

INVESTMENTS IN EDUCATION DEVELOPMENT

Quantum measurements in laser gravitation wave detectors

F.Ya.Khalili

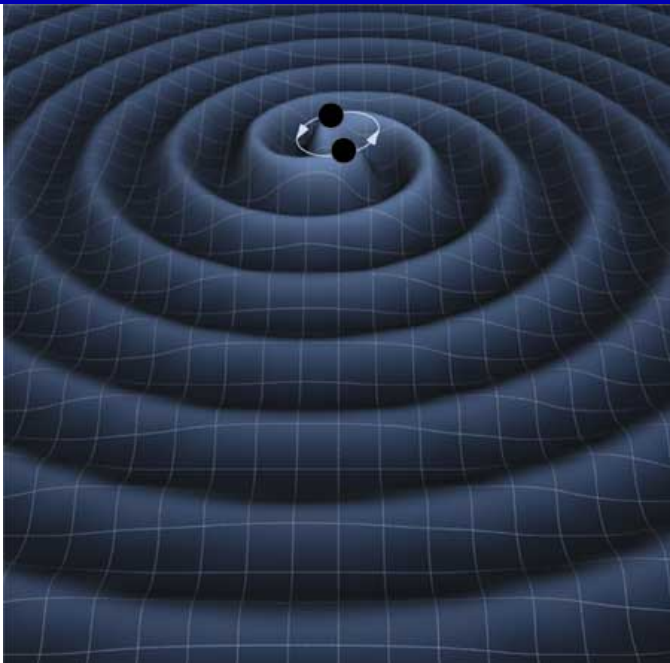
February 4, 2013

- ① Brief introduction into GW detectors
- ② Brief theory of optical position meters
- ③ Standard Quantum Limit
- ④ Quantum noises cross-correlation
- ⑤ Quantum speedmeter

- 1 Brief introduction into GW detectors
- 2 Brief theory of optical position meters
- 3 Standard Quantum Limit
- 4 Quantum noises cross-correlation
- 5 Quantum speedmeter

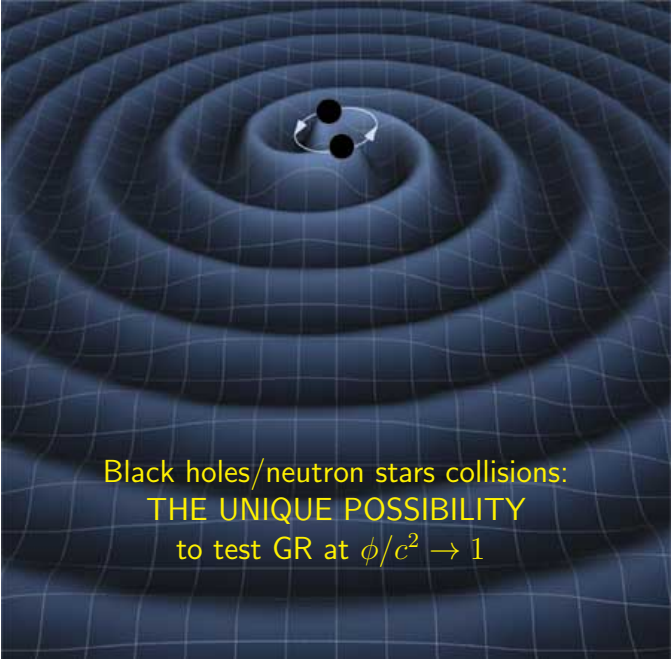
In some far far away galaxy...

<http://www.ligo.org/science.php>



In some far far away galaxy...

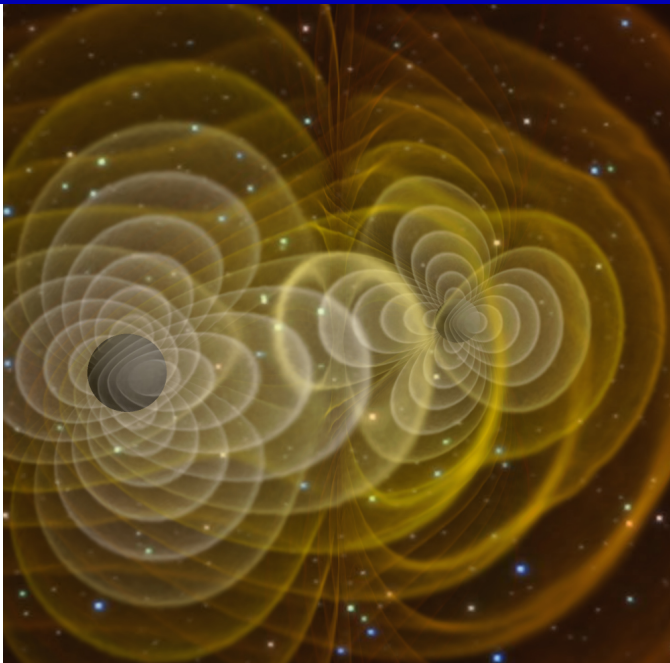
<http://www.ligo.org/science.php>



Black holes/neutron stars collisions:
THE UNIQUE POSSIBILITY
to test GR at $\phi/c^2 \rightarrow 1$

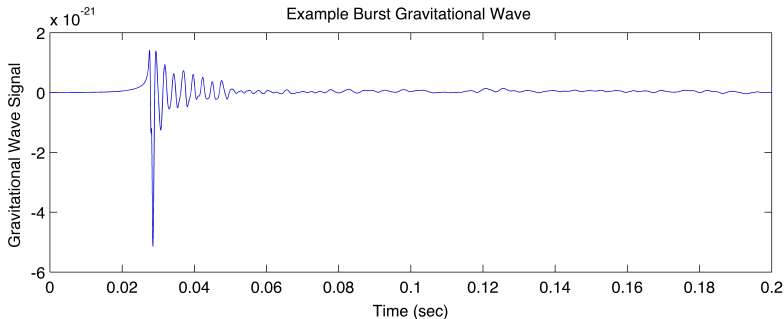
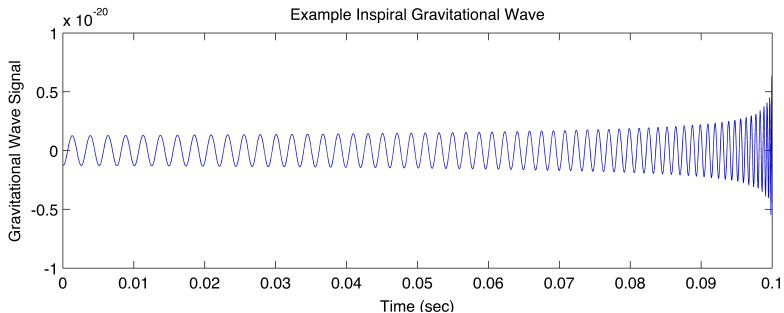
In some far far away galaxy...

<http://www.ligo.org/science.php>



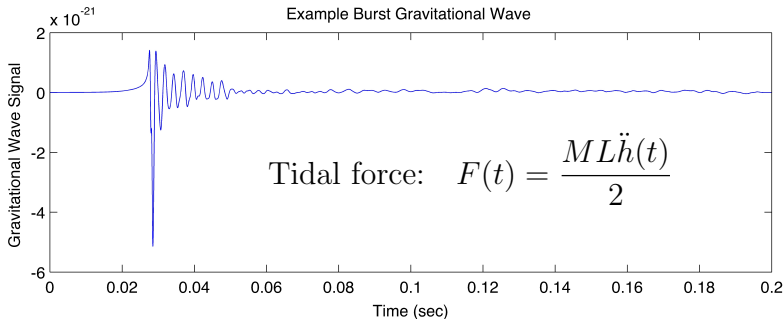
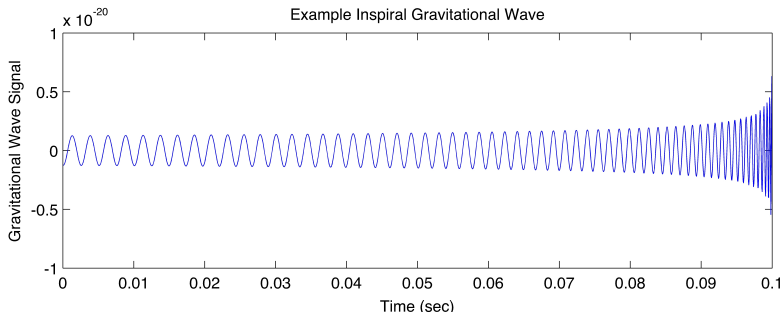
On Earth (some 100 000 000 years later)...

<http://www.ligo.org/science.php>



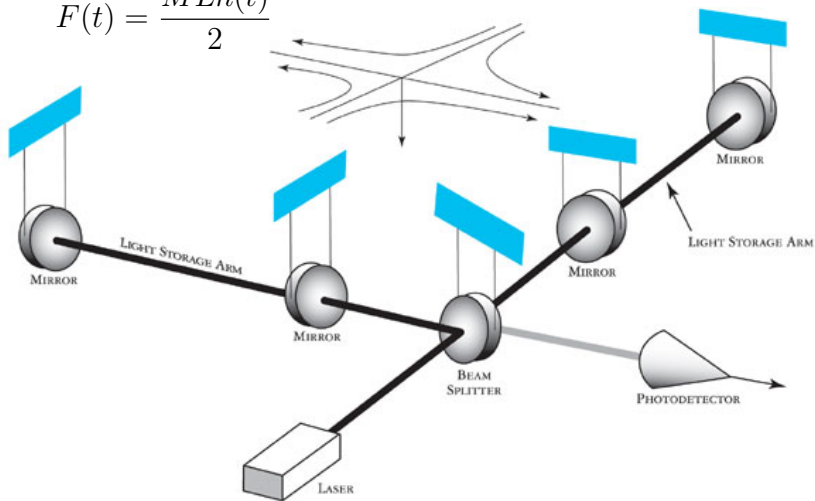
On Earth (some 100 000 000 years later)...

<http://www.ligo.org/science.php>



On Earth (some 100 000 000 years later)...

$$F(t) = \frac{ML\ddot{h}(t)}{2}$$



The old good Michelson...

... but **HUGE** one!



Actually, four of them



Closer look

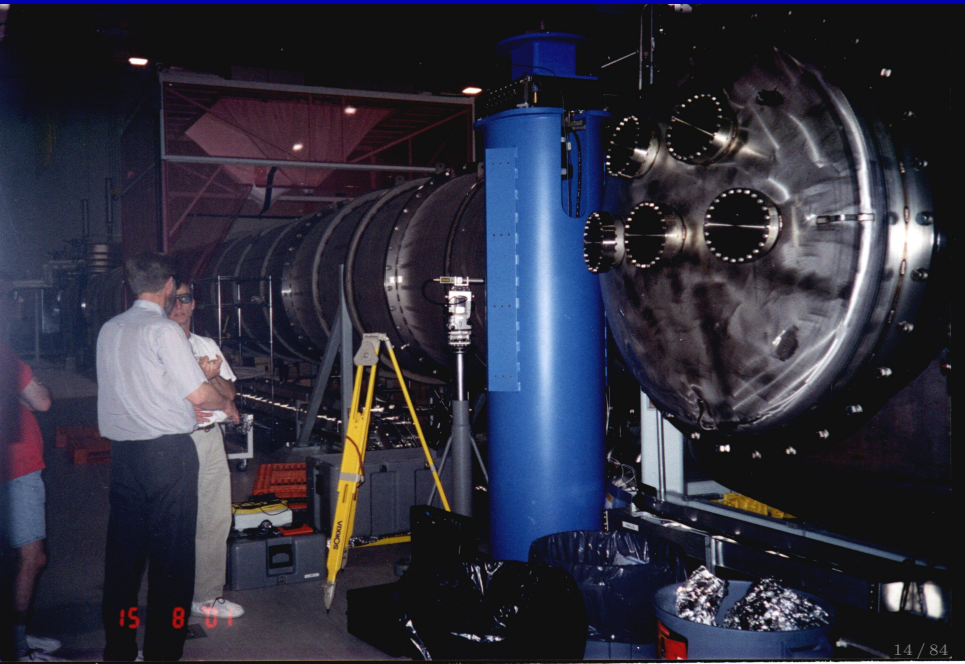


Closer look



14 8 01

Closer look



Weber's bar detector



Initial LIGO (2001-2009)

Sensitivity: $h \sim 10^{-21} \Rightarrow \delta x \sim 10^{-18} \text{ m}$

Prediction: ~ 0.5 events/year

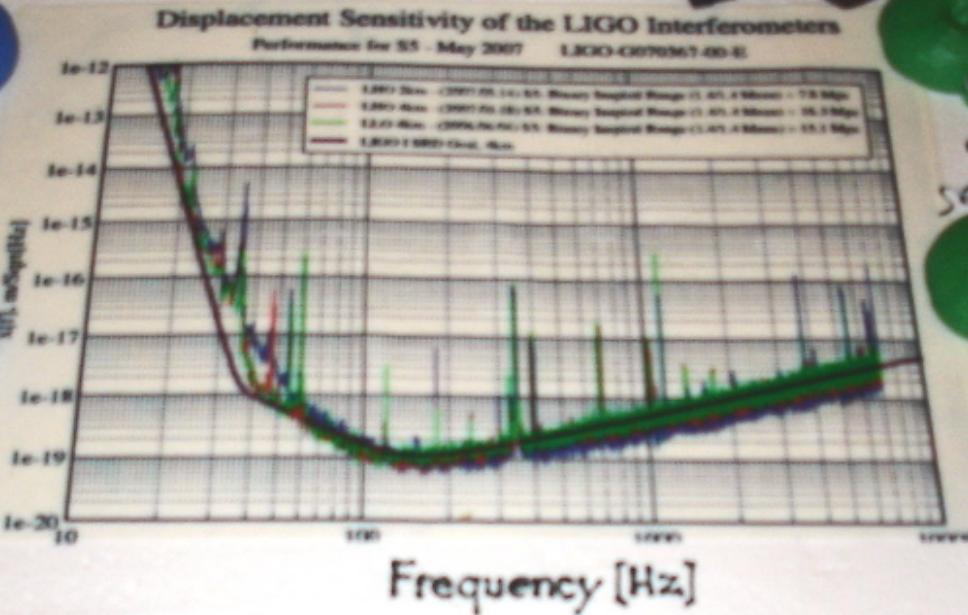
Results: **NONE**

Nevertheless...



13 5:31PM

Nevertheless...



Initial LIGO (2001-2009)

Sensitivity: $h \sim 10^{-21} \Rightarrow \delta x \sim 10^{-18} \text{ m}$

Prediction: ~ 0.5 events/year

Results: **NONE**

Advanced LIGO (2014)

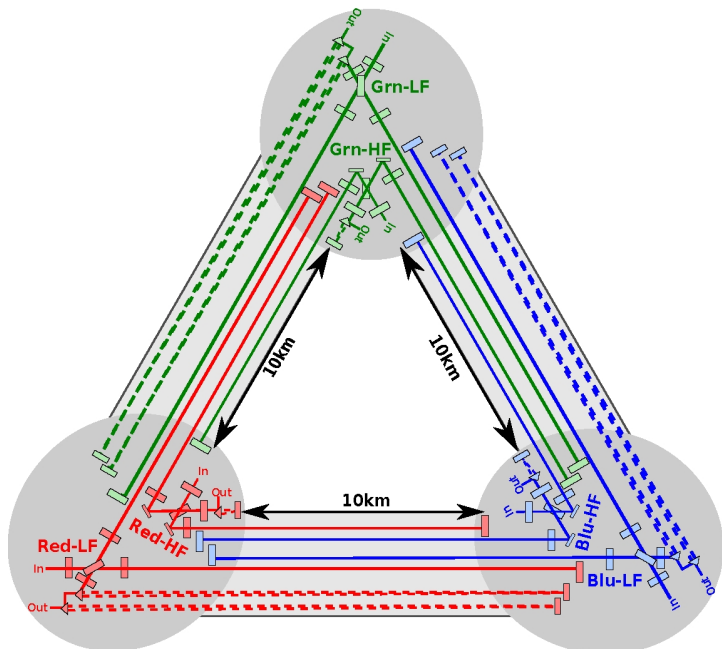
Sensitivity: $h \sim 10^{-22} \Rightarrow \delta x \sim 10^{-19} \text{ m}$

Prediction: up to $\sim 10^3$ events/year

A new hope

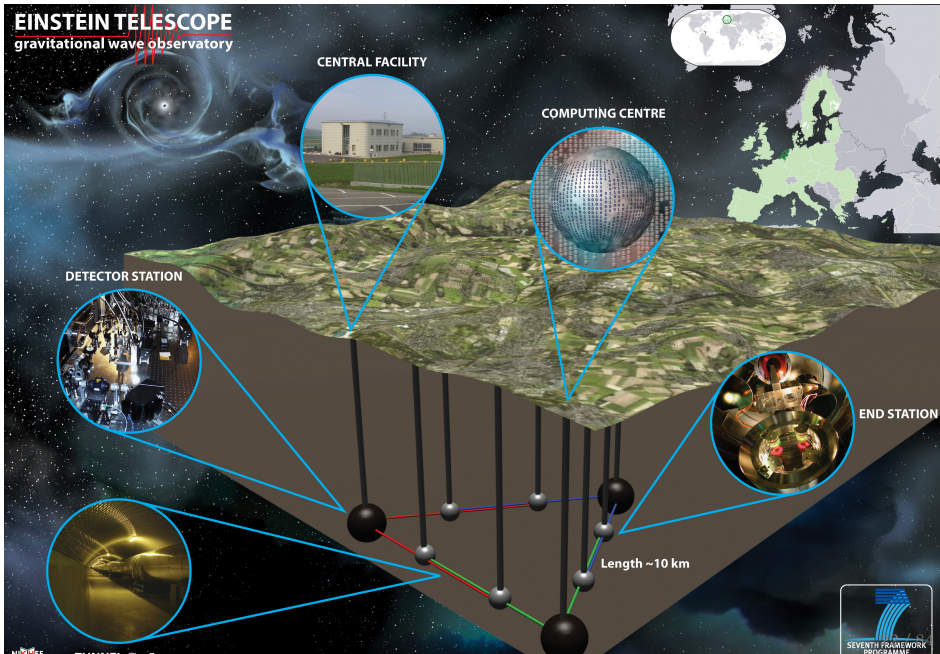


3rd generation (2020?): Einstein Telescope



3rd generation (2020?): Einstein Telescope

EINSTEIN TELESCOPE gravitational wave observatory



3rd generation (2020?): Einstein Telescope

EINSTEIN TELESCOPE gravitational wave observatory

CENTRAL FACILITY

COMPUTING CENTRE

DETECTOR STATION

Location: unknown yet...

- Abandoned mining region
- Seismic quiet
- Good restaurants...

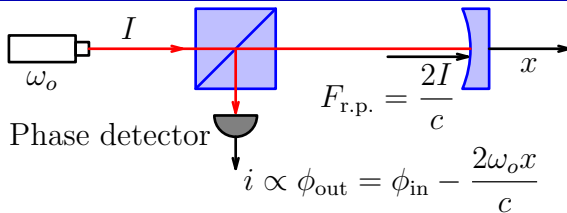
END STATION

Length ~10 km

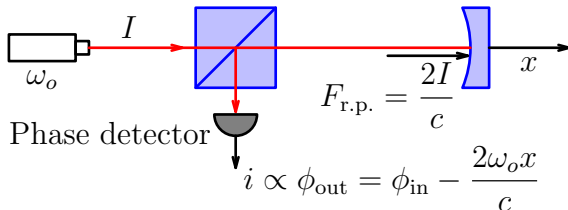
Questions?

- 1 Brief introduction into GW detectors
- 2 Brief theory of optical position meters
- 3 Standard Quantum Limit
- 4 Quantum noises cross-correlation
- 5 Quantum speedmeter

Optical position meter (simplified version)



Optical position meter (simplified version)



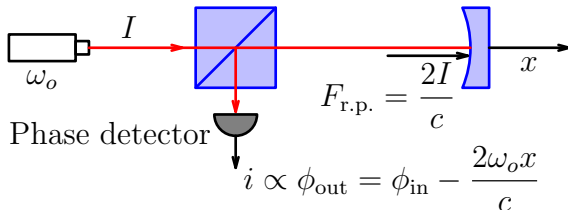
$$\hat{A}(t) = [A + \hat{a}_c(t)] \cos \omega_o t - \hat{a}_s(t) \sin \omega_o t$$

$$\approx [A + \hat{a}_c(t)] \cos[\omega_o t + \hat{\phi}(t)]$$

$$\hat{\phi}(t) = -\frac{\hat{a}_s(t)}{A}$$

$$S_c = \frac{e^{2r}}{2} \quad S_s = \frac{e^{-2r}}{2}$$

Optical position meter (simplified version)



$$\hat{A}(t) = [A + \hat{a}_c(t)] \cos \omega_o t - \hat{a}_s(t) \sin \omega_o t$$

$$\approx [A + \hat{a}_c(t)] \cos[\omega_o t + \hat{\phi}(t)]$$

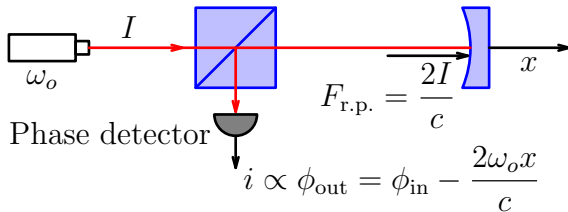
$$\hat{\phi}(t) = -\frac{\hat{a}_s(t)}{A}$$

$$S_c = \frac{e^{2r}}{2} \quad S_s = \frac{e^{-2r}}{2}$$

Phase noise: $S_\phi = \frac{S_s}{A^2} = \frac{\hbar \omega_o}{4\langle I \rangle} e^{-2r}$

Intensity noise: $S_I = (\hbar \omega_o)^2 A^2 S_c = \hbar \omega_o \langle I \rangle e^{2r}$

Optical position meter (simplified version)



Phase noise: $S_\phi = \frac{S_s}{A^2} = \frac{\hbar\omega_o}{4\langle I \rangle} e^{-2r}$

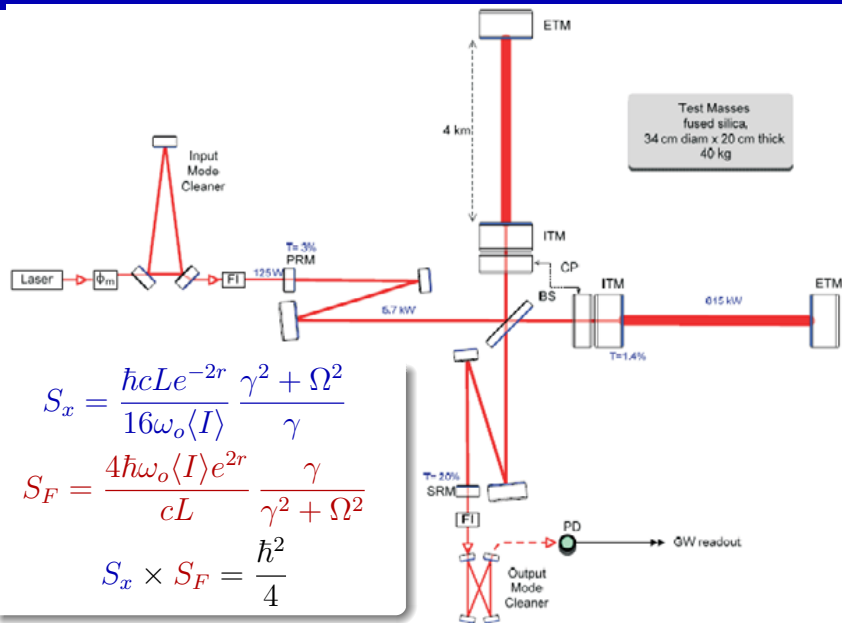
Intensity noise: $S_I = (\hbar\omega_o)^2 A^2 S_c = \hbar\omega_o \langle I \rangle e^{2r}$

Measurement noise: $S_x = \frac{c^2}{4\omega_o^2} S_\phi = \frac{\hbar c^2 e^{-2r}}{16\omega_o \langle I \rangle}$

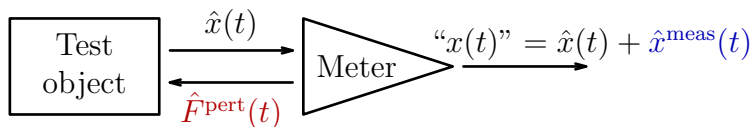
Back-action noise: $S_F = \frac{4}{c^2} S_I = \frac{4\hbar\omega_o \langle I \rangle e^{2r}}{c^2}$

$$S_x \times S_F = \frac{\hbar^2}{4}$$

Optical position meter (real-world version)



Linear position meter: abstract model



S_x : spectral density of measurement noise $\hat{x}^{\text{meas}}(t)$

S_F : spectral density of back action noise $\hat{F}^{\text{pert}}(t)$

$$S_x \times S_F = \frac{\hbar^2}{4}$$

Questions?

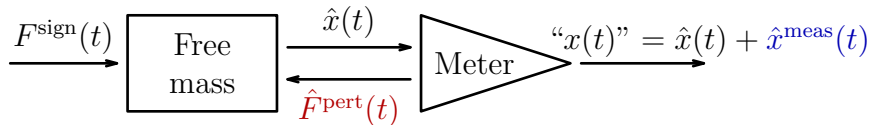
- ① Brief introduction into GW detectors
- ② Brief theory of optical position meters
- ③ **Standard Quantum Limit**
- ④ Quantum noises cross-correlation
- ⑤ Quantum speedmeter



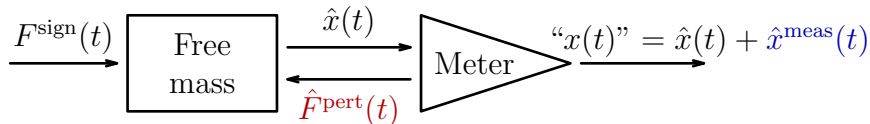
LENGTHY EQUATIONS AHEAD!

shortcut

Detection of classical force



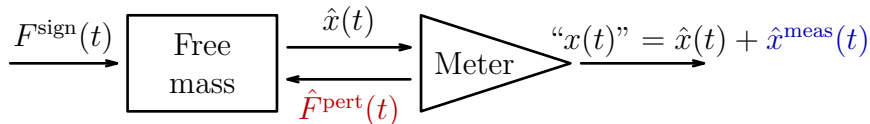
Detection of classical force



$$-m\Omega^2 \hat{x}(\Omega) = F^{\text{sign}}(\Omega) + \hat{F}^{\text{pert}}(\Omega)$$

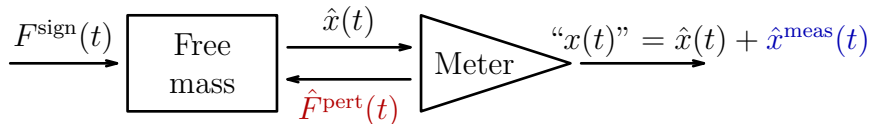
$$\text{"x}(\Omega)\text{"} = \hat{x}(\Omega) + \hat{x}^{\text{meas}}(\Omega)$$

Detection of classical force



$$\begin{aligned}
 -m\Omega^2 \hat{x}(\Omega) &= F^{\text{sign}}(\Omega) + \hat{F}^{\text{pert}}(\Omega) \\
 "x(\Omega)" &= \hat{x}(\Omega) + \hat{x}^{\text{meas}}(\Omega) \\
 &= \frac{F^{\text{sign}}(\Omega)}{-m\Omega^2} + \underbrace{\frac{\hat{F}^{\text{pert}}(\Omega)}{-m\Omega^2} + \hat{x}^{\text{sum}}(\Omega)}_{\text{the sum noise}}
 \end{aligned}$$

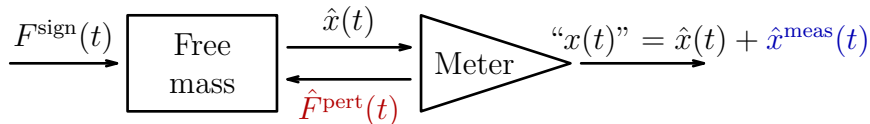
Detection of classical force



$$\begin{aligned}
 -m\Omega^2 \hat{x}(\Omega) &= F^{\text{sign}}(\Omega) + \hat{F}^{\text{pert}}(\Omega) \\
 "x(\Omega)" &= \hat{x}(\Omega) + \hat{x}^{\text{meas}}(\Omega) \\
 &= \frac{F^{\text{sign}}(\Omega)}{-m\Omega^2} + \underbrace{\frac{\hat{F}^{\text{pert}}(\Omega)}{-m\Omega^2} + \hat{x}^{\text{sum}}(\Omega)}_{\text{the sum noise}}
 \end{aligned}$$

$$S_{\text{sum}}(\Omega) = S_x + \frac{S_F}{m^2\Omega^4}$$

Detection of classical force

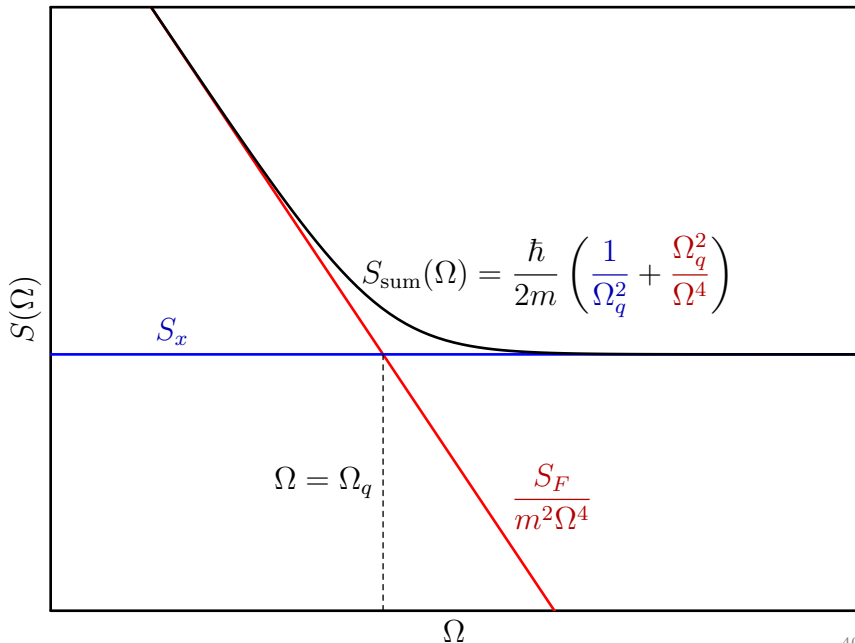


$$\begin{aligned}
 -m\Omega^2 \hat{x}(\Omega) &= F^{\text{sign}}(\Omega) + \hat{F}^{\text{pert}}(\Omega) \\
 "x(\Omega)" &= \hat{x}(\Omega) + \hat{x}^{\text{meas}}(\Omega) \\
 &= \frac{F^{\text{sign}}(\Omega)}{-m\Omega^2} + \underbrace{\frac{\hat{F}^{\text{pert}}(\Omega)}{-m\Omega^2} + \hat{x}^{\text{sum}}(\Omega)}_{\text{the sum noise}}
 \end{aligned}$$

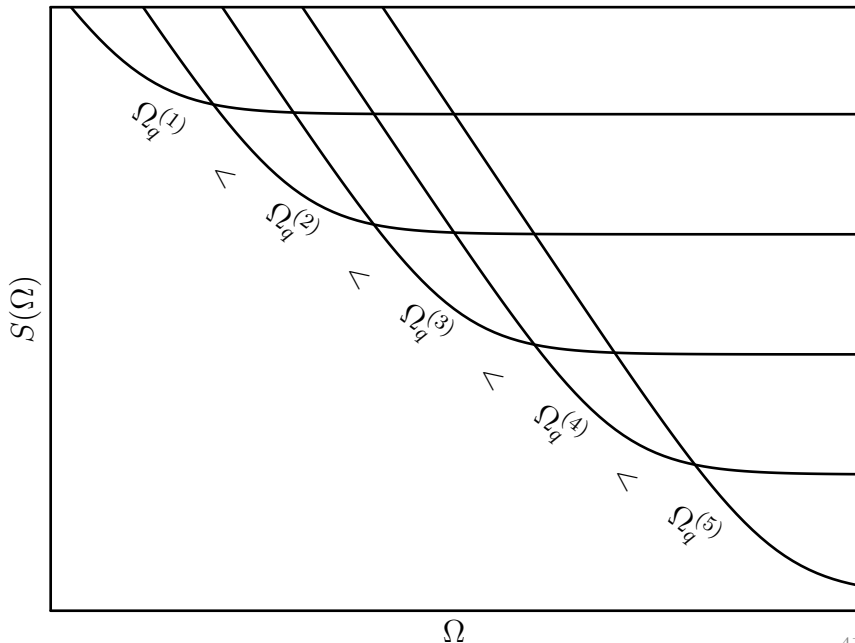
$$S_{\text{sum}}(\Omega) = S_x + \frac{S_F}{m^2\Omega^4} = \frac{\hbar}{2m} \left(\frac{1}{\Omega_q^2} + \frac{\Omega_q^2}{\Omega^4} \right)$$

$$\Omega_q = \left(\frac{S_F}{m^2 S_x} \right)^{1/4} \propto \sqrt{\frac{\langle I \rangle}{m}} e^r$$

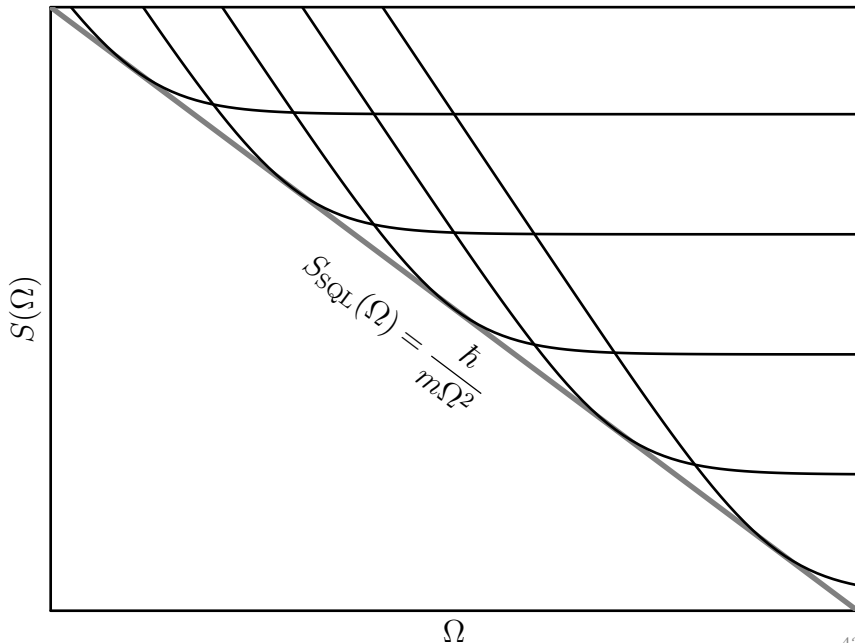
Sum noise spectral density



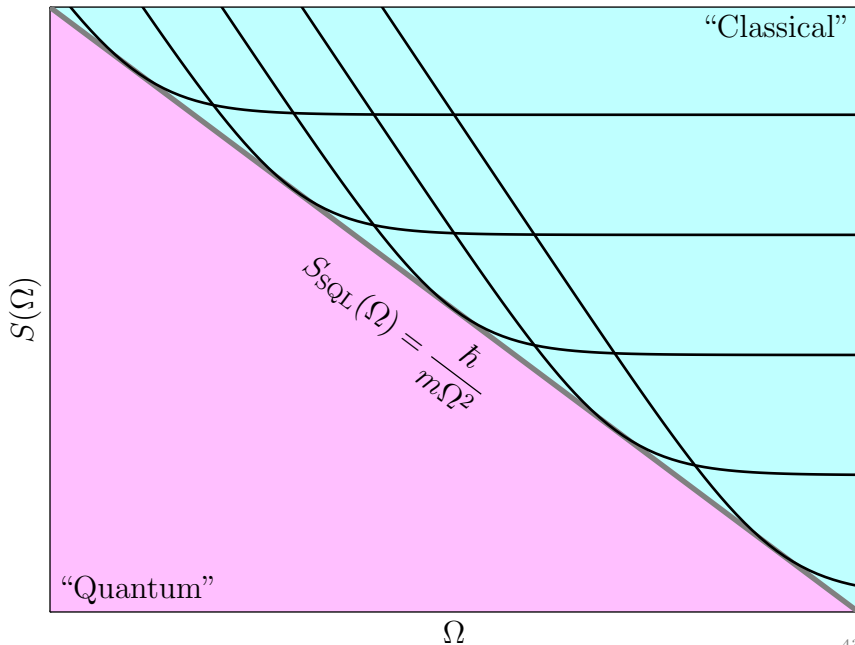
Sum noise spectral density



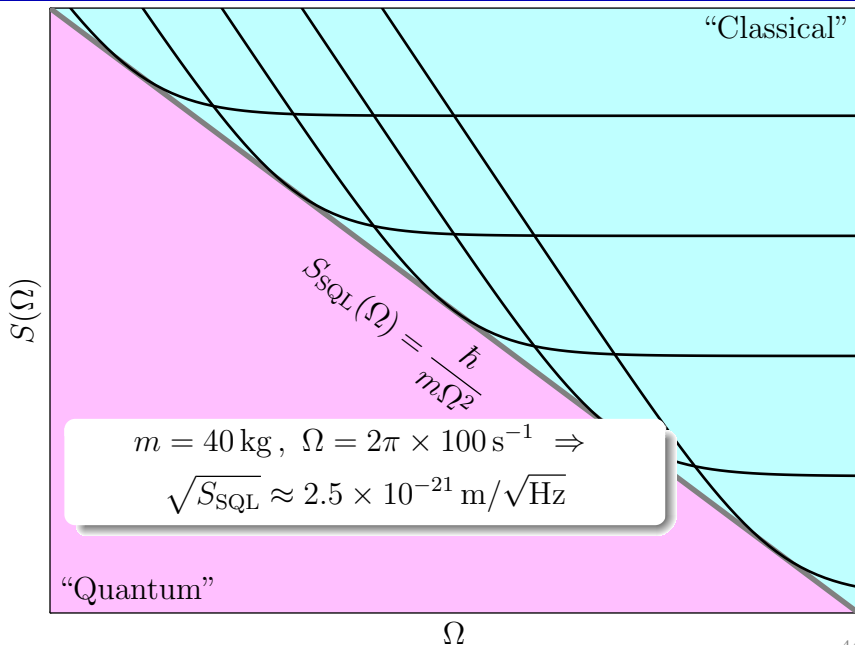
Standard Quantum Limit



Standard Quantum Limit



Standard Quantum Limit



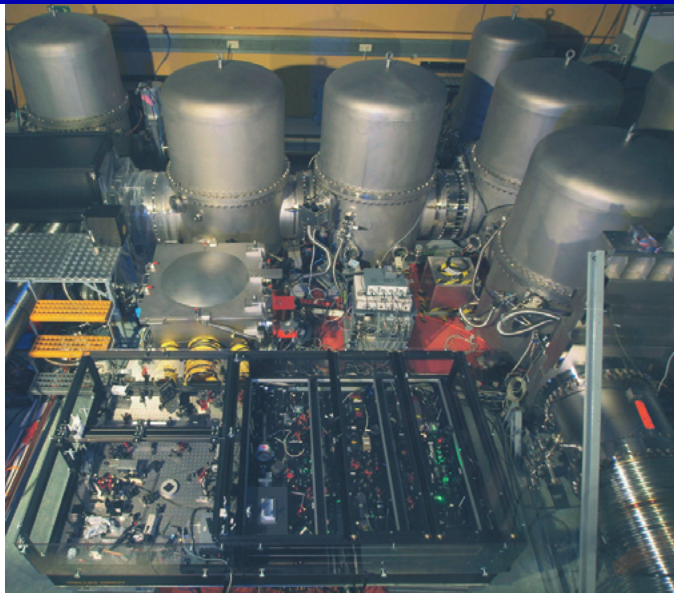
What about the light squeezing?

In all equations, we have only the combination

$$\langle I \rangle e^{2r}$$

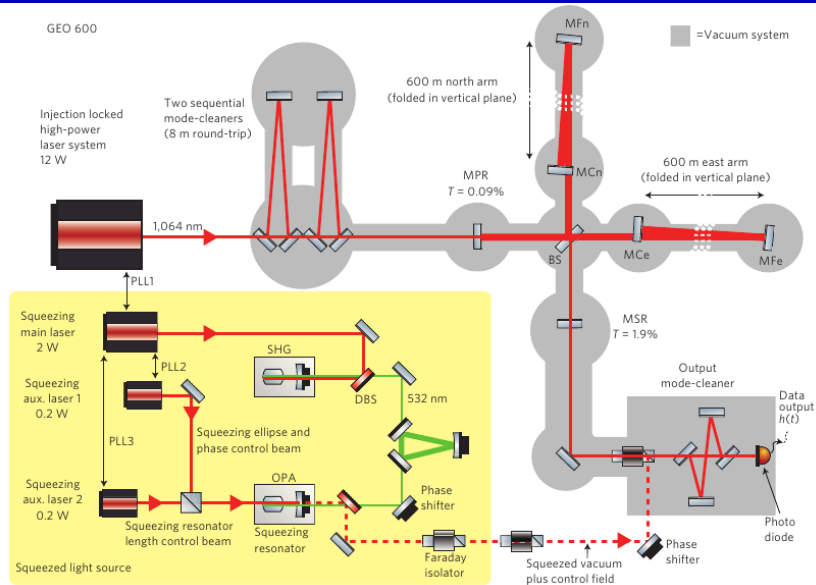
$\Rightarrow$ squeezing does not allow to overcome the SQL,
but allows to decrease $\langle I \rangle$

Squeezing in GEO-600



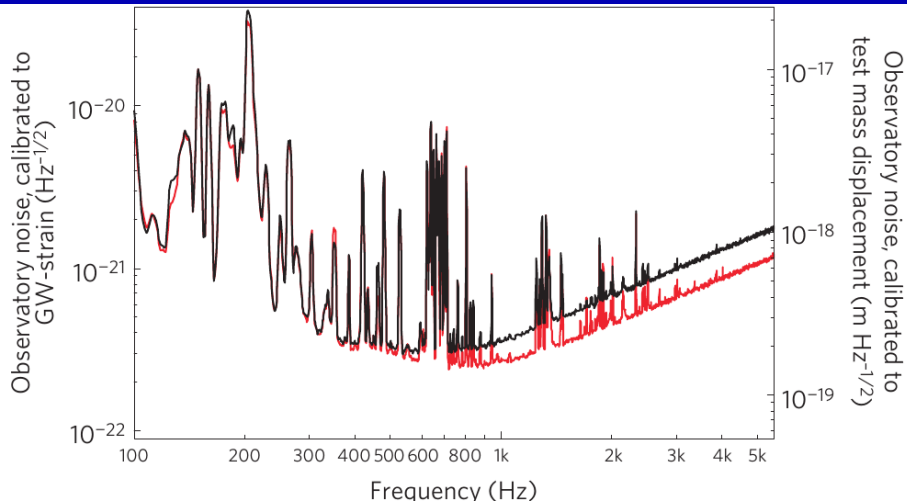
The LSC, Nature Physics **7**, 962 (2011)

Squeezing in GEO-600



The LSC, Nature Physics 7, 962 (2011)

Squeezing in GEO-600



The LSC, Nature Physics 7, 962 (2011)

How to overcome the SQL

Implicit assumptions that have been made:

- ① The scheme is stationary (invariant with respect to time shift).
- ② The noises are Markovian.
- ③ The noise are mutually uncorrelated.
- ④ The test object is a free mass.

How to overcome the SQL

Implicit assumptions that have been made:

- ① The scheme is stationary (invariant with respect to time shift).
- ② The noises are Markovian.
- ③ The noise are mutually uncorrelated.
- ④ The test object is a free mass.

Number of schemes based on violation of some of these assumptions has been proposed.

Only one successful experiment was performed 😞 (for now).

How to overcome the SQL

Implicit assumptions that have been made:

- ① The scheme is stationary (invariant with respect to time shift).
- ② The noises are Markovian.
- ③ The noise are mutually uncorrelated.
- ④ The test object is a free mass.

Number of schemes based on violation of some of these assumptions has been proposed.

Only one successful experiment was performed 😞 (for now).

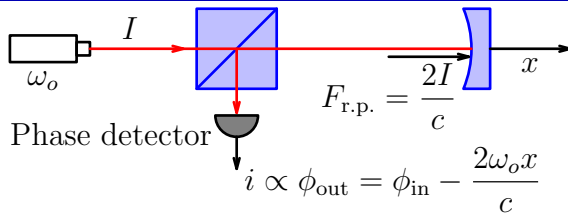
For the laser gravitation wave detectors, the way “2+3” is considered as the most promising.

Questions?

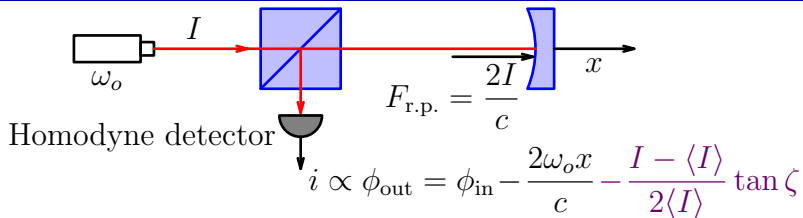
- 1 Brief introduction into GW detectors
- 2 Brief theory of optical position meters
- 3 Standard Quantum Limit
- 4 Quantum noises cross-correlation
- 5 Quantum speedmeter

shortcut

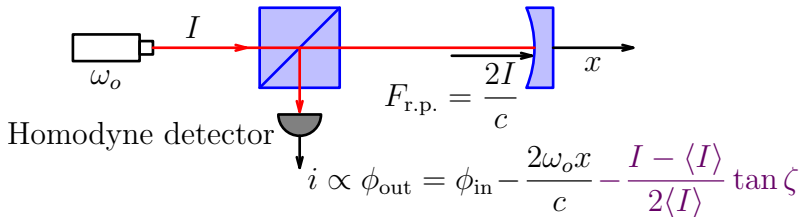
Optical meter with homodyne detector



Optical meter with homodyne detector



Optical meter with homodyne detector

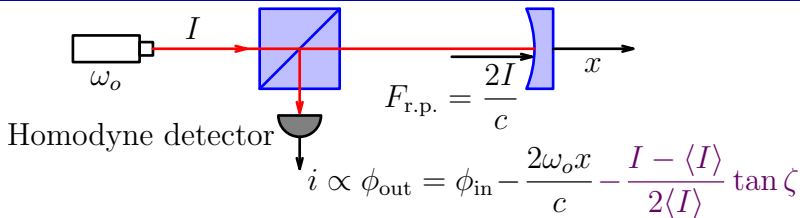


Measurement noise: $S_x = \frac{\hbar c^2}{16\omega_o \langle I \rangle \cos^2 \zeta}$

Back-action noise: $S_F = \frac{4}{c^2} S_I = \frac{4\hbar\omega_o \langle I \rangle}{c^2}$

Cross-correlation: $S_{xF} = \frac{\hbar}{2} \tan \zeta$

Optical meter with homodyne detector



Measurement noise: $S_x = \frac{\hbar c^2}{16\omega_o \langle I \rangle \cos^2 \zeta}$

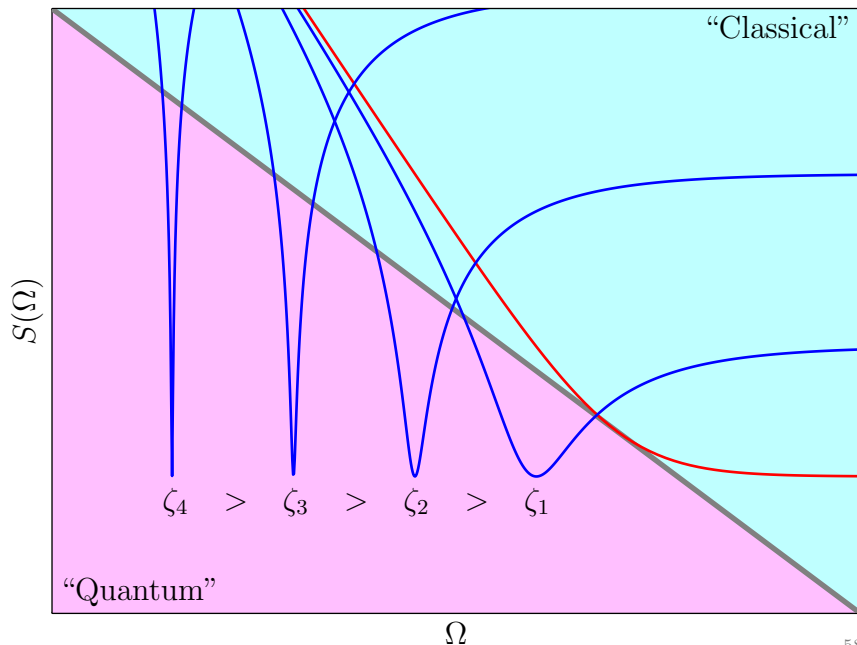
Back-action noise: $S_F = \frac{4}{c^2} S_I = \frac{4\hbar\omega_o \langle I \rangle}{c^2}$

Cross-correlation: $S_{xF} = \frac{\hbar}{2} \tan \zeta$

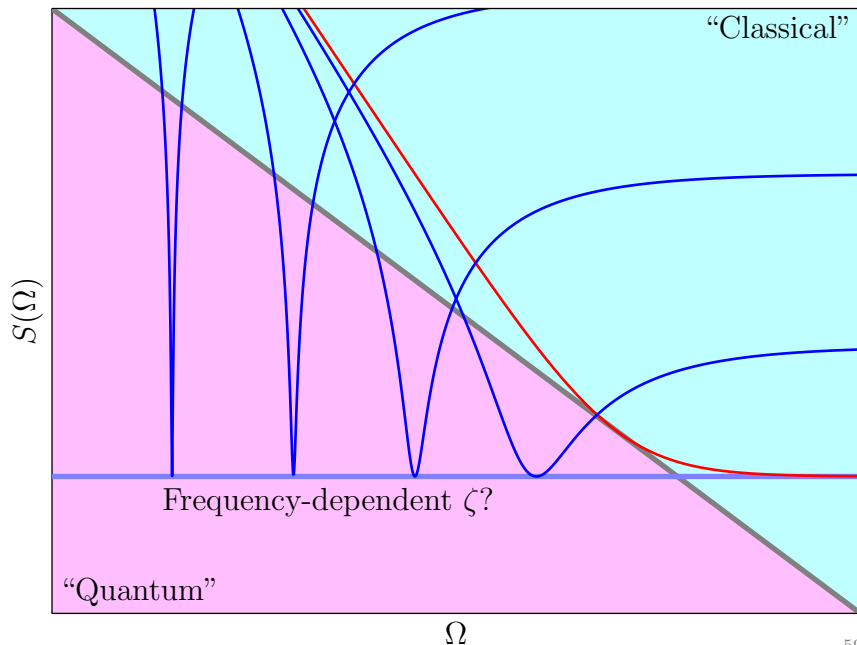
$$S_x \times S_F - S_{xF}^2 = \frac{\hbar^2}{4}$$

$$S_{\text{sum}}(\Omega) = S_x - \frac{2S_{xF}}{m\Omega^2} + \frac{S_F}{m^2\Omega^4}$$

Narrow-band gain



Broad band gain (?)



Broad band gain: options

$$S_{\text{sum}}(\Omega) = \frac{\hbar}{2m} \left(\frac{1}{\Omega_q^2 \cos^2 \zeta} - \frac{2}{\Omega^2} \tan \zeta + \frac{\Omega_q^2}{\Omega^4} \right)$$

$$\Omega^2 \tan \zeta = \Omega_q^2 \equiv \frac{2S_F}{\hbar m} \Rightarrow S_{\text{sum}} = \frac{\hbar}{2m\Omega_q^2}$$

$$\boxed{\Omega_q^2 \propto I \rightarrow \infty \Rightarrow S_{\text{sum}} \rightarrow 0}$$

Broad band gain: options

$$S_{\text{sum}}(\Omega) = \frac{\hbar}{2m} \left(\frac{1}{\Omega_q^2 \cos^2 \zeta} - \frac{2}{\Omega^2} \tan \zeta + \frac{\Omega_q^2}{\Omega^4} \right)$$

$$\Omega^2 \tan \zeta = \Omega_q^2 \equiv \frac{2S_F}{\hbar m} \Rightarrow S_{\text{sum}} = \frac{\hbar}{2m\Omega_q^2}$$

$$\boxed{\Omega_q^2 \propto I \rightarrow \infty \Rightarrow S_{\text{sum}} \rightarrow 0}$$

Option 1: variational readout

$$\tan \zeta \propto \frac{1}{\Omega^2} \quad \Omega_q^2 = \text{const}$$

 H.J.Kimble *et al*, Phys.Rev.D **65**, 022002 (2001)

Broad band gain: options

$$S_{\text{sum}}(\Omega) = \frac{\hbar}{2m} \left(\frac{1}{\Omega_q^2 \cos^2 \zeta} - \frac{2}{\Omega^2} \tan \zeta + \frac{\Omega_q^2}{\Omega^4} \right)$$

$$\Omega^2 \tan \zeta = \Omega_q^2 \equiv \frac{2S_F}{\hbar m} \Rightarrow S_{\text{sum}} = \frac{\hbar}{2m\Omega_q^2}$$

$$\boxed{\Omega_q^2 \propto I \rightarrow \infty \Rightarrow S_{\text{sum}} \rightarrow 0}$$

Option 1: variational readout

$$\tan \zeta \propto \frac{1}{\Omega^2} \quad \Omega_q^2 = \text{const}$$

 H.J.Kimble *et al*, Phys.Rev.D **65**, 022002 (2001)

Option 2: quantum speedmeter

$$\tan \zeta = \text{const} \quad \Omega_q^2 \propto \Omega^2$$

 V.B.Braginsky, F.Ya.Khalili, Phys.Lett.A **147**, 251 (1990)

Broad band gain: options

$$S_{\text{sum}}(\Omega) = \frac{\hbar}{2m} \left(\frac{1}{\Omega_q^2 \cos^2 \zeta} - \frac{2}{\Omega^2} \tan \zeta + \frac{\Omega_q^2}{\Omega^4} \right)$$

$$\Omega^2 \tan \zeta = \Omega_q^2 \equiv \frac{2S_F}{\hbar m} \Rightarrow S_{\text{sum}} = \frac{\hbar}{2m\Omega_q^2}$$

$$\boxed{\Omega_q^2 \propto I \rightarrow \infty \Rightarrow S_{\text{sum}} \rightarrow 0}$$

Option 1: variational readout

$$\tan \zeta \propto \frac{1}{\Omega^2} \quad \Omega_q^2 = \text{const}$$

 H.J.Kimble *et al*, Phys.Rev.D **65**, 022002 (2001)

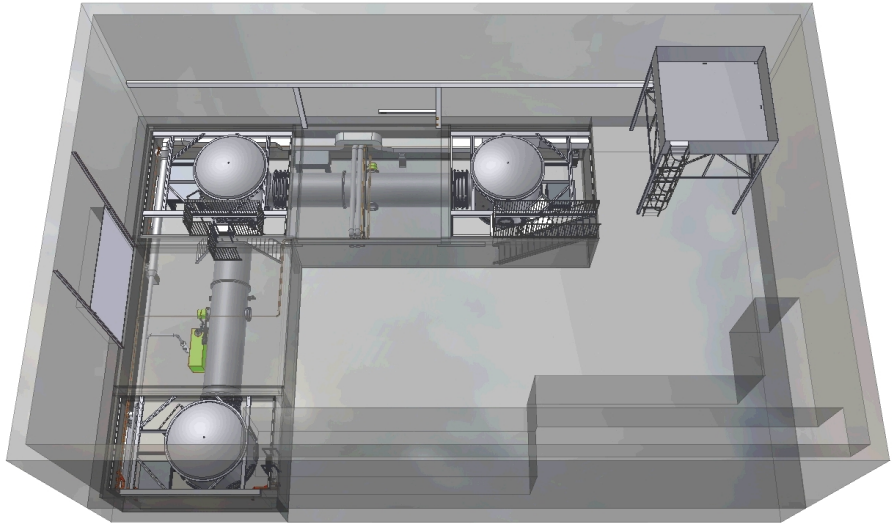
Option 2: quantum speedmeter

$$\tan \zeta = \text{const} \quad \Omega_q^2 \propto \Omega^2$$

 V.B.Braginsky, F.Ya.Khalili, Phys.Lett.A **147**, 251 (1990)

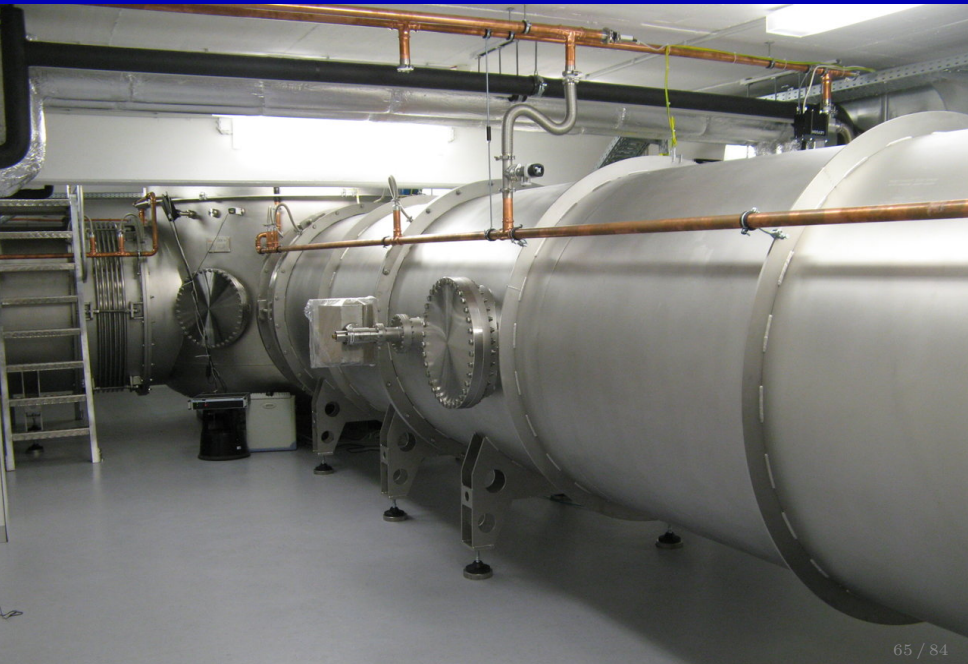
Optomechanical coupling $\Omega_q^2 \propto I \propto \Omega^2$: **is it possible?**

Hannover 10m prototype

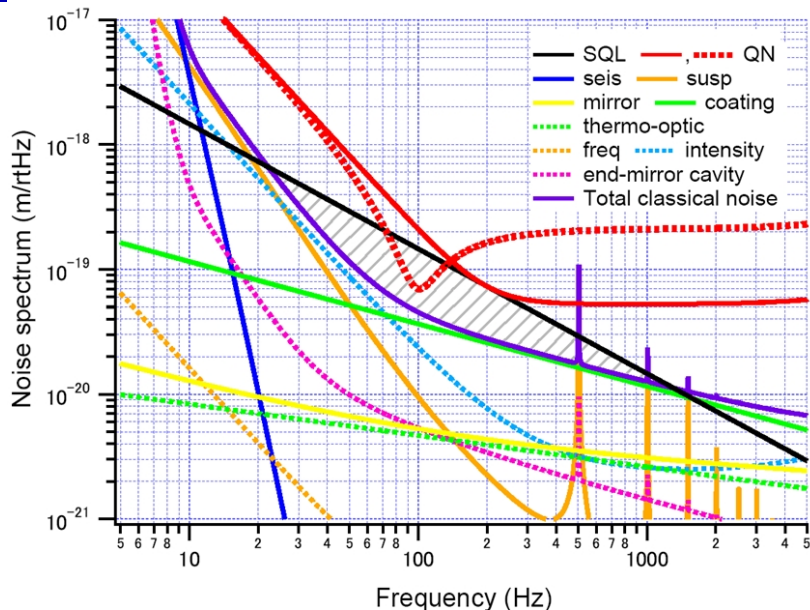


 S.Goßler *et al*, Class. Quantum Grav. **27**, 084023 (2010)

Hannover 10m prototype



Hannover 10m prototype



Questions?

- ① Brief introduction into GW detectors
- ② Brief theory of optical position meters
- ③ Standard Quantum Limit
- ④ Quantum noises cross-correlation
- ⑤ Quantum speedmeter

SQL: simplified consideration

Position measurement

$$\hat{x}(t) = \hat{x} + \frac{\hat{p}t}{m}$$

$$\Delta x \times \Delta p \geq \frac{\hbar}{2}$$

$$\Delta x(t) \geq \sqrt{\frac{\hbar t}{m}} \sim \sqrt{\frac{\hbar}{m\Omega}} \Rightarrow S_{\text{sum}} \geq \frac{[\Delta x(t)]^2}{\Omega} = \frac{\hbar}{m\Omega^2}$$

SQL: simplified consideration

Position measurement

$$\hat{x}(t) = \hat{x} + \frac{\hat{p}t}{m}$$

$$\Delta x \times \Delta p \geq \frac{\hbar}{2}$$

$$\Delta x(t) \geq \sqrt{\frac{\hbar t}{m}} \sim \sqrt{\frac{\hbar}{m\Omega}} \Rightarrow S_{\text{sum}} \geq \frac{[\Delta x(t)]^2}{\Omega} = \frac{\hbar}{m\Omega^2}$$

Momentum measurement

$$\hat{p}(t) = \text{const} \quad [\text{in the absence of } F(t)]$$

$\Rightarrow$ no uncertainty relation

$\Rightarrow$ no SQL

SQL: simplified consideration

Position measurement

$$\hat{x}(t) = \hat{x} + \frac{\hat{p}t}{m}$$

$$\Delta x \times \Delta p \geq \frac{\hbar}{2}$$

$$\Delta x(t) \geq \sqrt{\frac{\hbar t}{m}} \sim \sqrt{\frac{\hbar}{m\Omega}} \Rightarrow S_{\text{sum}} \geq \frac{[\Delta x(t)]^2}{\Omega} = \frac{\hbar}{m\Omega^2}$$

Momentum measurement

$$\hat{p}(t) = \text{const} \quad [\text{in the absence of } F(t)]$$

$\Rightarrow$ no uncertainty relation

$\Rightarrow$ no SQL

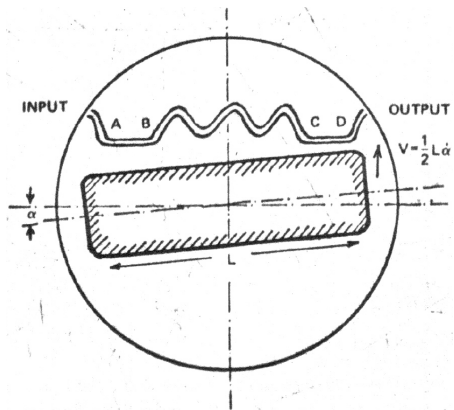
Velocity measurement

We can not measure momentum?

Let us measure velocity instead!

The idea

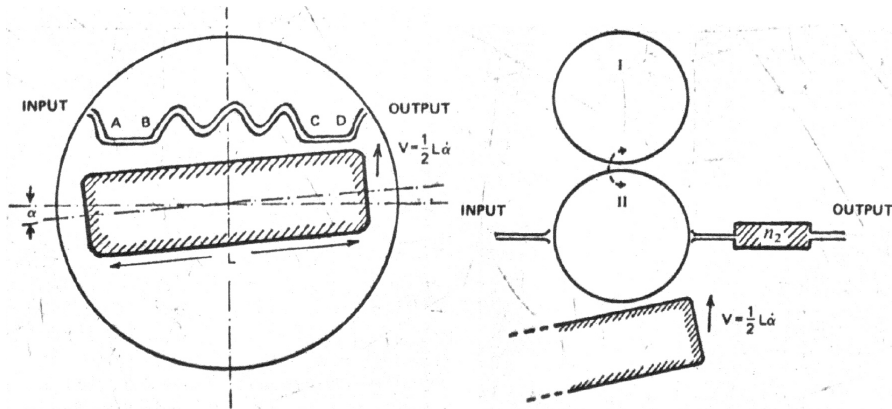
Light interacts with the probe **twice**:



V.B.Braginsky, F.Ya.Khalili, Phys.Lett.A 147, 251 (1990)

The idea

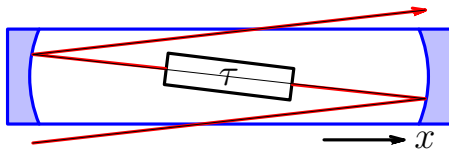
Light interacts with the probe **twice**:



V.B.Braginsky, F.Ya.Khalili, Phys.Lett.A 147, 251 (1990)

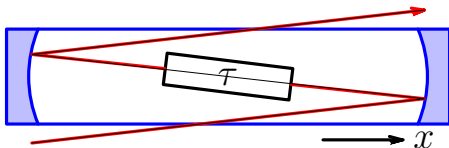
The idea

Light interacts with the probe **twice**:



The idea

Light interacts with the probe **twice**:

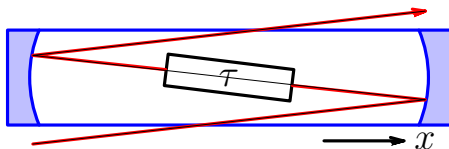


$$\phi_{\text{out}}(t) = \phi_{\text{in}}(t - \tau) + \frac{2\omega_o}{c} [x(t) - x(t - \tau)]$$

$$\text{(quantum SPEED meter!)} \quad \approx \phi_{\text{in}}(t - \tau) + \frac{2\omega_o\tau}{c} \frac{dx(t)}{dt}$$

The idea

Light interacts with the probe **twice**:



$$\phi_{\text{out}}(t) = \phi_{\text{in}}(t - \tau) + \frac{2\omega_o}{c} [x(t) - x(t - \tau)]$$

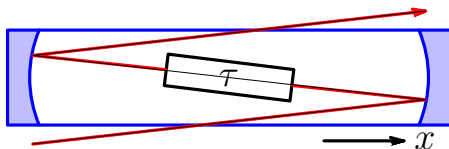
(quantum SPEED meter!) $\approx \phi_{\text{in}}(t - \tau) + \frac{2\omega_o\tau}{c} \frac{dx(t)}{dt}$

$$F_{\text{pert}}(t) = \frac{2}{c} [I(t + \tau) - I(t)] \approx \frac{2\tau}{c} \frac{dI(t)}{dt}$$

$$\Rightarrow p_{\text{pert}}(t) = \int_{-\infty}^t F_{\text{pert}}(t') dt' \approx \frac{2\tau I(t)}{c}$$

The idea

Light interacts with the probe **twice**:



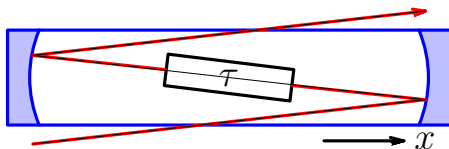
Fourier picture

$$\phi_{\text{out}}(\Omega) \approx \phi_{\text{in}}(\Omega) + \frac{2\omega_o\tau}{c} \times [-i\Omega x(\Omega)]$$

$$F_{\text{pert}}(\Omega) \approx \frac{2\tau}{c} \times [-i\Omega I(\Omega)]$$

The idea

Light interacts with the probe **twice**:



Spectral densities

$$\text{Measurement noise: } S_x = \frac{\hbar c^2}{16\omega_o \langle I \rangle e^{2r} \cos^2 \zeta} \times \frac{1}{\Omega^2 \tau^2} = \frac{S_v}{\Omega^2}$$

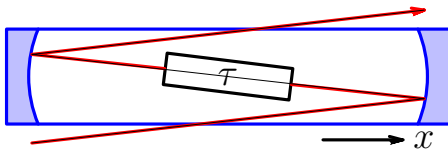
$$\text{Back-action noise: } S_F = \frac{4\hbar\omega_o \langle I \rangle e^{2r}}{c^2} \times \Omega^2 \tau^2 = S_p \Omega^2$$

$$\text{Cross-correlation: } S_{xF} = \frac{\hbar}{2} \tan \zeta = -S_{vp}$$

Note the requested frequency dependence!

The idea

Light interacts with the probe **twice**:

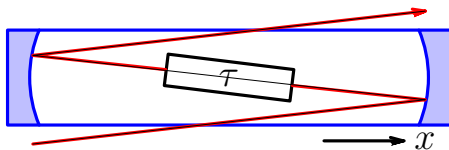


Detection of classical force

$$S_{\text{sum}} = \frac{1}{\Omega^2} \left(S_v + \frac{2S_{vp}}{m} + \frac{S_p}{m^2} \right); \quad S_v S_p - S_{vp}^2 = \frac{\hbar^2}{4}$$

The idea

Light interacts with the probe **twice**:



Detection of classical force

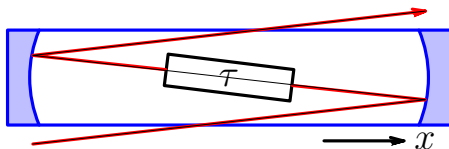
$$S_{\text{sum}} = \frac{1}{\Omega^2} \left(S_v + \frac{2S_{vp}}{m} + \frac{S_p}{m^2} \right); \quad S_v S_p - S_{vp}^2 = \frac{\hbar^2}{4}$$

$$S_{vp} = -\frac{S_p}{m}$$

$$\Rightarrow S_{\text{sum}} = \frac{\hbar^2}{4S_p\Omega^2} = \left[S_{\text{SQL}}(\Omega) = \frac{\hbar}{m\Omega^2} \right] \times \frac{mc^2}{16\omega_o \langle I \rangle e^{2r} \tau^2}$$

The idea

Light interacts with the probe **twice**:



Detection of classical force

$$S_{\text{sum}} = \frac{1}{\Omega^2} \left(S_v + \frac{2S_{vp}}{m} + \frac{S_p}{m^2} \right); \quad S_v S_p - S_{vp}^2 = \frac{\hbar^2}{4}$$

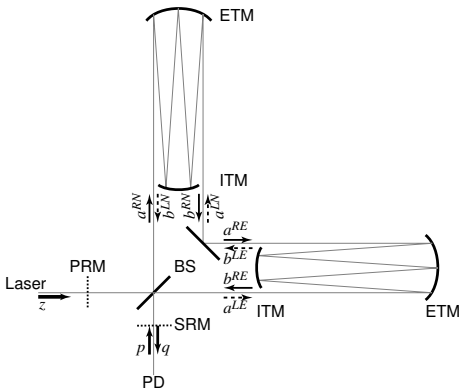
$$S_{vp} = -\frac{S_p}{m}$$

$$\Rightarrow S_{\text{sum}} = \frac{\hbar^2}{4S_p\Omega^2} = \left[S_{\text{SQL}}(\Omega) = \frac{\hbar}{m\Omega^2} \right] \times \frac{mc^2}{16\omega_o \langle I \rangle e^{2r} \tau^2}$$

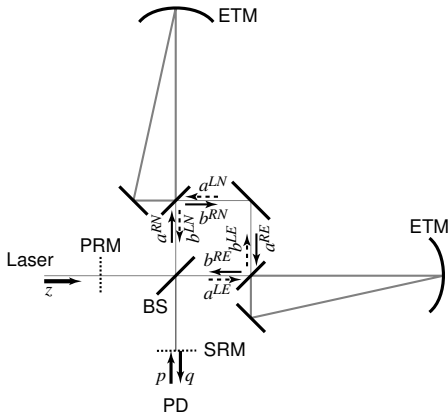
The factor $1/\tau^2$ calls for REALLY LARGE setups!

Practical variants for GW detectors: Sagnac interferometer

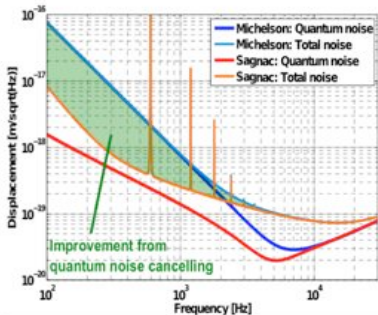
Delay lines implementation



Ring cavities implementation

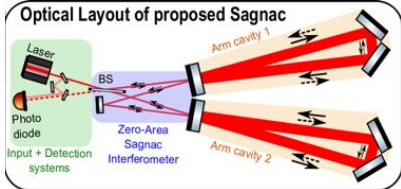


Overview of 1m speed meter experiment



- 1g mirrors suspended in monolithic fused silica suspensions.
- 1kW of circulating power. Arm cavities with finesse of 10000. 100ppm loss per roundtrip.
- Sophisticated seismic isolation + double pendulums with one vertical stage.
- Large beams to reduce coating noise.
- Armlength = 1m. Target better than $10^{-18}\text{m}/\sqrt{\text{Hz}}$ at 1kHz.
- No recycling, no squeezing, but plan to use homodyne detection.
- LOTS OF CHALLENGES! (*let me know if you want to help...*)

Optical Layout of proposed Sagnac



Questions?