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Nonequilibrium thermodynamics with feedback

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Outline

1. Nonequilibrium thermodynamics without feedback
   - Classical nonequilibrium processes
   - Quantum nonequilibrium processes
2. Nonequilibrium thermodynamics with feedback
   - Classical information thermodynamics
   - Quantum information thermodynamics

Introduction

Observation:

Thermodynamics describes equilibrium transformations

Challenge:

Generalization to arbitrary nonequilibrium processes

Motivation:

Far from equilibrium quantum regime accessible in recent cold-atom experiments

Thermodynamics: a short reminder

Equilibrium (nonequilibrium) processes:

Entropy: $\Delta S = Q/T + \Sigma$

Work: $W = \Delta F + W_{irr}$ ($F = U - TS$ = free energy)

with $\langle \Sigma \rangle \geq 0$ and $\langle W_{irr} \rangle \geq 0$ (Second law)

➔ Thermodynamics does not allow computation of Σ, W_{irr}

Nonequilibrium entropy production:

$$\Sigma = \beta(W - \Delta F) = \beta W_{irr} \qquad \beta = 1/(kT)$$

➔ Difference between total work and equilibrium work

Nonequilibrium entropy production

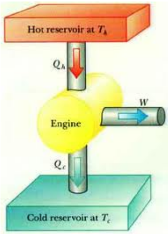
Maximum extractable work:

$$-\langle W\rangle = -\Delta F - kT\langle\Sigma\rangle \leq -\Delta F$$

➔ is reduced by nonequilibrium entropy production

Efficiency of thermodynamic devices:

$$\mathcal{E} = (1 - \frac{T_1}{T_2})Q - T_1\Sigma$$



➔ fundamental quantity

Large versus small systems

In large systems, fluctuations are negligible $(\sim 1/\sqrt{N})$

➔ W, Q and Σ are deterministic variables

In small systems, fluctuations are important

➔ W, Q and Σ are stochastic variables, in particular $\Sigma < 0$

➔ second law needs to be generalized

Here: we focus on work $P(W)$ and entropy $P(\Sigma)$ distribution

Generalizations of the second law

Fluctuation theorem:

Evans, Morris and Cohen PRL (1993)

$$\frac{P(\Sigma)}{P(-\Sigma)} = e^{\Sigma}$$

➔ negative fluctuations exponentially small: $P(-\Sigma) = P(\Sigma)e^{-\Sigma}$

Jarzynski equality:

Jarzynski PRL (1997)

$$\langle \exp(-\beta W) \rangle = \exp(-\beta \Delta F)$$

➔ equilibrium free energy from nonequilibrium work $P(W)$

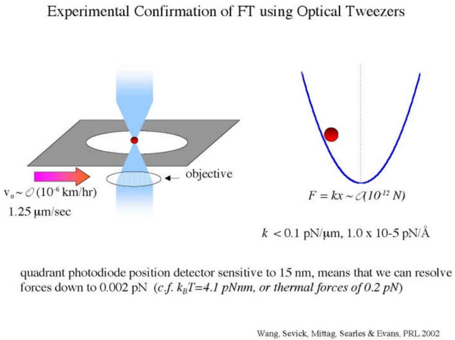
General form: $\langle e^{-\Sigma} \rangle = 1$ implies $\langle \Sigma \rangle \geq 0$

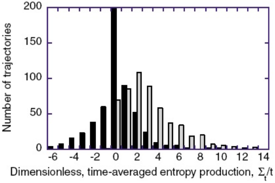
➔ valid far from equilibrium (beyond linear response)

Experiment: Fluctuation theorem

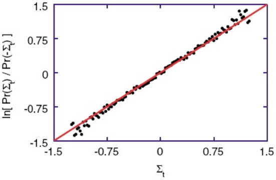
Colloidal particle in a driven optical trap:

Wang *et al.* PRL (2002)





Average over 540 trajectories:
t= .01s (black) and t=2s (grey)



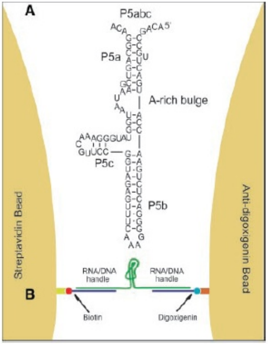
Latex beads d= 6,3 micron

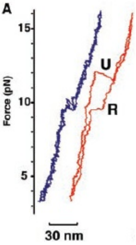
$$\Sigma = \beta W_{ir} = \beta \int_0^t ds\, v.F(s)$$

Experiment: Jarzynski equality

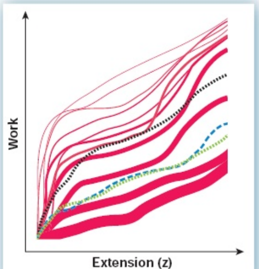
Stretching of single RNA molecule:

Liphardt *et al.* Science (2002)





Average over 40 pullings



red: $W(z) = \int_0^z dx\, F(x)$

green: $\Delta F = \langle W_{rev} \rangle$

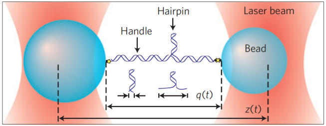
black: $\langle W \rangle > \Delta F$

blue: $\Delta F = -kT \ln \langle \exp(-\beta W) \rangle$

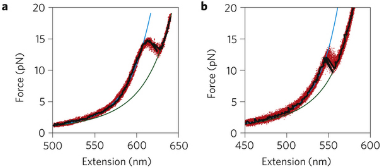
Usefulness of the Jarzynski equality

Free-energy surface reconstruction:

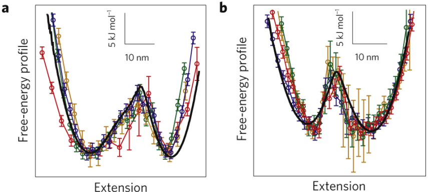
Gupta *et al.* Nature Phys. (2011)



About 2000-3000 force extension curves:



Yield free energy surface:



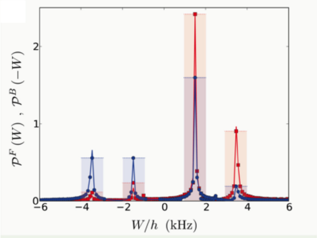
Quantum experiment: Crooks equality

Batalhão *et al.* PRL (2014)

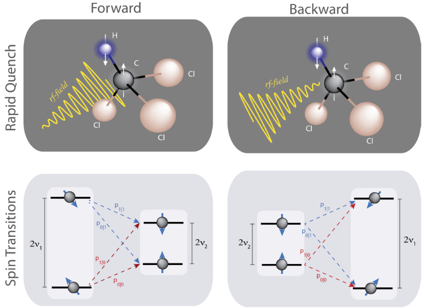
Crooks equality:

Crooks PRE (1999)

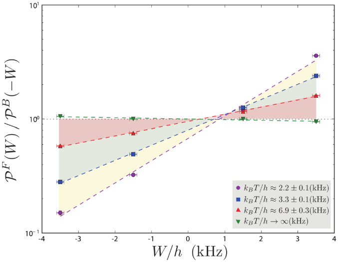
$$\frac{P_F(W)}{P_R(-W)} = e^{\beta(W-\Delta F)} = e^{\Sigma}$$



NMR system with B field quench:

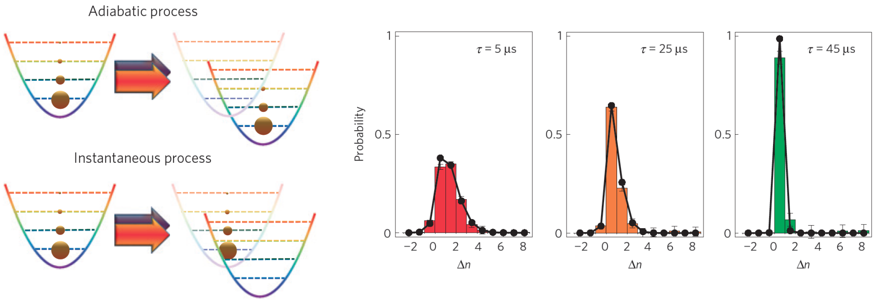


Quantum work distribution



Quantum experiment: Jarzynski equality

Shifted harmonic potential (trapped ion): An *et al.* Nature Phys. (2015)



Based on theory presented in Huber, Schmidt-Kaler, Deffner, Lutz PRL (2008)

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Information thermodynamics

What happens when a system is measured (feedback)?

- ➔ information is gained about the system
- ➔ second law needs to be generalized (again)

Generalized second law: Sagawa and Ueda, PRL (2010)

$$\langle e^{-(\Sigma - I)} \rangle = 1$$

Maximal extractable work: Sagawa and Ueda, PRL (2008)

$$-\langle W \rangle \leq -\Delta F + kT \langle I \rangle$$

where $\langle I \rangle = H(P) - \sum_i p_i H(\tilde{P}_i) =$ mutual information

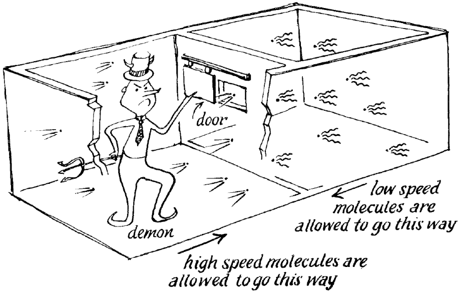
➔ more work can be extracted with feedback

(Maxwell's demon) Lutz and Ciliberto, Physics Today (2015)

Maxwell’s demon

Gas in a partitioned box

Maxwell 1867/1871



Information about particle (position/velocity) leads to sorting

➔ temperature difference (decrease of entropy without work)

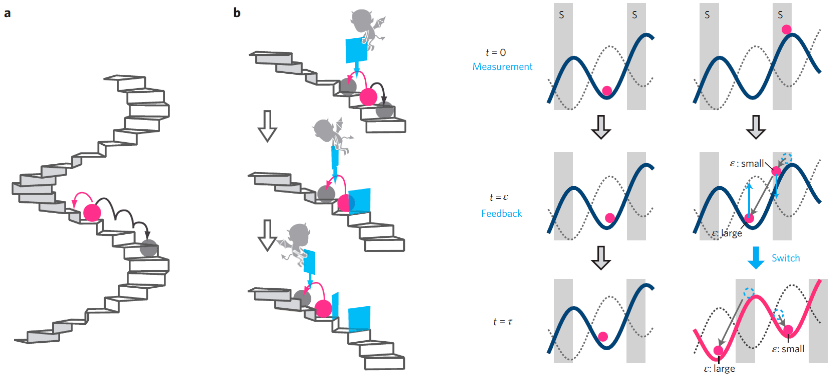
(could be used to run a heat engine and produce work)

➔ apparent violation of the second law

Classical experiment

Brownian particle in a tilted periodic potential

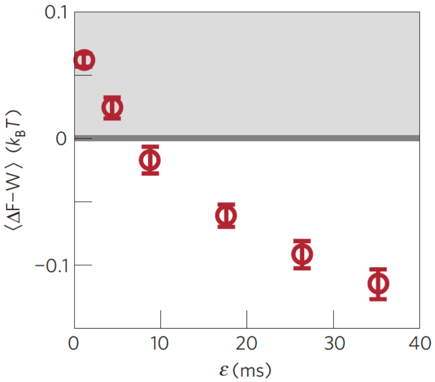
Toyabe *et al.* Nature Phys. (2010)



→ demonstration of information-to-work conversion

Classical experiment

Brownian particle in a tilted periodic potential Toyabe *et al.* Nature Phys. (2010)



➔ more work is extracted in white area

Quantum Maxwell's demon experiments

PRL 117, 240502 (2016) Selected for a Viewpoint in *Physics* PHYSICAL REVIEW LETTERS week ending 9 DECEMBER 2016

Experimental Rectification of Entropy Production by Maxwell's Demon in a Quantum System

Patrice A. Camati,[1] John P. S. Peterson,[2] Tiago B. Batalhão,[1] Kaonan Micadei,[1] Alexandre M. Souza,[2] Roberto S. Sarthour,[2] Ivan S. Oliveira,[2] and Roberto M. Serra[1,3,*]

PNAS

Observing a quantum Maxwell demon at work

Nathanaël Cottet[a,1], Sébastien Jezouin[a,1], Landry Bretheau[a], Philippe Campagne-Ibarcq[a], Quentin Ficheux[a], Janet Anders[b], Alexia Auffèves[c], Rémi Azouit[d,e], Pierre Rouchon[d,e], and Benjamin Huard[a,f,2]

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nature COMMUNICATIONS

ARTICLE

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Information-to-work conversion by Maxwell's demon in a superconducting circuit quantum electrodynamical system

Y. Masuyama[1], K. Funo[2], Y. Murashita[3], A. Noguchi[1], S. Kono[1], Y. Tabuchi[1], R. Yamazaki[1], M. Ueda[3,4] & Y. Nakamura[1,4]

- ➔ projective measurements destroy coherence
- ➔ can be explained with *classical* information

Quantum information

Information in classical measurements:

Shannon (1948)

$$\langle I \rangle_c = H(P) - \sum_i p_i H(\tilde{P}_i) \geq 0$$

where H is the Shannon entropy

➔ information always gained

Information in quantum measurements:

Groenewold (1971)

$$\langle I \rangle_q = S(\rho) - \sum_i p_i S(\tilde{\rho}_i)$$

where H is the von Neumann entropy

➔ can be negative for weak measurements because of backaction

➔ information sometimes lost: disturbance larger than gain Funo PRE 2013

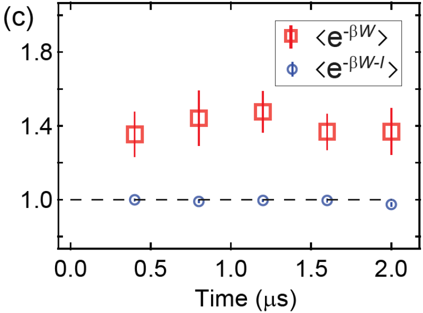
Quantum experiment

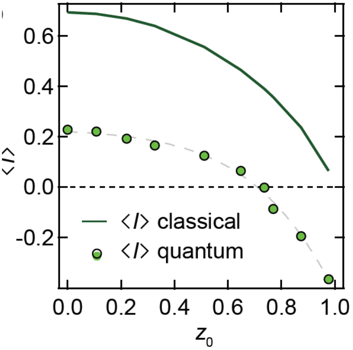
Periodically driven qubit in a superconducting circuit

Naghiloo *et al.*, arXiv:1802.07205

(1) Information: qubit (coherent) evolution weakly measured

(2) Feedback: qubit brought to ground state (work extraction)





➔ Maxwell's demon with *quantum* information $z_0 = \tanh(\beta\omega/2)$

Summary

- in small systems thermal/quantum fluctuations dominate
  - ➔ second law needs to be generalized (fluctuation theorems)
- measurements are not included in thermodynamics
  - ➔ second law needs to be generalized (FT with feedback: more work can be extracted)
- information can be lost due to quantum backaction
  - ➔ typical quantum signature