IRREVERSIBLE ENTROPY PRODUCTION IN QUANTM SYSTEMS OUT OF EQUILIBRIUM

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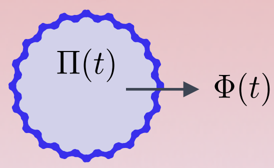
ABSTRACT

Finite-time transformations necessarily result in some production of entropy, which signals the irreversible nature of the process. Its quantification is fundamental for the characterisation of non-equilibrium processes. However, irreversible entropy production arising from quantum dynamics in mesoscopic quantum systems has not been experimentally investigated so far. We obtain an expression of the entropy production rate for the open dynamics of linearly coupled quantum oscillators. Furthermore, we test our result by measuring the rate of entropy produced by an open quantum system in a non-equilibrium steady state for two different experimental setups: a micro-mechanical resonator and a Bose-Einstein condensate. Each of them is coupled to a high finesse cavity and hence subject also to optical losses. Key features of our setups, such as the cooling of the mechanical resonator and signatures of a structural quantum phase transition in the condensate are reflected in the entropy production rates.

ENTROPY PRODUCTION

✶ Second Law: $\Delta S \geq \int \frac{\delta Q^{\text{in}}}{T}$



$$\frac{\mathrm{d}S}{\mathrm{d}t} = \Pi - \Phi$$

$\Pi(t)$ entropy production rate

$\Phi(t)$ entropy flux (outgoing)

✶ Non equilibrium steady state: $\frac{\mathrm{d}S}{\mathrm{d}t} = 0 \;\Rightarrow\; \Pi_s = \Phi_s$

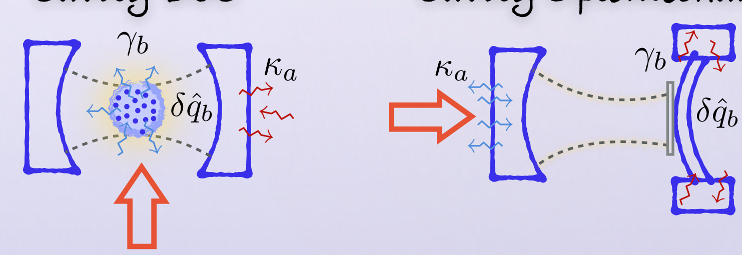
✶ For stochastic dynamics non-negativity of the entropy production rate follows from fluctuation theorem:

$$\langle \exp(-\Pi) \rangle = 1 \;\Rightarrow\; \langle \Pi \rangle \geq 0$$

✶ Nanoscopic/mesoscopic devices operating in the quantum regime: How do quantum fluctuations contribute to the production of entropy?

THE SYSTEM

Cavity BEC / Cavity Optomechanics

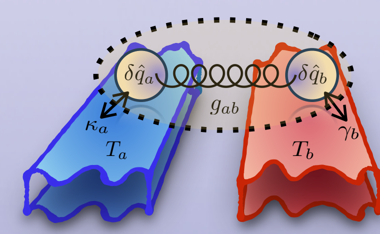


Mean field + fluctuations: Linearly coupled quantum oscillators

$$\hat{H} = \frac{\hbar\omega_a}{2}(\delta\hat{q}_a^2 + \delta\hat{p}_a^2) + \frac{\hbar\omega_b}{2}(\delta\hat{q}_b^2 + \delta\hat{p}_b^2) + \hbar g_{ab}\delta\hat{q}_a\delta\hat{q}_b,$$

Optical mode: $\hat{a},\ \omega_a$

Atomic/mechanical mode: $\hat{b},\ \omega_b$



$n_{T_a} = 0 \qquad n_{T_b} = (e^{\hbar\omega_b/k_B T_b} - 1)^{-1}$

THEORY

Main Ingredients:

✶ Quadratic Hamiltonian: Gaussian distribution in phase space

$$S(t) = \int \mathcal{W}(u,t)\log\mathcal{W}(u,t)du$$

We apply tools from classical stochastic processes to quantum fluctuations

✶ Symmetry of dynamical variables under time reversal:

$$f(u,t) = f^{\text{irr}}(u,t) + f^{\text{rev}}(u,t) \qquad f^{\text{rev}}(u,t) = -Ef^{\text{rev}}(Eu,t) \qquad f^{\text{irr}}(u,t) = Ef^{\text{irr}}(Eu,t)$$

Rate of irreversible entropy production of the system at the steady state

$$\boxed{\Pi_s = \underbrace{2\gamma_b\left(\frac{n_b + 1/2}{n_{T_b} + 1/2} - 1\right)}_{\mu_b} + \underbrace{4\kappa_a n_a}_{\mu_a}}$$

$n_b = \langle \delta\hat{b}^\dagger\delta\hat{b}\rangle_s$

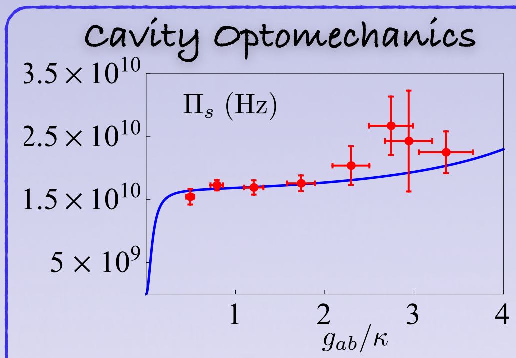
$n_a = \langle \delta\hat{a}^\dagger\delta\hat{a}\rangle_s$

Atomic/mechanical contribution μ_b — μ_a Optical contribution

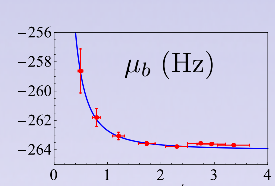
MEASUREMENTS

	$\omega_a/2\pi$ [MHz]	$\omega_b/2\pi$ [kHz]	$\gamma_b/2\pi$ [Hz]	$\kappa_a/2\pi$ [kHz]	λ_{p} [nm]	T [°K]	Other parameters
Optomechanics	1.27815	1278.15	264.1	435.849	1064	292	$m = 176$ ng
Cavity-BEC	15.13	8.3	see SI	1250	785.3	38×10^{-9}	$N = 10^5$

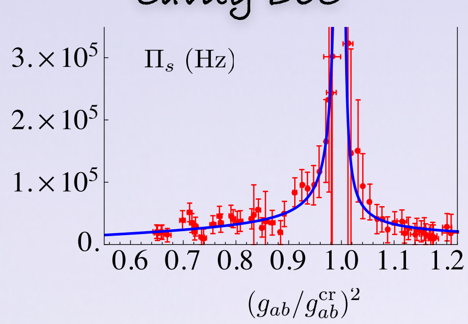
Cavity Optomechanics



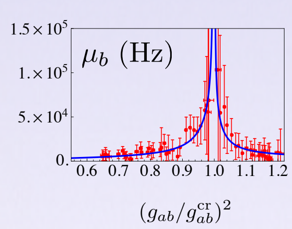
Mechanical contribution to Π_s:



Cavity BEC



Atomic contribution to Π_s:



Experimental assessment of the irreversible entropy production rate Π_s as a function of the coupling parameter. Blue lines show the theoretical predictions based on the values given in the table. Red dots show the experimental data. In the upper panel, the vertical error bars report statistical errors extracted from the fit, while the horizontal ones show experimental error on the values of the parameter. In the lower panel, the vertical and horizontal error bars report the statistical errors from the fit and the determination of the critical point, respectively.

FUTURE WORKS

✶ Entropy production in the time-resolved domain;

✶ Entropy production and bipartite correlations.

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