Pulsed Quantum Interaction Between Two Distant Mechanical Modes

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INVESTMENTS IN EDUCATION DEVELOPMENT

Abstract

Implementable setup which realises the quantum non-demolition interaction between mechanical modes of two optomechanical cavities through the optical mediator and classical feed-forward is proposed. We also explore the entanglement of these modes and propose ways to get maximum of the entanglement in this system. Our investigation might be considered as the important step toward merging quantum optics and quantum optomechanics by the reason of this protocol being already explored in the context of general methods of quantum optics.

Introduction

We propose a feasible way of implementation of quantum non-demolition (QND) interaction between mechanical modes of two distant optomechanical cavities. The QND interaction of two harmonic oscillators may be described by Hamiltonian $\mathcal{H}_{int} = \hbar g Q_1 Q_2$ with $Q_{1,2}$ being the quadrature (position or momentum) of the corresponding oscillator and g - interaction strength. After the interaction the quadratures Q remain unperturbed whereas the conjugated ones become displaced by a value proportional to gQ.

The proposed scheme is presented in the Fig. 1. The two mechanical modes interact by turns with a pulse of light via optomechanical QND coupling. The pulse is then detected and the result of the detection is used to displace the mechanical mode of the first cavity. The quadratures of the mechanical modes after the interactions and feed-forward are transformed in the same way, as if the modes were coupled via the QND interaction.

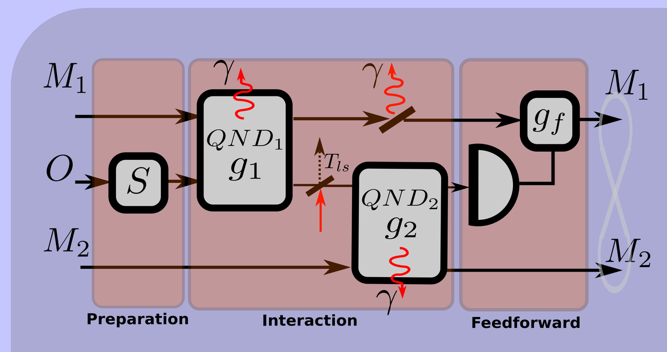
A proper strong presqueezing of the light pulse allows to eliminate its quadratures from the final quadratures of the mechanical modes that approach an ideal QND interaction between them.

To describe the interaction of the propagating light pulse with the optomechanical cavity we complement the Hamiltonian of the optomechanical interaction $\mathcal{H}_1 = -\hbar g_1 X_1 p_1$ with input-output relations (henceforth we denote with index "1" or "2" quantities corresponding to the respective cavity). The system is thus described by the following set of equations:

$$\begin{aligned} \dot{q}_1 &= -\tfrac{\gamma}{2} q_1 - g_1 X_1 + \xi_{q1}, \\ \dot{p}_1 &= -\tfrac{\gamma}{2} p_1 + \xi_{p1}, \\ \dot{X}_1 &= -\kappa X_1 + \sqrt{2\kappa} X^{in}, \\ \dot{Y}_1 &= -\kappa Y_1 + \sqrt{2\kappa} Y^{in} + g_1 p_1 \end{aligned} \qquad (1)$$

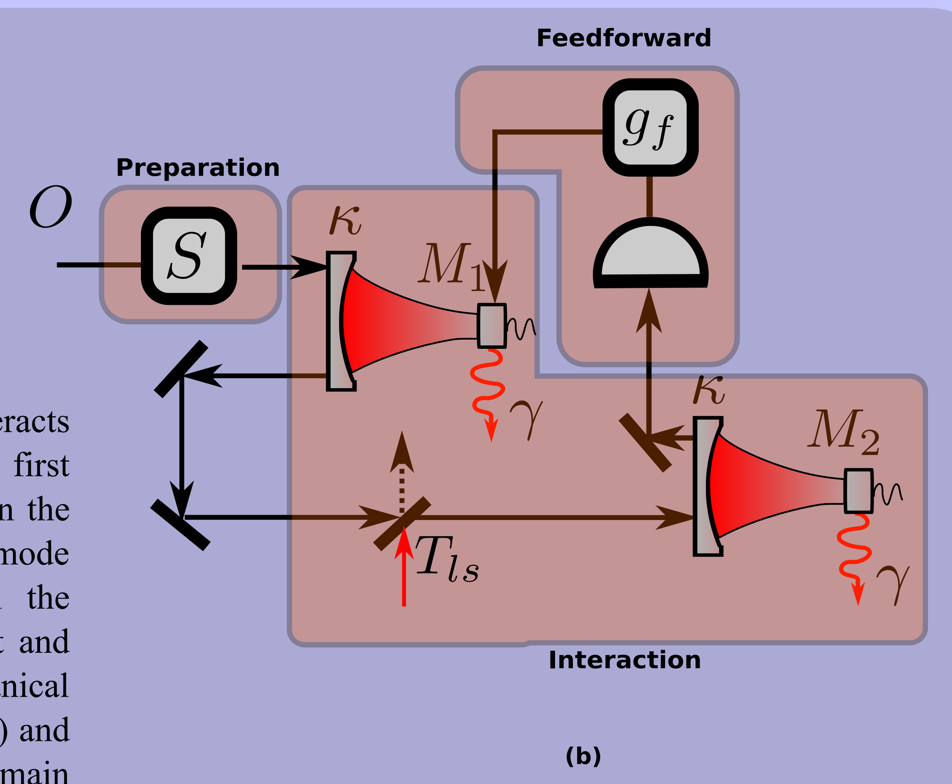
$$Q^{out} = \sqrt{2\kappa} Q - Q^{in}, \quad Q = X, Y$$

Here X^{in}, Y^{in} are the quadratures of the pulse with commutator $[X^{in}(t), Y^{in}(t')] = i\delta(t-t')$, $\xi_{q,p}$ are the quadratures of mechanical noise. κ and γ are respectively optical and viscous mechanical damping coefficients.



(a)

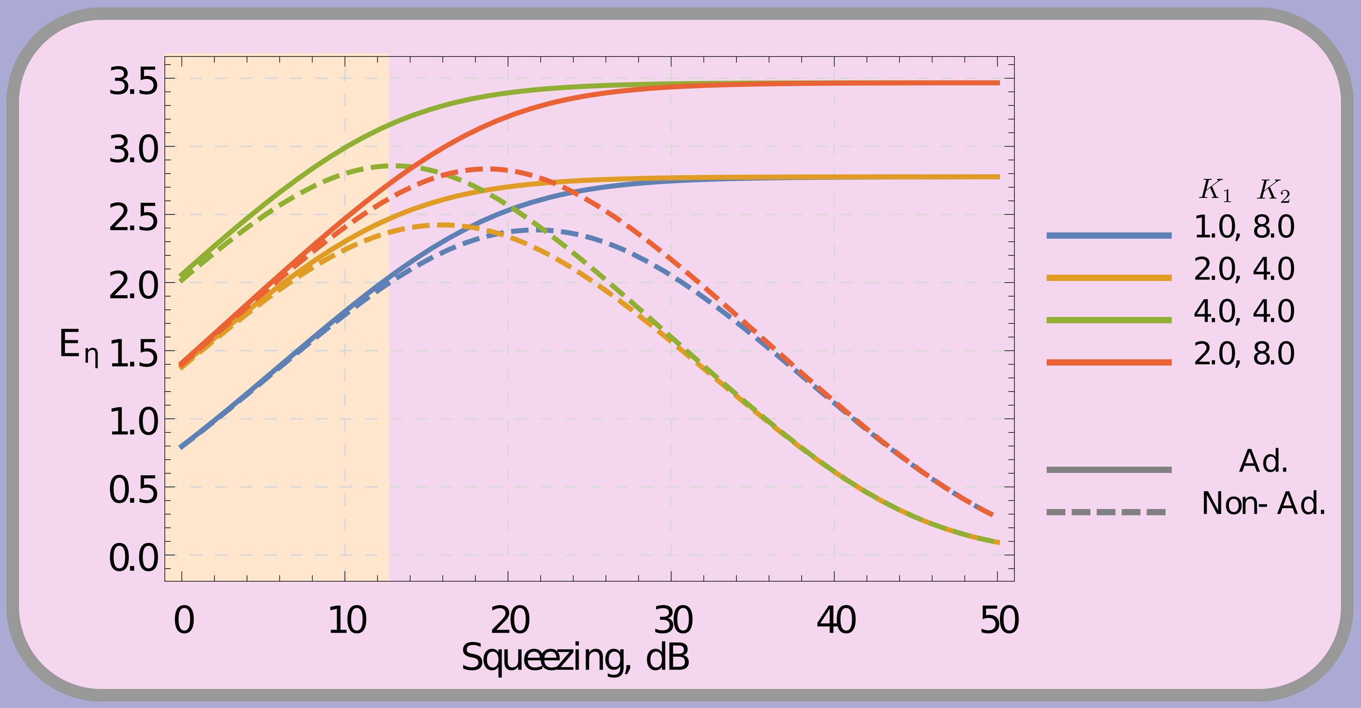
Simplified scheme. First mechanical mode (M_1) interacts with the squeezed (S) light mode (O) through the first QND_1 interaction with the coupling strength g_1. Then the light mode interacts with the second mechanical mode (M_2) through the second QND_2 interaction with the coupling strength g_2. Afterthis step light is read out and one of its quadratures is transferedto the first mechanical mode using the feed-forward technique g_f. Optical (η) and mechanical (γ) losses are sketched showing main decoherence processes.



(b)

Possible optomechanical implementation. Mechanical modes are presented here as the optomechanical cavity's ideally (optical decay rate κ) reflective mirrors.

Fig. 1



Entanglement between the two mechanical modes as a function of optical presqueezing. Solid lines correspond to adiabatic solution; dashed lines to solution with cavity mode. Different colors are used for different ratio of the gains K_1 and K_2. Losses are absent $T_{ls} = 1$.

Fig. 2

Adiabatic regime

As a first approximation we consider the system in adiabatic regime. Formally this corresponds to replacement of all the functions of time with their own versions averaged over the interval with duration τ^* such that $\kappa \gg 1/\tau^* \gg \gamma, g$ In this section we leave out the mechanical decoherence, setting $\gamma = 0, \xi_{q1} = \xi_{p1} = 0.$

The output field from the first cavity is delivered to the input of the second one through a purely lossy channel that performs an admixture of vacuum to the signal, therefore

$$Q_2^{in} = \sqrt{T_{ls}} Q_1^{out} + \sqrt{1 - T_{ls}} Q_{ls}, \quad Q = X, Y.$$

Here Q_{ls} are the quadratures of vacuum mode.

The optical output quadrature $\mathcal{X}_2^{out}$ is measured and the position of the mechanical mode of the first cavity is displaced so that the final value equals $q_1 = q_1(\tau) + K_f \mathcal{X}_2^{out}$.

$$\begin{aligned} q_1 &= q_1(0) + K_2 K_f q_2(0) - S\mathcal{X}^{in}\left(K_1 - K_f\sqrt{T_{ls}}\right) + \mathcal{X}_{ls} K_f \sqrt{1 - T_{ls}}, \\ p_1 &= p_1(0), \\ q_2 &= q_2(0), \\ p_2 &= p_2(0) - K_1 K_2 p_1(0)\sqrt{T_{ls}} - \mathcal{Y}^{in} \tfrac{K_2\sqrt{T_{ls}}}{S} - K_2\sqrt{1 - T_{ls}}\mathcal{Y}_{ls} \end{aligned} \qquad (2)$$

we have introduced $K_i = g_i\sqrt{\frac{2\tau}{\kappa}}$ and $\mathcal{Q}^k = \frac{1}{\sqrt{\tau}}\int_0^\tau Q^k(s)ds, \quad Q = X, Y, \quad k = in, out$ here.

To approach the ideal QND interaction of the wo mechanical modes with Hamiltonian $\mathcal{H}_{QND} = \hbar K_1 K_2 \tau^{-1} p_1 q_2$ one needs to fulfill a few conditions:

1) First, ensure low loss ($T_{ls} \to 1$) to get rid of the noisy mode Q_{ls} .

2) Second, pick a proper feed-forward gain $K_f = K_1/\sqrt{T_{ls}}$ and provide high squeezing $S \gg 1$ to suppress the optical mode $\mathcal{Y}^{in}$

For the feedforward $K_f = K_1$ the entanglement increases with squeezing infinitely. In the limit of moderately strong coupling ($K_{1,2} \gg 1$) the following approximation for the entanglement holds:

$$E_\eta \simeq -\ln \frac{1}{2K_1K_2}\sqrt{1 + \frac{K_2^2}{S^2}}. \qquad (3)$$

From this expression follows that although increase of both S and $K_{1,2}$ leads to stronger entanglement, it is more efficient to increase K_1 .

Robustness to imperfections

There are two sources of hindrance that we left out previously:

1) The intracavity modes that mediate interaction between the propagating pulse and the mechanical modes of interest
2) The interaction of mechanical modes with the thermal environment

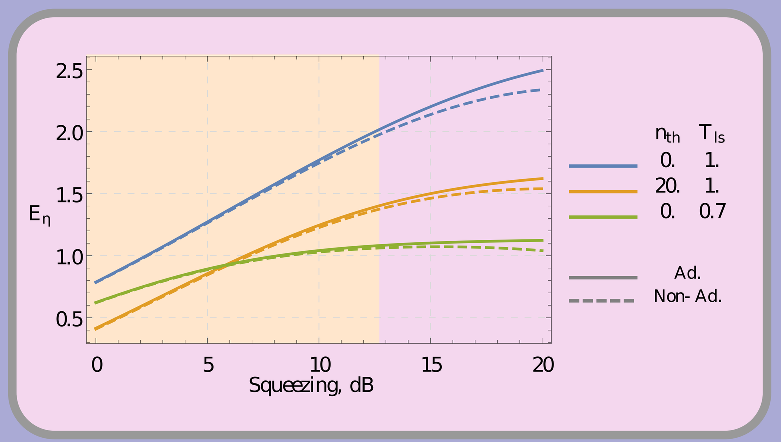


Fig.3

Entanglement as a function of squeezing in presence of mechanical bath and optical losses

To consider the effect of the intracavity modes we should solve the set of intial equation (1). The solution reads:

$$\begin{aligned} q_1 &= q_1(0) + \alpha_1 q_2(0) - S(K_1 - K_f)\tfrac{1}{\sqrt{\tau}}\int_0^\tau X_1^{in}(s)ds + SK_1\int_0^\tau X_1^{in}(s)(e^{-\kappa(\tau - s)}[1 - \\ &\quad 4\kappa(\tau - s)\tfrac{K_f}{K_1}])ds + \alpha_2 X_1(0) + \alpha_3 X_2(0), \\ p_1 &= p_1(0), \\ q_2 &= q_2(0), \\ p_2 &= p_2(0) + \beta_1 p_1(0) - \tfrac{K_2}{S}\tfrac{1}{\sqrt{\tau}}\int_0^\tau \beta_2(\tau, s) Y_1^{in}(s)ds - Y_1(0)\beta_3 - Y_2(0)\tfrac{g_2}{\kappa}(1 - e^{-\kappa\tau}) \end{aligned} \qquad (4)$$

Here we can see the influence of the intracavity fields $Q(0)$. As well the pulse quadratures Q^{in} can no longer be eliminated completely by a proper choice of K_f and high squeezing S . Moreover, in this case high squeezing apmlifies the noisy summand with X^{in} degrading the interface.

The thermal bath is represented in the equations (1) by Langevin force quadratures $\xi_{q,p}$.The logarithmic negativity in adiabatic regime with intracavity modes eliminated is approximately given by (here $K_1 = K_2 = K$):

$$E_\eta \simeq -\ln\frac{1}{2K^2}\sqrt{1 + \Gamma K^4 + \frac{K^2}{S^2}(1 + \Gamma K^4)}, \quad \Gamma = 2\gamma\tau n_{th}. \qquad (5)$$

The main means how the mechanical environment affects the entanglement is adding the thermal noise to the mechanical quadratures. Besides this the environment also creates small imbalance that prohibits the perfect cancellation of the optical mediator mode in q_1 by feedforward. The magnitude of this imbalance is however almost negligible.

Optimization

In prior sections we focused on approaching a pure QND interaction between the two mechanical modes.Therefore we assumed the feedforward to be adjusted in a way that helps to cancel most of the optical mediator quadrature X^{in}, i.e. $K_f = K_1/T_{ls}$. Now we aim for maximization of the entanglement between the two modes. We waive the constraint on K_f and numerically optimize the logarithmic negativity with respect to the optomechanical gains $K_{1,2}$, feedforward strength K_f , and the pulse duration τ given a limitation on the coupling strength. The results of the optimization are presented in Fig. 4.

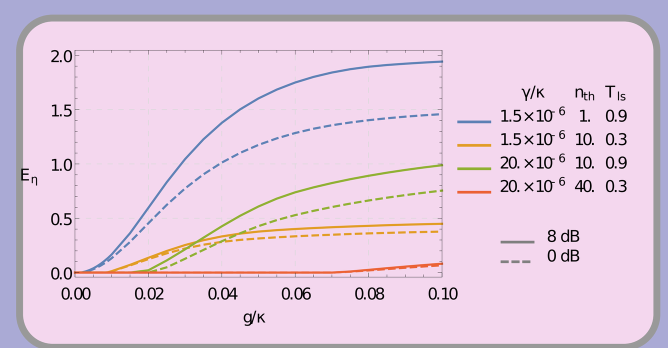


Fig. 4

Conclusion

We have considered the protocol of the QND interaction between two distant mechanical modes. We have shown that it is possible to implement such a scheme with currently achievable experimental parameters. By the appropriate choice of the feed-forward gain and the optical presqueezing it is feasible to approach quite close to the ideal QND interaction between two mechanical oscillators.

We also have investigated destructive sources such as thermal bath, optical losses and the influence of the intracavity field finding regions of parameters where the implementation of proposed interface is still applicable.

Finally we have maximized the entanglement between two modes varying the feed-forward strength, optomechanical gains and the pulse duration.