Visit to Tampere University of Technology

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european social fund in the czech republic
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MINISTRY OF EDUCATION, YOUTH AND SPORTS
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INVESTMENTS IN EDUCATION DEVELOPMENT

Basic Information

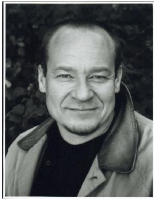
Tampere University of Technology

- founded in 1965
- branch of Helsinki University of Technology
- TUT consists of 5 faculties
  - Faculty of Automation, Mechanical and Materials
  - Faculty of Science and Environmental Engineering
  - Faculty of Built Environment
  - Faculty of Business and Technology Management
  - Faculty of Computing and Electrical Engineering
- about 20,000 students
- about 1,900 staff

Basic Information

Esko Turunen, Ph.D.

- associate professor at Department of Mathematics
- topics of research:
  - GUHA method
  - many-valued logics and their applications
  - fuzzy signal control
  - relational structures as knowledge instruments
  - algorithmic decision theory



Logic Group

- meeting with Logic Group (University of Tampere, School of Information Sciences)
- members:
  - Lauri Hella (group leader, professor)
  - Kerkko Luosto (university lecturer)
  - Antti Kuusisto (researcher)
  - Jonni Virtema (researcher)
  - Jevgeni Haigora (doctoral student)
- seminar: 14. 9. 2012, Eduard Bartl: *Generalized fuzzy relational equations*

Conducted Research

- generalized fuzzy relational equations
- complexity issues
- minimal solutions
- probabilistic algorithm for finding all minimal solutions based on greedy approach

Problem setting

- we focus on fuzzy relational equations (FRE)

Problem of FRE

$$R \odot S = T$$

$$\overset{R}{\begin{pmatrix} 0.1 & 1.0 & 0.6 & \\ 0.0 & 0.9 & 1.0 & \cdots \\ 0.3 & 0.7 & 0.7 & \\ & \vdots & & \ddots \end{pmatrix}} \odot \overset{S}{\begin{pmatrix} 0.5 & 0.1 & 0.0 & \\ 0.9 & 0.4 & 0.2 & \cdots \\ 1.0 & 0.4 & 0.7 & \\ & \vdots & & \ddots \end{pmatrix}} = \overset{T}{\begin{pmatrix} 0.1 & 0.1 & 0.2 & \\ 1.0 & 0.8 & 0.6 & \cdots \\ 0.5 & 0.8 & 0.1 & \\ & \vdots & & \ddots \end{pmatrix}}$$

- given R and T, determine S, or
- given S and T, determine R

Our generalization

- depending on the type of composition $\odot$ there are distinguished two types of FREs: sup-t-norm, inf-residuum type
- we show that sup-t-norm and inf-residuum type can be viewed as two particular instances of a single (more general) type of FRE

Problem setting

Main contribution of our approach

- we don't need to prove the results (such as criteria of solvability) for both types of FRE separately
- instead, we can work only with our general type of FRE

Other results

- the method of finding all minimal solutions
- better insight to complexity issues regarding the problem of finding a minimal solution (it turns out that this problem is interesting **NP**-hard optimazation problem)

Question

How to look at the sup-t-norm, and inf-resiuum FRE as a single type?

- based on *aggregation structure* introduced and developed by prof. Radim Belohlavek:

  Belohlavek, R., 2012, Sup-t-norm and inf-residuum are one type of relational product: unifying framework and consequences, Fuzzy Sets and Systems, 197, 45-48

Main idea behind the aggregation structure

- residuated lattice: an algebra with specific properties, where
  - multiplication $\otimes$ is included in this algebra
  - residuum $\rightarrow$ is not involved (playing the role of an additional operation bounded by adjointness property)
- but: if one looks the properties of $\otimes$ and $\rightarrow$, one can recognize many similarities (e.g., distributivity over infima and suprema, monotony in both arguments, etc.)
- what if we define general structure with two operations (playing the role of multiplication and residuum) such that the role of main and additional operation can be changed?

General composition of fuzzy relations

Using aggregation structure we define composition $\boxdot$ which generalizes sup-t-norm and inf-residuum composition

Definition

For an aggregation structure $\langle \mathbf{L}_1, \mathbf{L}_2, \mathbf{L}_3, \square \rangle$, and fuzzy relations $R \in L_1^{X \times Y}$ and $S \in L_2^{Y \times Z}$, let a fuzzy relation $R \circ S \in L_3^{X \times Z}$ be defined by

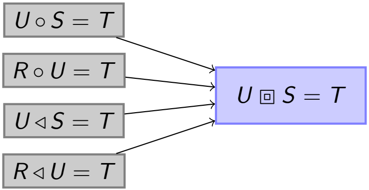
$$(R \boxdot S)(x, z) = \bigvee\nolimits_{3\, y \in Y} (R(x, y) \,\square\, S(y, z)).$$

Generalized FRE

- so we get single type of FRE:

$$U \boxdot S = T$$

- all types of FRE can be described by this single type



Minimal solutions

Vector-by-matrix equation

Theorem

Let $\langle \mathbf{L}_1, \mathbf{L}_2, \mathbf{L}_3, \Box \rangle$ be an aggregation structure with $\mathbf{L}_1$ being a chain, and $\hat{R} = (\hat{r}_1\ \hat{r}_2\ \ldots\ \hat{r}_n)$ be the greatest solution of a solvable vector-by-matrix equation. Then every minimal solution $\check{R} = (\check{r}_1\ \check{r}_2\ \ldots\ \check{r}_n)$ is of the form:

$$\check{r}_j = \begin{cases} \hat{r}_j, & \text{for } j \in J_{\mathrm{cov}}, \\ 0_1, & \text{otherwise,} \end{cases}$$

where J_{cov} is a minimal covering of the last row of the table $\mathfrak{B}$.