Time-Continuous Monitoring For Quantum Optomechanics: From Quantum State Engineering To Fundamental Parameter Estimation

M. G. Genoni

University College London

University of Milan

PBQ Workshop, Olomouc, 4th May 2016

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Quantum optomechanics

Motivations

- promising platforms for ultra-sensitive detectors thanks to the ability of mechanical systems to respond to optical, electrical or magnetic forces.
- ideal playground to test fundamental physics:
  "quantum-to-classical transition", non-linear corrections to Schroedinger equation, collapse models.
- mechanical oscillators as transducers (e.g. between light and microwaves)

Goals

- cool nanomechanical systems to their quantum ground states and to non-classical states (e.g. squeezed states)

Physical implementations

- moving mirrors
- membranes
- levitating nanospheres

Aspelmeyer et al., RMP **86**, 1391 (2014)

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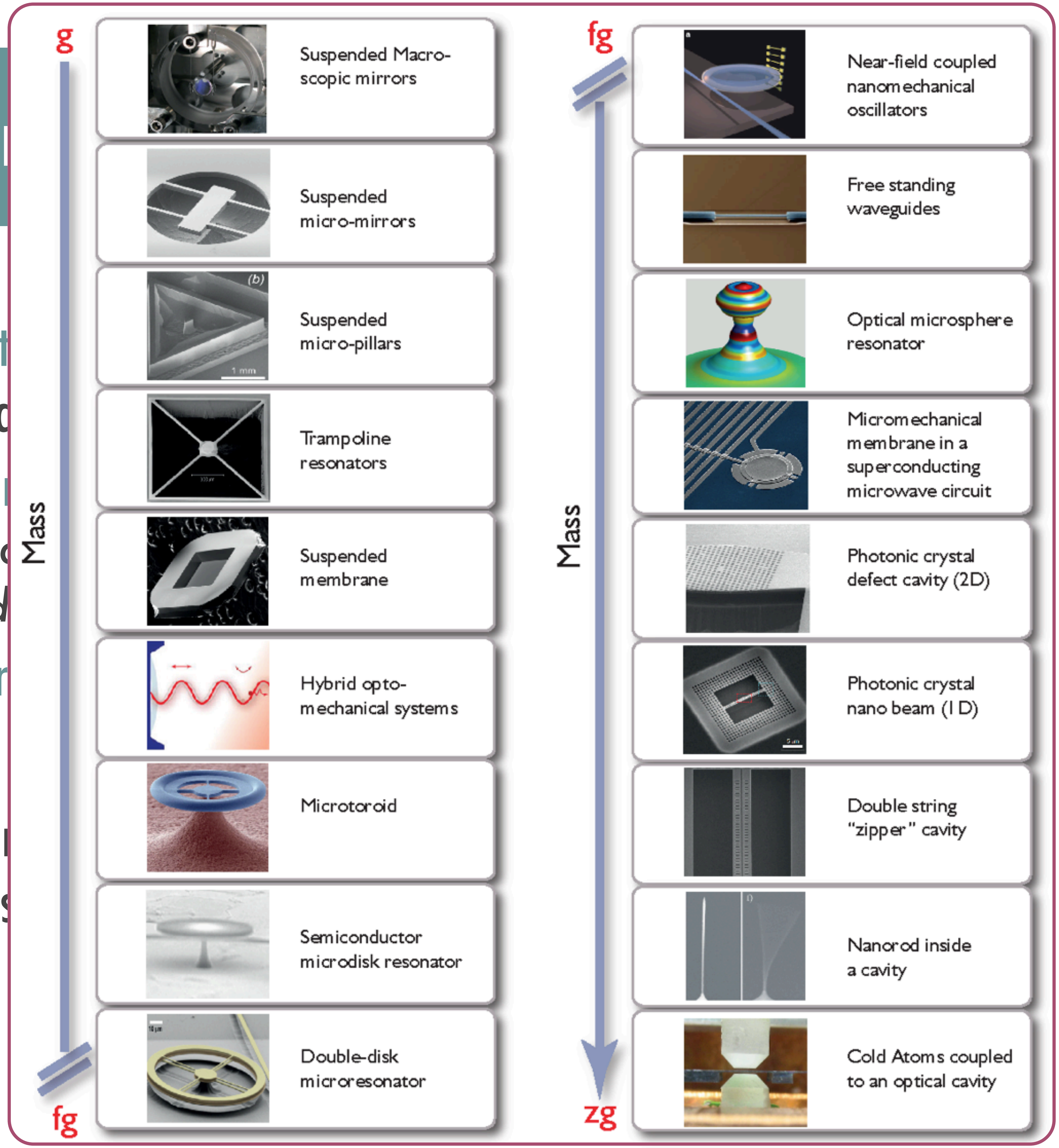

Aspelmeyer et al., RMP **86**, 1391 (2014)

Outline

- Quantum filtering and feedback control for quantum state-engineering
  - quantum cooling and squeezing generation for a levitated dielectric nanosphere via time-continuous measurement

with J. Zhang, J. Millen, P. Barker and A. Serafini

MGG et al., NJP **17**, 073019 (2015)

- Quantum filtering and feedback control for parameter estimation
  - unravelling the noise: the discrimination of wave-function collapse models under time-continuous measurements

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MGG et al., arXiv:1605:?????

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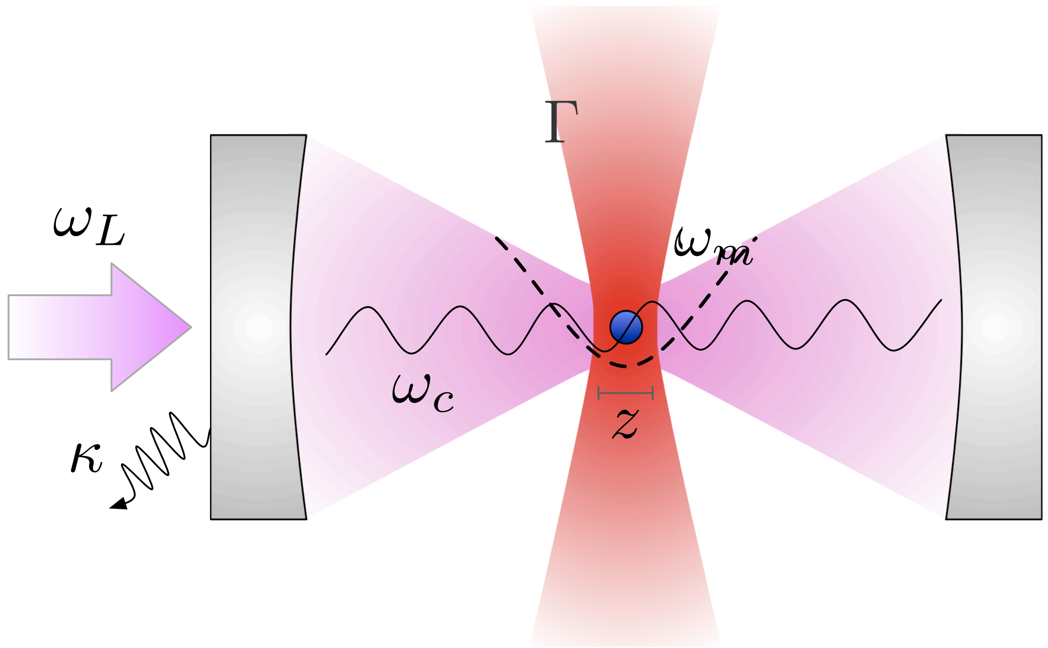
Quantum optomechanics with a levitated nanosphere



Hamiltonian (*linearized* assuming cavity strongly driven)

$$H = \omega_m b^\dagger b - \Delta a^\dagger a + g(a + a^\dagger)(b + b^\dagger)$$

$\Delta = \omega_L - \omega_c$ detuning from cavity resonance

Romero-Isart et al., PRA **83**, 013803 (2011)
Pflanzer et al., PRA **86**, 013802 (2012)

Master equation

$$\dot{\varrho} = i[\varrho, H] + \kappa \mathcal{D}[a]\varrho + \Gamma \mathcal{D}[b + b^\dagger]\varrho \qquad \mathcal{D}[A]\varrho = A\varrho A^\dagger - \frac{1}{2}\{A^\dagger A, \varrho\}$$

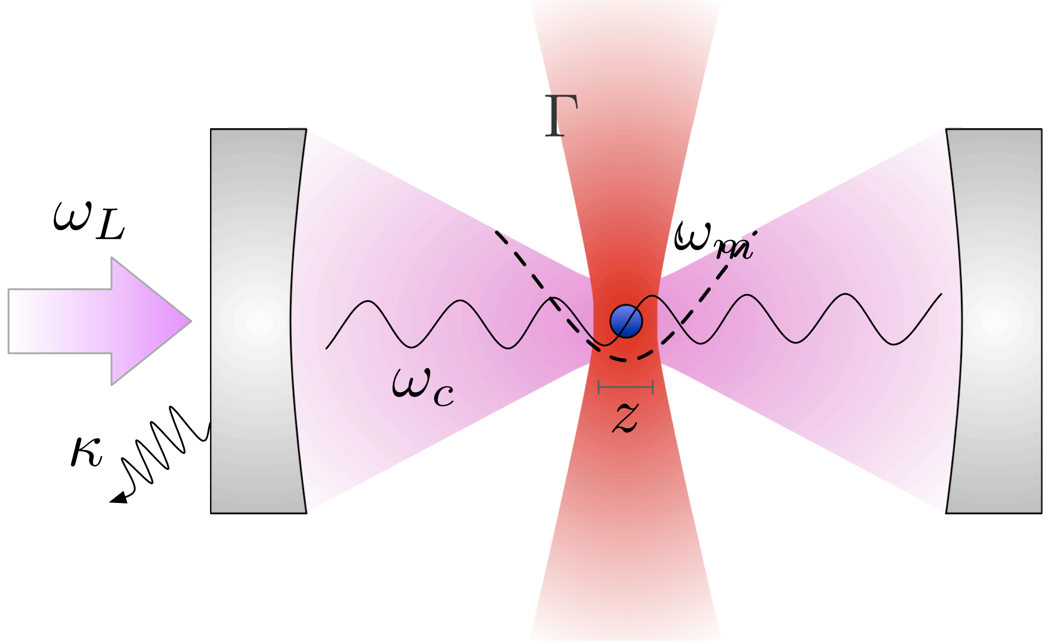
cavity loss *recoil heating (light scattering)*

Physical parameters ($\mathrm{kHz} \cdot 2\pi$)

$\omega_c \approx 2 \cdot 10^{11}$ $\omega_m \approx 9 \cdot 10^2$

$\kappa \approx 3 \cdot 10^2$ $\Gamma \approx 60$ $g \approx 8 \cdot 10^2$

Quantum optomechanics with a levitated nanosphere



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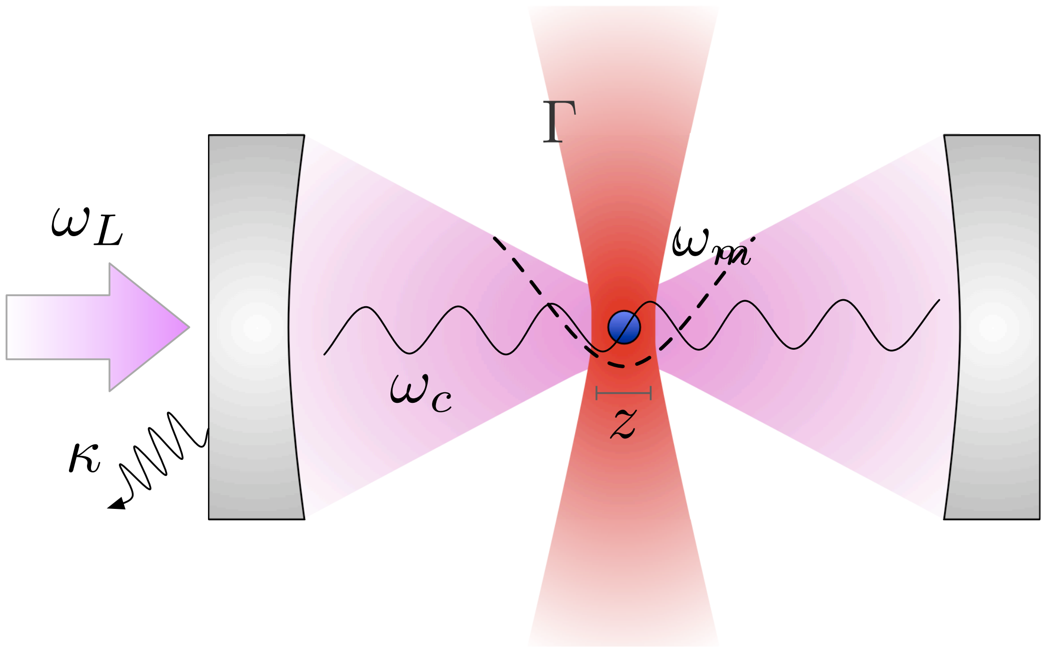
cavity loss *recoil heating (light scattering)*

- ✓ Quadratic Hamiltonian
- ✓ linear coupling to the environment

→ Evolution preserves Gaussianity of the state

Quantum optomechanics with a levitated nanosphere



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First and second moments evolution

$$\dot{\boldsymbol{\sigma}} = A\boldsymbol{\sigma} + \boldsymbol{\sigma} A^{\mathsf{T}} + D$$ covariance matrix

$$\langle \dot{\hat{R}} \rangle = A \langle \hat{R} \rangle$$ first moments vector

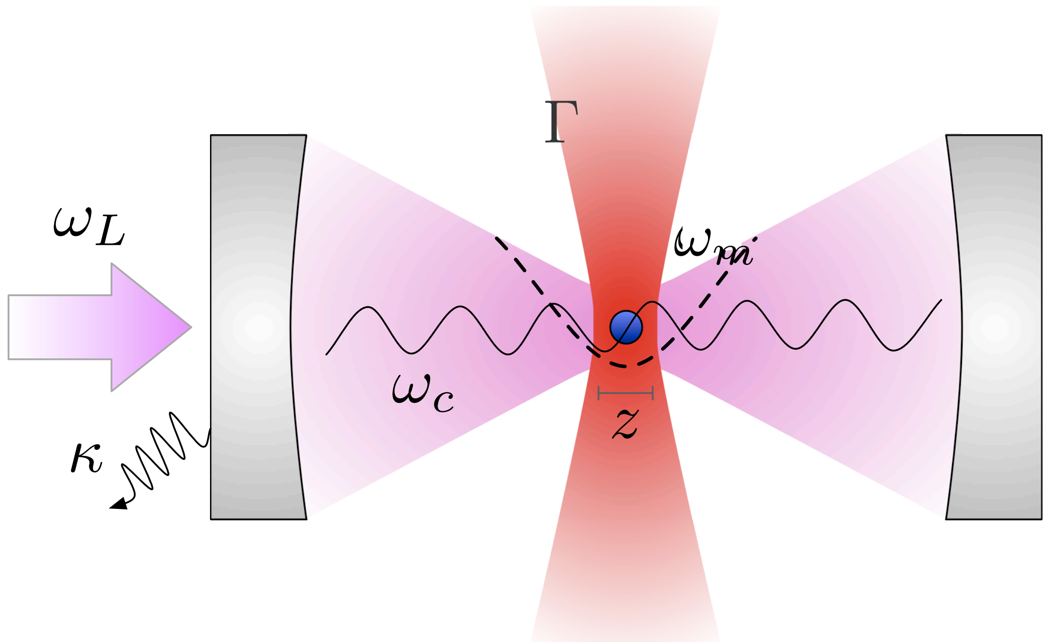
drift matrix

$$A = \begin{pmatrix} -\frac{\kappa}{2} & -\Delta & 0 & 0 \\ \Delta & -\frac{\kappa}{2} & -2g & 0 \\ 0 & 0 & 0 & \omega_m \\ -2g & 0 & -\omega_m & 0 \end{pmatrix}$$

diffusion matrix

$$D = \begin{pmatrix} \kappa & 0 & 0 & 0 \\ 0 & \kappa & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 4\Gamma \end{pmatrix}.$$

Quantum optomechanics with a levitated nanosphere



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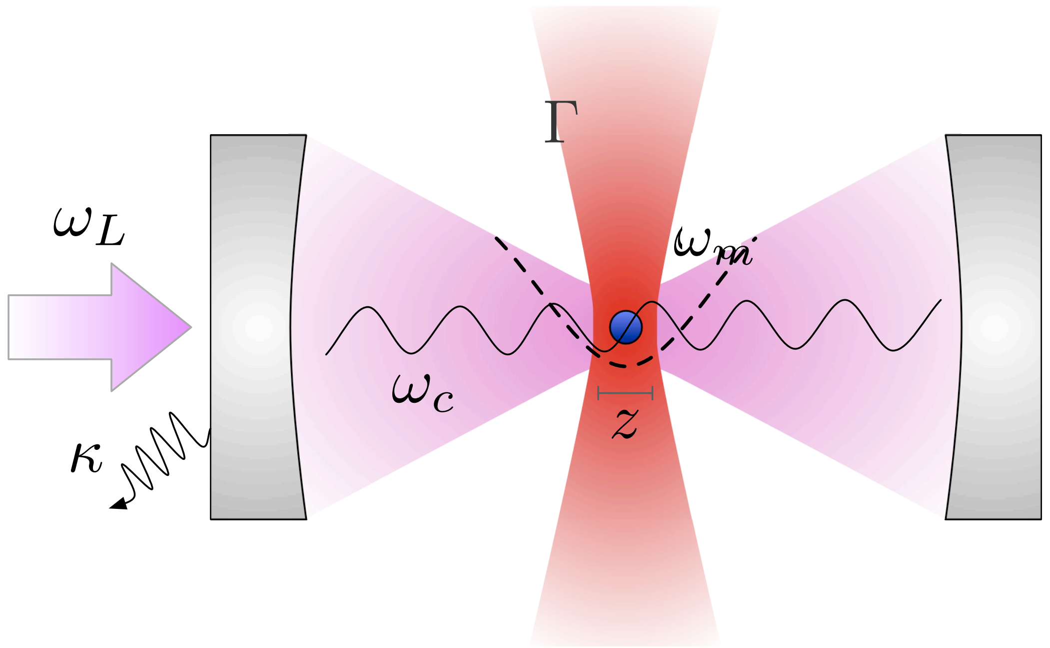
recoil heating

Quantum optomechanics with a levitated nanosphere



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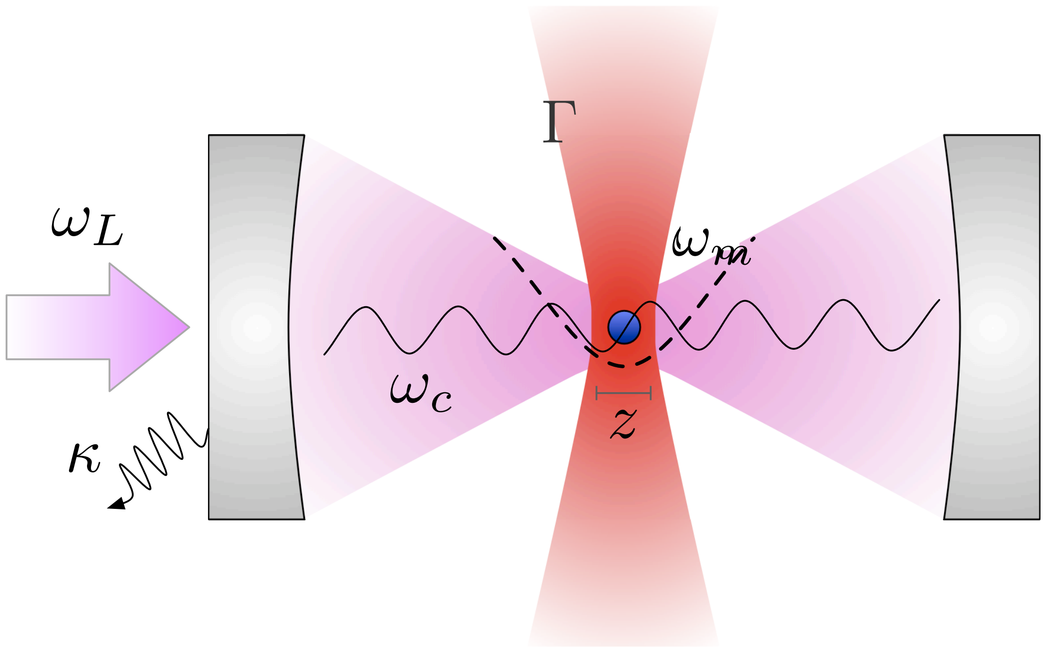
Stability condition

$$\mathrm{Re}[\mathrm{eigs}(A)] < 0$$

Quantum optomechanics with a levitated nanosphere



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Steady-state solution

$$A\boldsymbol{\sigma}_{\mathsf{ss}} + \boldsymbol{\sigma}_{\mathsf{ss}} A^{\mathsf{T}} + D = 0$$

figures of merit ($\boldsymbol{\sigma}_{\mathsf{ss}}^{(m)}$ covariance matrix of the oscillator only)

- average number of phonons $n_{\mathsf{ph}} = (\mathrm{tr}[\boldsymbol{\sigma}_{\mathsf{ss}}^{(m)}] - 2)/4$ (for zero first-moments)
- mechanical oscillator purity $\mu = 1/\sqrt{\det[\boldsymbol{\sigma}_{\mathsf{ss}}^{(m)}]}$
- mechanical oscillator quantum squeezing $\mathrm{dB} = 10 \log_{10} \xi\,, \ \ \xi = \min \mathrm{eig}[\boldsymbol{\sigma}_{\mathsf{ss}}^{(m)}]$

Sideband cooling

Interaction Hamiltonian (in interaction picture respect to $H_0 = \omega_m b^\dagger b - \Delta a^\dagger a$)

$$H_{\mathsf{int}} = g\left(ab^\dagger e^{i(\omega_m+\Delta)t} + a^\dagger b e^{-i(\omega_m+\Delta)t} + abe^{-i(\omega_m-\Delta)t} + a^\dagger b^\dagger e^{i(\omega_m-\Delta)t}\right)$$

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resonant for $\Delta = -\omega_m$

red-detuned cavity

beam-splitter like interaction

good for cooling by exchanging excitations

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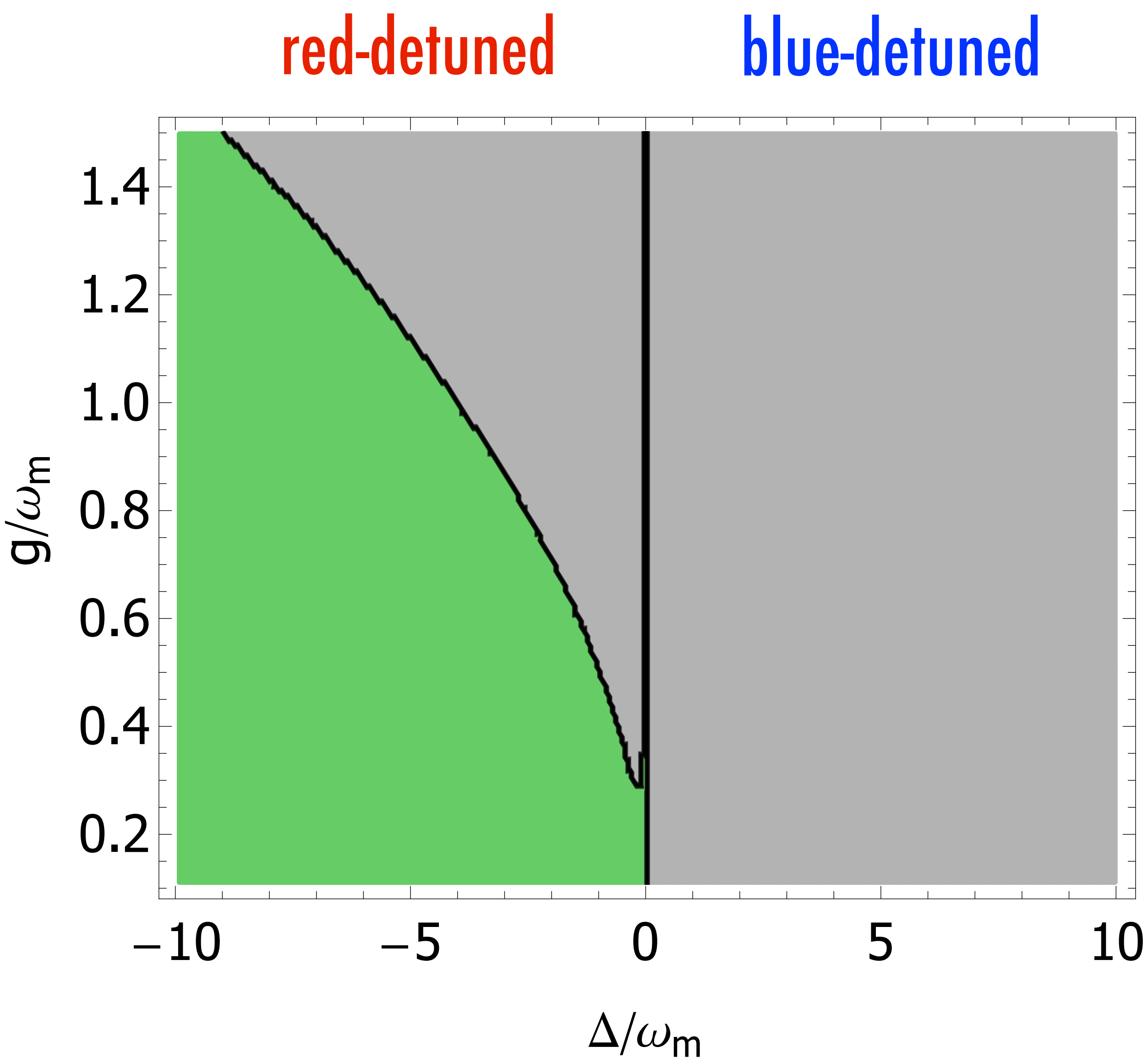
resonant for $\Delta = -\omega_m$
red-detuned cavity

beam-splitter like interaction
good for cooling by exchanging excitations

resonant for $\Delta = \omega_m$
blue-detuned cavity

two-mode squeezing interaction
good to create correlations between cavity and oscillator

Sideband cooling



Stability condition

$$\mathrm{Re}[\mathrm{eigs}(A)] < 0$$

unstable dynamics (no steady-state)

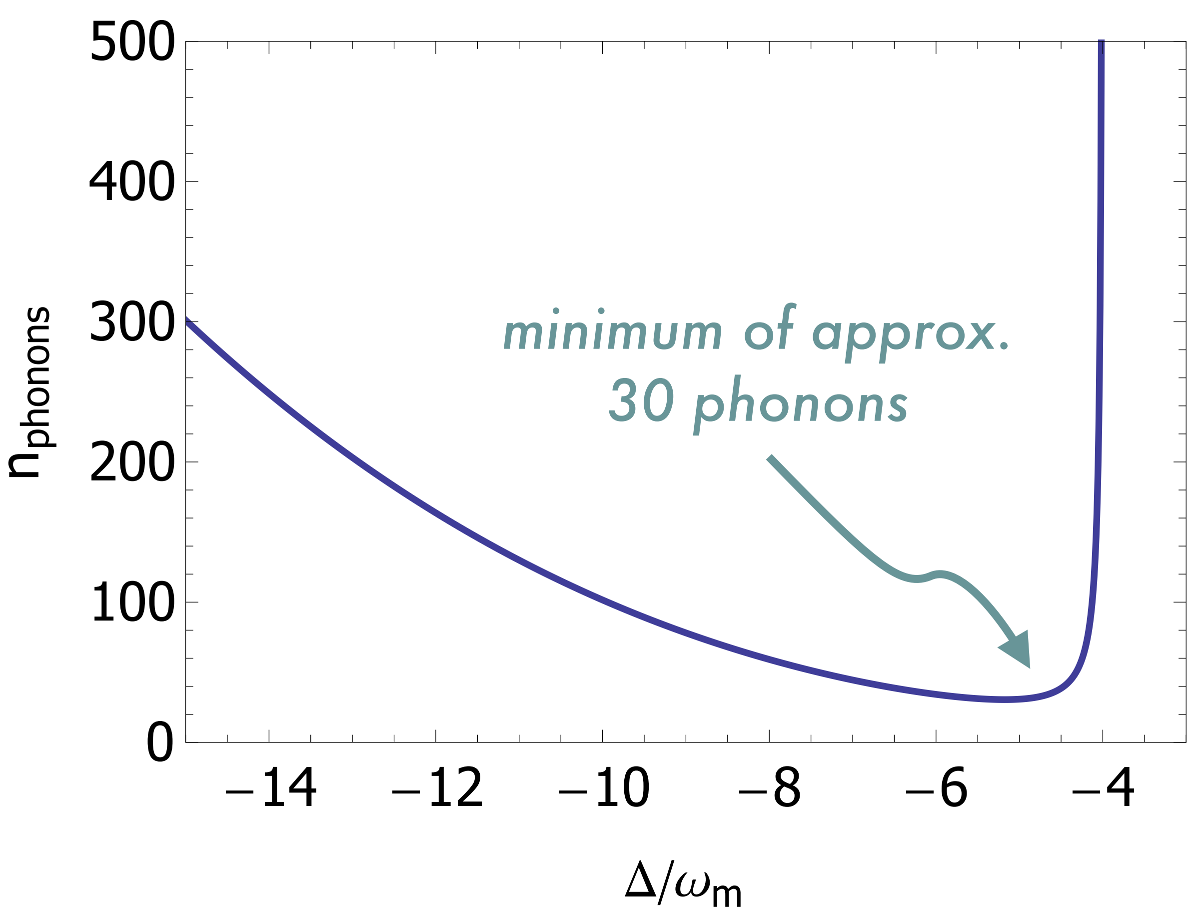
stable dynamics

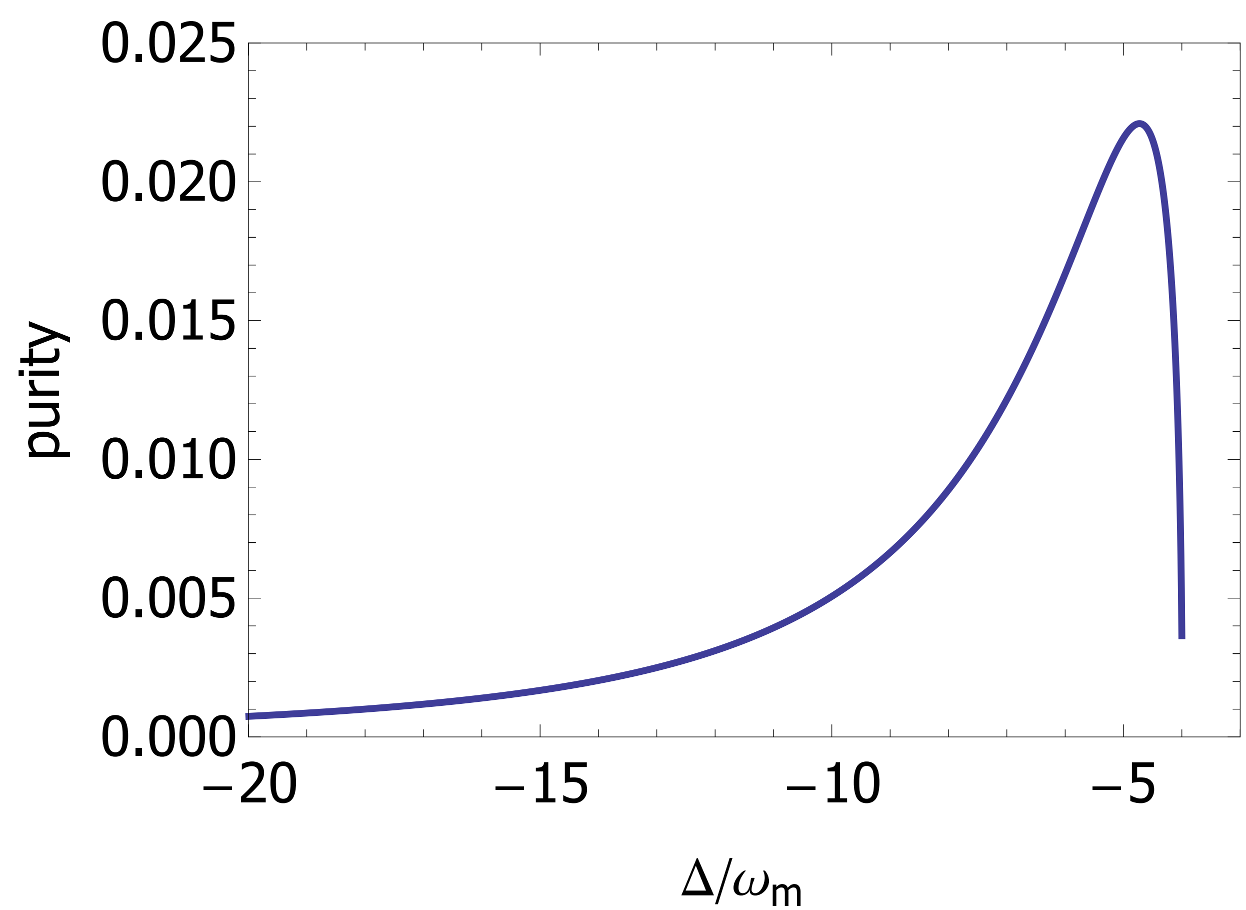
If the cavity is properly red-detuned, the dynamics has a steady-state

Sideband cooling

Steady-state properties

($g = \omega_m,\ \kappa = \omega_m/3,\ \Gamma = \omega_m/10$)





...can we improve the performances of sideband cooling with the help of time-continuous measurements?

Time-continuous measurements

Assumption: *It is possible to monitor (measure) the system on time-scales which are much shorter than the typical system's response time.*

$$d\varrho = i[\varrho, H]dt + \kappa\mathcal{D}[a]\varrho\, dt + \Gamma\mathcal{D}[b+b^\dagger]\varrho\, dt$$
$$+\sqrt{\kappa\eta_1}\,\mathcal{H}[ae^{i\phi}]\varrho\, dw_1 + \sqrt{\Gamma\eta_2}\,\mathcal{H}[b+b^\dagger]\varrho\, dw_2$$

homodyne measurement of the cavity mode — *position measurement of the oscillator*

- superoperator: $\mathcal{H}[A]\varrho = A\varrho + \varrho A^\dagger - \langle A + A^\dagger\rangle\varrho$
- measurement efficiencies, η_j and Wiener increments dw_j

Belavkin, Wiseman&Milburn, ...

Stochastic evolution for the 1st moments

Wiener "stochastic" increments

$$d\langle\hat{R}\rangle_c = A\langle\hat{R}\rangle_c\, dt + (\ldots\ldots)d\mathbf{w}$$

Deterministic evolution for the 2nd moments

$$\frac{d\boldsymbol{\sigma}}{dt} = \widetilde{A}\boldsymbol{\sigma} + \boldsymbol{\sigma}\widetilde{A}^{\mathsf{T}} - \boldsymbol{\sigma}BB^{\mathsf{T}}\boldsymbol{\sigma} + \widetilde{D}$$

Wiseman & Doherty, PRL **94**, 070405 (2005)
MGG et al., Contemporary Physics (2016)

Time-continuous measurements

Stochastic evolution for the 1st moments

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We can use feedback in order to set the stochastic part equal to zero and obtain a completely deterministic evolution for both first and second moments

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Optimal Feedback Control

It is enough to add a "linear" term in the Hamiltonian (displacement in phase space):

$$H_{\text{fb}} = H + \hat{R}^{\mathsf{T}}FJ(t)$$

F : optimal feedback matrix

J(t) : photocurrent (outcomes of time-continuous measurement)

- linear driving of the cavity field *("easy")*
- combination of impulses and shifting of the trapping potential for the mechanical oscillator *(not so easy)*

see: Wieczorek et al., PRL **114**, 223601 (2015)

Stabilization

Definition

A pair (C, Δ) is said detectable if

$$C\mathbf{x}_\lambda \neq 0 \quad \forall \mathbf{x}_\lambda \text{ with } \mathrm{Re}[\lambda] \geq 0 \,:\, \boldsymbol{\Delta}\mathbf{x}_\lambda = \lambda \mathbf{x}_\lambda$$

$$\dot{\boldsymbol{\sigma}} = \widetilde{A}\boldsymbol{\sigma} + \boldsymbol{\sigma}\widetilde{A}^{\mathsf{T}} - \boldsymbol{\sigma}BB^{\mathsf{T}}\boldsymbol{\sigma} + \widetilde{D}$$

A steady-state solution (in the quantum case) exists

$\Updownarrow$

Wiseman & Doherty, PRL **94**, 070405 (2005)

The pair $(B, \widetilde{A})$ is detectable

Our result

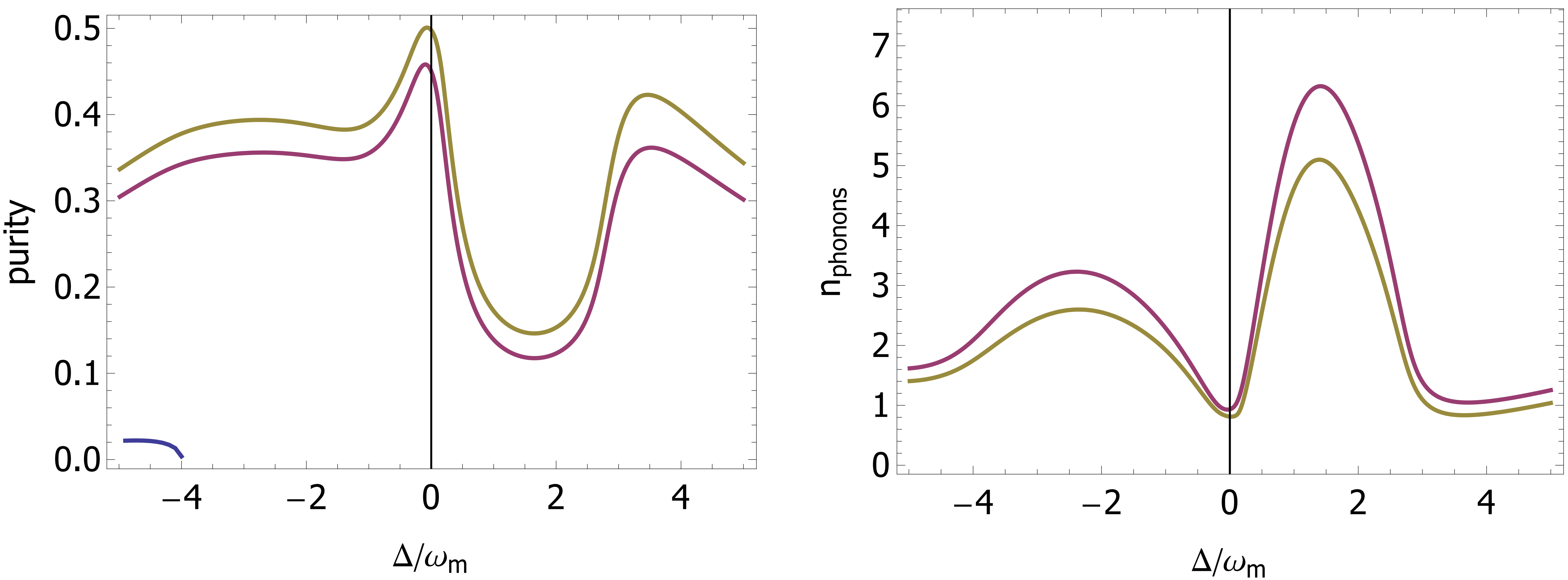
The system admits a steady-state solution for $\eta_1 \neq 0$ or $\eta_2 \neq 0$

Results:
sideband cooling + continuous measurements

Steady-state properties

homodyne measurement of the cavity field ($\eta_2 = 0,\ g = \omega_m,\ \kappa = \omega_m/3,\ \Gamma = \omega_m/10$)

$\eta_1 = 1$ $\quad$ $\eta_1 = 0.8$ $\quad$ $\eta_1 = 0$



- Great cooling enhancement via time-continuous measurement for all values of detuning (great news for experimentalists)
- Best performances near to resonance ($\Delta \approx 0$)

Results: sideband cooling + continuous measurements

Steady-state properties

homodyne measurement of the cavity field $(\,\eta_2 = 0,\; g = \omega_m,\; \kappa = \omega_m/3,\; \Gamma = \omega_m/10\,)$

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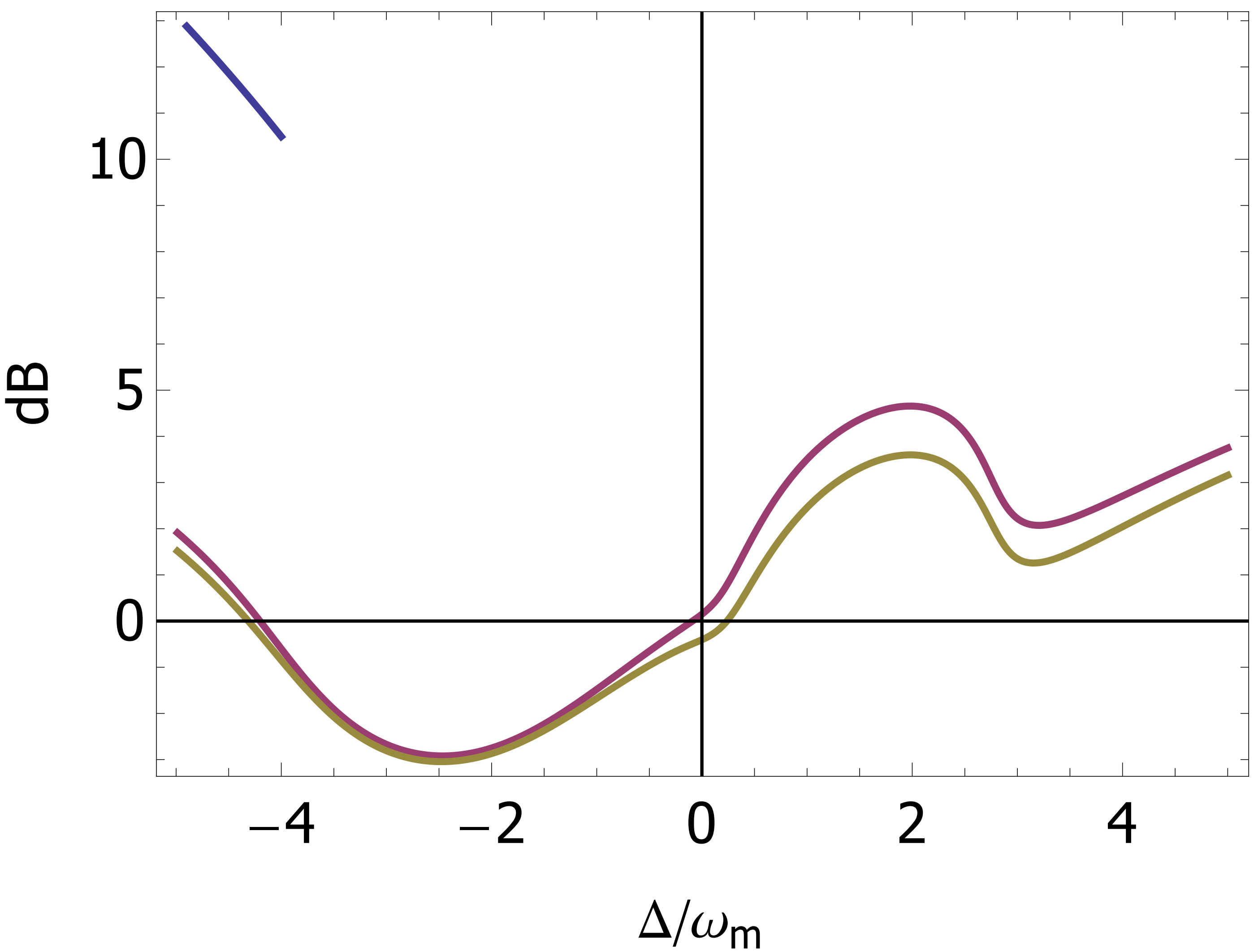


Quantum squeezing is obtained at steady state for a red-detuned cavity

3dB of squeezing are obtained for:

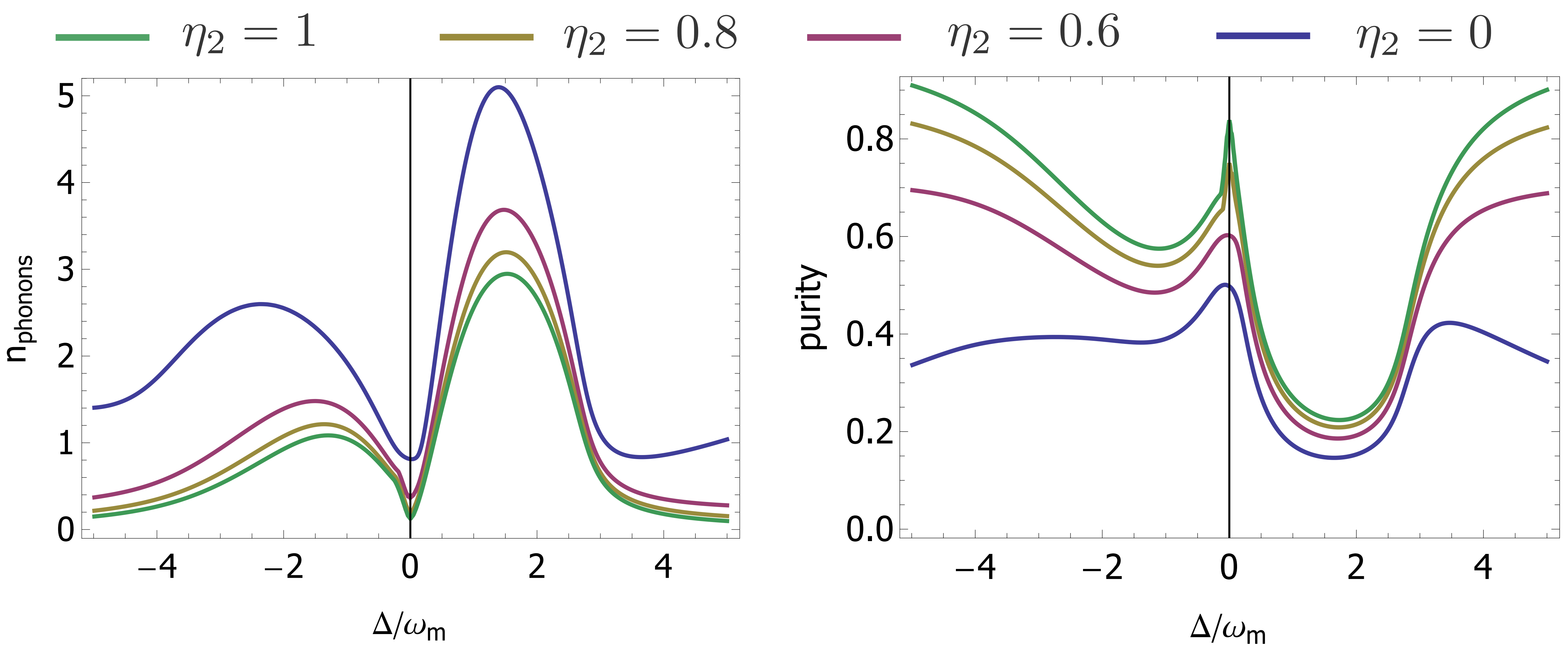
$\Delta/\omega_m \approx -2.5$

Results:
sideband cooling + continuous measurements

Steady-state properties

$(\ \eta_1 = 1, \ g = \omega_m, \ \kappa = \omega_m/3, \ \Gamma = \omega_m/10 \ )$

oscillator position + homodyne cavity field measurements



- as expected performances are simply further improved for simultaneous measurements.
- better cooling is obtained near resonance; larger squeezing for red-detuned cavity.

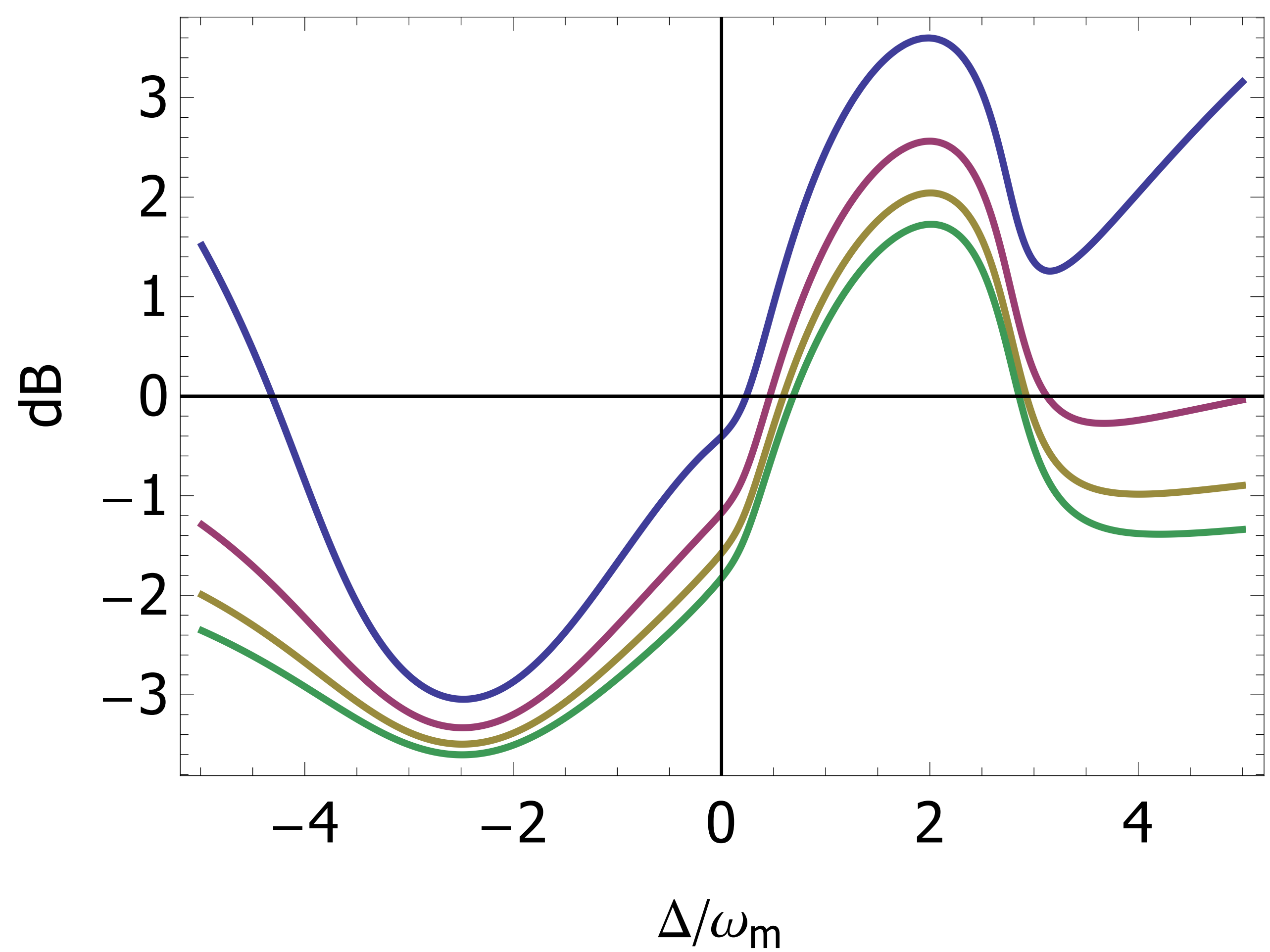
Results: sideband cooling + continuous measurements

Steady-state properties

$(\ \eta_1 = 1, \ g = \omega_m, \ \kappa = \omega_m/3, \ \Gamma = \omega_m/10 \ )$

oscillator position + homodyne cavity field measurements

$\eta_2 = 1$ $\quad \eta_2 = 0.8$ $\quad \eta_2 = 0.6$ $\quad \eta_2 = 0$



- ☑ as expected performances are simply further improved for simultaneous measurements.
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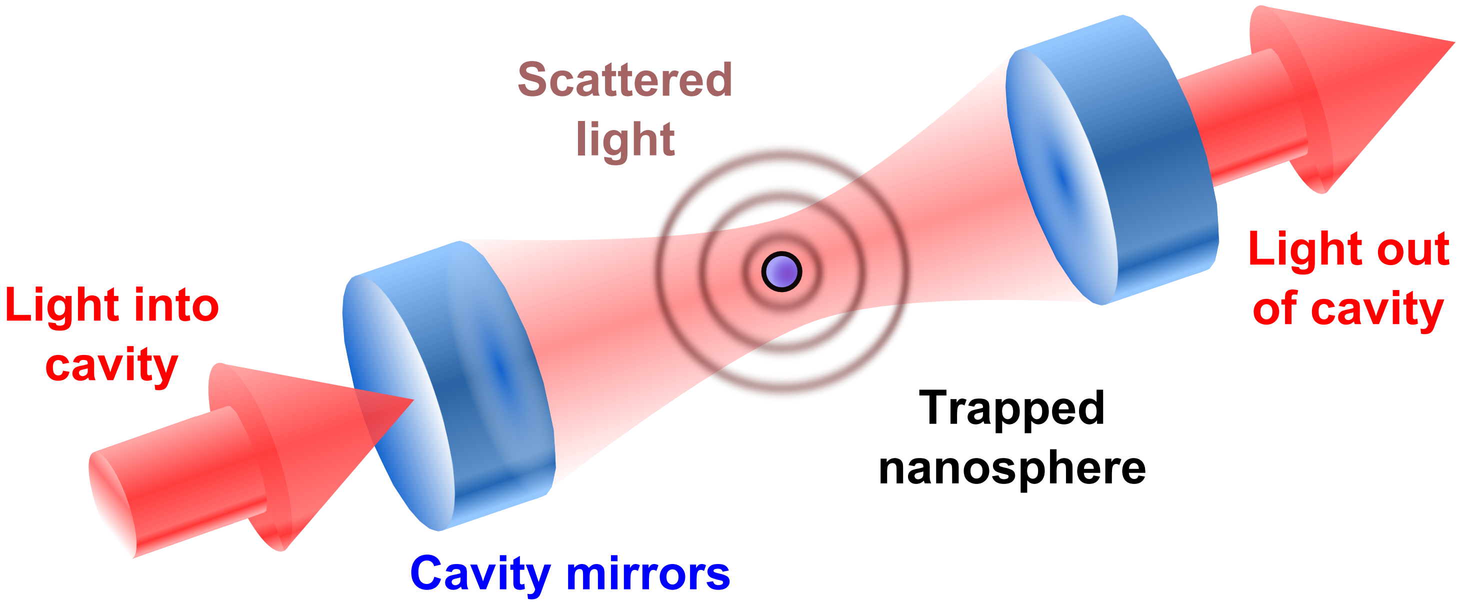
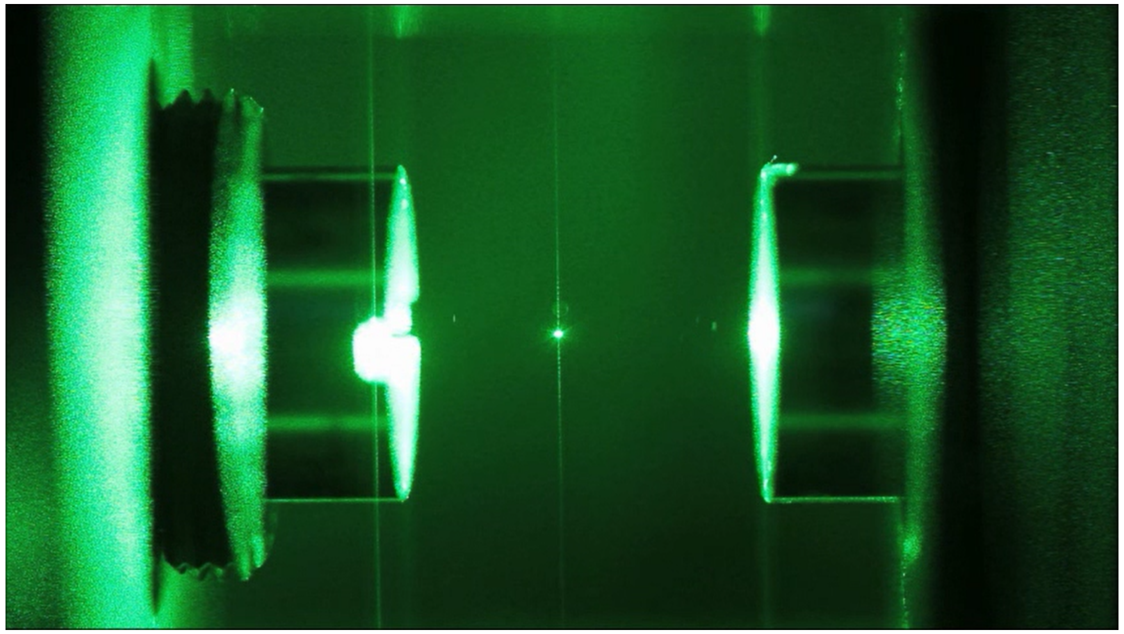
Results for UCL setup





Millen et al., PRL **114**, 123602 (2015)

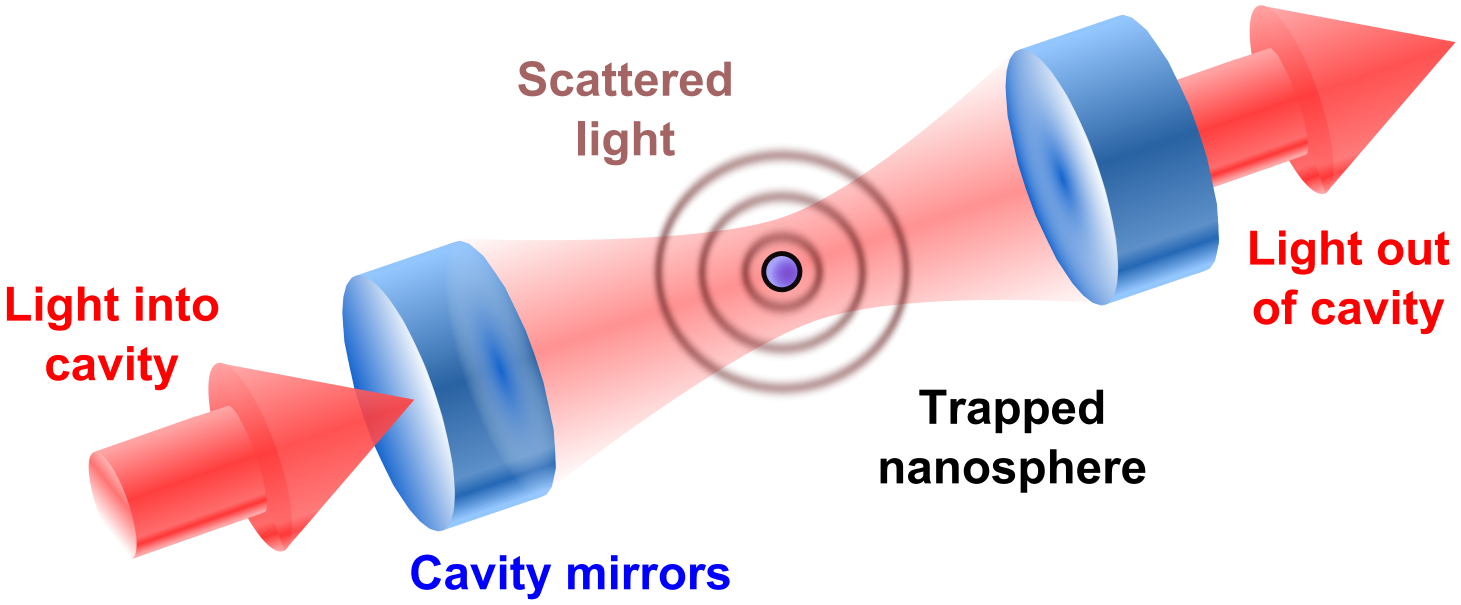
- no optical tweezers - nanosphere trapped by the cavity field only
- mechanical frequency and coupling constant depend on the number of photons in the cavity, and thus on the detuning Δ

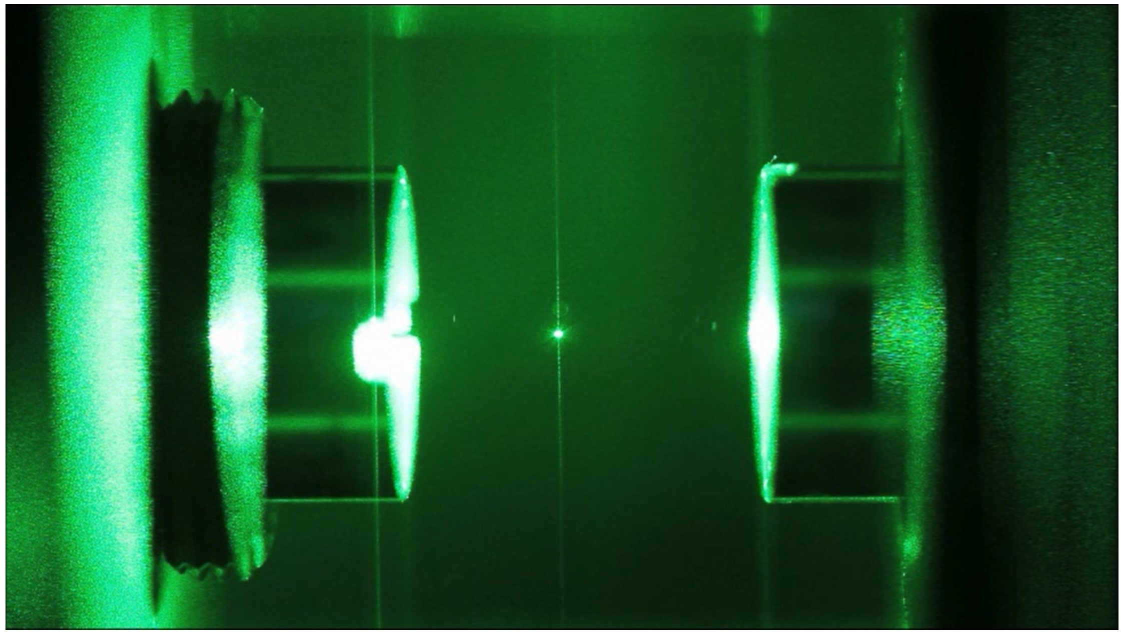
cavity photons $n_c \sim (\kappa^2/4 + \Delta^2)^{-1}$

mechanical frequency $\omega_m \sim \sqrt{n_c}$

coupling constant $g \sim \sqrt{n_c/\omega_m} \sim n_c^{1/4}$

Results for UCL setup





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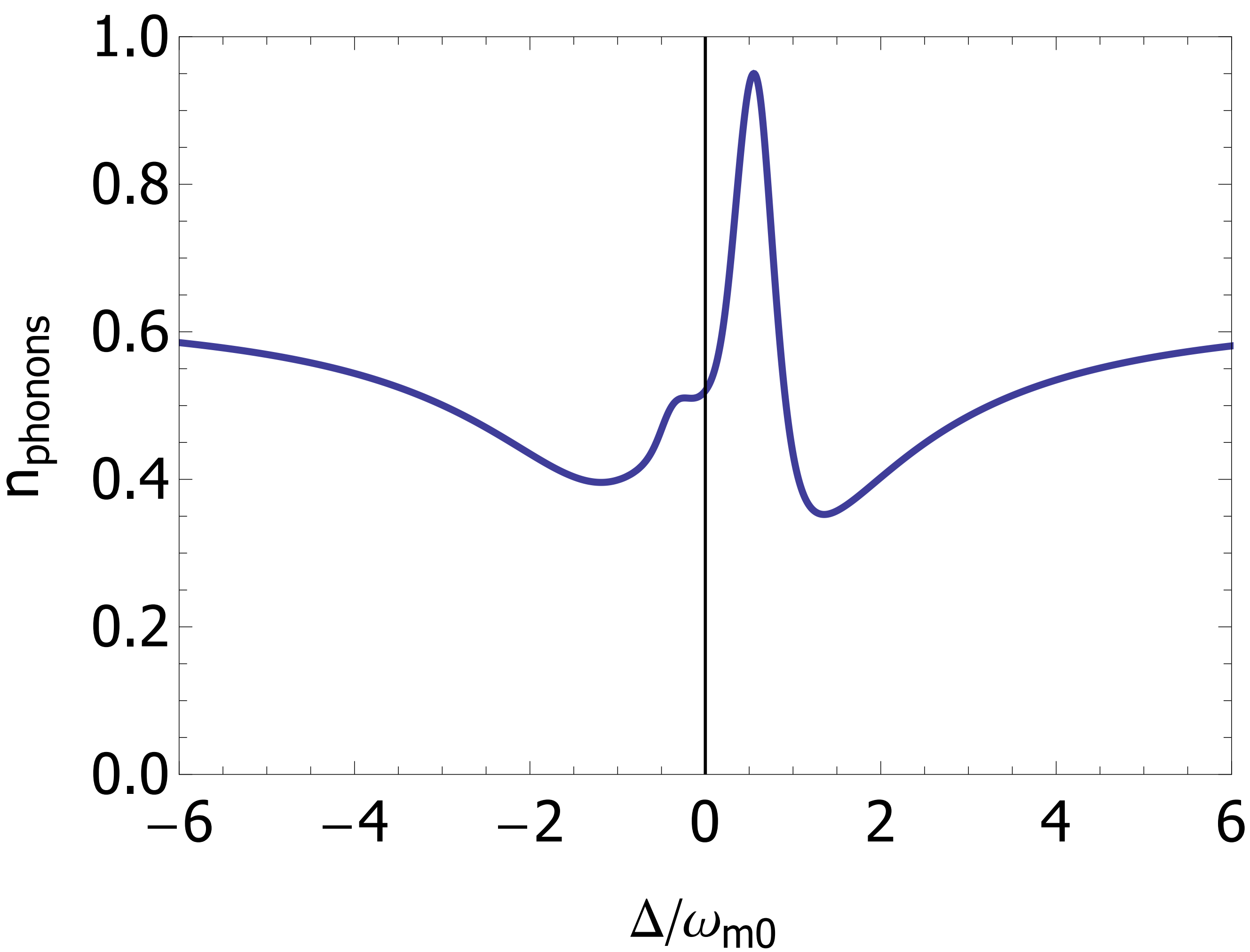
As the mechanical frequency changes with the detuning, we will focus on the actual position fluctuations (and not on the quantum squeezing), i.e.

$$\Delta X = \sqrt{\frac{\hbar\, \Delta x_m^2}{m\omega_m}} \qquad \text{where} \quad x_m = (b + b^\dagger)/\sqrt{2}$$

Results for UCL setup

Steady-state number of phonons



$$\omega_{m0}/2\pi = 48\,\mathrm{kHz}$$
$$g/2\pi = 24\,\mathrm{kHz}$$
$$\kappa/2\pi = 29\mathrm{kHz}$$
$$\Gamma = \omega_m/5$$

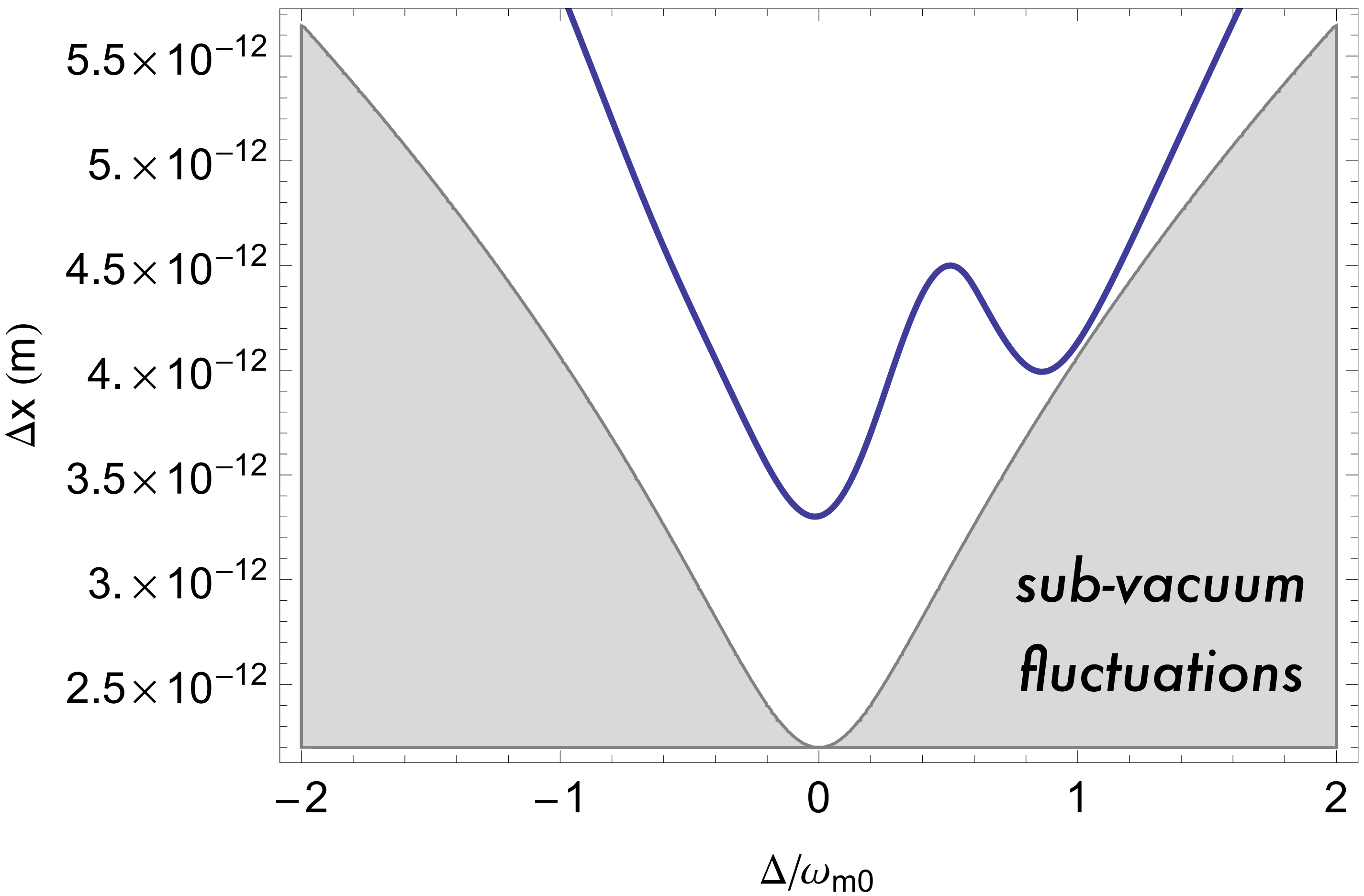
$\eta_1 = 0.9$ *homodyne efficiency*

$\eta_2 = 0.2$ *position meas. efficiency*

- ☑ less than one phonon at steady-state for all values of the detuning
- ☑ unexpectedly best cooling obtained for blue detuning $\Delta \approx 1$
- ☑ large detuning is not optimal for such a low efficiency

Results for UCL setup

Steady-state position fluctuations



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$\eta_1 = 0.9$ *homodyne efficiency*

$\eta_2 = 0.2$ *position meas. efficiency*

- ☑ smallest position fluctuations near resonance
- ☑ *almost* quantum squeezing for blue-detuned cavity ($\Delta \approx 1$)

Conclusions (part I)

- our results: sideband cooling + time-continuous measurement
  - ☑ stabilization
  - ☑ cooling
  - ☑ squeezing generation
- very good performances for nanosphere levitated by optical field in a high-finesse cavity (UCL experimental setup)

- OUTLOOKS (work in progress)
  - ☑ what if the feedback operations have some some constraints?
  - ☑ what about the entanglement between oscillator and cavity?

MGG et al., NJP **17**, 073019 (2015)

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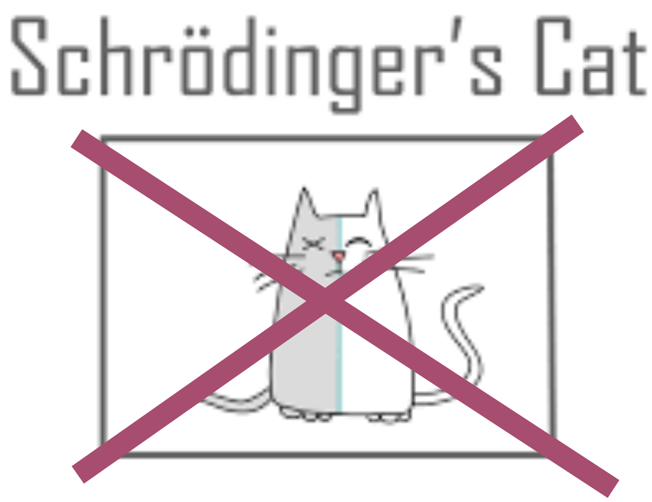
with O. S. Duarte and A. Serafini

MGG et al., arXiv:1605:?????

Collapse models

Correction to the Schroedinger equation added in order to "solve" the measurement problem and the absence of macroscopic quantum superposition, must

- be non-linear
- be stochastic
- be non unitary
- give rise to "amplification" mechanism
- not allow superluminal signaling



MODELS PROPOSED

- Ghirardi-Rimini-Weber model
- "Universal position localization"
- Continuous spontaneous localization (CSL)
- Gravity-induced collapses

NICE REVIEW!

A. Bassi et al, RMP **85**, 471 (2013)

Collapse models

How to Test These Models ?

- Test quantum superposition with matter-wave interferometry (large massive molecules)

Hornberger, RMP **84**, 157 (2012)

- Test quantum superposition with opto-mechanical systems

Romero-Isart, PRA **84**, 052121 (2011)
Pepper et al., PRL **109**, 023601 (2012)
Bateman et al., Nat. Comm. **5**, 4788 (2014)

- Non-interferometric tests with opto-mechanical systems (testing the induced decoherence)

Bahrami et al., PRL **112**, 210404 (2014)
Nimmrichter et al., PRL **113**, 020405 (2014)
Bateman et al., Nat. Comm. **5**, 4788 (2014)
Sekatski et al., PRL **112**, 080502 (2014)
Diosi, PRL **114**, 050403 (2015)
Goldwater et al., arXiv:1504.00790
Li et al., arXiv:1508.00466

CSL induced decoherence

CSL stochastic Schroedinger equation

$$d|\psi\rangle = \cdots + \left[K\sqrt{\lambda_{\text{csl}} r_C^2}\int d\mathbf{x}[M(\mathbf{x}) - \langle M(\mathbf{x})\rangle]\, dW(\mathbf{x}) - K^2\frac{\lambda_{\text{csl}} r_C^2}{2}\int d\mathbf{x}[M(\mathbf{x}) - \langle M(\mathbf{x})\rangle]^2\, dt\right]|\psi\rangle$$

λ_{csl} collapse rate

r_C width of localization process = 100nm

$M(\mathbf{x})$ “smeared” mass density operator

$dW(\mathbf{x})$ independent Wiener increments

Master-equation for the density operator under CSL (after stochastic average and for small oscillations compared to r_C)

$$\frac{d\varrho}{dt} = \Gamma_{\text{csl}}\mathcal{D}[\hat{x}]\varrho$$

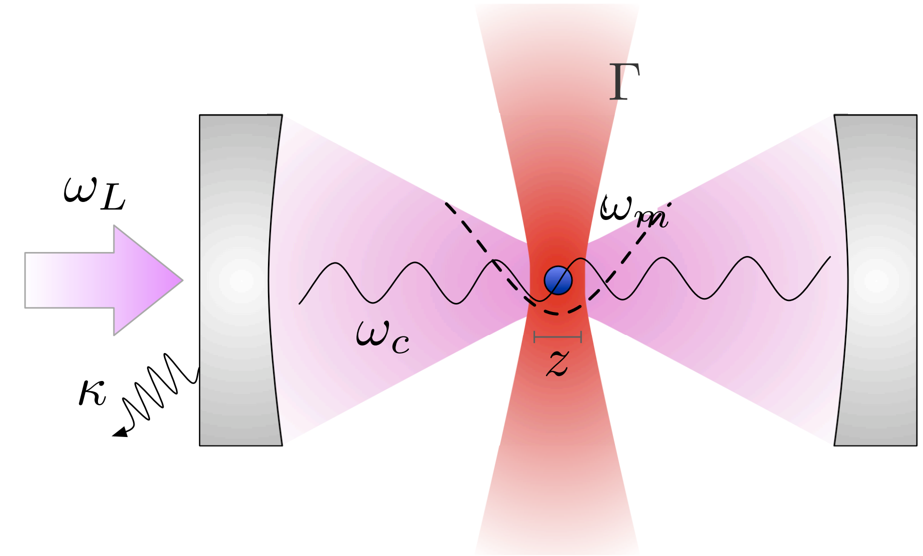
INDUCED MOMENTUM DIFFUSION

$$\Gamma_{\text{csl}} = \frac{\hbar}{m\omega_m}\frac{\lambda_{\text{csl}}}{r_c^2}\alpha$$

Nimmrichter et al., PRL **113**, 020405 (2014)

Levitated opto-mechanics under CSL

Reduced dynamics for the levitating nanosphere
(no cavity field or large detuning regime)



$$\dot{\varrho} = i[\varrho, \hat{H}_0] + (\Gamma_{\mathsf{env}} + \Gamma_{\mathsf{csl}})\mathcal{D}[\hat{x}]\varrho$$

$$\hat{H}_0 = \frac{\hbar\omega_m}{2}\left(\hat{x}^2 + \hat{p}^2\right)$$

Γ_{env} environmental decoherence (photon scattering, but also background gas collision and blackbody radiation)

Γ_{csl} fundamental (CSL induced) decoherence

How to better estimate/discriminate the CSL-induced decoherence ?

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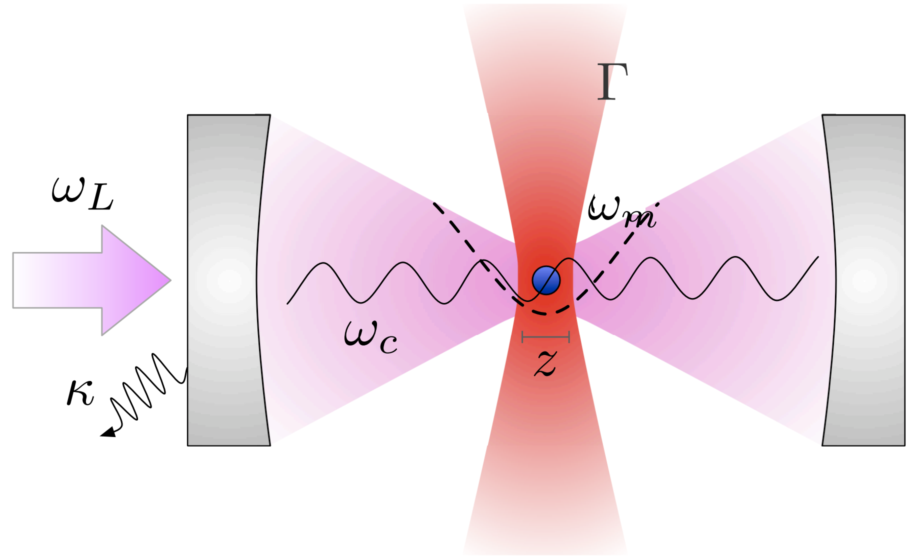

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How to better estimate/discriminate the CSL-induced decoherence ?

IDEA:
The environment can be, in principle, continuously monitored and thus one can *neutralize* the environmental decoherence, while CSL decoherence is unavoidable

CSL-decoherence estimation enhanced by continuous monitoring

Stochastic Master equation under continuous monitoring

$$d\varrho = i[\varrho, \hat{H}_0]\, dt + (\Gamma_{\text{env}} + \Gamma_{\text{csl}})\mathcal{D}[\hat{x}]\varrho\, dt + \sqrt{\Gamma_{\text{env}}\eta}\mathcal{H}[\hat{x}]\varrho\, dW$$

...as discussed before, thanks to monitoring and linear Markovian feedback, we obtain a deterministic (Gaussian) steady-state and we can investigate its properties...

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QUANTUM ESTIMATION THEORY

Quantum and Classical Cramér-Rao Bound

$$\mathrm{Var}(\Gamma_{\mathsf{csl}}) \geq \frac{1}{MF(\Gamma_{\mathsf{csl}})} \geq \frac{1}{MH(\Gamma_{\mathsf{csl}})}$$

M = nr of measurements

$F(\Gamma_{\mathsf{csl}})$ classical Fisher Information (FI) *(depends on the measurement)*

$H(\Gamma_{\mathsf{csl}})$ Quantum Fisher Information (QFI)

- *The QFI quantifies the ultimate precision we can get in the estimation of the parameter.*
- *There is always an optimal measurement (POVM) saturating the Quantum Cramér-Rao Bound*
- *We can "easily" evaluate analytically the QFI for Gaussian states*

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Stochastic evolution for the 1st moments

$$d\langle\hat{R}\rangle_c = A\langle\hat{R}\rangle_c\, dt + (\ldots\ldots)d\mathbf{w}$$

stochastic part can be set to zero via linear Markovian feedback

Deterministic evolution for the 2nd moments

$$\frac{d\boldsymbol{\sigma}}{dt} = \widetilde{A}\boldsymbol{\sigma} + \boldsymbol{\sigma}\widetilde{A}^{\mathsf{T}} - \boldsymbol{\sigma}BB^{\mathsf{T}}\boldsymbol{\sigma} + \widetilde{D}$$

PROBLEM: the optimal feedback depends on the parameter we want to estimate

SOLUTION: Multi-step adaptive method

e.g.: see monday's talk by Ullrik

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CSL-decoherence estimation enhanced by continuous monitoring

RESULTS QFI

$$H(\Gamma_{\text{csl}}) = \frac{3 + \frac{4\Gamma_{\text{env}}}{\Gamma_{\text{csl}}} - \frac{\omega_m}{\sqrt{4\Gamma_{\text{env}}(\omega_m^2 + \Gamma_{\text{env}} + \Gamma_{\text{csl}})}}}{8(\Gamma_{\text{env}} + \Gamma_{\text{csl}})(2\Gamma_{\text{env}} + \Gamma_{\text{csl}})}$$

Perfect monitoring $(\eta = 1)$

$$H(\Gamma_{\text{csl}} = 0) = \frac{(3+\eta)\sqrt{\omega_m^2 + 4\Gamma_{\text{env}}^2 \eta} - \omega_m(1-\eta)}{8\Gamma_{\text{env}}^2(1-\eta^2)\sqrt{\omega_m^2 + 4\Gamma_{\text{env}}^2\eta}}$$

No Collapses

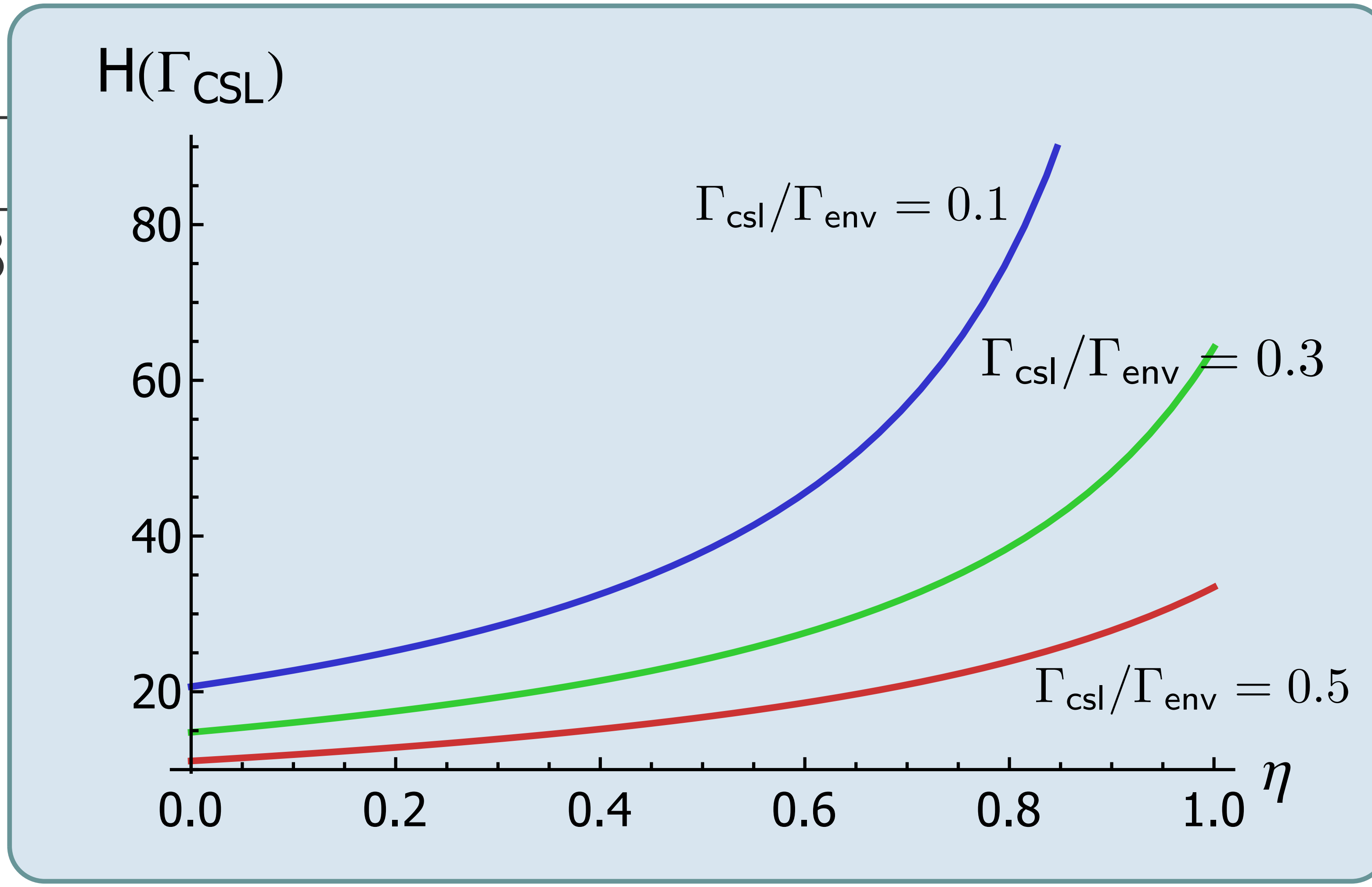
- ☑ QFI diverges (i.e. estimation with “infinite”precision) for perfect monitoring and no collapses ($\eta = 1$ and $\Gamma_{\text{csl}} = 0$)
- ☑ QFI increases monotonically with monitoring efficiency η
- ☑ QFI decreases monotonically with the collapse parameter Γ_{csl}

CSL-decoherence estimation enhanced by continuous monitoring

RESULTS QFI

$H(\Gamma_{csl}) = \frac{3 + \cdots}{8 \cdots}$ …g ($\eta = 1$)

$H(\Gamma_{csl} = 0) =$ …pses



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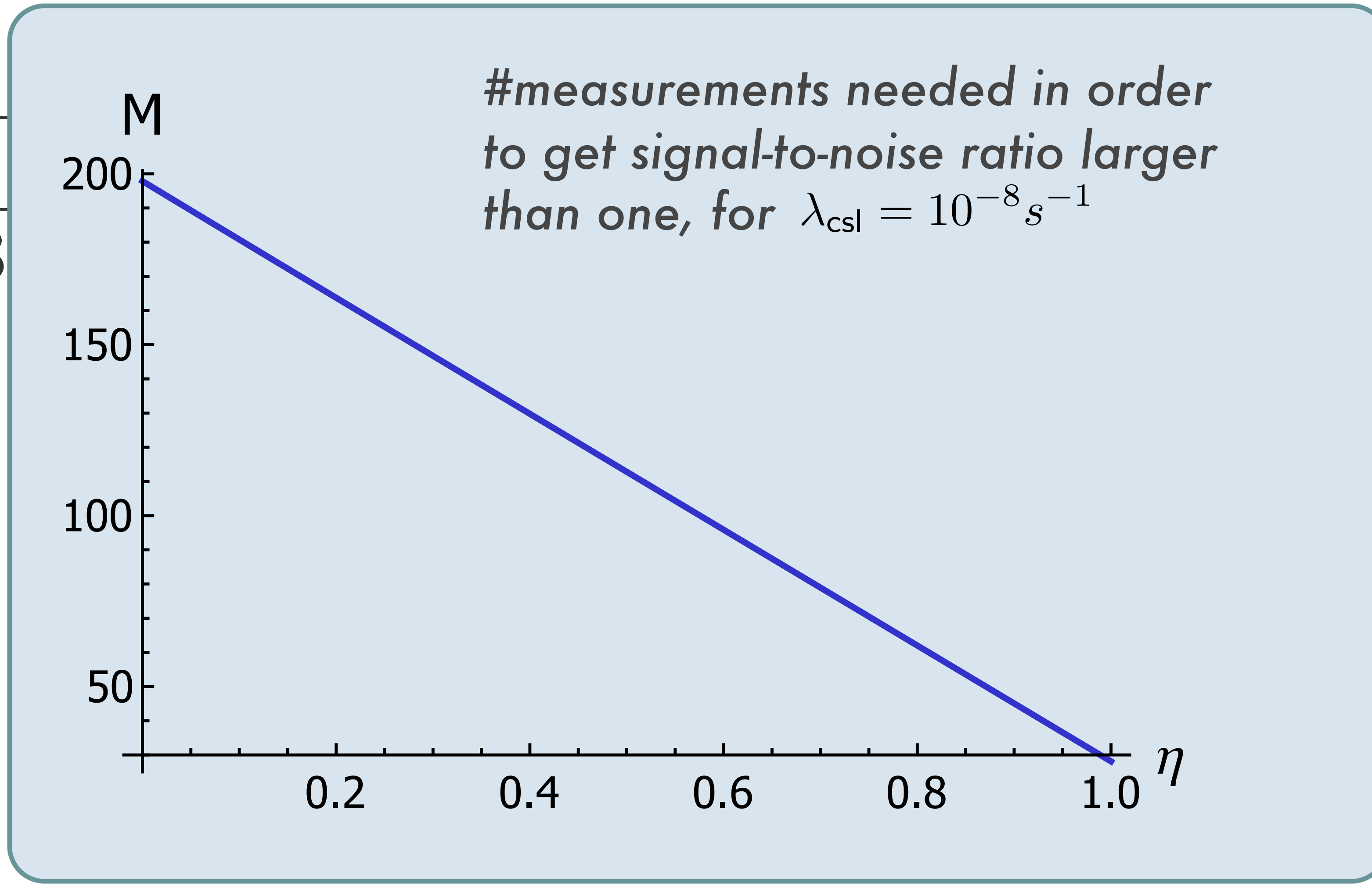

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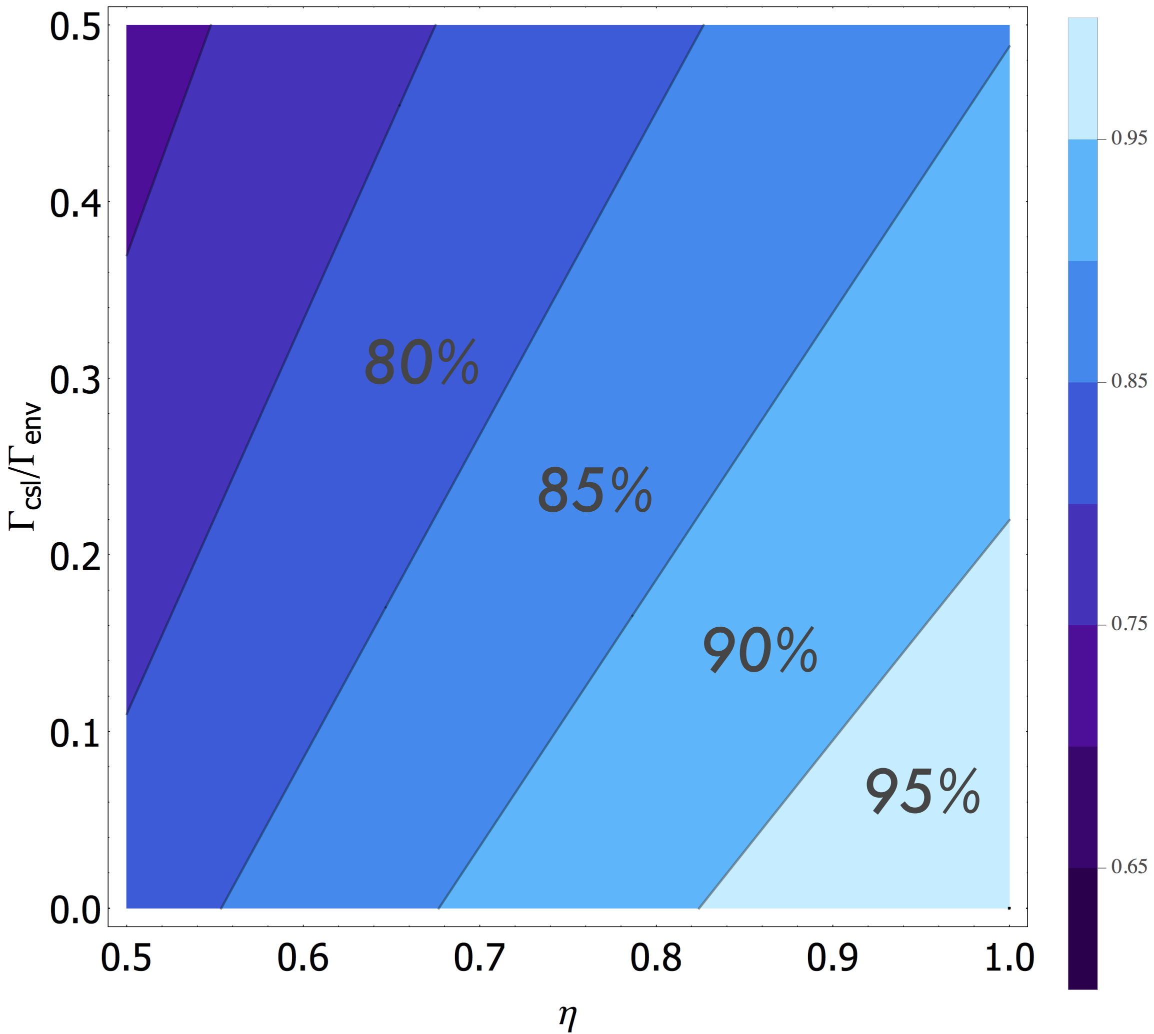
CSL-decoherence estimation enhanced by continuous monitoring

OPTIMAL MEASUREMENT

In the limit of perfect monitoring and no collapses, we are able to identify the optimal measurement that saturates the Quantum Cramér-Rao bound:

$$\Pi_0 = |\psi_{\mathrm{ss}}\rangle\langle\psi_{\mathrm{ss}}| \,,$$
$$\Pi_1 = \mathbb{1} - \Pi_0 \,.$$

DICHOTOMIC MEASUREMENT
(projection on the steady-state itself)



RATIO classical FI vs QFI

The measurement is still performing very well (ratio around 80%) also for smaller values of the monitoring efficiency and for larger values of the CSL-induced decoherence

Conclusions (part II)

- Monitoring the environment greatly enhance the possibility of estimating the decoherence induced by spontaneous collapses.
- In the limit of perfect monitoring and no collapses
  - ☑ QFI diverges, i.e. in principle absolute precision
  - ☑ Optimal dichotomic measurement corresponding to projection on the steady-state itself
- Optimal performances also in a different parameters regime

- OUTLOOKS (work in progress)
  - ☑ what about continuous monitoring of the cavity field?
  - ☑ what about more practical final measurements?

MGG et al., arXiv:1605:?????

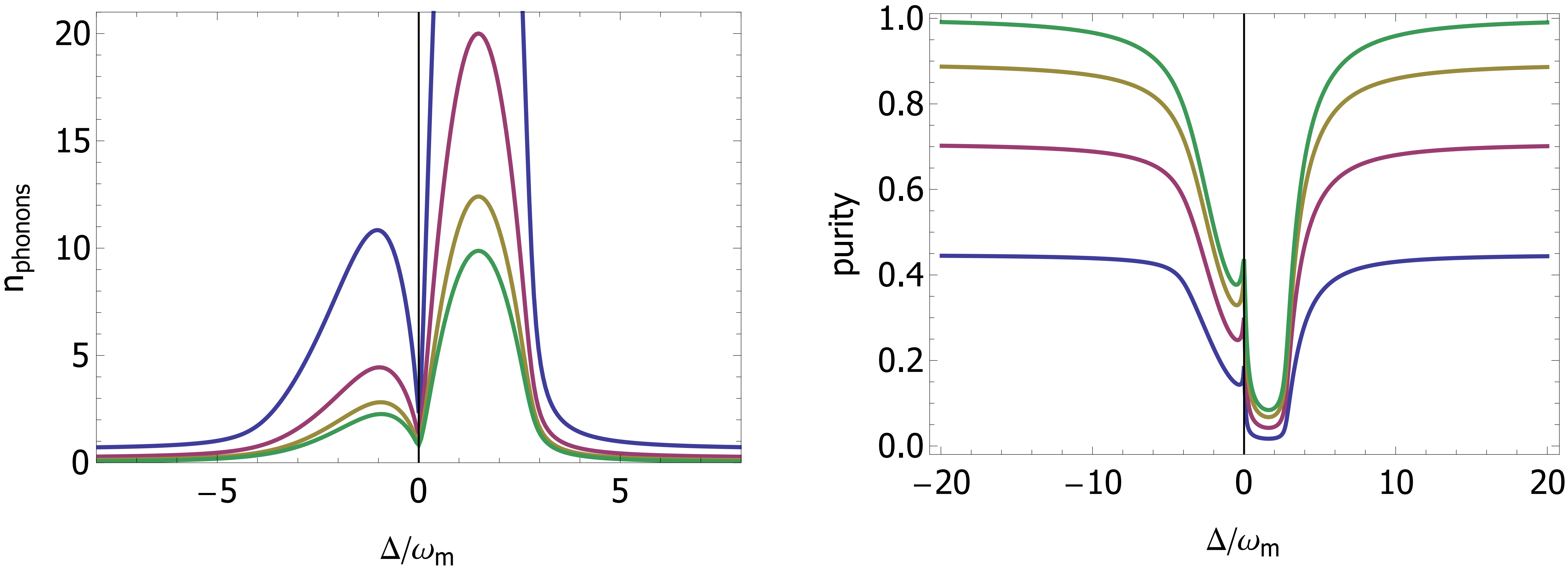
CSL-decoherence estimation enhanced by continuous monitoring

Results:
sideband cooling + continuous measurements

Steady-state properties

oscillator position measurement ($\eta_1 = 0,\ g = \omega_m,\ \kappa = \omega_m/3,\ \Gamma = \omega_m/10$)

$\eta_2 = 1$ $\eta_2 = 0.8$ $\eta_2 = 0.6$ $\eta_2 = 0.2$



- ☑ good performances in the case of large detuning (where the oscillator is practically decoupled from the cavity!)
- ☑ It is however difficult to obtain such large efficiencies for position measurement (light scattered in all the directions)

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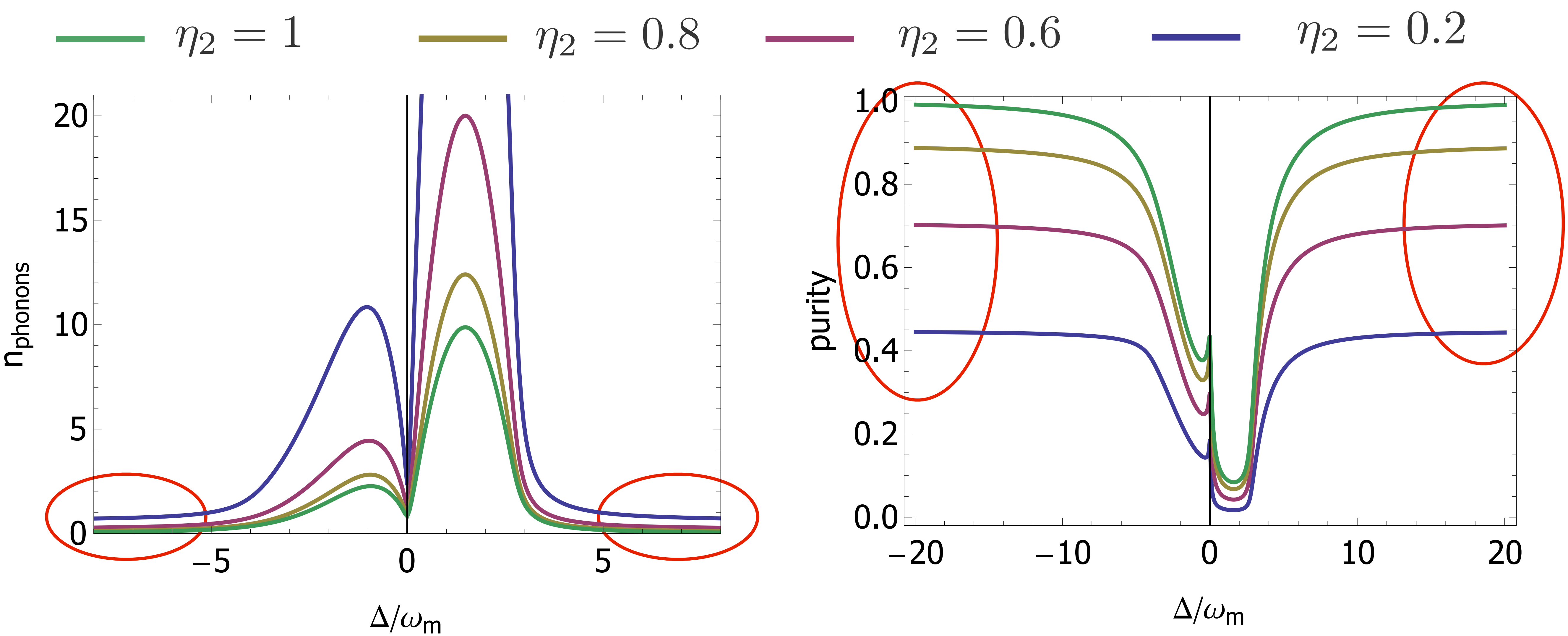

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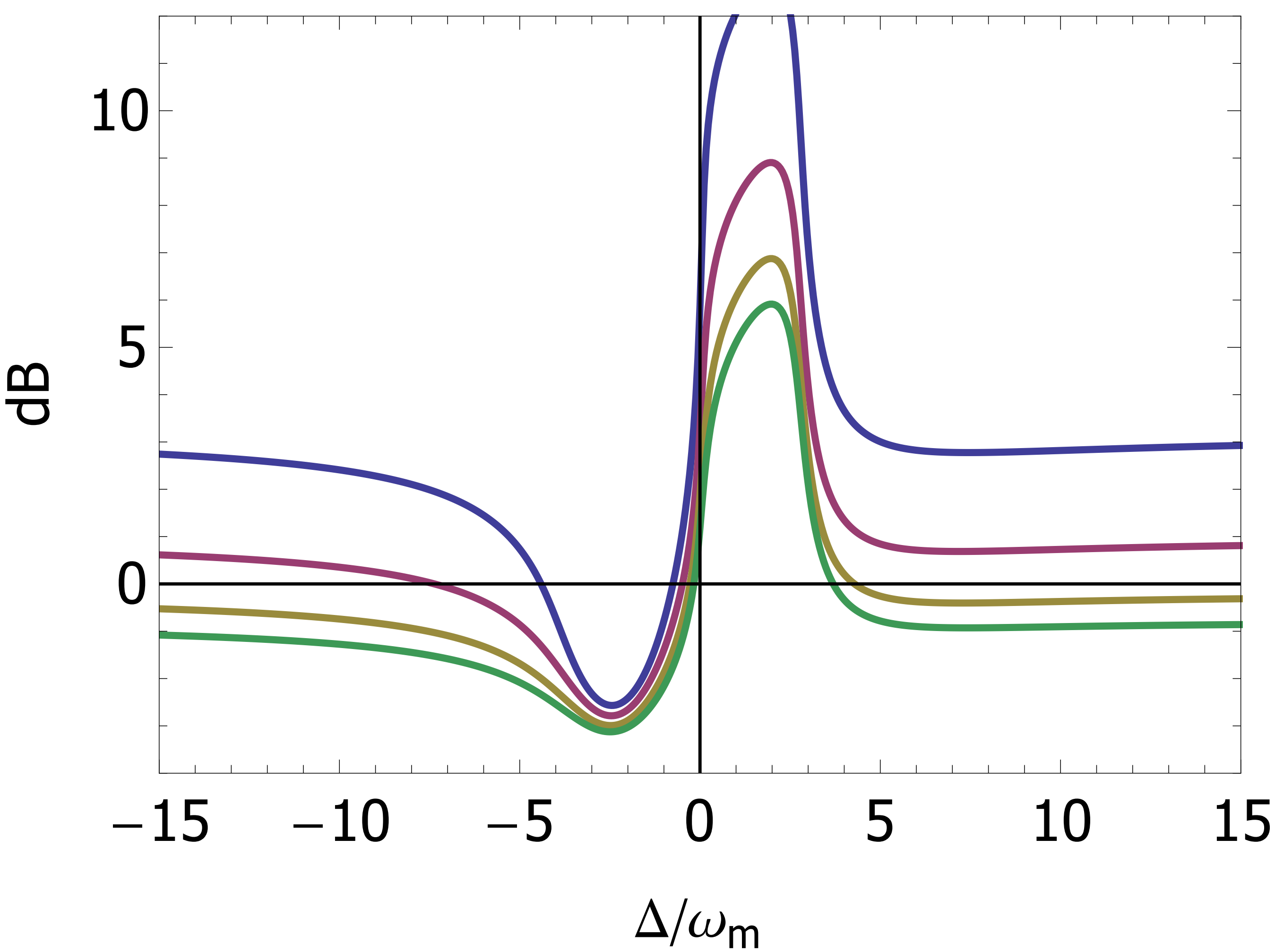
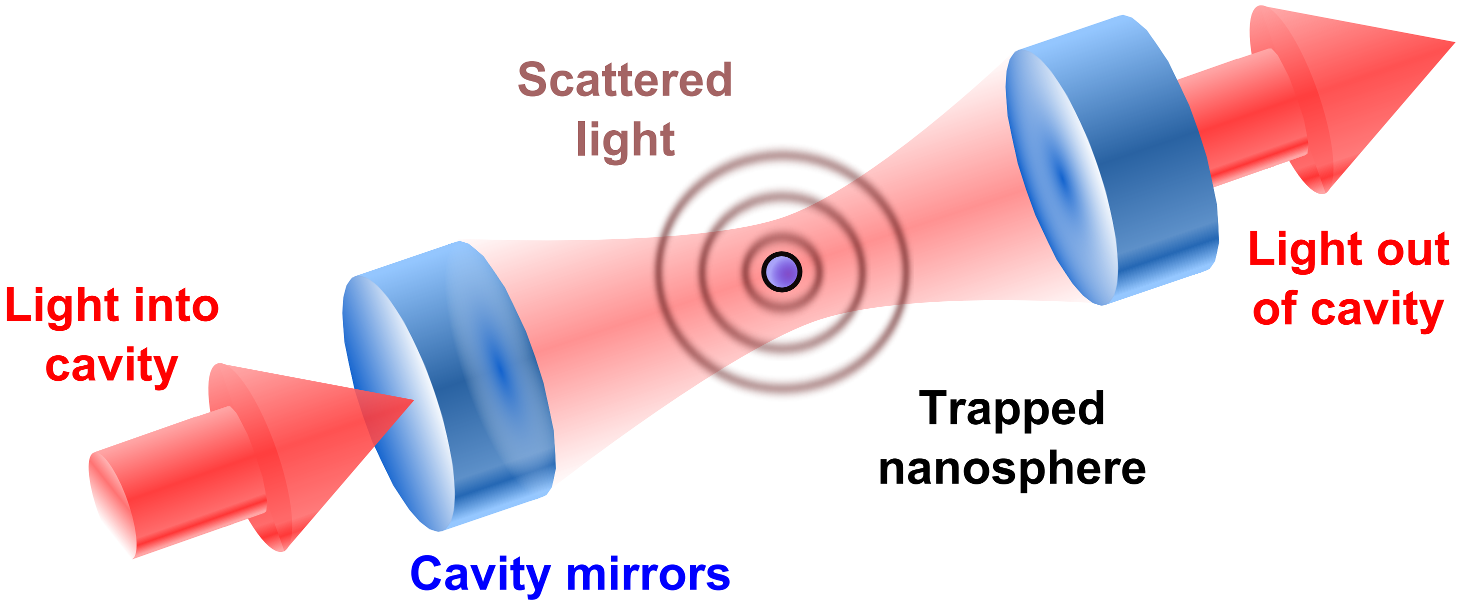


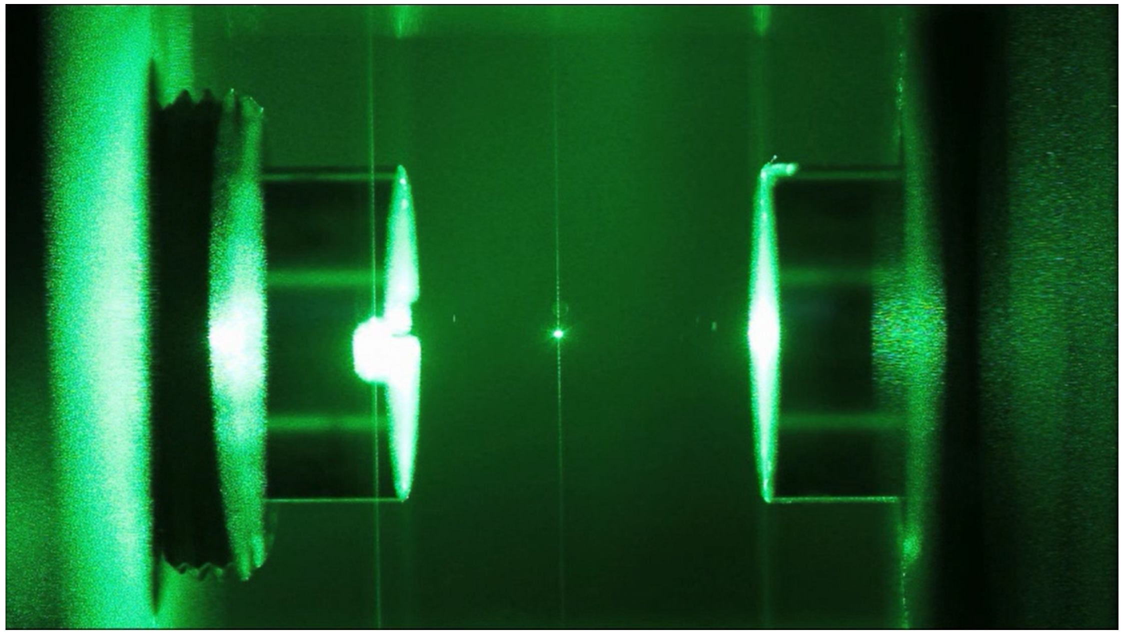
no squeezing for large detuning and low efficiency

larger quantum squeezing is still obtained for finite (negative) detuning

almost 4dB of squeezing for: $\Delta/\omega_m \approx -2.5$

Results for UCL setup





$$\omega_{m0}/2\pi = 48\,\mathrm{kHz}$$ *mech. frequency*

$$g/2\pi = 24\,\mathrm{kHz}$$ *coupling constant*

$$\omega_c/2\pi = 2.8 \cdot 10^{14}\ \mathrm{Hz}$$ *cavity frequency*

$$\kappa/2\pi = 29\mathrm{kHz}$$ *cavity loss*

$$\Gamma = \omega_m/5$$ *mechanical decoherence*

$$r = 200\,\mathrm{nm}$$ *nanosphere radius*

$$m = 7.35 \cdot 10^{-17}\ \mathrm{Kg}$$ *nanosphere mass*

$$L = 13\,\mathrm{mm}$$ *cavity length*

$$\mathcal{F} = 400000$$ *cavity finesse*