Nonclassical-State Generation in Macroscopic Systems [1]

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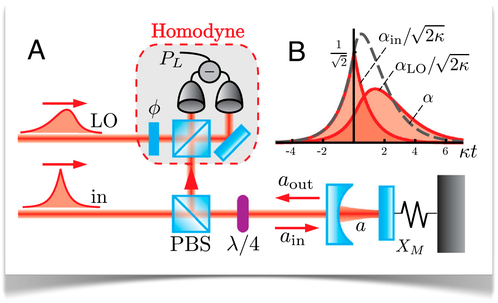
1. Introduction

• Nonclassical-state generation, e.g., the generation of Schrödinger cat states, is an important component throughout experimental quantum science for quantum information applications to probing the fundamentals of physics.

> Man kann auch ganz burleske Fälle konstruieren. Eine Katze wird in eine Stahlkammer gesperrt, zusammen mit folgender Höllenmaschine [2]

• One of the key tools currently investigated for quantum-state generation is quantum measurement, e.g., quadrature measurement in quantum optics, whereby a quantum state is conditionally created by a combination of Bayesian inference and quantum backaction.



[3]

• We investigate permutations of quantum nondemolition quadrature measurements and single-quanta addition or subtraction [4] to prepare superposition states in bosonic systems.

2. Idea

• Our idea is illustrated by the following simple intuition. A low-occupation thermal state has a Gaussian Wigner function. Single-quanta addition burns a hole in the centre of this distribution. Quadrature measurement about the origin then cuts a slice that resembles a cat state—a 'kitten' state.

• In the literature, single-quanta subtraction and squeezing has been proposed to create approximations to cat states. Our goal is to compare various permutations of quadrature measurement or squeezing and single-quanta addition or subtraction to determine which creates the 'most' non-classical state and which 'best' approximates a cat state.

• Experiment: two systems wherein our protocols are easily implementable are optomechanics [3, 4] and atomic spin ensembles.

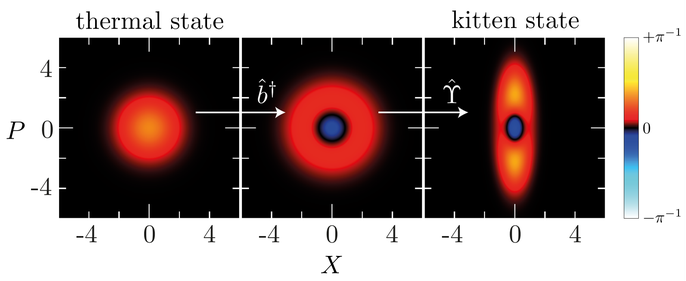


FIG. 1. Example quantum-state generation protocol.

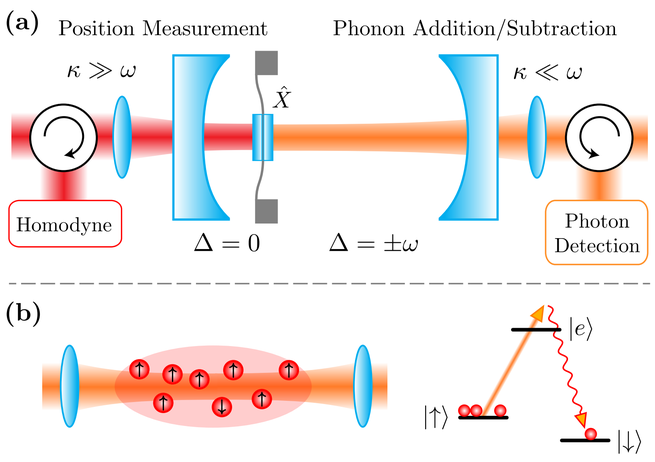


FIG. 2. Example systems that can implement our protocols. (a) An optomechanical system: on the left is a short cavity for quadrature measurement; on the right a long cavity for single-quanta addition or subtraction. (b) An atomic spin ensemble: quadrature measurement is achieved via polarization-based homodyne; single-quanta addition or subtraction via single-photon detection.

3. Numerical Examples

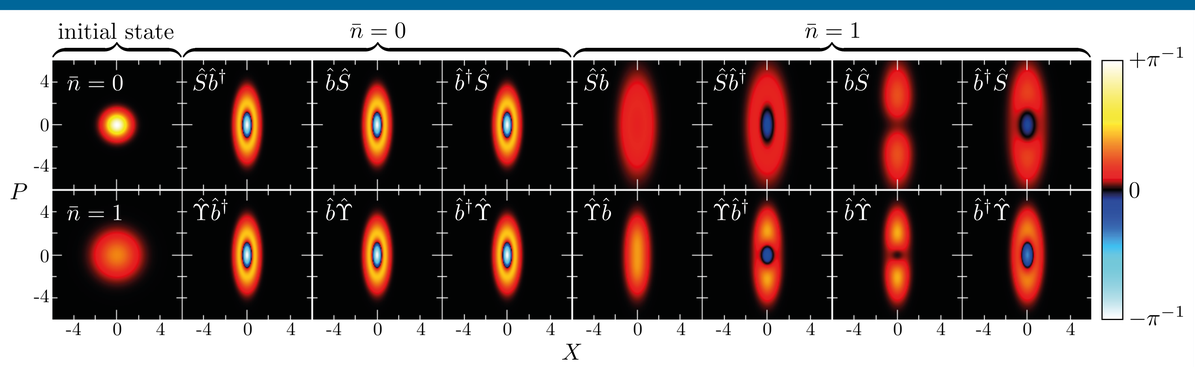


FIG. 3. Plots of example Wigner functions of states generated by our protocols.

4. Mathematical Description

• All our protocols are constructed from four basic operations:

$$\underbrace{\hat{b}^\dagger}_{\text{single-quanta addition}} \qquad \underbrace{\hat{b}}_{\text{single-quanta subtraction}}$$

$$\underbrace{\hat{\Upsilon} = \pi^{-1/4}\exp[-\tfrac{1}{2}(P_{\rm L} - \chi\hat{X})^2]}_{\text{quadrature measurement}} \qquad \underbrace{\hat{S} = \exp[\tfrac{1}{2}r(\hat{b}^2 - \hat{b}^{\dagger 2})]}_{\text{squeezing}}$$

We analysie the eight permutations of these operations (shown in Fig. 3).

• All eight protocols may be written in the following general forms:

$$\rho_{\hat{S}} \propto \hat{S}(r)(\nu\hat{b} + \mu\hat{b}^\dagger)\circ\rho_{\bar{n}} \qquad \rho_{\hat{\Upsilon}} \propto \hat{S}(\xi)\hat{D}(\zeta P_{\rm L})(\nu\hat{b} + \mu\hat{b}^\dagger + \lambda P_{\rm L})\circ\rho_{\bar{m}}$$

where n.b. $\bar{m} \leq \bar{n}$! In order for a comparison to be made we set $\xi = r$.

• The tool we use to describe the state is the Cahill function:

$$R_\tau(\beta) = \frac{1}{\pi\tau}\int d^2\beta' \exp(-|\beta-\beta'|^2/\tau)P(\beta') \quad \text{where} \quad \rho = \int d^2\beta P(\beta)|\beta\rangle\langle\beta|$$

$$\underbrace{P = R_{\tau=0}}_{\text{Glauber-Sudarshan function}} \qquad \underbrace{W = R_{\tau=1/2}}_{\text{Wigner function}} \qquad \underbrace{Q = R_{\tau=1}}_{\text{Husimi function}}$$

• In order to quantify the protocols we use three Figures of merit [5, 6]:

$$\underbrace{\delta = \frac{1}{2}\left(\int d^2\beta|W(\beta)| - 1\right)}_{\text{Wigner negativity}} \qquad \underbrace{\tau_{\rm inf} = \inf\{\tau : R_\tau \text{ is classical}\}}_{\text{nonclassical depth}}$$

$$\underbrace{F = \langle{\rm cat}|\rho|{\rm cat}\rangle = \frac{\pi}{N_{\rm cat}}[Q(\beta_{\rm cat}) + Q(-\beta_{\rm cat}) - 2\Re\exp(-2|\beta_{\rm cat}|^2)Q(\beta_{\rm cat})|_{\beta^*_{\rm cat}\mapsto-\beta^*_{\rm cat}}]}_{\text{fidelity with a cat state}}$$

5. Results

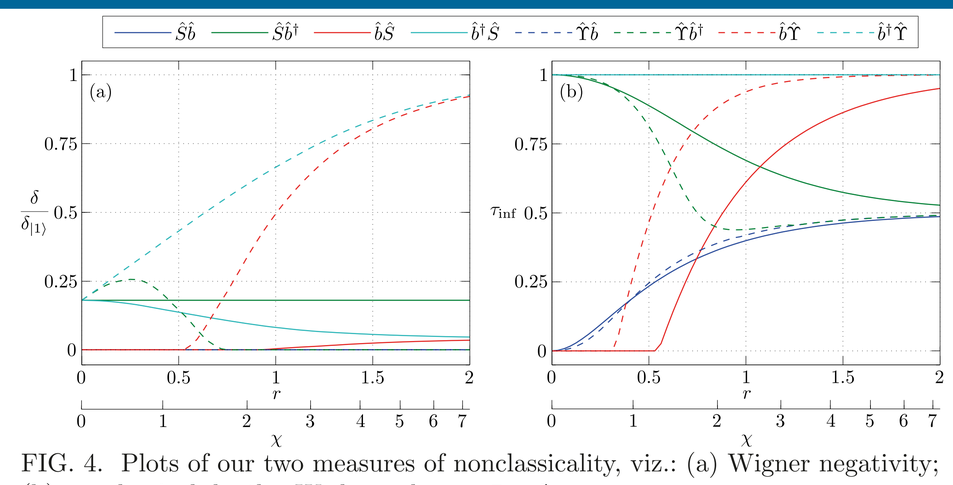


FIG. 4. Plots of our two measures of nonclassicality, viz.: (a) Wigner negativity; (b) nonclassical depth. We have chosen $\bar{n} = 1$.

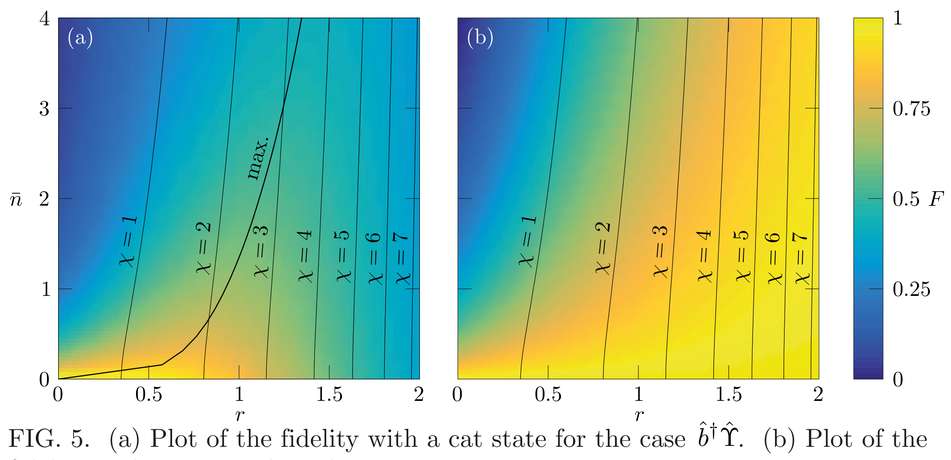


FIG. 5. (a) Plot of the fidelity with a cat state for the case $\hat{b}^\dagger\hat{\Upsilon}$. (b) Plot of the fidelity with a squeezed single-quanta state.

References

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FWF Der Wissenschaftsfonds.
CoQuS ComplexQuantumSystems
WWTF

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