



Optimal Unambiguous Discrimination of Two Incompatible Quantum Measurements



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1) Introduction

In this work we present an experimental setup of one-copy optimal unambiguous (probabilistic) discrimination of two, in general incompatible, Von Neumann projective measurements A and B. To be optimal, this quantum information processing task requires pairs of entangled particles and as part of the analyzing protocol conditionally performed unitary operation depending on the measured results on a probe particle. In our experiment we discriminate two polarization measurements A, B on a single photon.

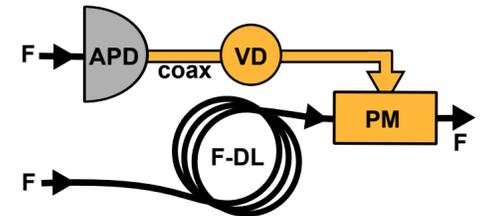
2) Optimality of the setup

Using the mathematical formalism of process positive-operator valued measure (PPOVM) [1] one can prove that the presented unambiguous discrimination [2] protocol for the Von Neumann measurements A,B is optimal.

[1] M. Ziman, Phys. Rev. A 77, 062112 (2008), G. Chiribella, G. M. D'Ariano, P. Perinotti, Phys. Rev. A 80, 022339 (2009)
[2] I. D. Ivanovic, Phys. Lett. A, 123:257 (1987), D. Dieks, Phys. Lett. A, 126:303 (1988), A. Peres, Phys. Lett. A, 128:19 (1988)

3) Feed-forward

The signal from the detector (TTL pulse, 5V, 30ns) is modified by a passive voltage divider and led directly to a phase modulator (half-wave voltage= 1.55V). It takes 17ns between photon detection and producing electronic signal, the TTL puls. Because of this, it is necessary to delay the 2nd photon.



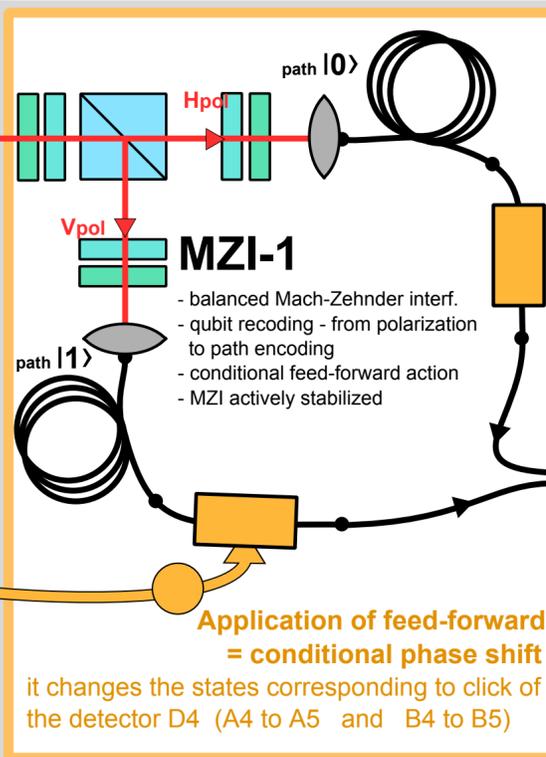
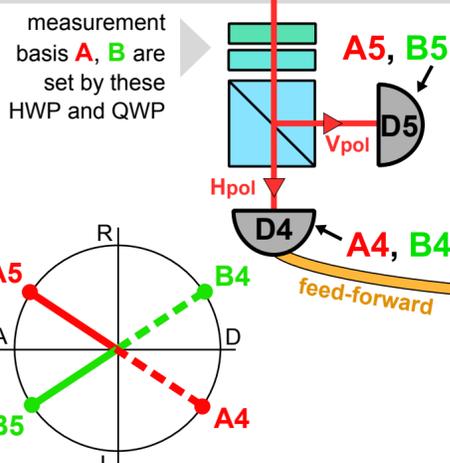
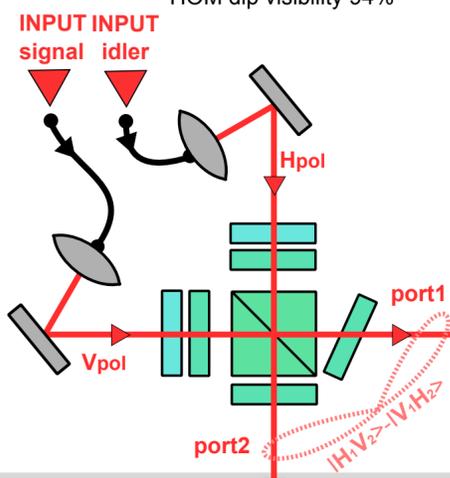
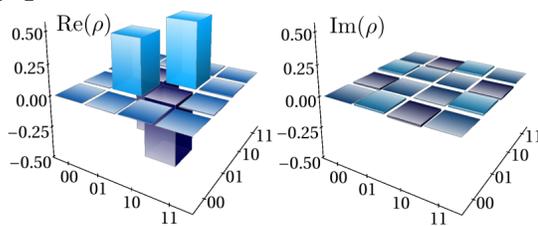
APD-single photon detector, coax-coaxial cable, VD-passive voltage divider, F-optical fiber, DL-delay line, PM-electro-optical phase modulator

4) Photon source

Our linear optical protocol requires photon pairs entangled in polarization. Photon pairs are generated by SPDC in BBO crystal, type II., degen. 810nm. Photons are entangled at non-polarizing beam splitter 50:50.

prepared state: $|H_1V_2\rangle - |V_1H_2\rangle$
purity 97.8%, fidelity 99.2%
HOM dip visibility 94%

reconstructed density matrix



LEGEND

BS beam splitter non-polarizing 50:50 - bulk	HWP half-wave plate	FS fiber splitter 50:50 polarization maintaining fibers	PM electro-optical phase modulator
PBS polarizing beam splitter - bulk	QWP quarter-wave plate	VRC variable ratio coupler pol. maintaining fibers from 100:0 to 0:100	coaxial cable & passive voltage divider
D single photon detector	C input / output coupling lens	light beam 810nm	pol. maintaining optical fiber PM780-HP

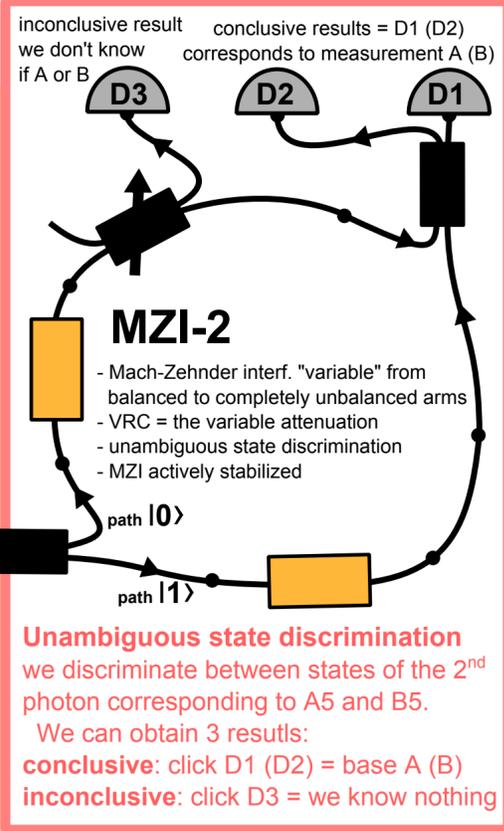
6) Photon analysis

Each polarization measurement, the "Black box" measurement in base A (B) on the probe photon can give two results, represented the two basis states A5 or A4 (B5 or B4). It corresponds to measurement of the probe photon by the detector D4 or D5.
Then the knowledge about the applied measurement A or B on probe photon is encoded into the 2nd photon of the pair.

detector D5 click - We can directly apply the unambiguous state discrimination on the second photon. We discriminate states corresponding to A5 and B5.

detector D4 click - The state of the second photon need to be modified (to get the same state as when the probe photon is detected by detector D5).

This modification is done by means of the electronic feed-forward. The states corresponding to A4, B4 are changed to states A5, B5. Then the unambiguous state discrimination is applied.

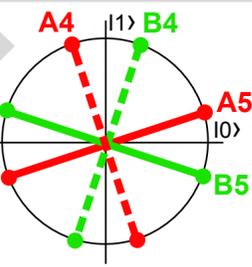


Unambiguous state discrimination
we discriminate between states of the 2nd photon corresponding to A5 and B5.
We can obtain 3 results:
conclusive: click D1 (D2) = base A (B)
inconclusive: click D3 = we know nothing

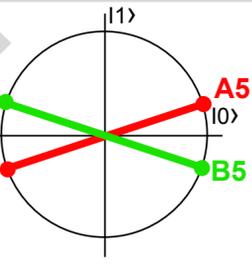
5) "Black box"

= Von Neumann measurement A or B
One of the two known Von Neumann measurements A, B is randomly chosen and apply on the probe particle of the entangled pair.
Then the state of the second particle of the entangled pair is analyzed.

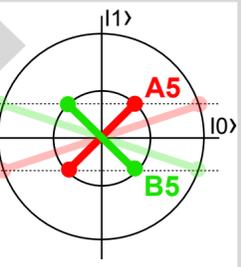
The four possible states of the 2nd photon after the measurement on the 1st -probe photon in the measurement bases A or B, corresponding to the states A4, A5, B4 and B5.



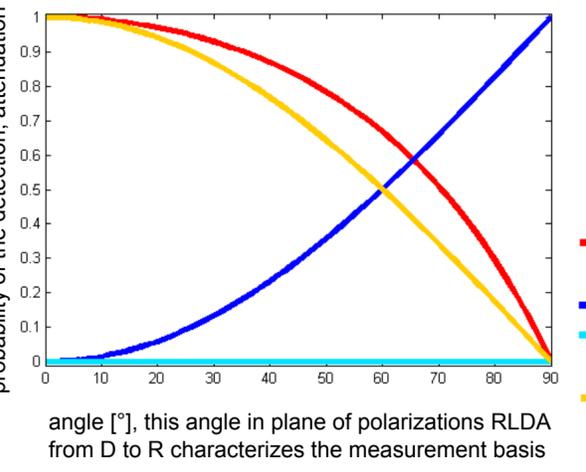
After the feed-forward action (in MZI-1) only the two non-orthogonal states A5, B5 remain.



The appropriate attenuation on one of the basis states (in MZI-2) makes the states A5, B5 orthogonal = the unambiguous state discrimination



The detection probability of the second photon and attenuation in MZI-2



7) Realization
Without loss of generality, we restrict only to the cases when the measurement bases A, B lie in the plane of circular and diagonals polarizations of the Poincaré sphere.

Legend of the graph
- attenuation in MZI-2 (1-t²)
- conclusive results:
- probability of detection at D1 (D2)
- probability of detection at D2 (D1)
- inconclusive result:
- probability of detection at D3

