# Non-equilibrium quantum thermodynamics of optomechanical systems

Mauro Paternostro

Queen's University Belfast

PBQ 2016

Olomouc, 2-4 May 2016

QTEQ QUANTUM TECHNOLOGY at QUEEN'S

# On the shoulders of Belfast's giants

John Stuart Bell

Joseph Larmor

Born in Belfast in 1824

Lord Kelvin

Belfast, Botanic Gardens

## Ask the expert!

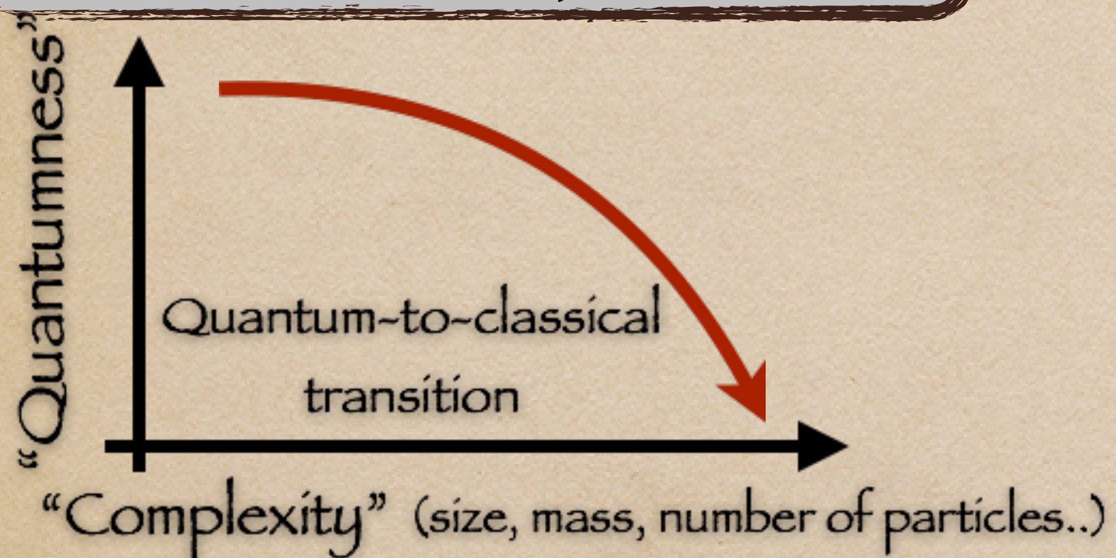
Matteo Brunelli

cf. Matteo's poster
(courtesy of Czech railways)

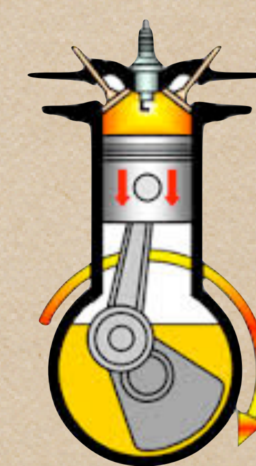
# Setting the context

## Thermodynamics of non-equilibrium quantum systems

Thermodynamics is a theory of inherently complex systems



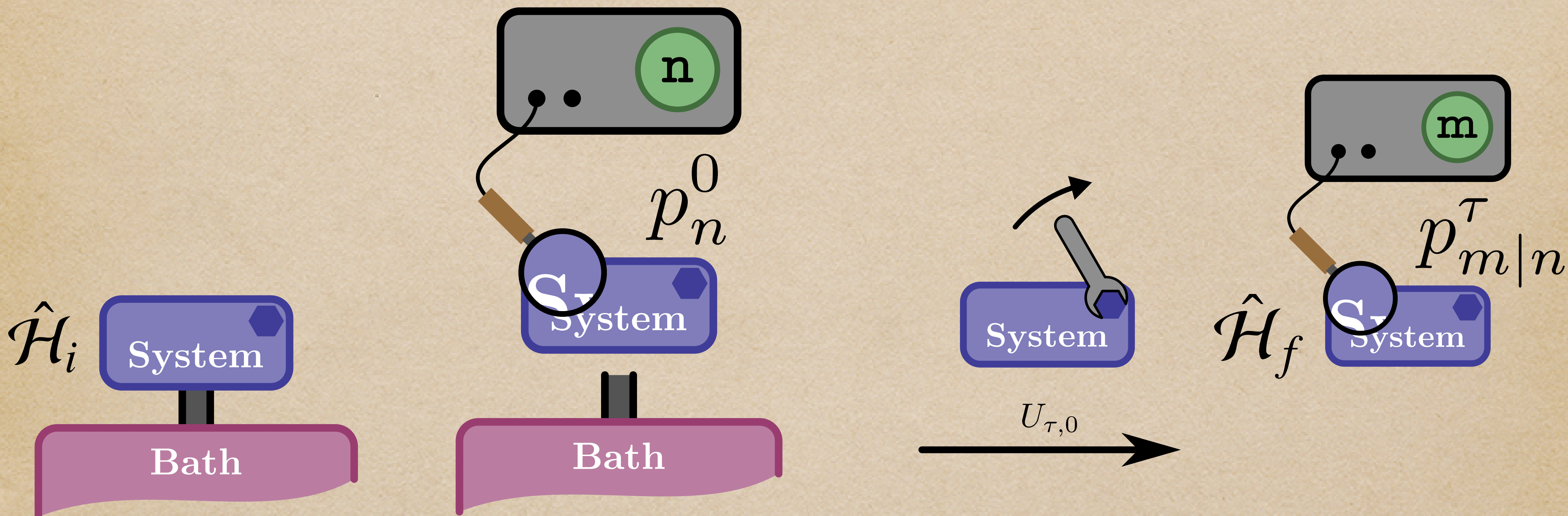

Optimisation of the performance of machines



Study of complex quantum phenomena through a thermodynamic approach?

# Setting the context

In quantum contexts: work is not an observable*




(a) $t < 0$ (b) $t = 0^+$ (c) $0 < t < \tau$ (d) $t = \tau$

Work Distribution

$$P_F(W) = \sum_{n,m} p_n^0 p^{\tau}_{m|n} \delta\left(W - (E'_m - E_n)\right)$$

*P. Talkner, E. Lutz, and P. Haenggi, Phys. Rev. E 75, 050102 (2007)

# The logic behind it

Work Distribution

$$P_F(W) = \sum_{n,m} p_n^0 p_{m|n}^\tau \delta\left(W - (E'_m - E_n)\right)$$

Characteristic function of Work Distribution

$$\chi_F(u) = \int dW e^{\imath u W} P_F(W)$$

Jarzinsky equality $\langle e^{-\beta W} \rangle = e^{-\beta \Delta F}$

free-energy change

Jarzynski, PRL 78 2690 (1997)

Tasaki-Crooks relation $\dfrac{P_F(W)}{P_B(-W)} = e^{\beta(W - \Delta F)}$

G. E. Crooks, PRE 60, 2721 (1999)

H. Tasaki, cond-mat/0009244 (2000)

M. Campisi, P. Haenggi, and P. Talkner, Rev. Mod. Phys. 83 (2011)

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# What's difficult with it

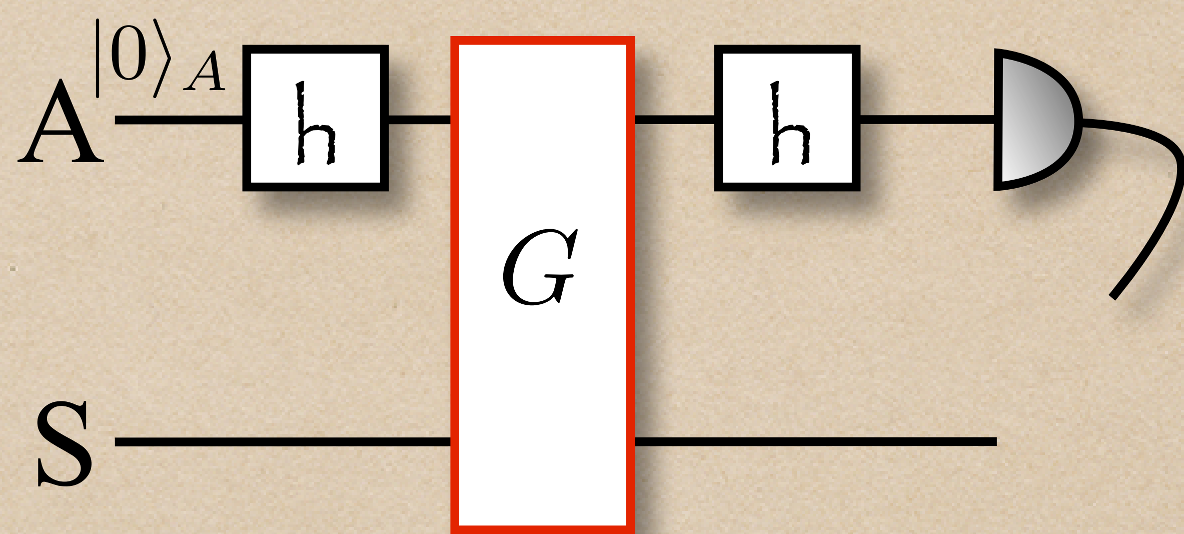
***"The major obstacle for the experimental verification of the work fluctuation relation is posed by the necessity of performing quantum projective measurements of energy"***

REVIEWS OF MODERN PHYSICS, VOLUME 83, JULY–SEPTEMBER 2011

**_Colloquium_: Quantum fluctuation relations: Foundations and applications**

Michele Campisi, Peter Hänggi, and Peter Talkner
*Institute of Physics, University of Augsburg, Universitätsstrasse 1, D-86135 Augsburg, Germany*

# Measuring work




$$\hat{G}(u,\tau) = \hat{U}_\tau e^{-i\hat{\mathcal{H}}_i u} \otimes |0\rangle\langle 0|_A + e^{-i\hat{\mathcal{H}}_f u}\hat{U}_\tau \otimes |1\rangle\langle 1|_A$$

$$\rho_A = (I + \alpha\hat{\sigma}_z + \beta\sigma_y)/2$$

$\mathrm{Re}\chi(u,\tau)$ $\mathrm{Im}\chi(u,\tau)$


R. Dorner, et al., Phys. Rev.Lett. 110, 230601 (2013)

L. Mazzola, G. De Chiara, and MP, Phys. Rev. Lett. 110, 230602 (2013)

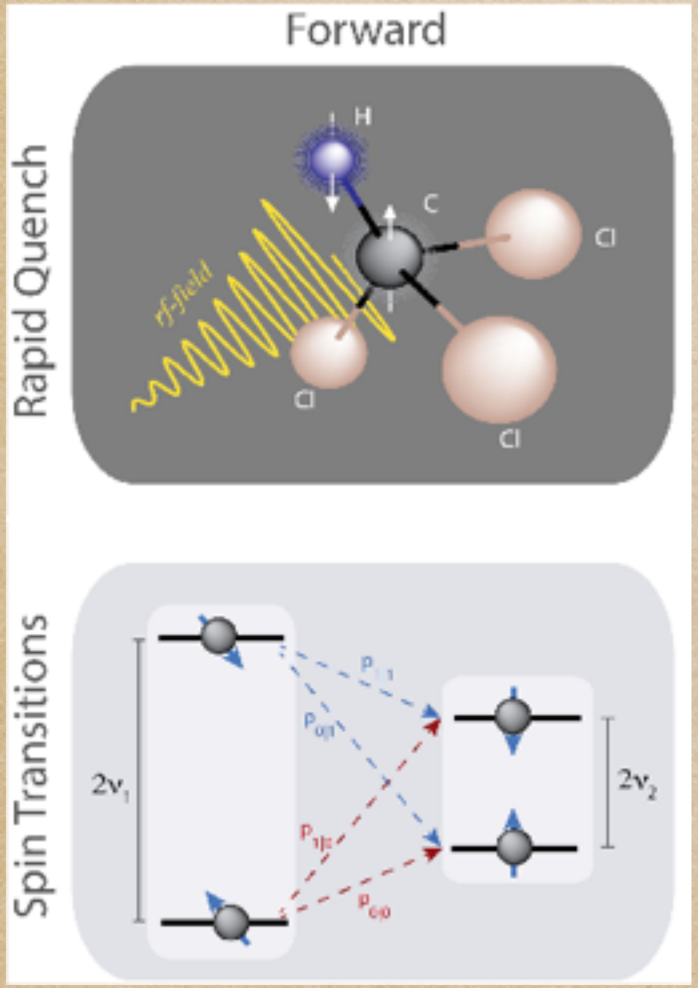
L. Mazzola, G. De Chiara, and MP, Int. J. Quant. Inf. (2014)

# Testing fluctuation theorem

PRL 113, 140601 (2014) PHYSICAL REVIEW LETTERS week ending 3 OCTOBER 2014

**Experimental Reconstruction of Work Distribution and Study of Fluctuation Relations in a Closed Quantum System**

Tiago B. Batalhão,[1] Alexandre M. Souza,[2] Laura Mazzola,[3] Ruben Auccaise,[2] Roberto S. Sarthour,[2] Ivan S. Oliveira,[2] John Goold,[4] Gabriele De Chiara,[3] Mauro Paternostro,[3,5] and Roberto M. Serra[1]




$$\mathcal{H}_t^{\mathrm{F}} = 2\pi\hbar\nu(t)\left(\sigma_x^{\mathrm{C}}\cos\phi(t) + \sigma_y^{\mathrm{C}}\sin\phi(t)\right)$$

$$\phi(t) = \pi t/(2\tau)$$

$$\nu(t) = \nu_0(1 - t/\tau) + \nu_\tau t/\tau$$

Fantastic framework to study (experimentally) reversibility (or lack thereof)

# Testing fluctuation theorem

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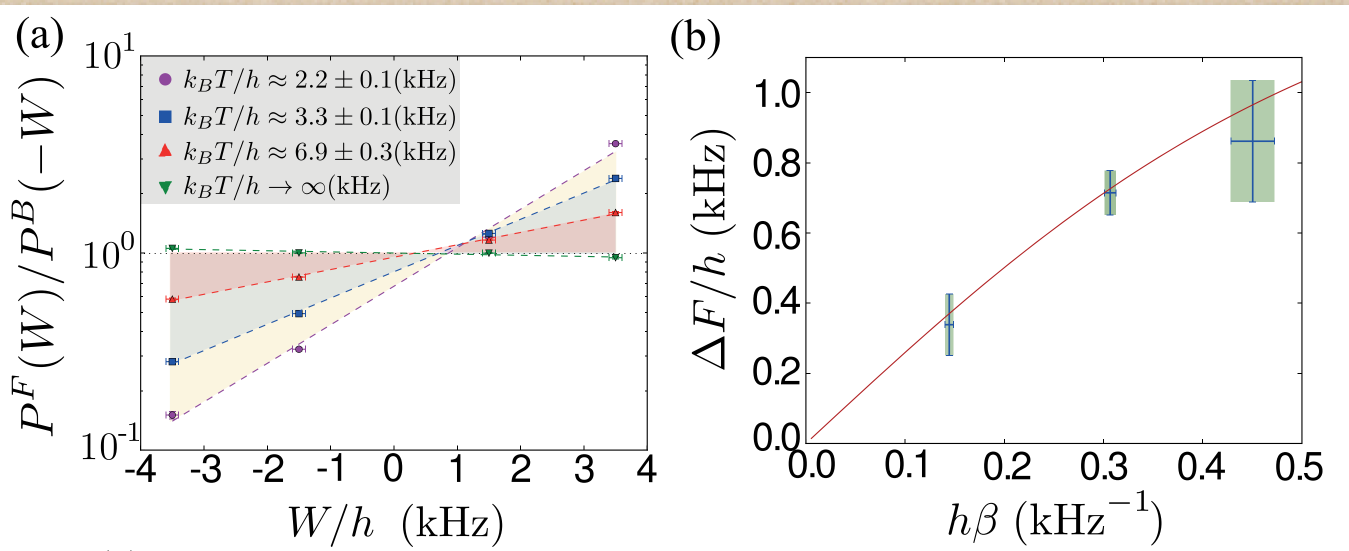



(c)

| $k_BT/h$ (kHz) | $2.2 \pm 0.1$ | $3.3 \pm 0.1$ | $6.9 \pm 0.3$ |
|---|---|---|---|
| $\log \langle e^{-\beta W} \rangle$ | $0.38 \pm 0.03$ | $0.20 \pm 0.02$ | $0.06 \pm 0.01$ |
| $\beta\Delta F$ | $0.39 \pm 0.06$ | $0.22 \pm 0.02$ | $0.05 \pm 0.01$ |
| Theory | $0.43 \pm 0.03$ | $0.22 \pm 0.01$ | $0.053 \pm 0.003$ |

Fantastic framework to study (experimentally) reversibility (or lack thereof)

# *Entropy production*

Jarzinsky equality $\langle e^{-\beta W} \rangle = e^{-\beta \Delta F}$

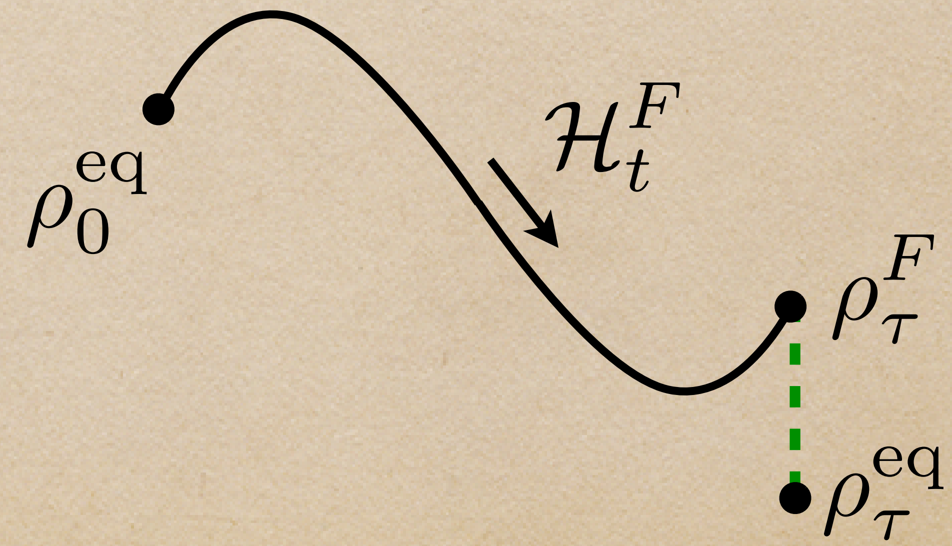
$$\langle W \rangle \geq \Delta F \longrightarrow \boxed{\langle \Sigma \rangle = \beta(\langle W \rangle - \Delta F) \geq 0}$$

(Clausius law)

$$\Sigma = \beta(W - \Delta F)$$

Entropy production

(Prigogine, de Groot)



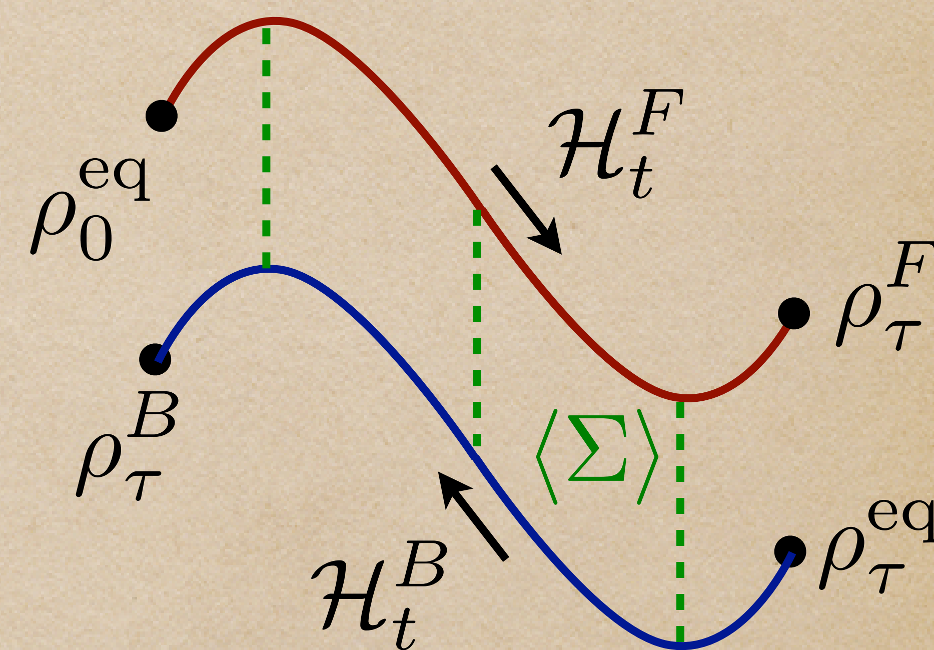

R. M. Velasco, L. S. Garcia-Colin, and F. J. Uribe, Entropy 13, 82 (2011)

# *Entropy production*

$$\frac{P_F(W)}{P_B(-W)} = e^{\beta(W-\Delta F)}$$

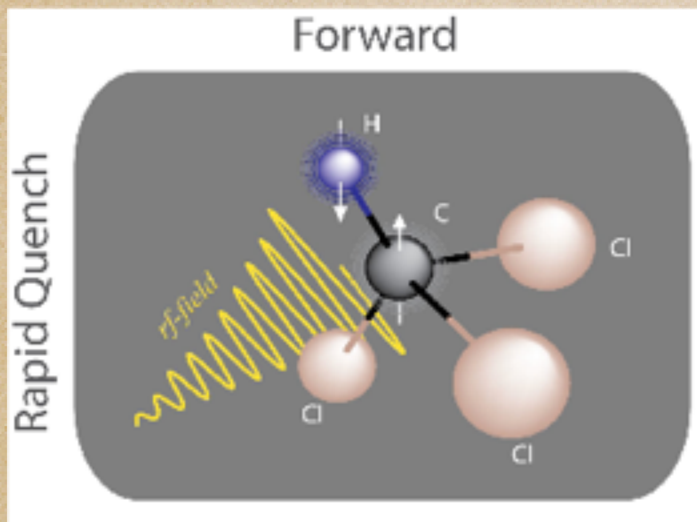
Tasaki-Crooks

$$\boxed{\begin{aligned}\langle \Sigma \rangle &= S\left(\rho_t^{\mathrm{F}} \| \rho_{\tau-t}^{\mathrm{B}}\right) \\ &= \mathrm{tr}\left[\rho_t^{\mathrm{F}}\left(\ln \rho_t^{\mathrm{F}} - \ln \rho_{\tau-t}^{\mathrm{B}}\right)\right]\end{aligned}}$$




J. Parrondo et al, NJP 11, 073008 (2009)
T. B. Batalhao, et al. arXiv:1502.06704 (2015)

# *Experimental assessment of entropy production*

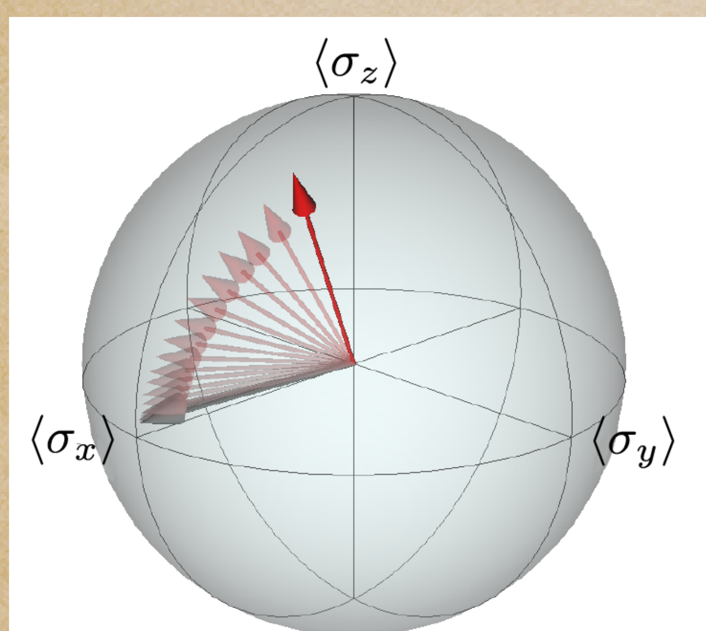


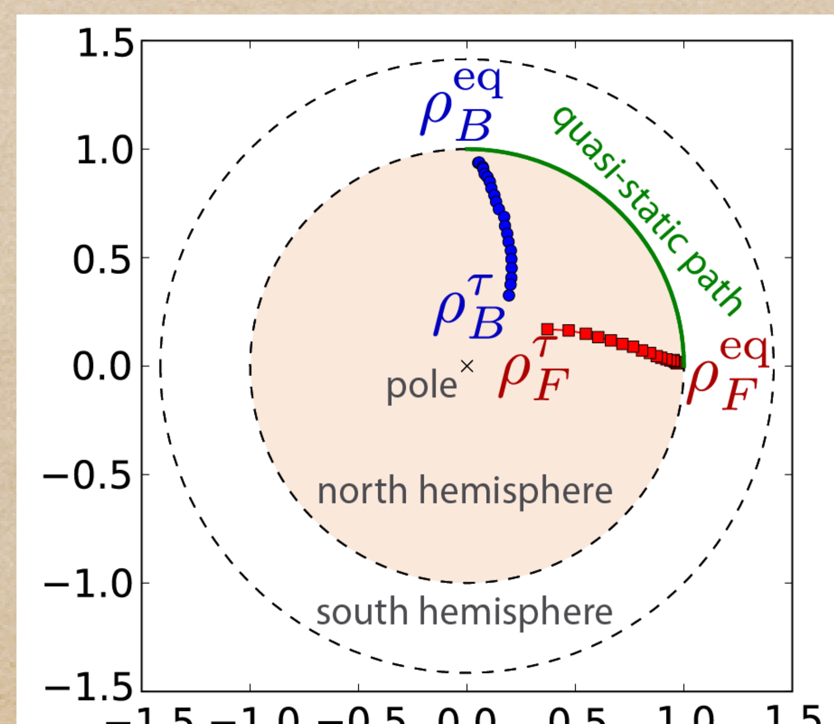

$$\mathcal{H}_t^{\mathrm{F}} = 2\pi\hbar\nu(t)\left(\sigma_x^{\mathrm{C}}\cos\phi(t) + \sigma_y^{\mathrm{C}}\sin\phi(t)\right)$$

$$\phi(t) = \pi t/(2\tau)$$

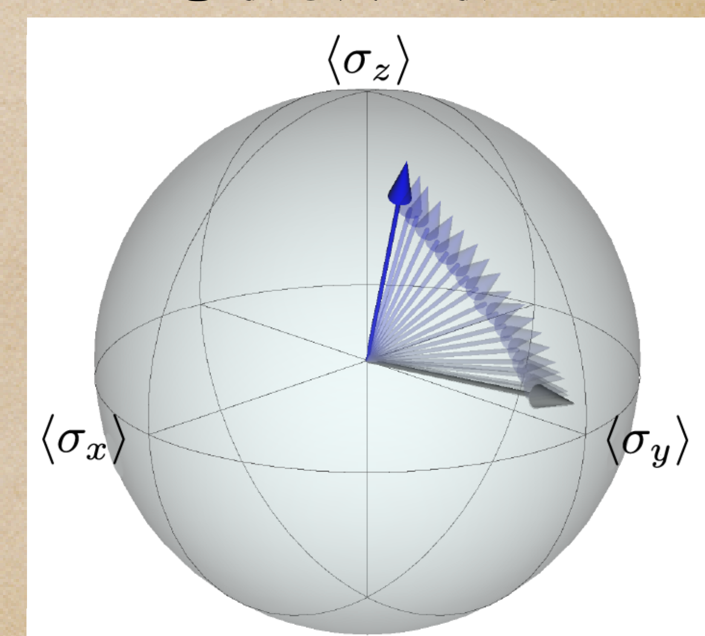
$$\nu(t) = \nu_0(1 - t/\tau) + \nu_\tau t/\tau$$

## Forward



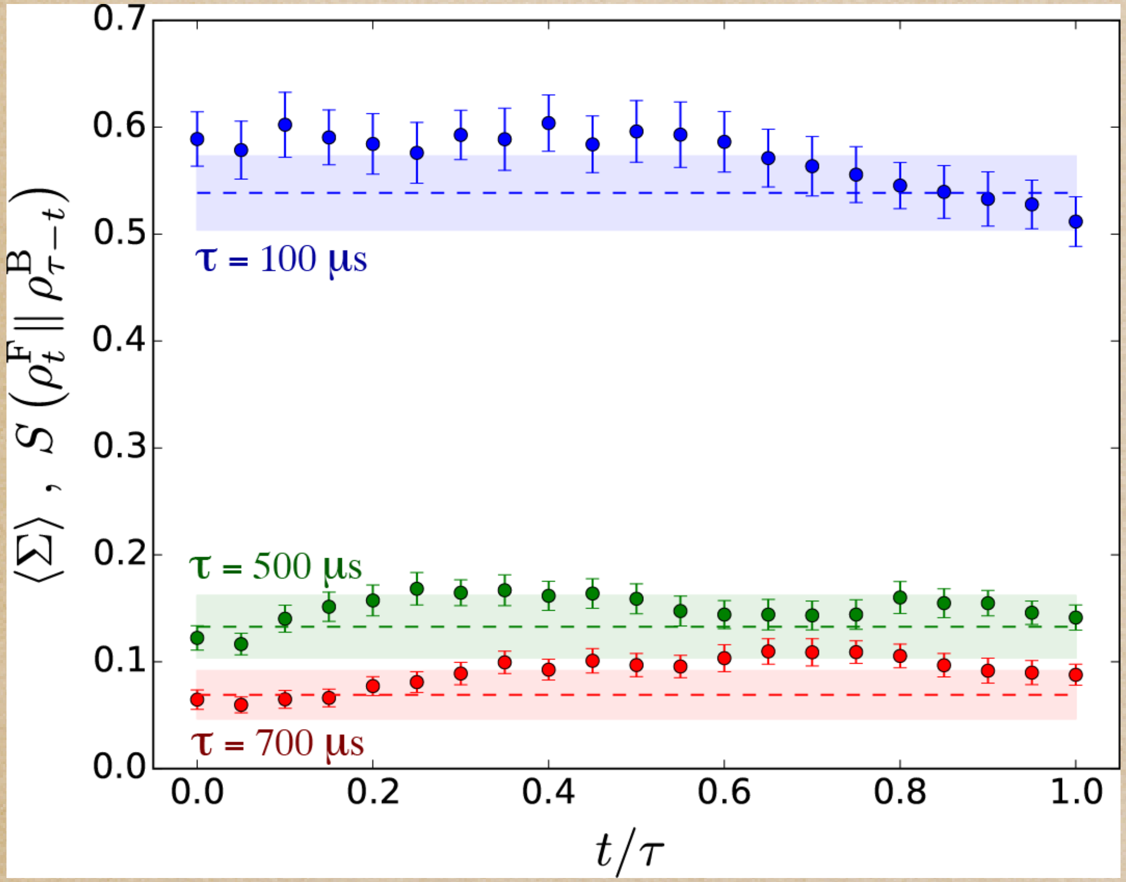



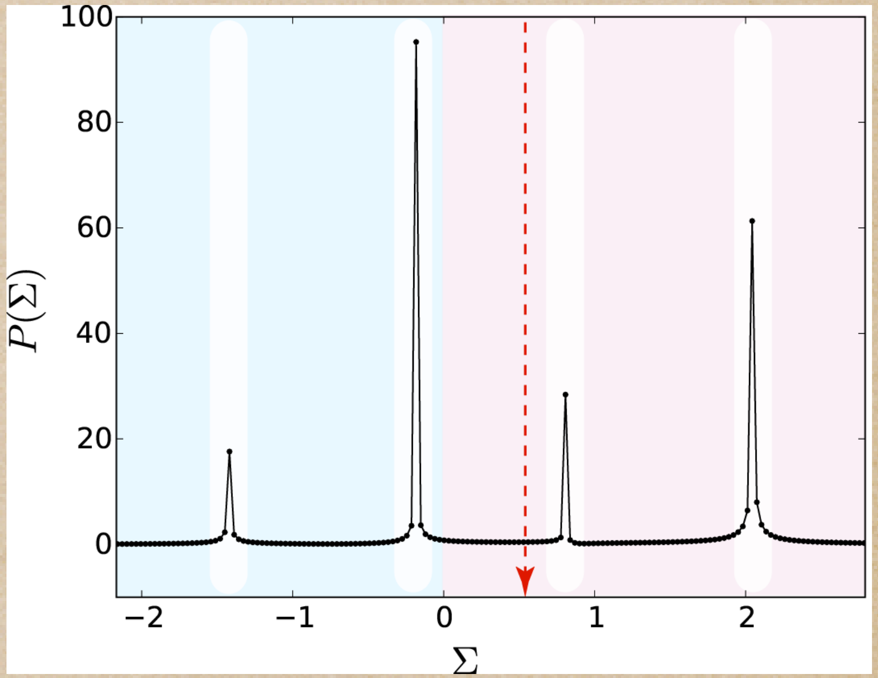

## Backward




T. B. Batalhao, et al. PRL 115, 190601 (2015)

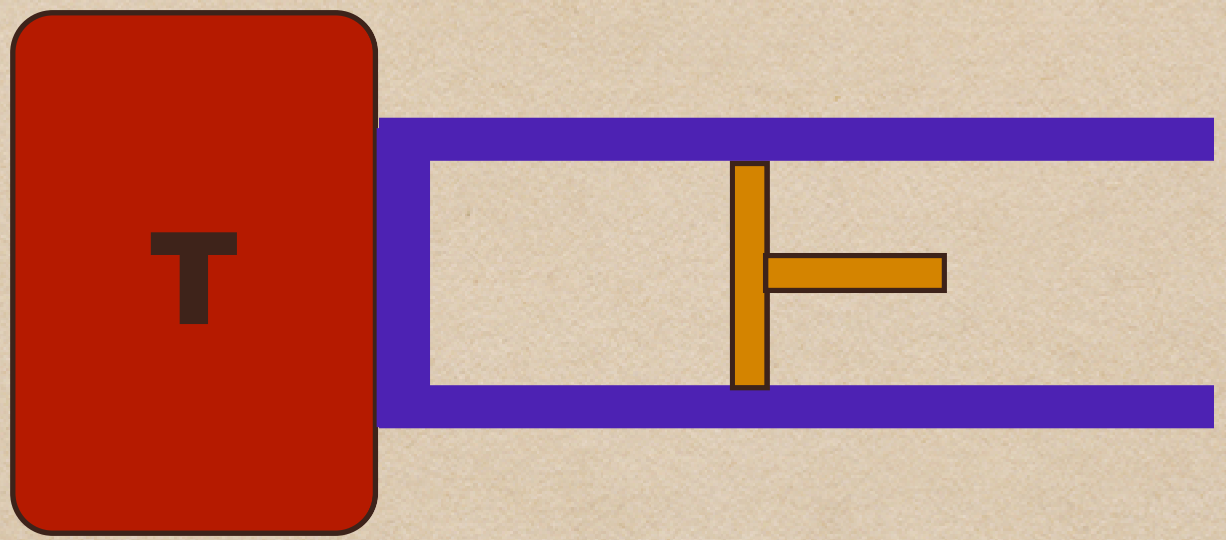
# Experimental assessment of entropy production

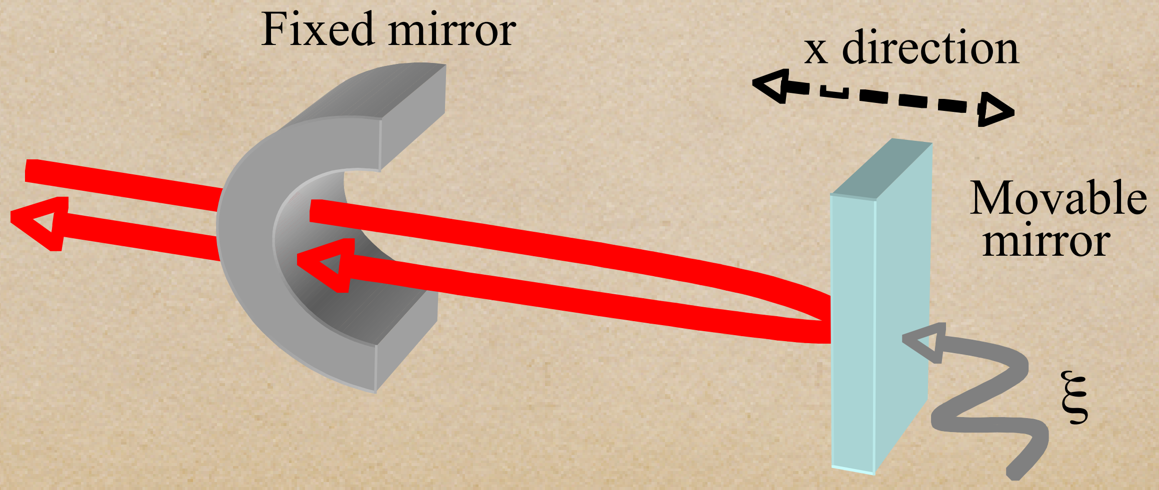


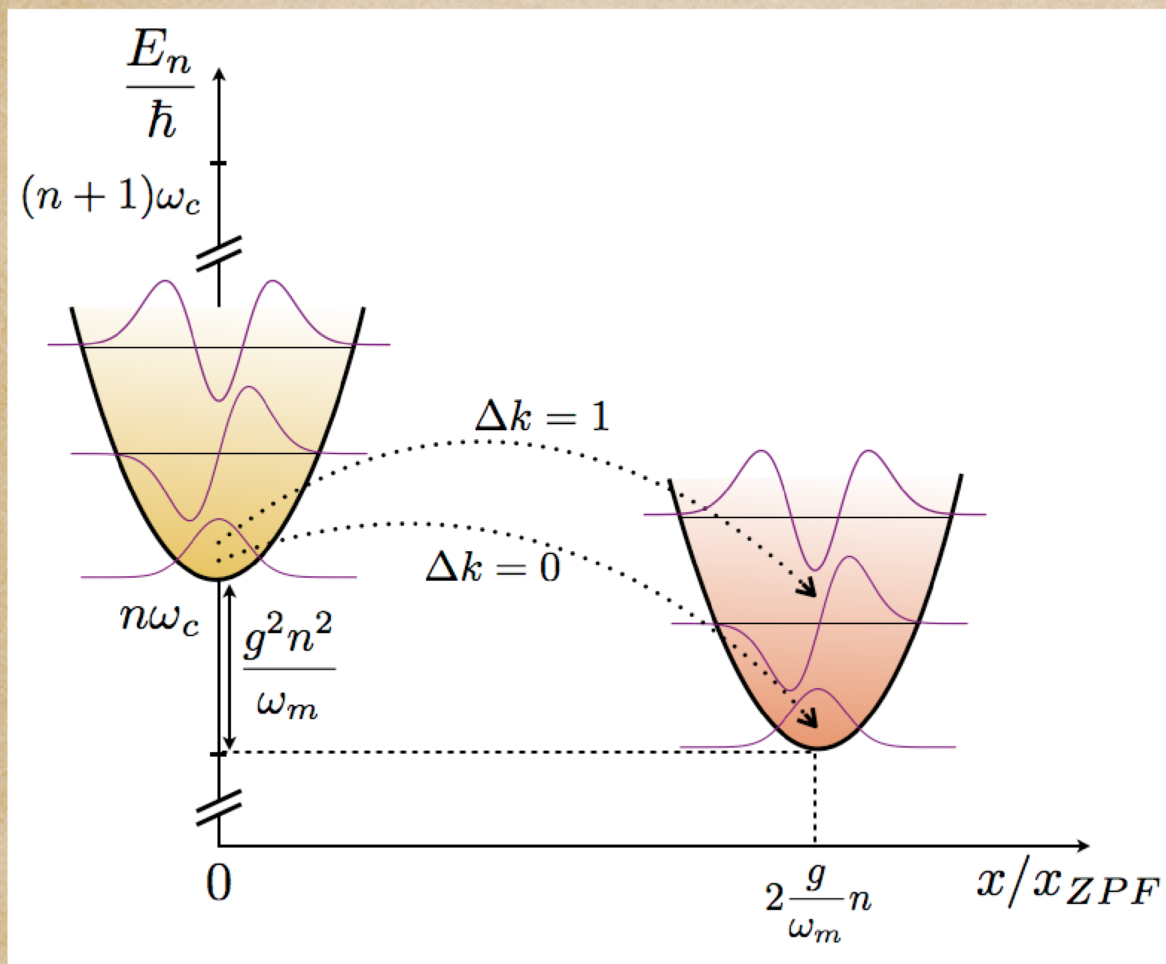

T. B. Batalhao, et al. PRL 115, 190601 (2015)

# Setting the context



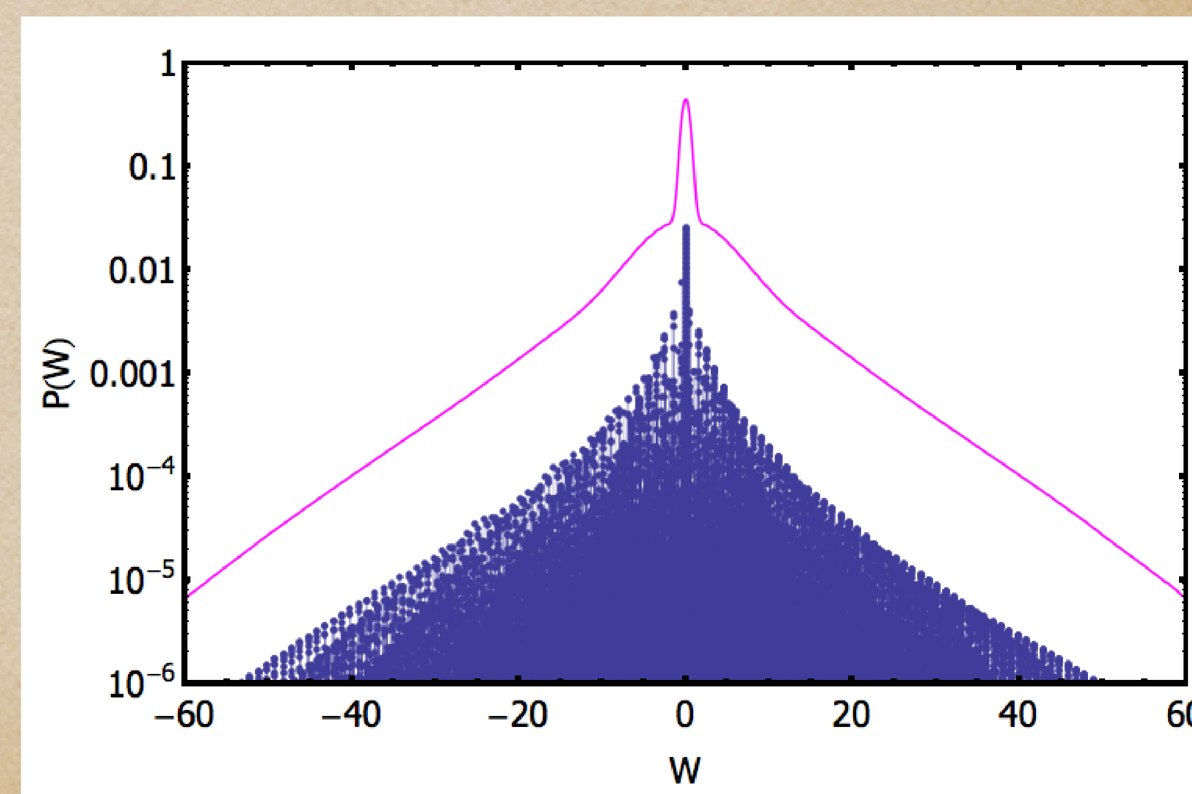



# Quenching the linear coupling




$$\hat{H}_{t=0} = \hbar\omega_c \hat{a}^\dagger \hat{a} + \hbar\omega_m(\hat{b}^\dagger \hat{b} + \tfrac{1}{2})$$

$$\hat{H}_{t>0} = \hat{H}_{t=0} + \hbar\, g\, \hat{a}^\dagger \hat{a}(\hat{b} + \hat{b}^\dagger)$$

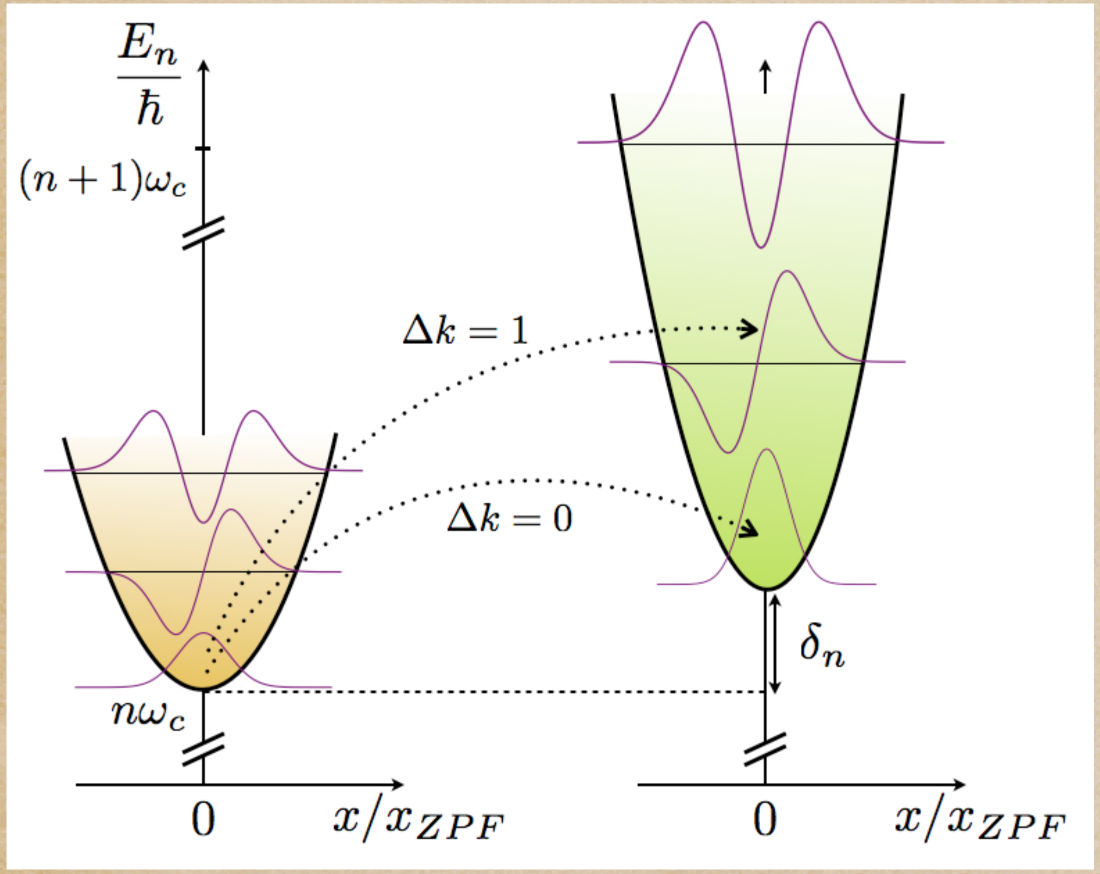


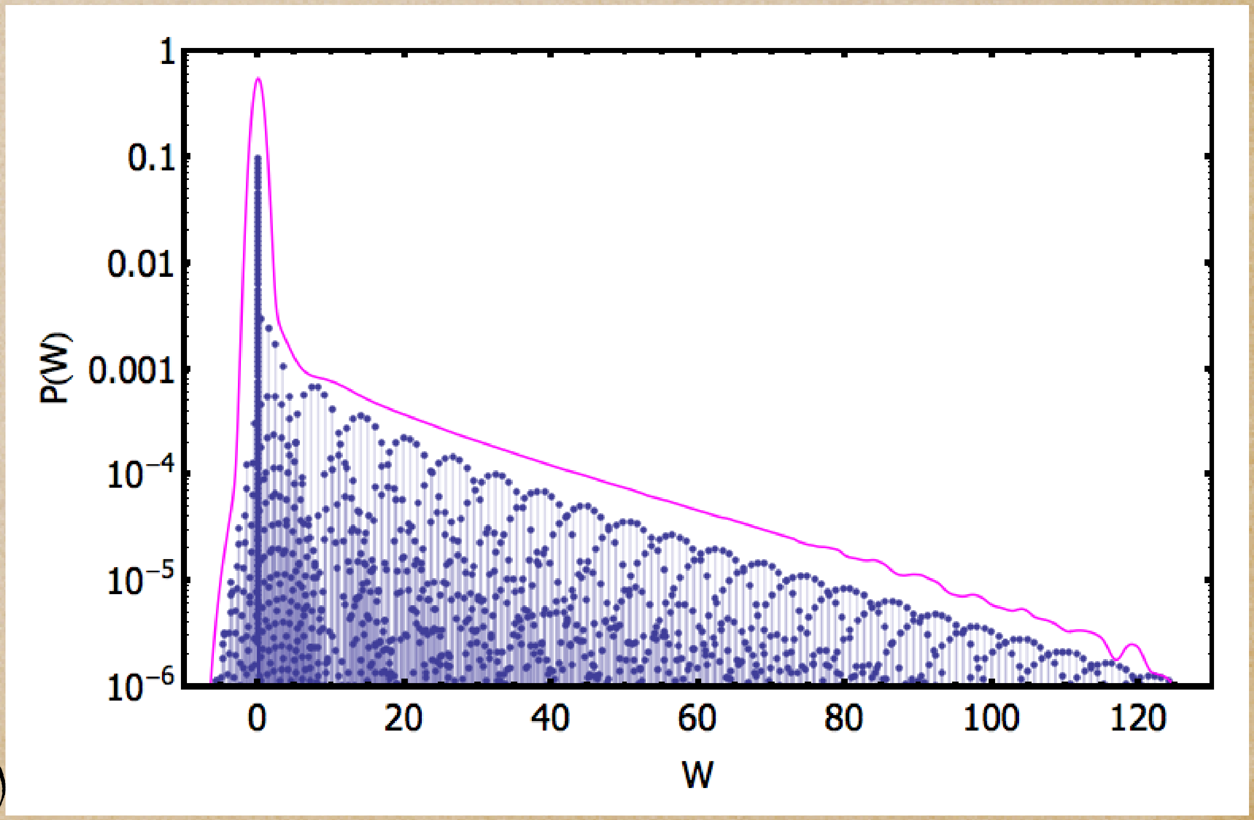

M. Brunelli et al., NJP 17, 035016 (2015)

# Quenching the linear coupling

$$\hat{H}_{t=0} = \hbar\omega_c \hat{a}^\dagger \hat{a} + \hbar\omega_m(\hat{b}^\dagger \hat{b} + \tfrac{1}{2})$$

$$\hat{H}_{t>0} = \hat{H}_{t=0} + \hbar\,\kappa\,\hat{a}^\dagger \hat{a}(\hat{b} + \hat{b}^\dagger)^2.$$




M. Brunelli et al., NJP 17, 035016 (2015)

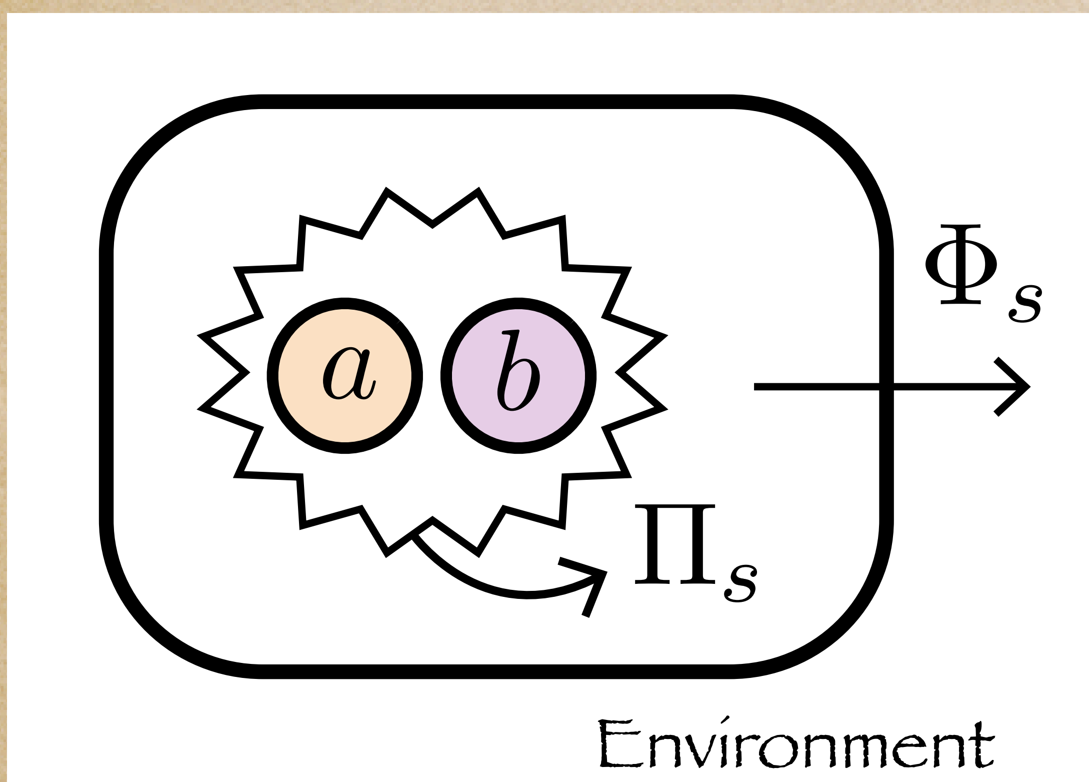
# Entropy production

$$\Sigma = \Delta\mathcal{S} - \int \frac{dQ}{T} \geq 0$$

$$\frac{\mathrm{d}\mathcal{S}}{\mathrm{d}t} = \Pi(t) - \Phi(t)$$

Entropy production rate

Entropy flux rate into the environment

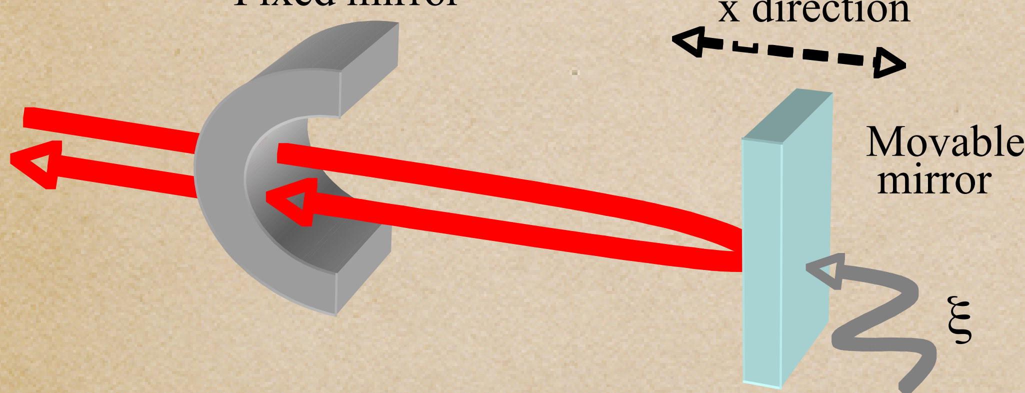



Collaboration with the groups of
M. Aspelmeyer & N. Kiesel (Vienna)
T. Esslinger & T. Donner (ETHZ)

M. Brunelli, et al. arXiv:1602.06958 (2016)

# Entropy production in mesoscopic quantum systems

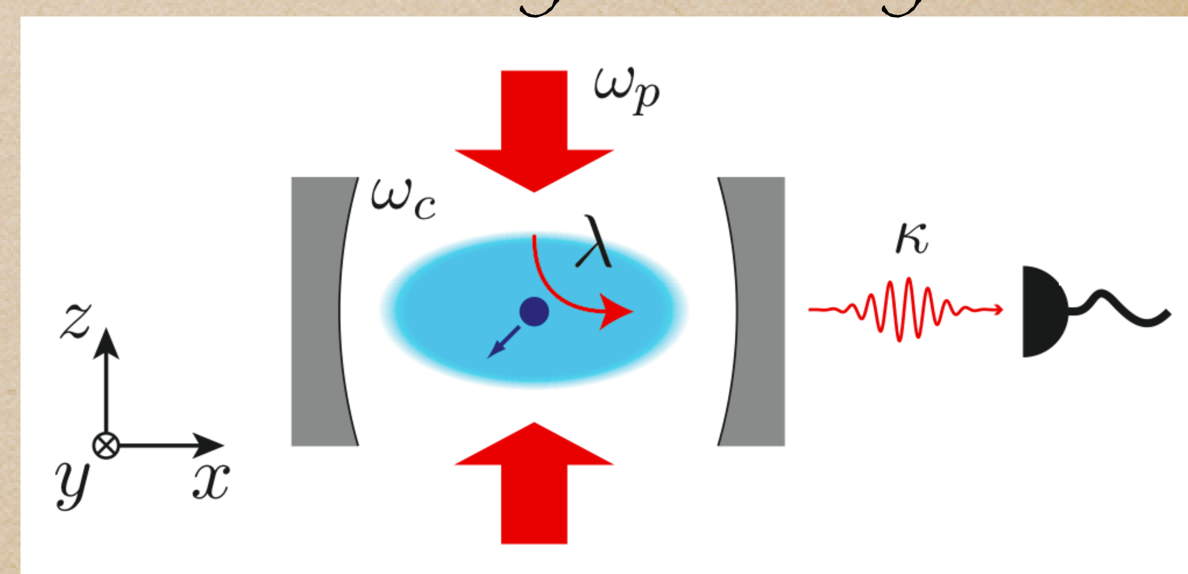
## Optomechanics




$$H = \frac{\hbar\omega}{2}(p^2 + q^2) + \hbar(\omega_c - gq)a^\dagger a$$
$$+ i\hbar\mathcal{E}(a^\dagger e^{-i\omega_0 t} - a e^{i\omega_0 t})$$

## Intra-cavity atomic systems

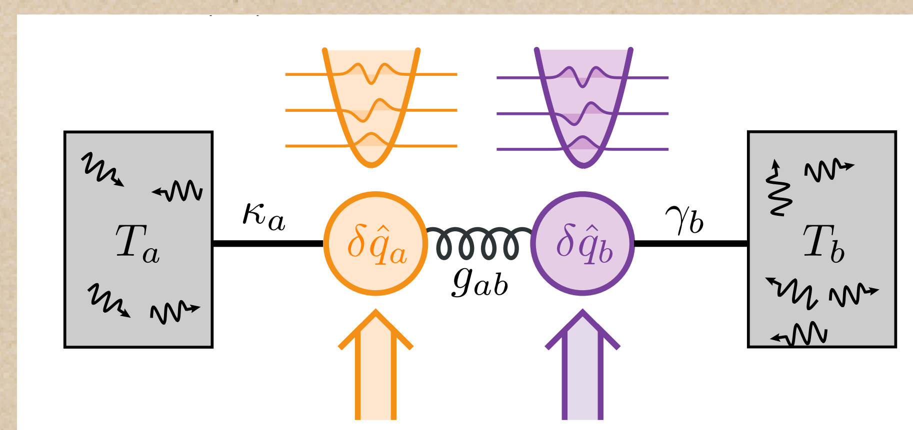
$$\hat{H} = \omega_0 \hat{J}_z + \omega \hat{a}^\dagger \hat{a} + \frac{2\lambda}{\sqrt{N}}\left(\hat{a} + \hat{a}^\dagger\right)\hat{J}_x$$

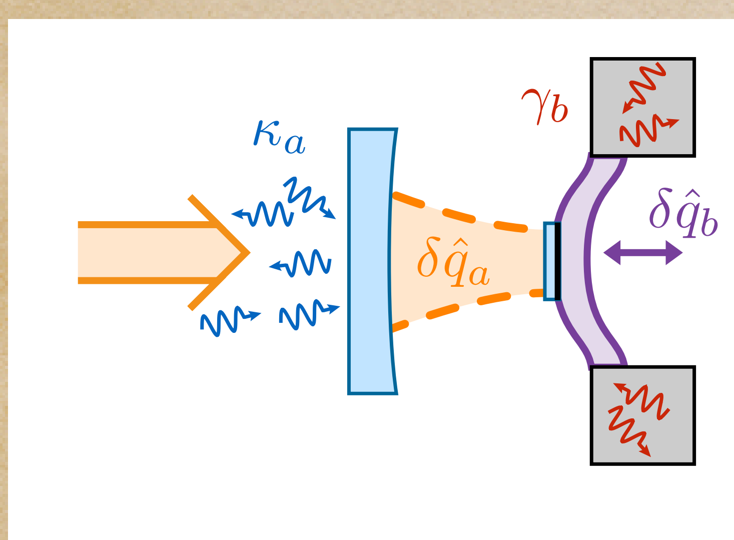



M. Brunelli, et al. arXiv:1602.06958 (2016)

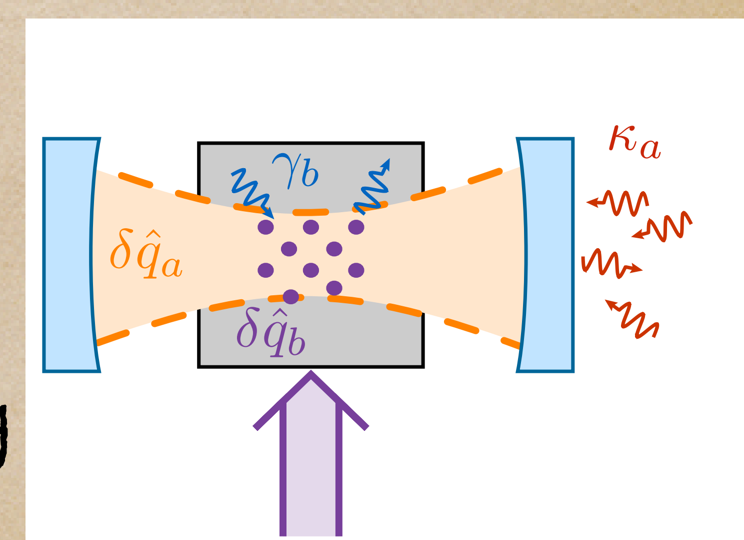
# Entropy production in mesoscopic quantum systems

$$\hat{H} = \frac{\hbar\omega_a}{2}(\delta\hat{q}_a^2 + \delta\hat{p}_a^2) + \frac{\hbar\omega_b}{2}(\delta\hat{q}_b^2 + \delta\hat{p}_b^2) + \hbar g_{ab}\delta\hat{q}_a\delta\hat{q}_b$$




General framework
for the study
of thermo-irreversibility




M. Brunelli, et al. arXiv:1602.06958 (2016)

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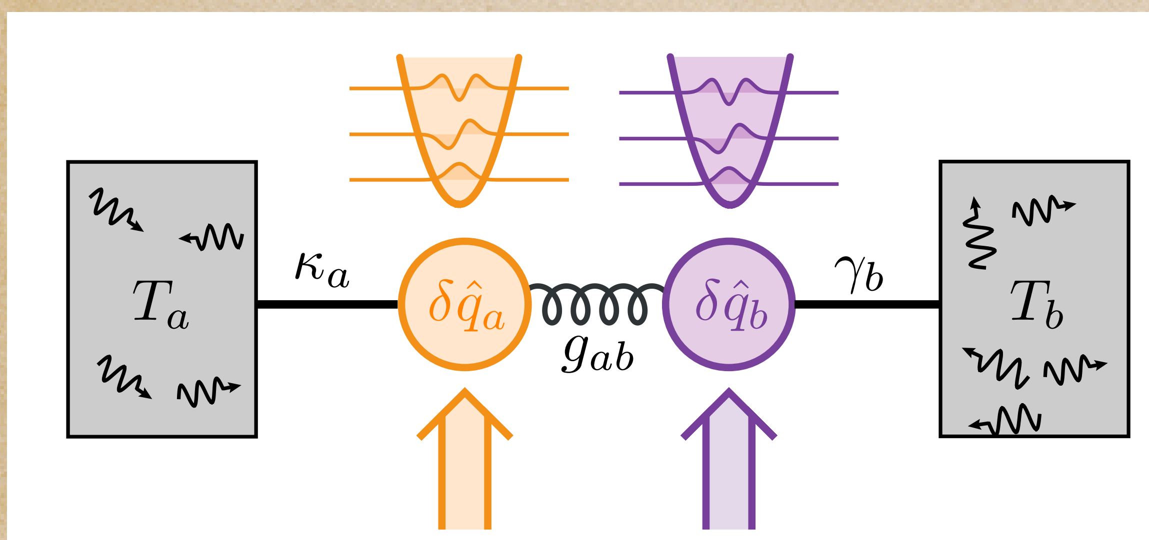
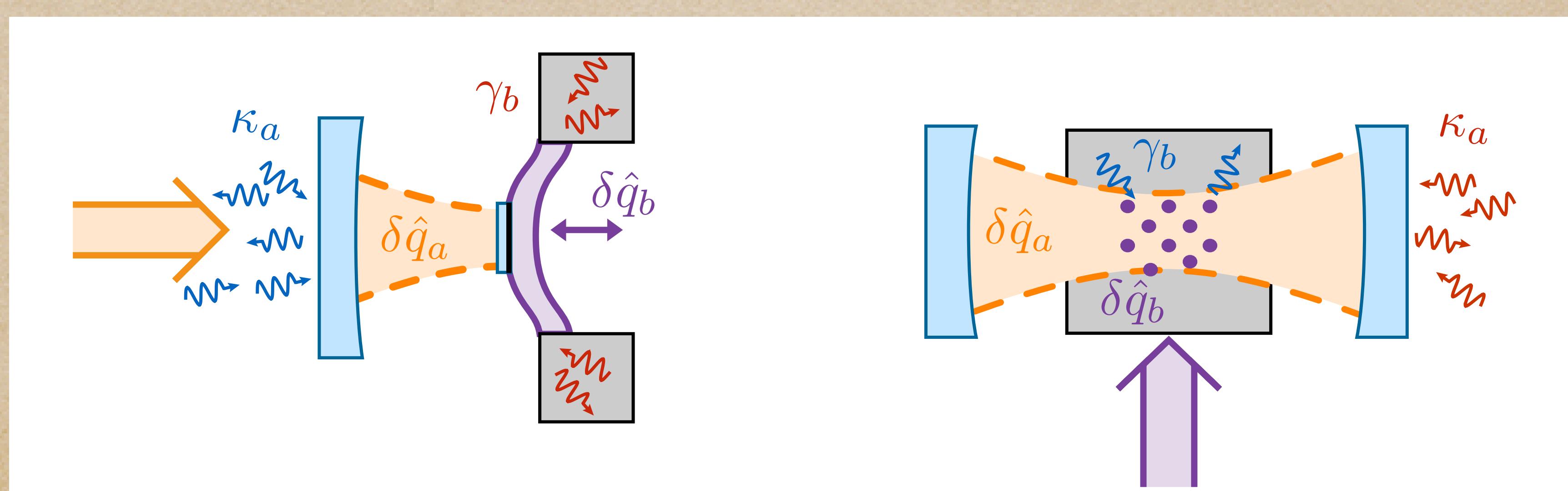
# Entropy production in mesoscopic quantum systems

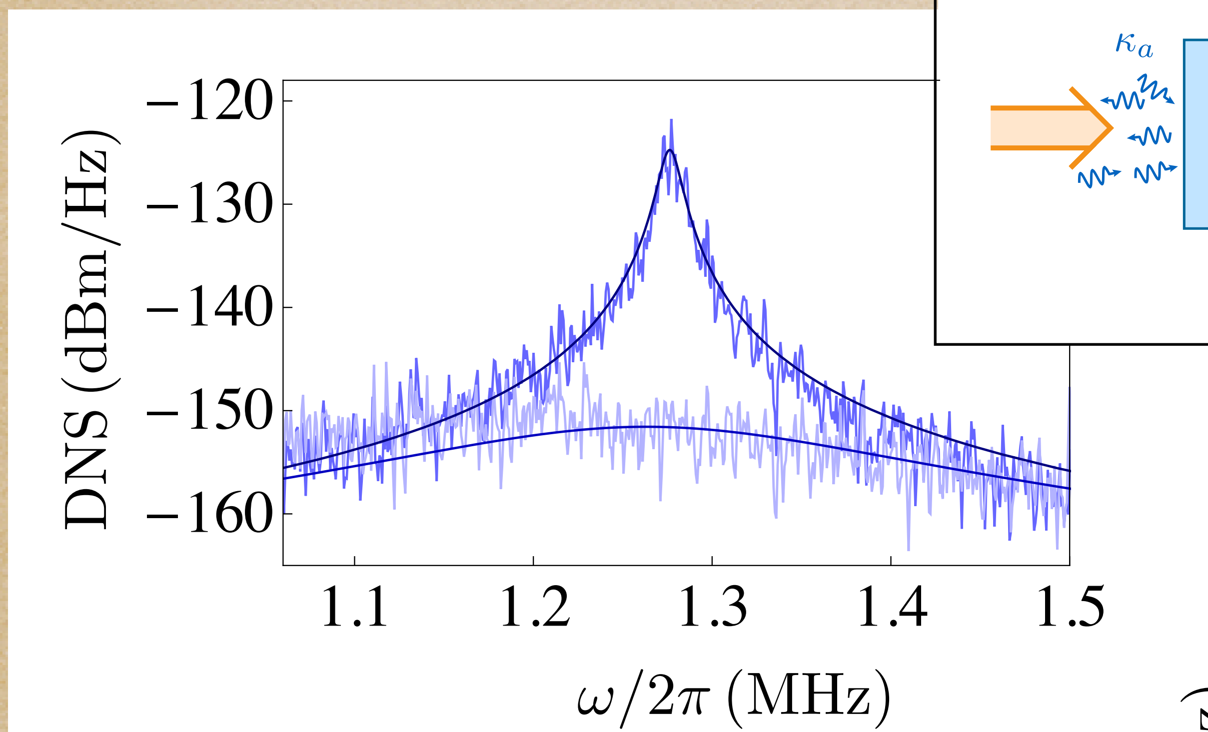
$$\hat{H} = \frac{\hbar\omega_a}{2}(\delta\hat{q}_a^2 + \delta\hat{p}_a^2) + \frac{\hbar\omega_b}{2}(\delta\hat{q}_b^2 + \delta\hat{p}_b^2) + \hbar g_{ab}\delta\hat{q}_a\delta\hat{q}_b$$

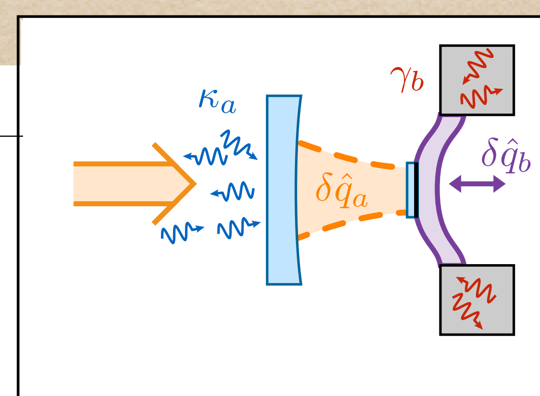


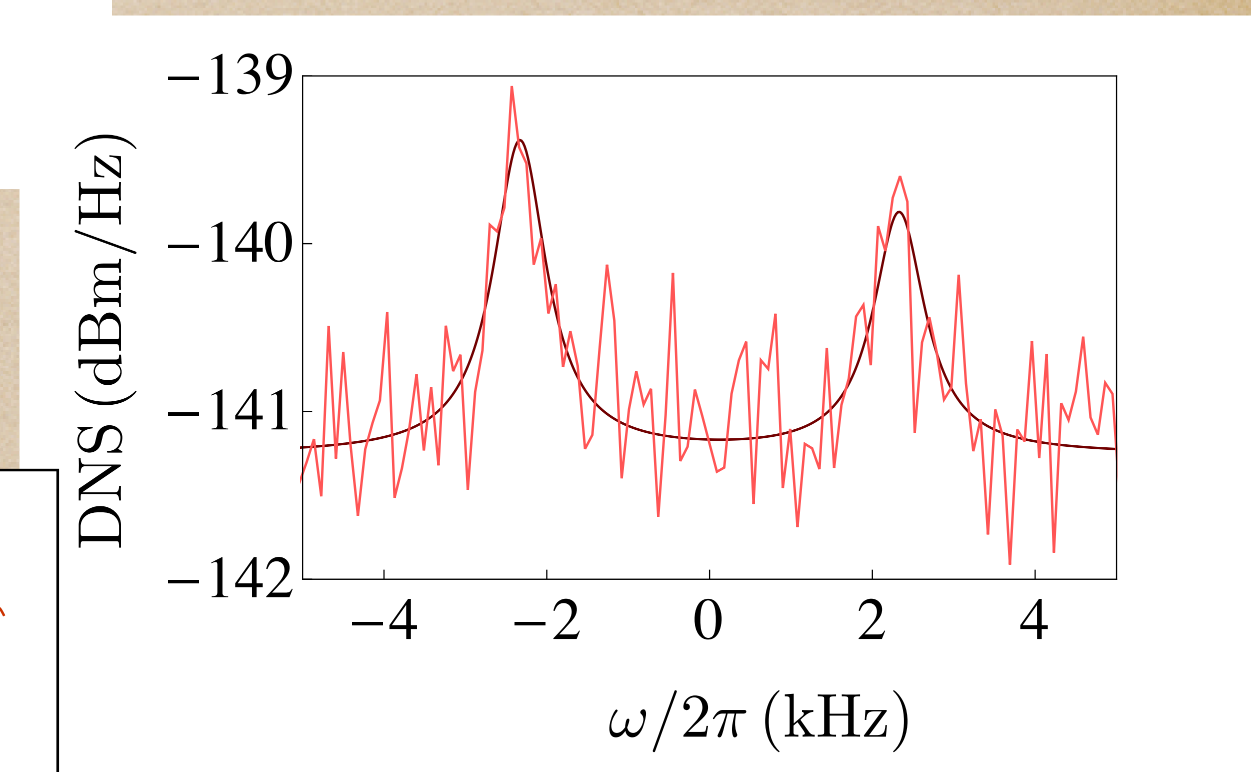

$$\Pi_s = 2\gamma_b\left(\frac{n_b + 1/2}{n_{T_b} + 1/2} - 1\right) + 4\kappa_a n_a$$

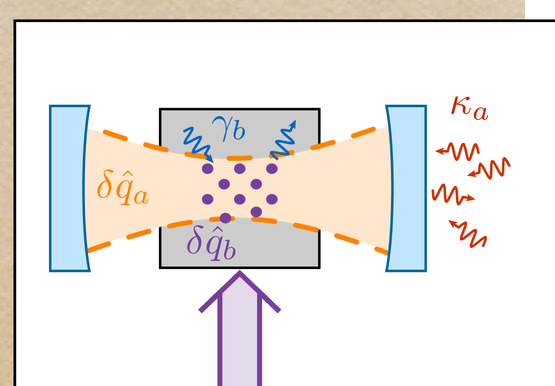


# Experimental DNS

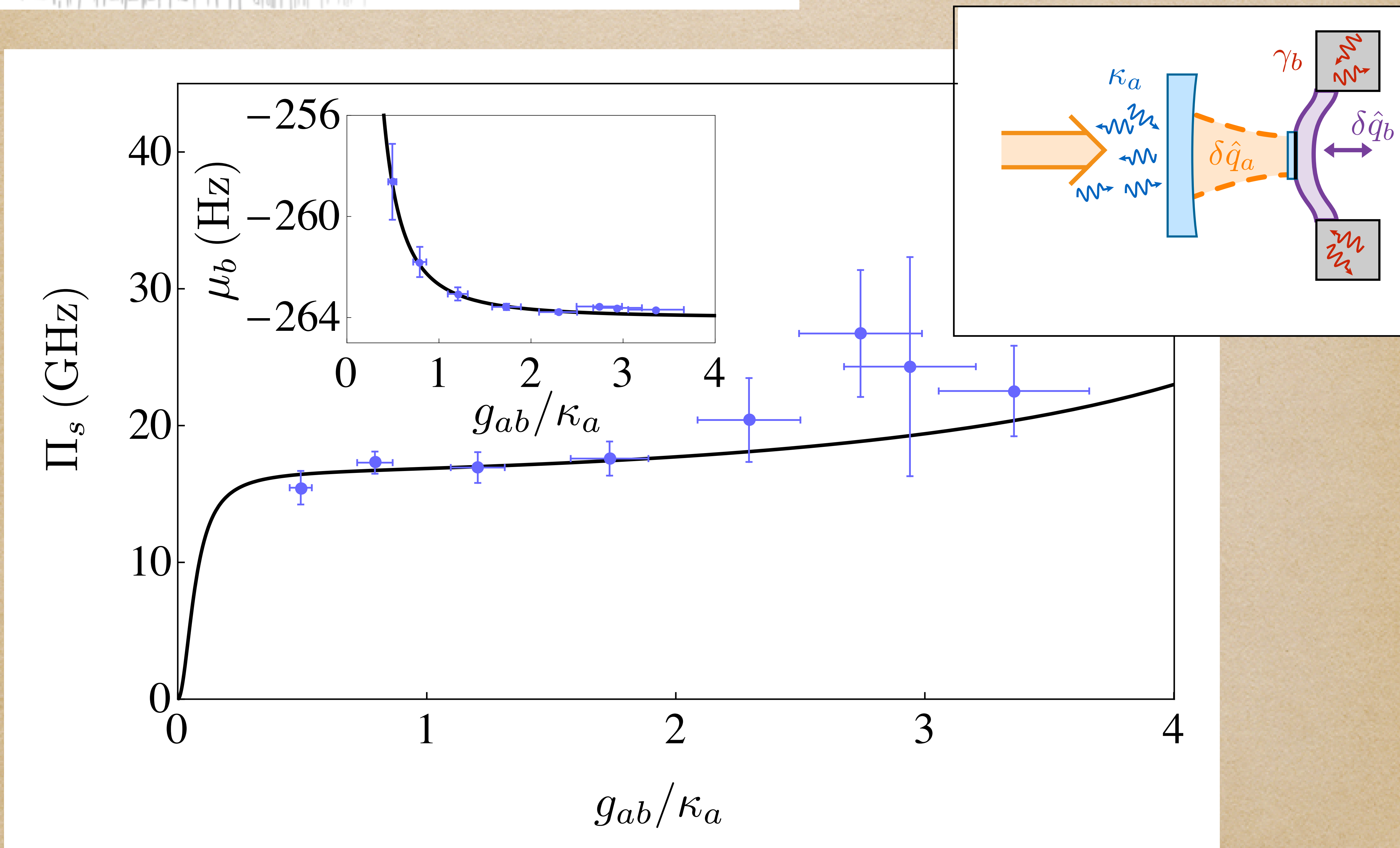


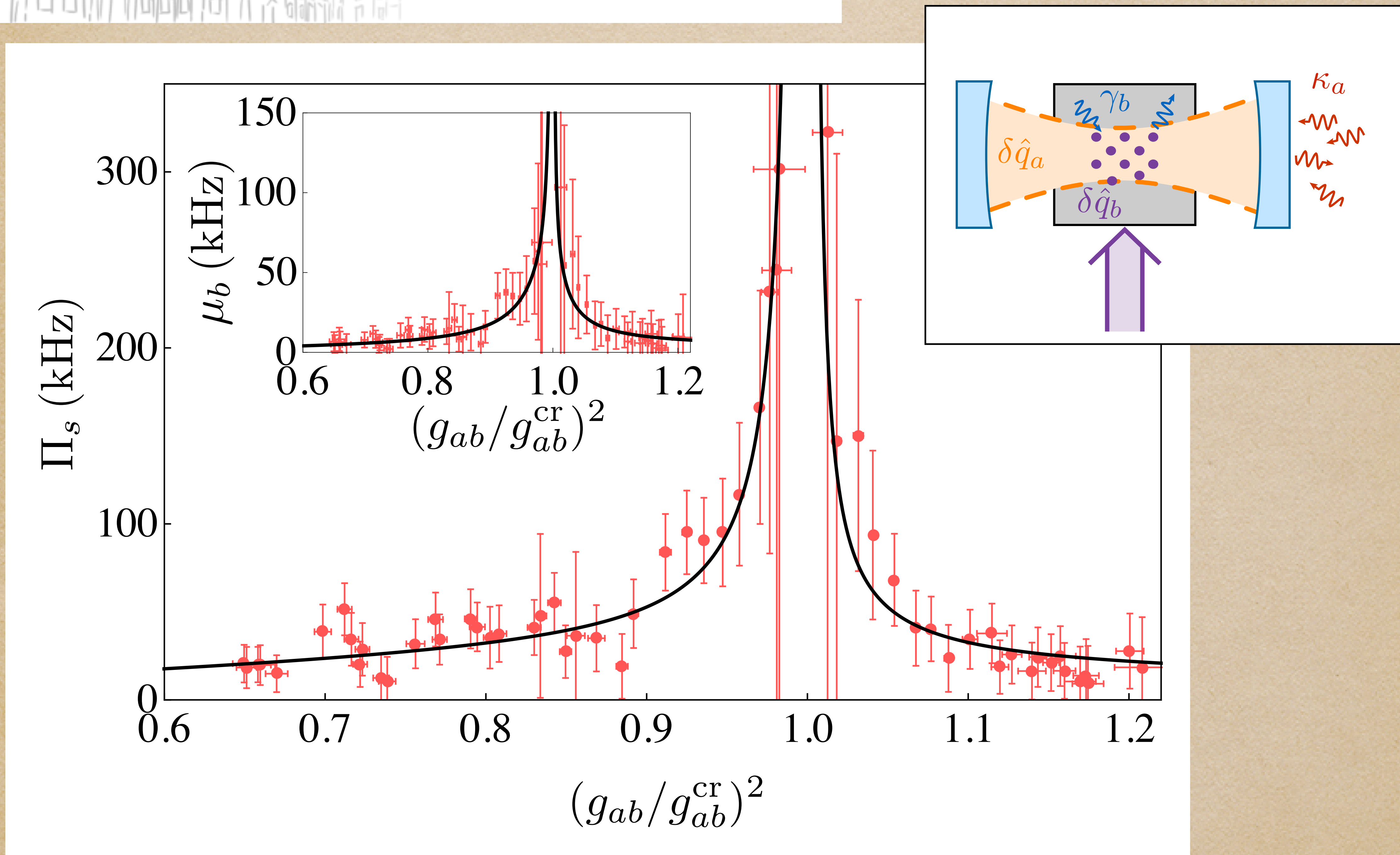



# Results




M. Brunelli, et al. arXiv:1602.06958 (2016)

# Results




M. Brunelli, et al. arXiv:1602.06958 (2016)

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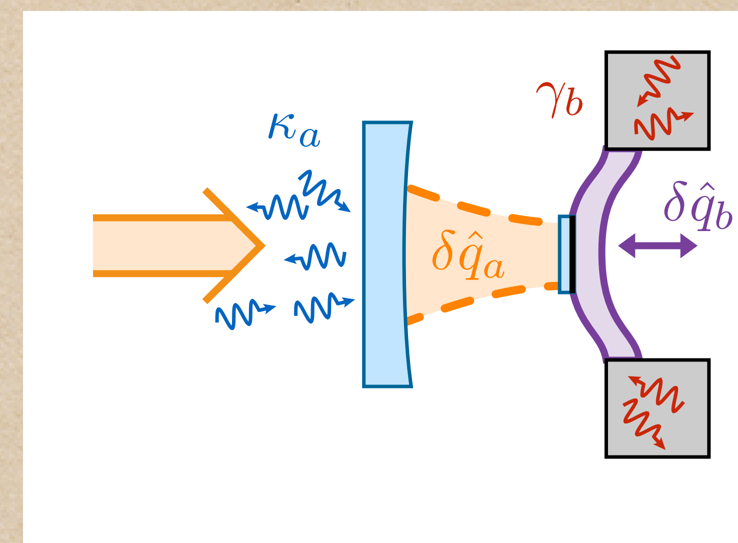
# Collaborations

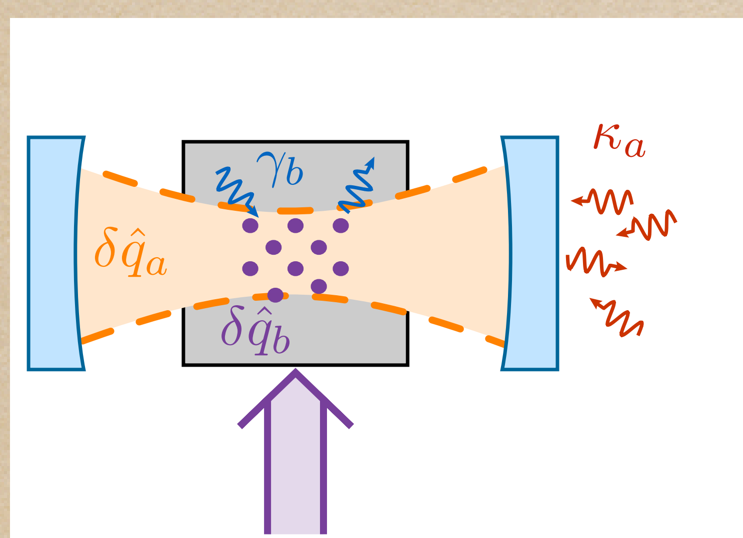
## Entropy & optomechanics

M. Brunelli, L. Fusco,

G. De Chiara, A. Ferraro (Belfast)

W. Wieczorek, N. Kiesel (Vienna)




## Entropy & ultracold atoms

M. Brunelli, L. Fusco,

G. De Chiara, A. Ferraro (Belfast)

R. Landig, T. Donner(ETH)

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# Kneeling before the Lord

Lord Kelvin


Quantum thermo meeting, Belfast August 2014

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# The Belfast crew

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*Go raibh maith agaibh*

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