# Spin squeezing and Schrödinger cat generation in atomic samples with Rydberg blockade

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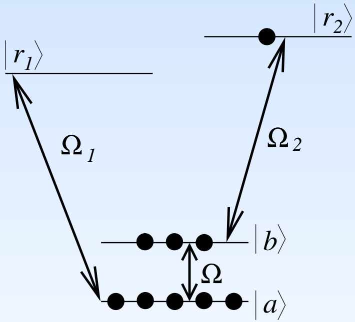
# Squeezing with Rydberg blockade

## Outlook

- Squeezing, spin squeezing
- Rydberg atoms and Rydberg blockade
- Jaynes-Cummings model
- Hamiltonian, dynamic squeezing, adiabatic squeezing
- Schrödinger cat generation
- Conclusion



# Squeezing with Rydberg blockade

**Squeezing**

Example: harmonic oscillator

- Hamiltonian:

$$H = \frac{p^2}{2m} + \frac{1}{2} m\omega^2 x^2$$

- Coherent states: saturate uncertainty relation

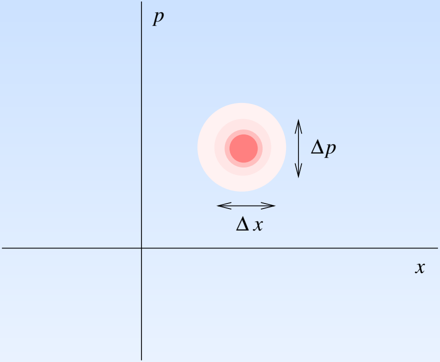
$$\begin{aligned}
\Delta x &= \sqrt{\frac{\hbar}{2m\omega}} \\
\Delta p &= \sqrt{\frac{\hbar m\omega}{2}} \\
\Delta x \Delta p &= \frac{\hbar}{2}
\end{aligned}$$

# Squeezing with Rydberg blockade

**Squeezing**

Example: harmonic oscillator

Coherent states: saturate uncertainty relation



**Squeezing**

Example: harmonic oscillator

Creation and annihilation operators:

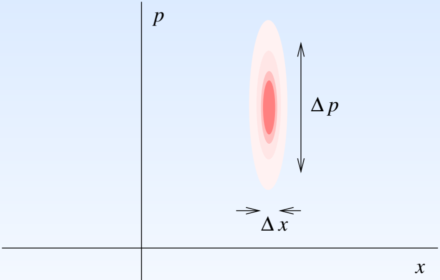
$$
\begin{aligned}
x &= \sqrt{\frac{\hbar}{2m\omega}}(a + a^\dagger) \\
p &= -i\sqrt{\frac{\hbar m\omega}{2}}(a - a^\dagger) \\
[x, p] &= i\hbar \\
[a, a^\dagger] &= 1 \\
H &= \hbar\left(a^\dagger a + \frac{1}{2}\right)
\end{aligned}
$$

# Squeezing with Rydberg blockade

**Squeezing**

Example: harmonic oscillator Squeezed coherent states: can also saturate uncertainty relation, but, e.g.:

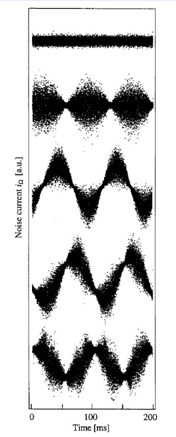
$$\begin{aligned}\Delta x &< \sqrt{\frac{\hbar}{2m\omega}} \\ \Delta p &> \sqrt{\frac{\hbar m\omega}{2}} \\ \Delta x \Delta p &= \frac{\hbar}{2}\end{aligned}$$

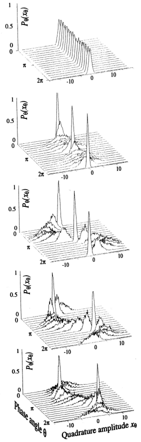


**Squeezing**

Example: harmonic oscillator

Squeezed coherent states: saturate uncertainty relation



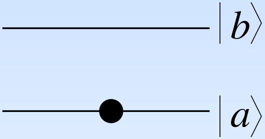




Obrázek: Noise squeezing [G. Breitenbach dissertation, 1998]

# Squeezing with Rydberg blockade

**Spin squeezing**

Example: two-level atoms



# Squeezing with Rydberg blockade

**Spin squeezing**

Example: two-level atoms

Single-atom operators:

$$\begin{aligned} S_x &= \frac{1}{2}(|a\rangle\langle b| + |b\rangle\langle a|), \\ S_y &= \frac{i}{2}(-|a\rangle\langle b| + |b\rangle\langle a|), \\ S_z &= \frac{1}{2}(|a\rangle\langle a| - |b\rangle\langle b|). \end{aligned}$$

# Squeezing with Rydberg blockade

**Spin squeezing**

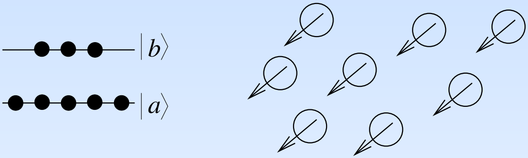
Example: two-level atoms

Many atoms:

$$
\begin{aligned}
\vec{J} &= \sum_k \vec{S}_k \\
J_x &= \frac{1}{2}(a^\dagger b + ab^\dagger), \\
J_y &= \frac{i}{2}(a^\dagger b - ab^\dagger), \\
J_z &= \frac{1}{2}(a^\dagger a - b^\dagger b), \\
[a, a^\dagger] &= [b, b^\dagger] = 1 \\
[J_x, J_y] &= -iJ_z
\end{aligned}
$$

**Spin squeezing**

Example: many two-level atoms

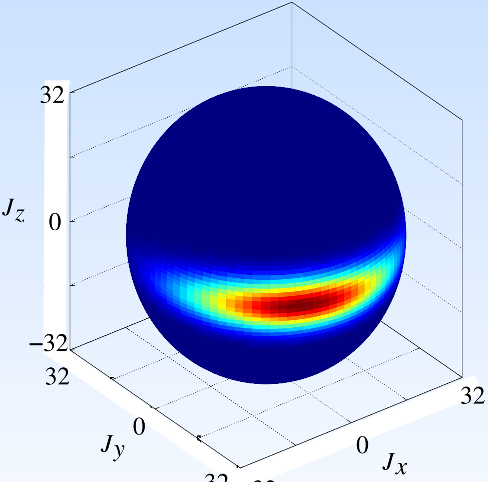


# Squeezing with Rydberg blockade

**Spin squeezing**

Many two-level atoms

Poincare sphere



# Squeezing with Rydberg blockade

**Spin squeezing**

Many two-level atoms

PHYSICAL REVIEW A VOLUME 50, NUMBER 1 JULY 1994

## Squeezed atomic states and projection noise in spectroscopy

D. J. Wineland, J. J. Bollinger, and W. M. Itano
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D. J. Heinzen
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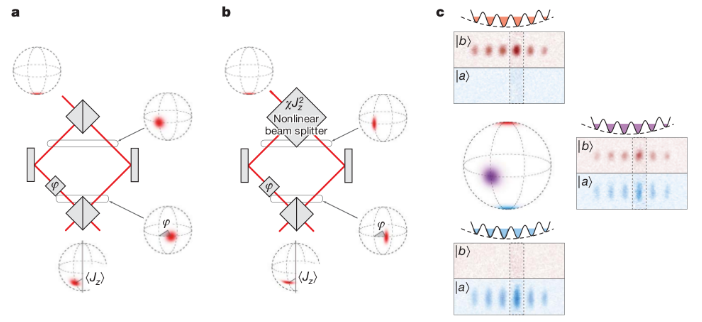
(Received 11 January 1994)

We investigate the properties of angular-momentum states which yield high sensitivity to rotation. We discuss the application of these "squeezed-spin" or correlated-particle states to spectroscopy. Transitions in an ensemble of $N$ two-level (or, equivalently, spin-$\frac{1}{2}$) particles are assumed to be detected by observing changes in the state populations of the particles (population spectroscopy). When the particles' states are detected with 100% efficiency, the fundamental limiting noise is projection noise, the noise associated with the quantum fluctuations in the measured populations. If the particles are first prepared in particular quantum-mechanically correlated states, we find that the signal-to-noise ratio can be improved over the case of initially uncorrelated particles. We have investigated spectroscopy for a particular case of Ramsey's separated oscillatory method where the radiation pulse lengths are short compared to the time between pulses. We introduce a squeezing parameter $\xi_R$ which is the ratio of the statistical uncertainty in the determination of the resonance frequency when using correlated states vs that when using uncorrelated states. More generally, this squeezing parameter quantifies the sensitivity of an angular-momentum state to rotation. Other squeezing parameters which are relevant for use in other contexts can be defined. We discuss certain states which exhibit squeezing parameters $\xi_R \simeq N^{-1/2}$. We investigate possible experimental schemes for generation of squeezed-spin states which might be applied to the spectroscopy of trapped atomic ions. We find that applying a Jaynes-Cummings–type coupling between the ensemble of two-level systems and a suitably prepared harmonic oscillator results in correlated states with $\xi_R < 1$.

# Squeezing with Rydberg blockade

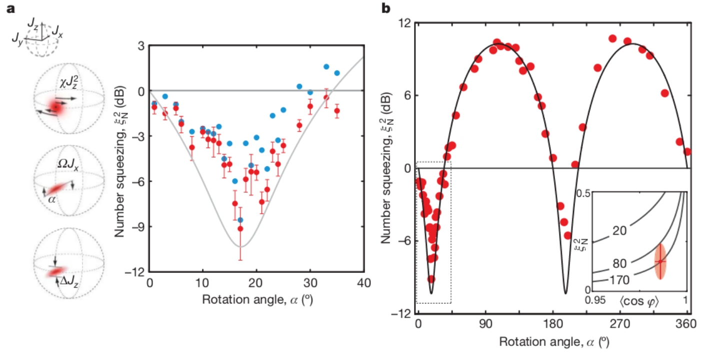
**Spin squeezing**

Gross et al., Nature 464, 1165 (2010)




$\sim 10^3$ atoms squeezed by $\sim 5$ dB in $\sim 10$ ms

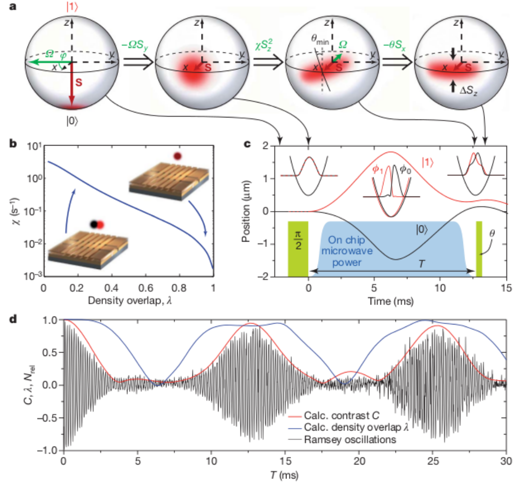
**Spin squeezing**
Gross et al., Nature 464, 1165 (2010)




$\sim 10^3$ atoms squeezed by $\sim 5$ dB in $\sim 10$ ms

**Spin squeezing**

Riedel et al., Nature 464, 1170 (2010)

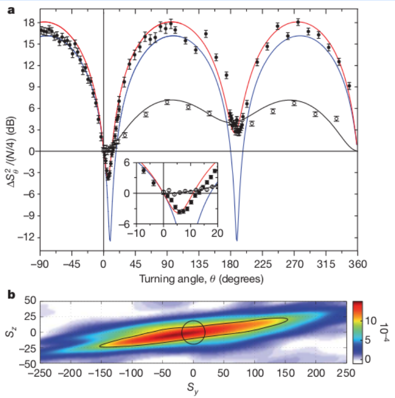



$\sim 10^3$ atoms squeezed by $\sim 5$ dB in $\sim 10$ ms

# Squeezing with Rydberg blockade

**Spin squeezing**

Riedel et al., Nature 464, 1170 (2010)



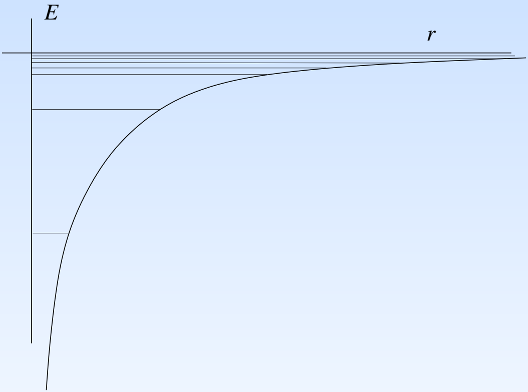

$\sim 10^3$ atoms squeezed by $\sim 5$ dB in $\sim 10$ ms

# Squeezing with Rydberg blockade

**Rydberg atom**

Excited atom with large principal number $n$

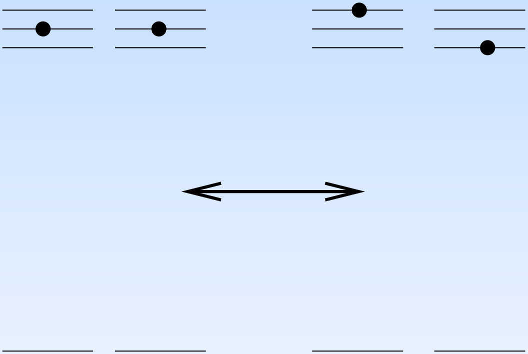
- size $\sim n^2$ ($\sim 0.3\ \mu$m for $n \approx 80$)
- lifetime $\sim n^3$–$n^{4.5}$ ($\sim 600\ \mu$s for $n \approx 80$)



# Squeezing with Rydberg blockade

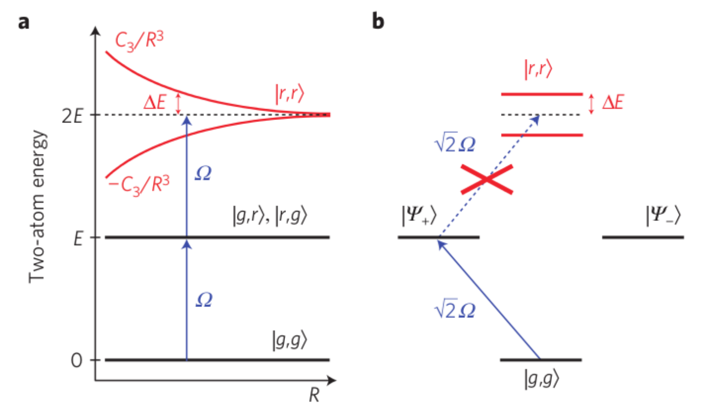
**Rydberg atom**

Rydberg blockade: resonance transitions



**Rydberg atom**

Rydberg blockade: resonance transitions




Gaetan et al., Nature Physics 5, 115 (2009)

# Squeezing with Rydberg blockade

**Jaynes - Cummings model**

A single two-level atom and a single-mode quantum field

$$\begin{aligned} H_{JC} &= g a^{+} \sigma_{-} + g^{*} a \sigma_{+} \\ \sigma_{+} &= |b\rangle\langle a| \\ \sigma_{-} &= |a\rangle\langle b| \end{aligned}$$

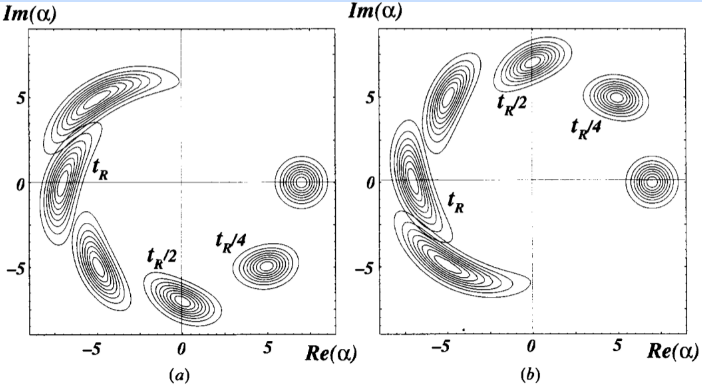
- Photon generation and atom deexcitation
- Photon absorption and atom excitation

# Squeezing with Rydberg blockade

**Jaynes - Cummings model**

A single two-level atom and a single-mode quantum field

Squeezing of the field




G. Banacloche, PRL 65, 3385 (1990); picture from JMO 40, 2361 (1993).

# Squeezing with Rydberg blockade

**Jaynes - Cummings model**

A single two-level atom and a single-mode quantum field

Schrödinger cat generation

PHYSICAL REVIEW A VOLUME 45, NUMBER 11 1 JUNE 1992

**Schrödinger-cat states in the resonant Jaynes-Cummings model: Collapse and revival of oscillations of the photon-number distribution**

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S. J. D. Phoenix

*BT Laboratories, Martlesham Heath, Ipswich IP5 7RE, England*

(Received 16 December 1991)

The Jaynes-Cummings model of optical resonance describes the simplest fully quantized interaction between two quantum systems of different nature: a two-level atom (fermionic system) and a quantized field mode (bosonic system). This interaction leads to extreme quantum entanglement of the atom and field. However, the model also predicts that, at precisely half of the revival time, the atom and field become asymptotically disentangled. This disentanglement becomes more exact as the coherent-state amplitude increases. In this paper we investigate the nature of the pure-field-state superposition generated at such times. We show that this superposition is of distinguishable states of the field with the same amplitude but opposite phase. Interference between these components leads to nonclassical oscillations in photon-number distributions and squeezing in quadratures of the field. The Schrödinger-cat states of the field are highly transient, and depend very sensitively on the interaction time, the initial intensity of the field, and the atom-field detuning.

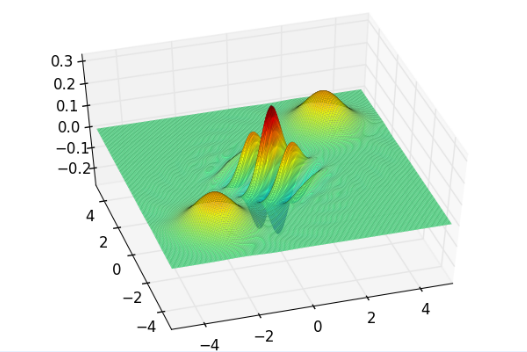
PACS number(s): 42.50.−p, 03.65.Bz, 42.52.+x

# Squeezing with Rydberg blockade

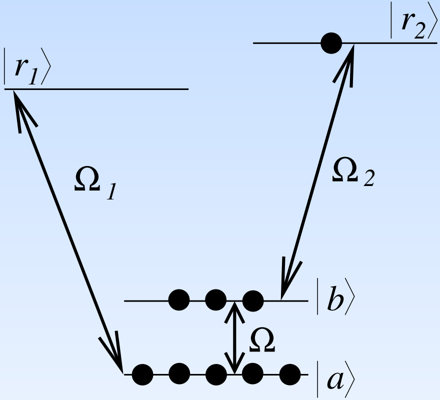
**Jaynes - Cummings model**

A single two-level atom and a single-mode quantum field

Schrödinger cat generation



**Spin squeezing and Schrödinger cat generation in atomic samples with Rydberg blockade**




T. Opatrný and K. Mølmer, PRA 86, 023845 (2012)

# Squeezing with Rydberg blockade

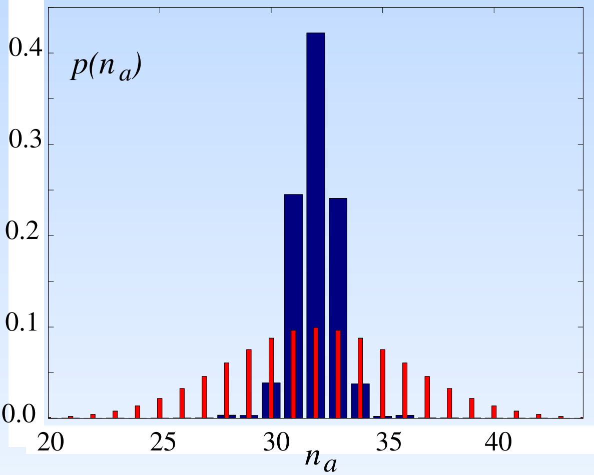
**Hamiltonian**

$$
\begin{aligned}
H_{JC1} &= \Omega_1 a\sigma_+^{(1)} + \Omega_1^* a^\dagger \sigma_-^{(1)} \\
H_{JC2} &= \Omega_2 b\sigma_+^{(2)} + \Omega_2^* b^\dagger \sigma_-^{(2)}
\end{aligned}
$$

- Initialize the state
- Act with the Hamiltonian
- Rotate the state

# Squeezing with Rydberg blockade

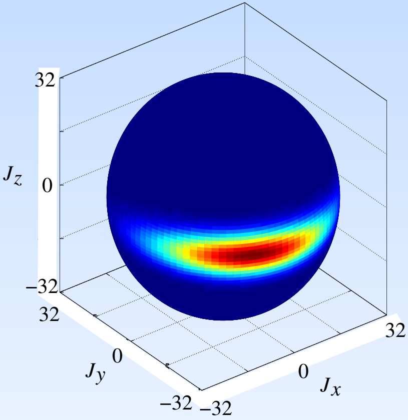
**Results**




Statistics of the atomic states $|a\rangle$ and $|b\rangle$ (64 atoms)

# Squeezing with Rydberg blockade

## Results




Q-function of the resulting state (64 atoms)

# Squeezing with Rydberg blockade

**Adiabatic squeezing: Hamiltonian eigenstates**

$$
\begin{aligned}
|\psi_{+,+}^{(n_a,n_b)}\rangle &= \frac{1}{2}\left(|n_a, n_b, 0, 0\rangle + |n_a - 1, n_b, 1, 0\rangle\right. \\
&+ \left.|n_a, n_b - 1, 0, 1\rangle + |n_a - 1, n_b - 1, 1, 1\rangle\right), \\
|\psi_{+,-}^{(n_a,n_b)}\rangle &= \frac{1}{2}\left(|n_a, n_b, 0, 0\rangle + |n_a - 1, n_b, 1, 0\rangle\right. \\
&- \left.|n_a, n_b - 1, 0, 1\rangle - |n_a - 1, n_b - 1, 1, 1\rangle\right), \\
|\psi_{-,+}^{(n_a,n_b)}\rangle &= \frac{1}{2}\left(|n_a, n_b, 0, 0\rangle - |n_a - 1, n_b, 1, 0\rangle\right. \\
&+ \left.|n_a, n_b - 1, 0, 1\rangle - |n_a - 1, n_b - 1, 1, 1\rangle\right), \\
|\psi_{-,-}^{(n_a,n_b)}\rangle &= \frac{1}{2}\left(|n_a, n_b, 0, 0\rangle - |n_a - 1, n_b, 1, 0\rangle\right. \\
&- \left.|n_a, n_b - 1, 0, 1\rangle + |n_a - 1, n_b - 1, 1, 1\rangle\right),
\end{aligned}
$$

# Squeezing with Rydberg blockade

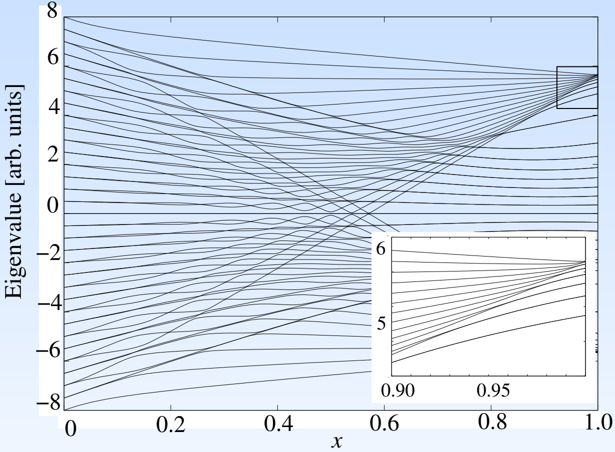
**Adiabatic squeezing: Eigenenergies**

$$
\begin{aligned}
E_{+,+}^{(n_a,n_b)} &= \Omega_{JC}\left(\sqrt{n_a}+\sqrt{n_b}\right), \\
E_{+,-}^{(n_a,n_b)} &= \Omega_{JC}\left(\sqrt{n_a}-\sqrt{n_b}\right), \\
E_{-,+}^{(n_a,n_b)} &= \Omega_{JC}\left(-\sqrt{n_a}+\sqrt{n_b}\right), \\
E_{-,-}^{(n_a,n_b)} &= \Omega_{JC}\left(-\sqrt{n_a}-\sqrt{n_b}\right).
\end{aligned}
$$

# Squeezing with Rydberg blockade

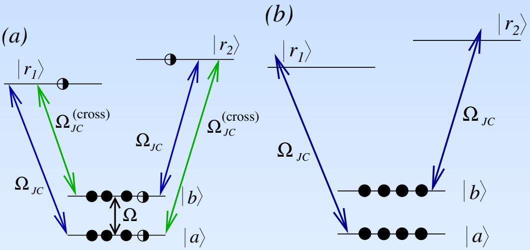
## Adiabatic squeezing: Combine Hamiltonian

$$H = uH_{JC} + (1-u)J_x$$



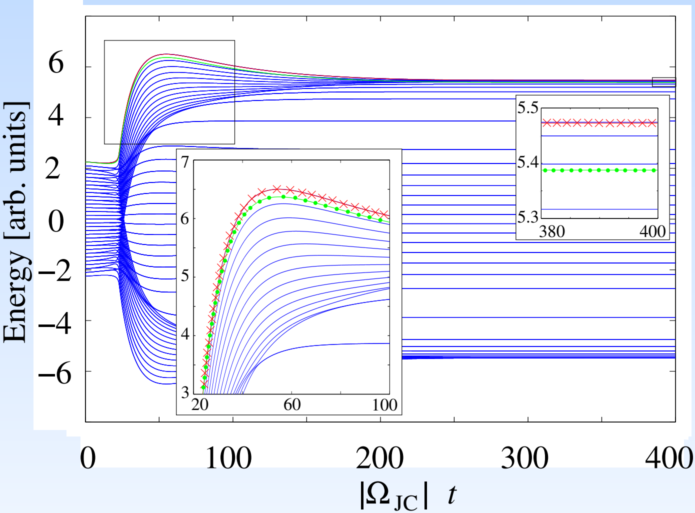
# Squeezing with Rydberg blockade

## Adiabatic squeezing

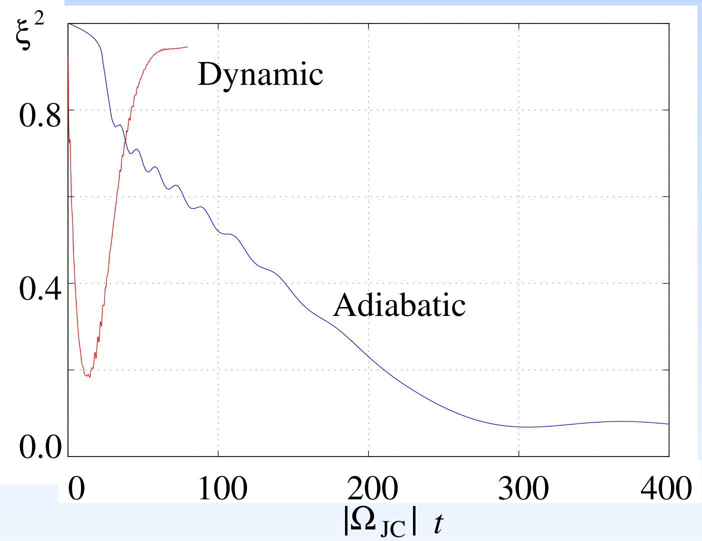


# Squeezing with Rydberg blockade

## Adiabatic squeezing



**Adiabatic squeezing**

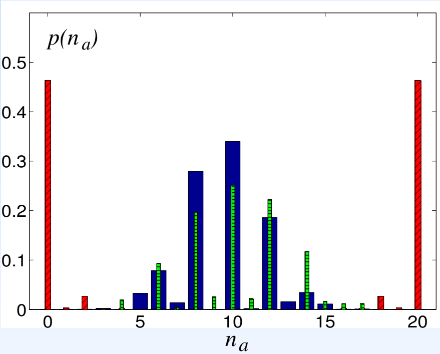


# Squeezing with Rydberg blockade

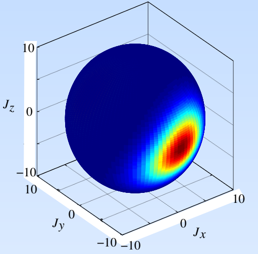
**Schrödinger cat generation**

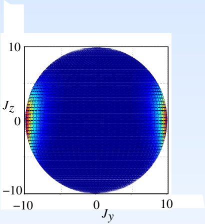
Hamiltonian $H_{JC1}$ with $\Omega_1 = \Omega_1^* = \Omega_{JC}$ switch on for

$$\tau = \begin{cases} \frac{\pi}{\Omega_{JC}}\sqrt{\frac{N}{2}} & \text{for } N \text{ even} \\ \frac{\pi}{\Omega_{JC}}\sqrt{\frac{N-1}{2}} & \text{for } N \text{ odd} \end{cases} .$$



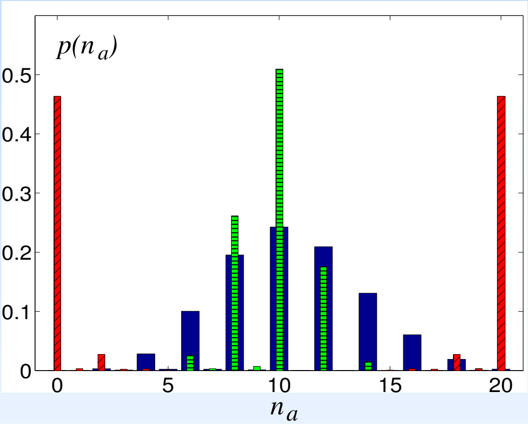
## Schrödinger cat generation



# Squeezing with Rydberg blockade

**Schrödinger cat generation**



# Squeezing with Rydberg blockade

**Summary**

- Squeezing and spin squeezing, consequences for metrology
- Rydberg blockade: strong nonlinearity in atom-field interaction
- Jaynes - Cummings model, can generate squeezing or Schrödinger cat states
- Atomic samples with driven transitions to Rydberg states: combine several JC interactions
- Dynamic and adiabatic squeezing, Schrödinger cat states in atomic samples

**Thanks for your attention!**