

Olomouc Seminar

Decoherence effects on quantum teleportation:

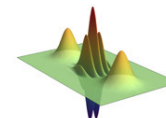
transfer of information encoded into particles and fields

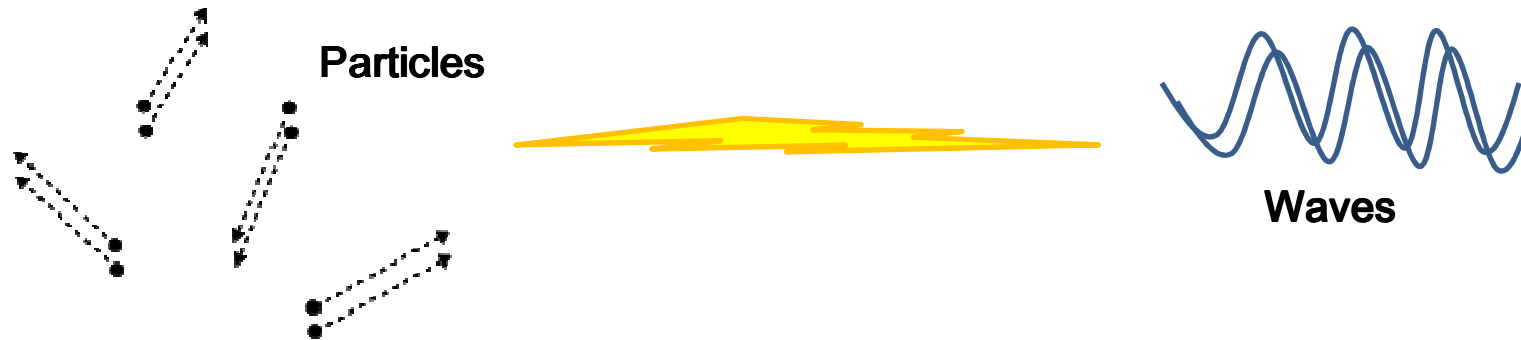
Kimin Park

Nov 5th, 2012



INVESTMENTS IN EDUCATION DEVELOPMENT





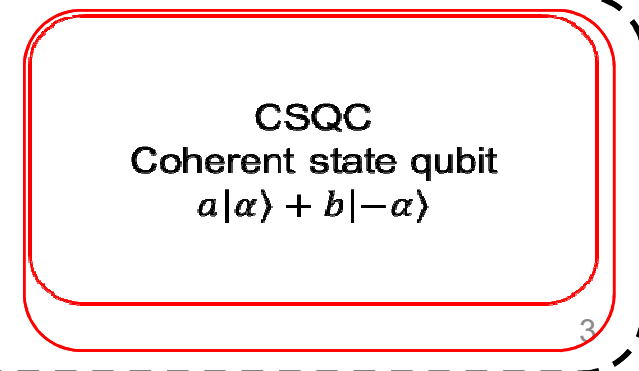
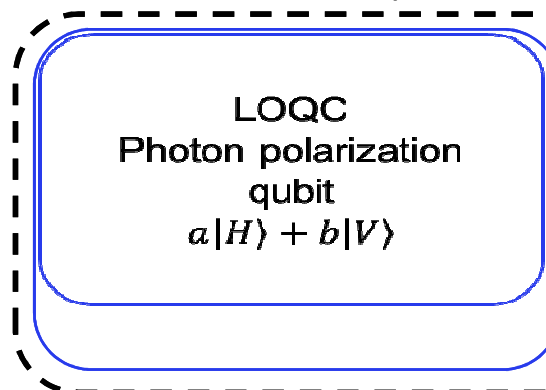
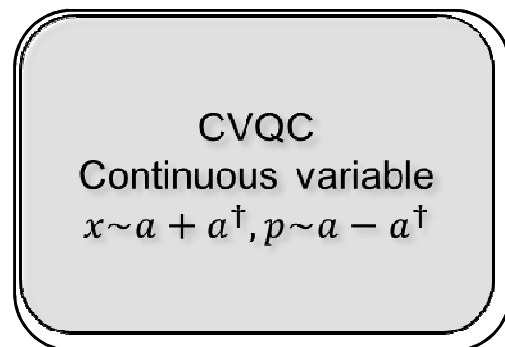
- Information transfer conveyed by particle and fields
- Environmental and device effect

Particles and fields

- Wave-particle duality: Bohr's Complementarity principle
- Wave-particle superposition (Tang et al.), Wave-particle conversion (Miwa et al.)
- *Particle-like* qubit: Photons $|H\rangle, |V\rangle$,
- *Field-like* qubit: coherent states $|\alpha\rangle, |-\alpha\rangle$ (Cochrane et al., Jeong and Kim, Ralph et al.),
Single-rail qubit $|1\rangle, |0\rangle$ (Lund and Ralph)

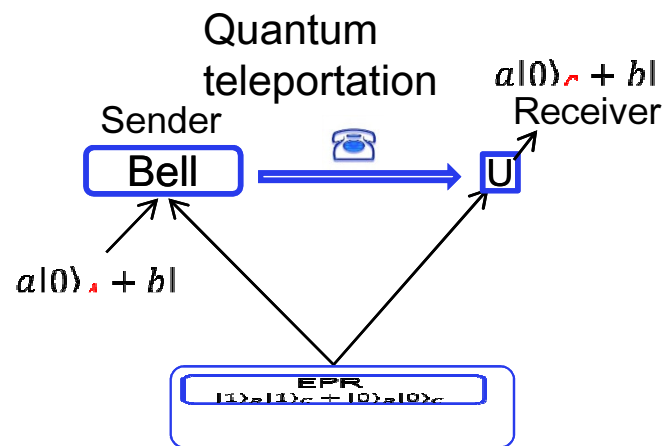
Optical QIP

(Discrete) variable

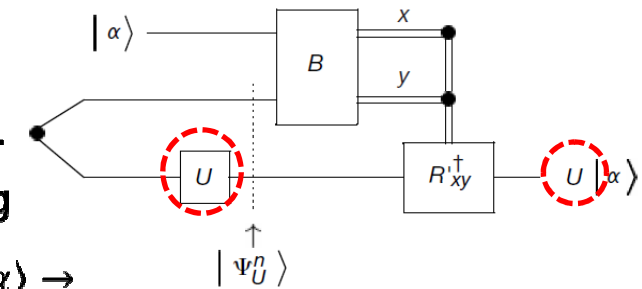


Information transfer (Communication)

	Classical Means	Quantum Means
Classical Information	Classical communication	<ul style="list-style-type: none"> • Superdense coding • Quantum Key Distribution (e.g. BB84, E91)
Quantum Information	<ul style="list-style-type: none"> • Measure and prepare • Direct sending 	<ul style="list-style-type: none"> • Teleportation (Bennett et al.) • Remote state preparation (Bennett et al.) • Interaction-mediated state transfer (e.g. Spin chain quantum wire)



- Difficult photon-photon coupling
- Off-line preparation & operation



Gate teleportation
(Gottesman *et al.*, Nature **402**, 390 (1999))

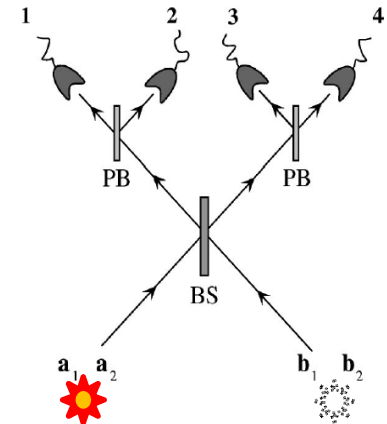
Bell measurement

Photon polarization Bell states

$$|B_{1,2}\rangle = \frac{1}{\sqrt{2}} (|H\rangle|H\rangle \pm |V\rangle|V\rangle)$$

$$|B_{3,4}\rangle = \frac{1}{\sqrt{2}} (|H\rangle|V\rangle \pm |V\rangle|H\rangle)$$

- When one or both photons are removed, “**failure**”.
- Success probability limited by 1/2.



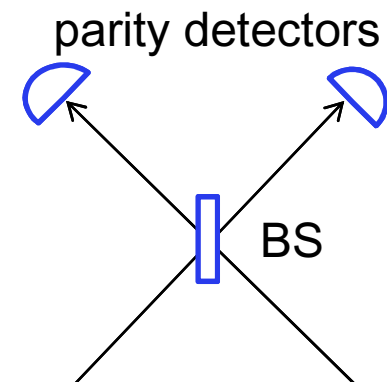
(Lütkenhaus et al.,
PRA 59, 3295 (1999))

Coherent state Bell states

$$|B_{1,2}\rangle = N_{1,2} (|\alpha\rangle|\alpha\rangle \pm |-\alpha\rangle|-\alpha\rangle)$$

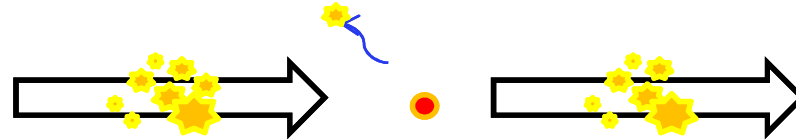
$$|B_{3,4}\rangle = N_{3,4} (|\alpha\rangle|-\alpha\rangle \pm |-\alpha\rangle|\alpha\rangle)$$

- no click: fail
- Success probability $\propto 1 - \frac{e^{-2\alpha^2}}{2}$ (**99.98%** for $\alpha = 2$)



(Jeong et al., PRA 64, 052308)

Decoherence



- One of the main obstacles in QIP
- Born-Markov approximation: Dissipation (photon-loss) by zero temperature, memory-less (time-local) bath
- $\frac{d\rho}{d\tau} = \gamma \sum_i a_i \rho a_i^\dagger - \frac{\gamma}{2} \sum_i (a_i^\dagger a_i \rho + \rho a_i^\dagger a_i),$
(γ : interaction strength, a_i : annihilation operator)
- Cause of the difference in field-like and particle-like qubits
- Probabilistic loss of qubit, decay of coherence.

$$|H\rangle\langle H| \rightarrow t^2 |H\rangle\langle H| + (1 - t^2) |0\rangle\langle 0|$$

$$|V\rangle\langle V| \rightarrow t^2 |V\rangle\langle V| + (1 - t^2) |0\rangle\langle 0|$$

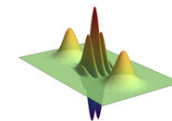
$$|H\rangle\langle V| \rightarrow t^2 |H\rangle\langle V| \quad (t = e^{-\frac{\gamma\tau}{2}})$$

- Continuous amplitude decay, exponential decay of coherence (faster decoherence for large α).

$$|\alpha\rangle\langle\alpha| \rightarrow |t\alpha\rangle\langle t\alpha|, |\alpha\rangle\langle-\alpha| \rightarrow e^{-2\alpha^2(1-t^2)} |t\alpha\rangle\langle -t\alpha|$$

Entangled Coherent States vs. Entangled Photon Pairs for Practical QIP

PHYSICAL REVIEW A 82, 062325 (2010)

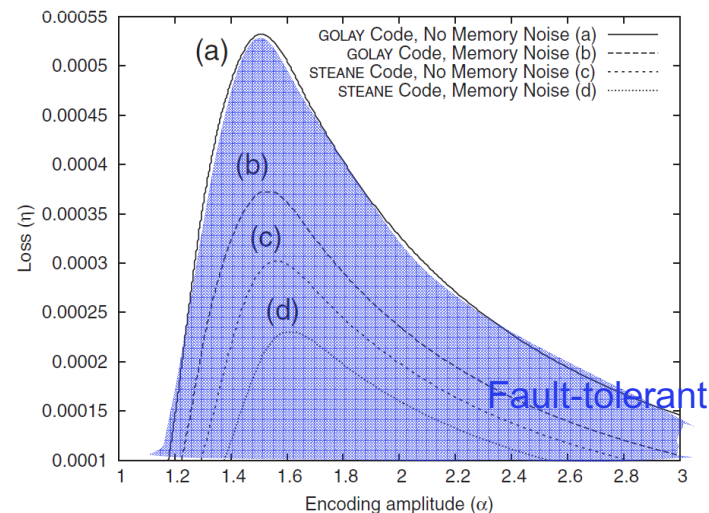
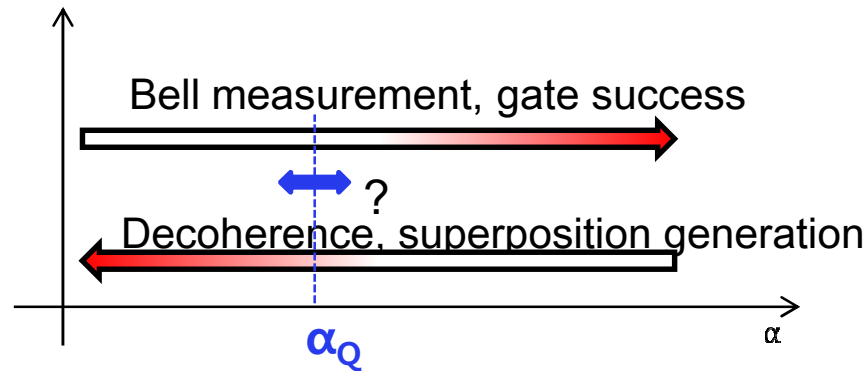


CMQC

Center for Macroscopic Quantum Control @ SNU



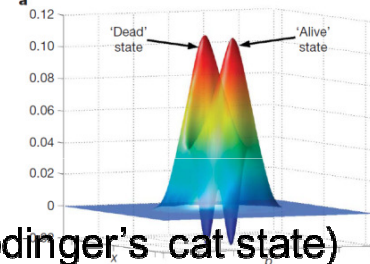
Motivation



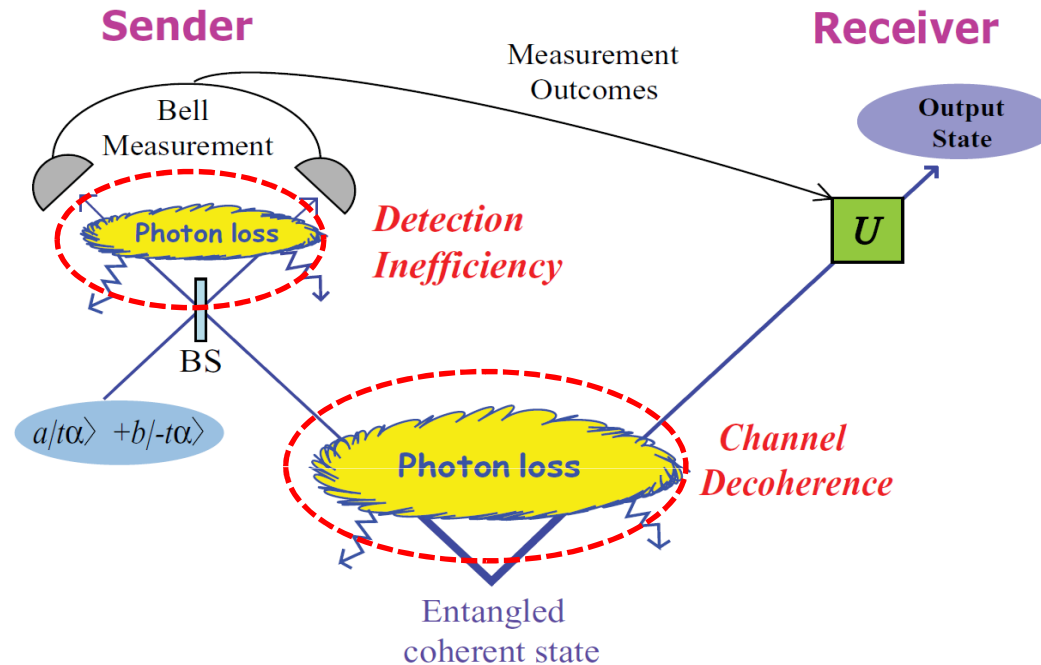
(Lund *et al.*, PRL **100**, 030503 (2008))

- Comparison of the *wave-like* and *field-like* information transfers
- Comparing LOQC with CSQC
- What α to use?
- Threshold error rate that allows **fault-tolerance**.
- $\alpha \approx 1.6$ is preferred.
- Coherent state superposition (Schrödinger's cat state)

$$|\alpha\rangle \pm |-\alpha\rangle$$
- Key resource in CSQC
- $|\alpha| = 1.76$ (Gerrits *et al.*, PRA 82, 031802 (2010)).
- Arbitrary qubit: $|\alpha| = 1$ (Neergaard-Nielsen *et al.*, PRL 105, 053602 (2010)).



Practical Teleportation

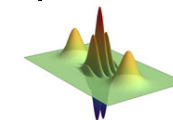


- **Teleportation under photon losses**
- Entangled Coherent state (\mathcal{C}):
- Entangled Single photon (\mathcal{S}):
- Entangled Photon pair (\mathcal{P}):

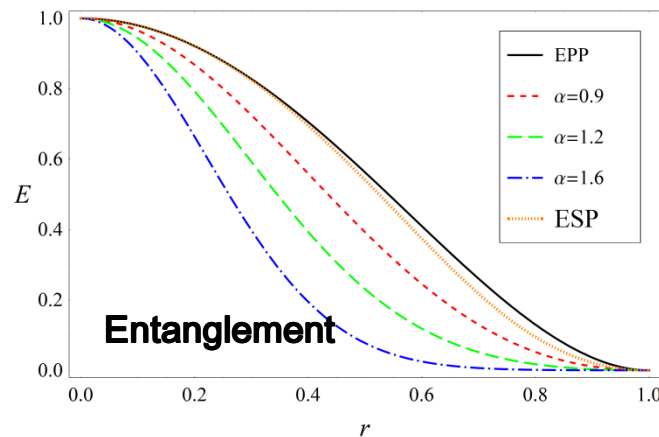
$$|\text{ECS}\rangle = N(|\alpha\rangle|-\alpha\rangle - |-\alpha\rangle|\alpha\rangle)$$

$$|\text{ESP}\rangle = \frac{1}{\sqrt{2}}(|1\rangle|0\rangle - |0\rangle|1\rangle)$$

$$|\text{EPP}\rangle = \frac{1}{\sqrt{2}}(|H\rangle|V\rangle - |V\rangle|H\rangle)$$



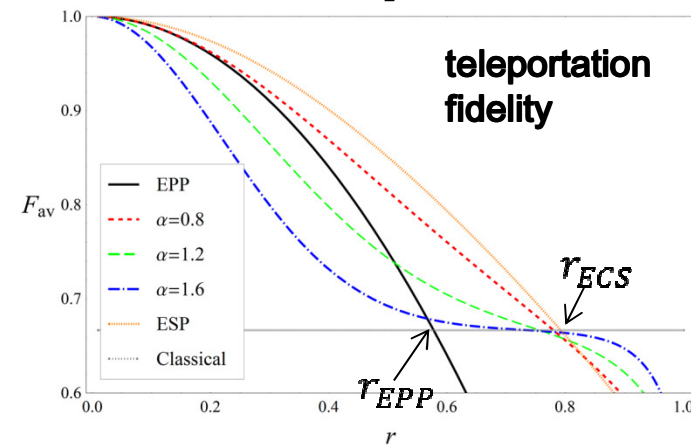
Entanglement vs. Teleportation fidelity



Normalized time $r = \sqrt{1 - e^{-\gamma\tau}}$

Negativity : $N = 2 \sum_i |\lambda_i|$ (λ_i : negative eigenvalues of PT)

BC



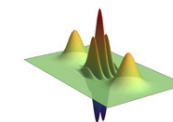
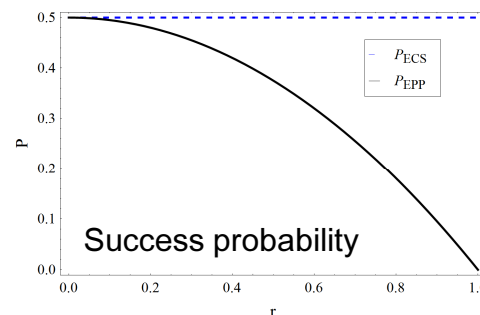
Quality of the teleportations compared.

For small $\alpha \lesssim 0.8$, $C > P$ (main result)

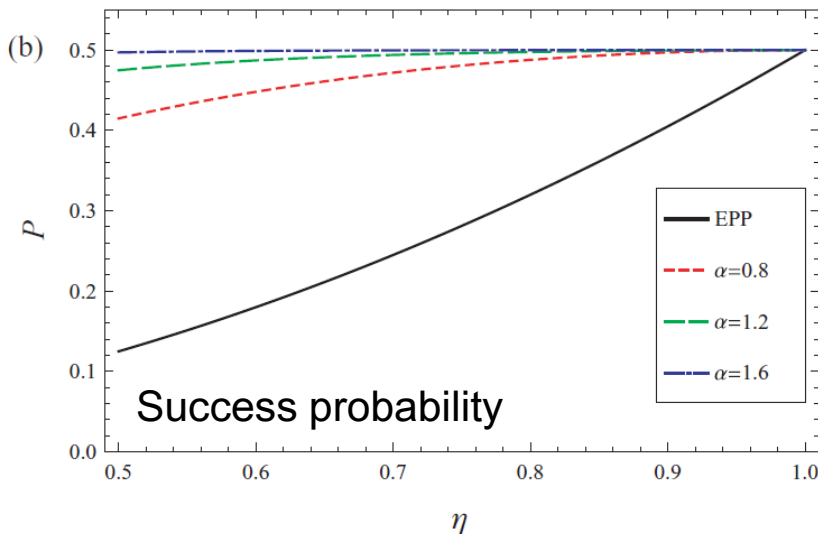
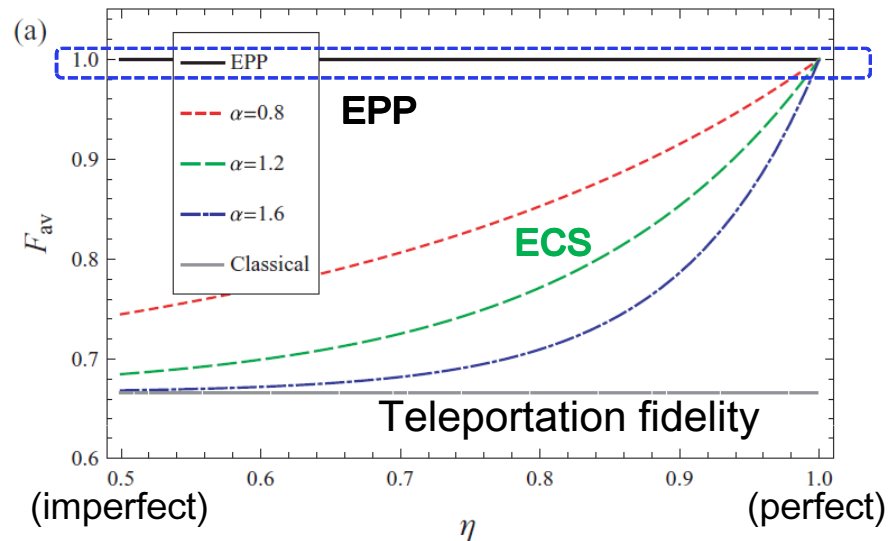
C stays over classical limit $2/3$ for longer.

overlap ($\langle \alpha | -\alpha \rangle$) + escape ($|H\rangle\langle H| \rightarrow |0\rangle\langle 0|$)

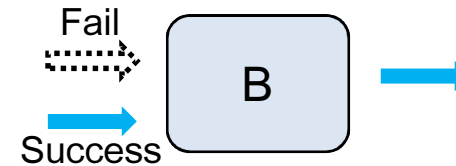
$C > P$ for success probability for all α .



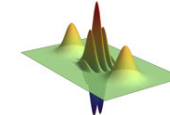
F vs. Detection efficiency



- η : **detector efficiency** (photon loss at the detector)
- Only **C** teleportation is easily “**contaminated**” by detection efficiency.
- Polarization Bell measurement **filters out the loss**.

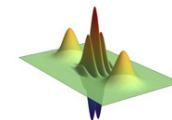


- Loss is less detectable for **C** Bell measurements.



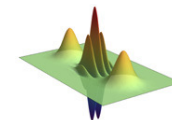
Section summary

- Particles transfers the information faithfully, and robust to inefficient detectors, while fields does it less faithfully, but more frequently.
- Entanglement: $P > C$
- Teleportation: $C > P$ ($\alpha \lesssim 0.8$: small amplitude),
 $C > P$ ($r \gtrsim 0.577$: strong decoherence, $\alpha \gtrsim 0.8$),
 $C < P$ ($r \lesssim 0.577$, $\alpha \gtrsim 0.8$),
- Overlap of C and escape filtering of P is responsible.
- Detection inefficiency: $P > C$
- P teleportation treats located errors more efficiently.

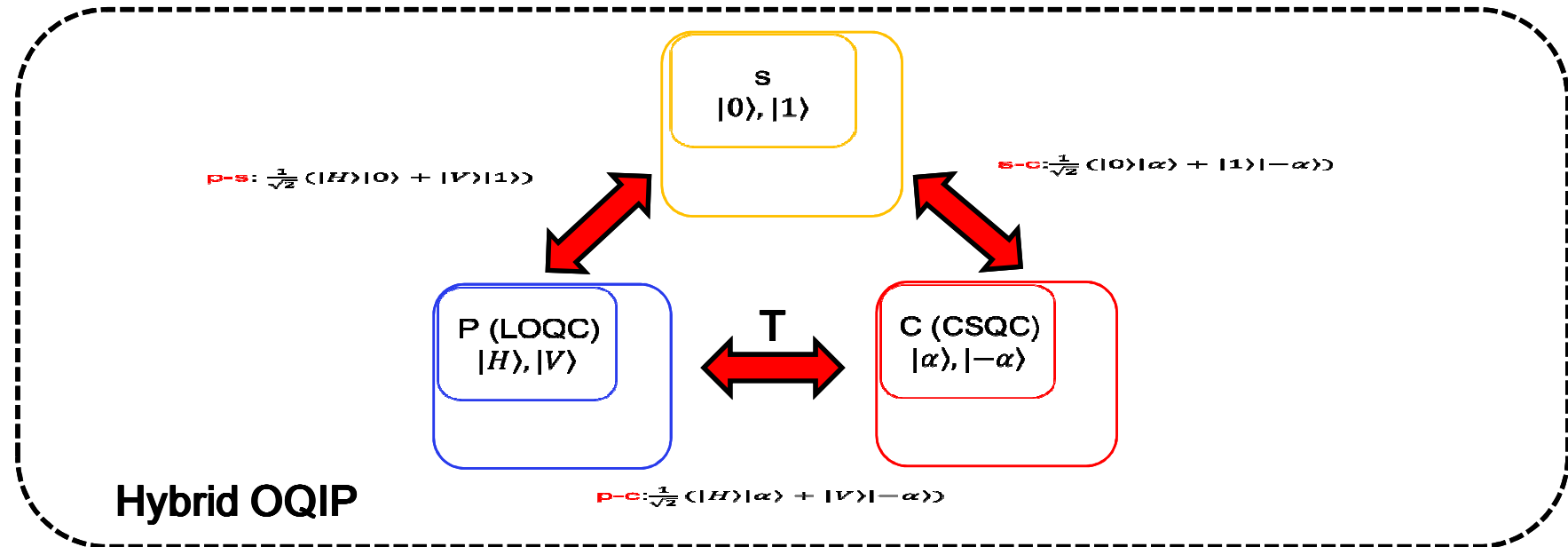


Quantum Teleportation between Particle-like and Field-like Qubits under Decoherence

Submitted to PRA



Motivation

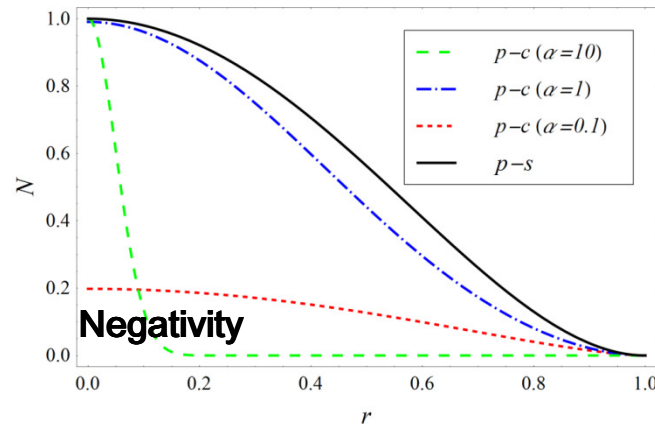


- **Difficulty** in completion of individual schemes \Rightarrow **Hybrid** computation
- **Teleportation** can transfer information between different systems (a quantum interface): How would information with one property transmitted to that with complementary property?
- **Different dynamics** for different qubits under the environmental effects.

Hybrid QIP

- Realization of a gate operation combining the qubits of different nature.
- Hybrid optical QIP using continuous-variable bus, controlled displacement atom-light interaction (van Loock et al., PRA 78, 022303 (2008))
- Polarization qubit CNOT gate by coherent state (Nemoto and Munro, PRL 93, 250502 (2004))
- Hybrid qubit $a|H\rangle|\alpha\rangle + b|V\rangle|-\alpha\rangle$ (Lee and Jeong, arXiv:1112.0825 (2011)) allows a near-deterministic QIP with relatively small resources.

Negativity vs. Teleportation



p-c entanglement is largest at $\alpha \sim 1$

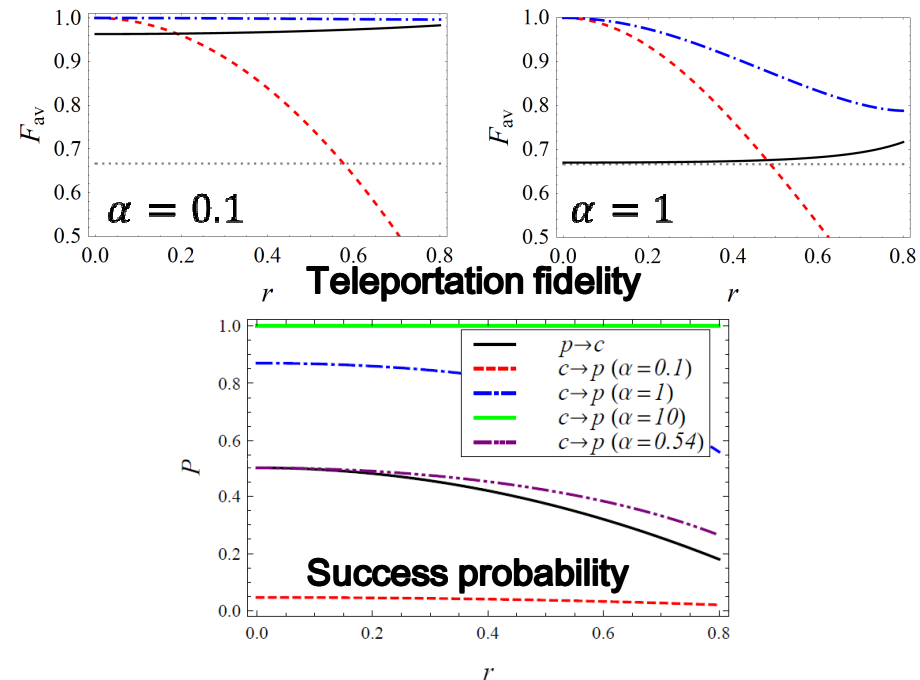
Productive for $\alpha \ll 1$:

$$|H\rangle|\alpha\rangle + |V\rangle|-\alpha\rangle \sim (|H\rangle + |V\rangle)|0\rangle$$

Large dephasing for $\alpha \gg 1$:

$$|\alpha\rangle\langle-\alpha| \rightarrow e^{-2\alpha^2}|\alpha\rangle\langle-\alpha|.$$

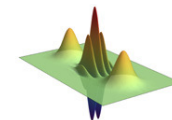
p-s > p-c



- Smaller fidelity for large α .
- **$p \rightarrow c > c \rightarrow p$** (difference in fidelity)
- **overlap** ($\langle \alpha | -\alpha \rangle = e^{-2\alpha^2}$) + photon loss filtering.
- $p \rightarrow c$ is always larger than classical while $c \rightarrow p$ drops below it.
- $c \rightarrow p$ succeeds more frequently than $p \rightarrow c$.

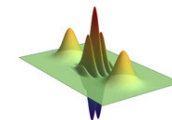
Section summary

- Hybrid optical QIP by **hybrid teleportation**.
- $p \rightarrow c,s > p \leftarrow c,s$
- For c and s, error remains in the qubit space, while not for p.
- Filtering and overlap is responsible for this.
- **Particle nature** of polarization qubit is the main cause of the difference.
- Can be interpreted as information transfer between microscopic and macroscopic objects.



Conclusions

- Particles transfers the information faithfully, and robust to inefficient detectors, while fields does it less faithfully, but more frequently.
- **A criteria for the choice of the amplitude** of the coherent state with regard to the decoherence effect is provided.
- **A hybrid strategy of optical QIP schemes** having different advantages and disadvantages is investigated under realistic situations.



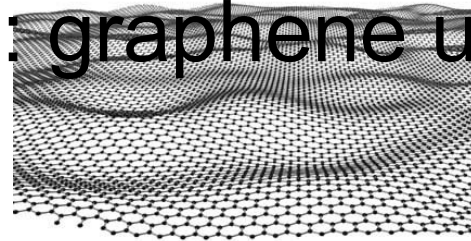
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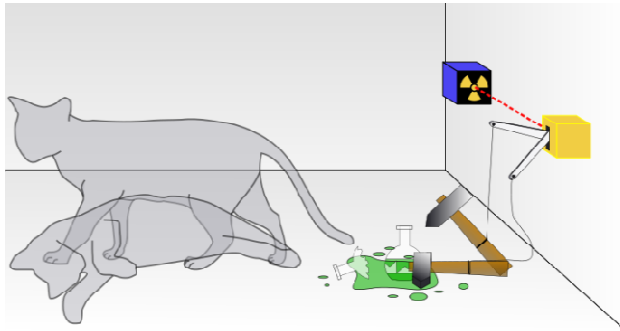
Future works

- Light-matter interfaced QIP (Yao and Belyanin, Phys. Rev. Lett. 108, 255503 (2012).): graphene under magnetic field
- Decoherence and Macro-realism (Kofler and Brukner, Phys. Rev. Lett. 99, 180403 (2007).)
- Interacting quantum-classical systems (Elze, Phys. Rev. A 85, 052109 (2012).)

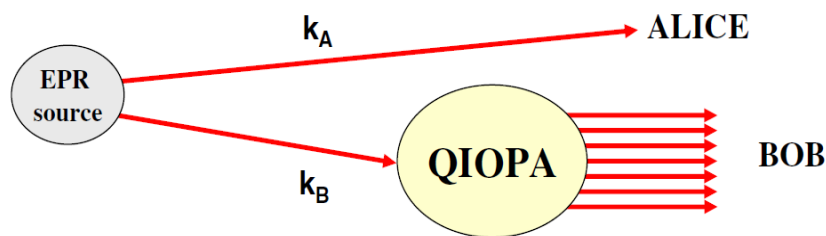


- **Thank you!**

micro-Macro entanglement



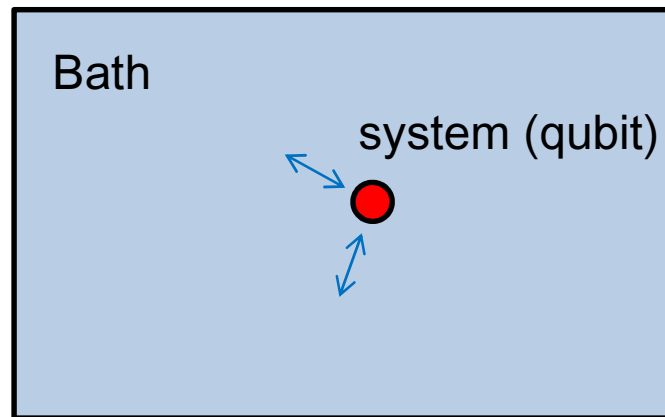
Schrödinger's cat.



de Martini *et al.*, PRL **100**, 253601 (2008).

- $\frac{1}{\sqrt{2}} (|H\rangle|\alpha\rangle + |V\rangle|-\alpha\rangle)$ may be interpreted as an micro-macro entanglement.
- $|\Sigma\rangle = \frac{1}{\sqrt{2}} (|H\rangle|\Phi^+\rangle + |V\rangle|\Phi^-\rangle)$ is generated via quantum-injected optical parametric amplification (QI-OPA).

Decoherence

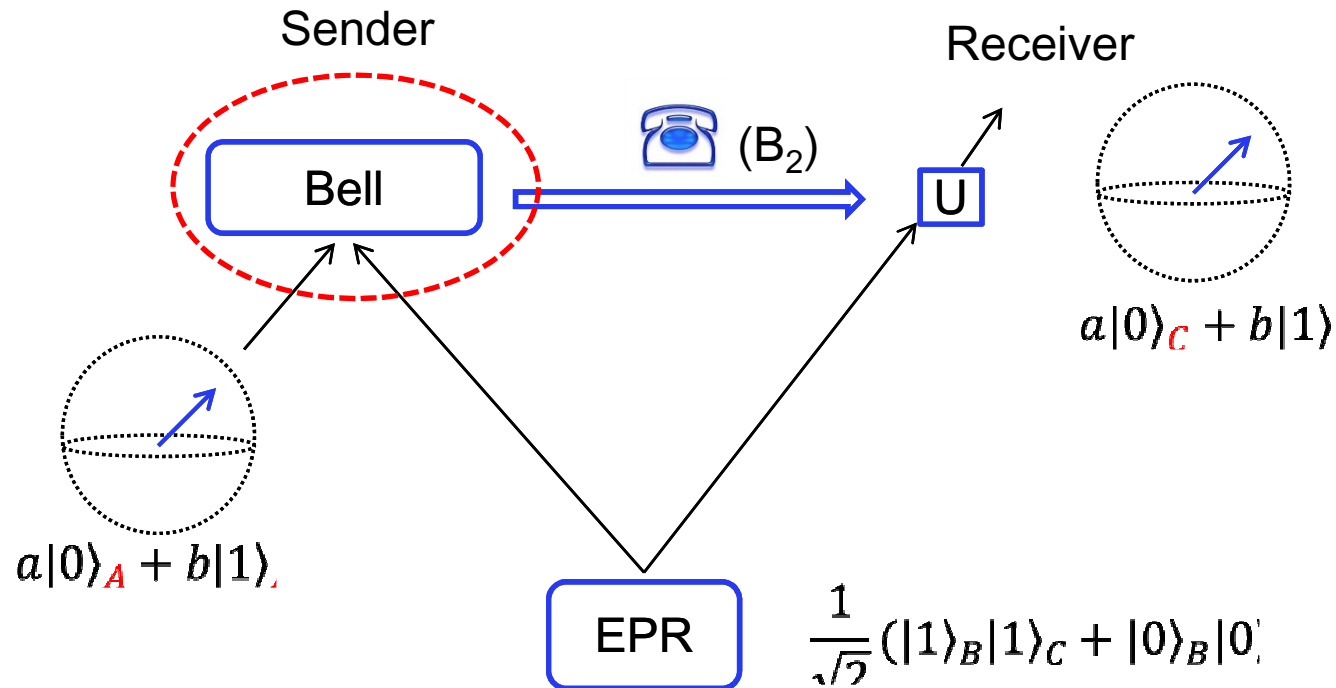


- $(a|0\rangle + b|1\rangle)_S |0\rangle_E \Rightarrow a|0\rangle|0\rangle + b|1\rangle|1\rangle$
- When only the system is observed, the reduced density matrix is seen as a mixture.

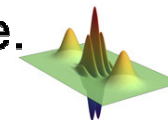
$$\rho = \begin{pmatrix} |a|^2 & 0 \\ 0 & |b|^2 \end{pmatrix}$$

Quantum teleportation

(Bennett *et al.*, PRL 70,1895 (1993))



- Central element of QIP
- Information is conveyed to another mode nonlocally.
- $\sim I$
- Need a Bell measurement and entangled state.



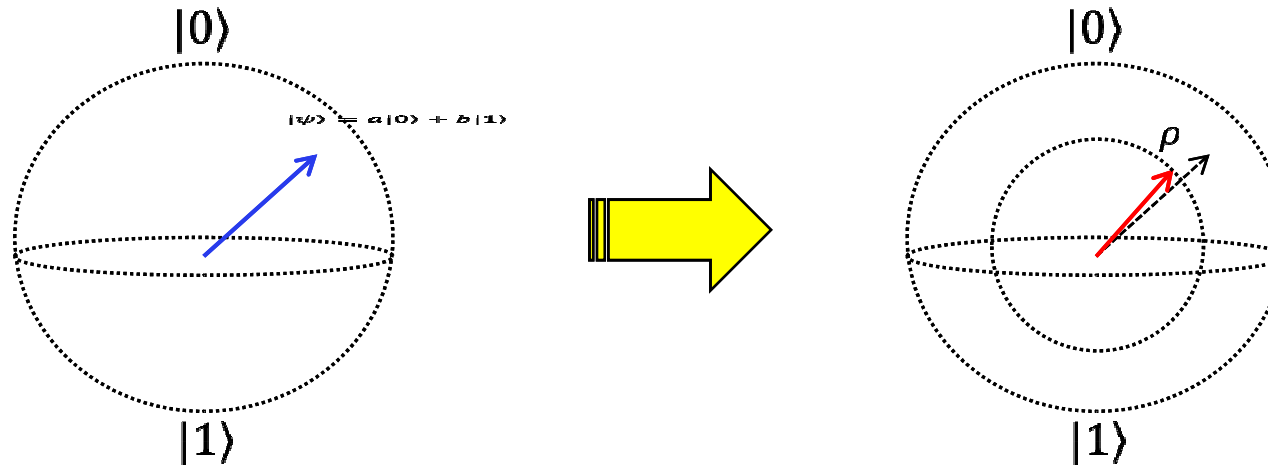
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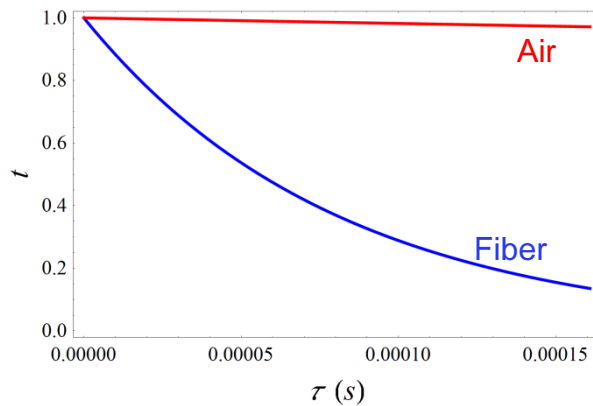
Teleportation fidelity



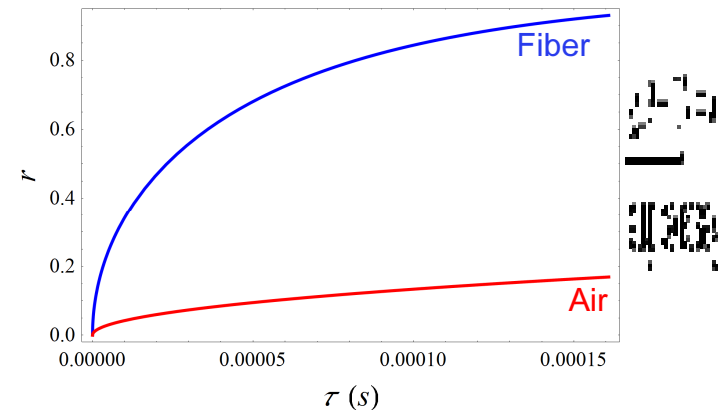
- Pure input state $|\psi\rangle \rightarrow$ Mixed output state ρ
- $F = \int d\psi \langle \psi | \rho | \psi \rangle$
- Classical limit (no-cloning bound): $2/3$

Attenuation

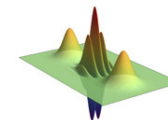
- Optical fiber: $\gamma_F \approx 1.2 \times 10^4 s^{-1}$, 0.18 dB/km (van Loock *et al.*, PRA 78, 062319 (2008))
- Air(rayleigh scattering): $\gamma_a \approx 1.8 \times 10^2 s^{-1}$ (Sneep and Ubachs, JQSRT 92, 293 (2005)).



Coherent amplitude damping

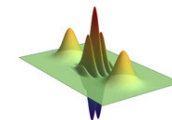


Time rescaling



Summary of last talk (reminder)

- Quantum aspect of information can bring **a significant improvement** in security and computation power.
- There exists **two (discrete) optical QIP schemes** using photon polarization and coherent states, which possess different advantages and disadvantages.
- **Teleportation** is an important element for both.



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Reminder

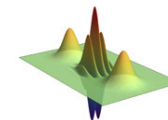
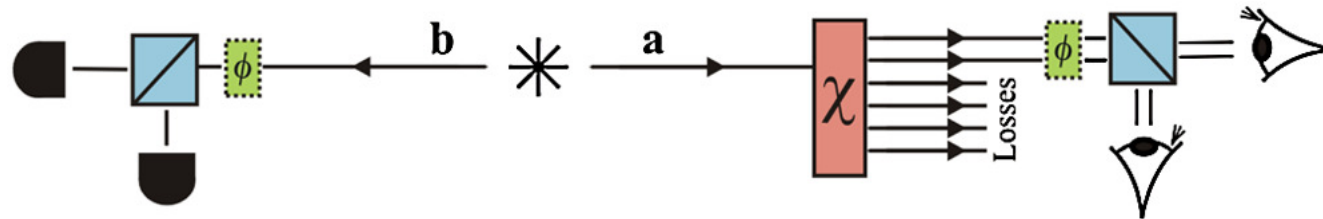
- Decoherence is caused by the interaction of an system with its environment, and is an phenomenon a system generally loses its quantum properties.
- **Dissipation** (photon loss) is the dominant decoherence mechanism for **optical** systems.
- A photon polarization qubit suffers an escape effect, while a coherent state qubit suffers a decay of its amplitude and dephasing.

Progress made since

- I wrote my thesis.
- Summarized a more tight backgrounds about hybrid QIPs.
- Made the analysis more clear.

micro-Macro entanglement

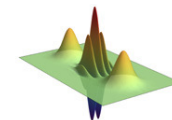
- Sekatski *et al.*, Phys. Rev. Lett. 103, 113601 (2009).
- $|\Sigma\rangle = \frac{1}{\sqrt{2}}(|H\rangle|\Phi^+\rangle + |V\rangle|\Phi^-\rangle)$ observed with human eyes shows a Bell-violation.



Teleportation fidelity and singlet fraction

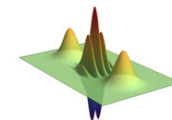
(Horodecki *et al.*, PRA 60, 1888 (1999))

- Average fidelity of quantum teleportation $f = \int \langle \psi | \Lambda(|\psi\rangle\langle\psi|) | \psi \rangle$ is related to maximal entangled fidelity of the channel $F(\rho^\Lambda) = \text{Tr}(|\Phi\rangle\langle\Phi| \rho^\Lambda)$.
- $f = \frac{dF(\rho^\Lambda)+1}{d+1}$, for which ρ^Λ should have a local reduced density matrix proportional to identity operator.



Entanglement and teleportation

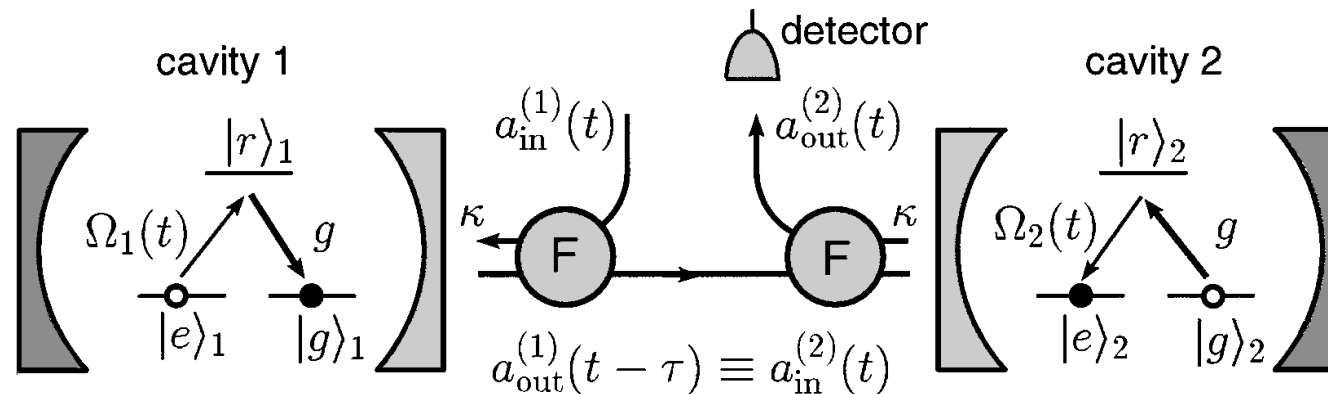
- A mixed state which does not violate Bell inequality may be useful for teleportation: $\rho = \frac{1}{8}I + \frac{1}{2}|\psi\rangle\langle\psi|$ (Popescu, PRL 72, 797 (1994)).
- A set of states that shows Bell nonlocality is a strict subset of non-separable states (Werner, PRA 40, 4277 (1989)).



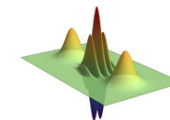
Contributions to the Field

- Suggested a new criteria for a choice of amplitude of coherent state qubit.
- Investigated the crucial element of a new strategy of hybrid QIP.

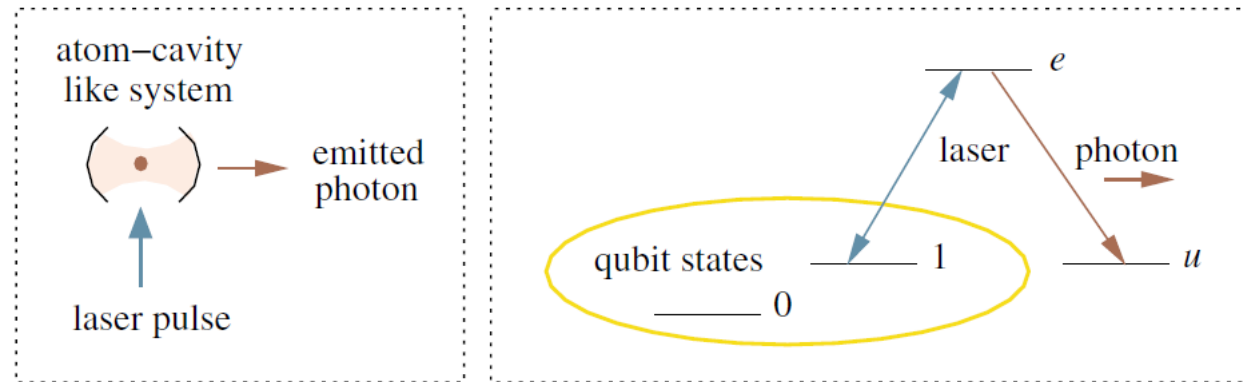
Hybrid QIP



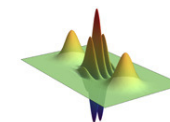
- Mostly centered on the realization of a gate operation utilizing the qubits of different nature.
- Cirac *et al.*, Phys. Rev. Lett 78, 3221 (1997).
- $(c_g|g\rangle + c_e|e\rangle)|g\rangle \rightarrow |g\rangle(c_g|g\rangle + c_e|e\rangle)$



Hybrid QIP

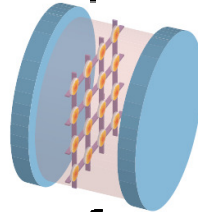


- Lim *et al.*, Phys. Rev. Lett. 95, 030505 (2005)
- Atom-cavity-like system (sources for the generation of single photons on demand)
- Two-qubit phase gate: $U_{CZ} = \text{diag}(1, 1, 1, -1)$

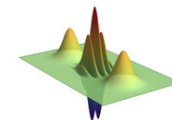


Hybrid QIP

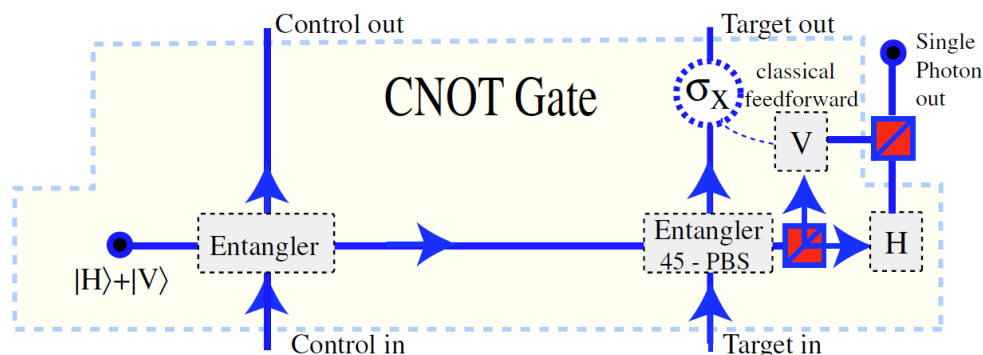
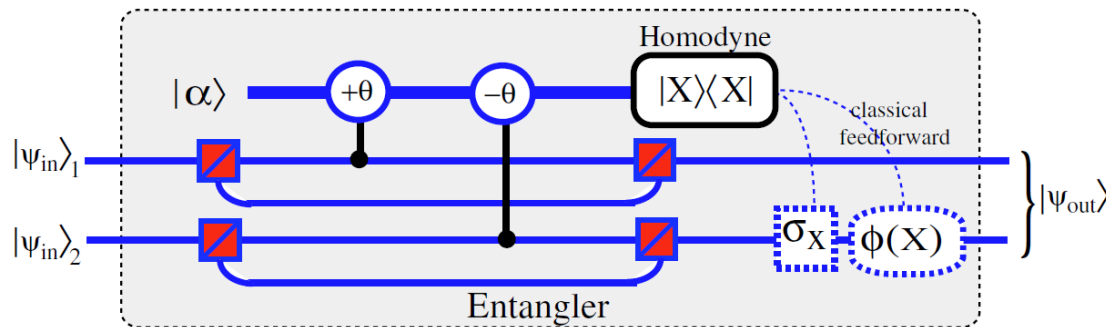
- Reliable reading and writing of topologically protected quantum memory using an atomic or photonic qubit (Jiang *et al.*, Nature Physics 4, 482 (2008)).



- Hybrid quantum computer of continuous and discrete quantum variables (Lloyd, arXiv: quant-ph/0008057 (2000)), utilizing the easiness of quantum Fourier transform in continuous variable QIP.
- Hybrid qubit in the form of $|0_L\rangle = (|H\rangle + |V\rangle)|\alpha\rangle$, $|1_L\rangle = (|H\rangle - |V\rangle)|\alpha\rangle$ may show advantages in universal gate operation and fault tolerance (Lee and Jeong, arXiv:1112.0825 (2011)).



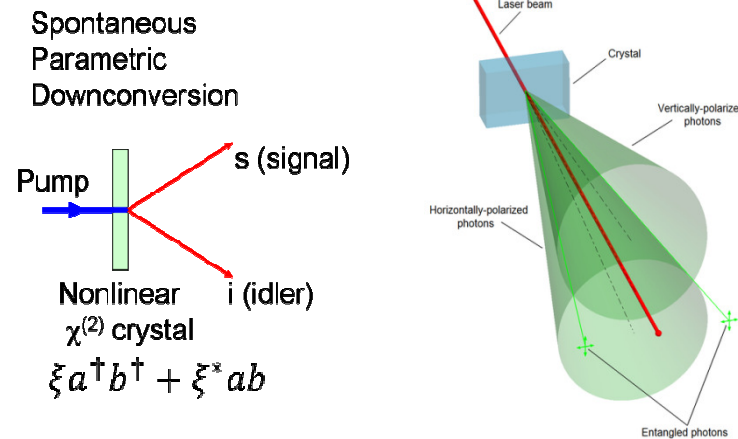
Hybrid QIP



- Nemoto *et al.*, Phys. Rev. Lett. 93, 250502 (2004).
- Near-deterministic CNOT gate using a part containing a coherent state.
- A weak cross-Kerr nonlinearity ($U = \exp[i\theta n_a n_p]$) with strong coherent state and

Entanglement generation

- ECS: $(|\alpha\rangle + |-\alpha\rangle)|0\rangle \xRightarrow{\text{BS}} \left| \frac{\alpha}{\sqrt{2}} \right\rangle \left| -\frac{\alpha}{\sqrt{2}} \right\rangle + \left| \frac{\alpha}{\sqrt{2}} \right\rangle \left| -\frac{\alpha}{\sqrt{2}} \right\rangle$
- EPP:



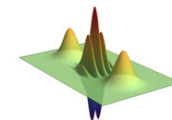
(Kwiat *et al.*, PRL **75**, 4337 (1995))

Entanglement generation

- p-c: $(|H\rangle + |V\rangle)|\alpha\rangle \xrightarrow{NL} |H\rangle|\alpha\rangle + |V\rangle|\alpha e^{-i\phi}\rangle$
(Gerry, PRA 59, 4095 (1999))

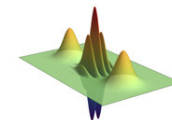
Polarization qubit and single-rail qubit

- Photon polarization (dual-rail for spatial-mode)
- $|H\rangle = |1\rangle|0\rangle$, $|V\rangle = |0\rangle|1\rangle$ ($|1\rangle = \hat{a}^\dagger|0\rangle$)
- qubit: $a|H\rangle + b|V\rangle$
- **Single-rail photon qubit**
- qubit: $a|0\rangle + b|1\rangle$
- Called a **Field-like** encoding together with coherent state qubit

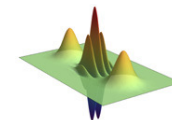


Research Interests

- Decoherence: quantum \rightarrow classical
- Hybrid QIP
- Measurement: weak measurement, phase estimation



Backgrounds: Quantum Information



CMQC

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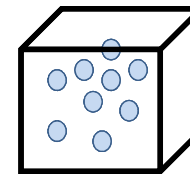


Quantum Information

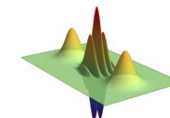
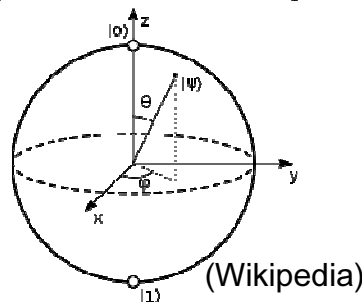
- “Information is physical.” (Landauer 1961)

$$S(\rho) = -\text{Tr}(\rho \ln \rho)$$

- “...the laws of information transmission are restricted or governed by the laws of physics...” (Galindo and Martin-Delgado, RMP (2002))
- “Quantum” aspect of information & “Informational” aspect of quantum mechanics (quantum simulator).



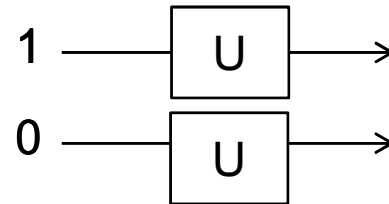
- **qubit** $a|0\rangle + b|1\rangle$ (atom, cavity, electron, quantum dot, photons...)



Advantages

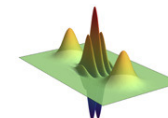
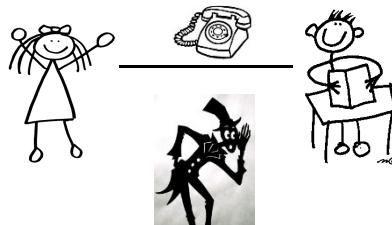
- **Exponential computation power** (quantum parallelism).

$$U(|0\rangle + |1\rangle) = U|0\rangle + U|1\rangle$$



- Factoring numbers (Shor), searching the database (Grover)...
- Unconditionally secure (quantum key distribution)

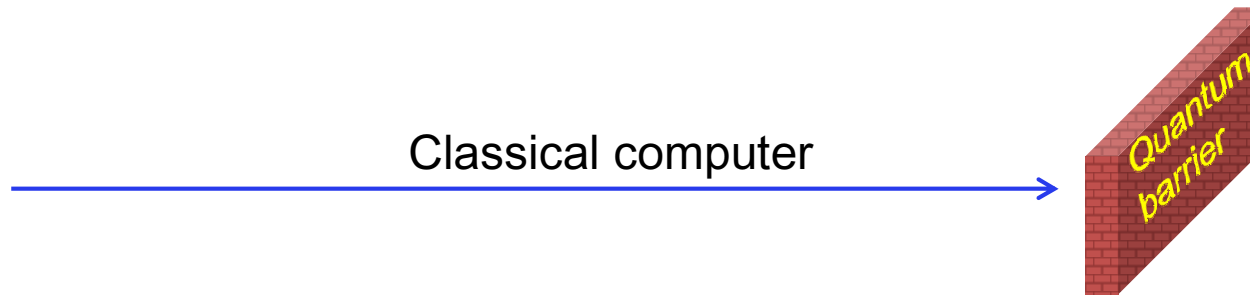
$|\psi\rangle|0\rangle \nrightarrow |\psi\rangle|\psi\rangle$ (no-cloning)



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FAQ



Q. Why should we use the “quantum” information?

A. The **resources**. 1 qubit $\equiv \infty$ bits to represent.

Q. Where is this quantum advantage from?

A. The **full space** of the state is exploited. As a price, we give up copying.

