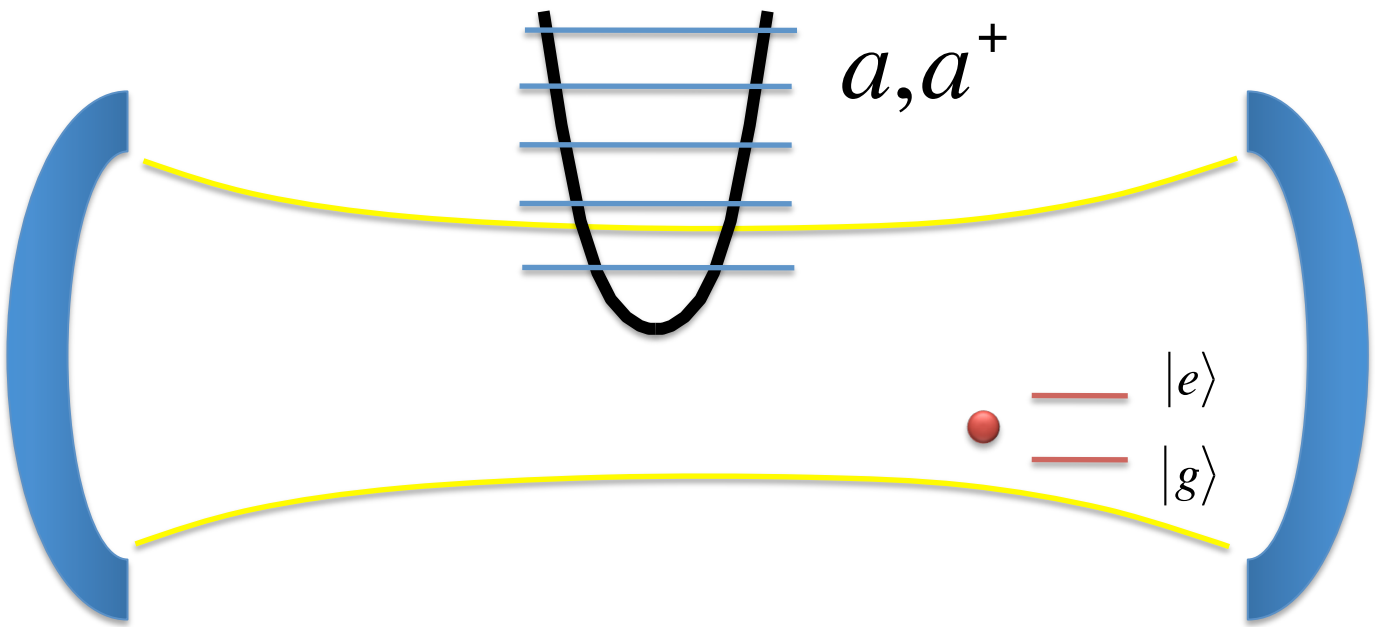
MPL Max Planck Institute for the science of light

Olomouc 15.4.2014

FAU FRIEDRICH-ALEXANDER UNIVERSITÄT ERLANGEN-NÜRNBERG

vectorial modes and „classical entanglement“

Quantum Optics



Maxwell's Equations

+ Field Quantization

+ Interaction with atoms ...

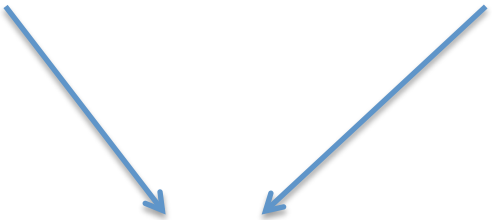
Vectorial Modes

CV Quantum states in spatial multimode

N. Treps et al., Science 301, 940 (2003)
K. Wagner et al., Science 321, 541 (2008),
V. Boyer et al, Science 321, 544 (2008)
M. Lassen et al., Phys. Rev. Lett., 102, 163602 (2009),
…

CV Quantum states in polarization

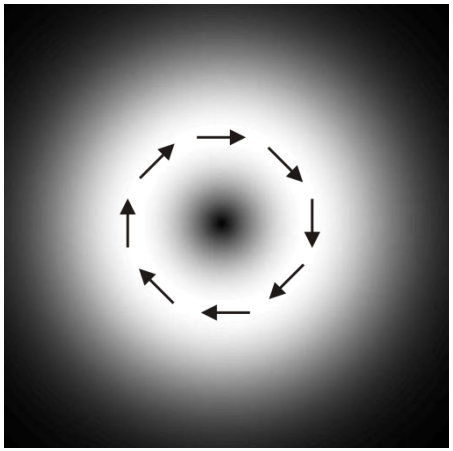
N. Korolkova et al., Phys. Rev. A, 65, 052306 (2002), W. Bowen et al., Phys. Rev.Lett. 89, 253601 (2002),
…

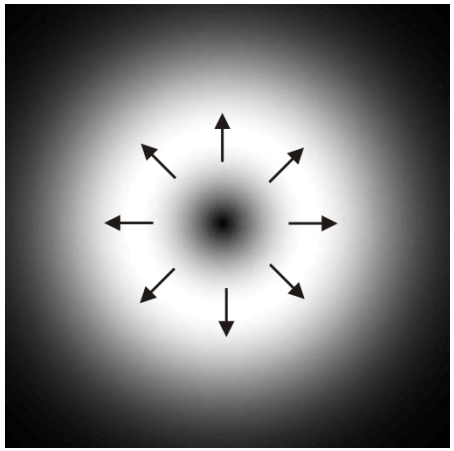


CV Quantum states in vectorial modes

Vectorial Modes

- Investigate nonclassical states of light in radial and azimuthal polarized modes, i.e. modes with complex spatial and complex polarisation pattern





classical applications, e.g. focusing R. Dorn et al. Phys. Rev. Lett., 91 (23), 233901 (2003)

- E.g. combine polarization and two orthogonal spatial modes

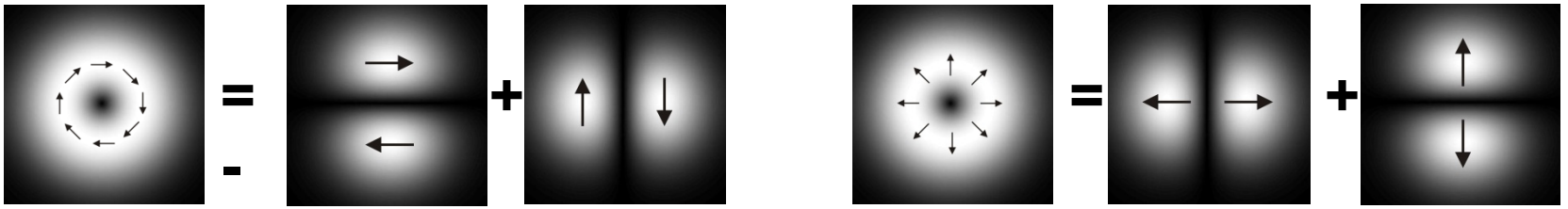
The Radial and Azimuthal Mode

- Modes can be described classically with the transverse mode functions:

$$\mathbf{u}^A = \left(-\mathbf{e}_H\,\psi_{01} + \mathbf{e}_V\,\psi_{10}\right)/\sqrt{2}, \quad \mathbf{u}^R = \left(\mathbf{e}_H\,\psi_{10} + \mathbf{e}_V\,\psi_{01}\right)/\sqrt{2}$$

- Or quantum mechanically with appropiate annihilation operators:

$$\hat{a}_A = \frac{1}{\sqrt{2}}\left(-\hat{a}_{H01} + \hat{a}_{V10}\right), \quad \hat{a}_R = \frac{1}{\sqrt{2}}\left(\hat{a}_{H10} + \hat{a}_{V01}\right)$$

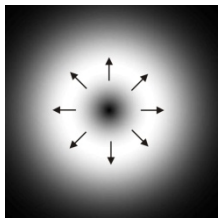


The classical radially polarized mode is not separable in its polarization and spatial degrees of freedom
- structural inseparability

C.E.R. Souza et al., Phys. Rev. Lett. 99, 160401 (2007), A. Holleczek et al, arXiv:1007.2528 (2010), A. Holleczek et al., Optics Express 19, 9714 (2011)

Structural Inseparability

The radially polarized mode can be classically written as:



$$\mathbf{u}_R(x,y,z) = \sum_{n=1}^{2} \sqrt{\lambda_n}\hat{e}_n v_n$$

with $\{\hat{e}_H, \hat{e}_V\} \equiv \{\hat{e}_1, \hat{e}_2\}$ and $\{\psi_{10}, \psi_{01}\} \equiv \{v_1, v_2\}$

and $\lambda_1, \lambda_2 = 1/2$

This classical state has the same tensor-product form of a two-dimensional bipartite maximally entangled quantum state with Schmitt rank:

$$K = 1/\left(\sum_{n=1}^{2} \lambda_n^2\right) = 2$$

The classical radially polarized mode is not separable in its polarization and spatial degrees of freedom.

A. Holleczek et al., Optics Express 19, 9714 (2011)

„classical entanglement“

Small (incomplete) history of classical entanglement analogues:

...
Greenberger et al. ...
Robert Spreeuw in 1998, Foundation of Physics, Vol 28, No. 3, pp. 361-374

C.E.R. Souza et al., Phys. Rev. Lett. 99, 160401 (2007)

A. Luis, Opt. Commun. 282, 3665 (2009)

E. Karimi, Phys. Rev. A 82, 022115 (2010)
C.V.S. Borges et al., Phys. Rev. A 82, 033833 (2010)
B.N. Simon et al., PRL 104, 023901 (2010)
A. Holleczek et al, arXiv:1007.2528 (2010)

A. Holleczek et al., Optics Express 19, 9714 (2011),
X.F. Qian, J.H. Eberly, Opt. Lett., 36, 4110 (2011)

...

is it useful?

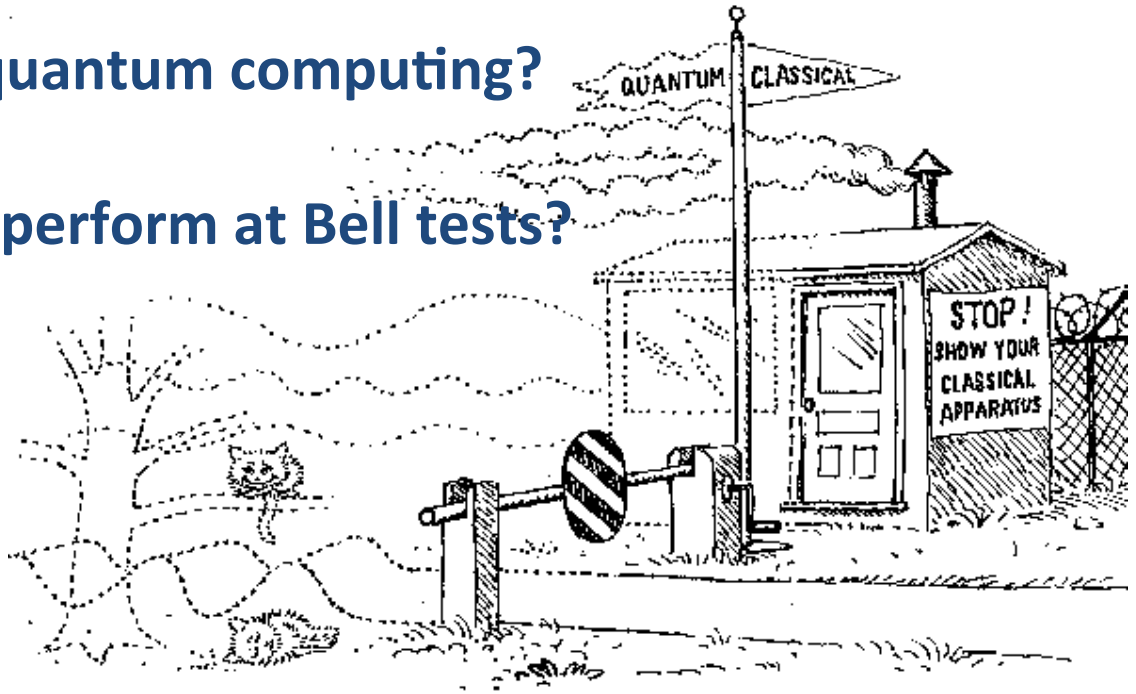
quantum – classical devide is not so obvious,

and deserves a closer look ...

What does non-classical mean?

What is really needed for quantum computing?

How do these correlations perform at Bell tests?



Bell test with classical states?

PHYSICAL REVIEW A **82**, 033833 (2010)

Bell-like inequality for the spin-orbit separability of a laser beam

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[1]*Universidade Federal Fluminense, Niterói, Brazil*
[2]*Universidade Federal Fluminense, Volta Redonda, Brazil*
(Received 11 November 2009; published 28 September 2010)

In analogy with Bell's inequality for two-qubit quantum states, we propose an inequality criterion for the nonseparability of the spin-orbit degrees of freedom of a laser beam. A definition of separable and nonseparable spin-orbit modes is used in consonance with the one presented in Phys. Rev. Lett. **99**, 160401 (2007). As the usual Bell's inequality can be violated for entangled two-qubit quantum states, we show both theoretically and experimentally that the proposed spin-orbit inequality criterion can be violated for nonseparable modes. The inequality is discussed in both the classical and quantum domains.

DOI: 10.1103/PhysRevA.82.033833 PACS number(s): 42.50.Tx, 42.25.Kb, 03.65.Ud, 03.65.Ta

classical „quantum computing“?

INSTITUTE OF PHYSICS PUBLISHING JOURNAL OF OPTICS B: QUANTUM AND SEMICLASSICAL OPTICS

J. Opt. B: Quantum Semiclass. Opt. **7** (2005) 288–292 doi:10.1088/1464-4266/7/9/009

Implementing the Deutsch algorithm with polarization and transverse spatial modes

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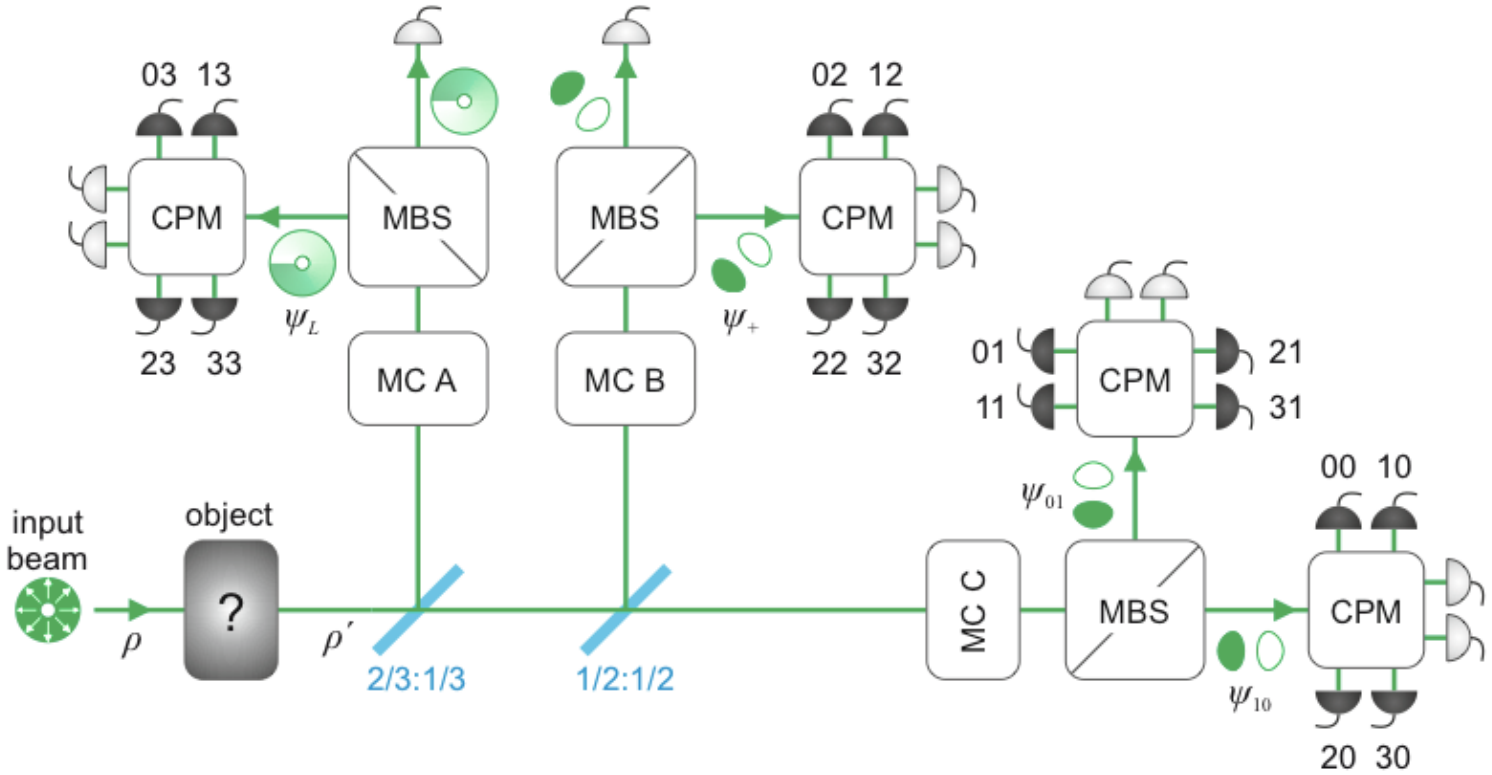
Online at stacks.iop.org/JOptB/7/288

Abstract
In this paper we implement a simple quantum algorithm using polarization and transverse spatial modes of the electromagnetic field as qubits. The transverse spatial modes used are the Laguerre–Gaussian and Hermite–Gaussian beams of order $N = 1$. With these two qubits carried by the same photon, we have implemented the Deutsch algorithm using a simple linear optical setup.

Advanced Measurements

Classical entanglement in polarization metrology:

F. Töppel et al., arXiv:1401.1543 (2014)



Müller matrix reconstruction with a single probe beam.

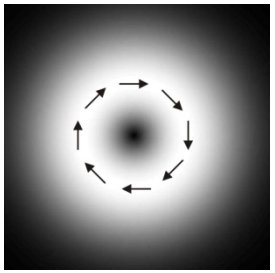
Squeezing the Azimuthal Mode

What if we squeeze vectorial modes?

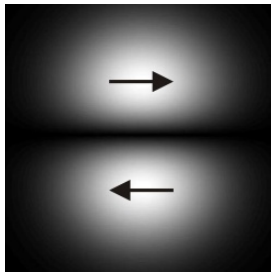
In case of an azimuthal polarized mode:

$$|\alpha,\zeta\rangle = \hat{D}_1(\alpha/\sqrt{2})\hat{S}_1(\zeta/2)\hat{D}_2(\alpha/\sqrt{2})\hat{S}_2(\zeta/2)\hat{S}_{12}(-\zeta/2)|0\rangle$$

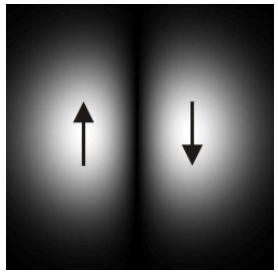
Squeezed



Squeezed

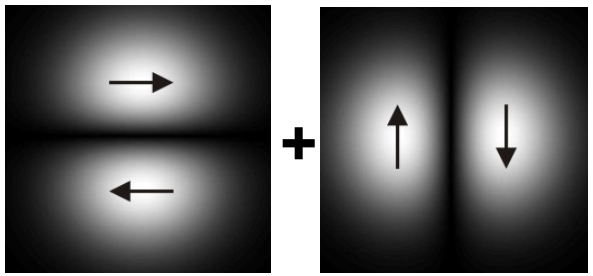


Squeezed



$$= \exp[(-\zeta^*\hat{a}_1\hat{a}_2 + \zeta\hat{a}_1^\dagger\hat{a}_2^\dagger)/2]$$

Entanglement



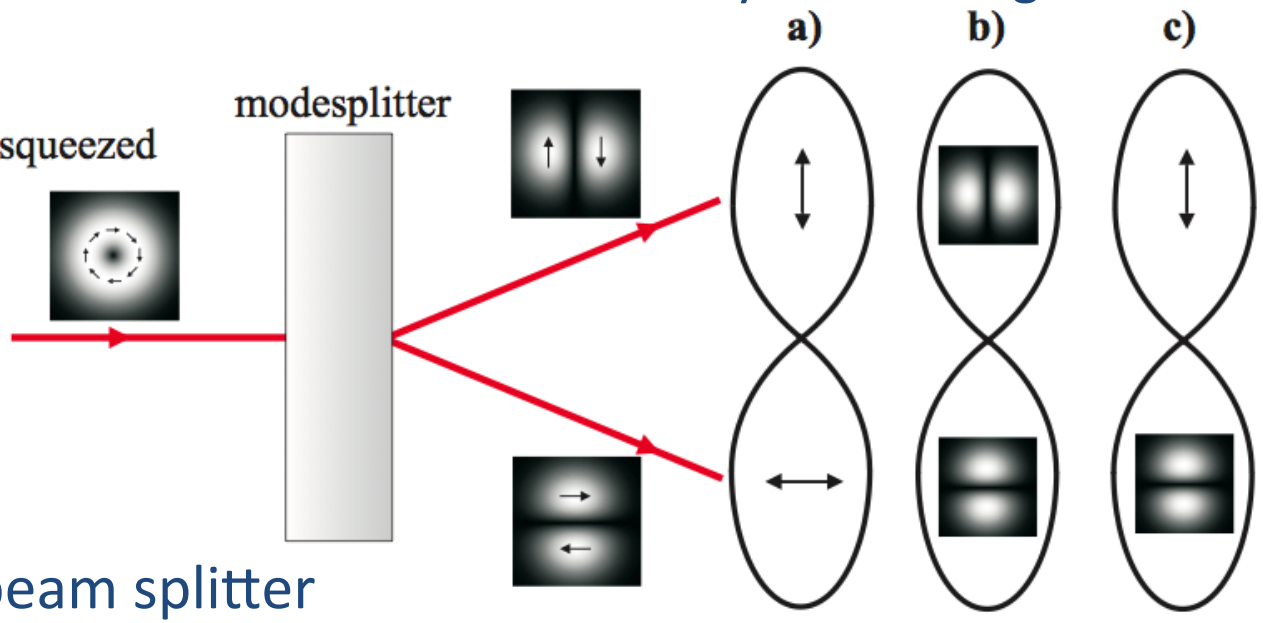
with *i*=1,2 stand for H01 and V10

CV Hybrid-Entanglement

- Hybrid-Entanglement is entanglement between two degrees of freedom (but we need two beams).

  M. Żukowski and A. Zeilinger, Phys. Lett. A 155, 69 (1991)

  E. Karimi et al., Phys. Rev. A 82, 022115 (2010)
- We have entanglement between the spatial and the polarization degree of freedom and therefore a CV hybrid-entangled state.



Modesplitter:

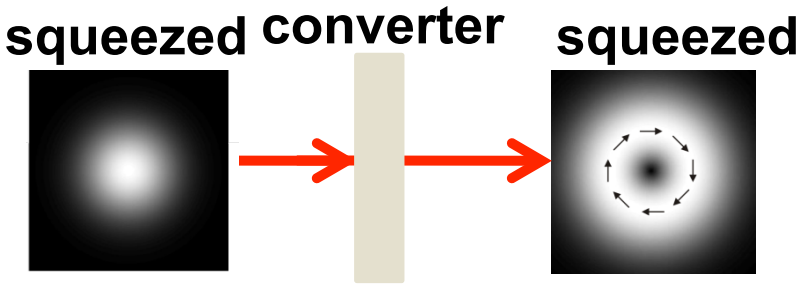
- Polarizing beam splitter
- Spatial modesplitter (special Mach-Zehnder interferometer)

Experimental Realization

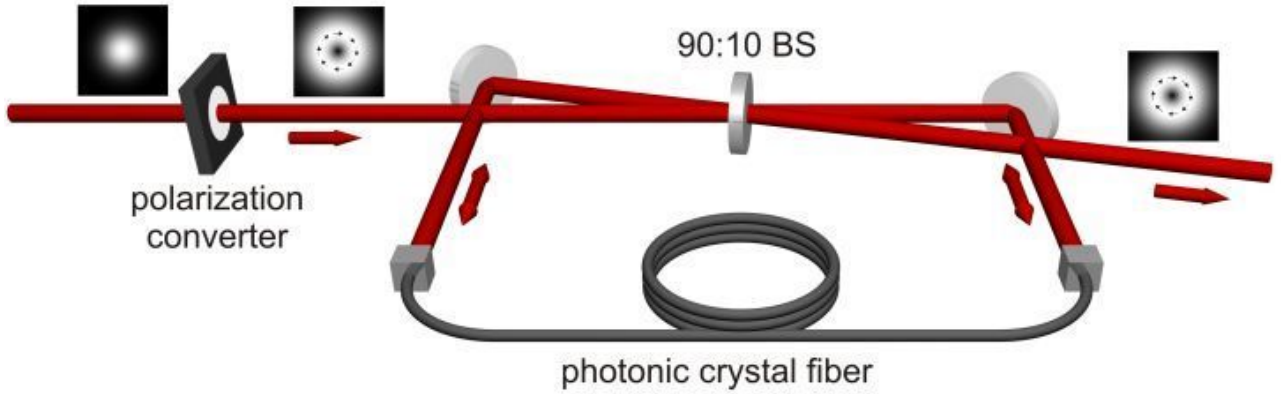
How to generate such a squeezed mode?

- Squeeze a Gaussian mode and use a mode converter

C. Gabriel et al.,
Eur. Phys. J. D 66, 172 (2012)



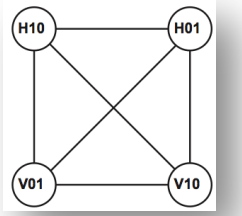
- Exploit nonlinear Kerr-effect in fibers which support these modes. Sagnac-loop can be used to measure amplitude squeezing



C. Gabriel et al., Phys. Rev. Lett., 106, 060502 (2011)

Hybrid Cluster States

Continuous variable cluster states:

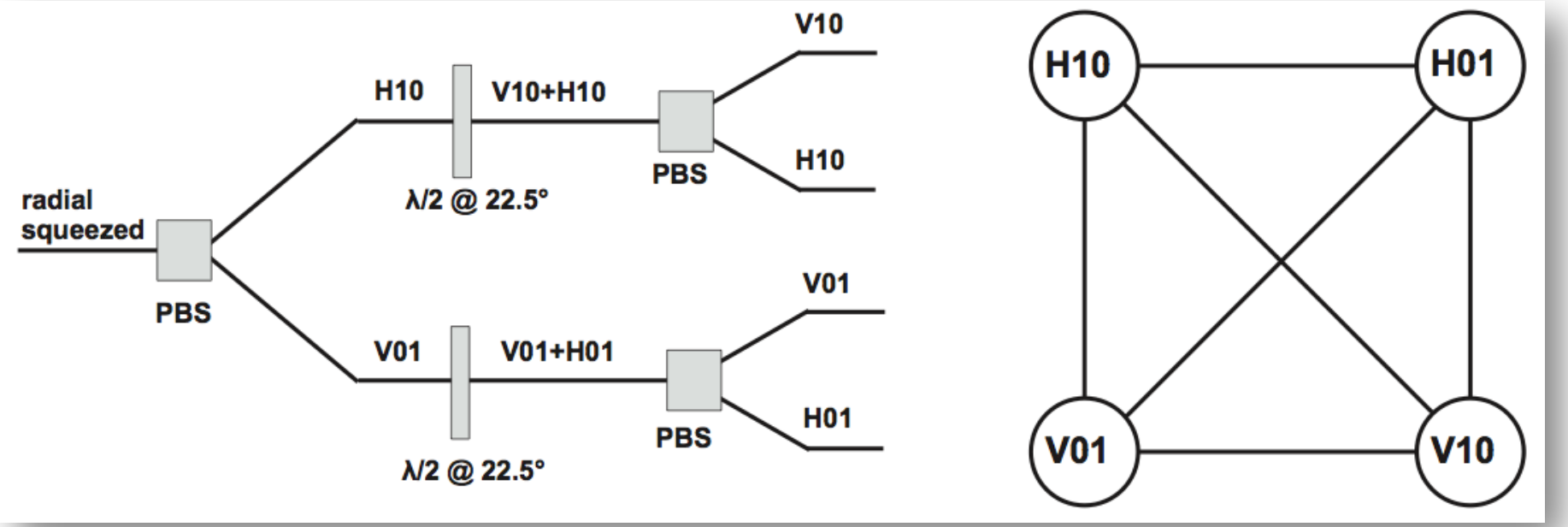


C. H. Bennett and D. P. DiVincenzo, Nature 404, 247 (2000).
N. Menicucci, P. van Loock, M. Gu, C. Weedbrook, T. Ralph,
M. Nielsen, Phys.Rev. Lett. 97, 110501 (2006).
N. Menicucci, S. Flammia, and O. Pfister, Phys. Rev. Lett. 101, 130501 (2008)
M. Yukawa, R. Ukai, P. van Loock, and A. Furusawa, Phys. Rev. A 78, 012301 (2008)

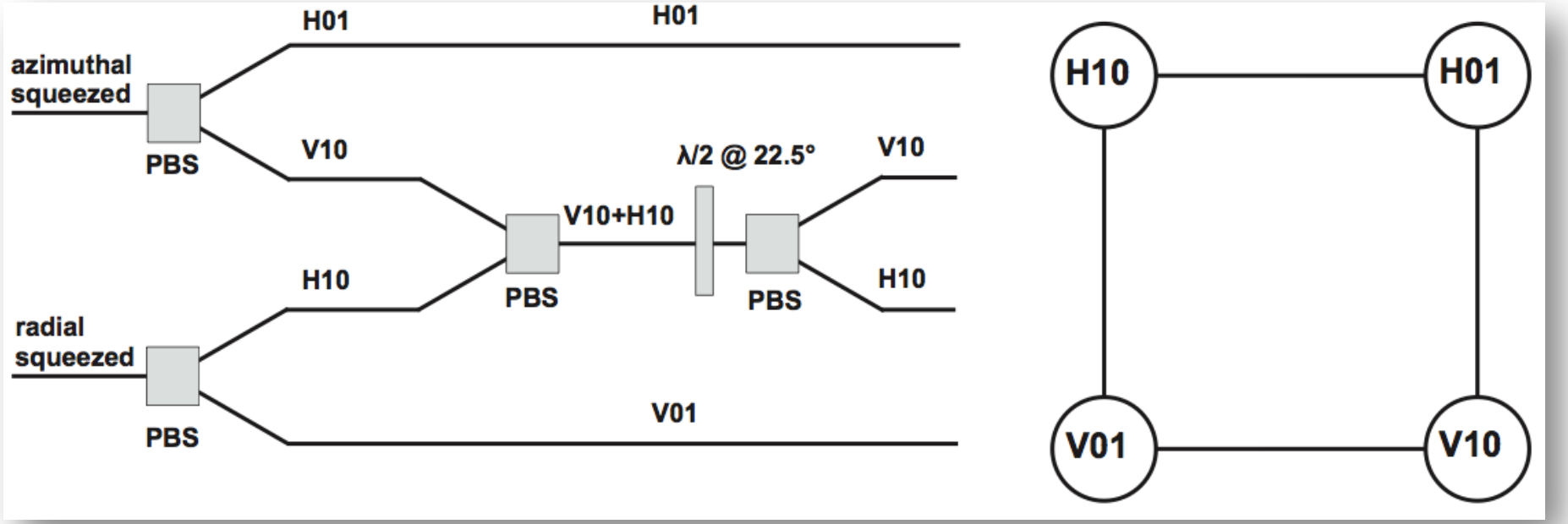
Convenient handling and discrimination of the cluster components:

- Use correlations in vectorial mode
- Select cluster modes in different basis modes

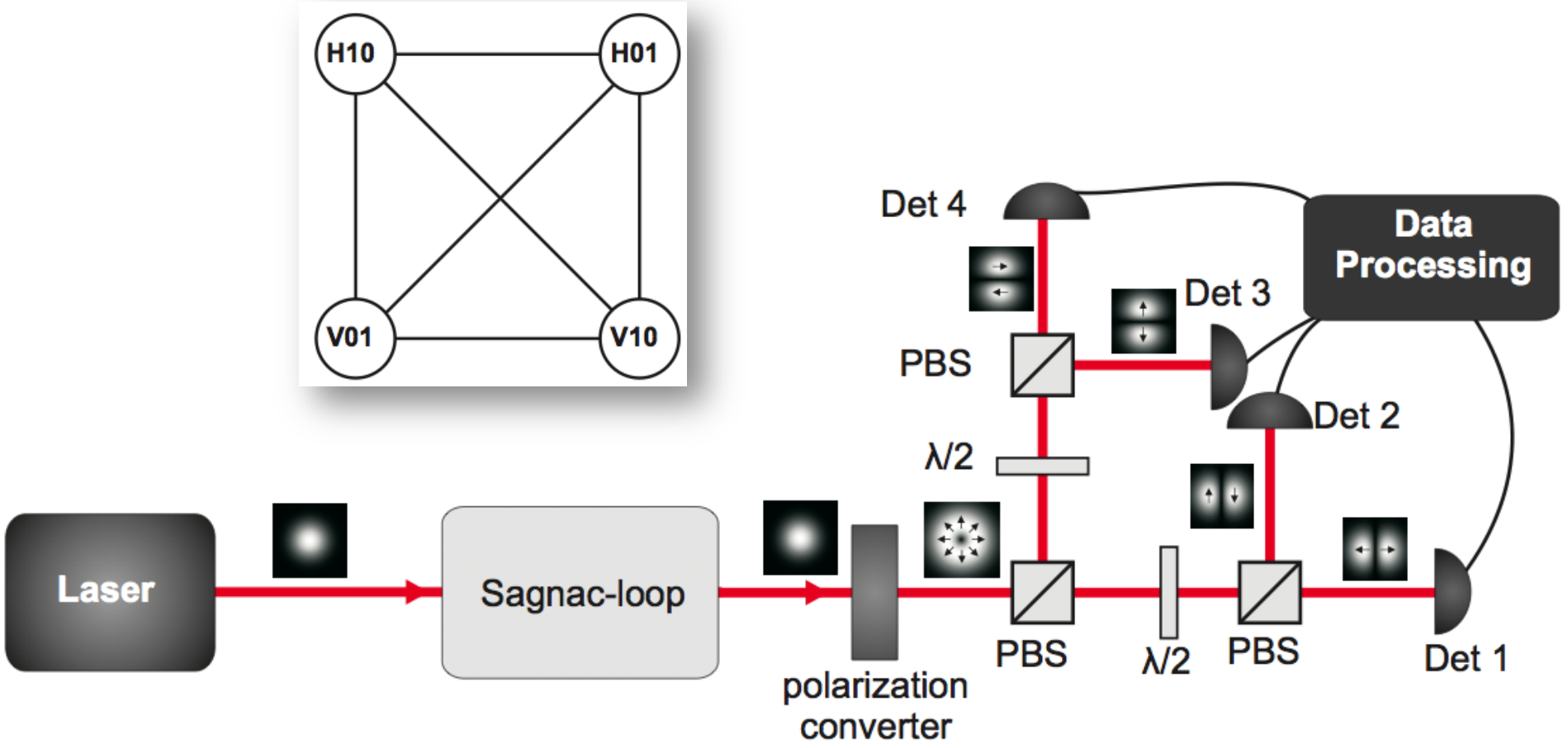
Hybrid Cluster States



Hybrid Cluster States



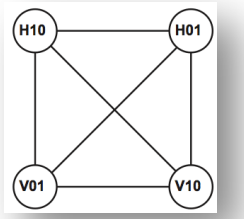
Hybrid Cluster - Experiment

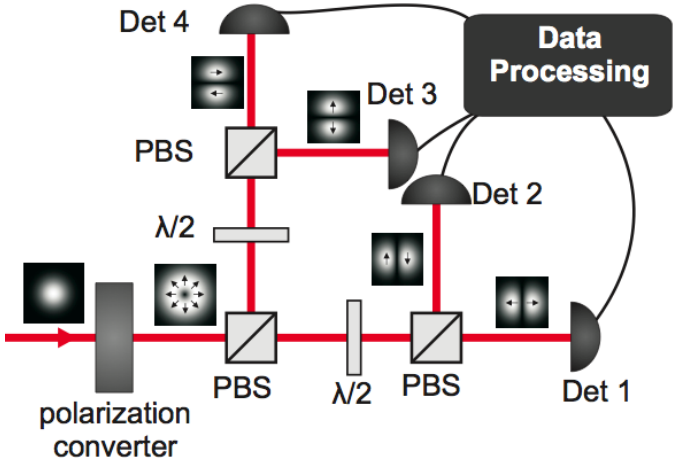


I. Rigas et al., arXiv:1210.5188

Hybrid Cluster - Experiment

Study amplitude quantum correlations in cluster state:



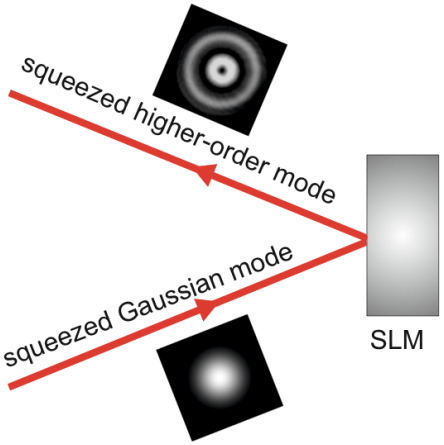


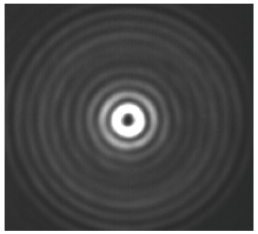
	Correlations [dB]
H01+H10	-0.8 (-0.8)
H01+V10	-0.8 (-0.8)
V01+H10	-0.8 (-0.8)
V01+V10	-0.7 (-0.8)
H01+V01	-0.8 (-0.8)
V10+H10	-0.7 (-0.8)
H01-H10	0.0 (0.0)
H01-V10	0.1 (0.0)
V01-H10	0.1 (0.0)
V01-V10	0.1 (0.0)
H01-V01	0.1 (0.0)
V10-H10	0.0 (0.0)

I. Rigas et al., arXiv:1210.5188

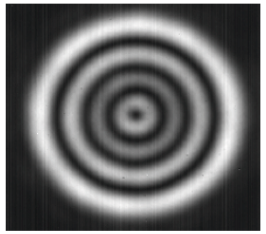
Squeezing Optical Modes

Squeezing arbitrary spatial modes with help of a spatial light modulator.



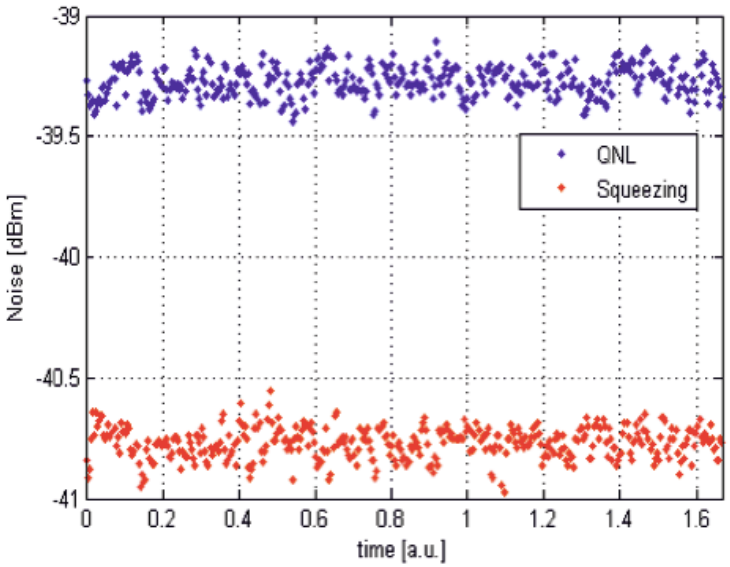


Bessel beam
(n=2)

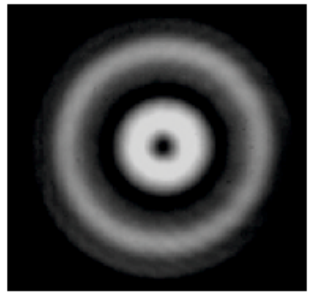


LG_{31}

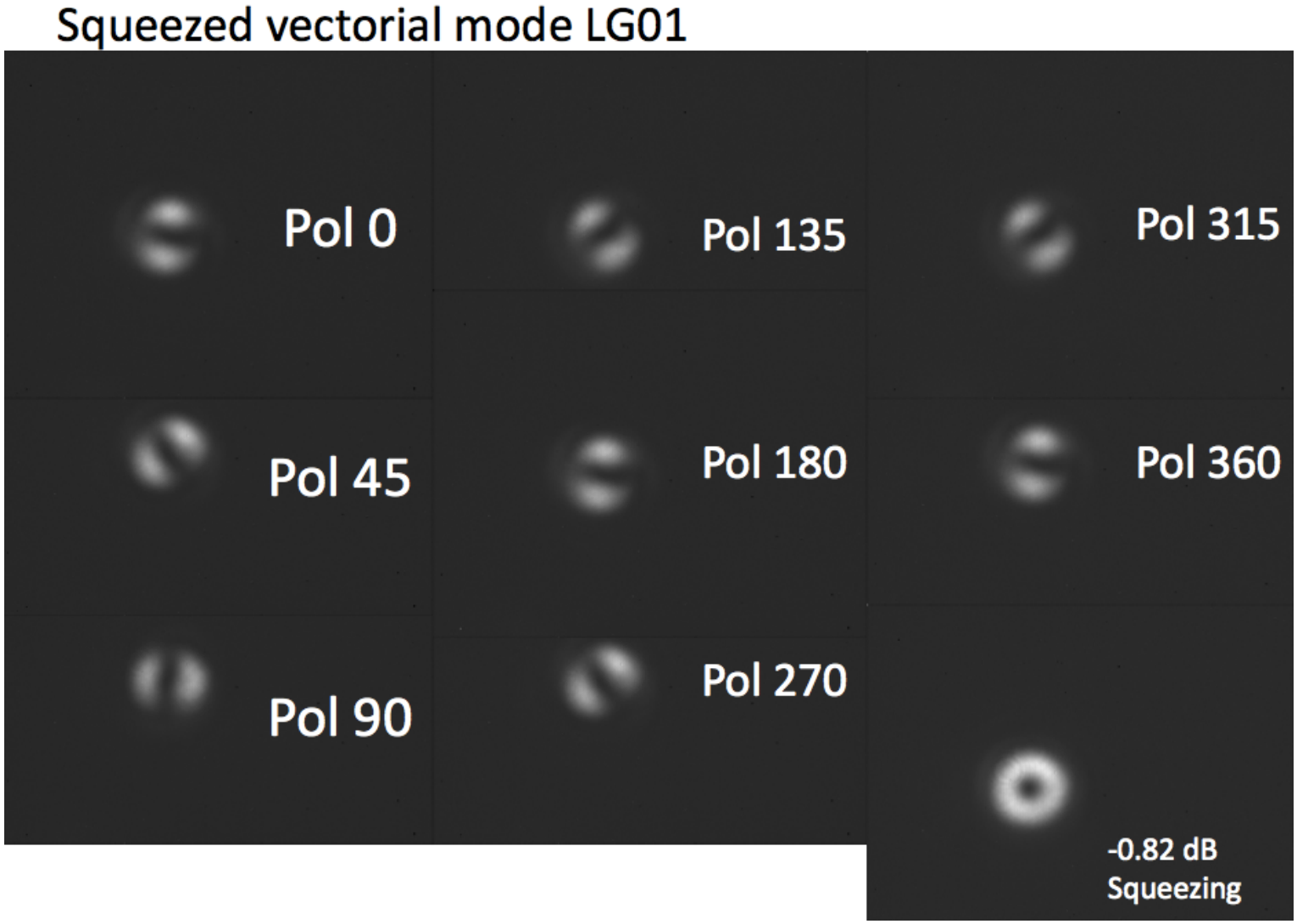
arbitrary patterns



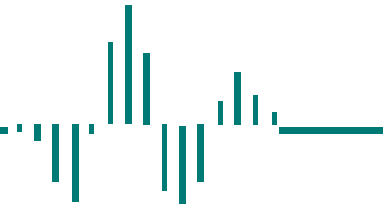
Spatial mode profile LG_{11}:



Squeezing Vectorial Modes



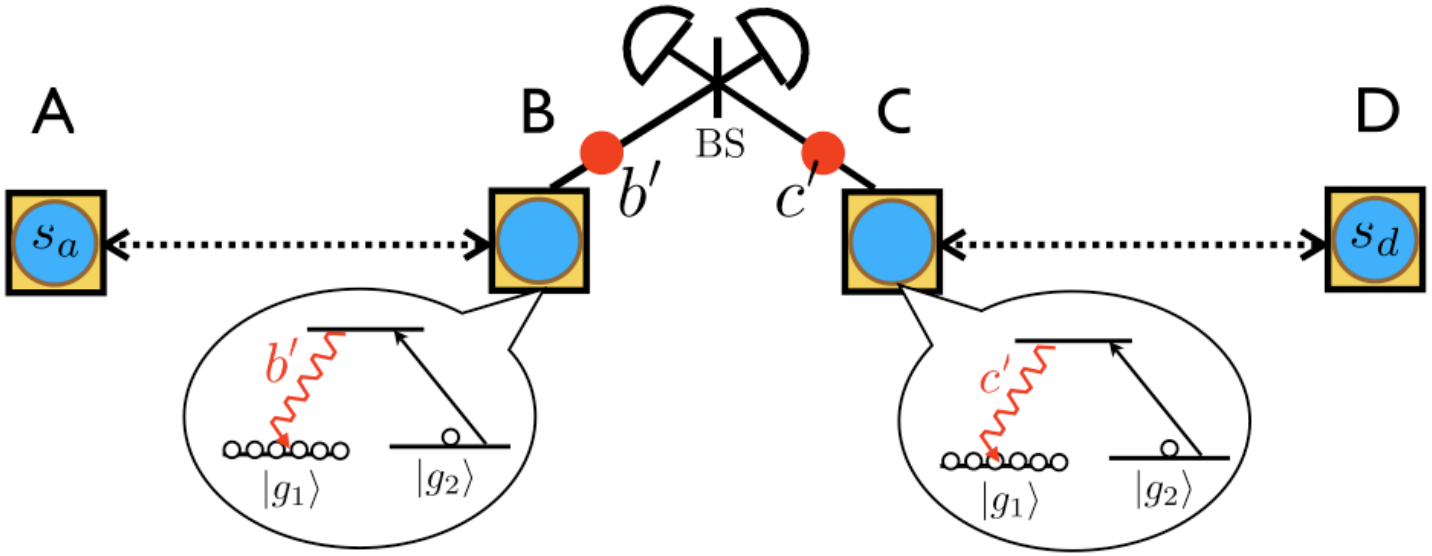
Conclusion & Outlook



- Classical structural inseparability in cylindrically polarized modes (spin-orbit coupled)
- “classical entanglement” shows interesting correlation properties also in classical states
- Squeezing in such modes leads to entanglement of different degrees of freedom, cluster state generation
- solutions to Maxwell equations can be rich – study quantum optics in vectorial modes

whispering gallery resonator source of single mode single photons

A Challenge in Quantum Optics



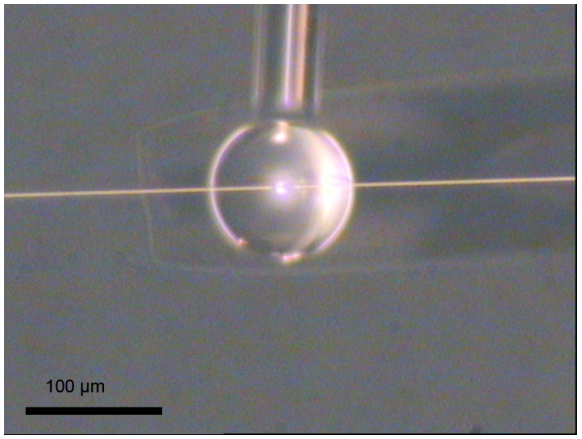
N. Sangouard et al., Rev. Mod. Phys. 83 (1) 33 (2011)

Efficient coupling:

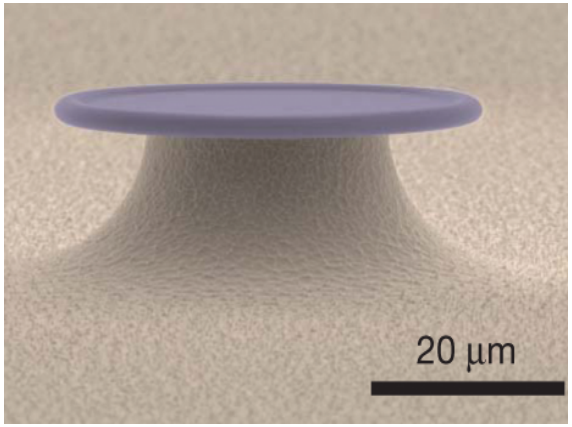
Requirements in different degrees of freedom, often: match central wavelength and bandwidth!

→ Versatile Sources of Quantum States of Light

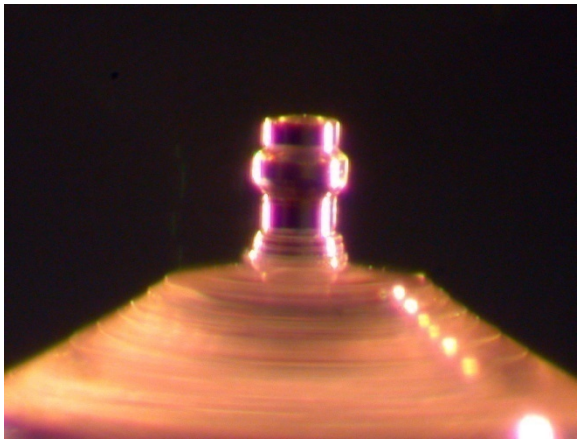
Whispering Gallery Resonators



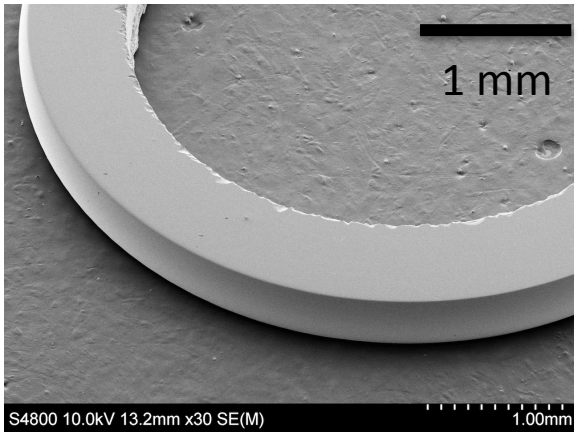
MPI for the Science of Light



Vahala group, Caltech



Jet Propulsion Laboratory



MPI for the Science of Light

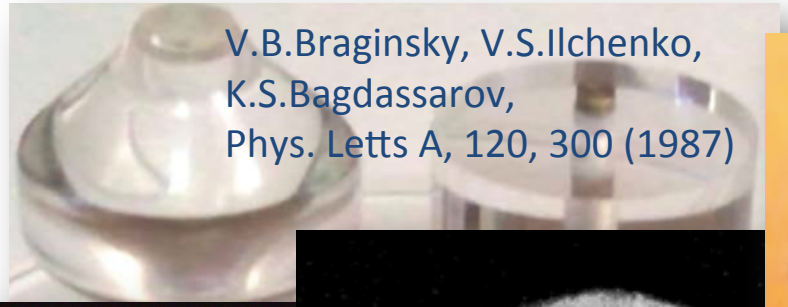
amorphous materials (e.g., fused silica) can be shaped by the surface tension in a reflow process. *Only third order nonlinearity!*

crystalline materials (e.g. Lithium Niobate) typically have lower losses **and may possess second order nonlinearity**, but they need to be polished mechanically.

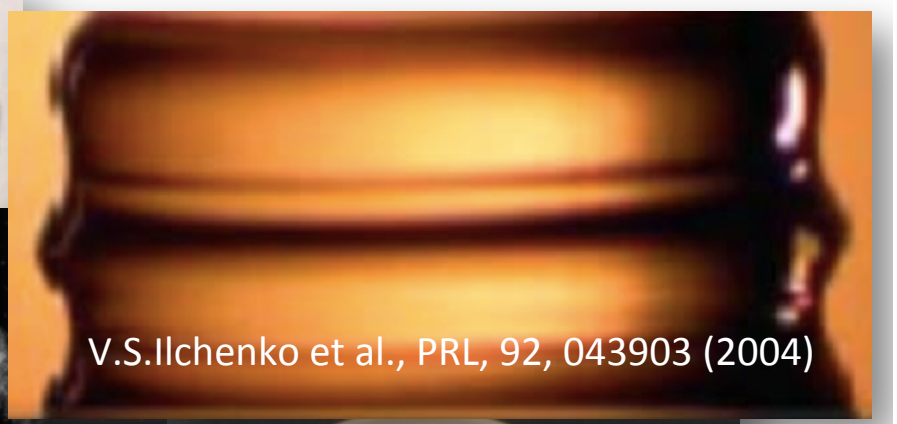
Whispering gallery resonators are coupled via evanescent field.

Crystalline WGRs

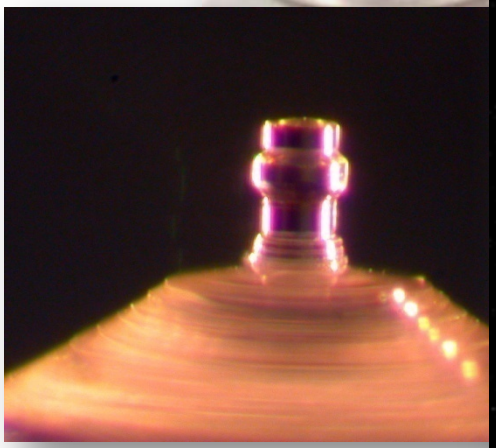
Very early work: Lasing in WGRs, Bell Labs, 1961 (!)
Braginsky group – Moscow State University
Maleki group – Jet Propulsion Laboratory / OEWaves
. . .



V.B.Braginsky, V.S.Ilchenko, K.S.Bagdassarov, Phys. Letts A, 120, 300 (1987)



V.S.Ilchenko et al., PRL, 92, 043903 (2004)



Jet Propulsion Laboratory

C. G. B. Garrett, W. Kaiser, and W. L. Bond, Phys. Rev. 124, 1807 (1961)

MPL Erlangen

Why use WGM resonator?

Q — cavity field build-up

figure of merit for nonlinear processes:

V — optical energy density

$$\frac{Q}{V}$$

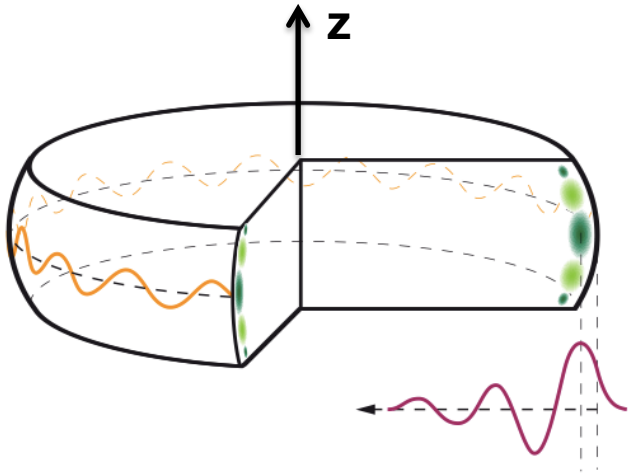
Q > 10^8 for Lithium Niobate crystalline disk resonator

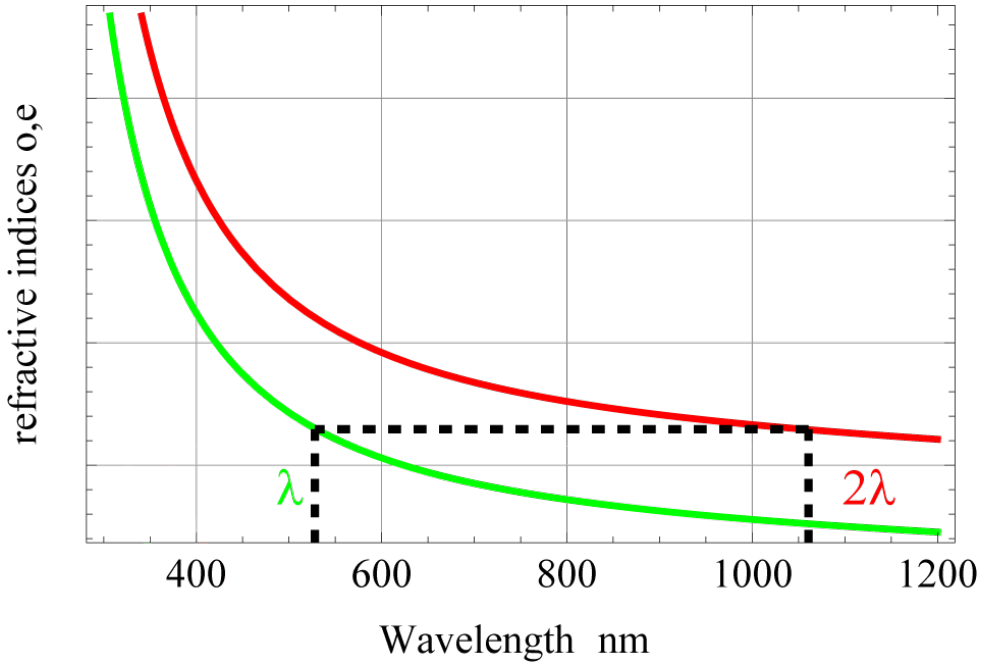
- high Q values in WGM resonators.
- WGM resonators have good stability
- output coupling can be variable
- resonator works in large wavelength regime

Second Harmonic Generation (SHG):
J. Fürst et al., *Phys. Rev. Lett.* **104**, 153901 (2010)

Lithium Niobate offers strong second order nonlinearity!
Natural (temperature) phase matching with uniaxial (z-cut) crystal with Type I phase matching.

Wavelength tuning





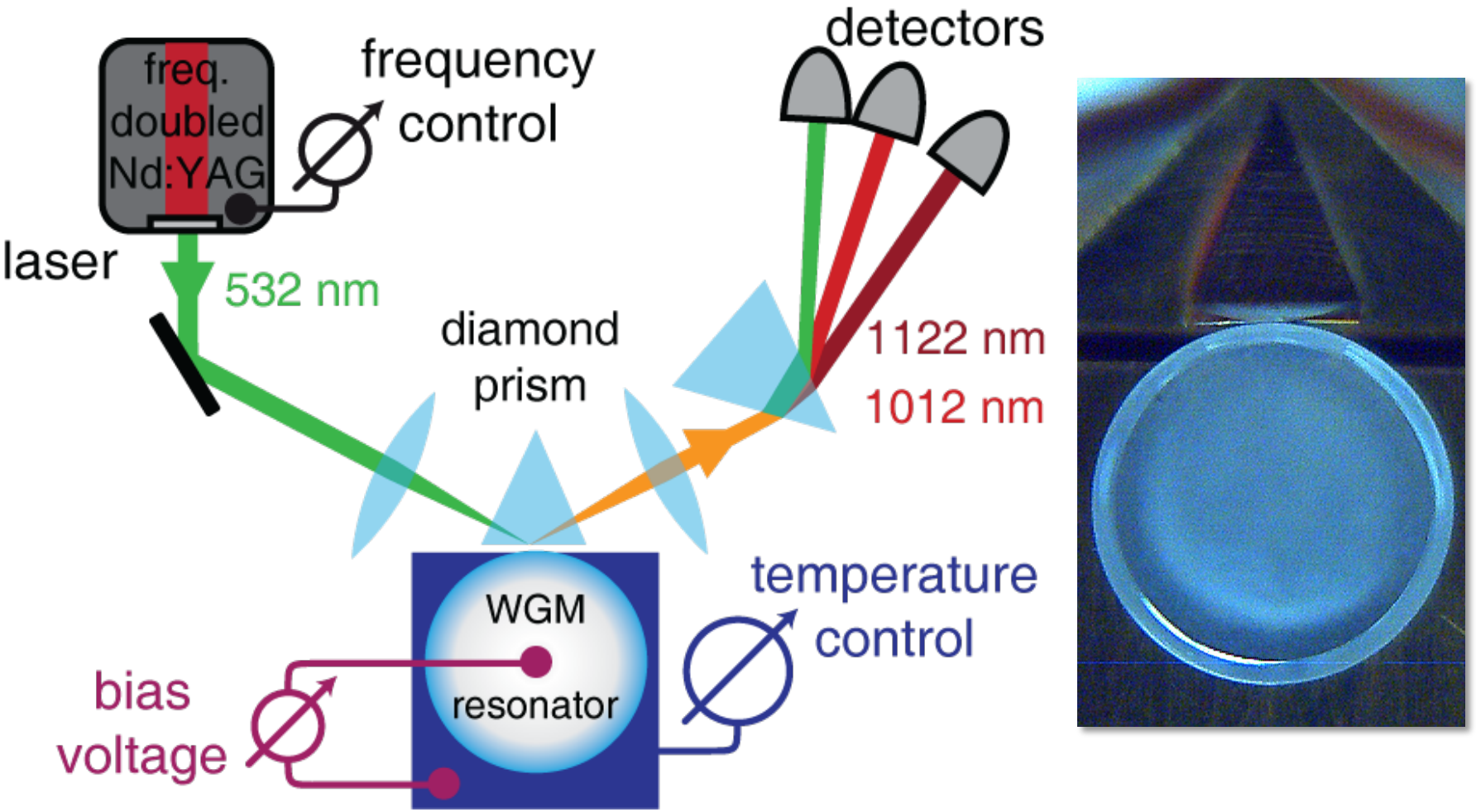
- Type I natural phase matching
- z-cut crystal – phase matching of WGR modes

Effective index of refraction depends on:
Temperature, Mg doping, disk size, voltage

Alternative: periodic poling

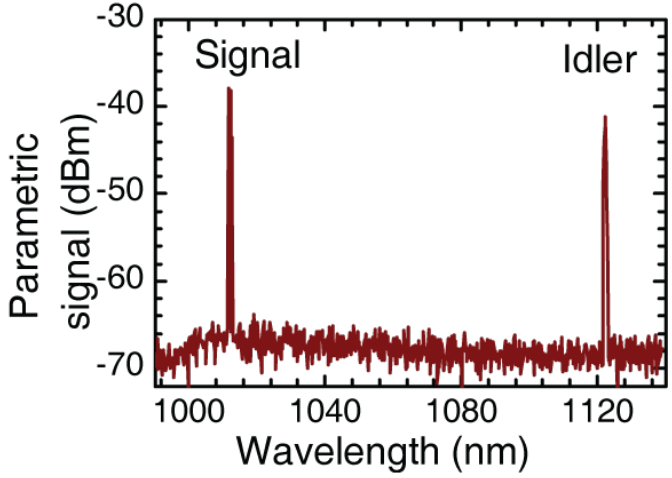
K. Sasagawa et al., Appl. Phys. Expr. 2, 122401 (2009)
T. Beckmann et al., PRL 106, 143903 (2011)

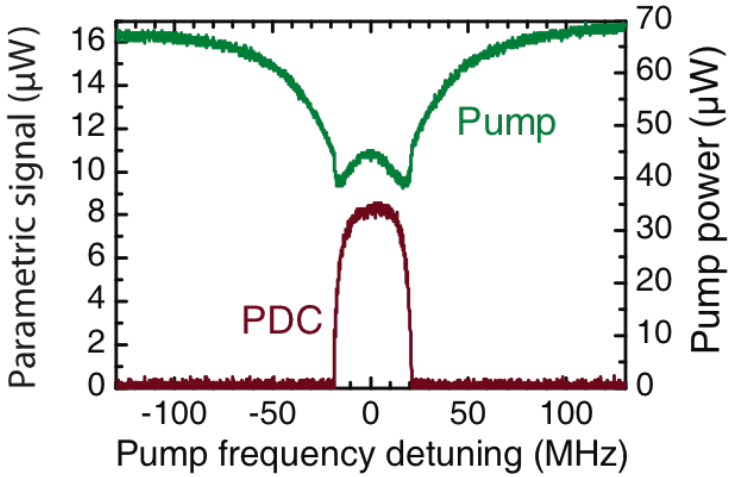
Optical Parametric Oscillator



J. U. Fürst et al., Phys. Rev. Lett. 105, 263904 (2010)

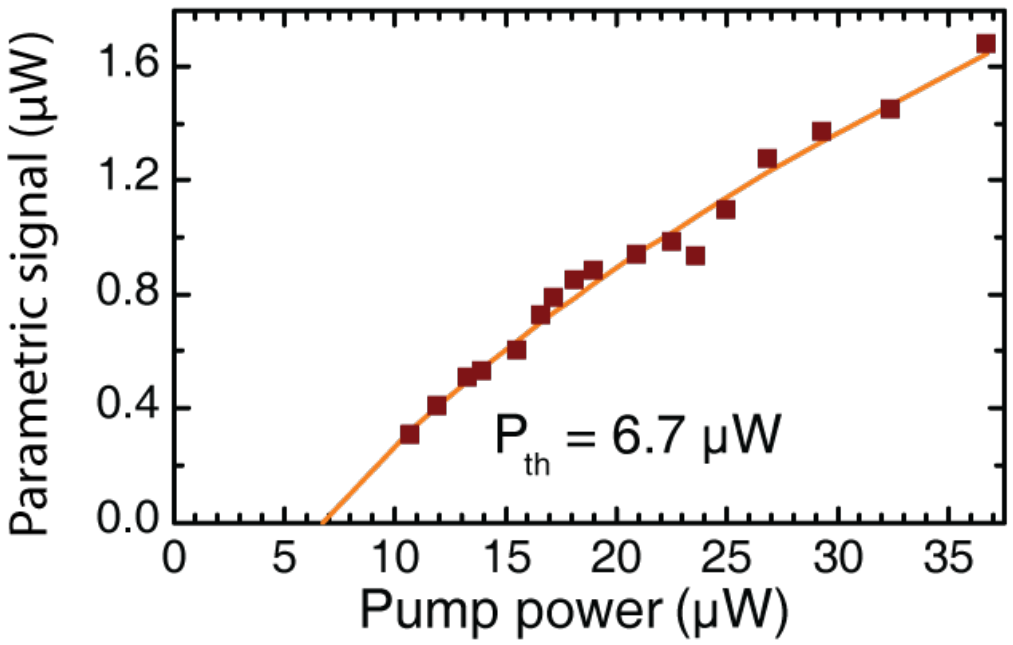
Parametric Downconversion





OPO output

$Q=3.4\times10^7$

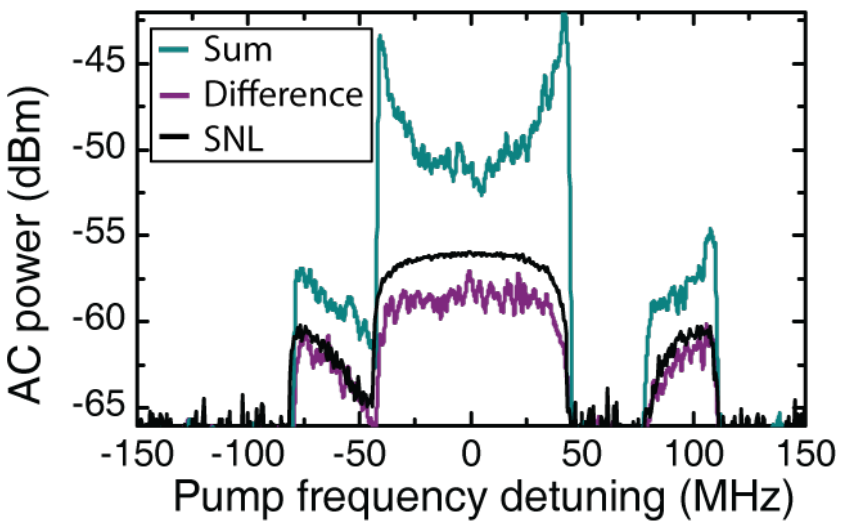


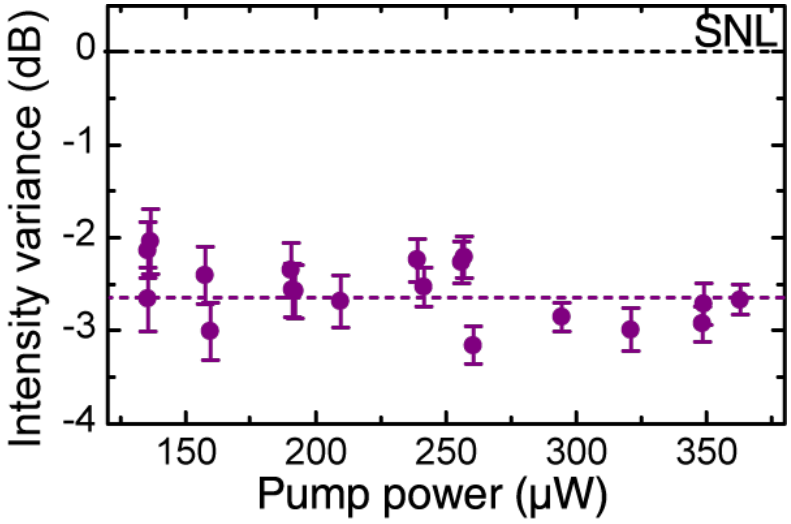
OPO with extremely low threshold!

J. U. Fürst et al., Phys. Rev. Lett. 105, 263904 (2010)

Two-mode suqeezing

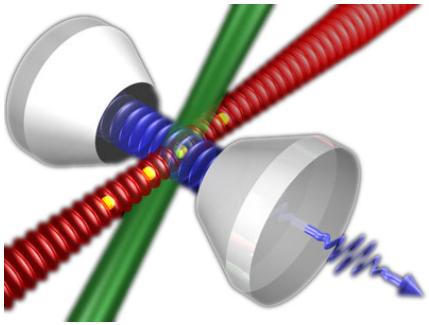
J. U. Fürst et al., Phys. Rev. Lett. 106, 113901 (2011)



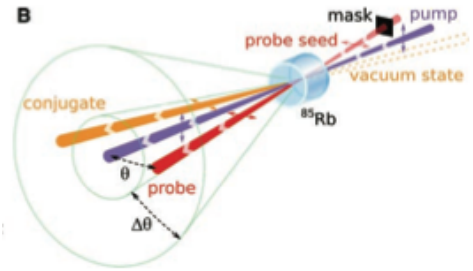


Quantum correlated beams of light from whispering gallery disk resonator.

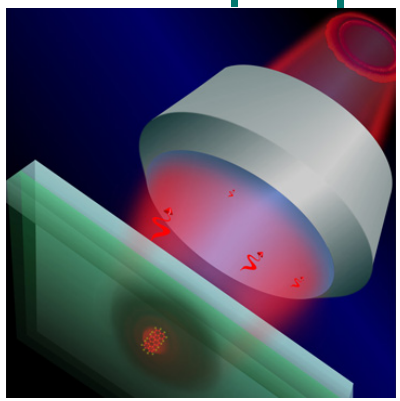
Single Photon Generation



G. Rempe, MPQ, Munich



P. Lett group, NIST Maryland
C. Kurtsiefer group, CQT



V. Sandoghdar, MPL, Erlangen

Four wave mixing

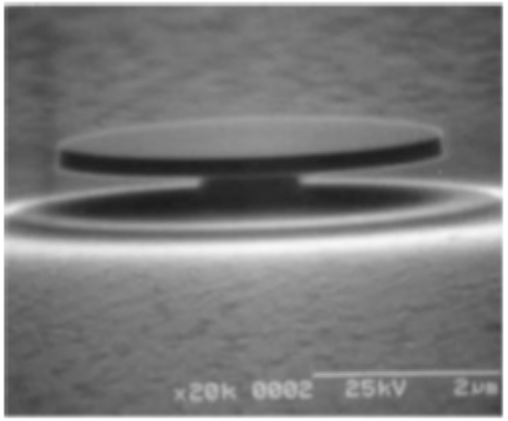
Atoms

Molecules

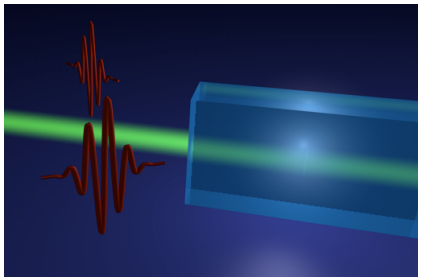
Quantum Dots

NV - Centers

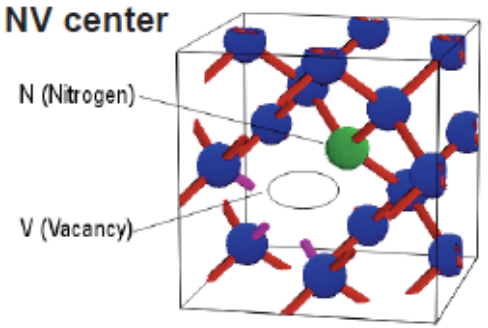
SPDC



A. Imamoğlu group, California



Ou, Lu, Indianapolis, Indiana
O. Benson, Berlin



C. Kurtsiefer, H. Weinfurter,
LMU, Munich

Whispering Gallery Modes

- Radial part: spherical Bessel functions

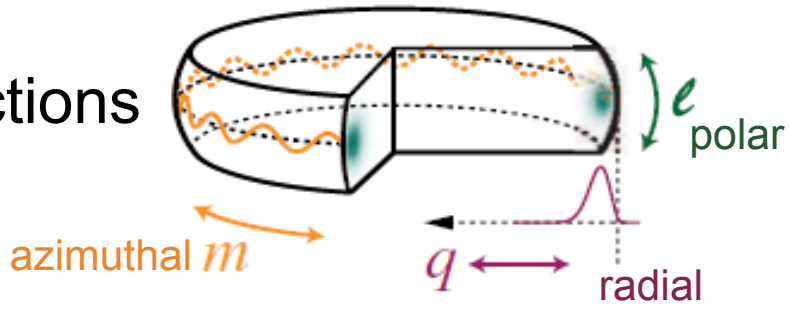
$$j(k_q r)$$

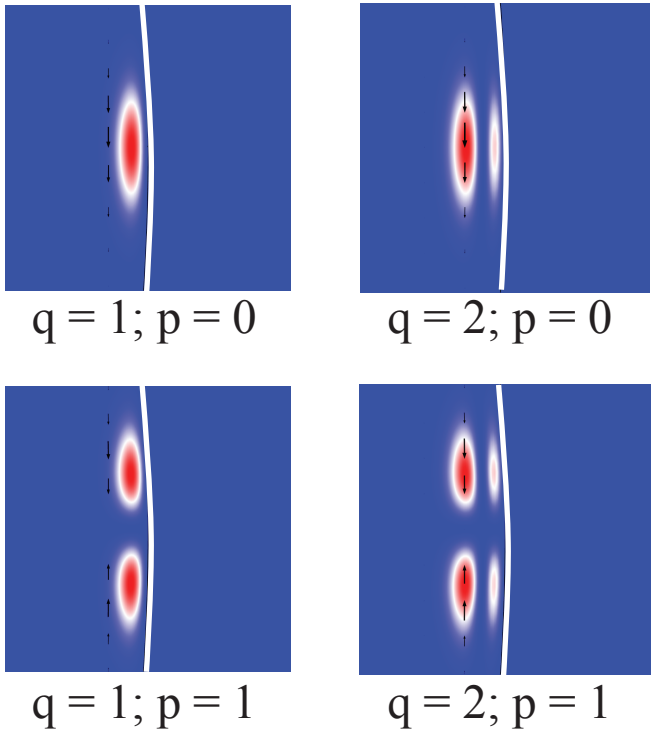
- Angular part: spherical harmonics

$$Y_{lm}(\theta, \varphi) \qquad [1]$$

- Describing mode indices

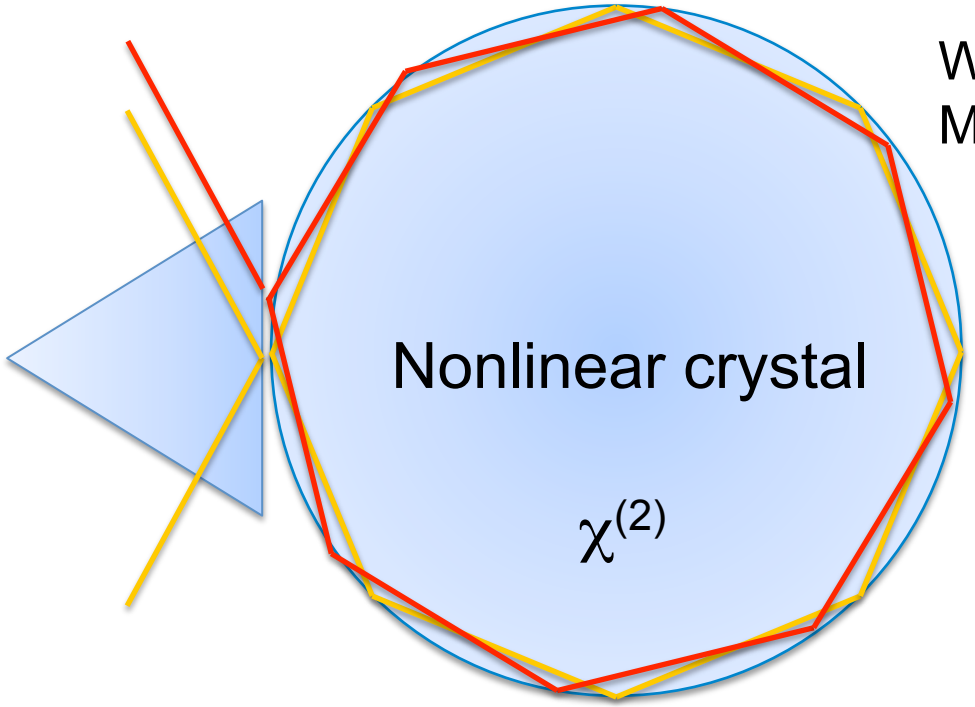
q	radial
m	azimuthal
l	polar
$(l-m)=p$	angular





[1] P. Debye, Ann. Physik, 30, 57 (1909)
[2] J. U. Fürst et al., Phys. Rev. Lett. ***105****, 263904 (2010)*

Phase Matching in WGM



Whispering Gallery Mode Resonator (WGMR)

Energy conservation

$$\omega_p = \omega_s + \omega_i$$

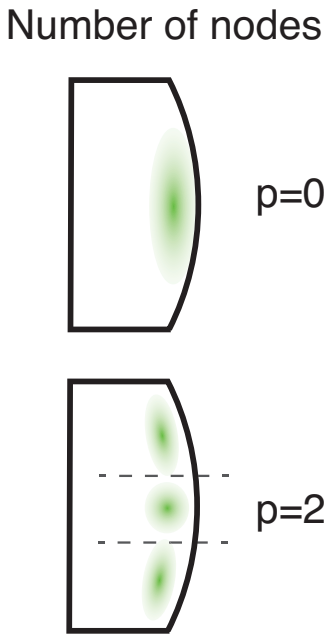
Momentum conservation

$$\vec{k}_p = \vec{k}_s + \vec{k}_i + \vec{k}_p = 0$$

$$|l_s - l_i| \leq l_p \leq l_s + l_i$$

$$l_s + l_i + l_p = 2\mathbb{N}$$

Single Photon Generation

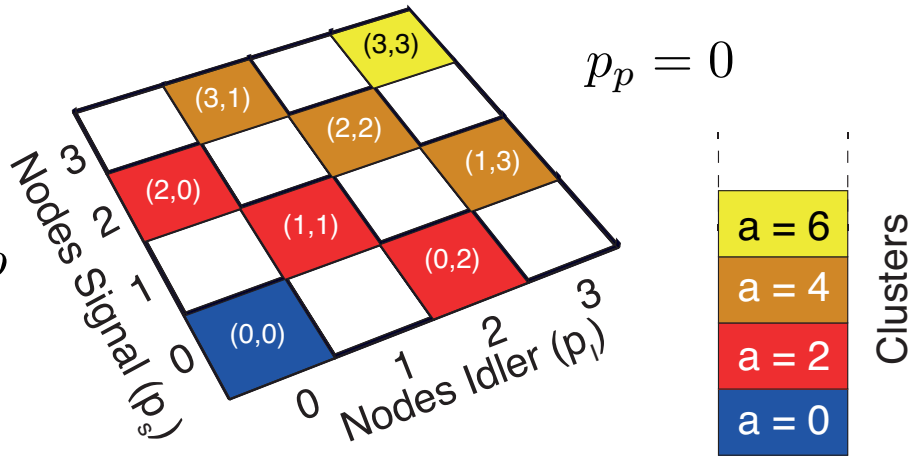


Momentum conservation

$$\underbrace{(m_s + l_s - m_i + l_i)}_{\in 2\mathbb{N}_0} m_p \underbrace{(p_s - p_i)}_{a} l = 0$$

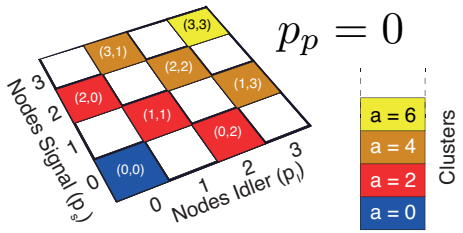
a: Cluster number

Modes per cluster $k = p_p + a + 1$



Single Photon Generation

Momentum conservation

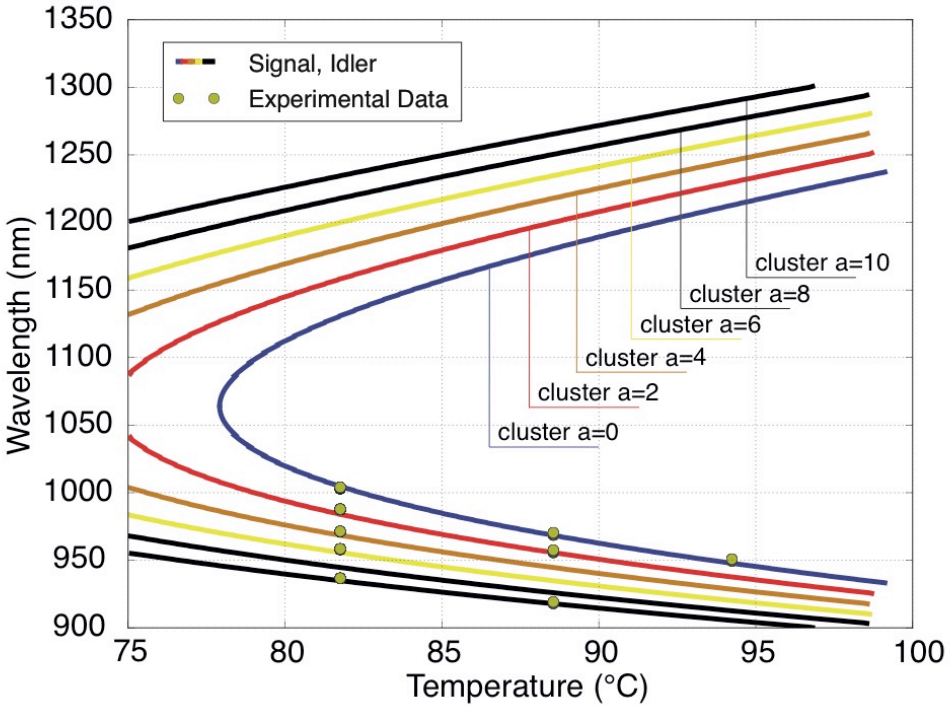


$$p_p = 0$$

Energy conservation

$$\omega_p = \omega_s + \omega_i$$

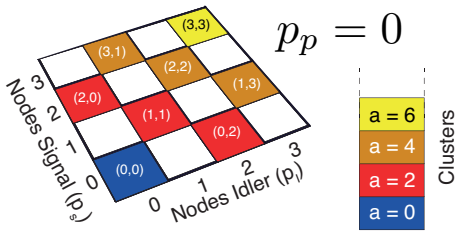
$$\omega = \underbrace{\frac{c}{nR}}_{\mathrm{FSR}} \left(l - \alpha_q \left(\frac{l}{2} \right)^{1/3} + p \left(\sqrt{\frac{R}{r}} - 1 \right) + \frac{1}{2} \sqrt{\frac{R}{r}} \right) \quad [1]$$



[1] M.L. Gorodetsky, Selected Topics in Quantum Electronics, 12, 33-39 (2006)

Single Photon Generation

Momentum conservation



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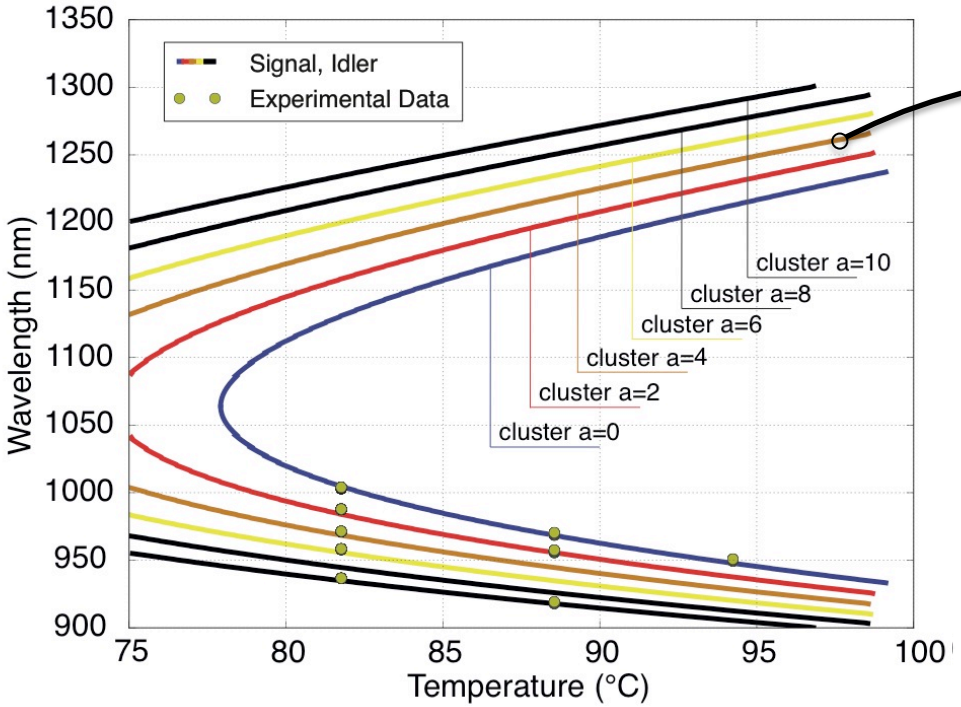
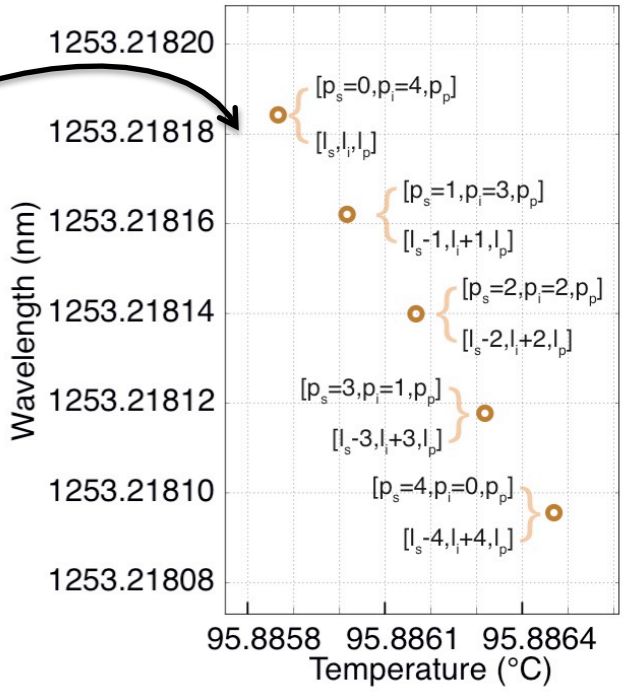


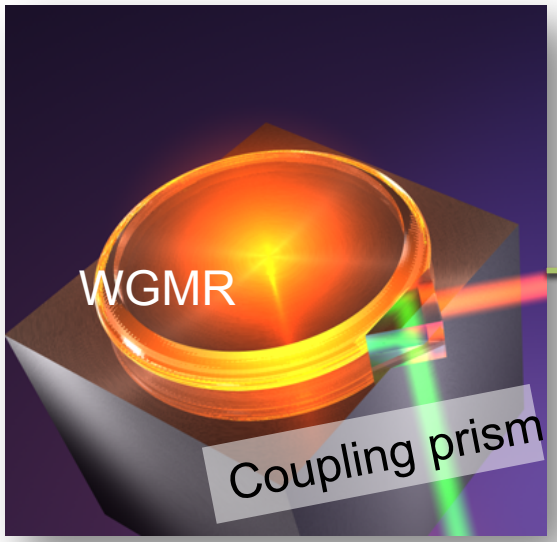


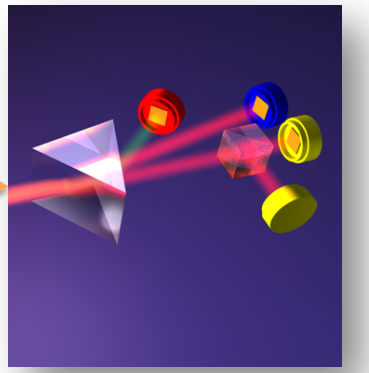
[1] M.L. Gorodetsky, Selected Topics in Quantum Electronics, 12, 33-39 (2006)

Single Photon Generation

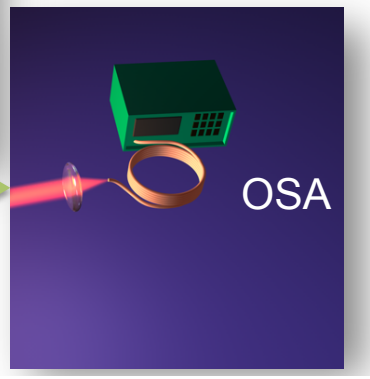
Photon Statistics

Source



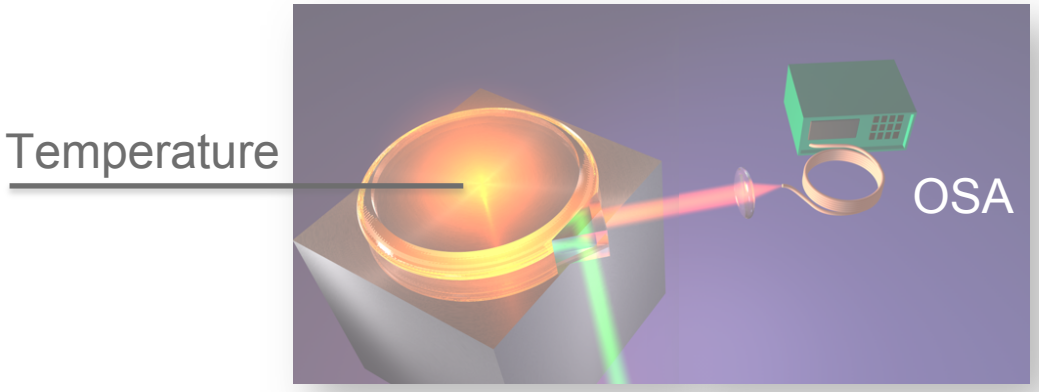


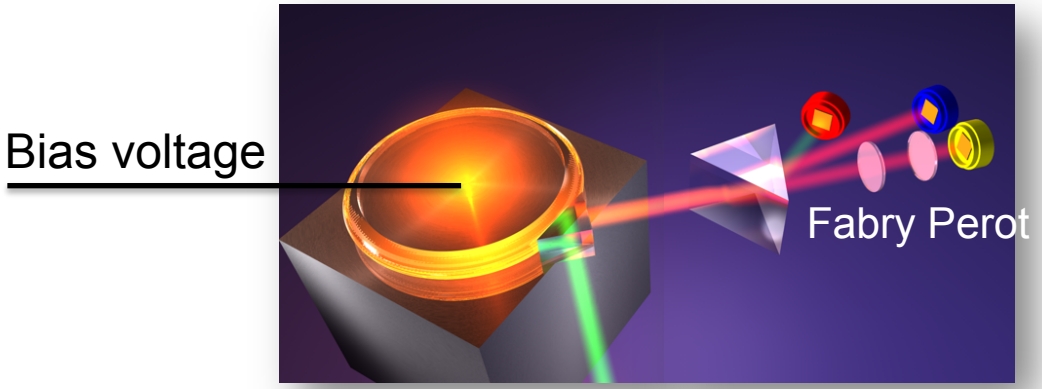
Wavelength Measurements

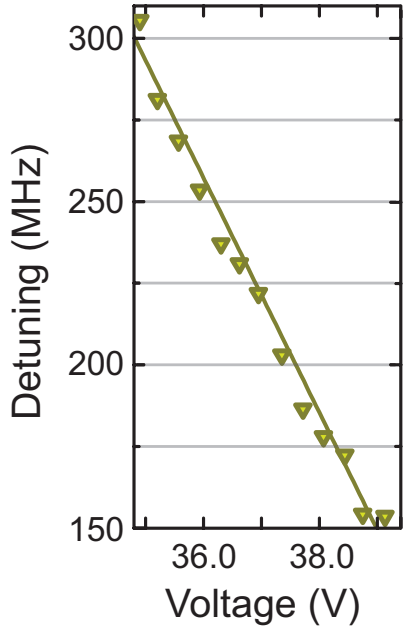


$LiNbO_3$
Pump at 532 nm (P ≈ 1 μW)
Natural phase matching (Type I)
Q ≈ 10^7
δv 4-25 MHz (@1064 nm)

Single Photon Generation

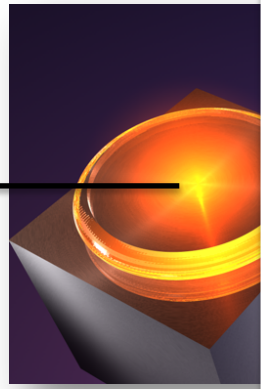




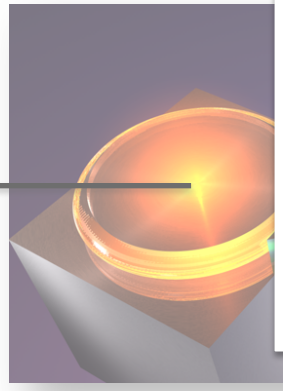


M.Förtsch et al., Nature Communications **4**, 1818 (2013)

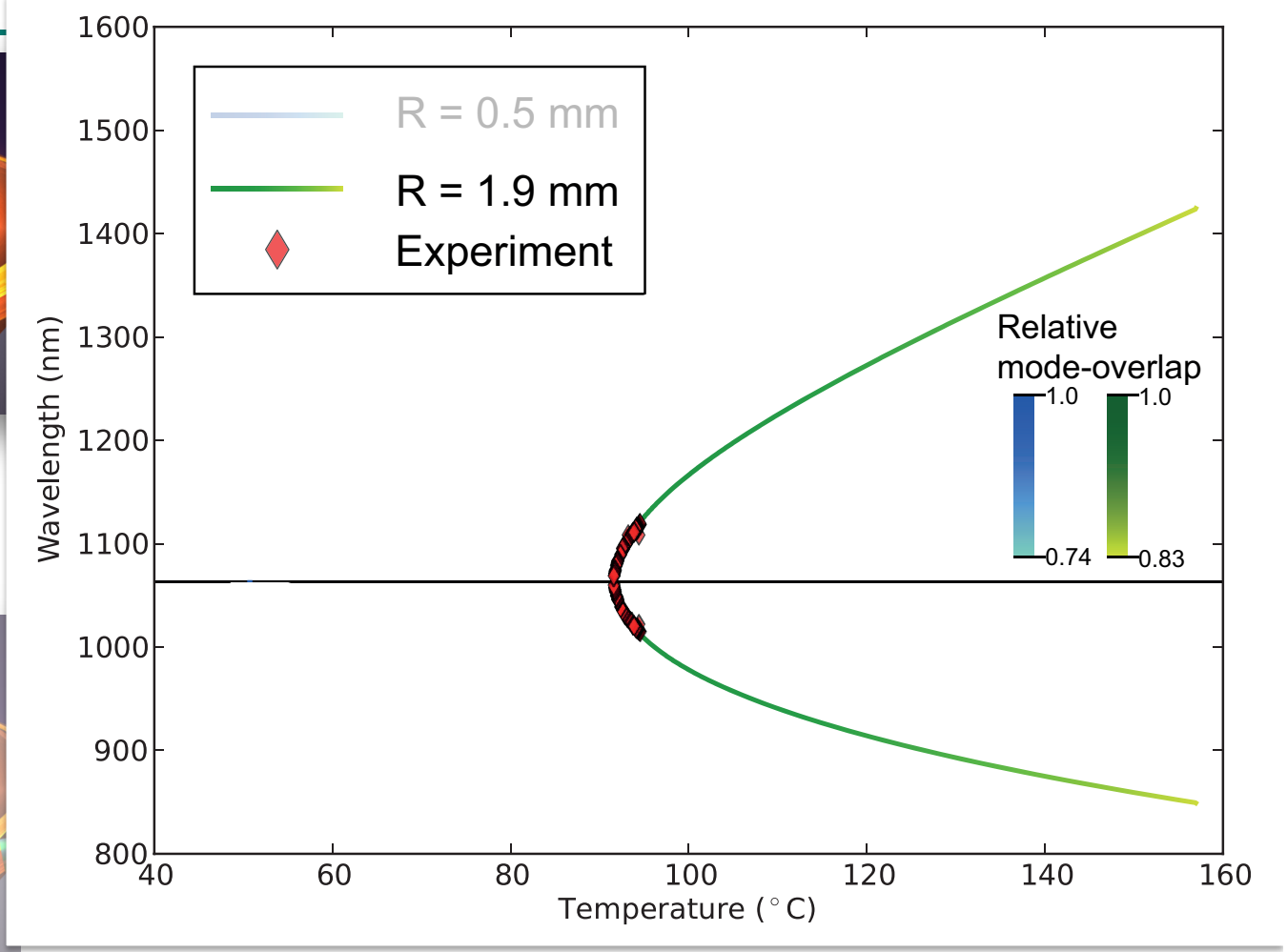
Single Photon Generation

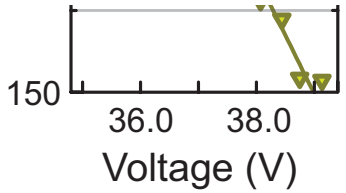


Temperature



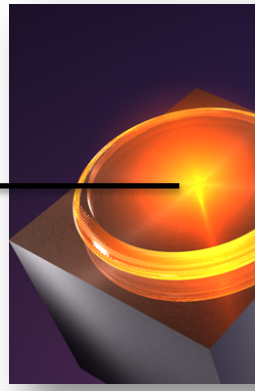
Bias voltage

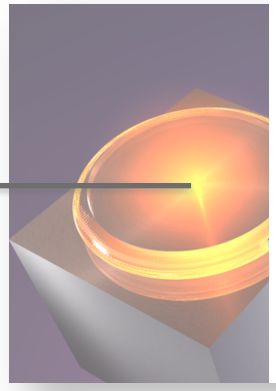


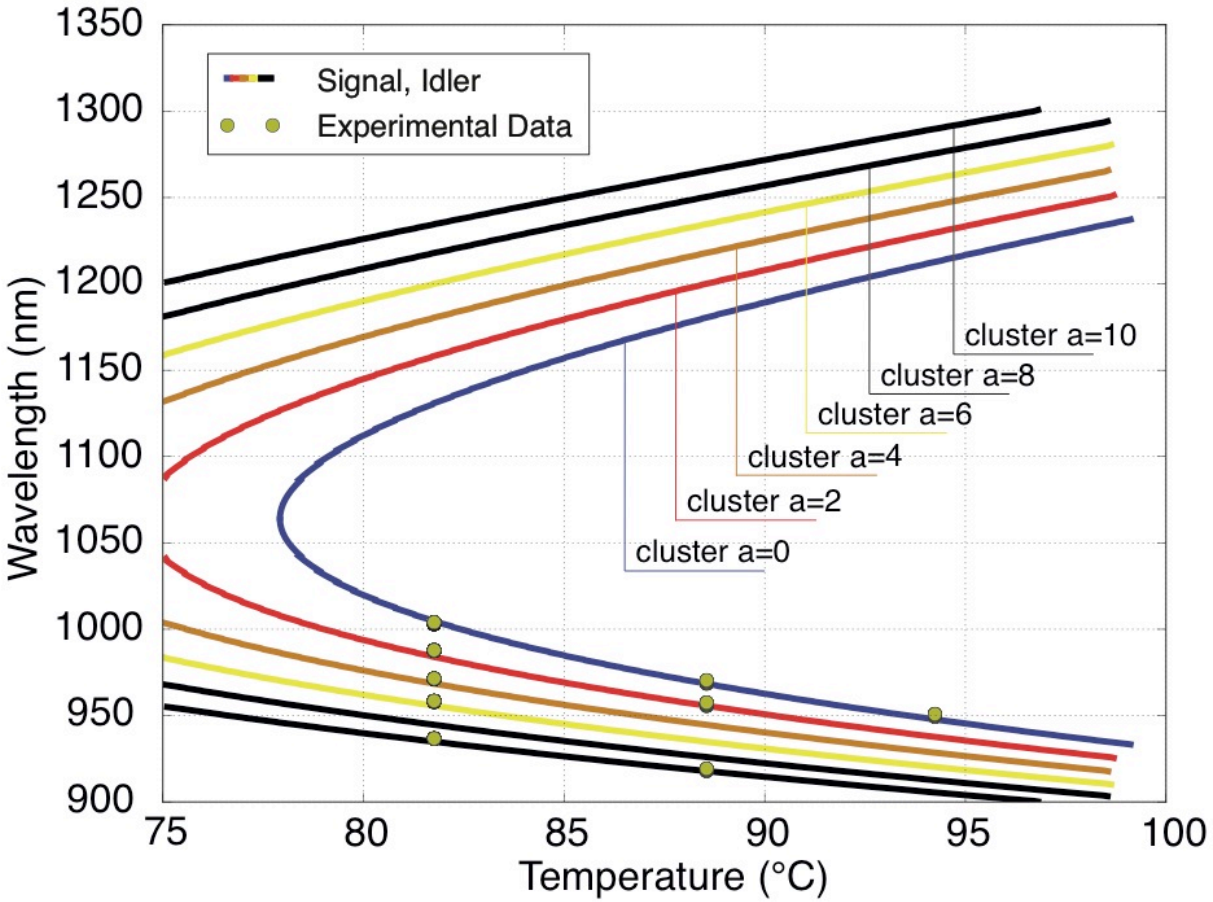


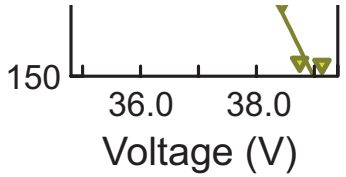
M.Förtsch et al., Nature Communications **4**, 1818 (2013)

Single Photon Generation

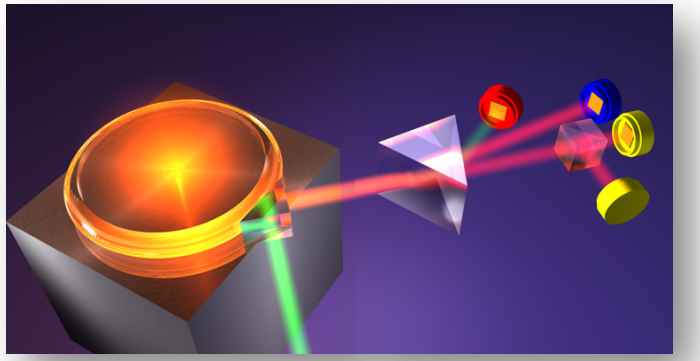




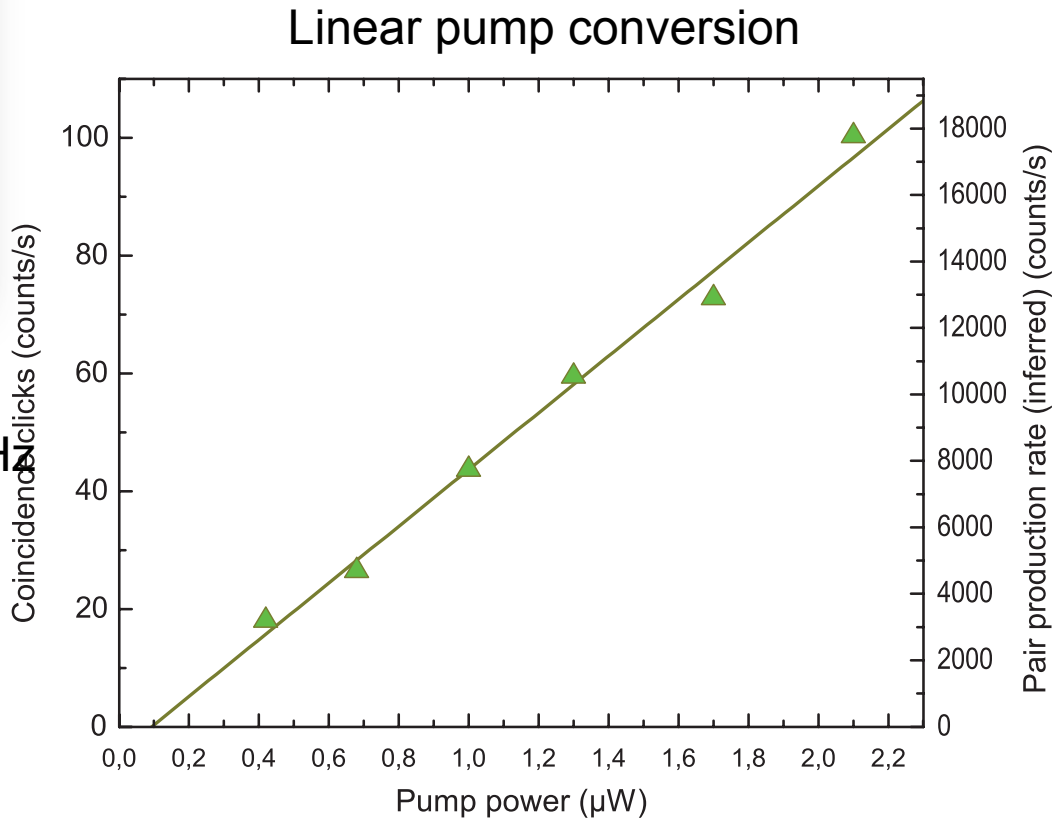




Single Photon Generation

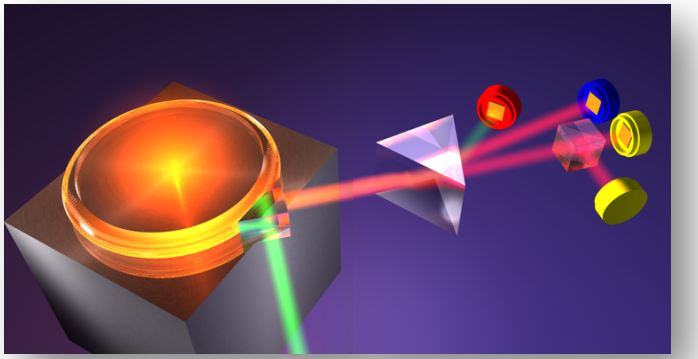


Efficiency:
1.3 x 10^7 pairs/(s mW) per 13 MHz

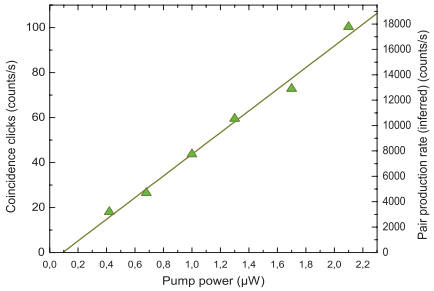


M.Förtsch et al., Nature Communications **4**, 1818 (2013)

Single Photon Generation



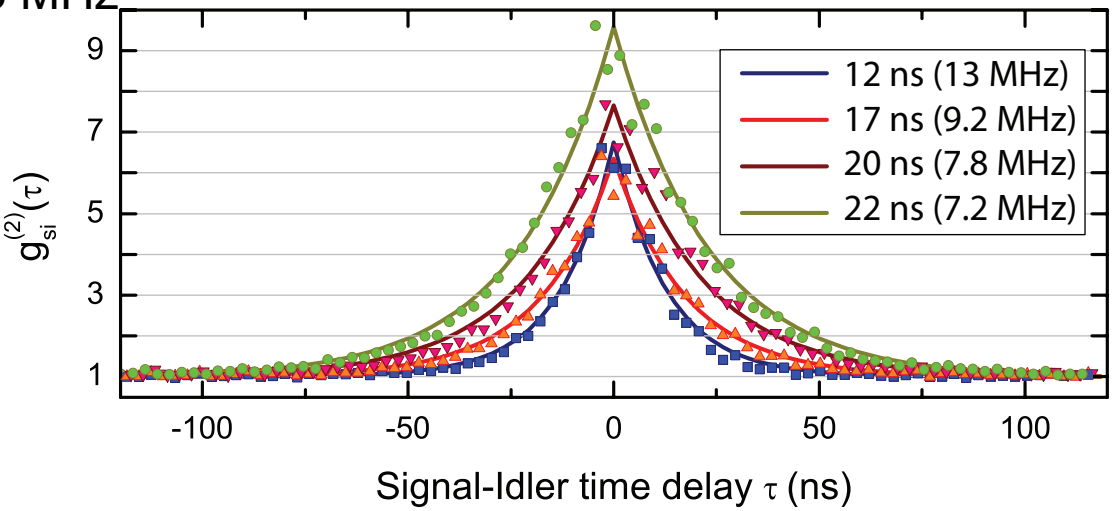
Linear pump conversion



Efficiency:
1.3 x 10^7 pairs/(s mW) per 13 MHz

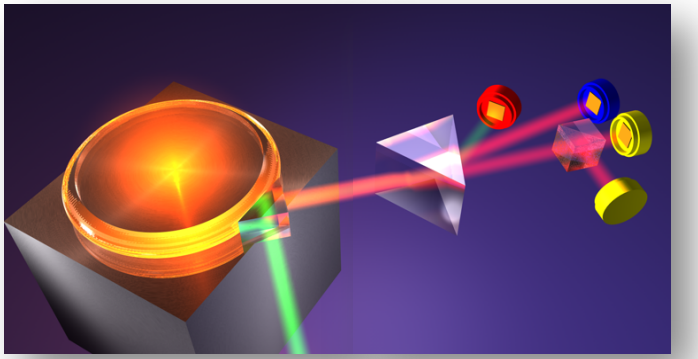
Tunable Bandwidth
7 – 13 MHz

Cross – correlation function

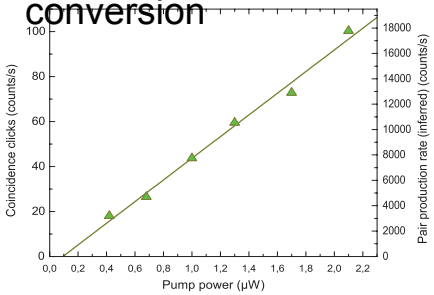


M.Förtsch et al., Nature Communications **4**, 1818 (2013)

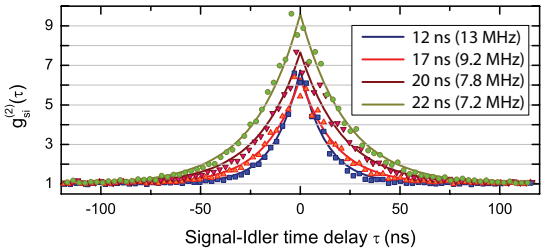
Single Photon Generation



Linear pump conversion



Cross – correlation function

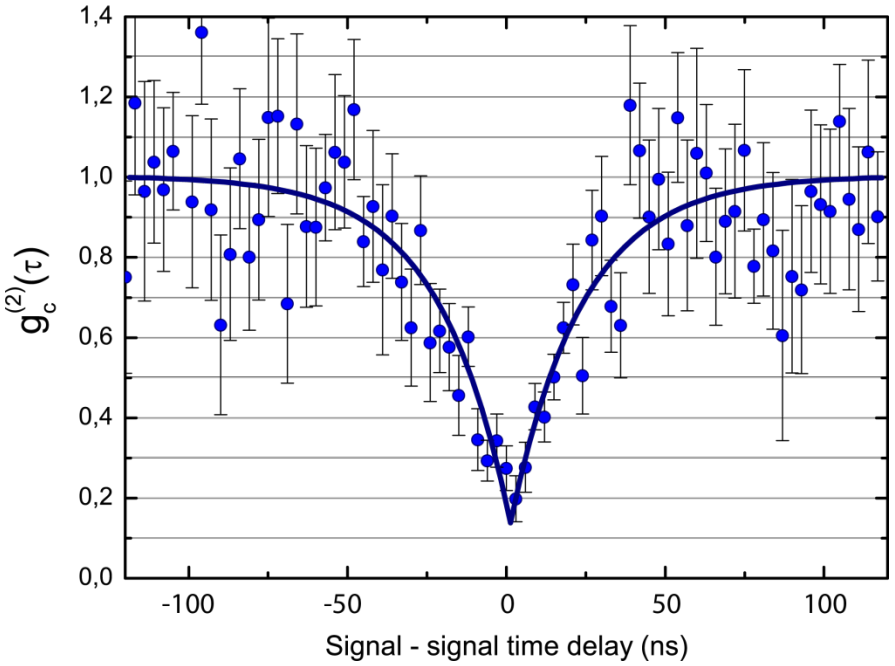


Efficiency:
1.3 x 10^7 pairs/(s mW) per 13 MHz

Tunable Bandwidth:
7 – 13 MHz

Heralded Single Photons

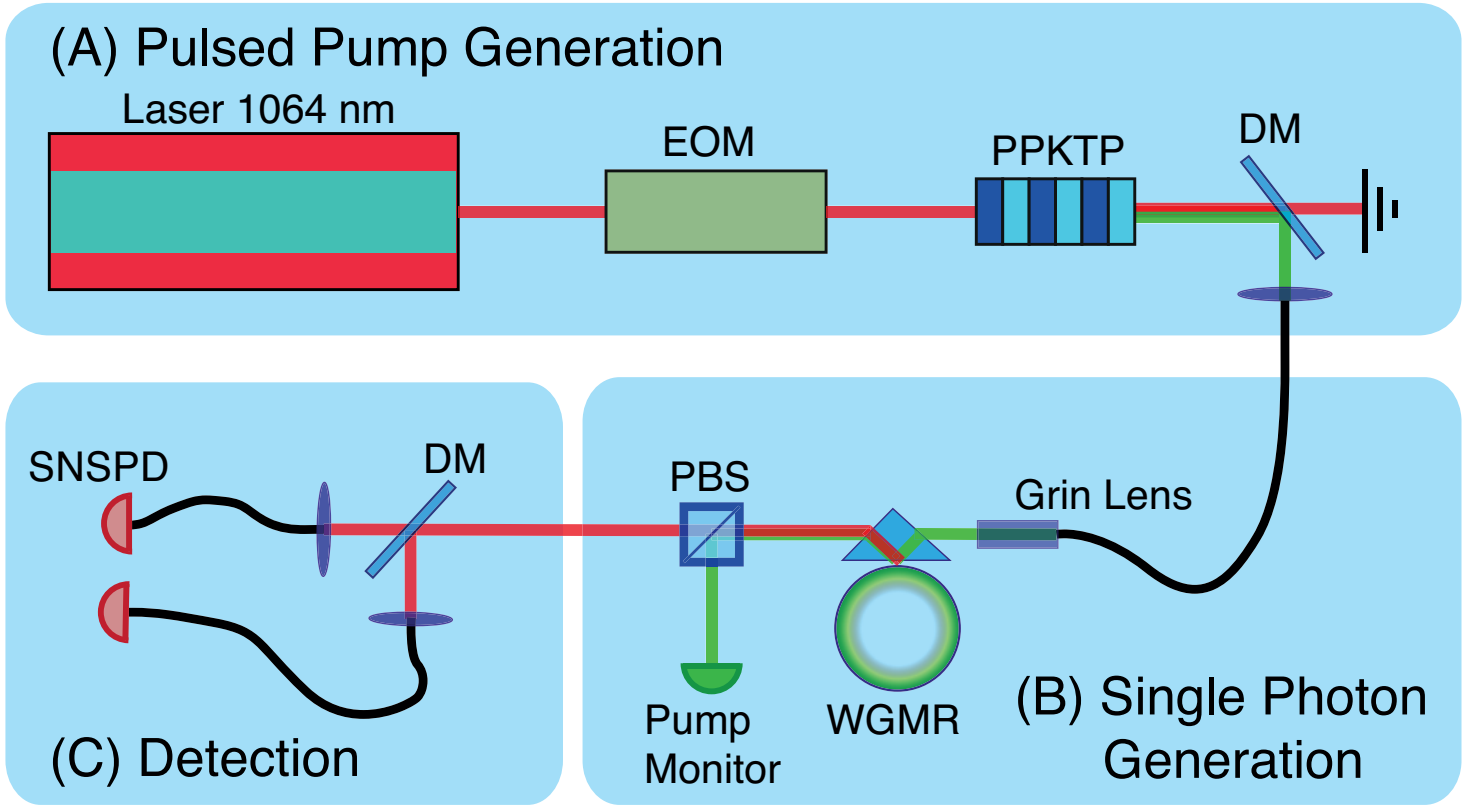
Heralded cross – correlation function



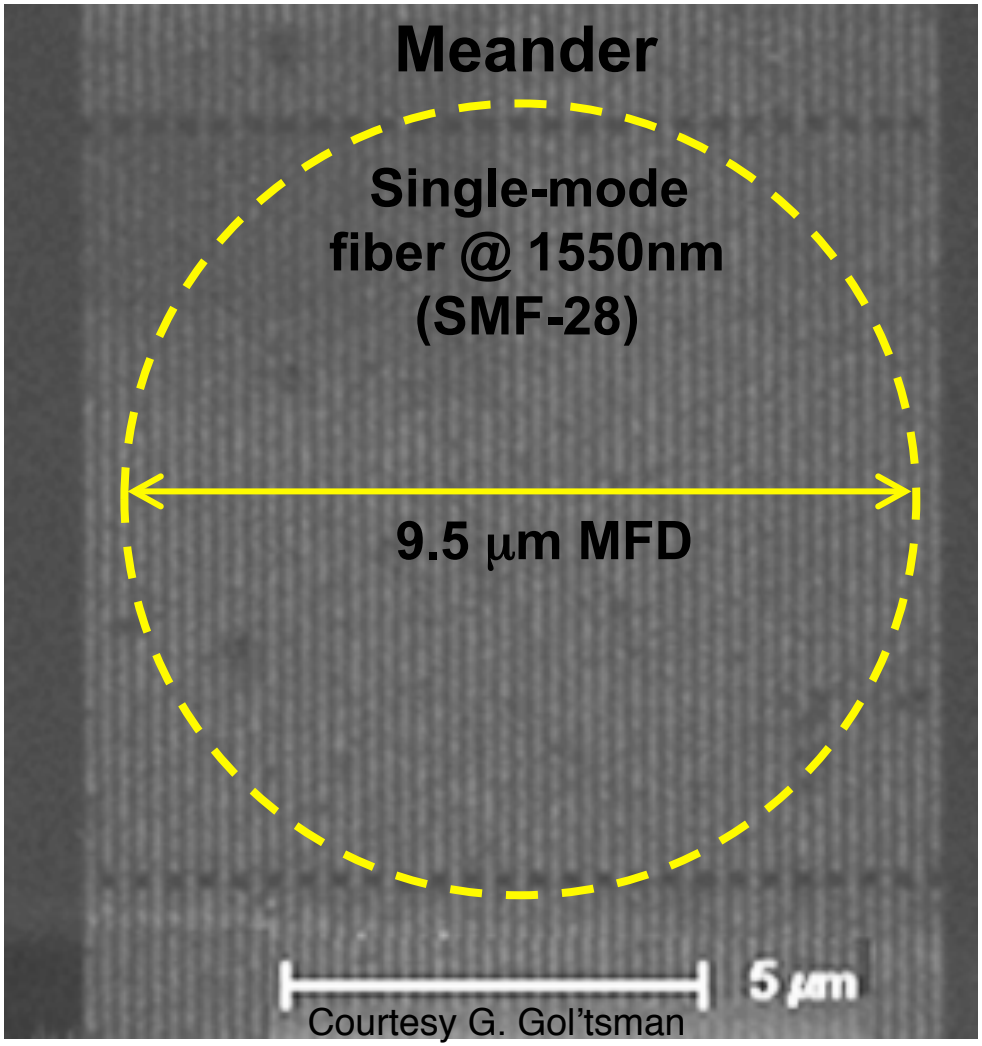
M.Förtsch et al., Nature Communications **4**, 1818 (2013)

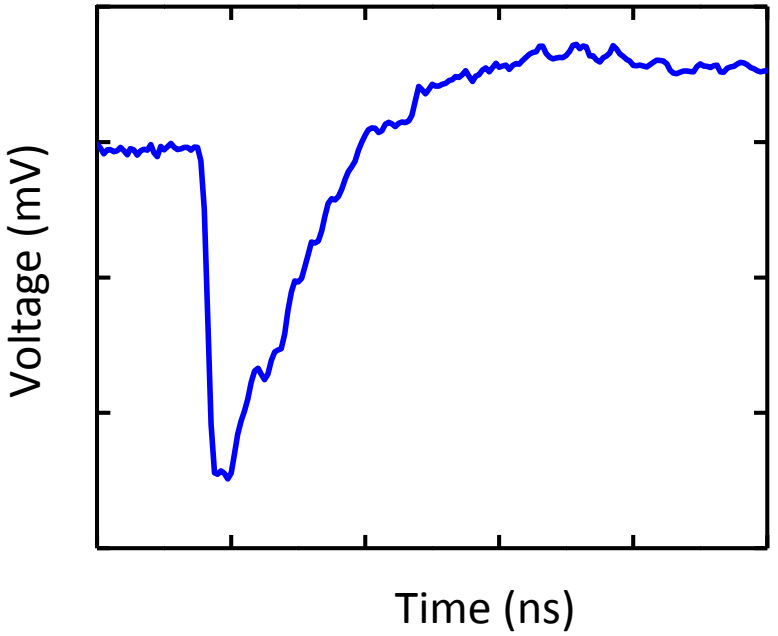
Single Photon Generation

pulsed operation and superconducting detectors



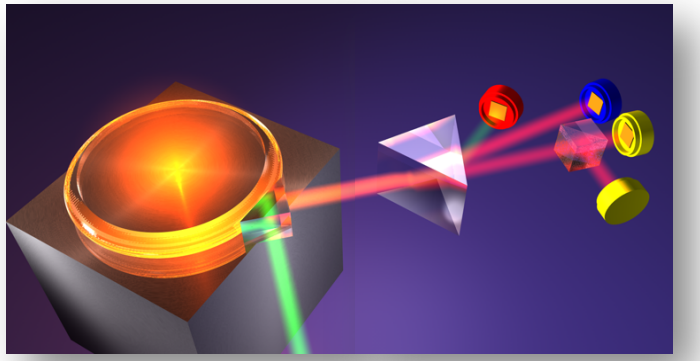
SC Nanowire Detectors @ NIST

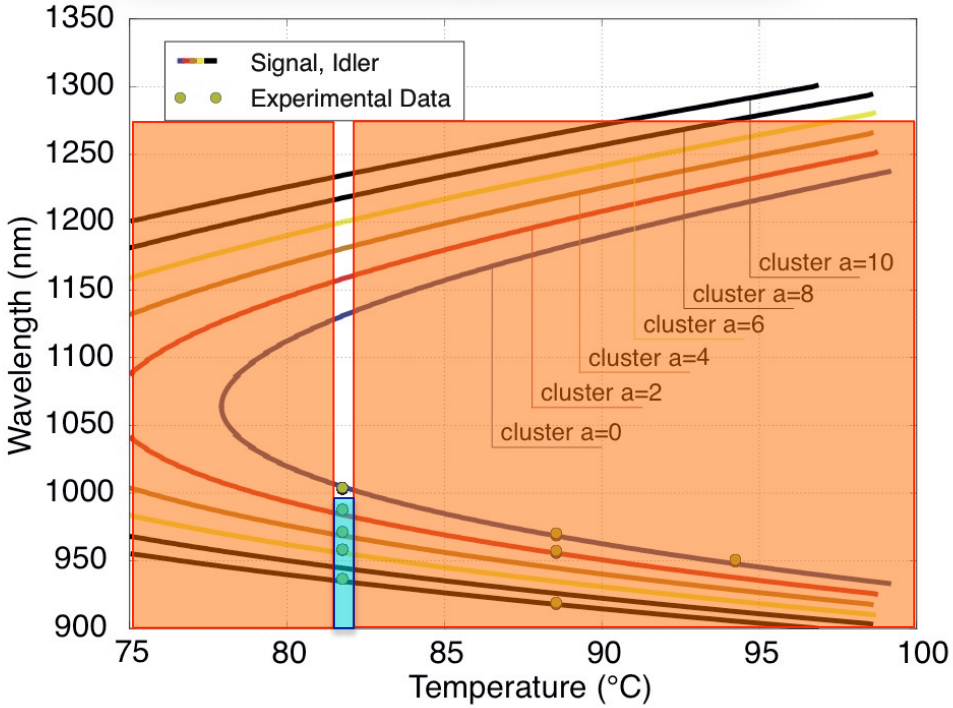


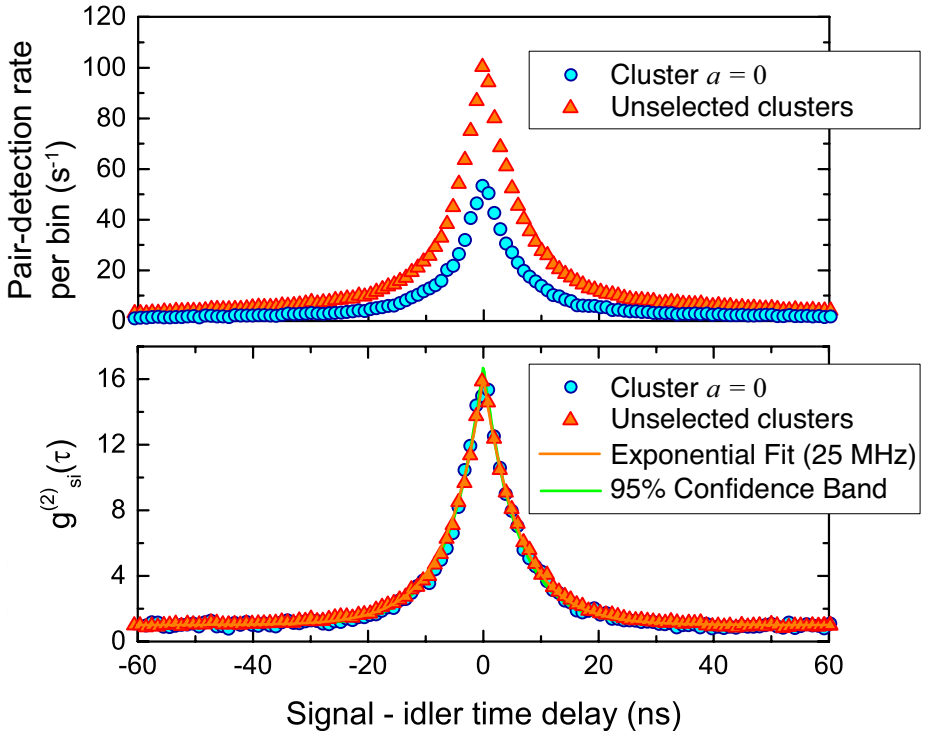


- >1 K operation
- Cheap electronics @ room temp.
- Fast

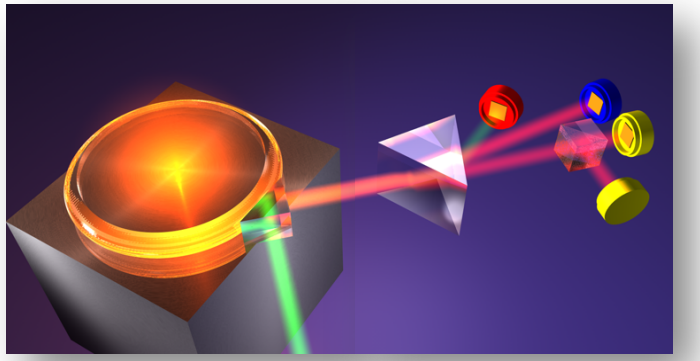
Single Photon Generation

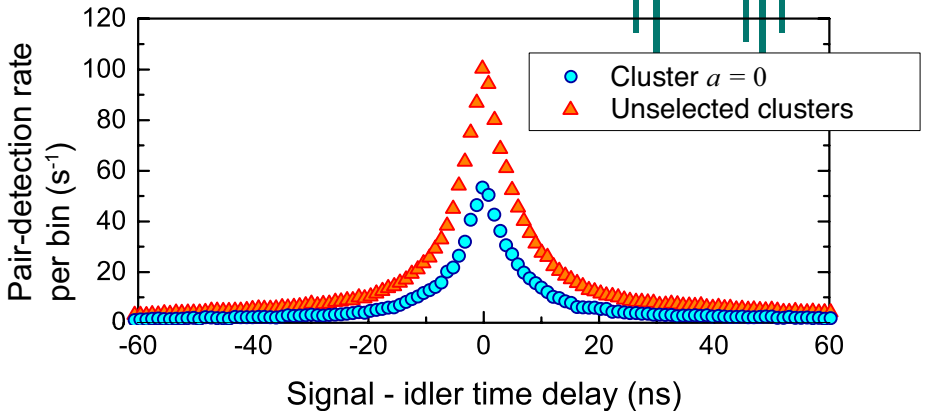


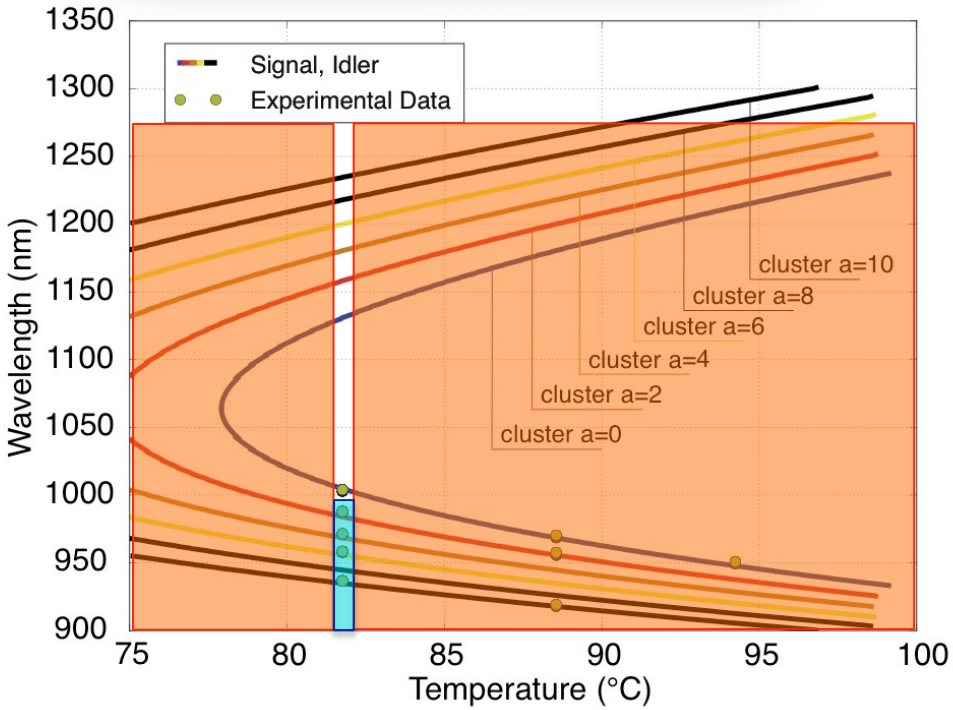




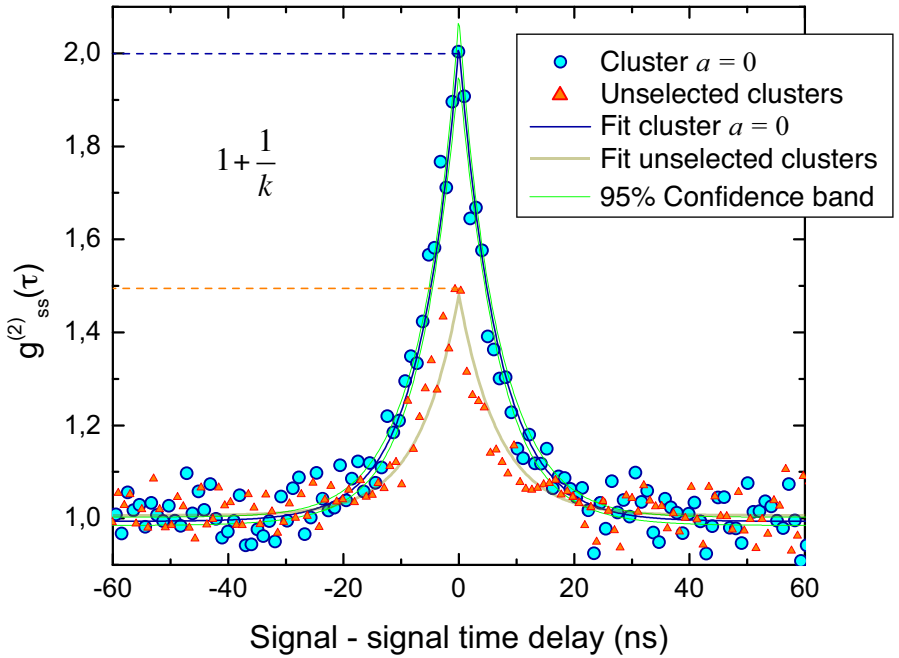
Single Photon Generation



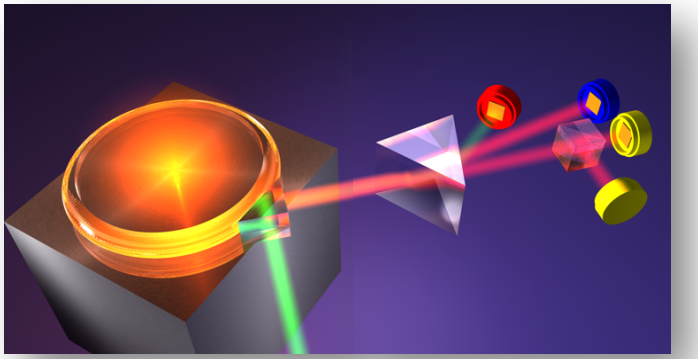




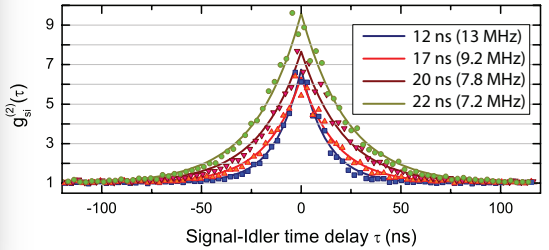
Autocorrelation function



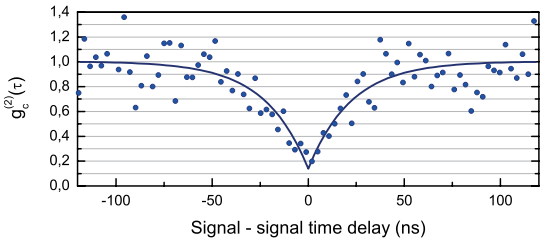
Single Photon Generation



Cross – correlation function



Heralded cross – correlation



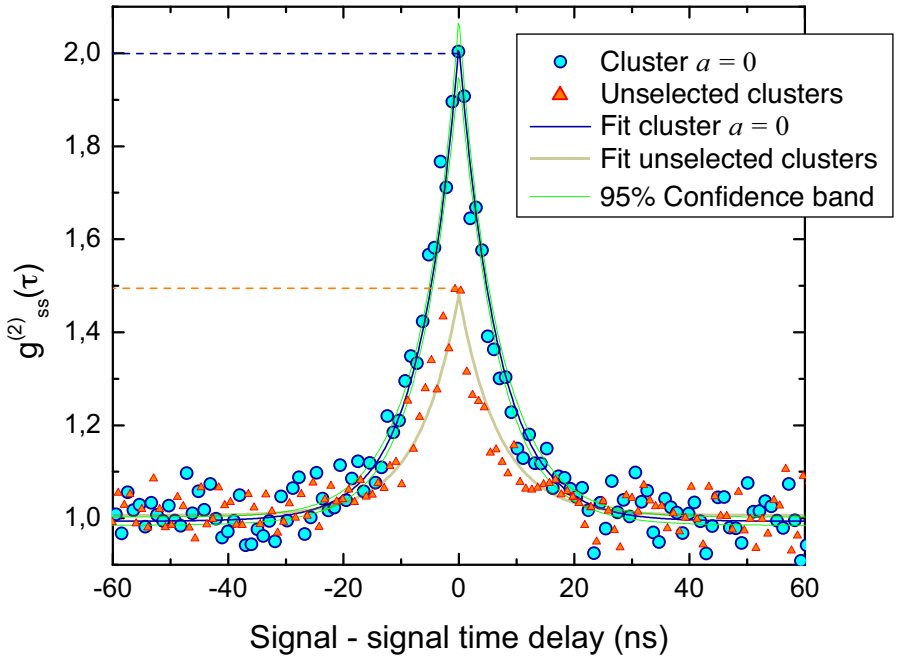
Efficiency:
1.3 x 10^7 pairs/(s mW) per 13 MHz

Tunable Bandwidth:
4 – 25 MHz

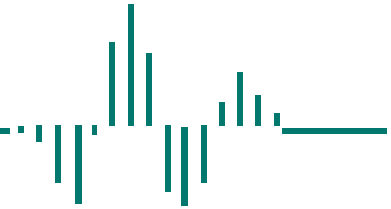
Heralded Single Photons

True single mode operation

Autocorrelation function



Conclusion & Outlook



- WGM resonators as versatile sources of narrow-band non-classical light
- efficient and still highly tunable in center wavelength and bandwidth
- angular momentum phase matching enables single mode operation
- couple to different physical systems, investigate different materials

QIV group – Division Leuchs

Thank you!

Josef Fürst, Vanessa Chille, Marion Semmler, Harald Schwefel, Florian Sedlmeir, Maria Chekhova, Ulrik L. Andersen, Elisabeth Giacobino, Dmitry Strekalov, Christian Gabriel