AUTONOMOUS GENERATION OF SINGLE STEADY-STATE COHERENCES BY SYSTEM-BATH INTERACTION

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OUTLINE OF THE TALK

- Open Quantum Systems
  - Quantum coherence
  - Coherence trapping
  - Autonomous generation of steady-state coherences by system-bath interaction
- Outlooks and conclusions

OPEN QUANTUM SYSTEMS

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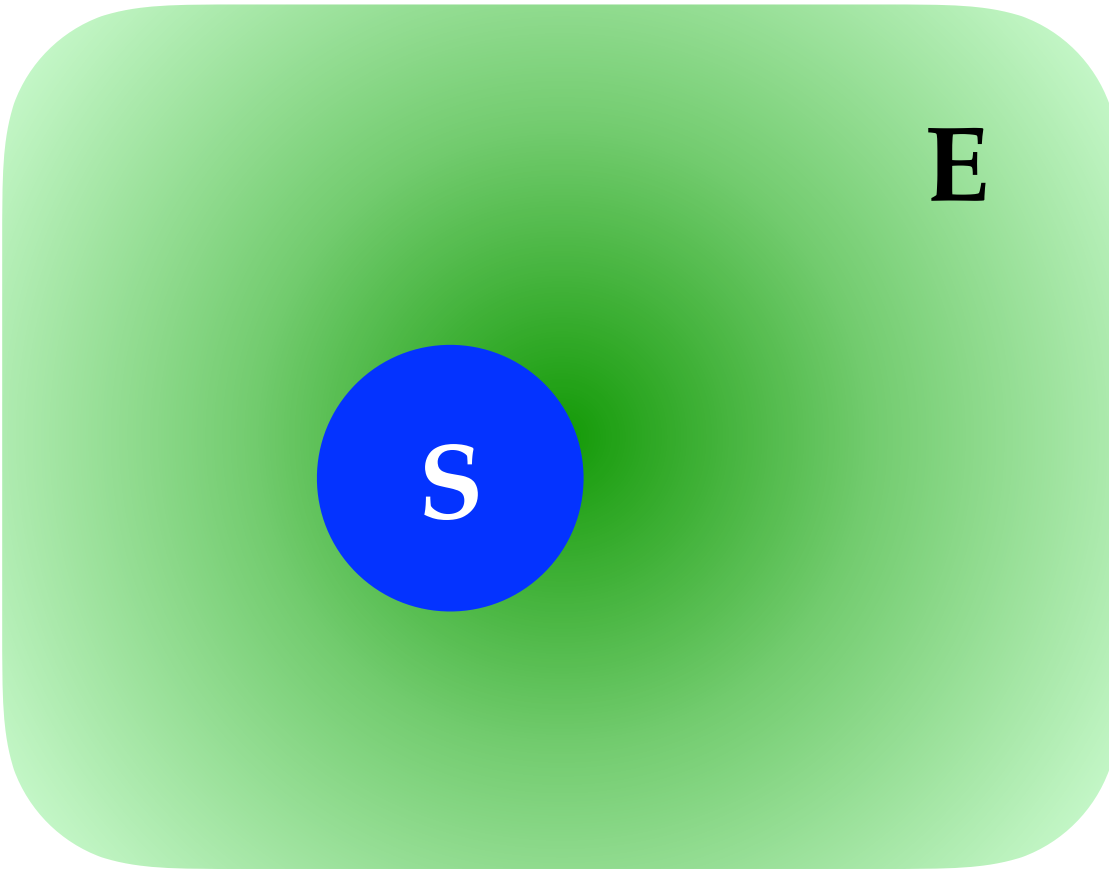


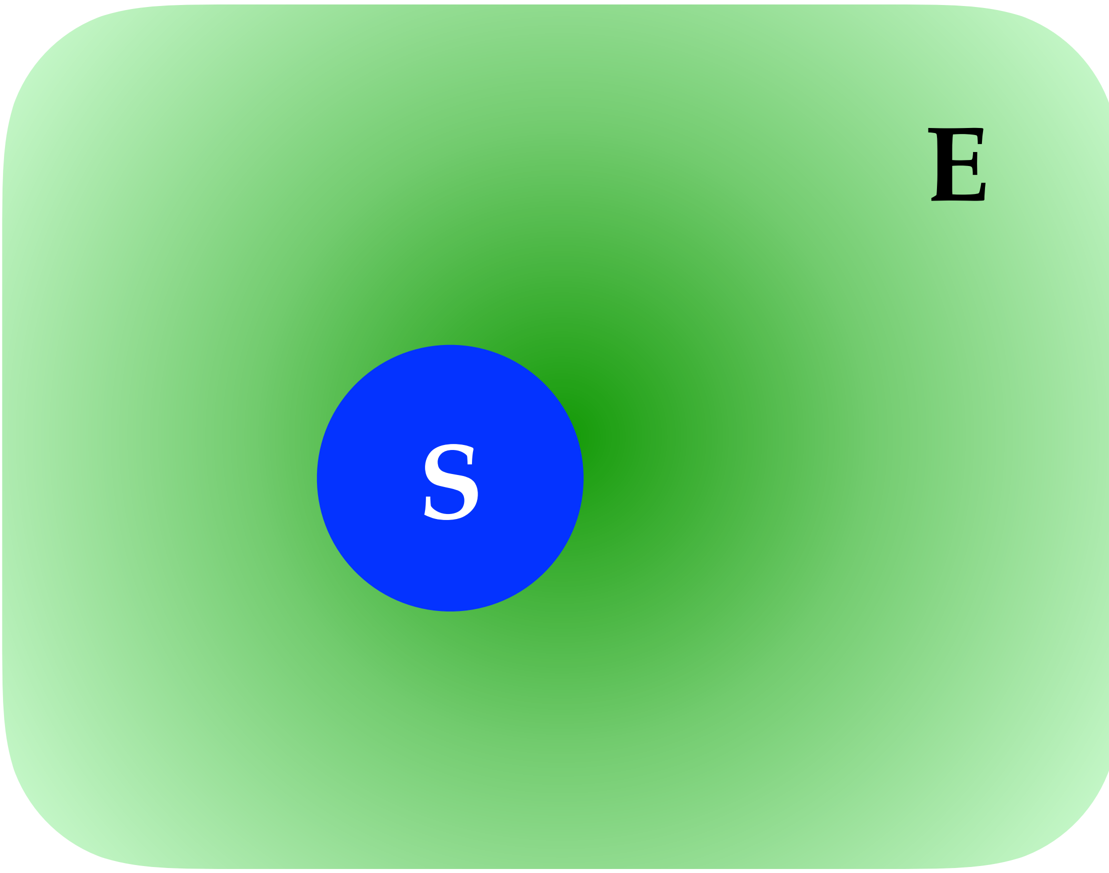
Composite system $\rho_{\mathrm{SE}} \in \mathcal{S}(\mathcal{H}_{\mathrm{SE}})$

System of interest $\rho_S = \mathrm{Tr}_E\left[\rho_{\mathrm{SE}}\right] \in \mathcal{S}(\mathcal{H}_S)$

Environment $\rho_E = \mathrm{Tr}_S\left[\rho_{\mathrm{SE}}\right] \in \mathcal{S}(\mathcal{H}_E)$

$$\mathcal{S}(\mathcal{H}) = \{\rho \in \tau(\mathcal{H}) | \rho \geq 0, ||\rho||_1 = 1\}$$

OPEN QUANTUM SYSTEMS



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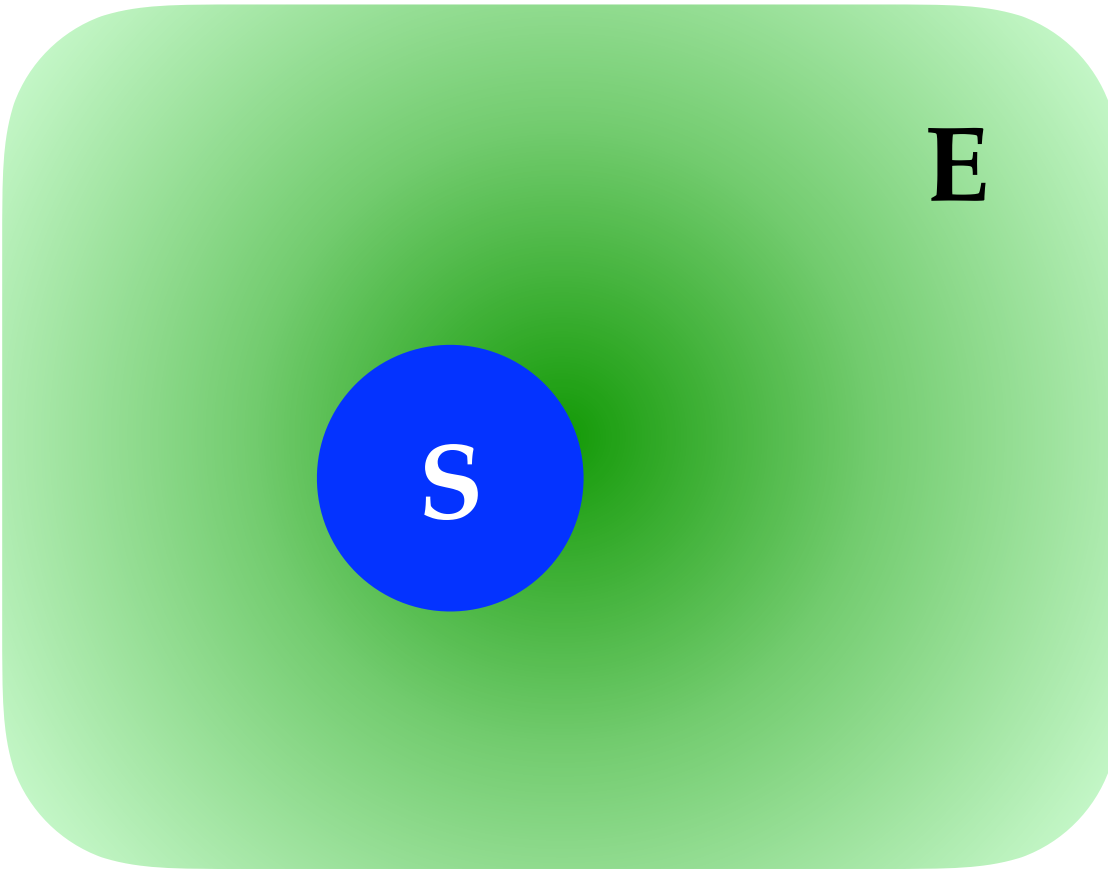
Initial correlations

$$\rho_{\mathrm{SE}}(t_0) \xrightarrow{\mathcal{U}(t,t_0)} \rho_{\mathrm{SE}}(t) = \mathcal{U}(t,t_0)\rho_{\mathrm{SE}}(t_0)\mathcal{U}^\dagger(t,t_0)$$

$$\downarrow \mathrm{Tr}_E \qquad\qquad ? \qquad\qquad \downarrow \mathrm{Tr}_E$$

$$\rho_S(t_0) \longrightarrow \rho_S(t)$$

OPEN QUANTUM SYSTEMS



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No initial correlations

$$\rho_{\mathrm{S}}(t_0) \otimes \rho_E(t_0) \xrightarrow{\mathcal{U}(t,t_0)} \rho_{\mathrm{SE}}(t) = \mathcal{U}(t,t_0)\rho_{\mathrm{SE}}(t_0)\mathcal{U}^\dagger(t,t_0)$$

$\downarrow \mathrm{Tr}_E$ $\qquad\qquad$ $\downarrow \mathrm{Tr}_E$

$$\rho_S(t_0) \xrightarrow{\Lambda(t,t_0)} \rho_S(t)$$

Quantum Dynamical Map

OPEN QUANTUM SYSTEMS

$$\Lambda \,:\, \mathcal{S}(\mathcal{H}_S) \rightarrow \mathcal{S}(\mathcal{H}_S)$$

$$\rho_S(t) = \Lambda(t, t_0)\rho_S(t_0)$$

Properties:

- **Linear**
- **Completely positive**
- **Trace - preserving**

OPEN QUANTUM SYSTEMS

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Time-dependent **GKSL** master equation

$$\frac{d}{dt}\rho_S(t) = -i\,[\mathcal{H}(t)\,,\,\rho_S(t)] + \sum_{k=1}^{N^2-1}\gamma_k(t)\left(\sigma_k(t)\rho_S(t)\sigma_k^\dagger(t) - \frac{1}{2}\{\sigma_k^\dagger(t)\sigma_k(t), \rho_S(t)\}\right)$$

OPEN QUANTUM SYSTEMS

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$\exists\, t \;\; \gamma_k(t) < 0$

Non-Markov dynamics

$\forall\, t \;\; \gamma_k(t) \geq 0$

Markov dynamics

$\gamma_k \geq 0$ (constants)

Semigroup dynamics

A. Rivas, S.F. Huelga, M.B. Plenio, PRL **105**, 050403 (2010)

QUANTUM COHERENCE

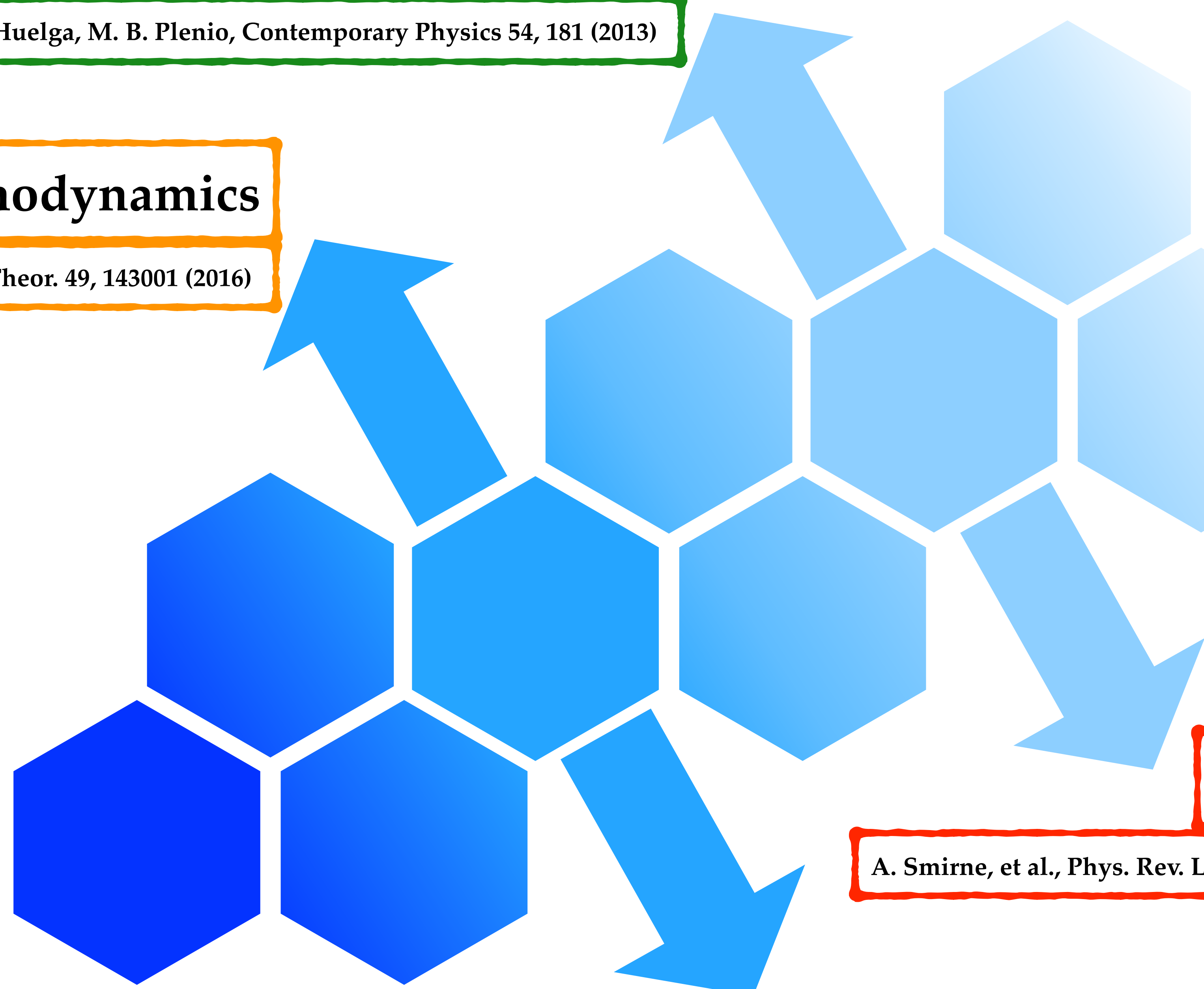
QUANTUM COHERENCE

Quantum Biology

S. F. Huelga, M. B. Plenio, Contemporary Physics 54, 181 (2013)

Thermodynamics

J. Goold et al. , Journ. Phys. A: Math. and Theor. 49, 143001 (2016)



Metrology

A. Smirne, et al., Phys. Rev. Lett. 116, 120801 (2016)

Quantum features (entanglement,…)

A. Streltsov et al., Phys. Rev. Lett. 115, 020403 (2015)

Quantum Coherence

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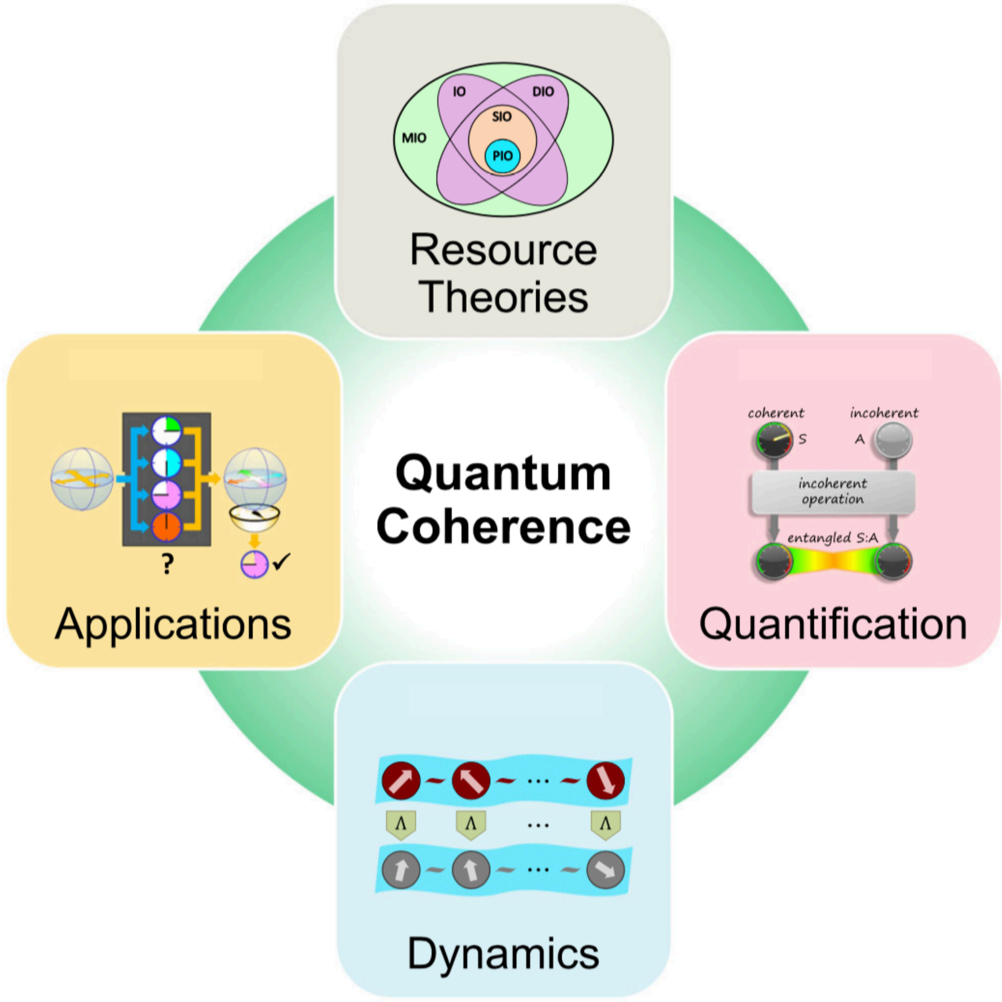

A. Streltsov, G. Adesso, and M. B. Plenio, Rev. Mod. Phys. **89**, 041003 (2017)

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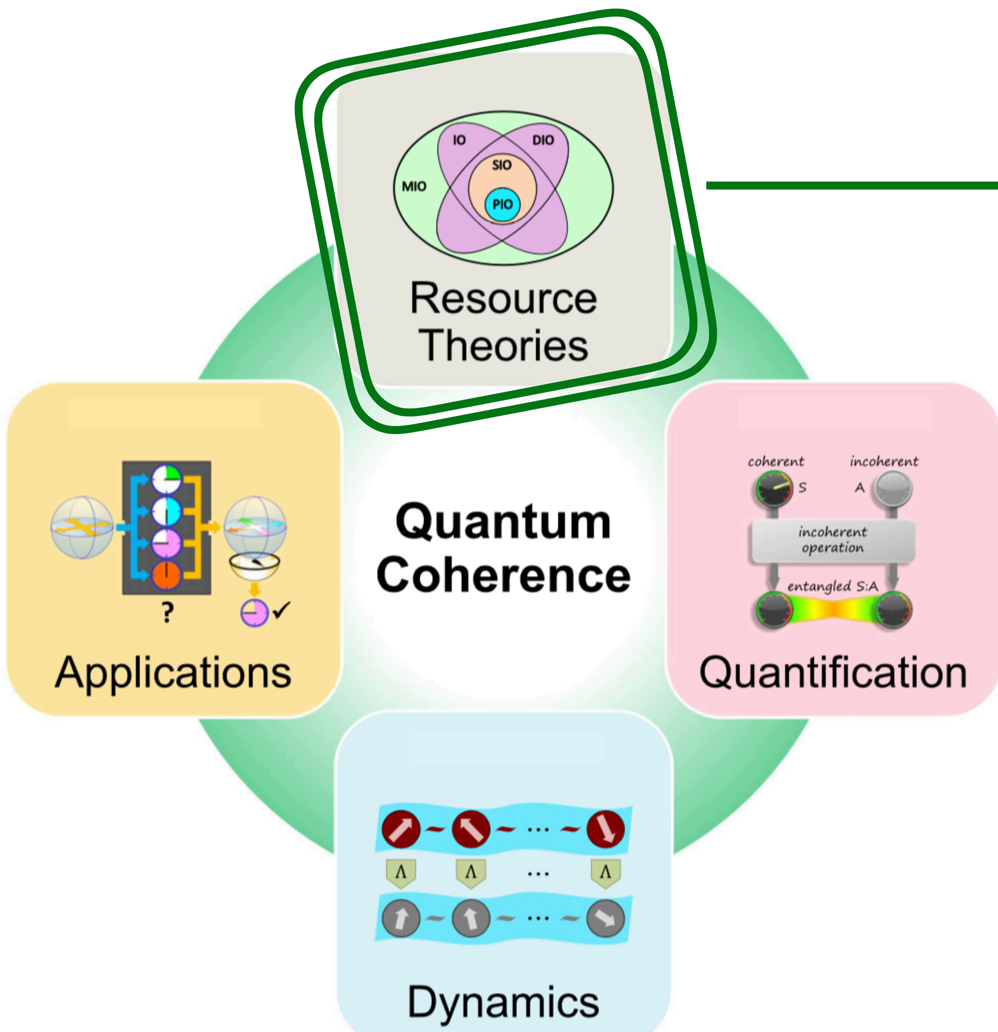


A. Streltsov, G. Adesso, and M. B. Plenio, Rev. Mod. Phys. **89**, 041003 (2017)

ORTHONORMAL BASIS $\{|j\rangle\}_{j=0,\dots,d-1}$

FREE STATES $\rho = \sum_j p_j |j\rangle\langle j|$ e.g.

$$\rho_S = \begin{pmatrix} \bullet & 0 \\ 0 & \bullet \end{pmatrix}$$

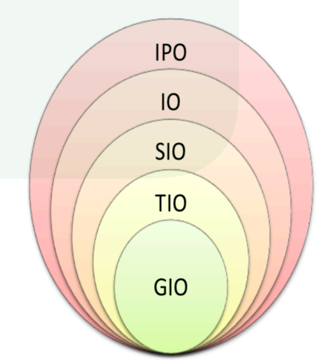
Thermal state

$$\rho_S = \begin{pmatrix} \bullet & 0 \\ 0 & \bullet \end{pmatrix}$$

Maximally mixed state

FREE OPERATIONS $\Lambda : \mathcal{S}(\mathcal{H}_S) \to \mathcal{S}(\mathcal{H}_S)$

Operations that map free states into free states



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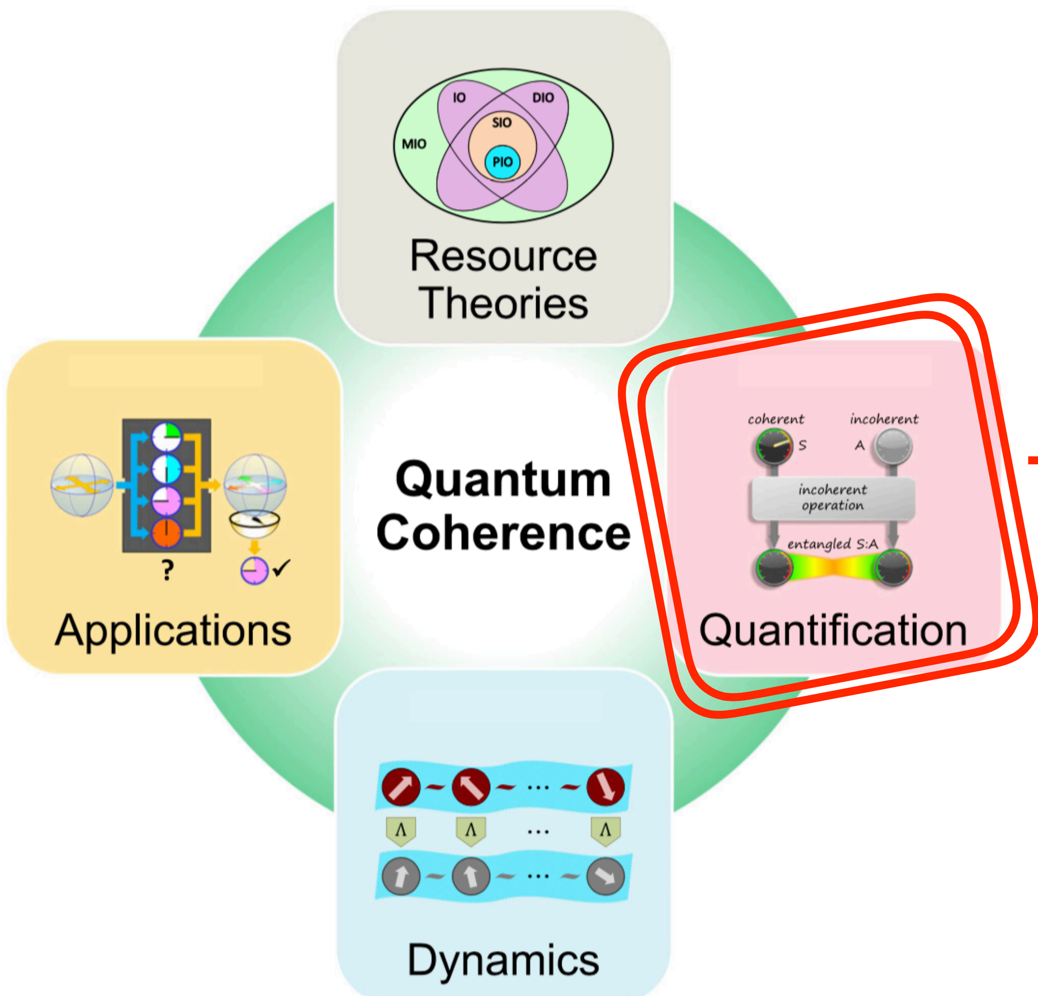

A. Streltsov, G. Adesso, and M. B. Plenio, Rev. Mod. Phys. **89**, 041003 (2017)

Coherence quantifier $\mathcal{C}$

defined w.r.t. a fixed orthonormal basis $\{|j\rangle\}_{j=0,\ldots,d-1}$

C1) Non-negativity $\mathcal{C}(\rho) \geq 0$ (=0 for free states)

C2) Monotonicity $\mathcal{C}(\Lambda[\rho]) \leq \mathcal{C}(\rho)$ (Λ free operation)

C3) Strong monotonicity $\mathcal{C}(\rho) \geq \sum_j p_j \mathcal{C}(\rho_j)$

$$\Lambda[\rho] = \sum_j \Omega_j \rho \Omega_j^\dagger \qquad \rho_j = p_j^{-1} \mathrm{Tr}\left[\Omega_j \rho \Omega_j^\dagger\right]$$

C4) Convexity $\sum_j p_j \mathcal{C}(\rho_j) \geq \mathcal{C}\left(\sum_j p_j \rho_j\right)$

QUANTUM COHERENCE

Entanglement quantifier E

defined w.r.t. a fixed bipartition $A:B$

E1) Non-negativity $E_{A:B}(\rho) \geq 0$ (=0 for separable states)

E2) Monotonicity $E_{A:B}(\Lambda[\rho]) \leq E_{A:B}(\rho)$ (Λ LOCC)

E3) Strong monotonicity $E_{A:B}(\rho) \geq \sum_j p_j E_{A:B}(\rho_j)$

$$\Lambda[\rho] = \sum_j \Omega_j \rho \Omega_j^\dagger \qquad \rho_j = p_j^{-1}\mathrm{Tr}\left[\Omega_j \rho \Omega_j^\dagger\right]$$

E4) Convexity $\sum_j p_j E_{A:B}(\rho_j) \geq E_{A:B}\left(\sum_j p_j \rho_j\right)$

Vedral et al., 1997; Vedral and Plenio, 1998

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Baumgratz, Cramer, and Plenio, 2014; Levi and Mintert, 2014)

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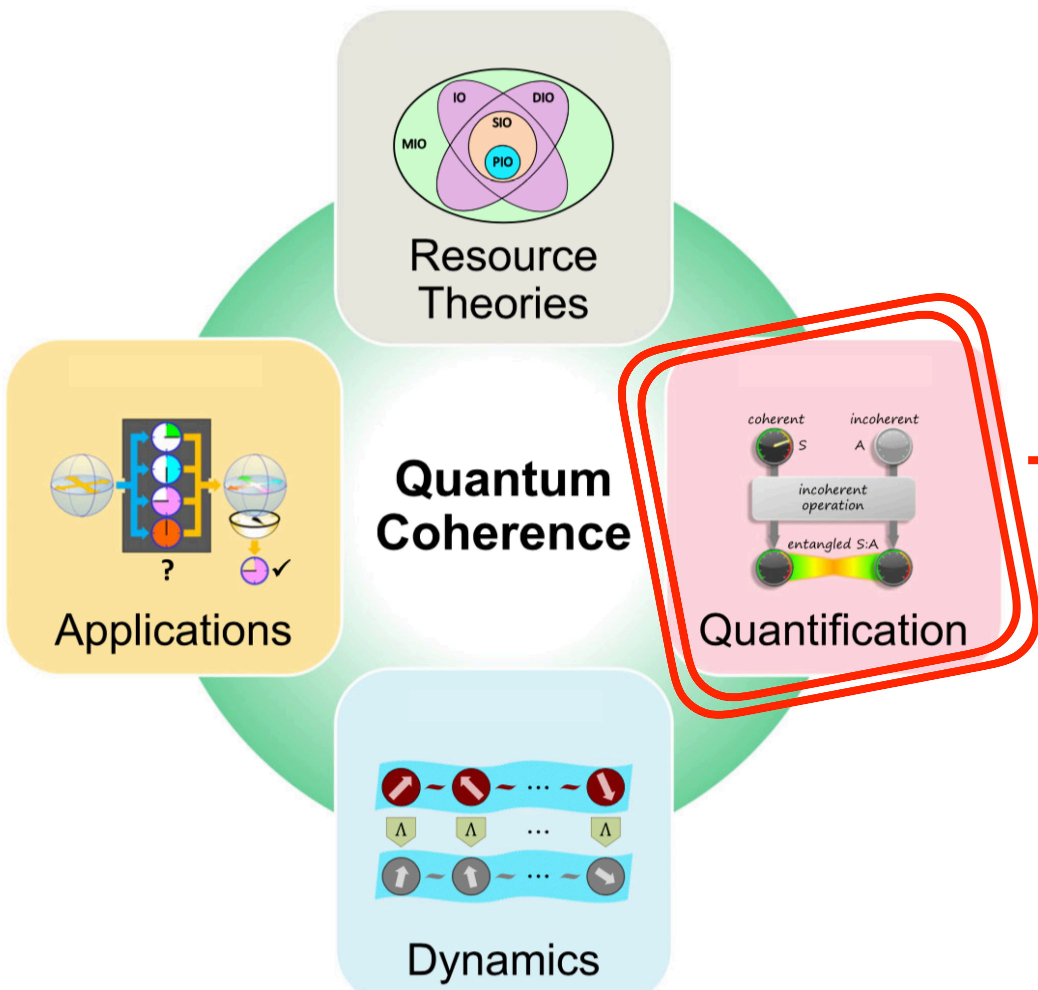
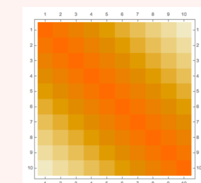


A. Streltsov, G. Adesso, and M. B. Plenio, Rev. Mod. Phys. **89**, 041003 (2017)

Coherence quantifier $\mathcal{C}$

- **l_1- norm**



$$\mathcal{C}_{l_1}(\rho) = \min_{\sigma \in \mathcal{I}} ||\rho - \sigma||_{l_1} = \sum_{j \neq k} |\rho_{jk}|$$

- **Relative entropy of coherence**

$$\mathcal{C}_{\mathrm{rel}}(\rho) = \min_{\sigma \in \mathcal{I}} S(\rho||\sigma) = S(\Delta[\rho]) - S(\rho)$$

$$\Delta[\rho] = \sum_{j=0}^{d-1} |j\rangle\langle j|\rho|j\rangle\langle j|$$

- **Many more!! Geometric coherence, coherence of formation, robustness of coherence, entanglement-based coherence, coherence of assistance, …**

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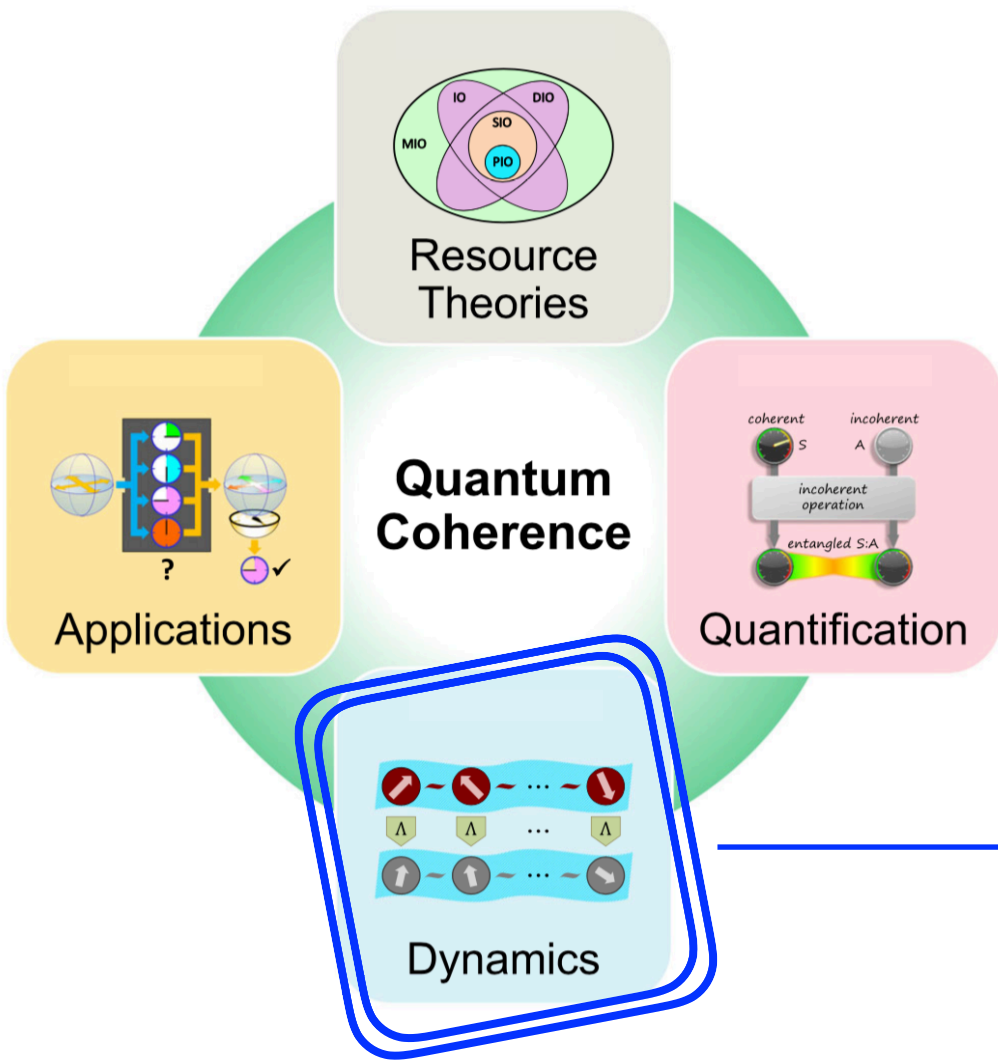
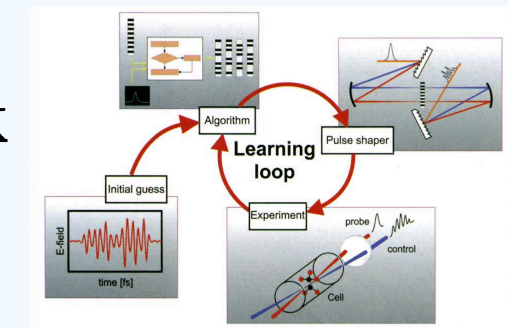


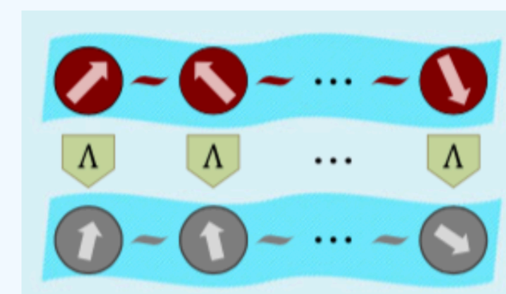
A. Streltsov, G. Adesso, and M. B. Plenio, Rev. Mod. Phys. **89**, 041003 (2017)

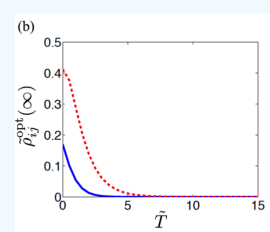
- **Quantum Feedback Control**



R. Rabitz, D. Vivie-riedle, M. Motzkus, and K. Kompa, Science 288, 824 (2000)

- **Coherence freezing**





T. R. Bromley, M. Cianciaruso, G. Adesso, PRL **114**, 210401 (2015)

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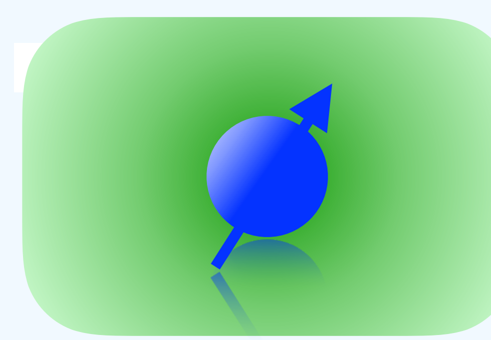

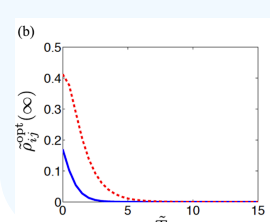

C. Addis, G. Brebner, P. Haikka, S. Maniscalco, PRA **89**, 024101 (2014)

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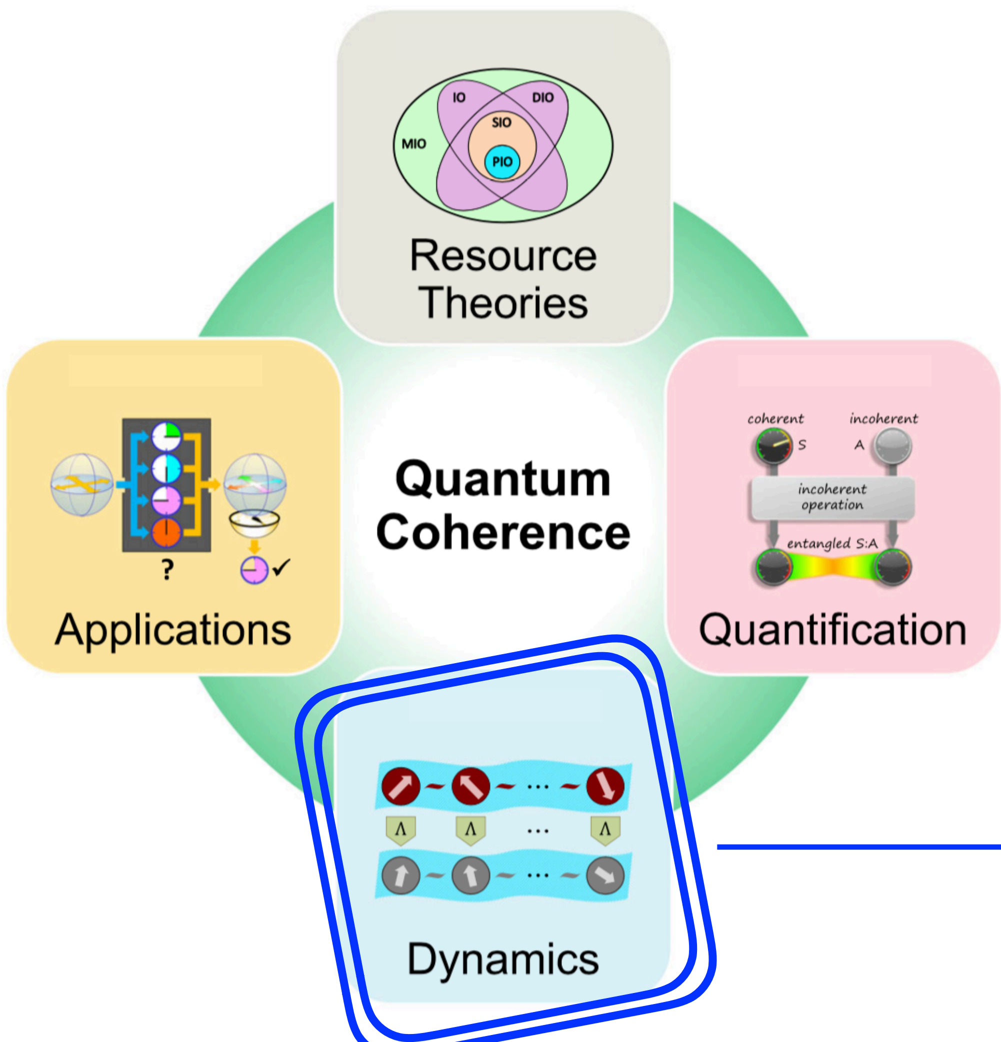

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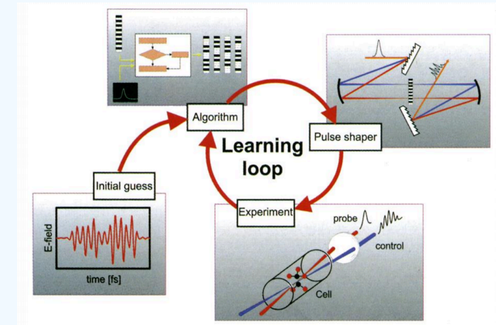

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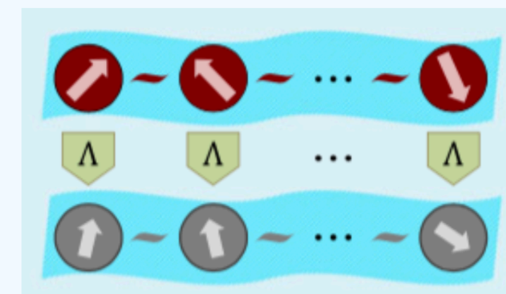

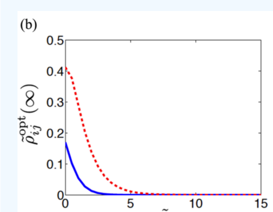

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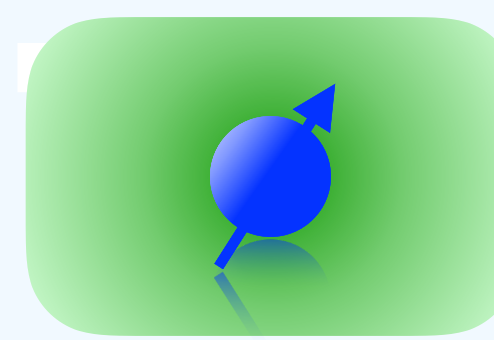

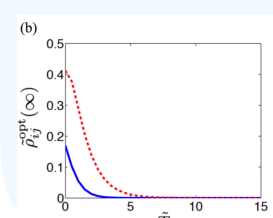

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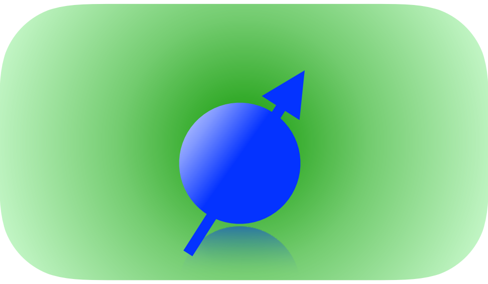

C. Addis, G. Brebner, P. Haikka, S. Maniscalco, PRA **89**, 024101 (2014)

Pure-dephasing spin-boson model $\mathcal{H} = \frac{\omega_0}{2}\sigma_z + \sum_k \omega_k b_k^\dagger b_k + \sigma_z \otimes B_E$ $\quad B_E = \sum_k g_k \left(b_k + b_k^\dagger\right)$

The energy of the system does not change in time $\rho_{jj}(t) = \rho_{jj}(0)$

The coherence get damped by the interaction with the bath $\rho_{jk}(t) = e^{-\int_0^t d\tau\gamma(\tau)}\rho_{jk}(0)$

$$\gamma(t) = \int_0^{+\infty} d\omega J(\omega) \coth\left(\frac{\omega}{2T}\right)\frac{\sin(\omega t)}{\omega}$$

QUANTUM COHERENCE

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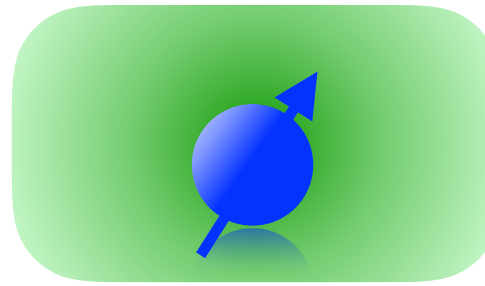

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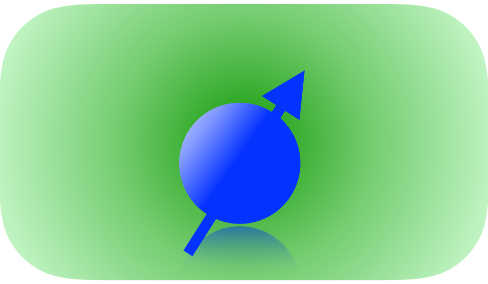
Spectral density $J(\omega) = \lambda \frac{\omega^s}{\Omega^{s-1}} e^{-\omega/\Omega}$

- s → Ohmicity parameter
- Ω → cut-off frequency
- λ → (weak) coupling strength

$s < 1$	$s = 1$	$s > 1$
Sub-ohmic	Ohmic	Super-ohmic

QUANTUM COHERENCE

Coherence trapping



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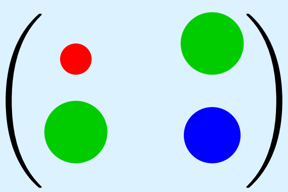
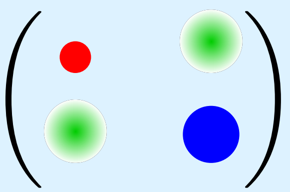
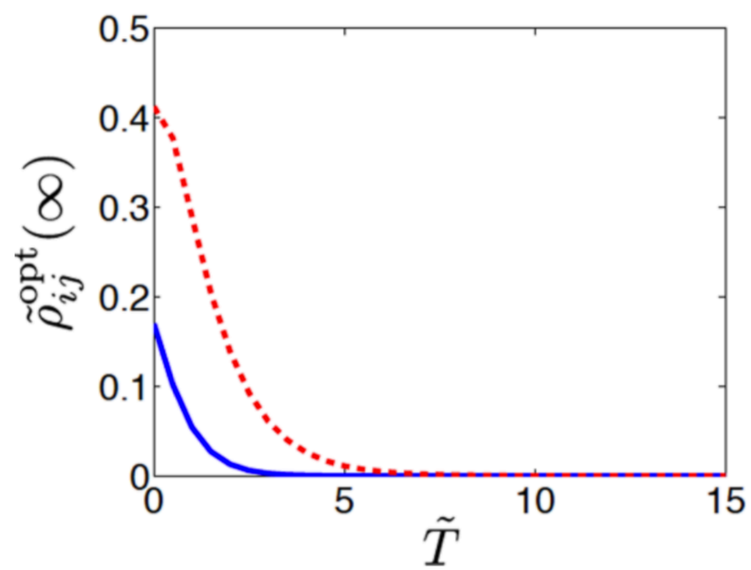
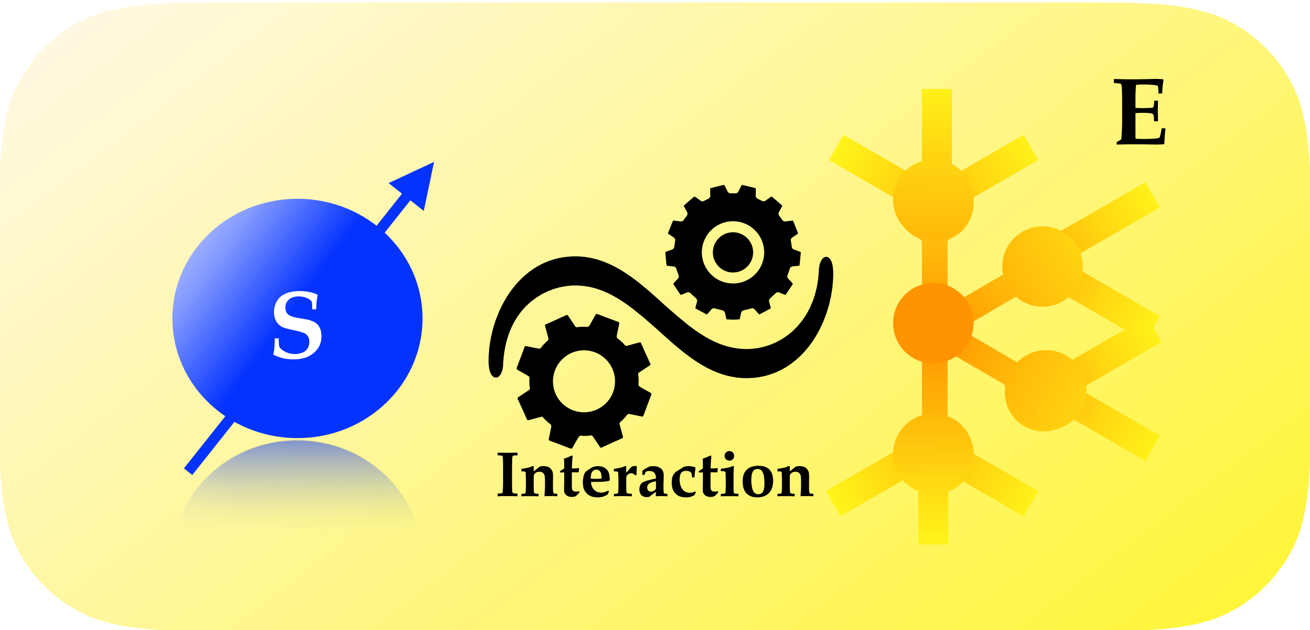
For a sufficiently **coherent initial state**, one can engineer the bath (typically going to low T and super-Ohmic s > 2 regime) to make a **fraction** of the coherences survive

$\rho_S(0) =$  $\longmapsto \rho_S(t) =$ 



AUTONOMOUS GENERATION OF STEADY-STATE COHERENCE

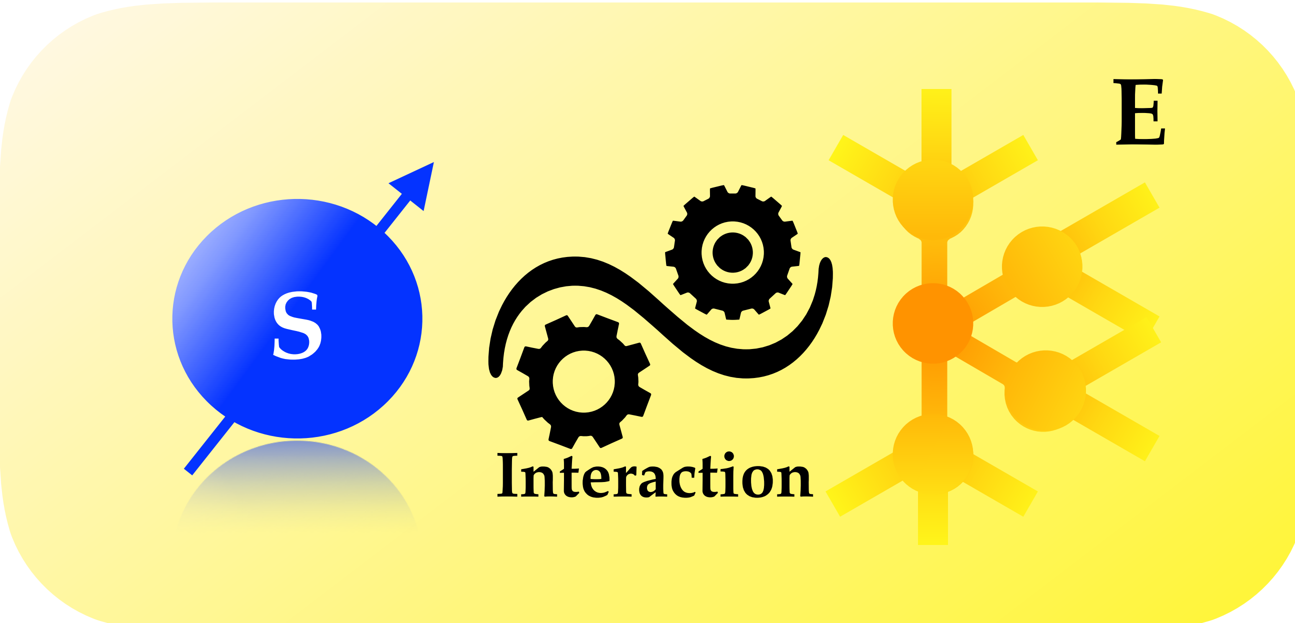
AUTONOMOUS GENERATION OF SSC



G. Guarnieri, M. Kolar, R. Filip, arXiv:1802.08283

$$\mathcal{H}_{\mathrm{S}} = \frac{\omega_0}{2}\sigma_z \qquad \mathcal{H}_{\mathrm{E}} = \sum_k \omega_k b_k^\dagger b_k$$

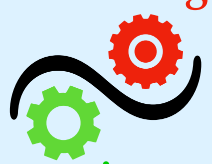
AUTONOMOUS GENERATION OF SSC



G. Guarnieri, M. Kolar, R. Filip, **arXiv:1802.08283**

$$\mathcal{H}_\mathrm{S} = \frac{\omega_0}{2}\sigma_z \qquad \mathcal{H}_\mathrm{E} = \sum_k \omega_k b_k^\dagger b_k$$

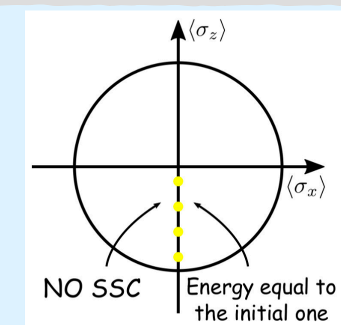
Non-commuting term



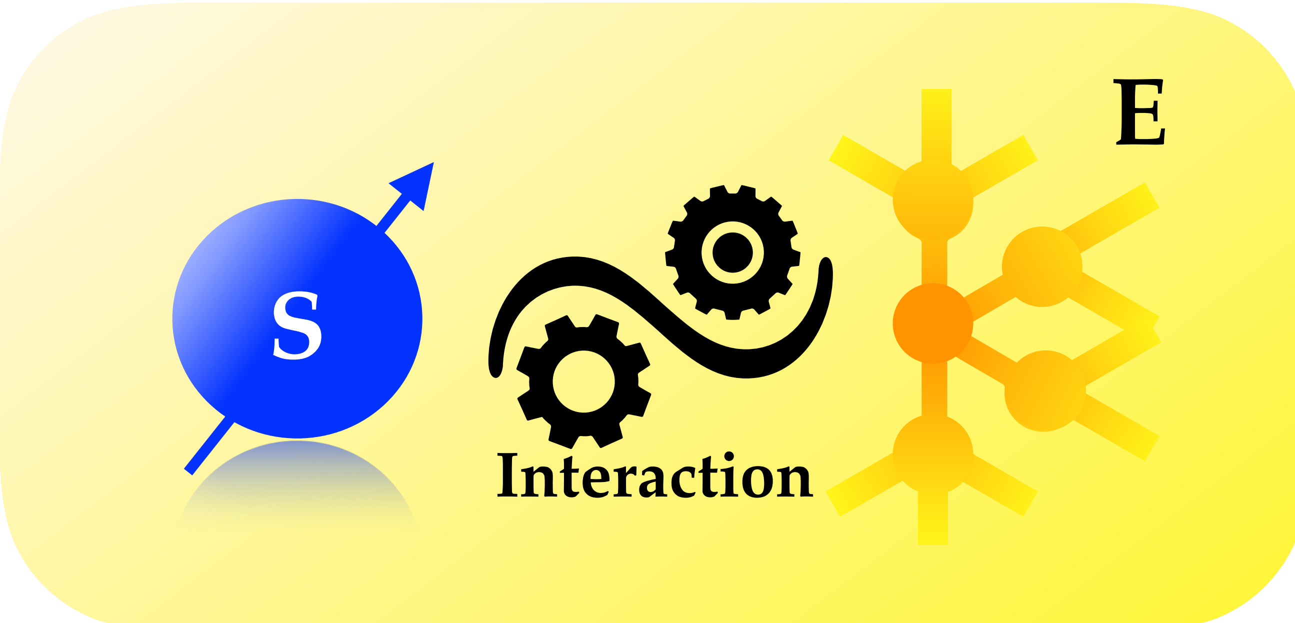
Commuting term

$$\mathcal{H}_\mathrm{SE} = \sigma_z \otimes B_E$$

$$\overline{\rho}_S = \begin{pmatrix} \bullet & 0 \\ 0 & \bullet \end{pmatrix}$$



AUTONOMOUS GENERATION OF SSC



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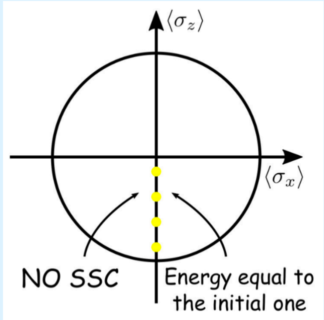
Non-commuting term

Commuting term

$$\mathcal{H}_{\rm SE} = \sigma_z \otimes B_E$$

$$\bar{\rho}_S = \begin{pmatrix} \bullet & 0 \\ 0 & \bullet \end{pmatrix}$$

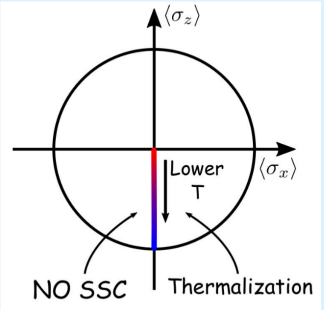


Non-commuting term

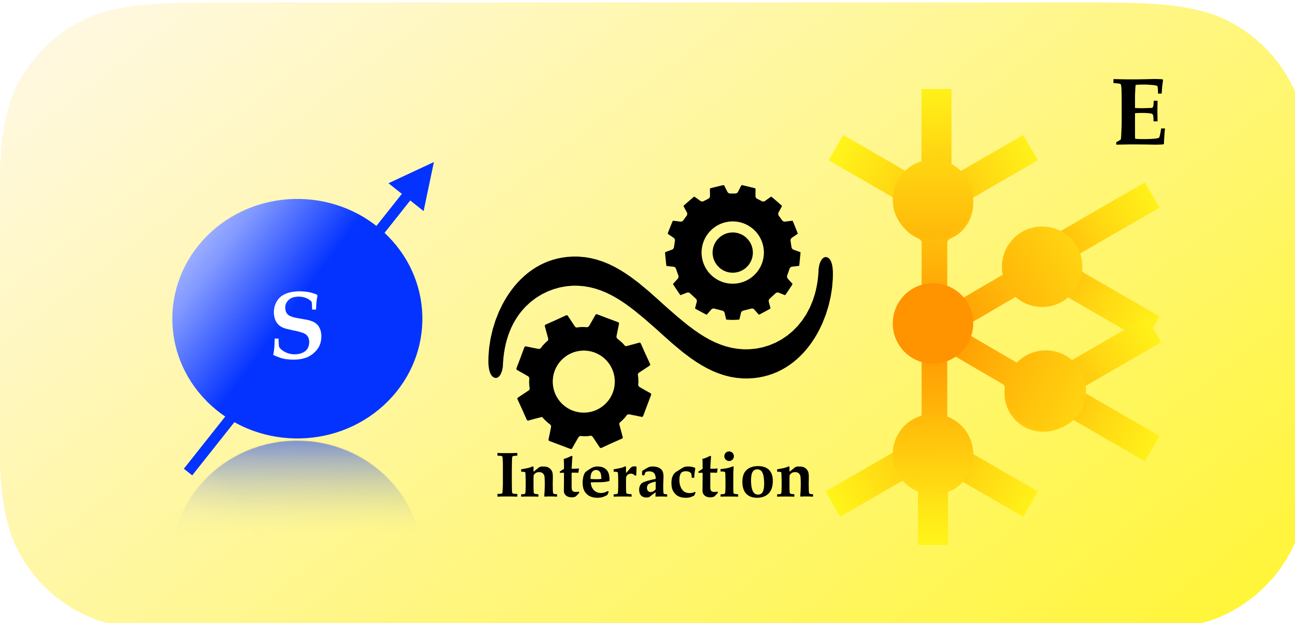
Commuting term

$$\mathcal{H}_{\rm SE} = \sigma_x \otimes B_E$$

$$\bar{\rho}_S = \begin{pmatrix} \bullet & 0 \\ 0 & \bullet \end{pmatrix}$$



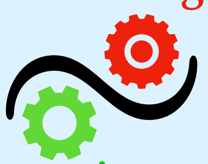
AUTONOMOUS GENERATION OF SSC



G. Guarnieri, M. Kolar, R. Filip, arXiv:1802.08283

$$\mathcal{H}_{\mathrm{S}} = \frac{\omega_0}{2}\sigma_z \qquad \mathcal{H}_{\mathrm{E}} = \sum_k \omega_k b_k^\dagger b_k$$

Non-commuting term



Commuting term

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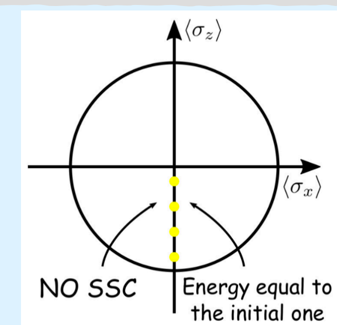

Non-commuting term

Commuting term

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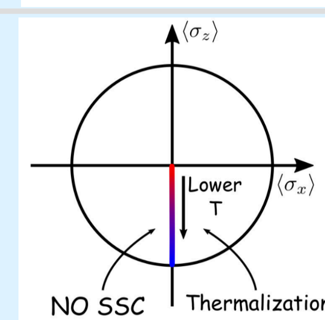
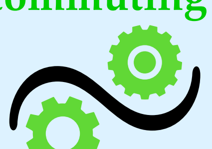


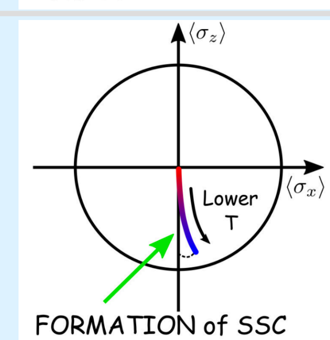
Non-commuting term



Commuting term

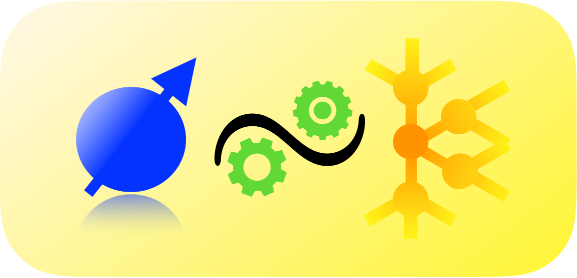
$$\mathcal{H}_{\mathrm{SE}} = (f_1\sigma_z + f_2\sigma_x) \otimes B_E$$

$$\bar{\rho}_S = \begin{pmatrix} \bullet & \bullet \\ \bullet & \bullet \end{pmatrix}$$



AUTONOMOUS GENERATION OF SSC

First Model



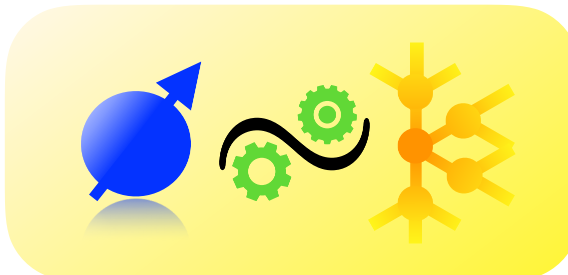
G. Guarnieri, M. Kolar, R. Filip, arXiv:1802.08283

$$\mathcal{H} = \frac{\omega_0}{2}\sigma_z + \sum_k \omega_k b_k^\dagger b_k + (f_1\sigma_z + f_2\sigma_x) \otimes B_E \qquad B_E = \sum_k g_k \left(b_k + b_k^\dagger\right) \qquad \rho_\beta = \frac{e^{-\beta\mathcal{H}_E}}{\mathrm{Tr}_E\left[e^{-\beta\mathcal{H}_E}\right]}$$

Weak coupling, no initial correlations → Time-dependent **GKSL** master equation

AUTONOMOUS GENERATION OF SSC

First Model



G. Guarnieri, M. Kolar, R. Filip, arXiv:1802.08283

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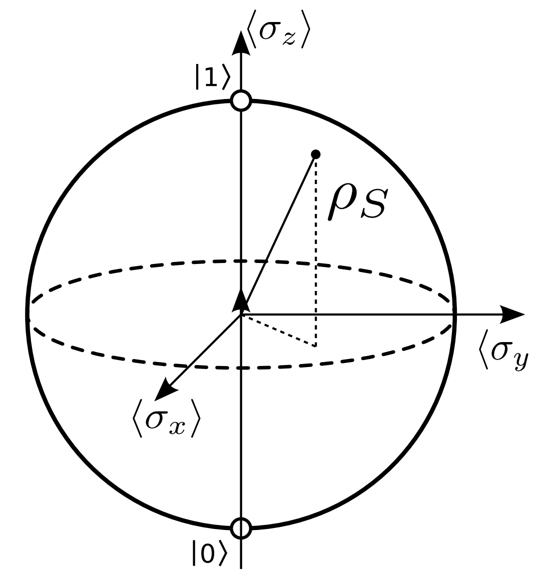
Weak coupling, no initial correlations → Time-dependent **GKSL** master equation

$$\overline{v}_1 = \frac{f_1 f_2\left[\Delta_1 \tanh\left(\frac{\omega_0}{2T}\right) + 4\lambda\Omega\Gamma(s) + \Delta_2\right]}{\omega_0 + f_2^2\Delta_1}$$

$$\overline{v}_2 = 0$$



$$\Delta_1 = -2\int_0^{+\infty} d\omega\, J(\omega)\coth\left(\frac{\omega}{2T}\right)\left[\mathcal{P}\frac{1}{\omega+\omega_0} - \mathcal{P}\frac{1}{\omega-\omega_0}\right]$$

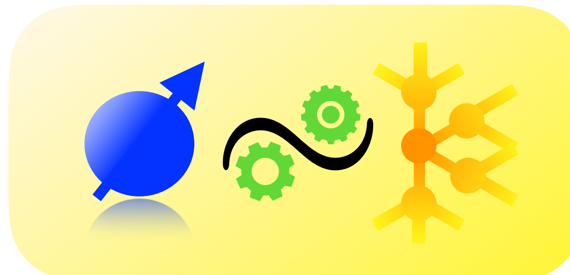
$$\Delta_2 = -2\int_0^{+\infty} d\omega\, J(\omega)\left[\mathcal{P}\frac{1}{\omega+\omega_0} + \mathcal{P}\frac{1}{\omega-\omega_0}\right]$$

$$J(\omega) = \lambda\,\frac{\omega^s}{\Omega^{s-1}}e^{-\omega/\Omega}$$

Spectral Density

AUTONOMOUS GENERATION OF SSC

First Model



G. Guarnieri, M. Kolar, R. Filip, arXiv:1802.08283

$$\mathcal{H} = \frac{\omega_0}{2}\sigma_z + \sum_k \omega_k b_k^\dagger b_k + (f_1\sigma_z + f_2\sigma_x) \otimes B_E \qquad B_E = \sum_k g_k \left(b_k + b_k^\dagger\right) \qquad \rho_\beta = \frac{e^{-\beta\mathcal{H}_E}}{\mathrm{Tr}_E\left[e^{-\beta\mathcal{H}_E}\right]}$$

Weak coupling, no initial correlations → Time-dependent **GKSL** master equation

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Autonomous generation of SSC independently of the initial state

$$\Delta_1 = -2\int_0^{+\infty} d\omega\, J(\omega) \coth\left(\frac{\omega}{2T}\right)\left[\mathcal{P}\frac{1}{\omega+\omega_0} - \mathcal{P}\frac{1}{\omega-\omega_0}\right]$$

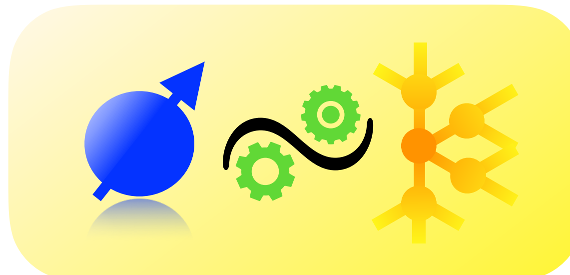
$$\Delta_2 = -2\int_0^{+\infty} d\omega\, J(\omega)\left[\mathcal{P}\frac{1}{\omega+\omega_0} + \mathcal{P}\frac{1}{\omega-\omega_0}\right]$$

$$J(\omega) = \lambda\,\frac{\omega^s}{\Omega^{s-1}}e^{-\omega/\Omega}$$

Spectral Density

AUTONOMOUS GENERATION OF SSC

First Model

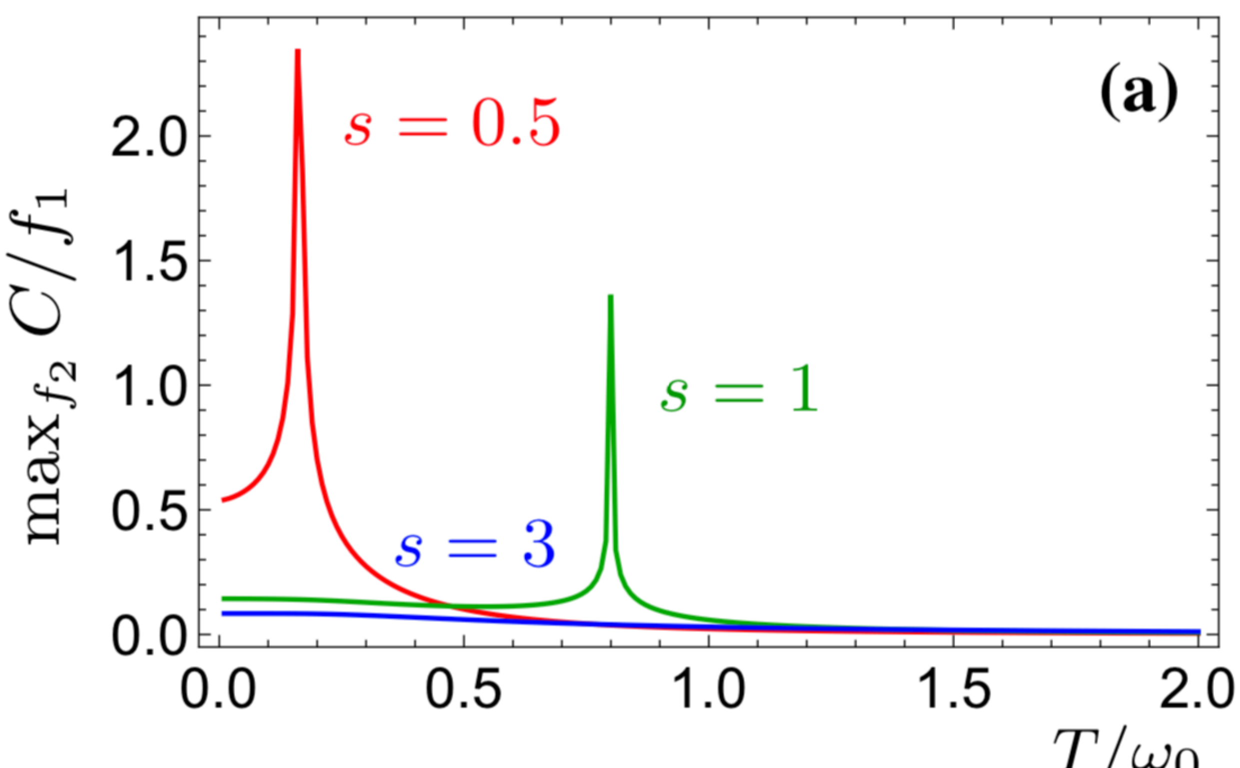


G. Guarnieri, M. Kolar, R. Filip, arXiv:1802.08283

l_1- measure of coherence

maximized over the two coupling strengths

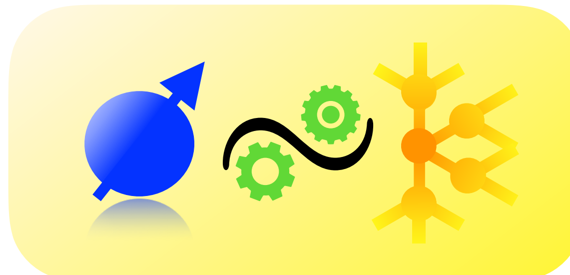
$$\max_{f_2} \frac{\mathcal{C}_{l_1}}{f_1} = \left| \frac{\left[\Delta_1 \tanh\left(\frac{\omega_0}{2T}\right) + 4\lambda\Omega\Gamma(s) + \Delta_2\right]}{2\sqrt{\omega_0 \Delta_1}} \right|$$



General amplification of the SSC for decreasing bath temperature

AUTONOMOUS GENERATION OF SSC

First Model



G. **Guarnieri**, M. Kolar, R. Filip, arXiv:1802.08283

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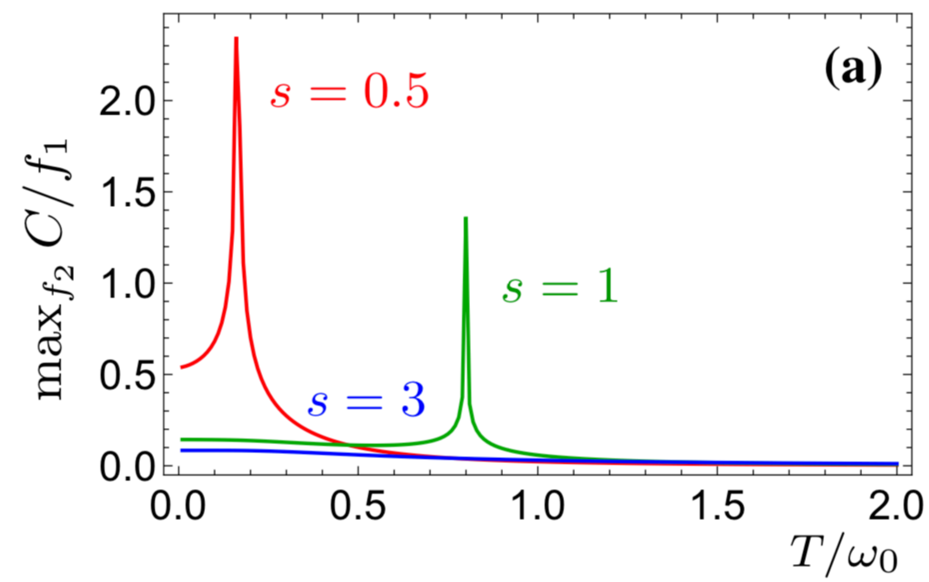
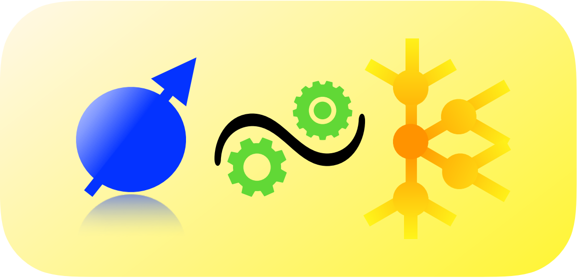
Ohmic (s=1) and sub-Ohmic (s<1) bath



AUTONOMOUS GENERATION OF SSC

First Model



G. Guarnieri, M. Kolar, R. Filip, **arXiv:1802.08283**

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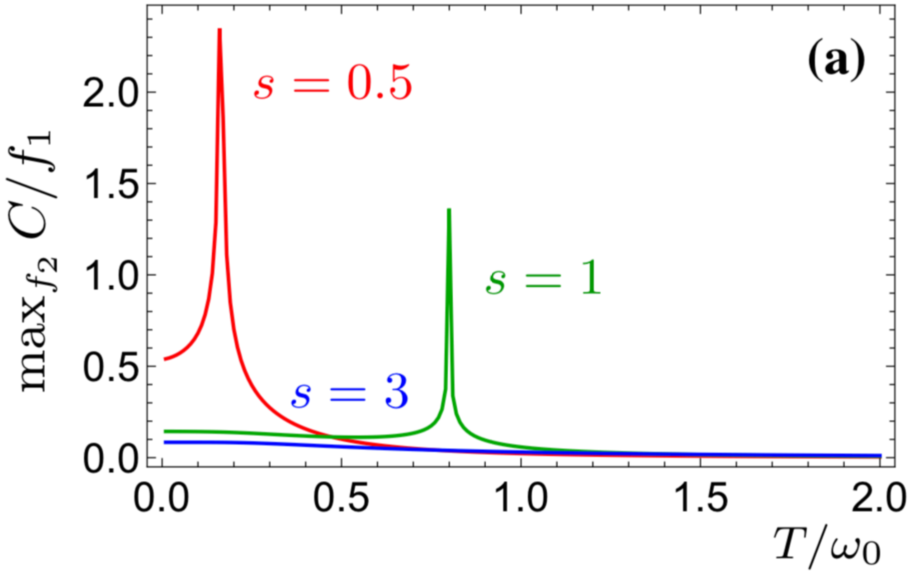
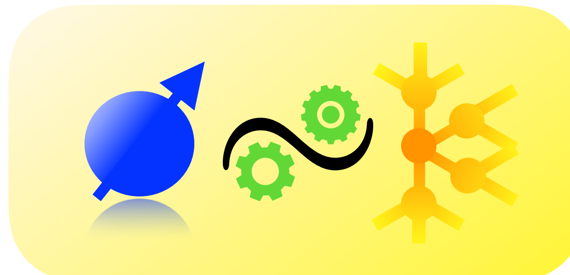


Ohmic (s=1) and sub-Ohmic (s<1) bath

First order pole at $\omega = \omega_0$ for $\Delta_1 = -2\int_0^{+\infty} d\omega\, J(\omega) \coth\left(\frac{\omega}{2T}\right) \left[\mathcal{P}\frac{1}{\omega+\omega_0} - \mathcal{P}\frac{1}{\omega-\omega_0}\right]$

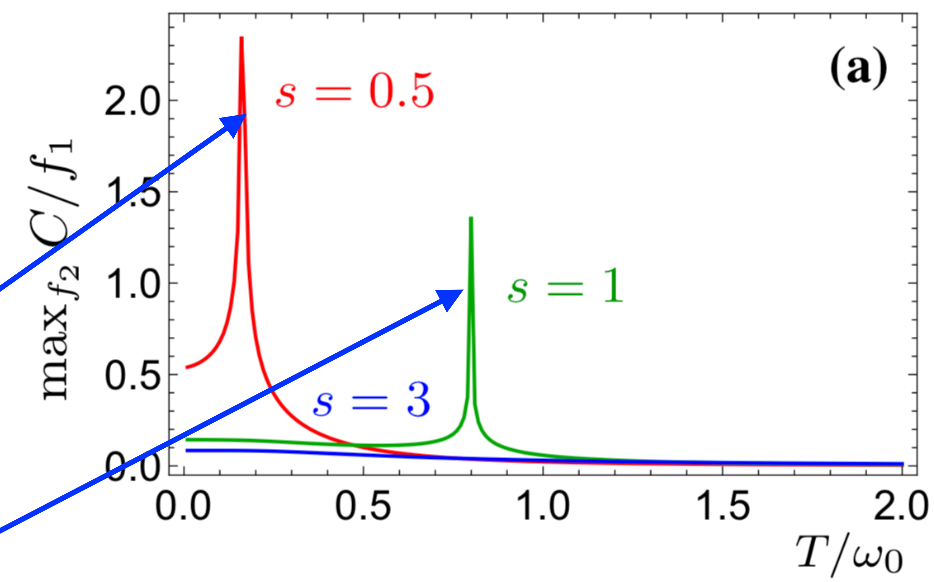
AUTONOMOUS GENERATION OF SSC

First Model



G. Guarnieri, M. Kolar, R. Filip, **arXiv:1802.08283**

$$\max_{f_2} \frac{\mathcal{C}_{l_1}}{f_1} = \left| \frac{\left[\Delta_1 \tanh\left(\frac{\omega_0}{2T}\right) + 4\lambda\Omega\Gamma(s) + \Delta_2\right]}{2\sqrt{\omega_0 \Delta_1}} \right|$$



Ohmic (s=1) and sub-Ohmic (s<1) bath

First order pole at $\omega = \omega_0$ for $\Delta_1 = -2\int_0^{+\infty} d\omega\, J(\omega) \coth\left(\frac{\omega}{2T}\right) \left[\mathcal{P}\frac{1}{\omega+\omega_0} - \mathcal{P}\frac{1}{\omega-\omega_0}\right]$

$J_{\text{eff}}(\omega, \Omega, T)$ Effective spectral density

Spike of the amount of SSC on the **Resonance Curve** $\partial_\omega J_{\text{eff}}(\omega,\Omega,T)|_{\omega=\omega_0} = 0$

AUTONOMOUS GENERATION OF SSC

Resonance condition $\partial_\omega J_{\text{eff}}(\omega, \Omega, T)|_{\omega=\omega_0} = 0$: what is it ?

If we define $\omega_{\max}$ to be the dominant environmental mode, namely the one satisfying

$$\partial_\omega J_{\text{eff}}(\omega, \Omega, T)|_{\omega=\omega_{\max}} = 0$$

then the resonance condition means the match $\omega_0 = \omega_{\max}$ and thus the system

mainly interacts on resonance with a locally flat spectrum

AUTONOMOUS GENERATION OF SSC

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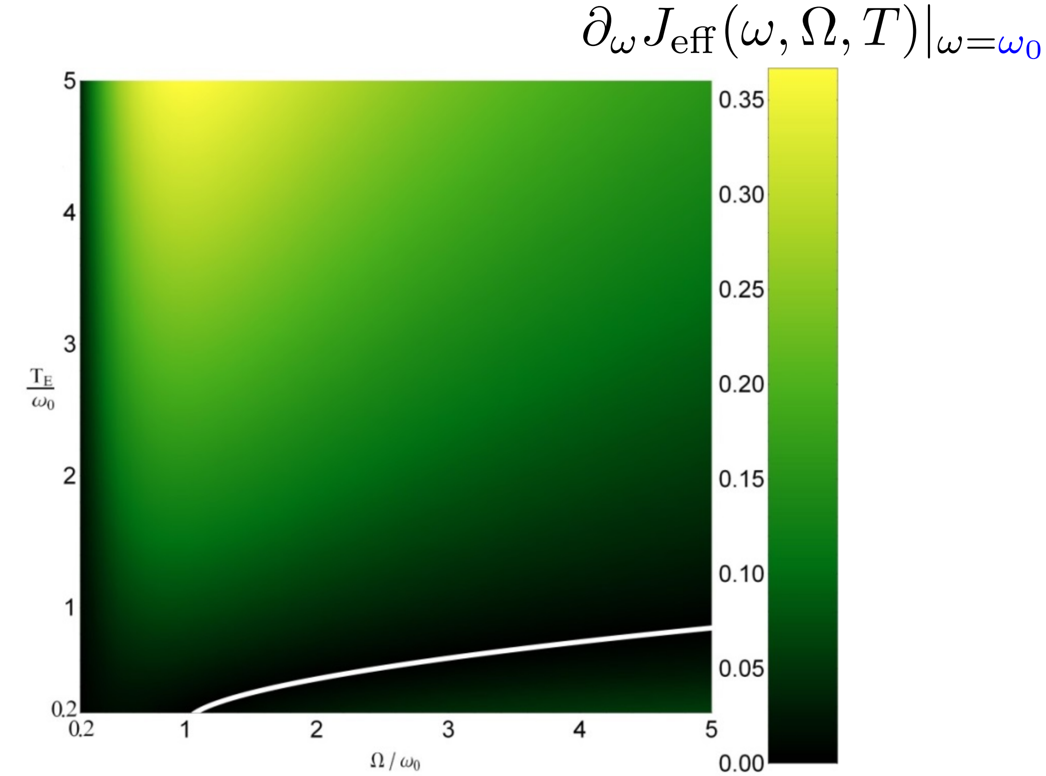
e.g. for an Ohmic bath

$$\Omega_{\text{res}}(T)=\frac{T}{T/\omega_0-\text{Cosech}\,(\omega_0/T)}.$$

G. Guarnieri, C. Uchiyama, B. Vacchini, PRA **93**, 012118 (2016)



AUTONOMOUS GENERATION OF SSC

Resonance condition $\partial_\omega J_{\text{eff}}(\omega, \Omega, T)|_{\omega=\omega_0} = 0$: what is it ?

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$$\partial_\omega J_{\text{eff}}(\omega, \Omega, T)|_{\omega=\omega_{\text{max}}} = 0$$

then the resonance condition means the match $\omega_0 = \omega_{\text{max}}$ and thus the system mainly interacts on resonance with a locally flat spectrum

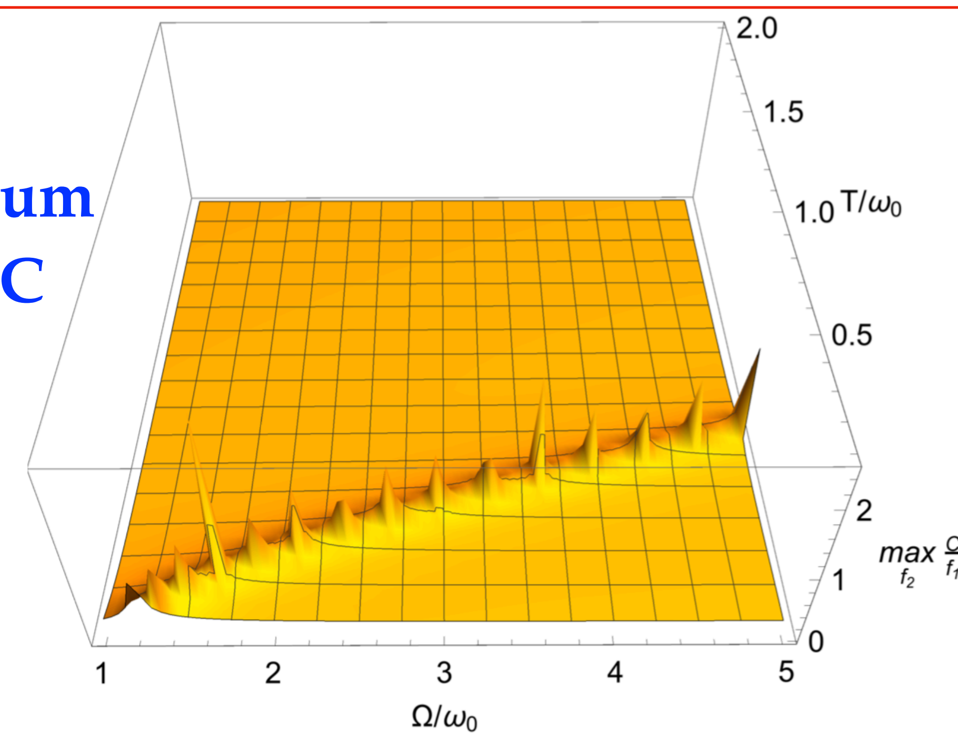
G. Guarnieri, M. Kolar, R. Filip, arXiv:1802.08283

e.g. for an Ohmic bath

$$\Omega_{\text{res}}(T) = \frac{T}{T/\omega_0 - \text{Cosech}\,(\omega_0/T)}.$$

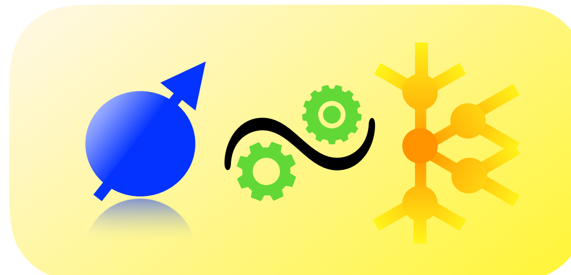
G. Guarnieri, C. Uchiyama, B. Vacchini, PRA **93**, 012118 (2016)

Maximum of SSC



AUTONOMOUS GENERATION OF SSC

Second Model



G. Guarnieri, M. Kolar, R. Filip, **arXiv:1802.08283**

$$\mathcal{H} = \frac{\omega_0}{2}\sigma_z + \sum_k \omega_k b_k^\dagger b_k + f_1 \sigma_z \otimes B_E + f_2 \left(\sigma_+ \otimes b_E + \sigma_- \otimes b_E^\dagger\right)$$

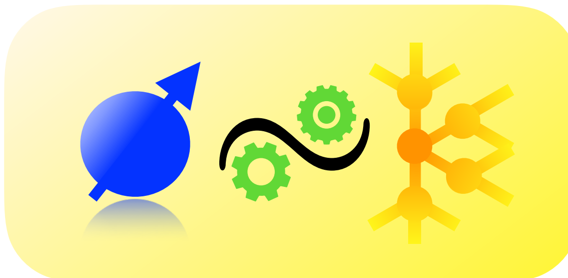
$$b_E = \sum_k g_k b_k$$

$$B_E = b_E + b_E^\dagger$$

Weak coupling, no initial correlations → Time-dependent **GKSL** master equation

AUTONOMOUS GENERATION OF SSC

Second Model



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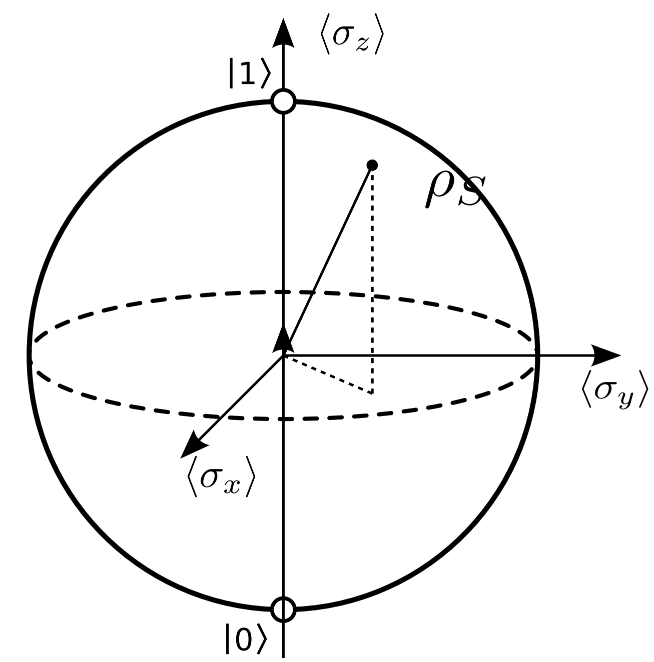
$$B_E = b_E + b_E^\dagger$$

Weak coupling, no initial correlations → Time-dependent **GKSL** master equation

$$\overline{v}_1 = 2 f_1 f_2 \alpha(\omega_0, T, s) \mathrm{Re}\left[\frac{1}{(\omega_0 + f_2^2 \delta_1) - i\left(\pi f_2^2 J_{\text{eff}}(\omega_0, T)\right)}\right]$$

$$\overline{v}_2 = 2 f_1 f_2 \alpha(\omega_0, T, s) \mathrm{Im}\left[\frac{1}{(\omega_0 + f_2^2 \delta_1) - i\left(\pi f_2^2 J_{\text{eff}}(\omega_0, T)\right)}\right]$$

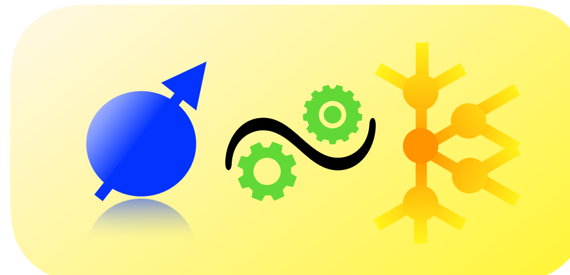


$$\alpha(\omega_0, T, s) \equiv \delta_1 \tanh\left(\frac{\omega_0}{2T}\right) + \delta_2 + \lambda\Omega\Gamma(s)$$

$$\delta_1 = \mathcal{P}\int_0^{+\infty} d\omega\, \frac{J_{\text{eff}}(\omega, T)}{\omega - \omega_0} \qquad \delta_2 = \mathcal{P}\int_0^{+\infty} d\omega\, \frac{J(\omega)}{\omega - \omega_0}$$

AUTONOMOUS GENERATION OF SSC

Second Model



G. Guarnieri, M. Kolar, R. Filip, **arXiv:1802.08283**

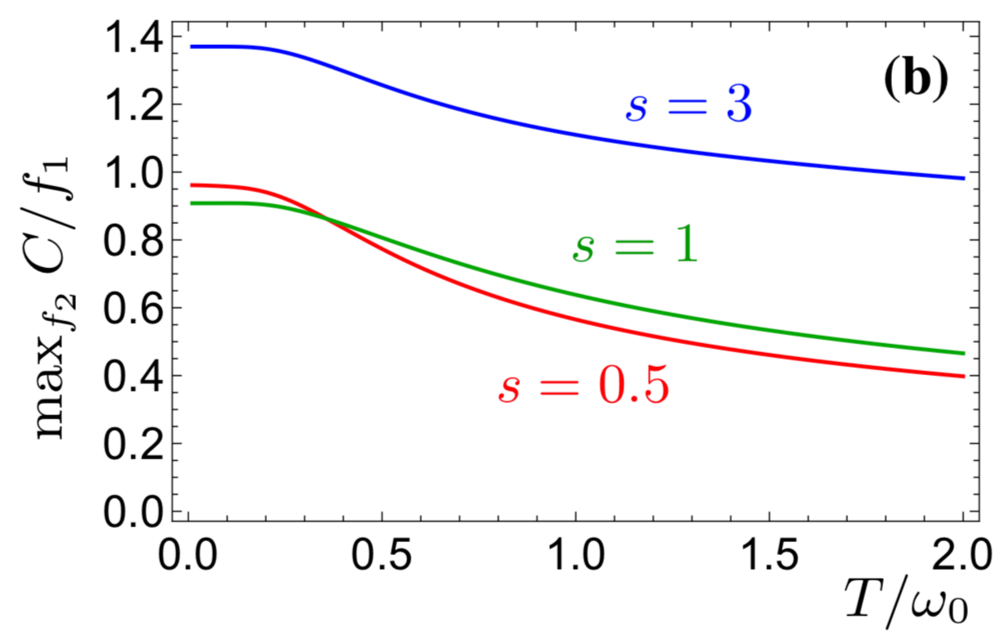
$$\mathcal{H} = \frac{\omega_0}{2}\sigma_z + \sum_k \omega_k b_k^\dagger b_k + f_1 \sigma_z \otimes B_E + f_2 \left(\sigma_+ \otimes b_E + \sigma_- \otimes b_E^\dagger\right)$$

$$b_E = \sum_k g_k b_k$$

$$B_E = b_E + b_E^\dagger$$

Weak coupling, no initial correlations → Time-dependent **GKSL** master equation

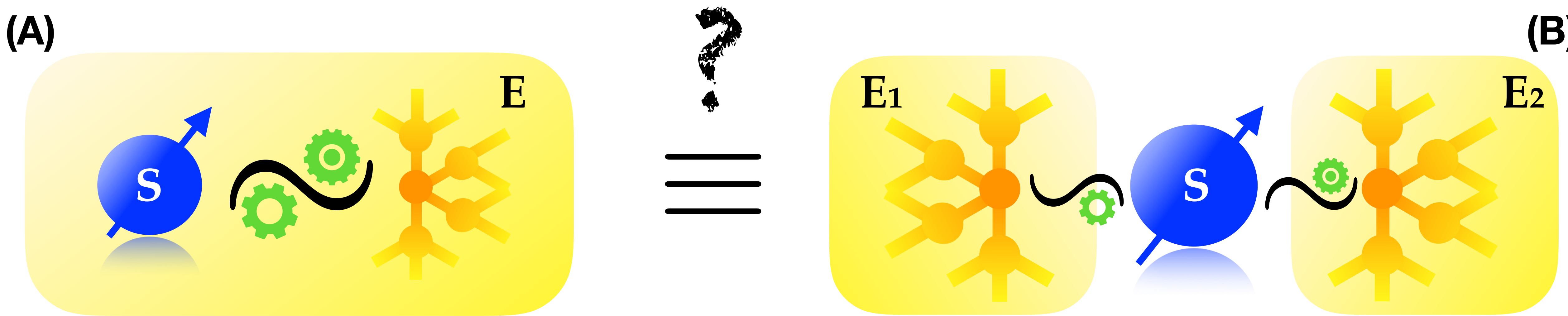
$$\mathcal{C}_{l_1} = \frac{|2 f_1 f_2 \alpha(\omega_0, T, s)|}{\sqrt{\left(\omega_0 + f_2^2 \delta_1\right)^2 + \left(\pi f_2^2 J_{\text{eff}}(\omega_0, T)\right)^2}}$$



General amplification of the SSC for decreasing bath temperature

AUTONOMOUS GENERATION OF SSC

Splitting of the interaction to two independent baths



$$\mathcal{H} = \mathcal{H}_S + \sum_{i=1}^{2} \mathcal{H}_{E_i} + \mathcal{H}_{SE_1} + \mathcal{H}_{SE_2} = \frac{\omega_0}{2}\sigma_z + \sum_{i=1}^{2}\sum_{k_i} \omega_{k_i} b_{k_i}^{\dagger} b_{k_i} + f_1 \sigma_z \otimes B_{E_1} + f_2 \sigma_x \otimes B_{E_2}$$

Weak coupling, no initial correlations

$$\rho(0) = \rho_S(0) \otimes \rho_{\beta_1} \otimes \rho_{\beta_2}$$

$$\rho_{\beta_i} = \frac{e^{-\beta_i \mathcal{H}_{E_i}}}{\mathrm{Tr}_{E_i}\left[e^{-\beta_i \mathcal{H}_{E_i}}\right]}$$

G. Guarnieri, M. Kolar, R. Filip, arXiv:1802.08283

AUTONOMOUS GENERATION OF SSC

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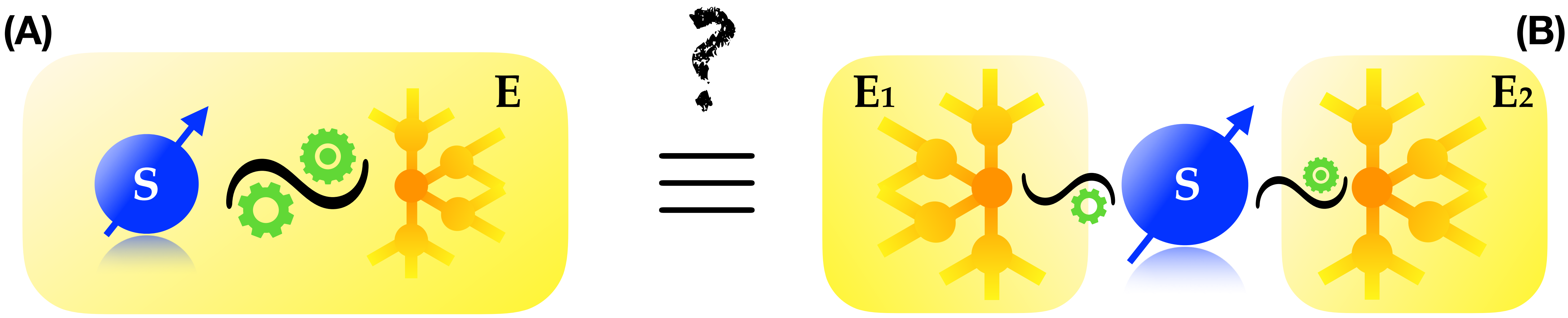


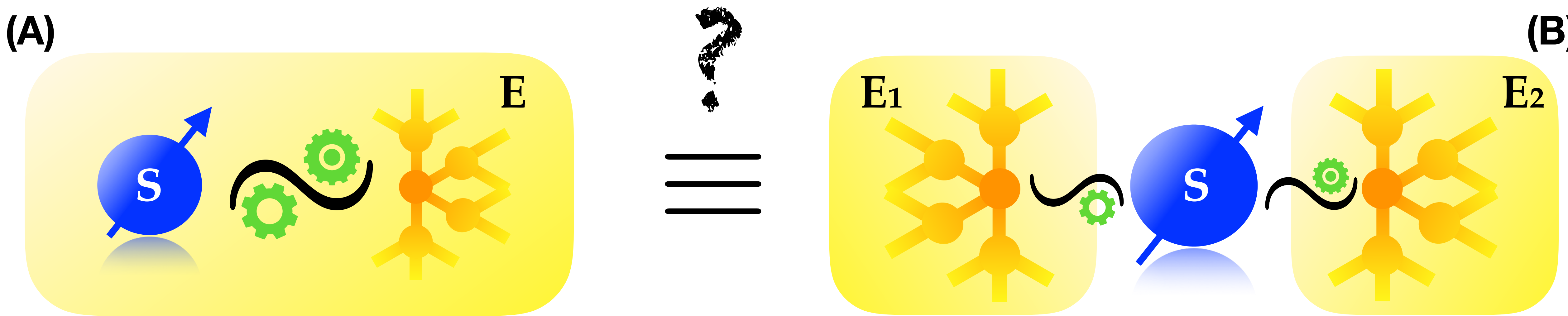
$$\mathcal{H} = \mathcal{H}_S + \sum_{i=1}^{2} \mathcal{H}_{E_i} + \mathcal{H}_{SE_1} + \mathcal{H}_{SE_2} = \frac{\omega_0}{2}\sigma_z + \sum_{i=1}^{2}\sum_{k_i} \omega_{k_i} b_{k_i}^\dagger b_{k_i} + f_1 \sigma_z \otimes B_{E_1} + f_2 \sigma_x \otimes B_{E_2}$$

$$\mathrm{Tr}_{E_i}\left[\mathcal{H}_{SE_i}(t_1)\mathcal{H}_{SE_i}(t_2)\ldots\mathcal{H}_{SE_i}(t_{2n+1})\rho_{\beta_i}\right] = 0, \qquad \forall n, \; i = 1, 2$$

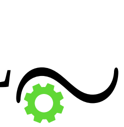
G. Guarnieri, M. Kolar, R. Filip, arXiv:1802.08283

AUTONOMOUS GENERATION OF SSC

Splitting of the interaction to two independent baths



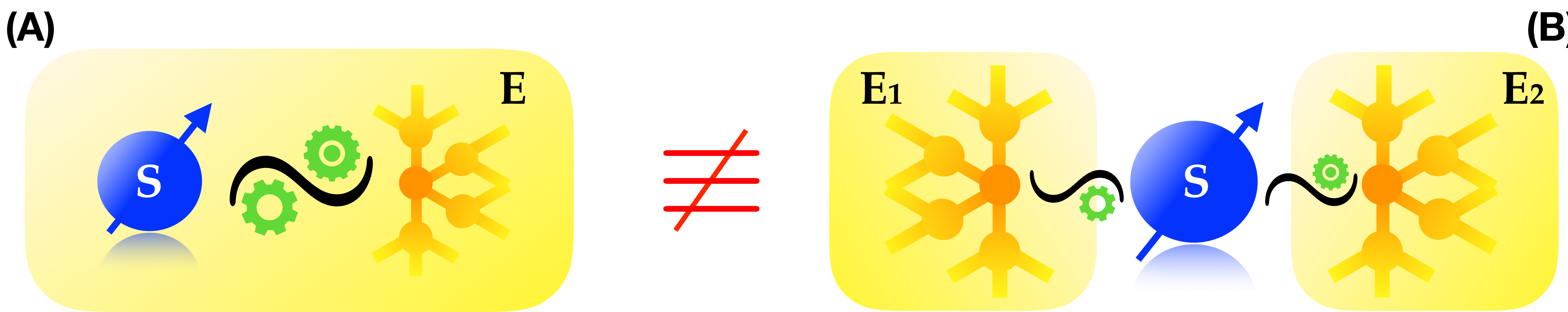
$$\mathrm{Tr}_{E_i}\left[\mathcal{H}_{SE_i}(t_1)\mathcal{H}_{SE_i}(t_2)\ldots\mathcal{H}_{SE_i}(t_{2n+1})\right]=0, \qquad \forall n,\ i=1,2$$

$\mathrm{GKSL}_{\mathbf{(B)}} = \mathrm{GKSL}$  $+\ \mathrm{GKSL}$ 

G. Guarnieri, M. Kolar, R. Filip, arXiv:1802.08283

AUTONOMOUS GENERATION OF SSC

Splitting of the interaction to two independent baths



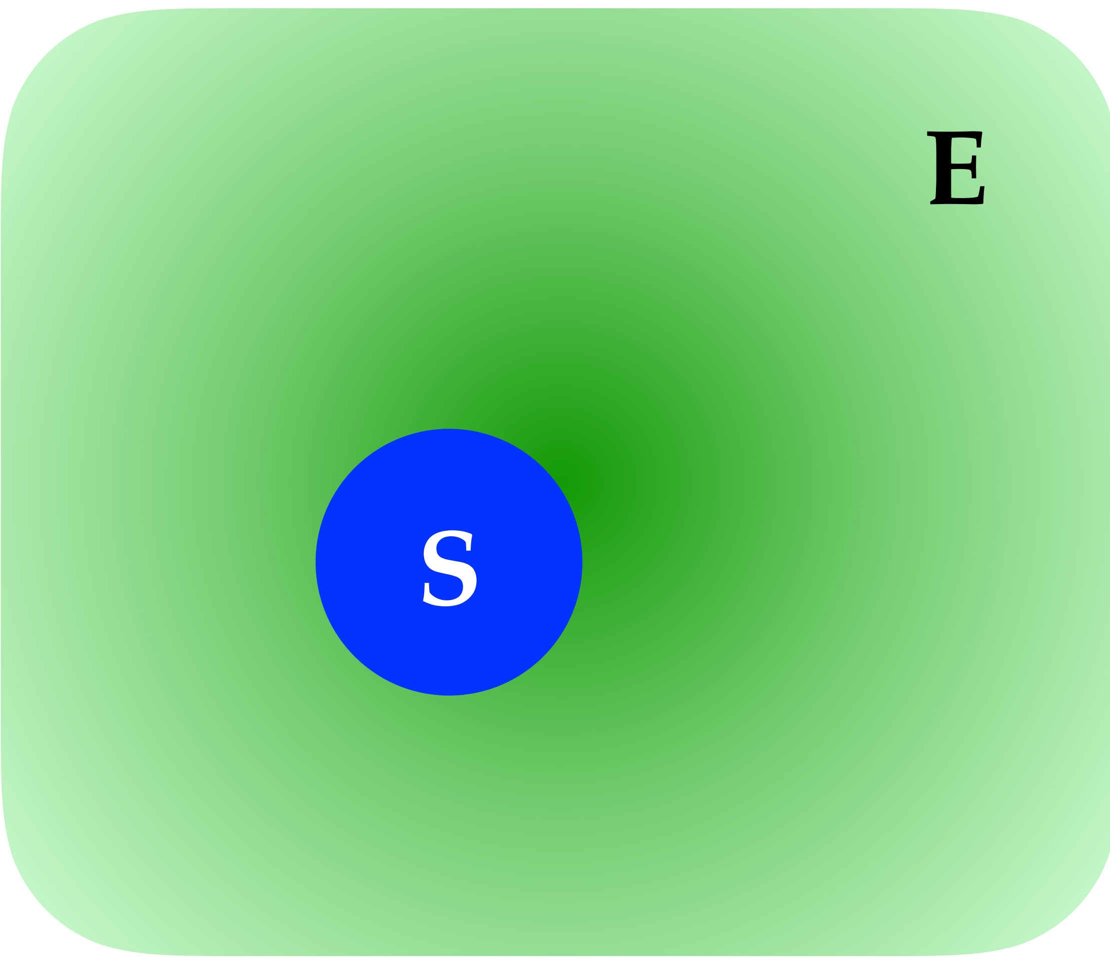
$$\mathrm{Tr}_{E_i}\left[\mathcal{H}_{SE_i}(t_1)\mathcal{H}_{SE_i}(t_2)\dots\mathcal{H}_{SE_i}(t_{2n+1})\right]=0, \qquad \forall n,\ i=1,2$$

$$\mathrm{GKSL}_{\mathbf{(B)}} = \mathrm{GKSL}_{\sim} + \mathrm{GKSL}_{\sim} \neq \mathrm{GKSL}_{\mathbf{(A)}}$$

G. Guarnieri, M. Kolar, R. Filip, arXiv:1802.08283

AUTONOMOUS GENERATION OF SSC

Alternative approach: **Equilibration Picture**



If the system is coupled to a large thermal bath,
then it equilibrates to
the local reduced state of the total Gibbs state

$$\bar{\rho}_S = \mathrm{Tr}_E \left[\frac{e^{-\beta \mathcal{H}}}{\mathrm{Tr}_{SE} \left[e^{-\beta \mathcal{H}} \right]} \right]$$

$$\mathcal{H} = \mathcal{H}_S + \mathcal{H}_E + \mathcal{H}_{SE}$$

$$\bar{\rho}_S \neq \frac{e^{-\beta \mathcal{H}_S}}{\mathrm{Tr}_S \left[e^{-\beta \mathcal{H}} \right]}$$

G. W. Ford, J. T. Lewis, R. F. O'Connell, PRL **55**, 2273 (1985)

T. Mori, S. Miyashita, JPSJ **77**, 1 (2008)

AUTONOMOUS GENERATION OF SSC

Alternative approach: **Equilibration Picture**

PROS *	CONS *
P1) Easy numerical computability even for more complex systems	C1) Not straightforward extension to multiple heat baths
P2) Access to corrections to system's energy induced by the SSC	C2) No information about the dynamical properties
P3) Access to the strong coupling regime	

** with respect to the GKSL analysis

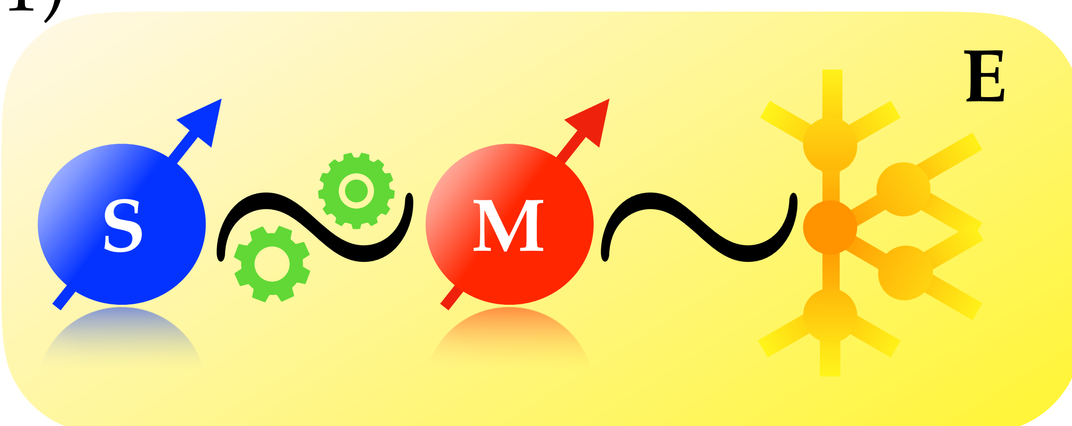
G. Guarnieri, M. Kolar, R. Filip, arXiv:1802.08283

AUTONOMOUS GENERATION OF SSC

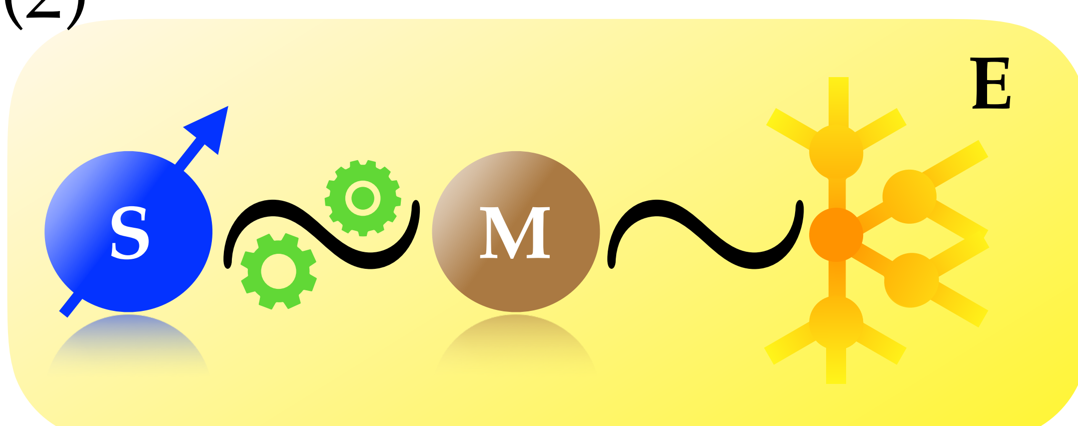
Equilibration Picture

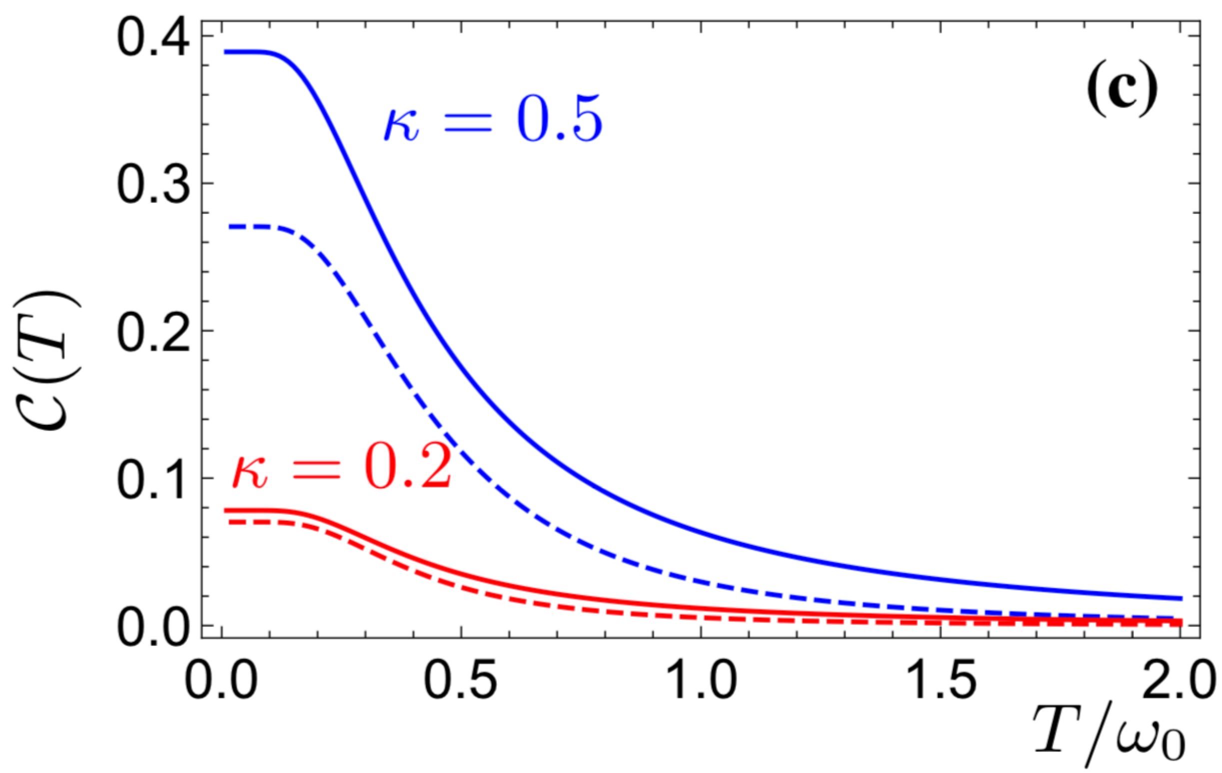
G. Guarnieri, M. Kolar, R. Filip, arXiv:1802.08283

(1)



(2)





Autonomous generation of SSC with the same properties as in the other two models considered before

CONCLUSIONS AND OUTLOOKS

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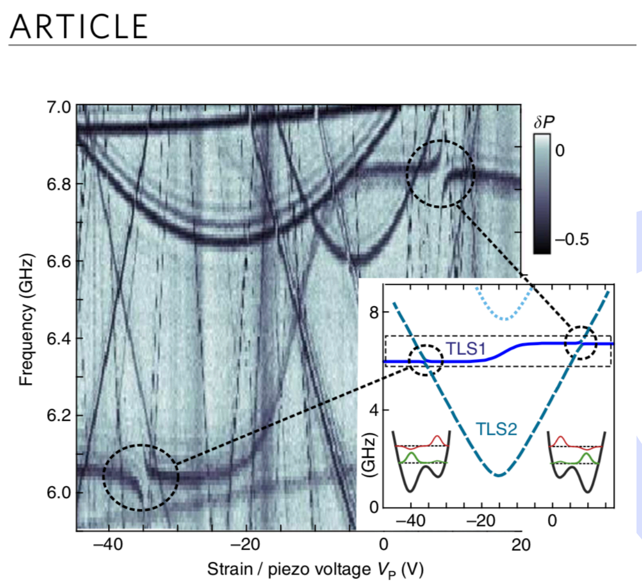
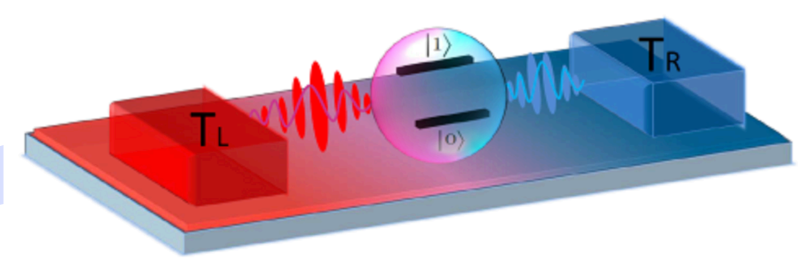


J. Lisenfeld, et al., Nat. Commun. 6 182 (2015)



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FOR MANY MORE INFORMATION AND DETAILS, SEE arXiv:1802.08283

THANK YOU
FOR YOUR ATTENTION

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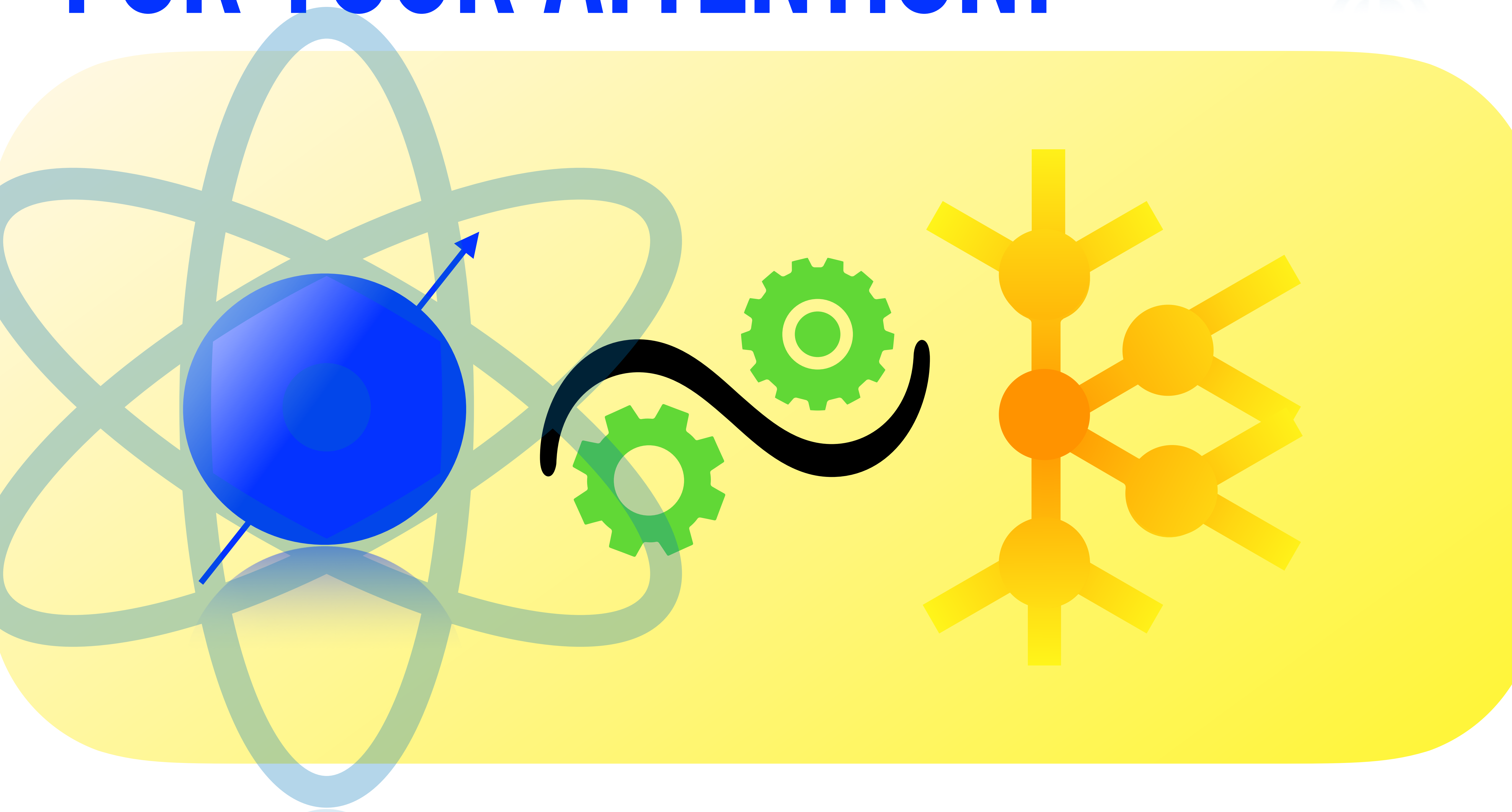

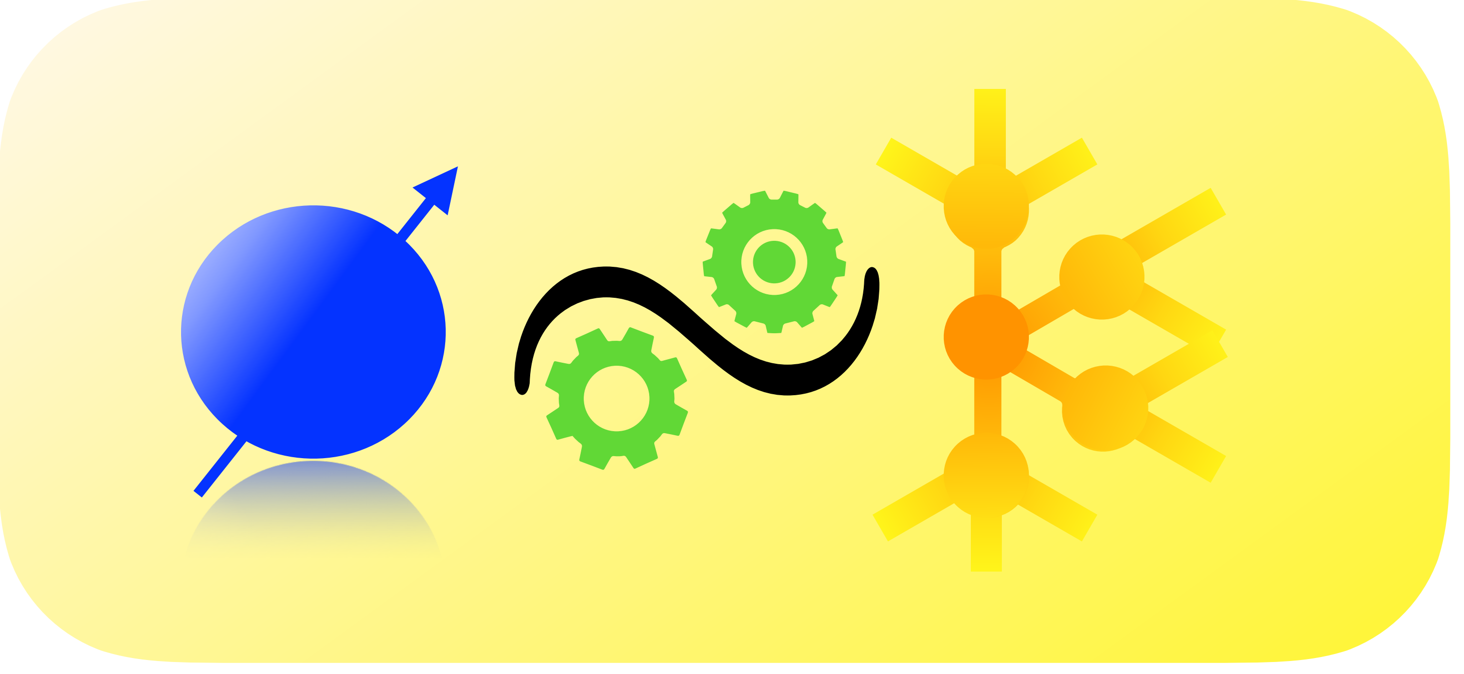

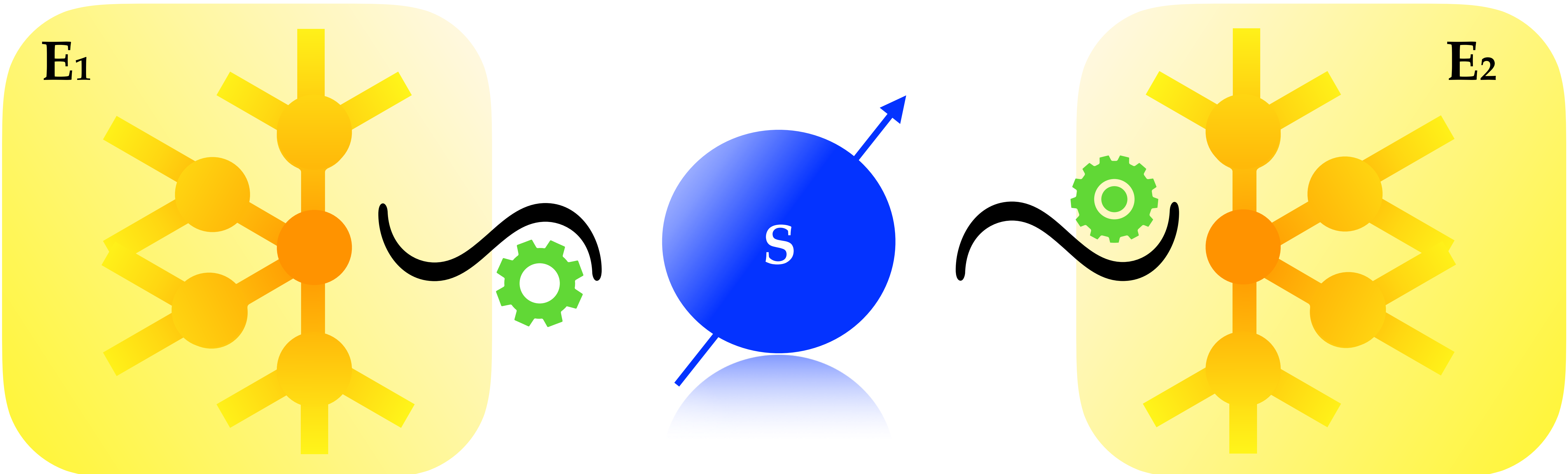
E1
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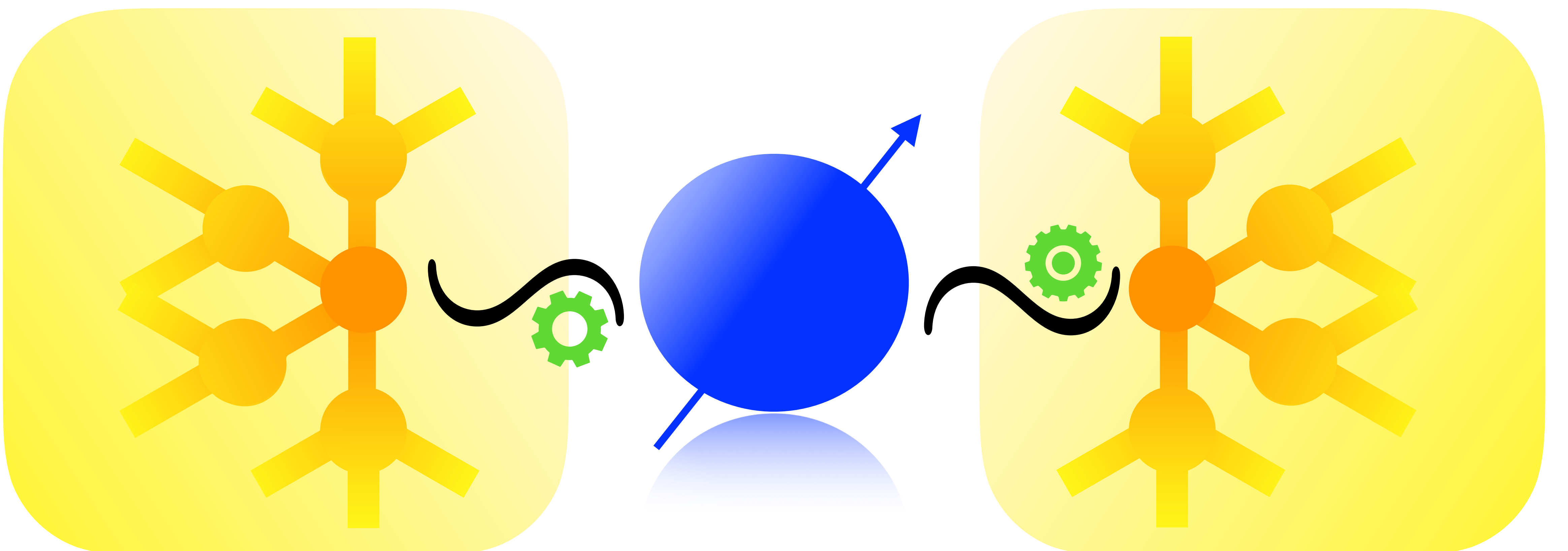


AUTONOMOUS GENERATION OF SSC

Resonance condition: what is it ?

If we define $\omega_{\max}$ to be the dominant environmental mode, namely the one satisfying

$$\partial_\omega J_{\text{eff}}(\omega, \Omega, T)|_{\omega=\omega_{\max}} = 0$$

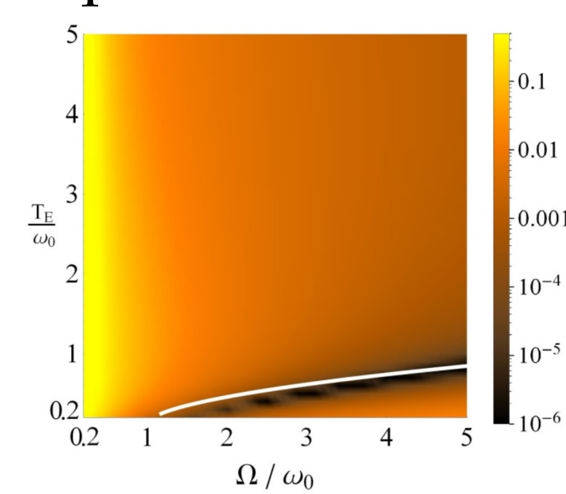
then the resonance condition means the match $\omega_0 = \omega_{\max}$ and thus the system

mainly interacts on resonance with a locally flat spectrum

= locally equivalent to white noise

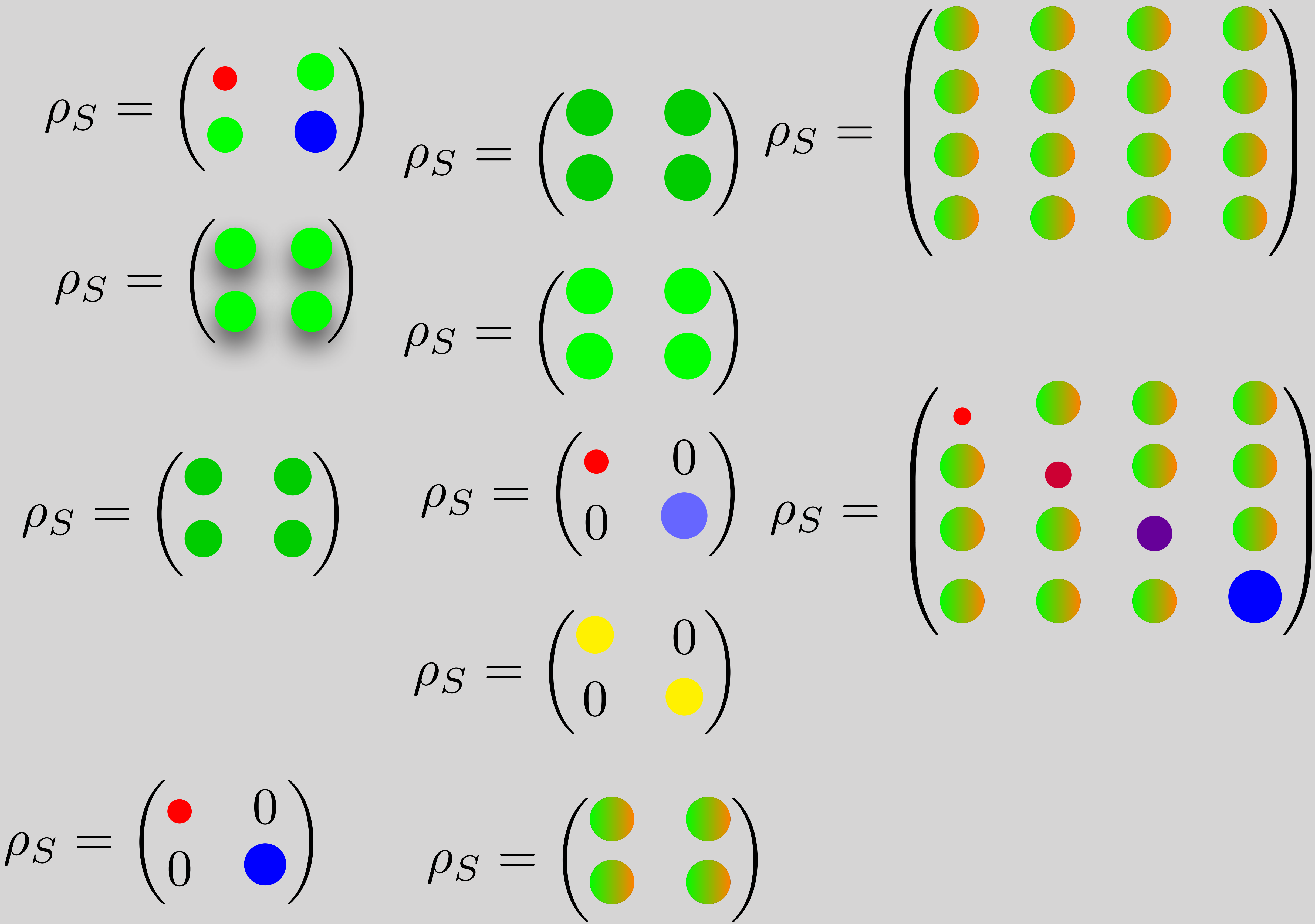
e.g. for an Ohmic bath

$$\Omega_{\text{res}}(T) = \frac{T}{T/\omega_0 - \text{Cosech}\,(\omega_0/T)}.$$



G. Guarnieri, C. Uchiyama, B. Vacchini, PRA **93**, 012118 (2016)

G. Clos, H. – P. Breuer, PRA **86**, 012115 (2012)


$\rho_S =$
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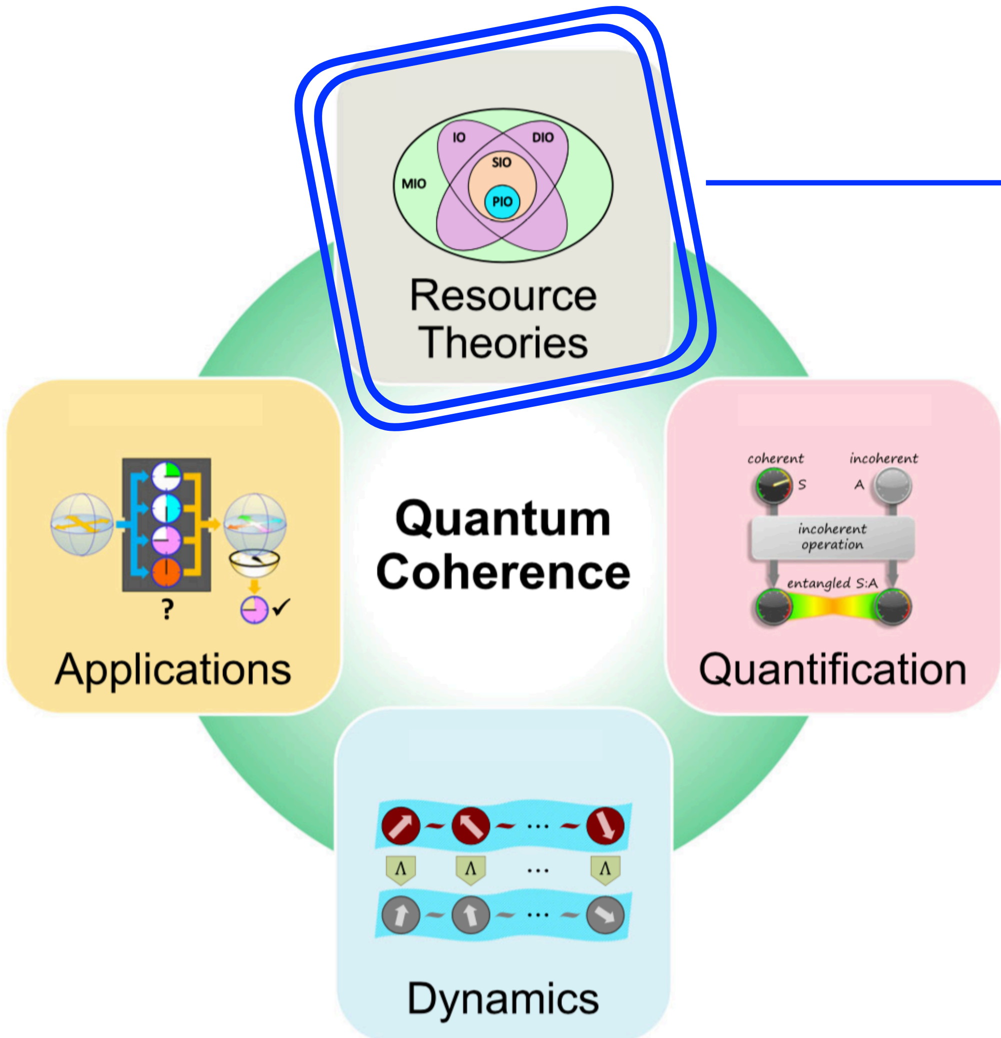
QUANTUM COHERENCE



Orthonormal basis $\{|j\rangle\}_{j=0,\dots,d-1}$

FREE STATES $\rho = \sum_j p_j |j\rangle\langle j|$

$$\rho_S = \begin{pmatrix} \bullet & 0 \\ 0 & \bullet \end{pmatrix} \qquad \rho_S = \begin{pmatrix} \bullet & 0 \\ 0 & \bullet \end{pmatrix}$$

Thermal state

Maximally mixed state

A. Streltsov, G. Adesso, and M. B. Plenio, Rev. Mod. Phys. **89**, 041003 (2017)

OUTLINE

- Open Quantum Systems Overview
- Quantum coherences
- Coherence trapping
- Autonomous generation of steady-state coherences by system-bath interaction
- Applications
- Outlooks and conclusions