Mezinárodní centrum pro informaci a neurčitost

Registrační číslo: CZ.1.07/2.3.00/20.0060

evropský sociální fond v ČR

EVROPSKÁ UNIE

MINISTERSTVO ŠKOLSTVÍ, MLÁDEŽE A TĚLOVÝCHOVY

OP Vzdělávání pro konkurenceschopnost

INVESTICE DO ROZVOJE VZDĚLÁVÁNÍ

Tento projekt je spolufinancován Evropským sociálním fondem a státním rozpočtem České republiky.

Centre for Quantum Engineering
and Space-Time Research

Leibniz
Universität
Hannover

Light Matter Interface

Klemens Hammerer

Leibniz University Hanover

Institute for Theoretical Physics

Institute for Gravitational Physics
Albert Einstein Institute

QUEST

Summer School on Quantum Physics and Quantum Information – Olomouc July 2012

Quantum Information Processing and Communication

"… it seems that the laws of physics present no barrier to reducing the size of computers until bits are the size of atoms, and quantum behavior holds sway."

Richard P. Feynman (1985)

	Computation	Communication
Theory	QSimulation QAlgorithms Complexity Error Correction Entanglement	Bell Inequalities Complexity QCryptography Quantum Repeaters Light Matter Interface
Implementation	Ion Traps NMR Optical Lattices SQUIDS Qdots Linear Optics	Atomic Ensembles CavityQED Free Space Space based

European and US roadmaps in the field of Quantum:Information:

QIPC – Strategic report on current status, visions and goals for research in Europe,
P. Zoller *et al.* (precisely, 37 others), Eur.Phys.J. D 36, 203 (2005)

A Quantum Information Science and Technology Roadmap,
R. Hughes *et al.* (only 17), http://qist.lanl.gov

Quantum Communication

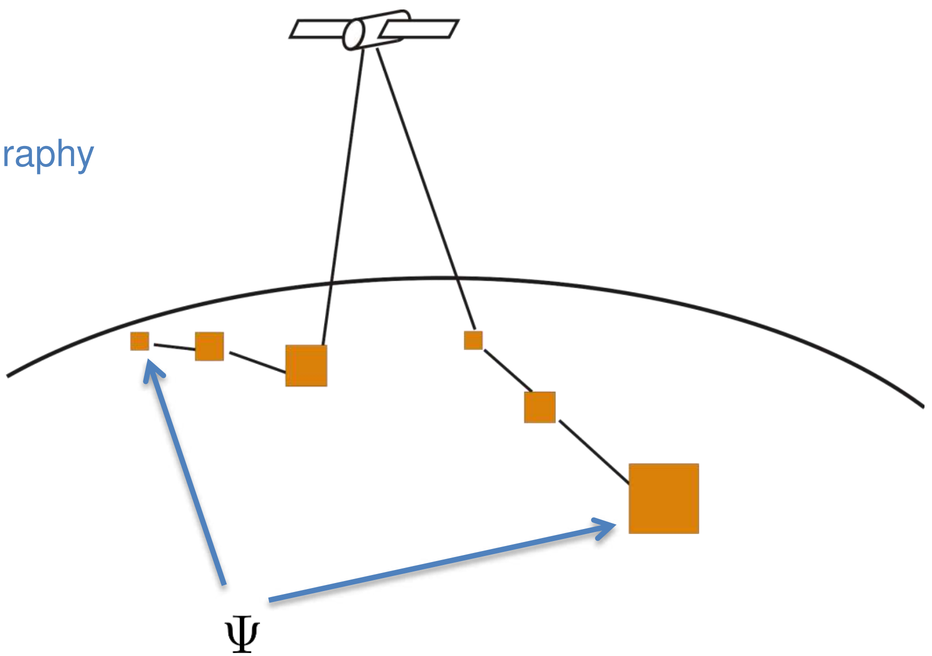
"…is the art of transferring a quantum state from one place to another."
Gisin, *Quantum Communication*, Nature Phys. 1, 165 (2007)

Quantum Cryptography & Key Distribution
Gisin, Rev. Mod. Phys. 74, 145 (2002)

Frédéric Grosshans' talks on Quantum cryptography

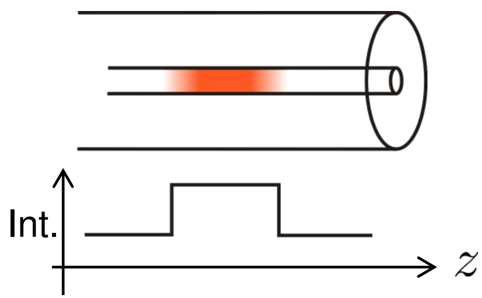
Quantum Communication Complexity
Brassard, Found. Phys. 70, 1593 (2003)

Tests of Quantum Theory
Bell Inequalities, GHZ theorem…



Encoding of Quantum Information

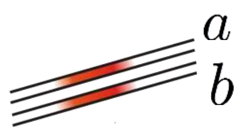
light is the natural long distance carrier of quantum information



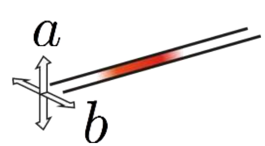
$$|1\rangle = a^\dagger|0\rangle \qquad [a, a^\dagger] = 1$$

$$|n\rangle \sim \left(a^\dagger\right)^n|0\rangle$$

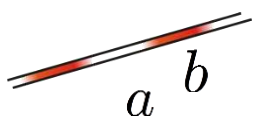
✓ dual rail qubits



✓ polarization qubits



✓ time bin qubits



$$\begin{aligned} |\psi\rangle &= \alpha|1\rangle_a + \beta|1\rangle_b \\ &= \alpha|0_L\rangle + \beta|1_L\rangle \end{aligned}$$

✓ continuous variable states

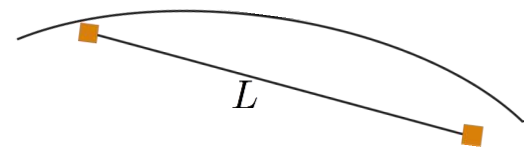
$$|\alpha\rangle \sim \sum_{n=0}^{\infty} \frac{\alpha^n}{n!}|n\rangle$$

Peter van Loock's talks on Quantum Information with Gaussian states

Direct Quantum Communication

✓ direct transmission is limited by absorption length of fibre

$$|1\rangle \longrightarrow \eta|1\rangle\langle 1| + (1-\eta)|0\rangle\langle 0|$$



$$|\alpha\rangle \longrightarrow |\sqrt{\eta}\alpha\rangle \qquad \eta = \exp(-L/L_{abs})$$

$$L_{abs} \simeq 100\text{km}$$

✓ acceptable damping is a question of signal rate of source

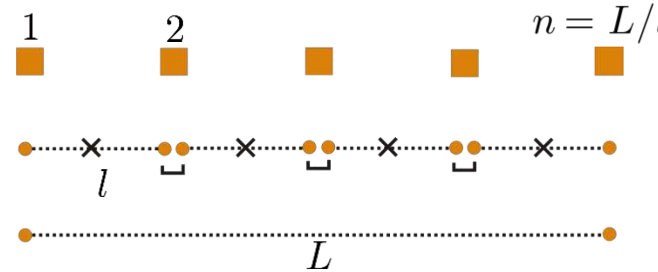
signal rate of 10^{10} Hz; loss of 0.2dB/km (fiber loss @ 1550nm)
for a Q cryptographic protocol limited by transmission losses,
and in order to have a secret bit rate of 1Hz: L < 500km
(below 500km is „only" an engineering problem)

✓ longer, intercontinental distances can not be bridged directly

Direct Long Distance Quantum Communication?

✓ naïve idea:

- ➢ break long distance in short segments
- ➢ distribute entanglement over segments
- ➢ connect via entanglement swapping



✓ impact of loss on entanglement distribution for **qubits**

$$|\psi\rangle \sim |01\rangle + |10\rangle \longrightarrow \eta_l |\psi\rangle\langle\psi| + (1-\eta_l)|vac\rangle\langle vac|$$

success probability for entanglement swapping

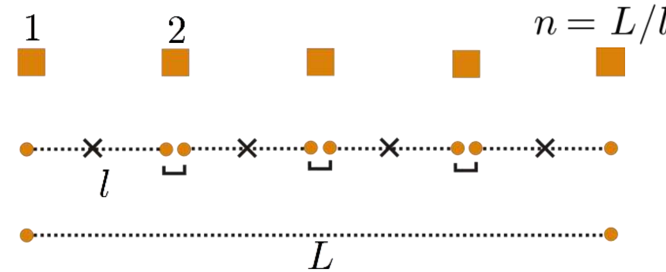
$$\sim \eta_l = \exp(-l/L_{abs}) \qquad l = L/N$$

overall success probability

$$\sim \eta_l^n = \exp(-L/L_{abs}) = \eta$$

Direct Long Distance Quantum Communication?

✓ impact of loss on entanglement distribution
for **continuous variable** two mode squeezed state



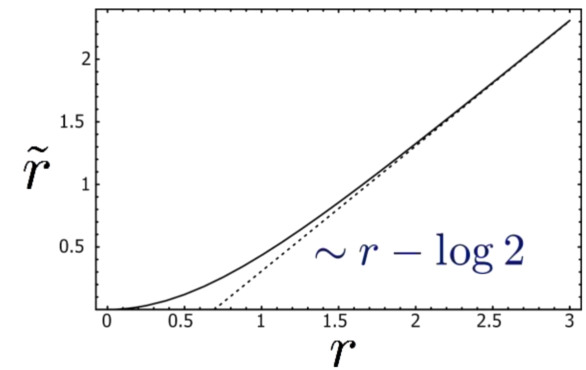
$$|r\rangle \sim \sum_n \mathrm{th}(r)^n |n, n\rangle \longrightarrow \rho$$

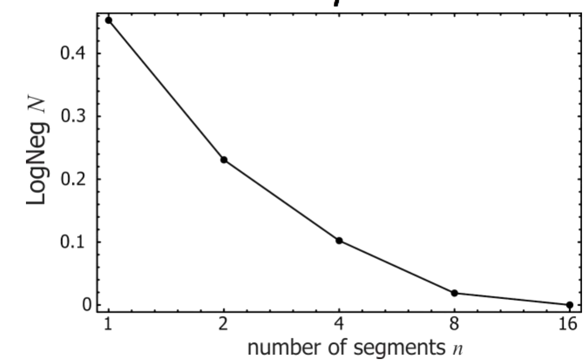
ρ is entangled for all η

entanglement swapping is deterministic

but entanglement is not conserved
(not even for zero loss)

$$\lim_{r' \to \infty} {}_{23}\langle r' | \Big(|r\rangle_{12} |r\rangle_{34} \Big) \to |\tilde{r}\rangle_{14}$$





→ direct transmission is best!

Side note: Space Based Quantum Communication

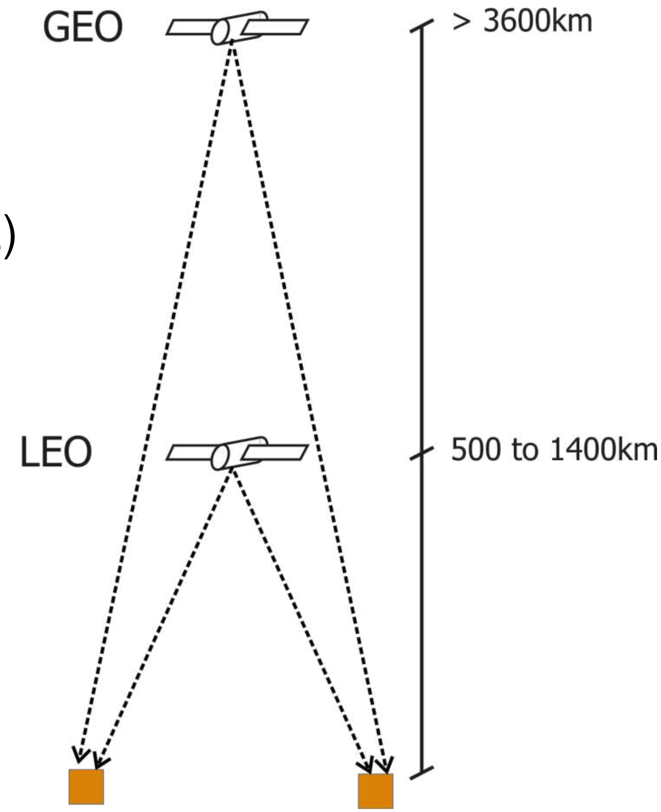
…less naïve ideas:

✓ Satellite based QC

tolerable damping 60dB
(allows for BI violation, source rate $5 \cdot 10^5$Hz,
for detection efficiency 30%, dark counts 10^3Hz)

Aspelmeyer et.al., quant-ph/0305105

800 nm $\boxed{\text{1550 nm}}$	ground-based receiver	LEO receiver	GEO receiver
ground based transmitter		27.4 dB $\boxed{\text{26.3 dB}}$	64.5 dB $\boxed{\text{63.4 dB}}$
LEO transmitter	6.4 dB $\boxed{\text{12.2 dB}}$	28.5 dB $\boxed{\text{33.6 dB}}$	52.9 dB $\boxed{\text{58.6 dB}}$
GEO transmitter	43.6 dB $\boxed{\text{49.3 dB}}$	52.9 dB $\boxed{\text{58.6 dB}}$	53.9 dB $\boxed{\text{59.7 dB}}$

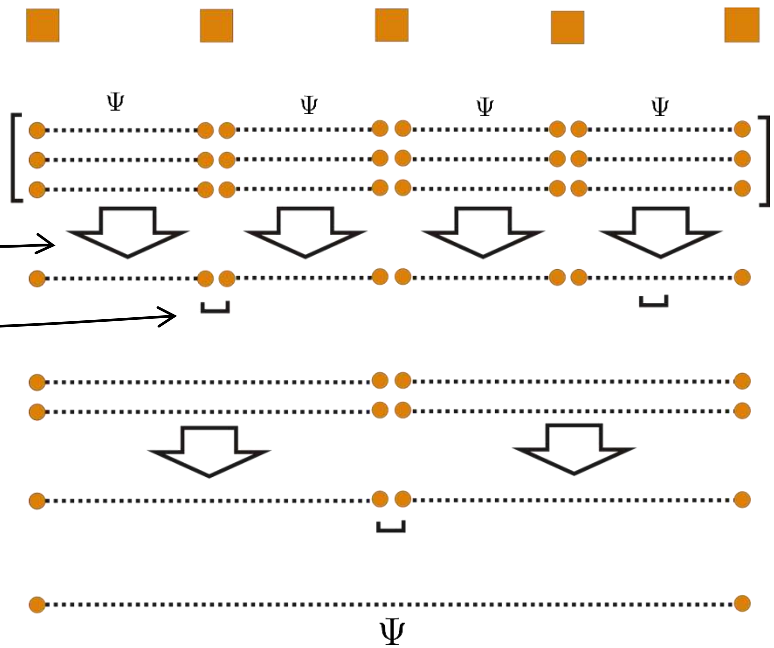


Violation of CHSH-inequality on Canary Islands over 144km, loss of 64dB
atmospheric absorption same as for ground-space distance
Fedrizzi et al., Nature Physics, 5, 389-392 (2009)

Repeater Based Quantum Communication

✓ **Quantum Repeater**:

- ➢ distribute many pairs of entangled states
- ➢ purify entanglement
- ➢ swap entanglement
- ➢ repeat…

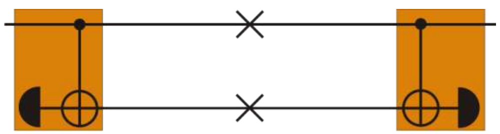


Briegel, PRL 81, 5932 (1998)
Dür, PRA 59, 169 (1999)

Quantum Repeater

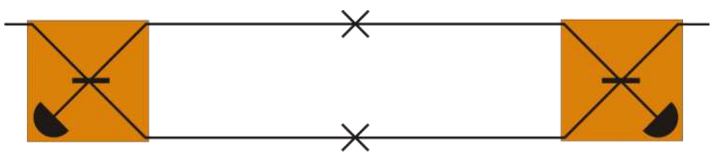
✓ entanglement purification is a probabilistic operation

- ➢ qubits



Bennett, PRL 76, 722 (1996)

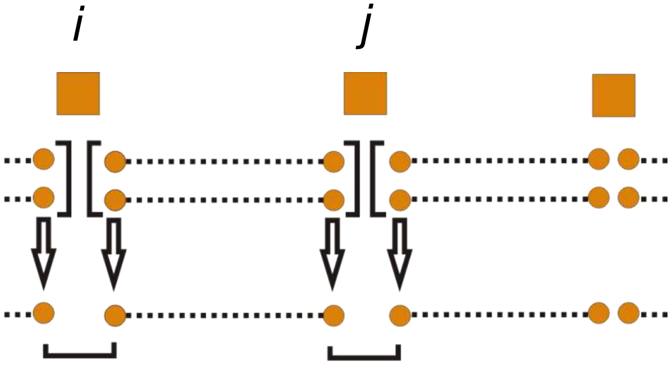
- ➢ cv



Browne, PRA 67, 062320 (2003)

✓ depends on correlated measurement outcomes at different sites

operation at station *i* depends on outcome at station *j*



✓ requires a quantum memory at each site to store and process states

✓ errors in local operations and finite memory time is critical

Briegel, PRL 81, 5932 (1998) Dür, PRA 59, 169 (1999) Hartmann, quant-ph/0610113

Light Matter Interface and Quantum Memory

✓ light is the natural long distance carrier of quantum information

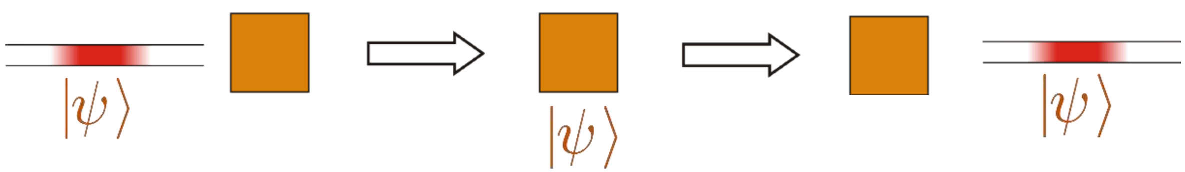
✓ long lived ground states of matter provide natural storage medium

✓ requires efficient light-matter interface

- for generation of entanglement

$$|\Psi\rangle \neq |\psi\rangle|\phi\rangle$$

- for storing and releasing states of light



→ requires strong coupling of light and matter

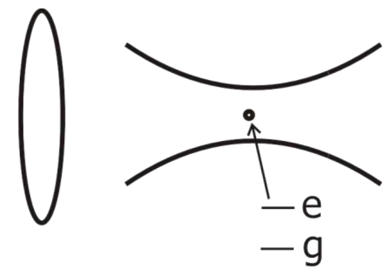
Strong Coupling of Light and Matter

✓ naïve idea: focus single photon pulse on free trapped atom

size of atom $\sigma = 3\lambda^2/2\pi$

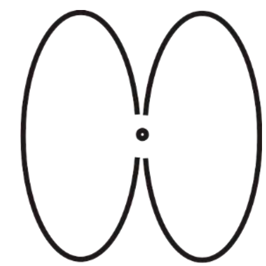
size of beam A

probability for absorption/scattering $\sim \sigma/A$



✓ but: for $\sigma \sim A$ modefunction of beam matches dipole pattern poorly

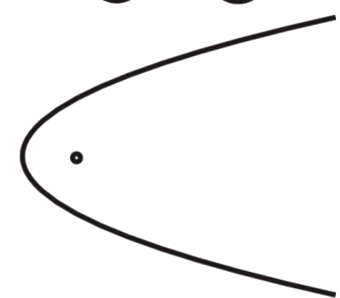
van Enk, PRA 61, 051802 (2000), PRA 63, 023809 (2001)



✓ optimal mode would be an icoming dipole field

time reversal of atomic decay

✓ sidemark: recent theoretical study with atom in deep parabolic mirror; almost 4π irradiation

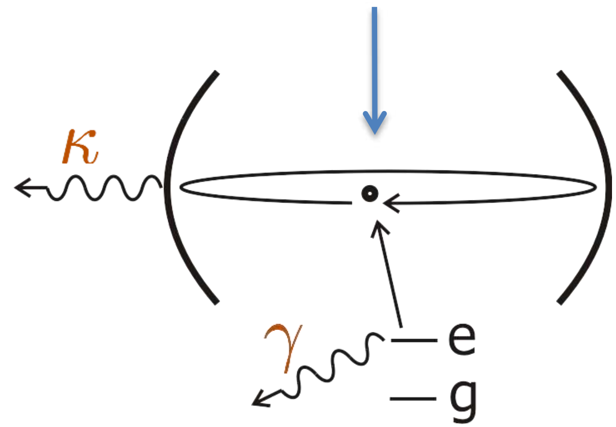


Stobinska, EPL 86, 14007 (2009)

✓ strong coupling with single atom in free space seems to be impractical

Strong Coupling in Cavity QED

photon meets atom F times



better ideas:

✓ **cavity QED**:

- coupling of atom to EM field

$$V = g\left(|e\rangle\langle g|a + |g\rangle\langle e|a^\dagger\right)$$

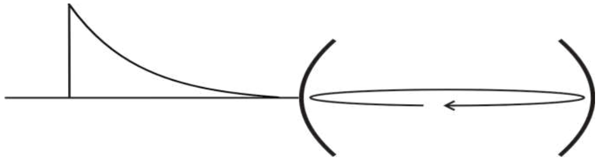
$$g^2 = (\sigma/A)F\gamma\kappa$$

- strong coupling $g^2 \gg \gamma\kappa$ possible for large Finesse $F \sim c/L\kappa$

- coupling single photons into high Finesse cavity is non-trivial

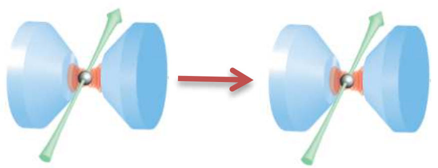
  mode of incoming photon $\stackrel{!}{=}$ mode of outgoing photon

  time reversal of cavity decay



  Cirac, PRL 78, 3221 (1997)

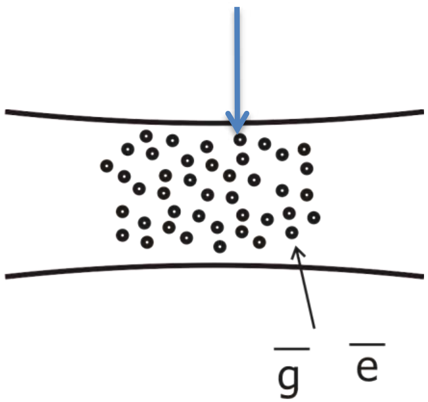
- Experiments: entanglement of two atoms
  fidelity 84%, success rate 0.2%



Ritter *et al.*, Nature 484, 195-200 (2012)

Collective States in Atomic Ensembles

photon meets N atoms once



✓ **atomic ensemble**:

use many scatterers to enhance coupling

light couples to collective states

$$|0\rangle = |g\,g\,g\ldots g\rangle$$

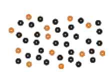
$$S^+ = \sum_i |e\rangle_i\langle g|$$

$$S_z = \tfrac{1}{2}\sum_i \sigma_z^i$$

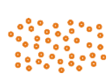
$$|1\rangle = \frac{1}{\sqrt{N}}\left(|e\,g\ldots g\rangle + |g\,e\ldots g\rangle + \cdots + |g\ldots e\rangle\right)$$

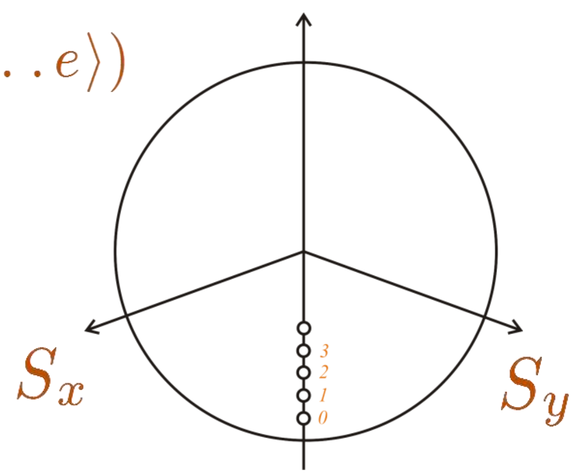
$$|2\rangle \sim \left(S^+\right)^2|0\rangle$$

$$|n\rangle \sim \left(S^+\right)^n|0\rangle$$



$$|N\rangle = |e\,e\,e\ldots e\rangle$$





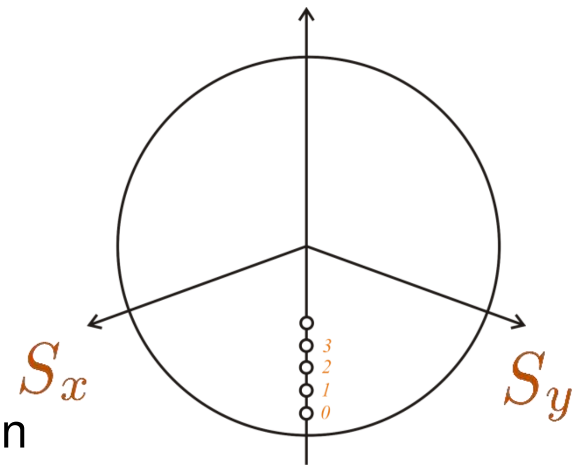
Atomic Mode

✓ reduction to single mode

for large numer of atoms N and large polarizations $n \ll N$

$$b^\dagger \simeq S^+/\sqrt{N} \qquad |n\rangle = 1/\sqrt{n!}\,(b^\dagger)^n|0\rangle \qquad S_z = \tfrac{1}{2}\sum_i \sigma_z^i$$

$$[b, b^\dagger] = [S^-, S^+]/N = -2S_z/N \simeq 1$$



✓ rigorous formulation:

➢ Group contraction

Arecchi, PRA 6, 2211 (1972)

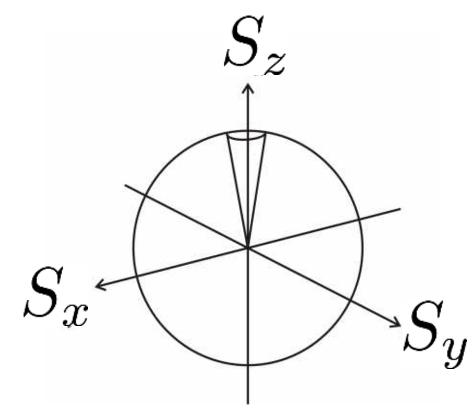
➢ Holstein Primakoff-transformation and approximation

Holstein, PR 58, 1098 (1940)

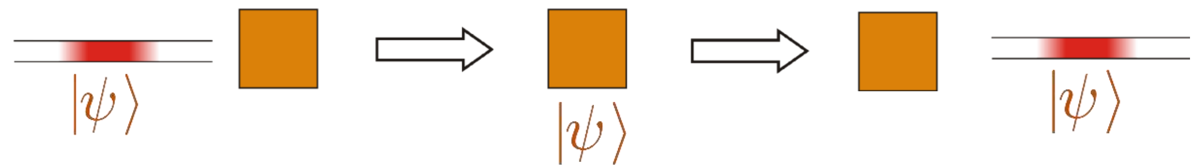
$$S^+ = \sqrt{N}\sqrt{1 - b^\dagger b/N}\, b$$

$$S^- = \sqrt{N}\, b^\dagger \sqrt{1 - b^\dagger b/N}$$

$$S_z = N/2 - b^\dagger b$$



Light Matter Interface



$$|\psi\rangle = \sum_n c_n |n\rangle$$

Photons

$$|\psi\rangle = \sum_n c_n |n\rangle$$

spin excitations

Roadmap

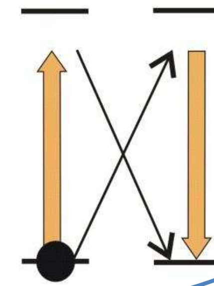
Quantum Communication

Quantum Repeater

Light-Matter interface & Quantum Memory

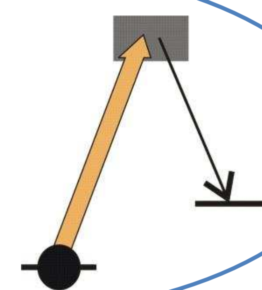
Faraday interaction

Polzik
Deutsch
Mitchell
Takeuchi



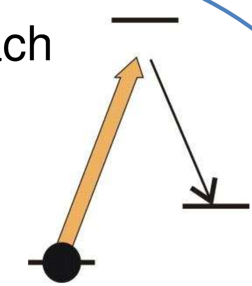
Photon echo

Gisin
Kröll…



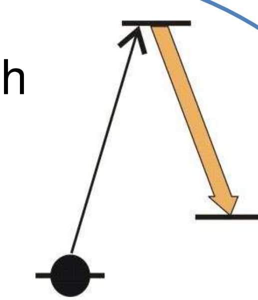
Raman approach

Kimble
Kuzmich
Vuletic
Lukin
Schmiedmayer
Pan…



EIT approach

Hau
Lukin
Kuzmich
Lam…



Roadmap

Quantum
Communication

Quantum
Repeater

Light-Matter interface & Quantum Memory

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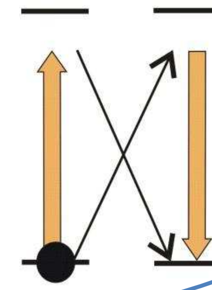

Photon echo

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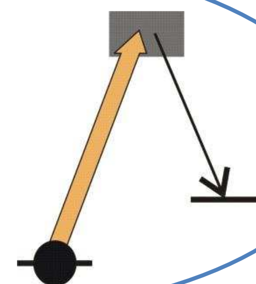

Raman approach

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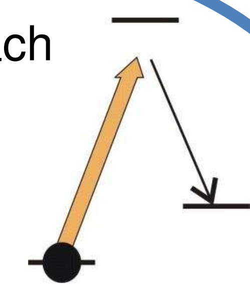

EIT approach

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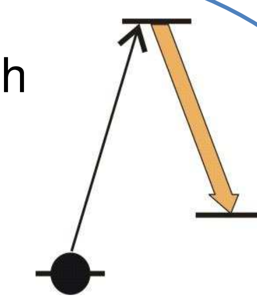
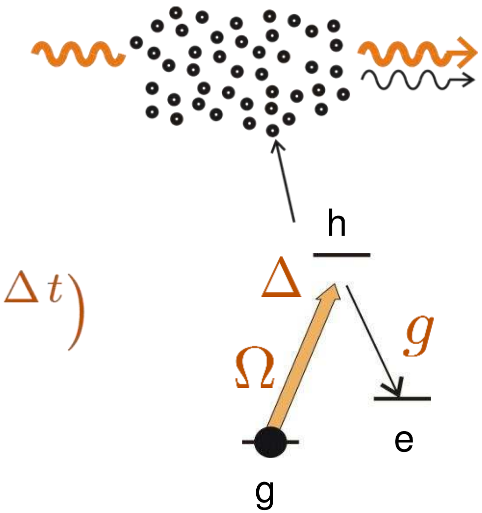


Interaction of Atoms and Light I: Raman Scattering



✓ effective interaction

$$H = \Omega \sum_i \left(|h\rangle_i \langle g| e^{i\Delta t} + |g\rangle_i \langle h| e^{-i\Delta t}\right)$$

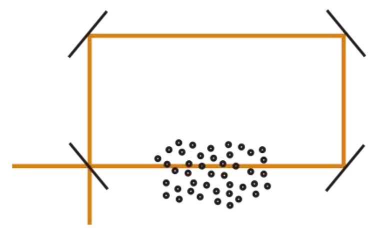
$$+g \sum_i \left(|h\rangle_i \langle e| a e^{i\Delta t} + |e\rangle_i \langle h| a^\dagger e^{-i\Delta t}\right)$$

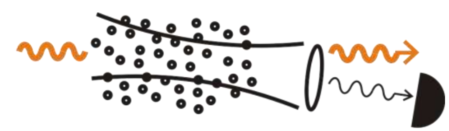
$$H = \frac{g\Omega}{\Delta} \sum_i \left(|e\rangle_i \langle g| a^\dagger + |g\rangle_i \langle e| a\right)$$

$$= \frac{g\Omega\sqrt{N}}{\Delta} \left(b^\dagger a^\dagger + ba\right)$$

✓ light mode a refers to

- ➢ low Finesse ring cavity
- ➢ ‘detector mode’





✓ decoherence due to scattering into other modes $\sim (\Omega/\Delta)^2 \gamma$

✓ interaction enhanced by $\sqrt{N}$
description reduced to two harmonic oscillators

Raman Scattering - Write

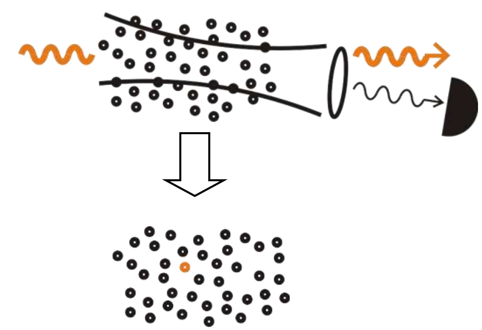
✓ interaction $H \sim b^\dagger a^\dagger + ba$ creates entanglement

$$\exp(-iHt)|0,0\rangle \sim \sum_n \mathrm{th}(r)^n |n,n\rangle = |r\rangle$$

✓ for ‘short’ interaction

Raymer, JMO 51, 1739 (2004),
Wasilewski, PRA 72, 063816 (2006)

$$\begin{aligned}\exp(-iH\,\delta t)|0,0\rangle &\simeq (1 - iV\,\delta t)|0,0\rangle \\ &= |0,0\rangle + \sqrt{p_w}\,|1,1\rangle\end{aligned}$$



detection of photon ‘writes’ a single excitation on atoms:

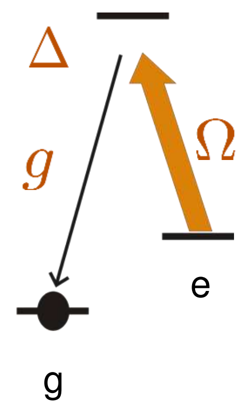
✓ branching ratio of scattering into ‘good’ and ‘bad’ modes

$$\boxed{\sim (\sigma/A)N = d \quad \text{...optical depth}}$$

Duan, PRA 66, 023818 (2002)

Raman Scattering - Read

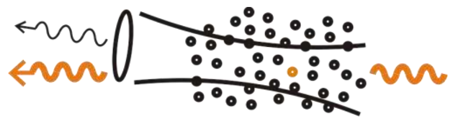
✓ Single excitation can be read out



$$H = \frac{g\Omega}{\Delta}\sum_i \left(|g\rangle_i\langle e|a^\dagger + |e\rangle_i\langle g|a\right)$$
$$= \frac{g\Omega\sqrt{N}}{\Delta}\left(ba^\dagger + b^\dagger a\right)$$

atomic excitation is released into light mode

$$|1\rangle_a|0\rangle_l \rightarrow |0\rangle_a|1\rangle_l$$



✓ can be resonant $\Delta = 0$

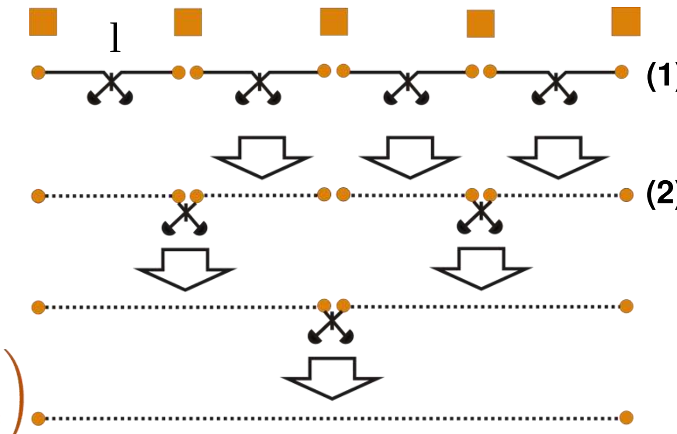
✓ happens with finite efficiency

$$|1\rangle_a|0\rangle_l \rightarrow |0\rangle_a\langle 0| \otimes \left[p_r|1\rangle_l\langle 1| + (1-p_r)|0\rangle_l\langle 0|\right]$$

inefficiency $1 - p_r \sim 1/d$

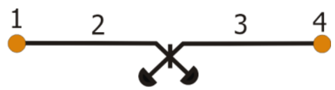
Gorshkov, PRL 98, 123601 (2007)
PRA 76, 033804 (2007)
PRA 76, 033805 (2007)
PRA 76, 033806 (2007)

DLCZ – Quantum Repeater



Duan, Lukin, Cirac, Zoller, Nature 414, 413 (2001)

✓ entanglement creation **(1)**:



$$\left(|0,0\rangle_{12}+\sqrt{p_w}|1,1\rangle_{12}\right)\left(|0,0\rangle_{34}+\sqrt{p_w}|1,1\rangle_{34}\right)$$

$$\xrightarrow{\text{click}} |1,0\rangle_{14}\pm e^{i\phi}|0,1\rangle_{14}=|\Psi\rangle_{14}$$

success probability

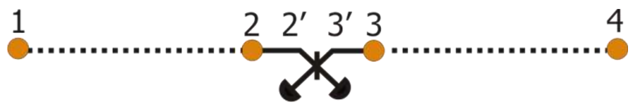
$$p_w\eta_l \qquad \eta_l=\exp(-l/L_{abs})$$

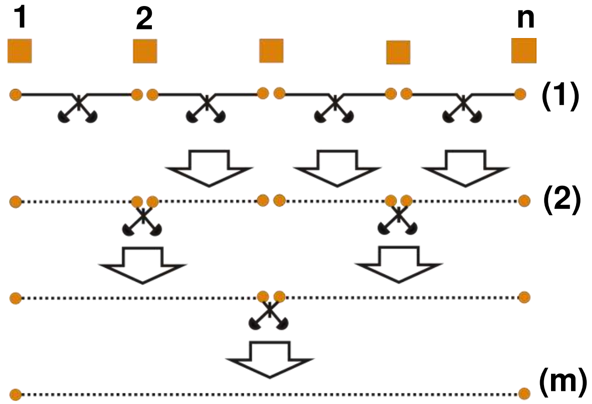
waiting time until state **(2)** is reached

$$t/p_w\eta_l\sim(l/c)\exp(l/L_{abs})/p_w$$

DLCZ – Quantum Repeater

✓ entanglement connection **(2)**





$$|\Psi\rangle_{12}|\Psi\rangle_{34}$$

$$\xrightarrow{\text{read}} \Big(p_r|\Psi\rangle_{12'}\langle\Psi| + (1-p_r)\text{rubbish}\Big)\Big(p_r|\Psi\rangle_{3'4}\langle\Psi| + (1-p_r)\text{rubbish}\Big)$$

$$\xrightarrow{\text{click}} |\Psi\rangle_{14}$$

'in-built entanglement purification'

success probability at each level $\sim p_r^2$

success probability for connection up to level (m)

$$\sim p_r^{2m} = p_r^{2\log_2(L/l)} \qquad n = L/l = 2^m$$

✓ overall waiting time $(l/c)\exp(l/L_{abs})/p_w p_r^{2\log_2(L/l)}$

subexponential scaling in L

DLCZ – Quantum Repeater

✓ other imperfections/noise sources

- finite detection efficiency: same effect as finite readout efficiency p_r

- detector dark counts:

$$\xrightarrow{\text{click}} c|0,0\rangle\langle 0,0| + (1-c)|\Psi\rangle\langle\Psi|$$

'effectively maximally entangled state', 'a posteriori entanglement'

van Enk, PRA 75, 052318 (2007)

- higher order excitations in entanglement generation step

$$\exp(-iH\,\delta t)|0,0\rangle \simeq |0,0\rangle + \sqrt{p_w}|1,1\rangle + p_w|2,2\rangle$$

lead to 'infidelity' with $|\psi\rangle$ on the order of p_w

higher fidelity can be bought at the price of longer waiting times

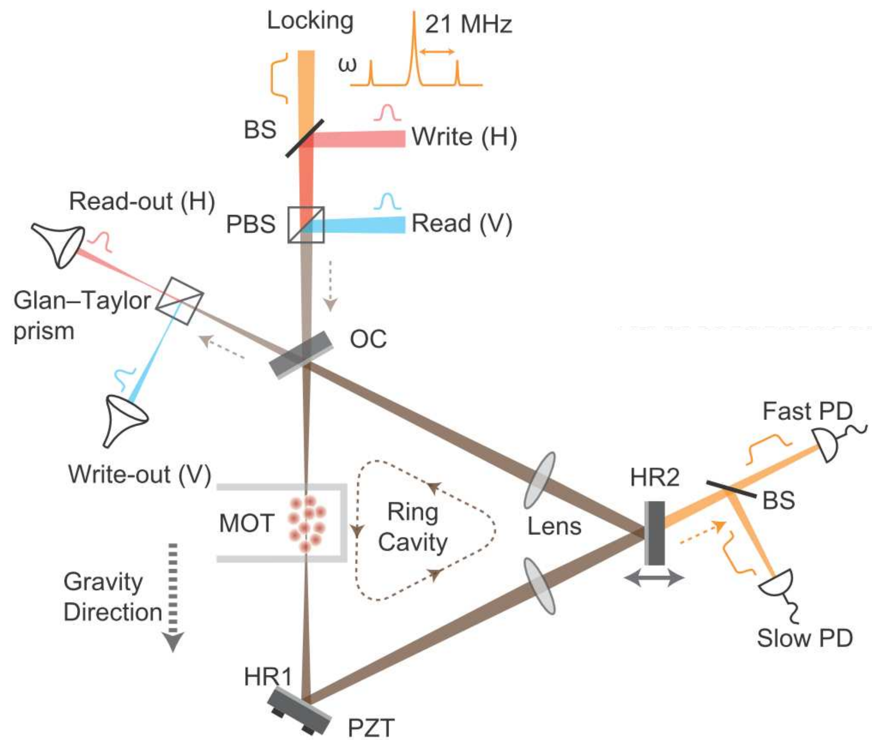
✓ is tolerable and does not change sub-exponential scaling in distance

Raman approach & DLCZ Repeater: State of the art and further reading

✓ experiment:

Bao *et al.* Nat. Phys. 8, 517 (2012)

retrieval efficiency of 73%
after 3.2 ms



✓review:

Sangouard *et* al., Rev. Mod. Phys. 83, 33 (2011)
Simon *et al.* , Eur. Phys. J. D 58, 1 (2010)

Roadmap

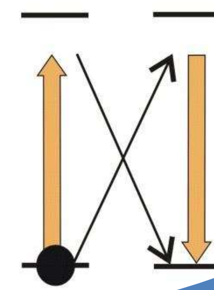
Quantum Communication

Quantum Repeater

Light-Matter interface & Quantum Memory

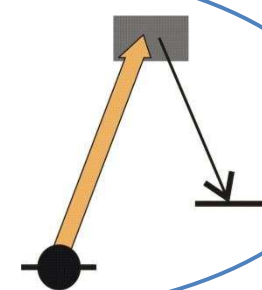
Faraday interaction

Polzik
Deutsch
Mitchell
Takeuchi



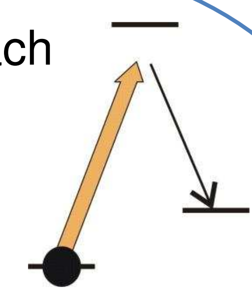
Photon echo

Gisin
Kröll…



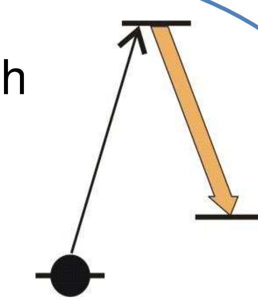
Raman approach

Kimble
Kuzmich
Vuletic
Lukin
Schmiedmayer
Pan…



EIT approach

Hau
Lukin
Kuzmich
Lam…



Interaction of Atoms and Light II: Faraday Interaction

✓ Faraday interaction = Raman write + Raman read

$$H = \frac{g\Omega\sqrt{N}}{\Delta}\left(ba^\dagger + b^\dagger a\right) + \frac{g\Omega\sqrt{N}}{\Delta}\left(b^\dagger a^\dagger + ba\right)$$

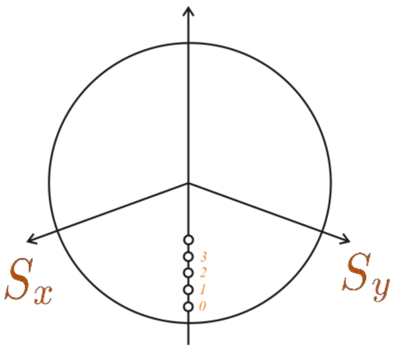
$$\sim (a + a^\dagger)(b + b^\dagger)$$

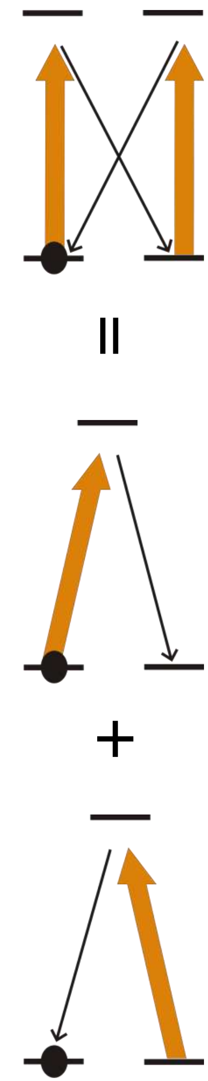
$$\sim x_a x_b$$

✓ atomic ‘quadratures’

$$x_b = (b + b^\dagger)/\sqrt{2} \sim S_- + S_+ \sim S_x$$

$$p_b = -i(b - b^\dagger)/\sqrt{2} \sim S_y$$





✓ state of light & matter after interaction: Heisenberg picture

$$U = \exp(-iHt) = \exp(-i\kappa x_a x_b)$$

$$x_a^{\text{out}} = U x_a^{\text{in}} U^\dagger = x_a^{\text{in}} \qquad p_a^{\text{out}} = p_a^{\text{in}} + \kappa x_b^{\text{in}}$$

$$x_b^{\text{out}} = x_b^{\text{in}} \qquad p_b^{\text{out}} = p_b^{\text{in}} + \kappa x_a^{\text{in}}$$

Faraday Interaction

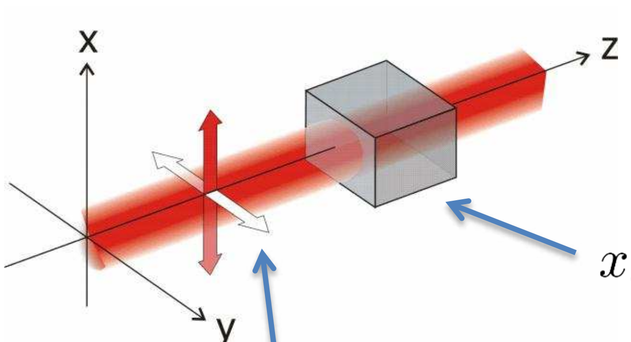
$$x_a^{\mathrm{out}} = x_a^{\mathrm{in}} \qquad p_a^{\mathrm{out}} = p_a^{\mathrm{in}} + \kappa x_b^{\mathrm{in}}$$

$$x_b^{\mathrm{out}} = x_b^{\mathrm{in}} \qquad p_b^{\mathrm{out}} = p_b^{\mathrm{in}} + \kappa x_a^{\mathrm{in}}$$

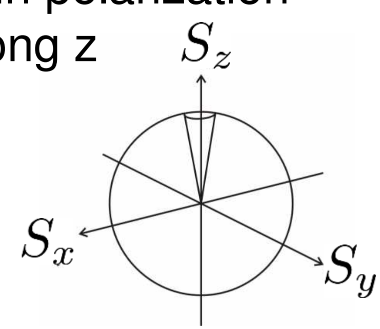
Stark shift

polarization of light $\rightleftarrows$ polarization of atoms

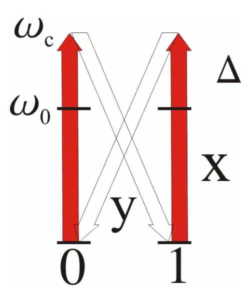
birefringence



$x_b,\ p_b$ spin projections orthogonal to mean spin polarization along z



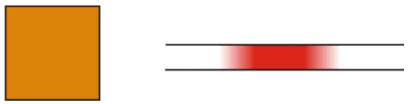
$x_a,\ p_a$ amplitude/phase quadrature of linear y-polarized field



Faraday Interaction

$$x_a^{\text{out}} = x_a^{\text{in}} \qquad\qquad p_a^{\text{out}} = p_a^{\text{in}} + \kappa x_b^{\text{in}}$$

$$x_b^{\text{out}} = x_b^{\text{in}} \qquad\qquad p_b^{\text{out}} = p_b^{\text{in}} + \kappa x_a^{\text{in}}$$



$$|\Psi\rangle \neq |\psi\rangle|\phi\rangle$$

✓ statistics of initial state $|0\rangle|0\rangle$

$$\langle 0|x^{\text{in}}|0\rangle = 0 \qquad \langle (x^{\text{in}})^2\rangle = (\Delta x^{\text{in}})^2 = 1$$

same for p

✓ statistics of final state

$$\langle 0|p_a^{\text{out}}|0\rangle = 0$$

$$\langle (p_a^{\text{out}})^2\rangle = \langle (p_a^{\text{in}})^2\rangle + 2\kappa\langle p_a^{\text{in}} x_b^{\text{in}}\rangle + \kappa^2\langle (x_b^{\text{out}})^2\rangle = 1 + \kappa^2$$

$$\langle x_a^{\text{out}} p_b^{\text{out}}\rangle = \langle x_a^{\text{in}} p_b^{\text{in}}\rangle + \kappa\langle (x_a^{\text{in}})^2\rangle = \kappa \qquad \Rightarrow |\Psi\rangle \neq |\psi\rangle|\phi\rangle$$

→ interaction creates entanglement

Intermezzo: Heisenberg picture

- ✓ Input/Output relations

$$x_a^{\text{out}} = x_a^{\text{in}} \qquad p_a^{\text{out}} = p_a^{\text{in}} + \kappa x_b^{\text{in}}$$

$$x_b^{\text{out}} = x_b^{\text{in}} \qquad p_b^{\text{out}} = p_b^{\text{in}} + \kappa x_a^{\text{in}}$$

determine state completely, e.g. in Wigner function

$$R = (x_a,\, p_a,\, x_b,\, p_b)^T$$

$$R^{\text{out}} = S R^{\text{in}}$$

where $S = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & \kappa & 0 \\ 0 & 0 & 1 & 0 \\ \kappa & 0 & 0 & 1 \end{pmatrix}$

Wigner function transforms as

$$W^{\text{in}}(\vec{r}) \quad \rightarrow \quad W^{\text{out}}(\vec{r}) = W^{\text{in}}(S\vec{r})$$

- ✓ e.g. for a quantum memory we want to achieve exchange of quantum states

$$W_1^{\text{in}}(\vec{r}_a)W_2^{\text{in}}(\vec{r}_b) \quad \rightarrow \quad W^{\text{out}}(\vec{r}) = W_1^{\text{in}}(\vec{r}_b)W_1^{\text{in}}(\vec{r}_a)$$

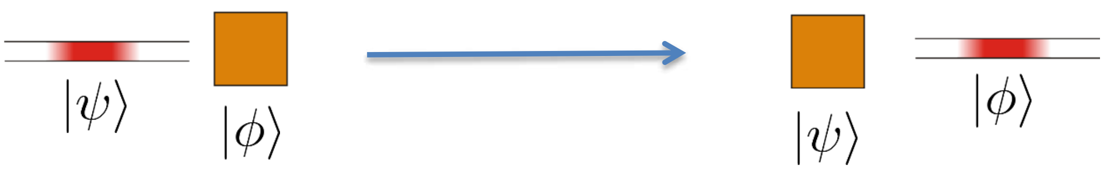
Intermezzo: Heisenberg picture

✓ for a quantum memory we want to achieve in Schrödinger picture

$$W_1^{\text{in}}(\vec{r}_a)W_2^{\text{in}}(\vec{r}_b) \quad \rightarrow \quad W^{\text{out}}(\vec{r}) = W_1^{\text{in}}(\vec{r}_b)W_1^{\text{in}}(\vec{r}_a)$$

which is achieved through the S-matrix

$$S = \begin{pmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{pmatrix}$$



equivalent to input/output relation in Heisenberg picture

$$R^{\text{out}} = SR^{\text{in}}$$

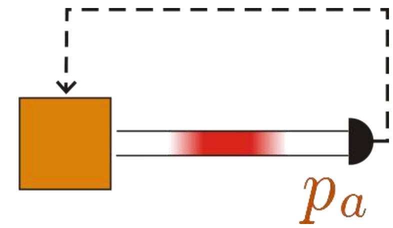
$$x_a^{\text{out}} = x_b^{\text{in}} \qquad x_b^{\text{out}} = x_a^{\text{in}}$$

$$p_a^{\text{out}} = p_b^{\text{in}} \qquad p_b^{\text{out}} = p_a^{\text{in}}$$

Spin Squeezing & Noise

$$x_a^{\text{out}} = x_a^{\text{in}} \qquad p_a^{\text{out}} = p_a^{\text{in}} + \kappa x_b^{\text{in}}$$

$$x_b^{\text{out}} = x_b^{\text{in}} \qquad p_b^{\text{out}} = p_b^{\text{in}} + \kappa x_a^{\text{in}}$$



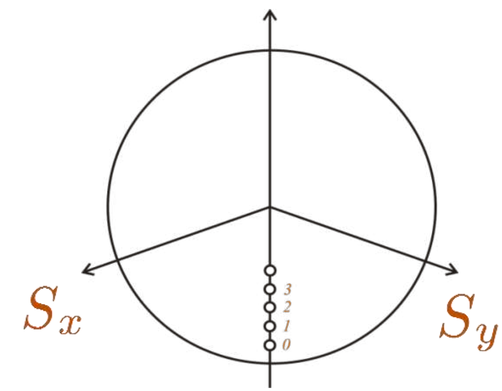
✓ application: QND measurement of spin and spin squeezing

$$\Delta x_b^2\Big|_{p_a} = (\Delta x_b^{\text{out}})^2 - \frac{\langle x_b^{\text{out}} p_a^{\text{out}}\rangle^2}{(\Delta p_a^{\text{out}})^2} = 1 - \frac{\kappa^2}{1+\kappa^2} = \frac{1}{1+\kappa^2} < 1$$

$$\Delta p_b^2 = 1 + \kappa^2$$

Kuzmich PRL 85 1594 (2000)

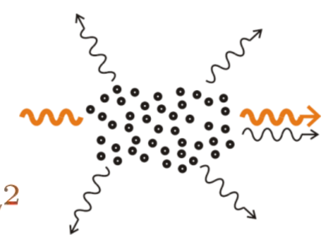
$S_z = \frac{1}{2}\sum_i \sigma_z^i$



✓ including atomic decay due to spontaneous emission

$$p_b^{\text{out}} = \sqrt{1-\eta}\,(p_b^{\text{in}} + \kappa x_a^{\text{in}}) + \sqrt{\eta}\, f$$

$$x_b^{\text{out}} = \sqrt{1-\eta}\, x_b^{\text{in}} + \sqrt{\eta}\, f$$



$$\kappa = \frac{\sqrt{N_{\text{at}} N_{\text{ph}}}\,\sigma\gamma}{A\Delta} \qquad \eta = \frac{N_{\text{ph}}\sigma\gamma^2}{A\Delta^2}$$

$$\boxed{\kappa^2 = \eta \frac{\sigma}{A} N_{\text{at}} = \eta d}$$

$$\Delta x_b^2\Big|_{p_a} = 1 - \frac{(1-\eta)\eta d}{1+\eta d} > \frac{1}{\sqrt{d}}$$

Vasilyev *et al.* J Phys B 45 124007 (2012)

Quantum Memory

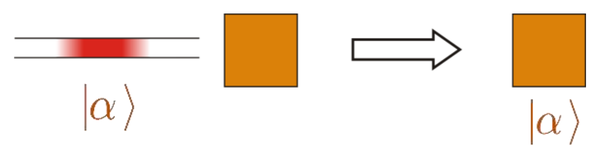
$$x_a^{\text{out}} = x_a^{\text{in}} \qquad p_a^{\text{out}} = p_a^{\text{in}} + x_b^{\text{in}}$$

$$x_b^{\text{out}} = x_b^{\text{in}} \qquad p_b^{\text{out}} = p_b^{\text{in}} + x_a^{\text{in}}$$

$$\kappa = 1$$

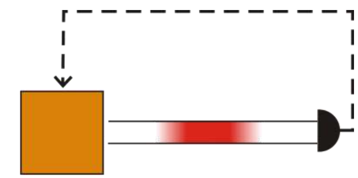
✓ for Q memory we would like to see

$$x_b^{\text{out}} = x_a^{\text{in}} \qquad p_b^{\text{out}} = p_a^{\text{in}}$$



✓ entanglement, measurement and feedback: ‘E-Feed-protocols’

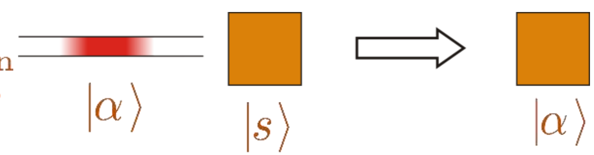
measure p_a and displace x_b by measurement result



$$x_b^{\text{final}} = x_b^{\text{out}} - p_a^{\text{out}} = -p_a^{\text{in}}$$

$$p_b^{\text{final}} = p_b^{\text{out}} = p_b^{\text{in}} + x_a^{\text{in}} \to x_a^{\text{in}}$$

squeeze p_b^{in}



✓ experiment

average fidelity of storage for coherent states

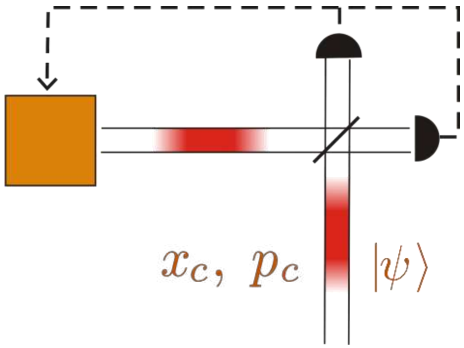
$$F \sim 66.7 \pm 1.7\% > 0.5\%$$

Julsgaard, Sherson, Cirac, Fiurasek, Polzik, Nature 432, 482 (2004)

Teleportation of Light on Atoms

$$x_a^{\text{out}} = x_a^{\text{in}} \qquad p_a^{\text{out}} = p_a^{\text{in}} + x_b^{\text{in}}$$

$$x_b^{\text{out}} = x_b^{\text{in}} \qquad p_b^{\text{out}} = p_b^{\text{in}} + x_a^{\text{in}}$$



✓ combine scattered (entangled) light with input light

measure $x_a^{\text{out}} + x_c \qquad p_a^{\text{out}} - p_c$

✓ feedback $x_b^{\text{final}} = x_b^{\text{out}} - (p_a^{\text{out}} - p_c)$ squeeze p_a^{in}

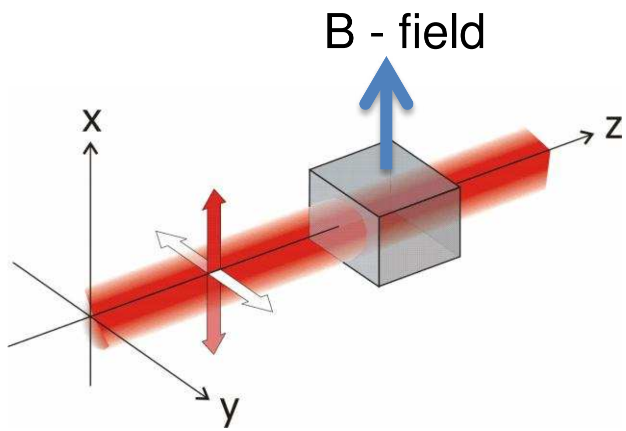
$$= x_b^{\text{in}} - (p_a^{\text{in}} + x_b^{\text{in}} - p_c) = -p_a^{\text{in}} + p_c \to p_c$$

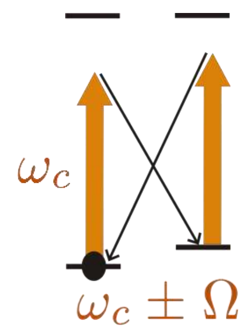
$$p_b^{\text{final}} = p_b^{\text{out}} - (x_a^{\text{out}} + x_c)$$

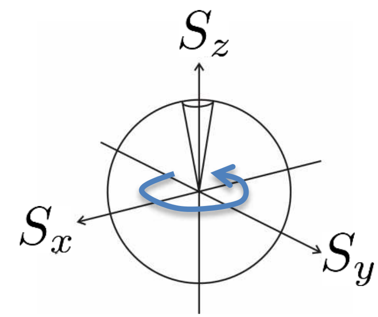
$$= p_b^{\text{in}} + x_a^{\text{in}} - (x_a^{\text{in}} + x_c) = p_b^{\text{in}} - x_c \to -x_c$$

squeeze p_b^{in}

Faraday Interaction in Magnetic Field







Ω Larmor frequency

Phase quadrature of light reads out *both* spin projections

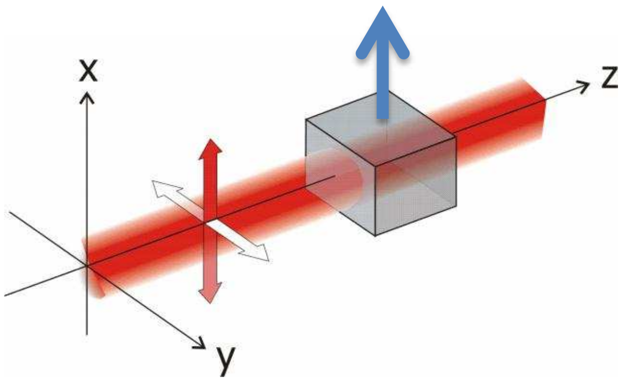
$$
\begin{aligned}
p_a^{\mathrm{out}}(t) &= p_a^{\mathrm{in}}(t) + \kappa x_b(t) \\
&= p_a^{\mathrm{in}}(t) + \kappa \left[\cos(\Omega t) x_b + \sin(\Omega t) p_b\right]
\end{aligned}
$$

Technical advantage: AC Signal at Larmor frequency (~300kHz)
But: No QND measurement anymore!

Measurement of two noncommuting observables implies back action

x_a amplitude quadrature $\rightarrow$ p_b spin projection
p_b spin projection $\rightarrow$ x_b spin projection
x_b spin projection $\rightarrow$ p_a phase quadrature
thus: x_a amplitude quadrature $\rightarrow$ p_a phase quadrature

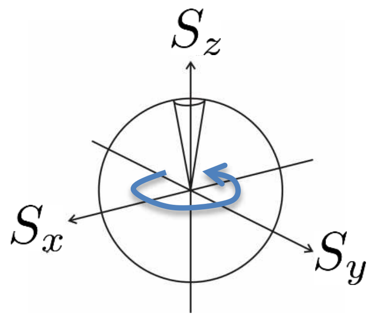
Back Action in Faraday Interaction



$$H = \Omega b^\dagger b - \kappa(b+b^\dagger)(a+a^\dagger)$$

Larmor frequency

$x_a x_b$ - coupling



Ω Larmor frequency

cavity a reads out spin projection x_b

$$\dot{p}_c = i[H, p_c] = \kappa x_b$$

(+ cavity decay + noise)

spin projection x_b evolves as

$$\dot{x}_b = -\Omega p_b$$

spin projection p_b receives amplitude noise x_a

$$\dot{p}_b = \Omega x_b - \kappa x_a$$

back action of light onto light! $\sim \kappa^2$

Limits sensitivity of spin measuement: Standard quantum limit

Teleportation in Magnetic Field

✓ interaction of light & atoms in an external magnetic field

input/output relations

ω_c

Atoms:
$$X^{\text{out}} = X^{\text{in}} + \frac{\kappa}{\sqrt{2}} p_{\text{c}}^{\text{in}}$$
$$P^{\text{out}} = P^{\text{in}} + \frac{\kappa}{\sqrt{2}} p_{\text{s}}^{\text{in}}$$

Cosine modulation mode:
$$x_{\text{c}}^{\text{out}} = x_{\text{c}}^{\text{in}} + \frac{\kappa}{\sqrt{2}} P^{\text{in}} + \left(\frac{\kappa}{2}\right)^2 p_{\text{s}}^{\text{in}} + \frac{1}{\sqrt{3}} \left(\frac{\kappa}{2}\right)^2 p_{\text{s},1}^{\text{in}}$$
$$p_{\text{c}}^{\text{out}} = p_{\text{c}}^{\text{in}}$$

Sine modulation mode:
$$x_{\text{s}}^{\text{out}} = x_{\text{s}}^{\text{in}} - \frac{\kappa}{\sqrt{2}} X^{\text{in}} - \left(\frac{\kappa}{2}\right)^2 p_{\text{c}}^{\text{in}} - \frac{1}{\sqrt{3}} \left(\frac{\kappa}{2}\right)^2 p_{\text{c},1}^{\text{in}}$$
$$p_{\text{s}}^{\text{out}} = p_{\text{s}}^{\text{in}}$$

back-action of light!

Hammerer, Polzik, Cirac
PRA 72, 053213 (2005)
PRA 74, 064301 (2006)

Teleportation Experiment

✓ for coherent states drawn from a Gaussian distribution with $\bar{n} = 2,5,10,20,200$

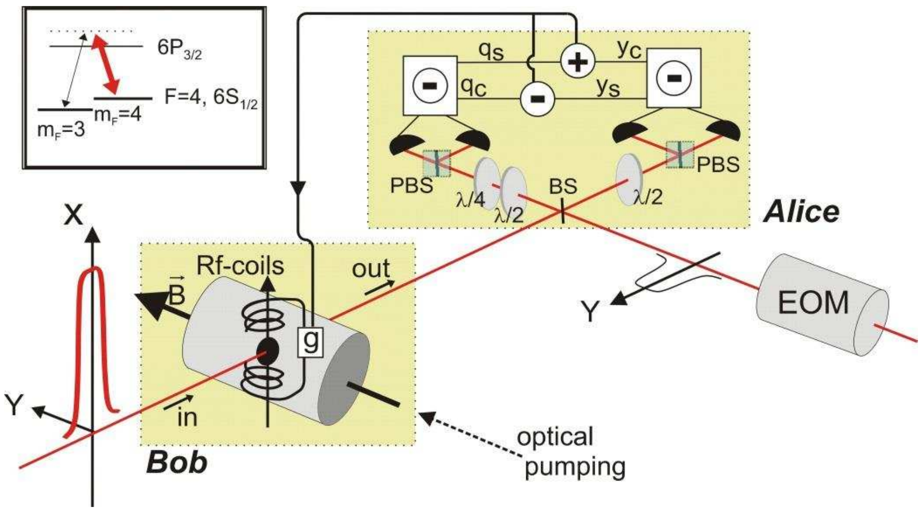
$$F_2 = 0.64 \pm 0.02 \qquad F_2^{class} = 0.60$$

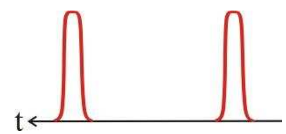
$$F_5 = 0.60 \pm 0.02 \qquad F_5^{class} = 0.54$$

$$F_{10} = 0.59 \pm 0.02 \qquad F_{10}^{class} = 0.52$$

$$F_{20} = 0.58 \pm 0.02 \qquad F_{20}^{class} = 0.51$$

$$F_{200} = 0.56 \pm 0.03 \qquad F_{200}^{class} = 0.50$$





1st entangling pulse

2nd verifying pulse

Sherson, Krauter, Olsson, Julsgaard, KH, Cirac, Polzik, Nature 443, 557 (2006)

Further results and reading

✓ Faraday interaction

Quantum memory for entangled two-mode squeezed states

Jensen *et al.* Nature Physics 7 13 (2011)

Steady State Entanglement of two atomic spins

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✓ Hybrid approaches towards long distance QC unifying CV protocols and DLCZ-like architecture (e.g. photon counting)

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Thank you