Quantumness Witnesses

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based on work in collaboration with
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seminal idea

Set A of observables of a system.

We do not know whether system is classical or quantum

Theorem (Alicki, Van Ryn):

(i) $\mathcal{A}$ *is commutative. To wit, for any pair* $X, Y \in \mathcal{A}$,

$$[X, Y] := XY - YX = 0.$$

(ii) For any pair $X, Y \in \mathcal{A}$ *with* $X \geq 0$ *and* $Y \geq 0$,

$$\{X, Y\} := XY + YX \geq 0.$$

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quantumness witness (QW)

$$X, Y \in \mathcal{A}$$

$$X, Y \geq 0$$

$$Q_{\mathrm{AVR}} = \{X, Y\}$$

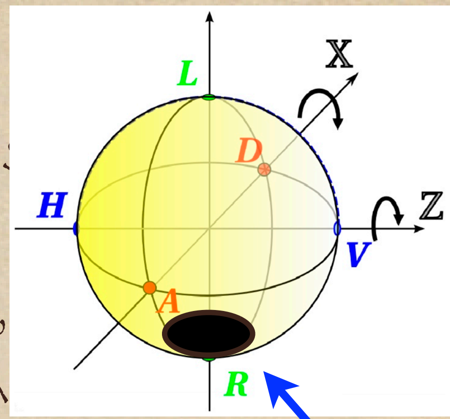
if Q is not positive definite then Q is a QW

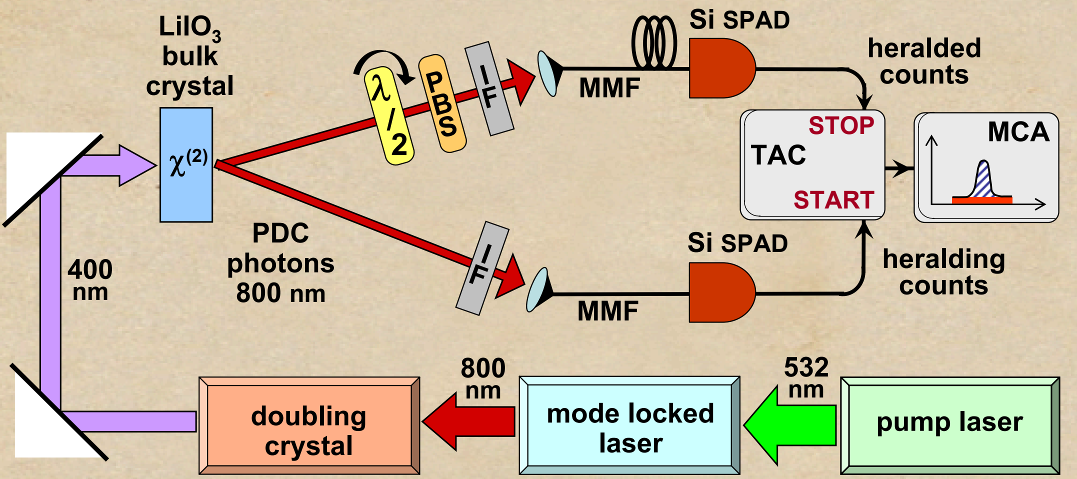
- EXAMPLE

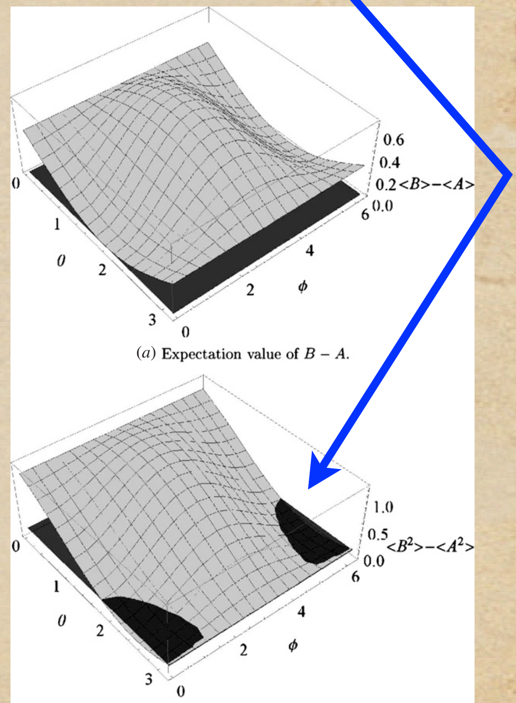
- is a photon quantum?
- check its anticommutator!

experiments (Turin)

G. Brida, I.P. Degiovanni, M. Genovese, V. Schettini,
and A. Migdall, Optics Express 16, 11750 (2008).
G. Brida, I.P. Degiovanni, M. Genovese, F. Piacentini,
N. Gisin, S.V. Polyakov, and A. Migdall, Phys. Rev. A .







(a) Expectation value of $B - A$.

(b) Expectation value of $B^2 - A^2$.

comments

- Alicki and Van Ryn, J. Phys. A: Math. Theor. 41, 062001 (2008)
  Alicki, Piani and Van Ryn, J. Phys. A: Math. Theor. 41, 495303 (2008)
- many definitions of classicality
- semiclassical limit
- Wigner, Sudarshan, Glauber...
- interesting? obvious?

interesting (related?) work

Horodecki's 2008

Kiesel, Vogel, Hage and Schnabel 2011

Chen and Deng 2009

Piani and Adesso 2011

Filippov and Man'ko 2009

Andersson, Bergou, Jex 2005

Jamiolkowski 2012

Fiurasek 2012

Perina and Krepelka 2011

(Quantumness and Entanglement Witnesses)

Facchi, Pascazio, Vedral, Yuasa, J. Phys. A (2012)

QW vs EW

We say that an observable $Q \in \mathcal{A}$ is a *quantumness witness* (QW) if

- for any classical state ρ one gets $\rho(Q) \geq 0$
- there exists a (quantum) state σ such that $\sigma(Q) < 0$.

Every EW is also a QW

idea:

(i) $\mathcal{A}$ *is commutative. To wit, for any pair* $X, Y \in \mathcal{A}$,

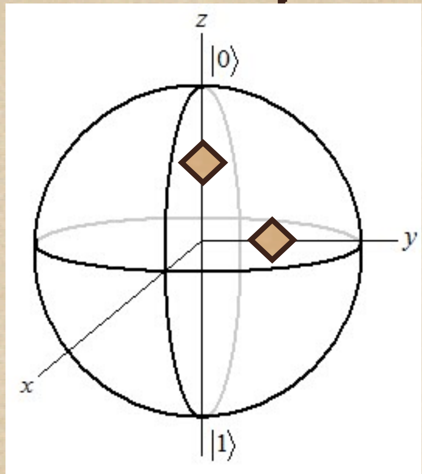
$$[X, Y] := XY - YX = 0.$$

(ii) *For any pair* $X, Y \in \mathcal{A}$ *with* $X \geq 0$ *and* $Y \geq 0$,

$$\{X, Y\} := XY + YX \geq 0.$$

- extend previous definition to states
- after all, states are density matrices
- therefore states are positive operators
- arXiv:1201.1212 [quant-ph]
  "Measuring quantumness via anticommutators"
  Phil. Trans. R. Soc. A 370 (2012) 4810
  "Classical to quantum in large number limit"
  Modi, Fazio, Pascazio, Vedral, Yuasa

example



$$\rho = \frac{1}{2}(1 + p\sigma_y)$$

$$\rho' = \frac{1}{2}(1 + p\sigma_z)$$

$$\{\rho, \rho'\} \geq 0 \leftrightarrow p \leq 1/\sqrt{2}$$

in general

$$\rho = \frac{1}{2}(1 + x \cdot \sigma)$$

$$\rho' = \frac{1}{2}(1 + x' \cdot \sigma)$$

$$\{\rho, \rho'\} \geq 0 \Longleftrightarrow x^2 + x'^2 + x \cdot x' \leq 1$$

rephrase in terms of unambiguous state discrimination and probabilistic cloning (Bergou, Buzek, Hillery, Herzog)

ρ_1

states

ρ_2

$[\rho_1, \rho_2] = ?$ $[\rho_1, \rho_2] \neq 0$

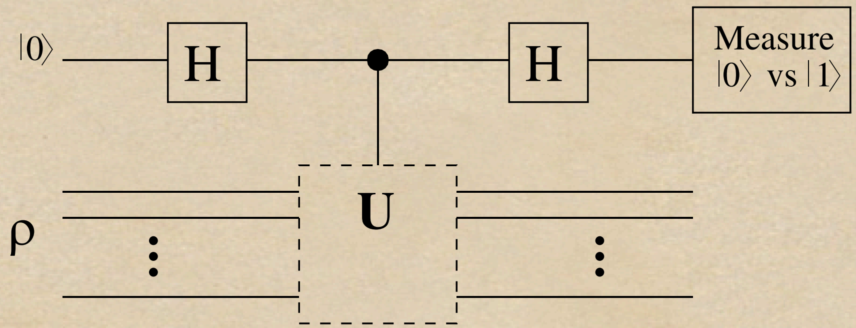
$\{\rho_1, \rho_2\}$ assume it can be measured

as well as...

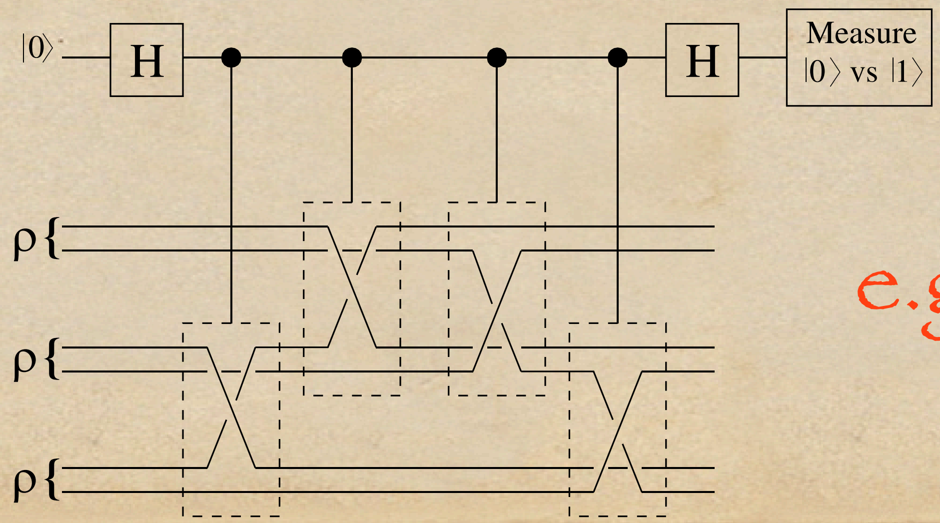
$\{\rho_1, \rho_2\}, \{\rho_1, \rho_1\}, \{\rho_2, \rho_2\}, \{\{\rho_1, \rho_2\}, \rho_1\}, \ldots$

what does it mean to measure an anticommutator?

interference!



general scheme



e.g. $tr[(\rho^{T_2})^3]$

interesting ideas by:

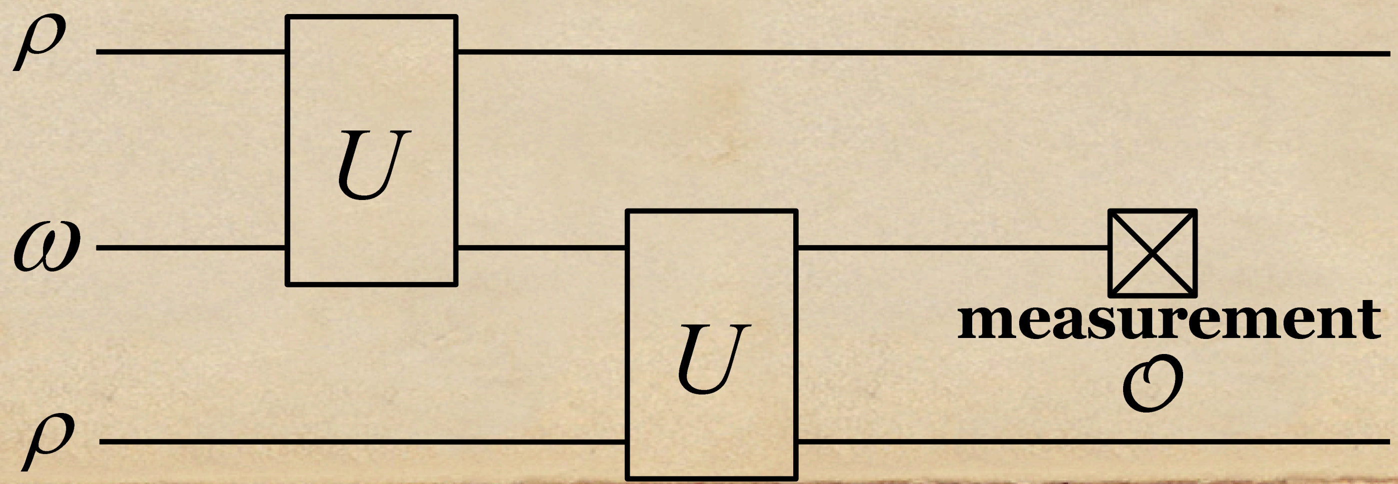
Carteret, PRL 94, 040502 (2005)

Sjoqvist, Pati, Ekert, Anandan, Ericsson, Oi, Vedral, PRL 85, 2845 (2000)

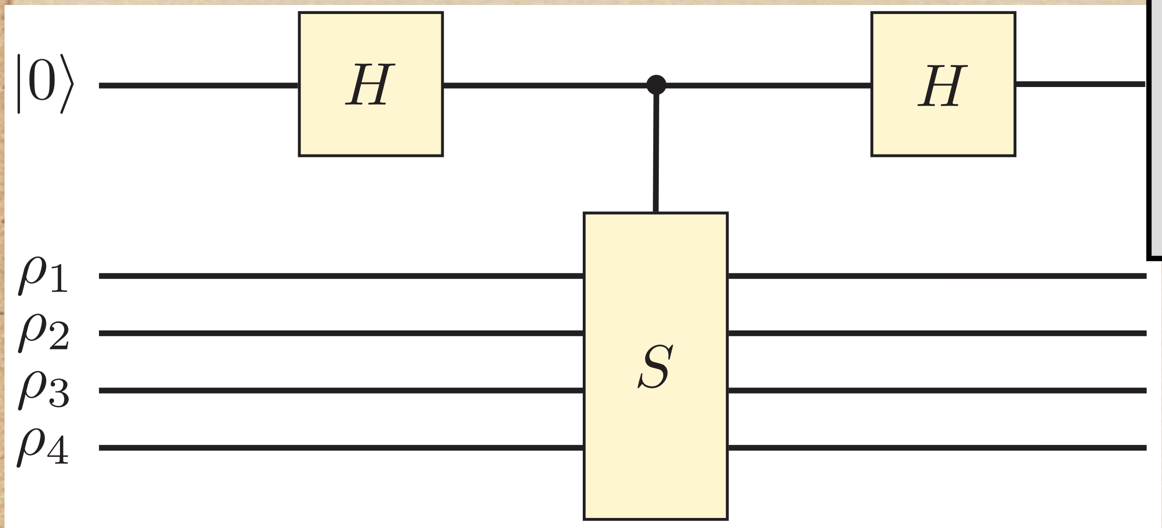
Adamson, Shalm, Steinberg PRA 75, 012104 (2007)

Bechmann-Pasquinucci, Huttner, Gisin, PLA 242, 198 (1998)

Nakazato, Tanaka, Yuasa, Florio, Pascazio, PRA 85, 042316 (2012): measure purity



in general



interference between $|0\rangle$ and $|1\rangle$

$$\text{visibility} \sim \mathrm{tr}(\rho_1 \rho_2 \rho_3 \rho_4)$$

valid for quDits!

(reminder)

ρ_1

states

ρ_2

$[\rho_1, \rho_2] =?$

$[\rho_1, \rho_2] \neq 0$

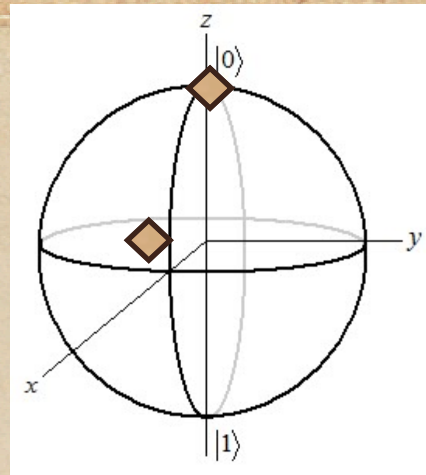
discord

$$D(A|B) = 0 \quad \Longleftrightarrow \quad [\rho_{B|i}, \rho_{B|j}] = 0, \quad \forall i, j$$

where $\rho_{B|i} = Tr_A[\Lambda_A^i \otimes \mathcal{I}_B(\rho_{AB})]$

(operational meaning)

first observation



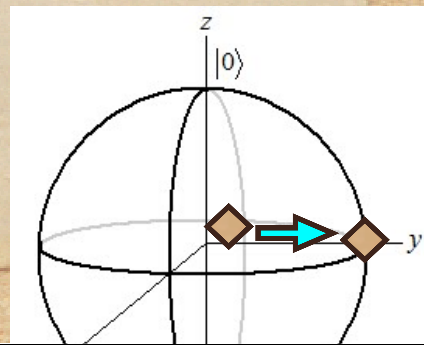
ρ_1 pure

$[\rho_1, \rho_2] \neq 0$ ⇒ $\{\rho_1, \rho_2\}$ not positive definite

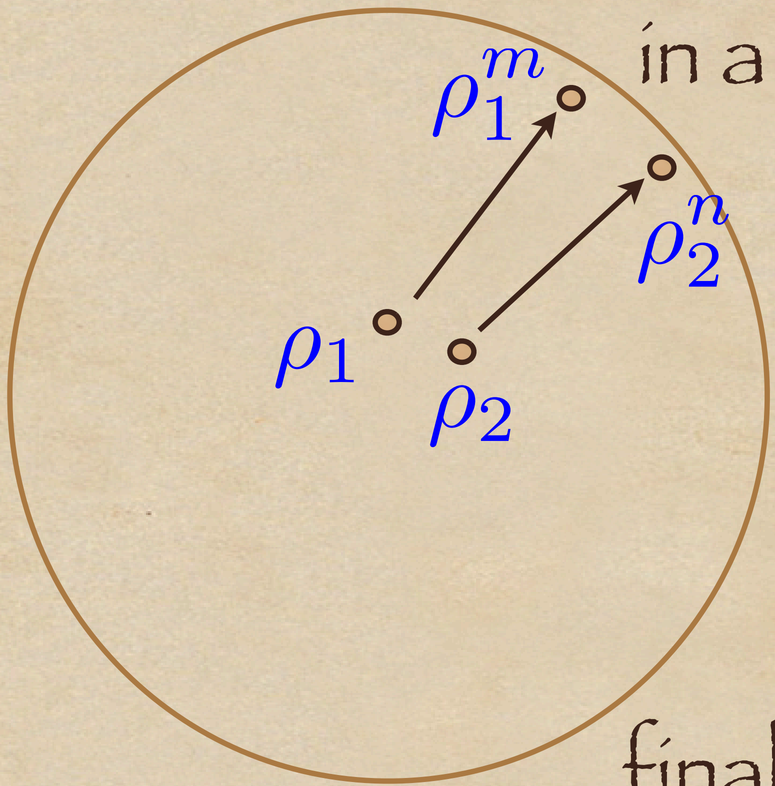
second observation

$$\rho = \sum_i \lambda_i |i\rangle\langle i| \Rightarrow \frac{\rho^n}{tr\rho^n} \to |0\rangle\langle 0|$$

max eigenvalue not degenerate



therefore



in a finite number of steps

... finally $\{\rho_1^m, \rho_2^n\}$
should become non-positive!

but:

$\{\rho_1^m, \rho_2^n\}$ non-positive means quantum

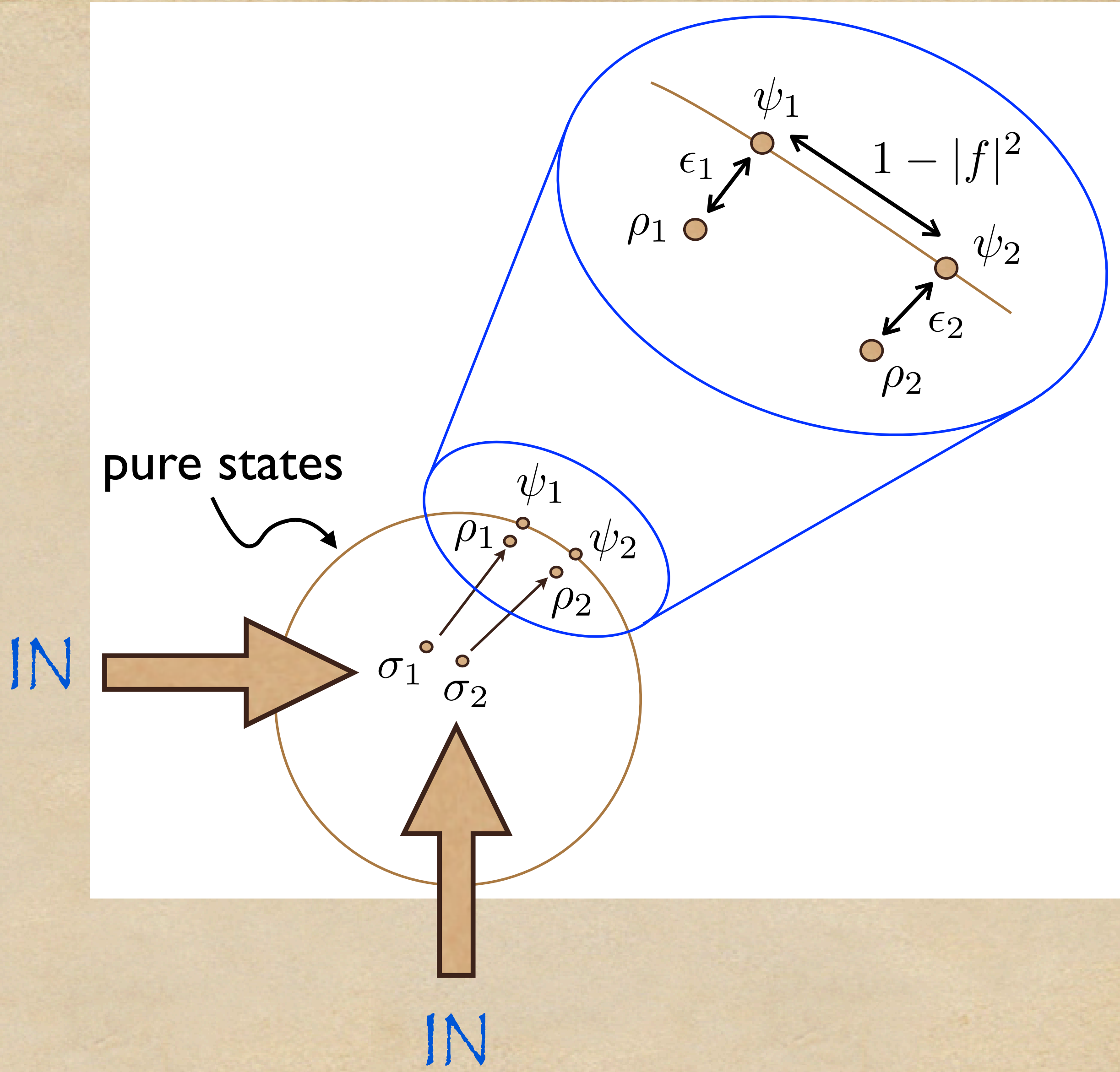
we are ready to state our final result

precept (Stephen Hawking)

Someone told me that each equation
I included in the book would halve the sales.
I therefore resolved not to have any
equations at all. In the end, however,
I did put in one equation, Einstein's famous
equation, E=mc^2. I hope that this will not
scare off half of my potential readers.

true for physics audience:

replace ~~equation~~ ⇒ theorem



pure states
IN
IN
σ_1
σ_2
ρ_1
ρ_2
ψ_1
ψ_2
ϵ_1
ϵ_2
$1 - |f|^2$

mixed states ρ_1, ρ_2

ϵ-close to pure states

$$\rho_i = (1-\epsilon_i)|\psi_i\rangle\langle\psi_i| + \epsilon_i\sigma_i \quad (\sigma_i|\psi_i\rangle = 0)$$

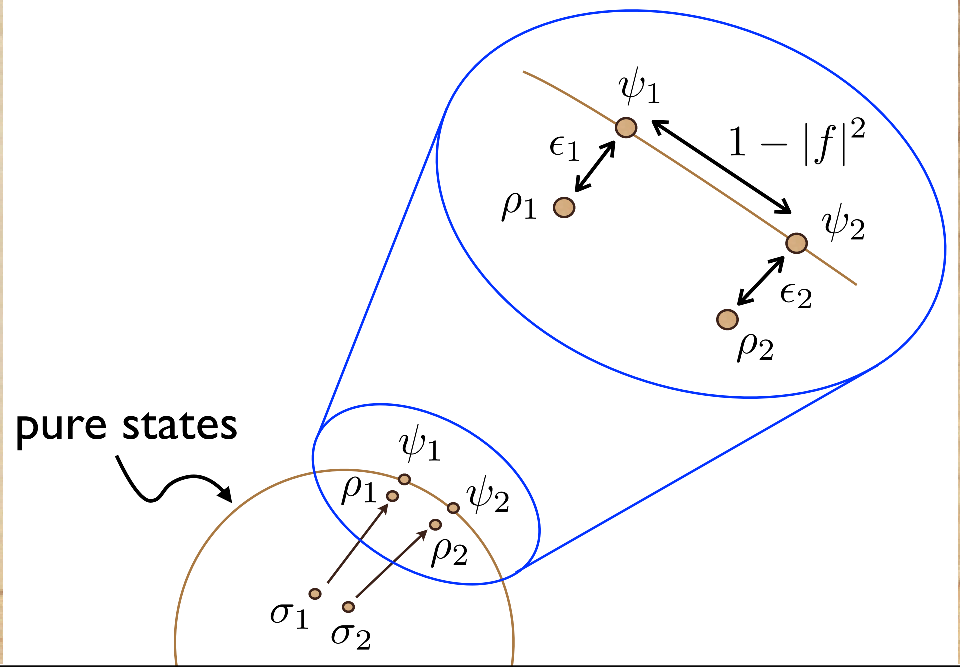
$$f = \langle\psi_1|\psi_2\rangle$$

$$\epsilon_1 g_1 + \epsilon_2 g_2 < (1-|f|^2)/2$$

$$g_1 = \langle\psi_2|\sigma_1|\psi_2\rangle$$

$$g_1 = \langle\psi_2|\sigma_1|\psi_2\rangle$$

$$\{\rho_1, \rho_2\} \not\geq 0$$



reminder

- bring to light quantumness by measuring anticommutators
- in fact: only anticommutators
- but in general one pays a price: high-order interference necessary
- for some states, it can be difficult to bring quantumness to light