Optimal covariant processing of quantum gates

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Palacky University Olomouc

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european social fund in the czech republic

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MINISTRY OF EDUCATION, YOUTH AND SPORTS

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Outline

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Axiomatic approach to state transformations

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Processing quantum transformations: Quantum Supermaps

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Higher Order Quantum Computation

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- Higher Order Quantum Computation

Processing unitary transformations

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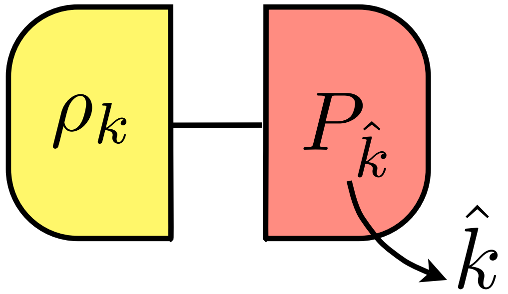
Future perspectives

State transformations

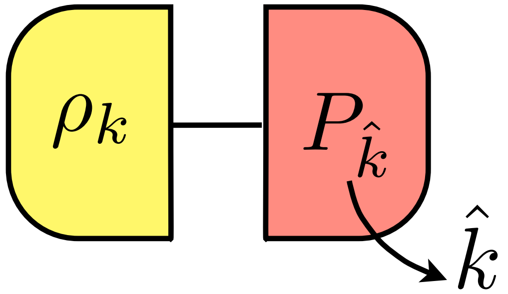
State transformations

state estimation

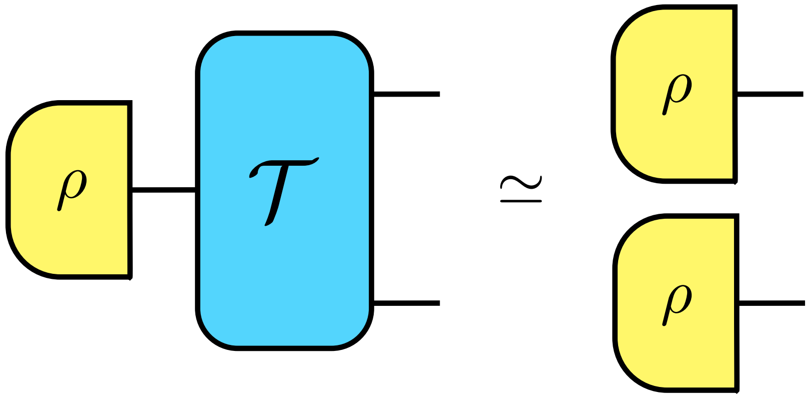


State transformations

state estimation

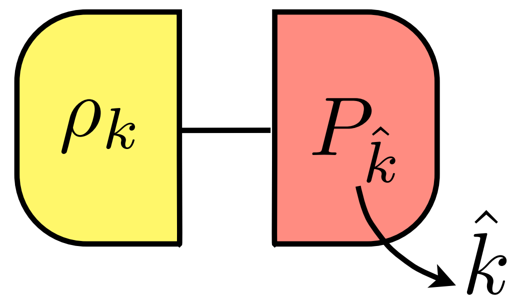


quantum cloning

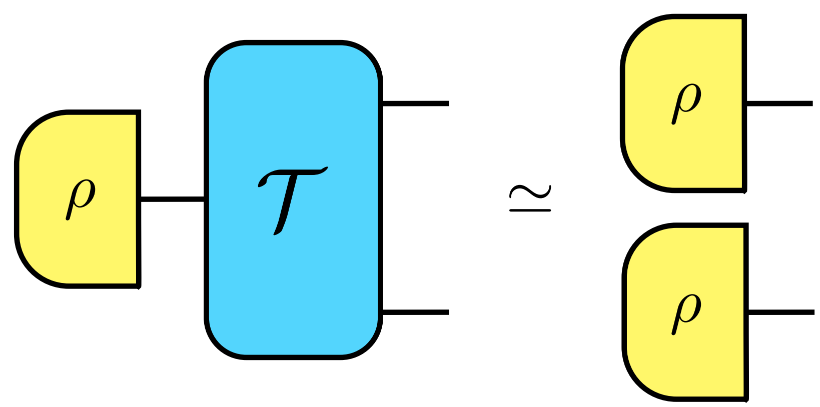


State transformations

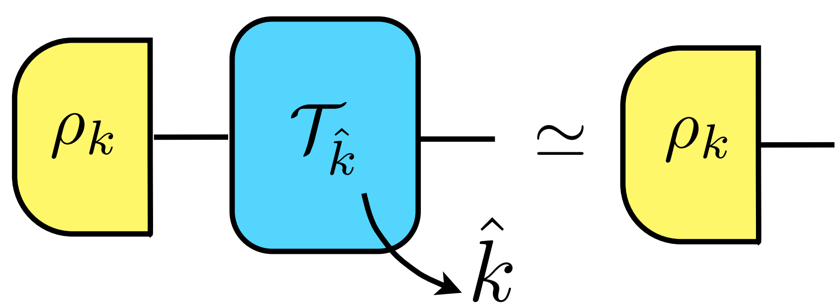
state estimation



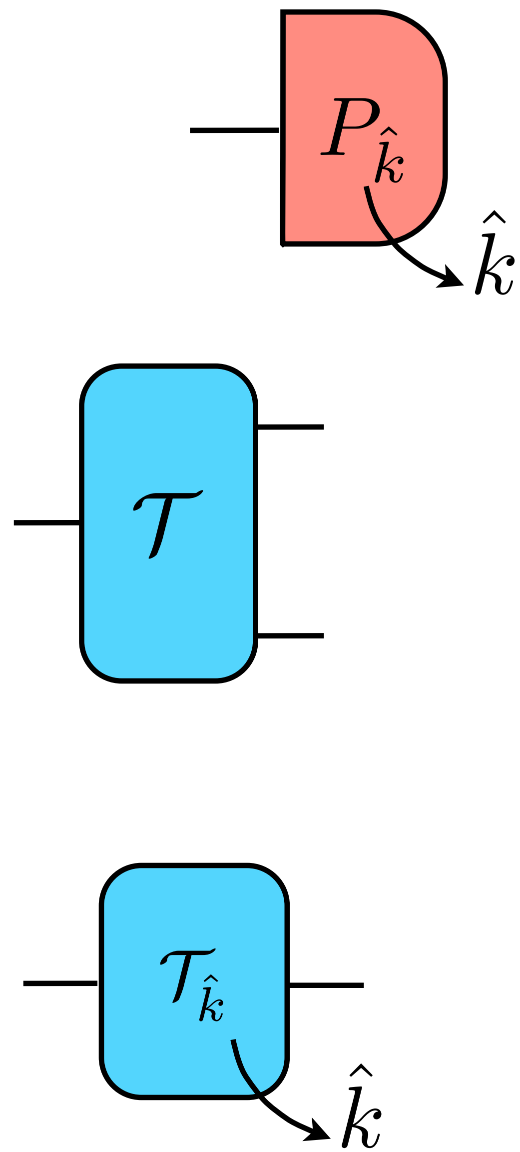
quantum cloning



information
disturbance
tradeoff



State transformations

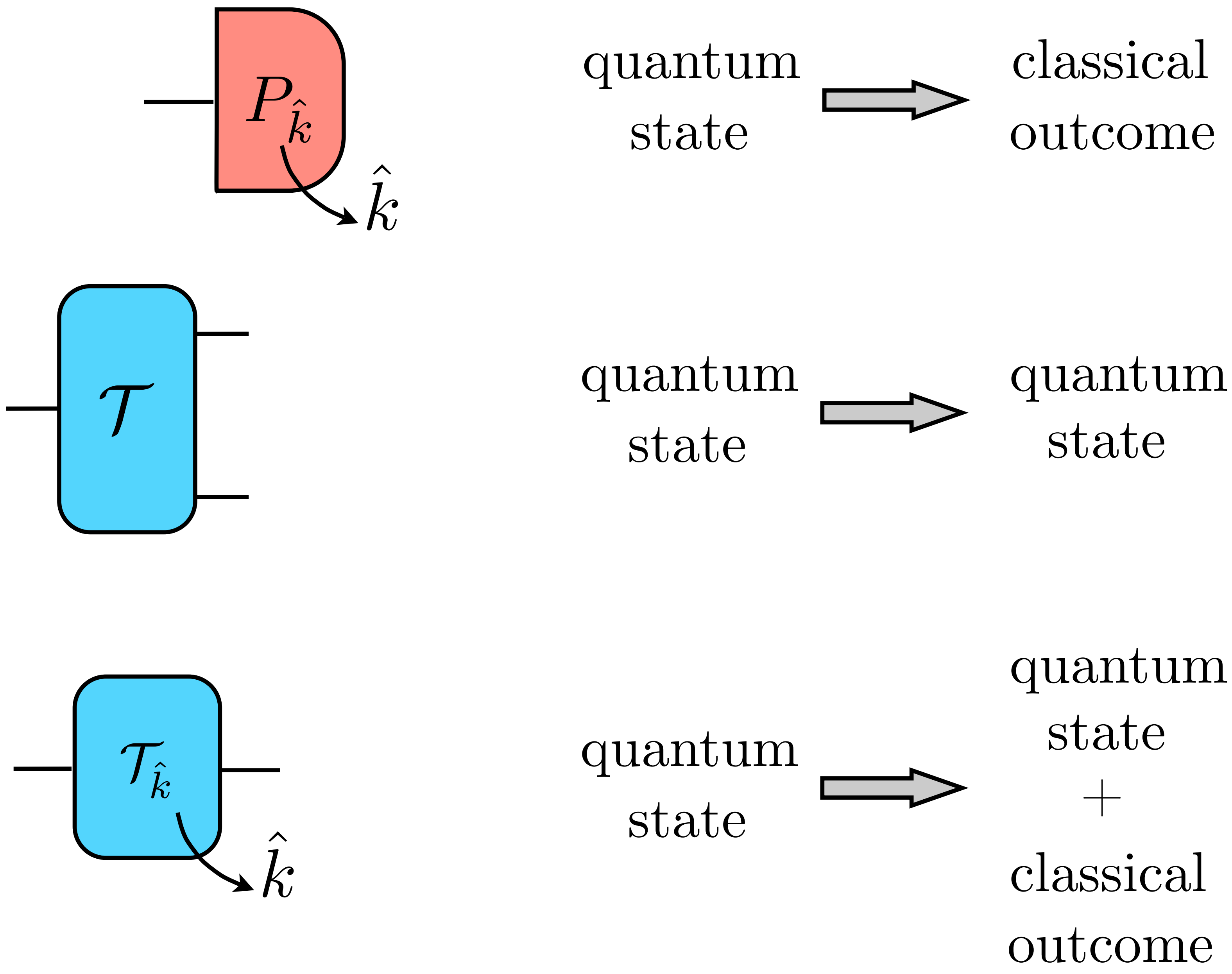


quantum state ⇒ classical outcome

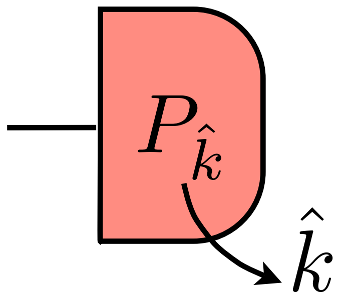
quantum state ⇒ quantum state

quantum state ⇒ quantum state + classical outcome

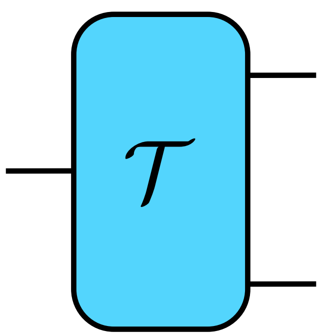
State transformations



State transformations

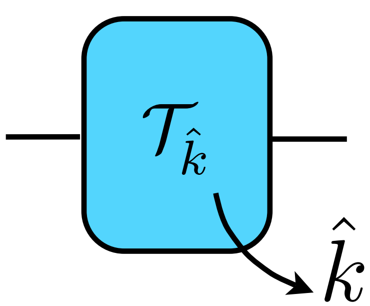


quantum state ⇒ classical outcome



quantum state ⇒ quantum state

The most general state transformation?

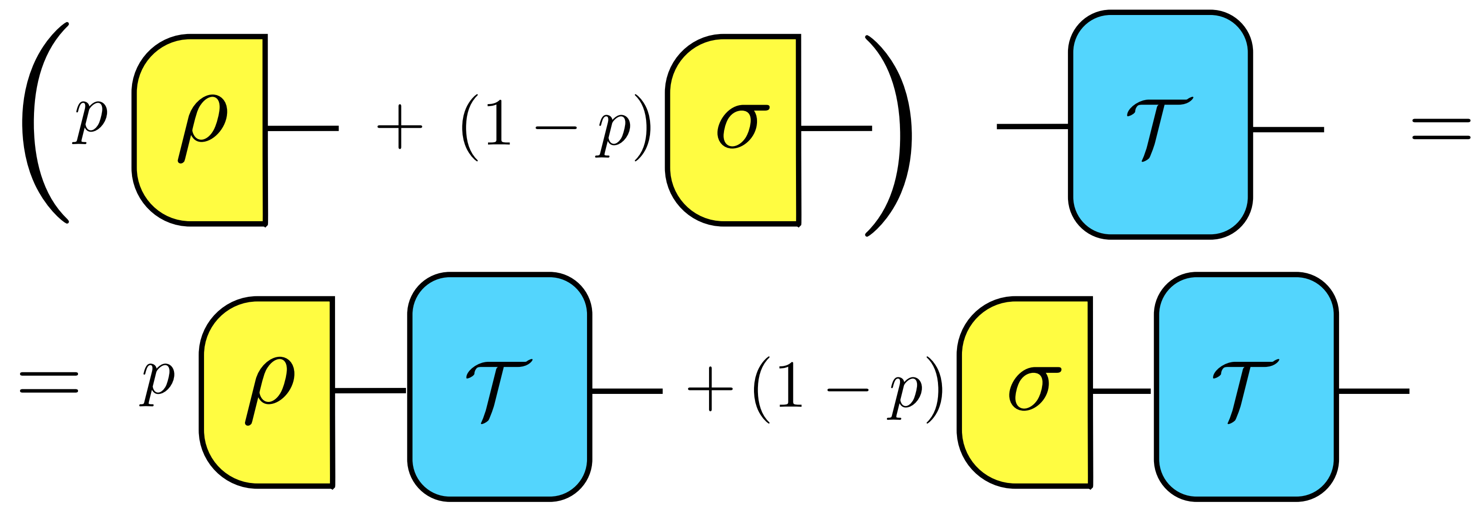


quantum state ⇒ quantum state + classical outcome

Admissibility conditions (deterministic)

Admissibility conditions (deterministic)

linearity



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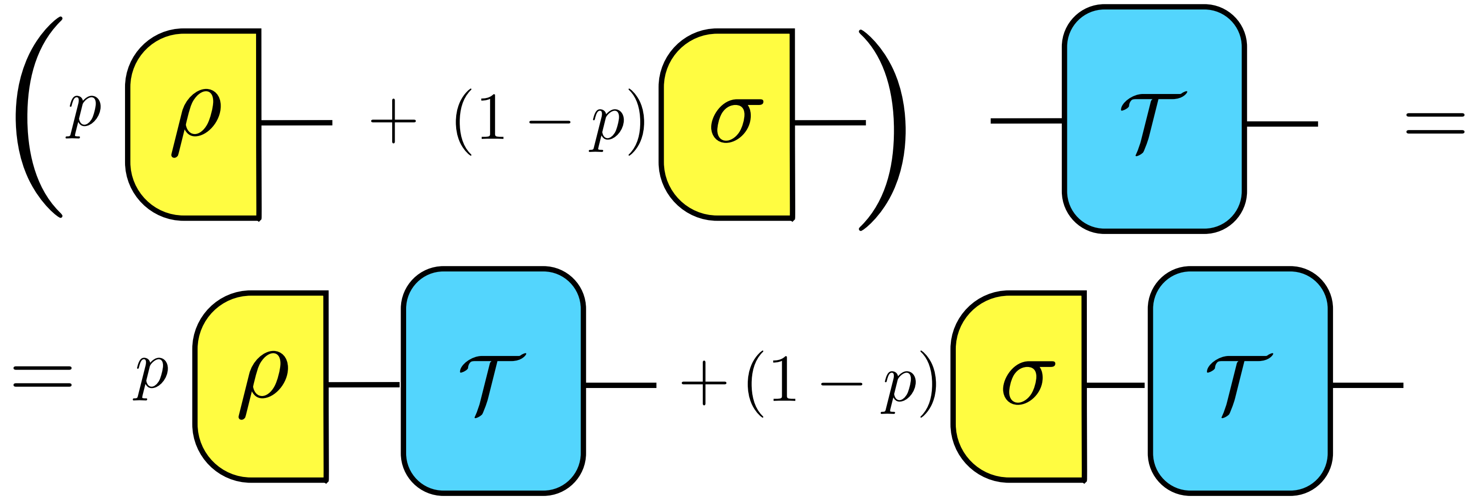
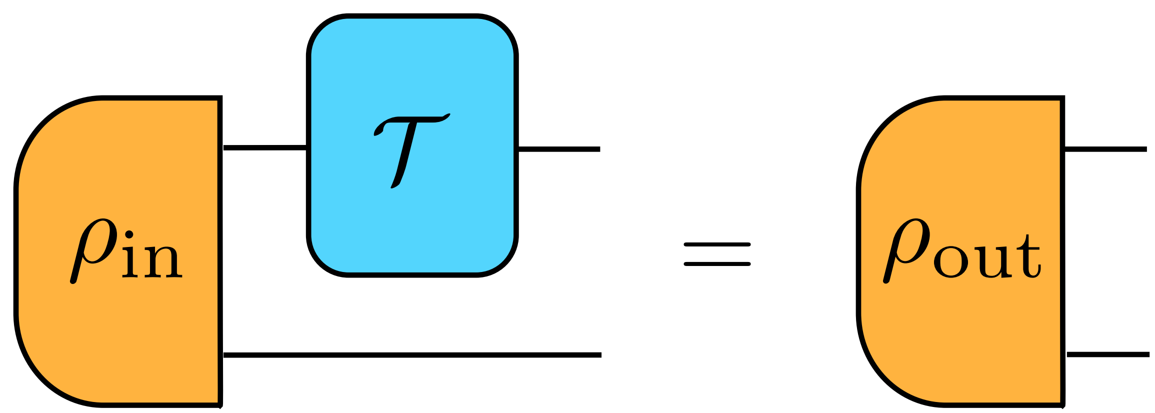


complete positivity



Admissibility conditions (deterministic)

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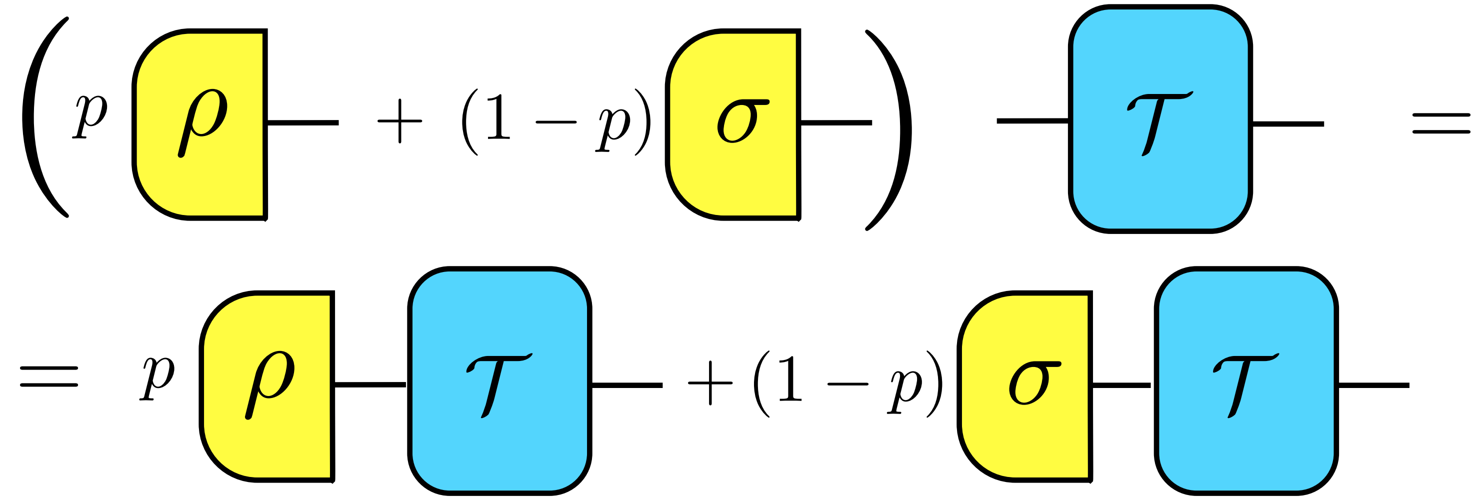

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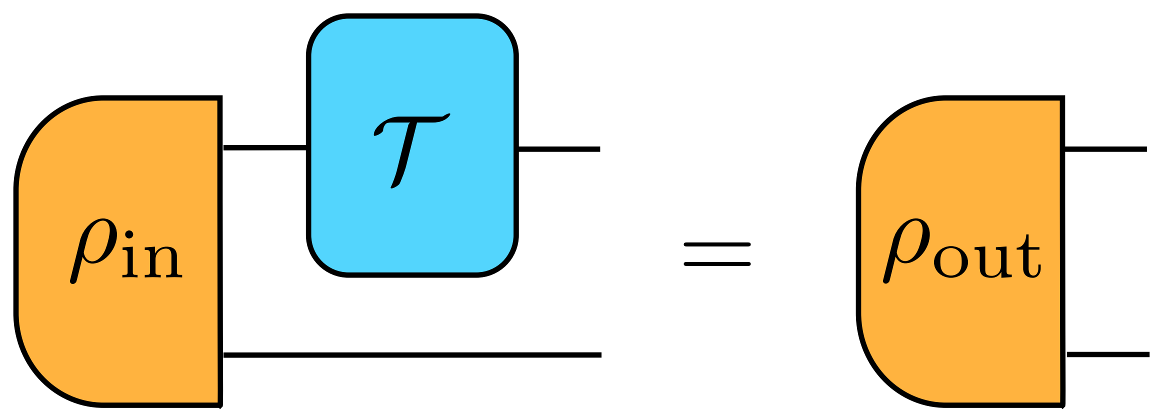
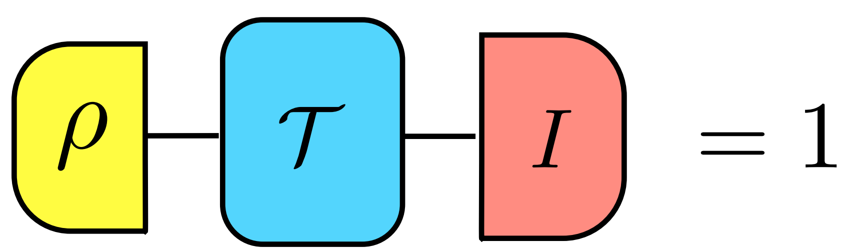
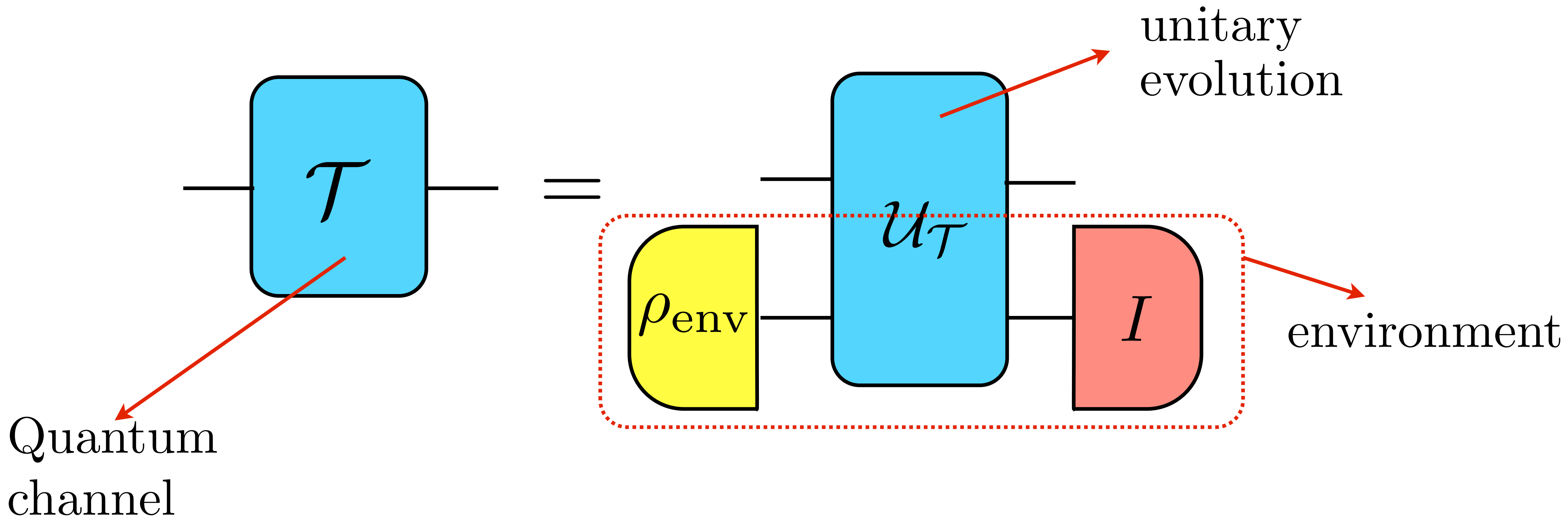


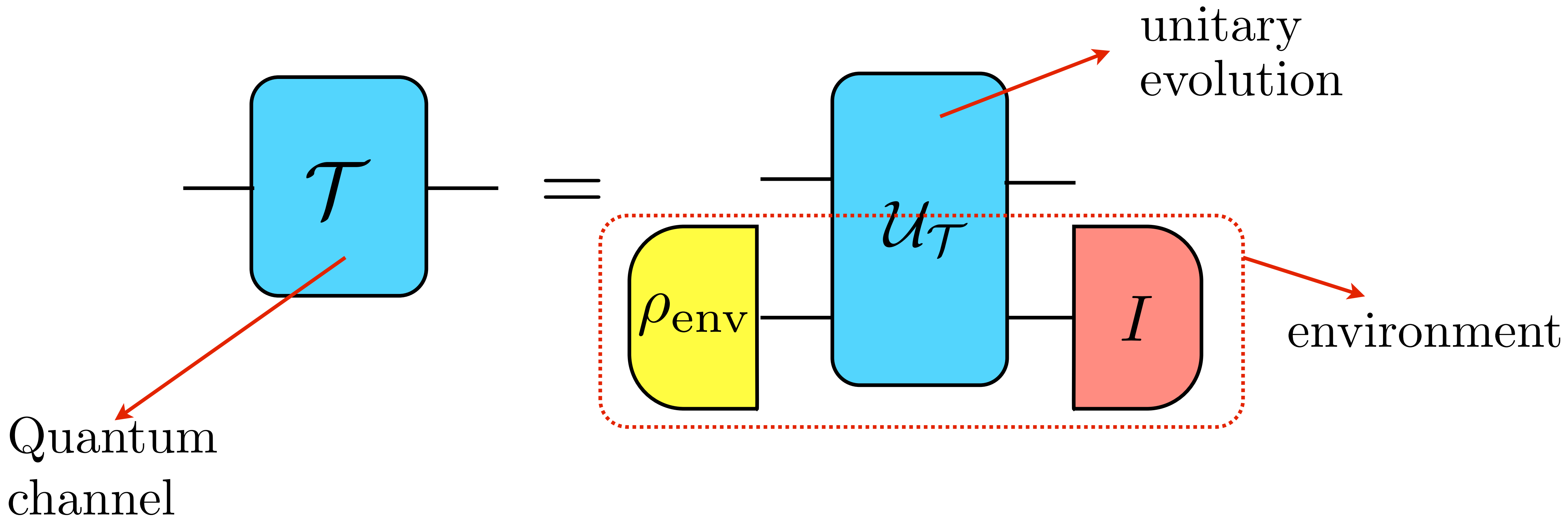
normalization



Realization theorem



Realization theorem

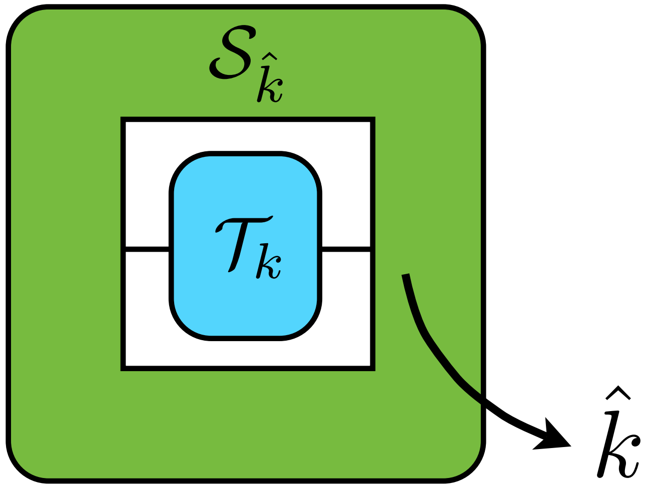


Deeper understanding of the probabilistic structure of Quantum Mechanics

Transformations as carriers of information

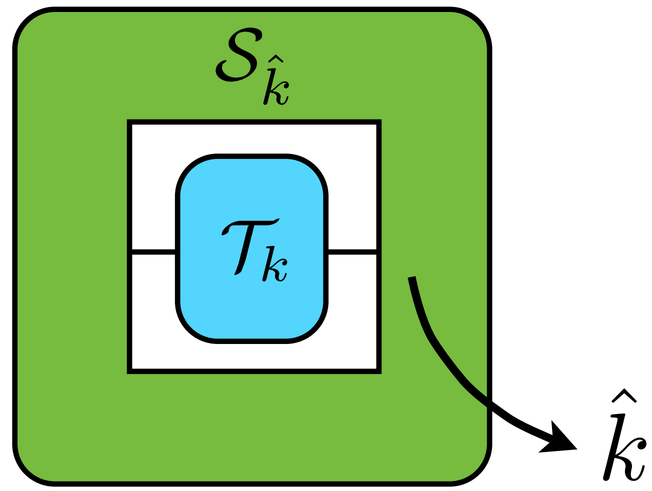
Transformations as carriers of information

channel
estimation

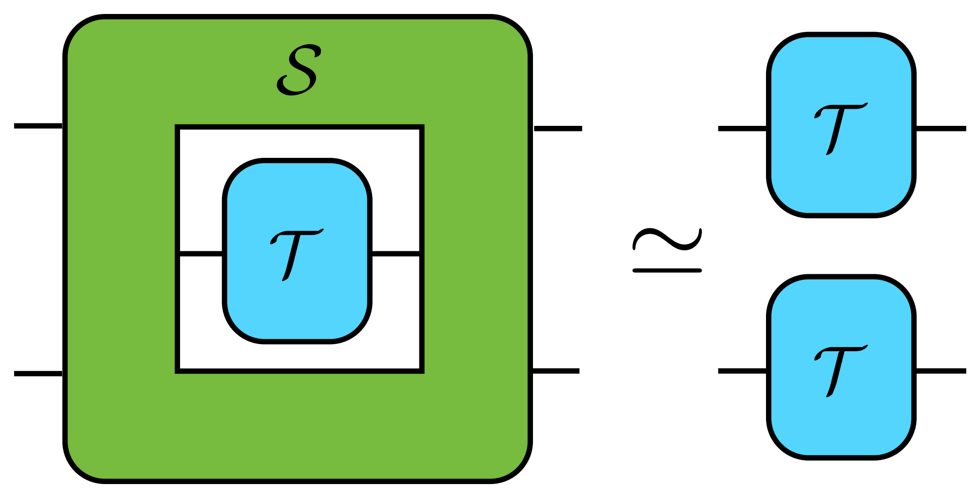


Transformations as carriers of information

channel estimation



cloning



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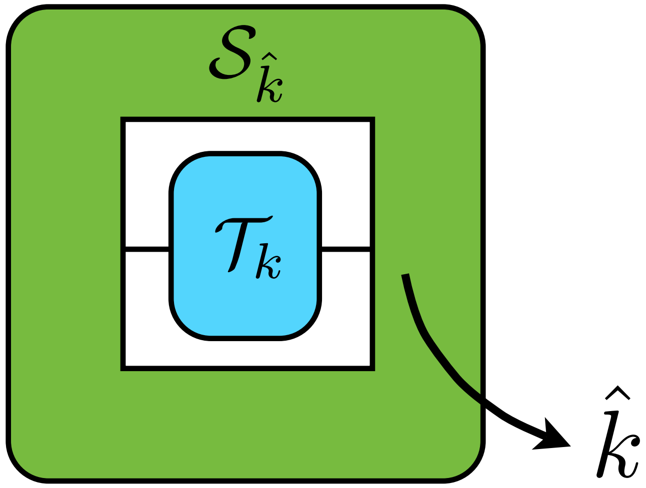

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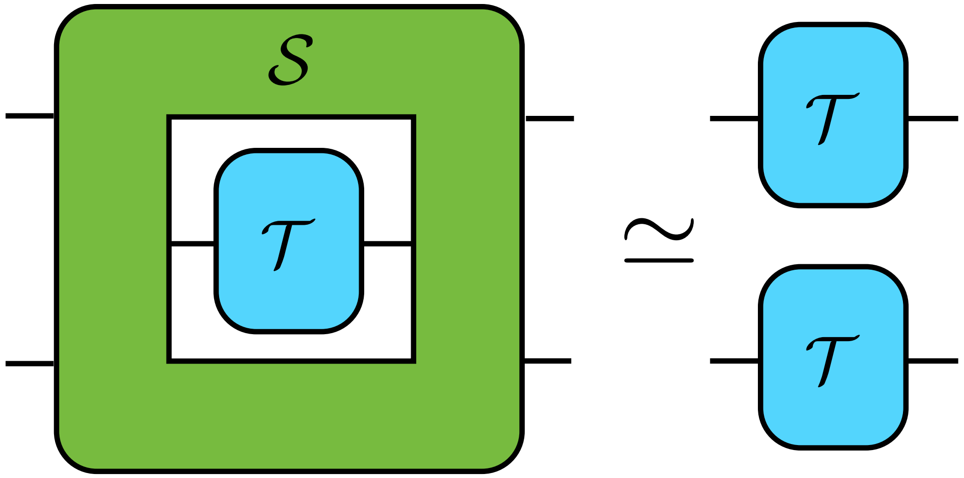
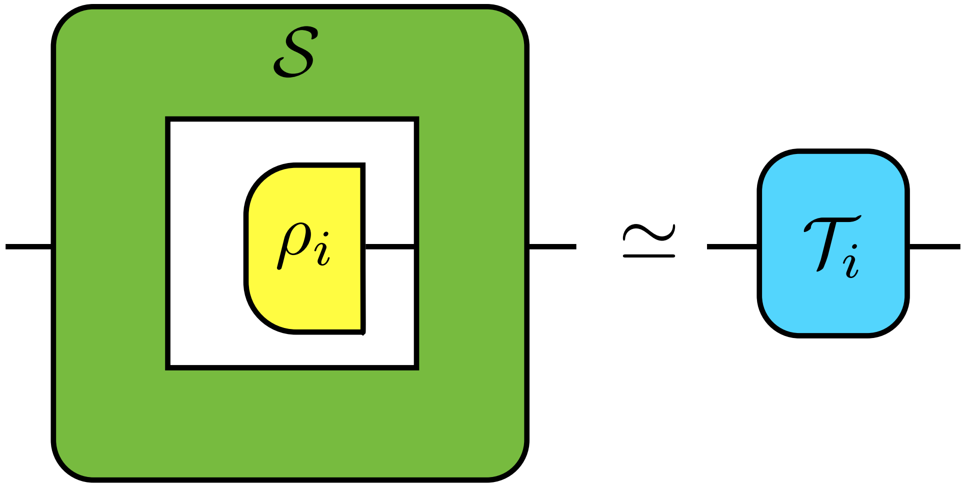
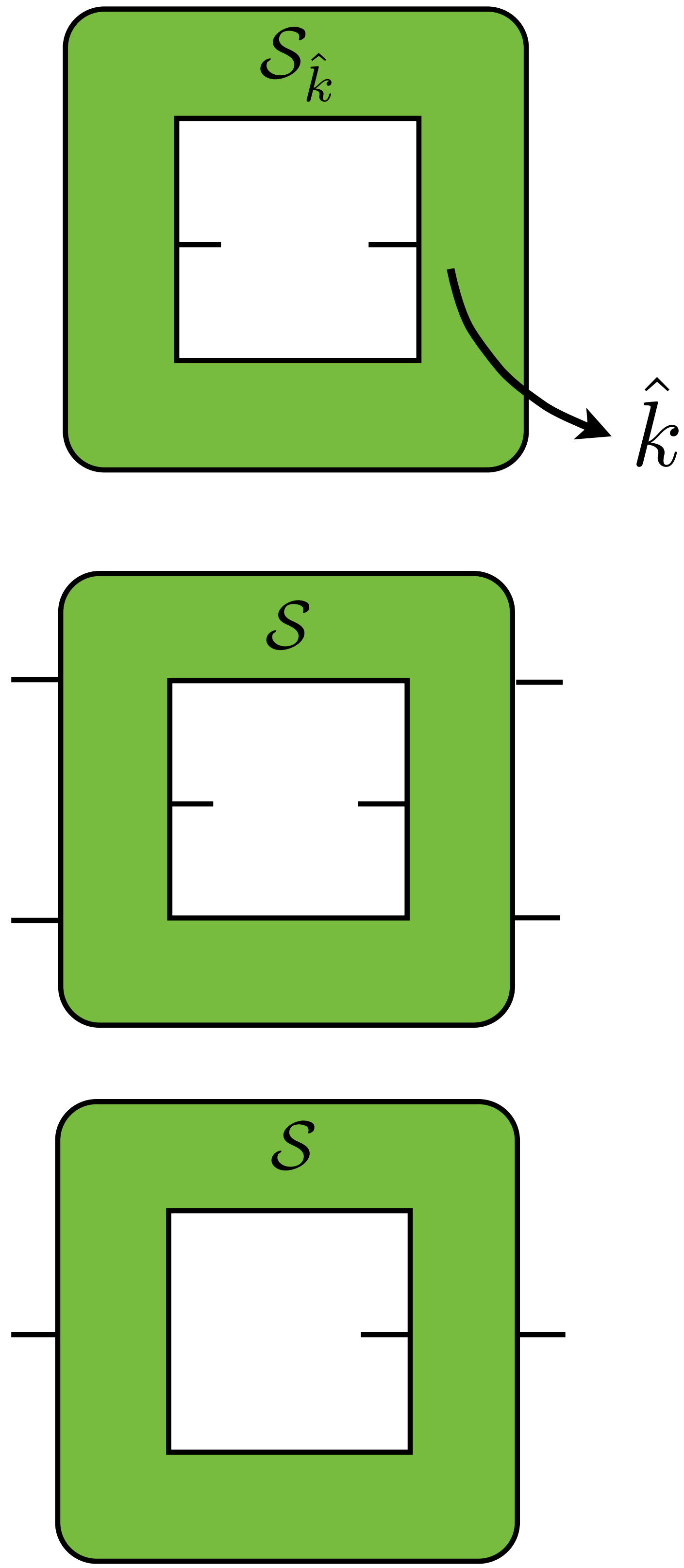


programmable channels



Transformations as carriers of information

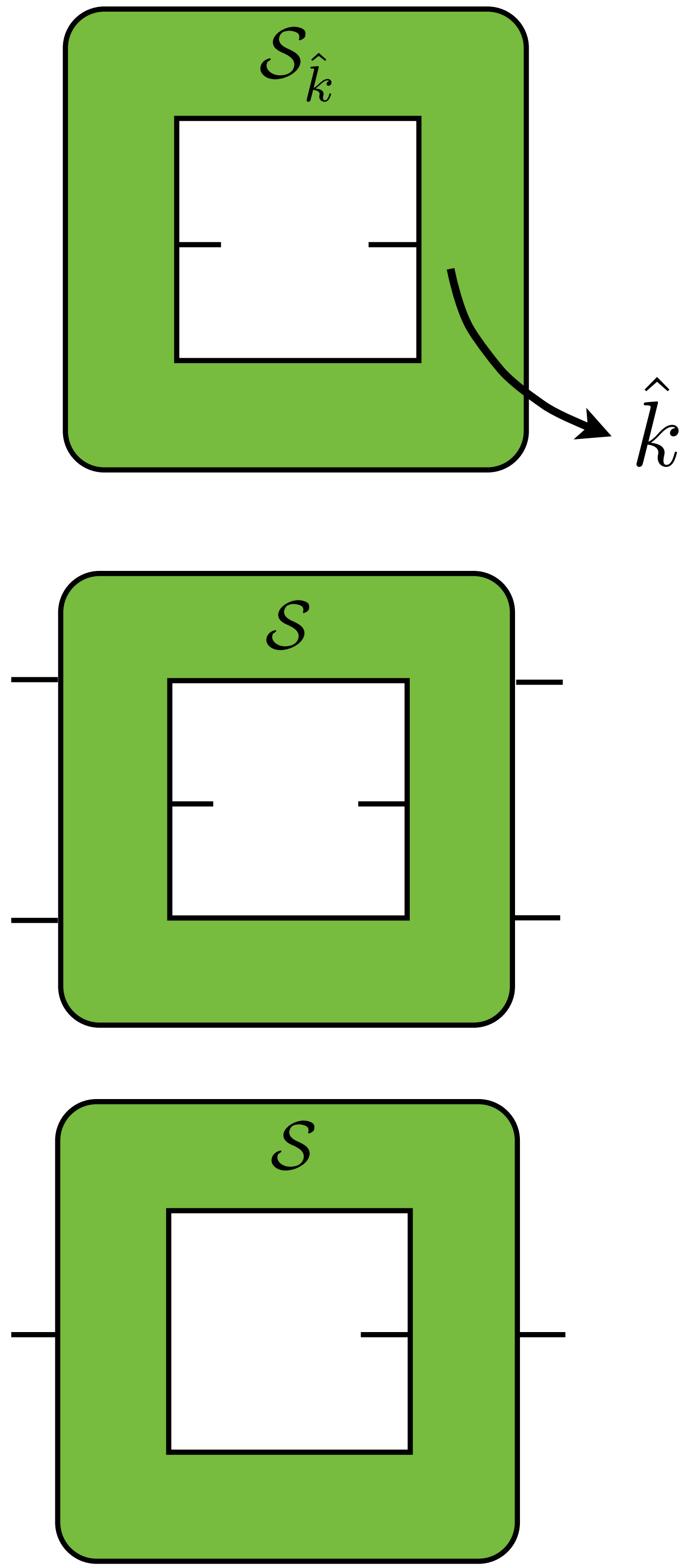


quantum channel → classical outcome

quantum channel → quantum channel

quantum state → quantum channel

Transformations as carriers of information



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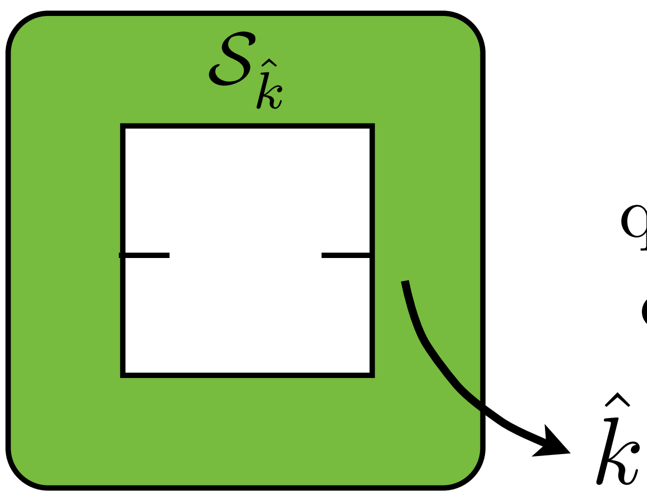
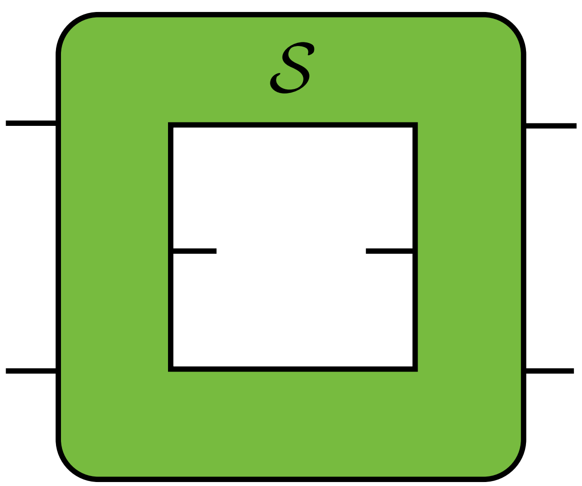
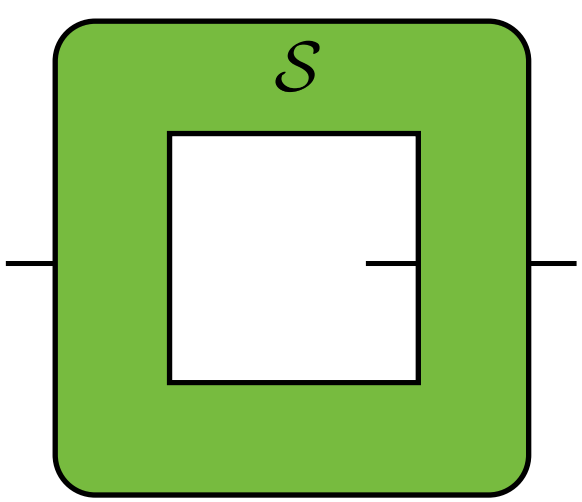


quantum channel → classical outcome



quantum channel → quantum channel



quantum state → quantum channel

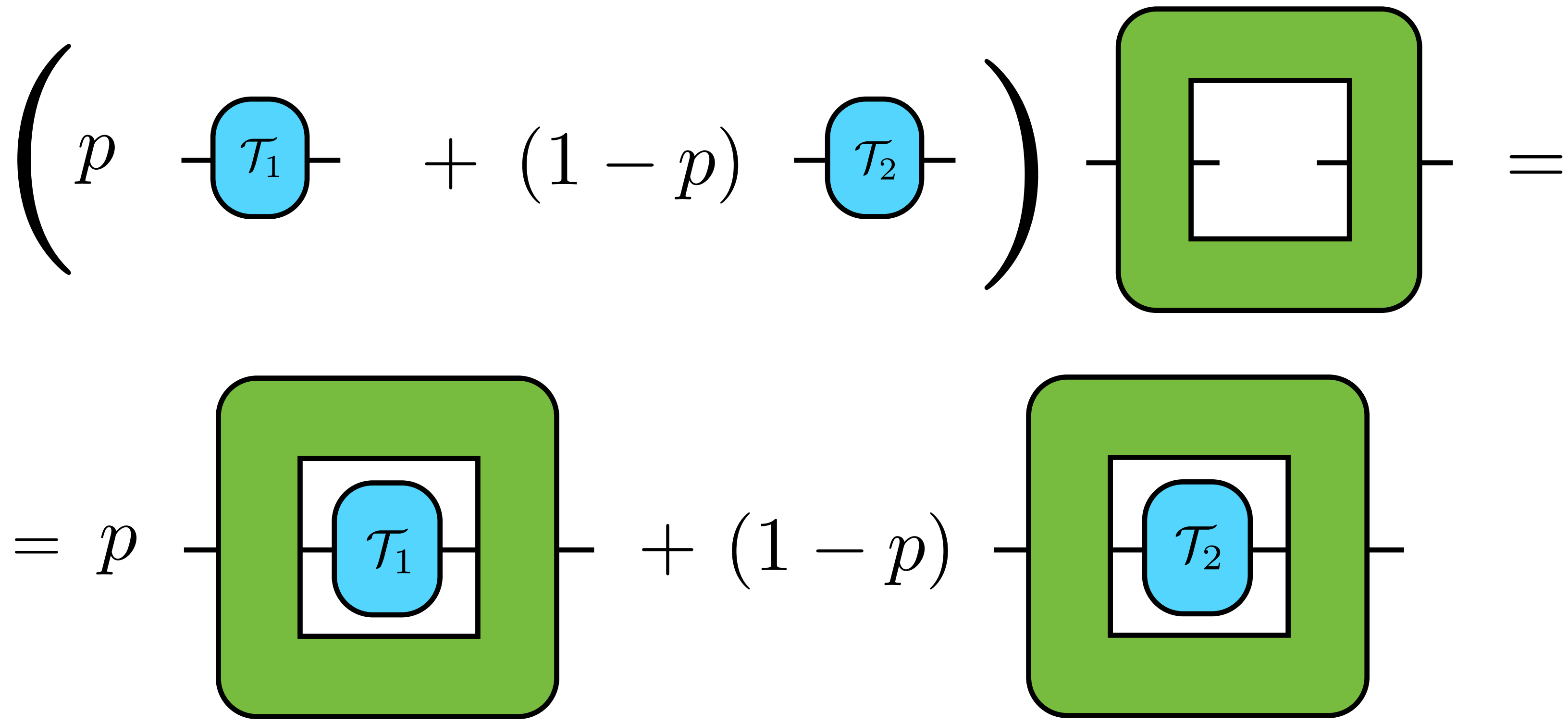
transformations of transformations:

Quantum "Supermaps"

Admissibility conditions (deterministic)

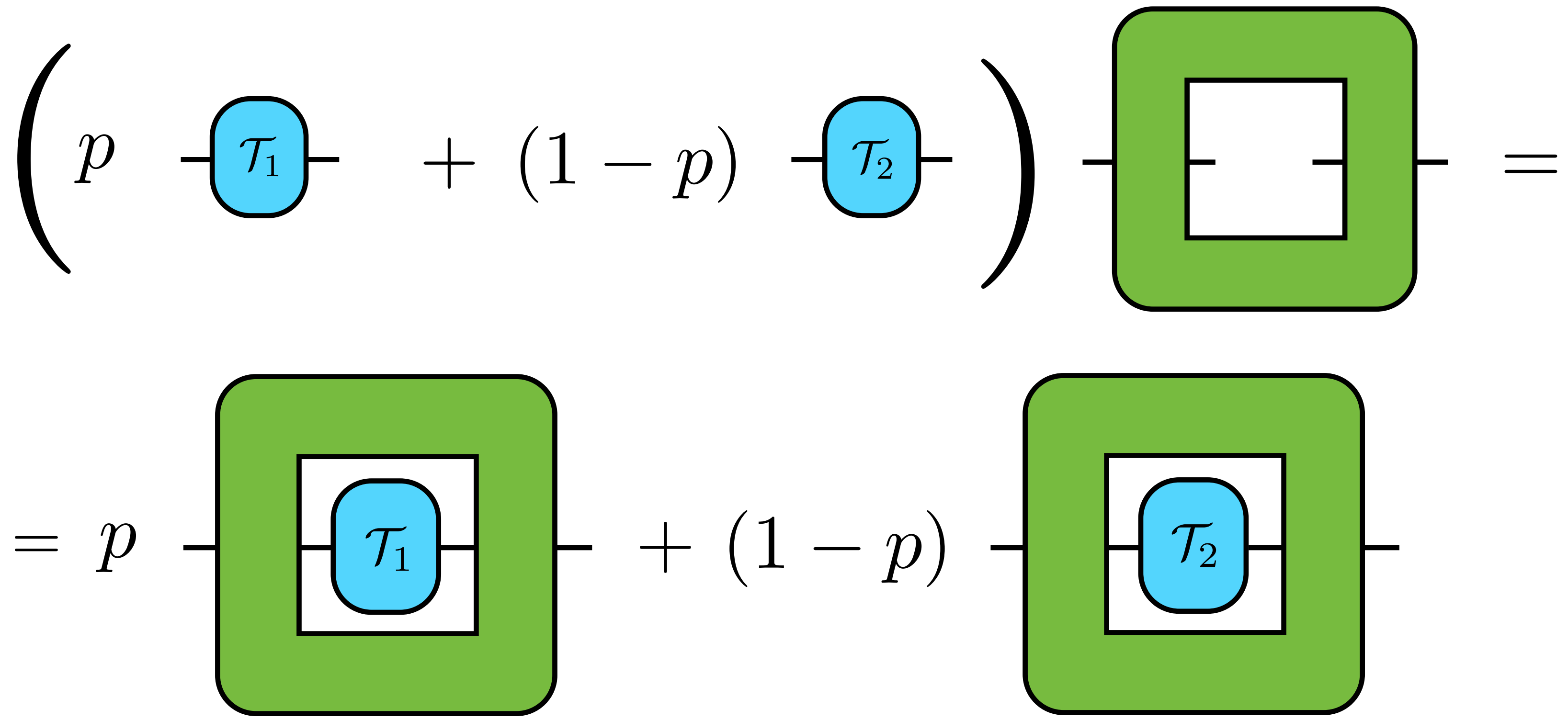
Admissibility conditions (deterministic)

linearity

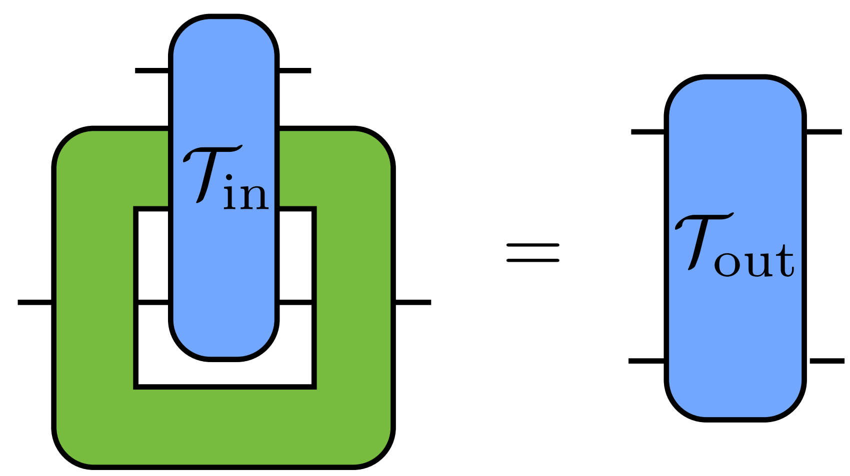


Admissibility conditions (deterministic)

linearity



complete positivity



Admissibility conditions (deterministic)

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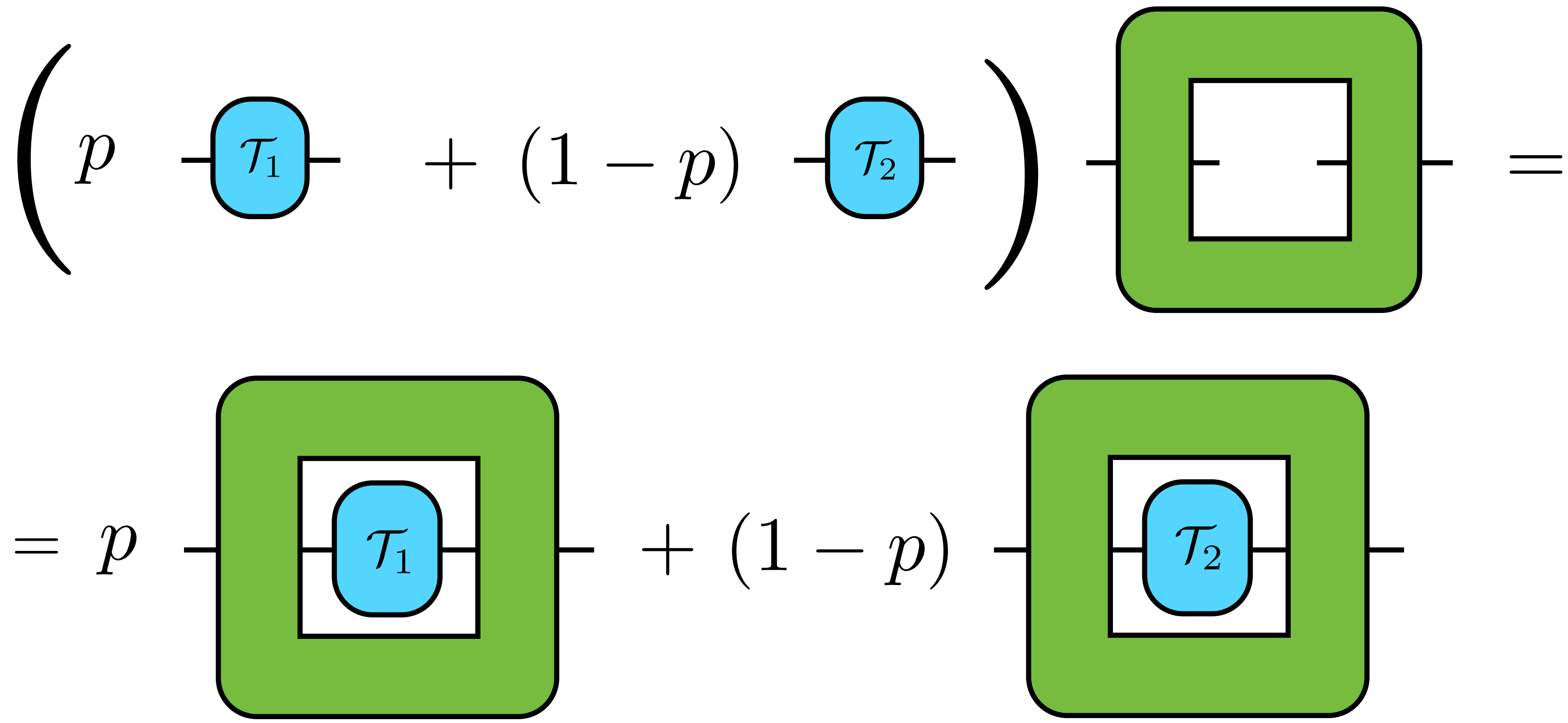

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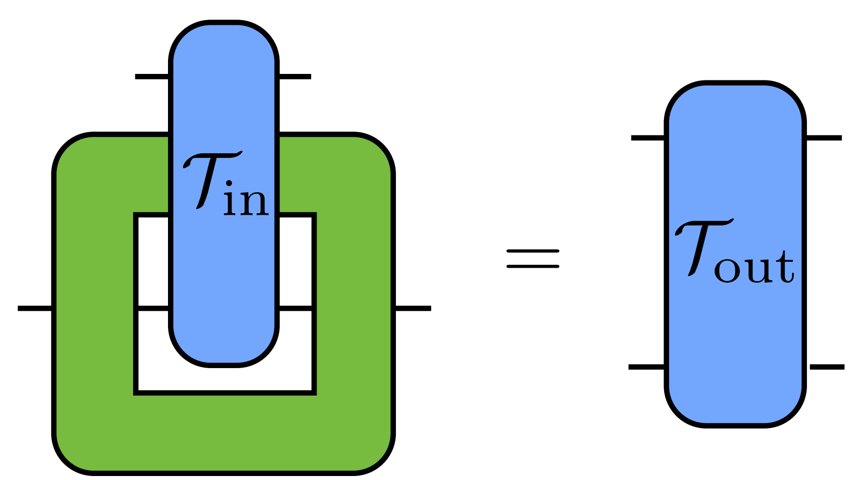
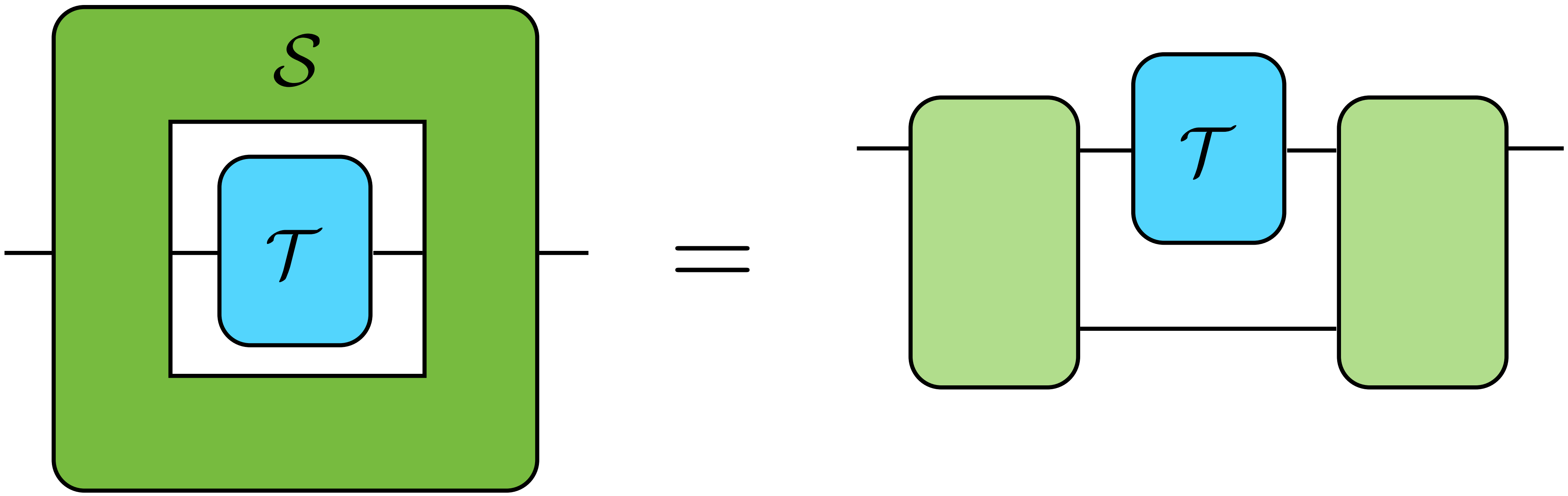


normalization

channels into channels

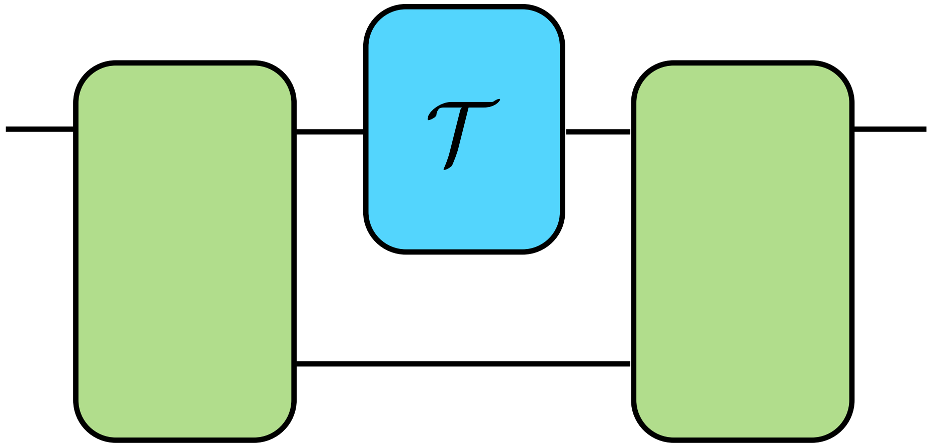
Realization theorem



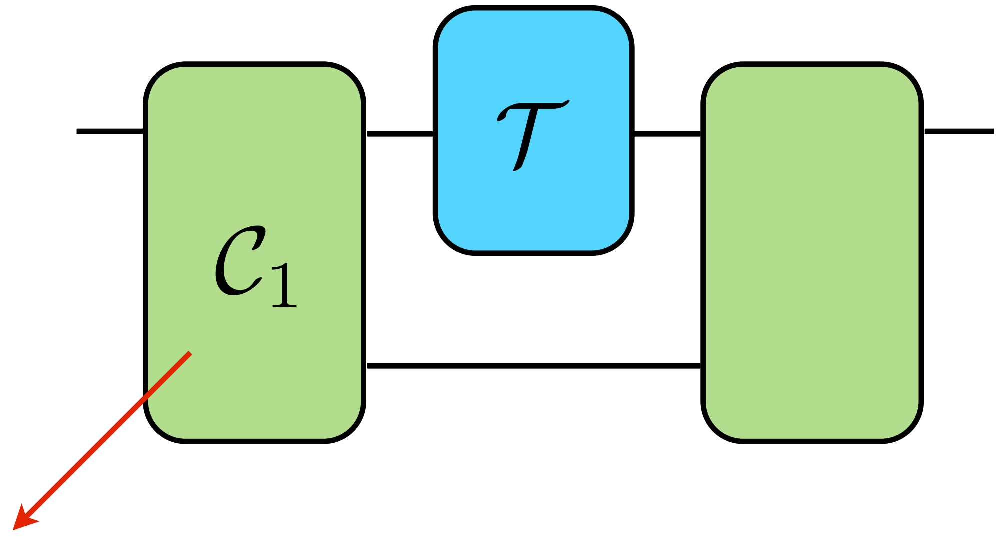
deterministic supermap

quantum circuit

Realization theorem

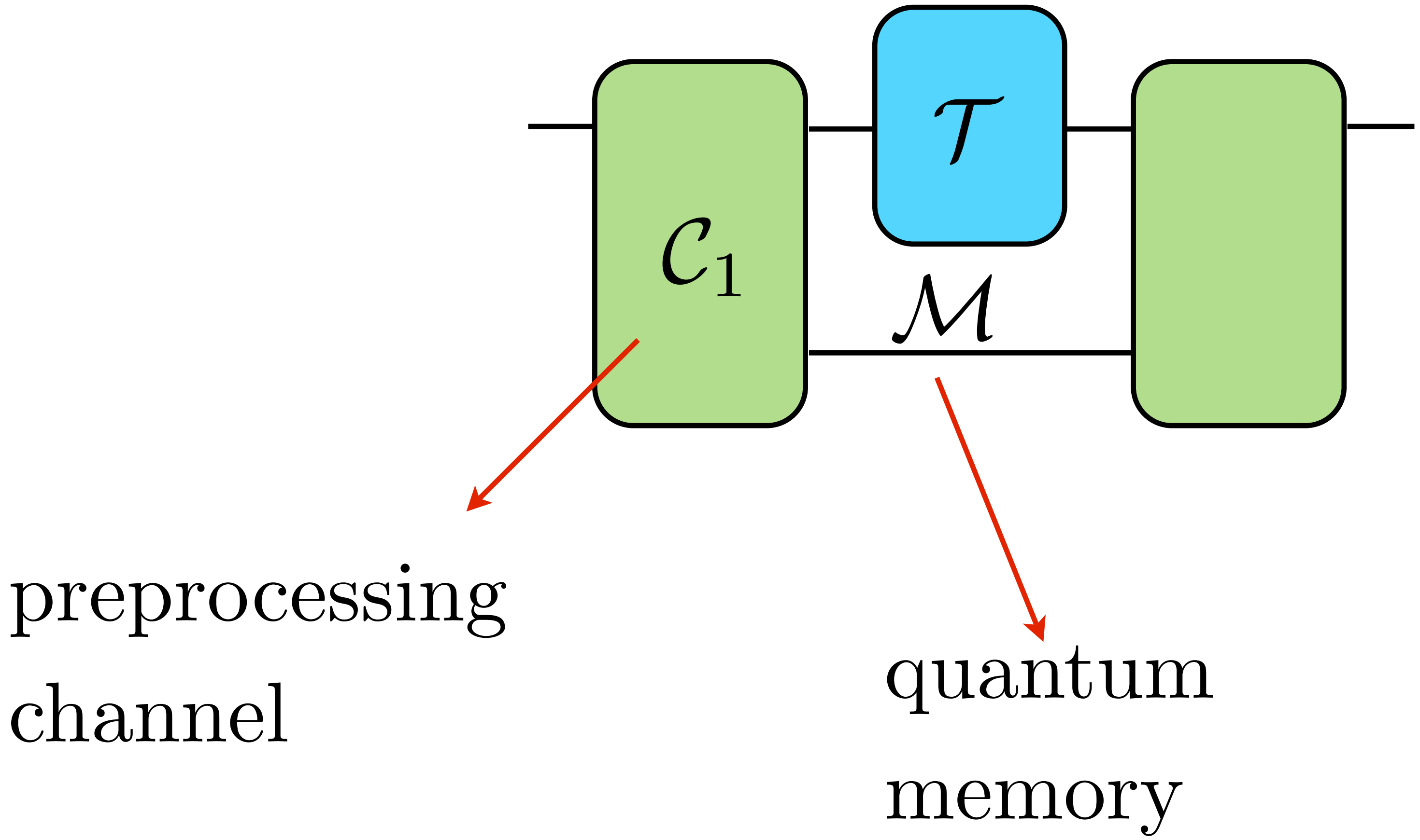


Realization theorem

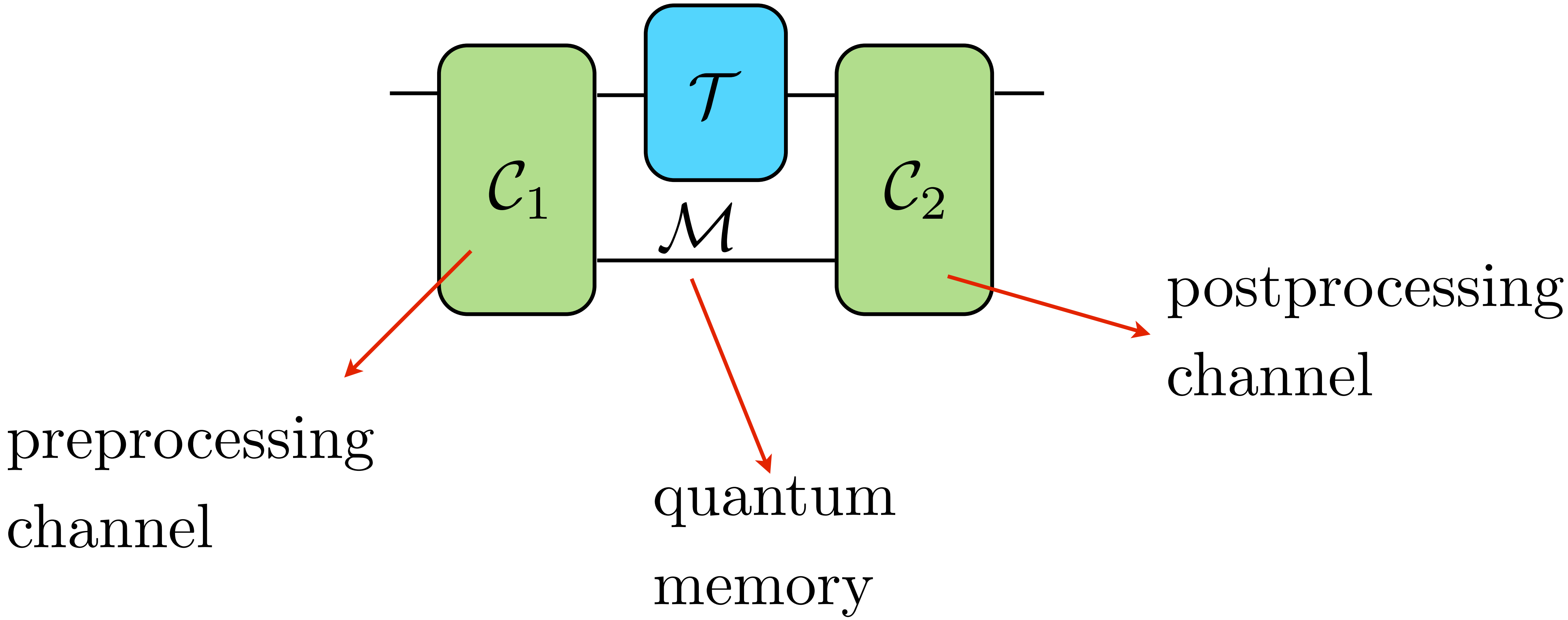


preprocessing
channel

Realization theorem

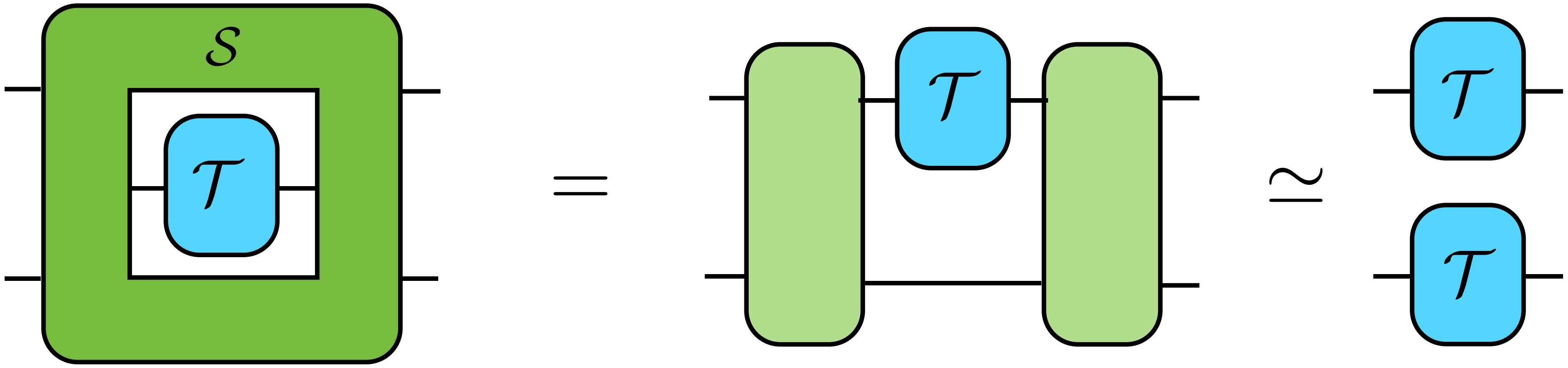


Realization theorem



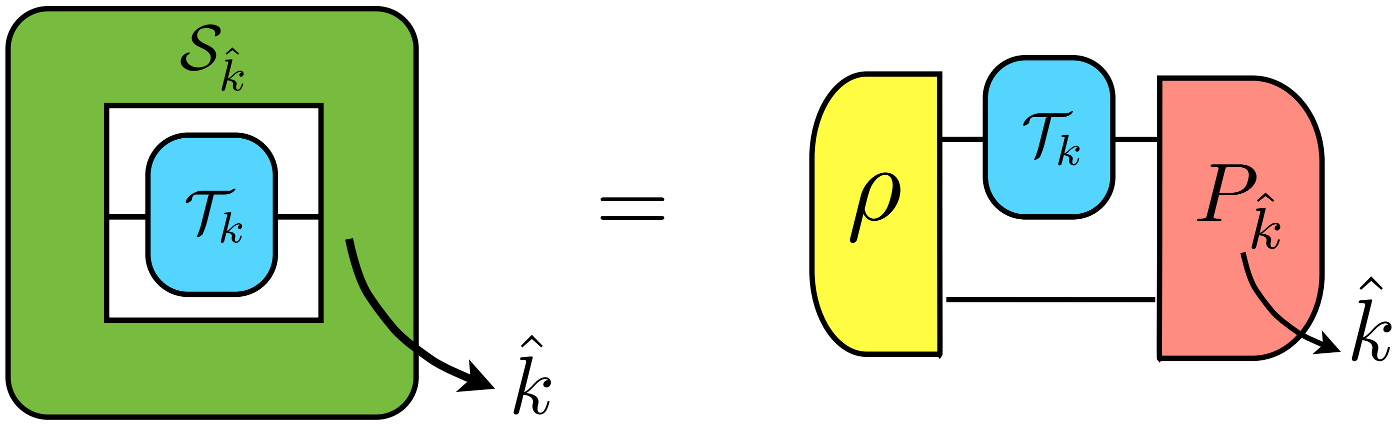
Realization theorem

cloning

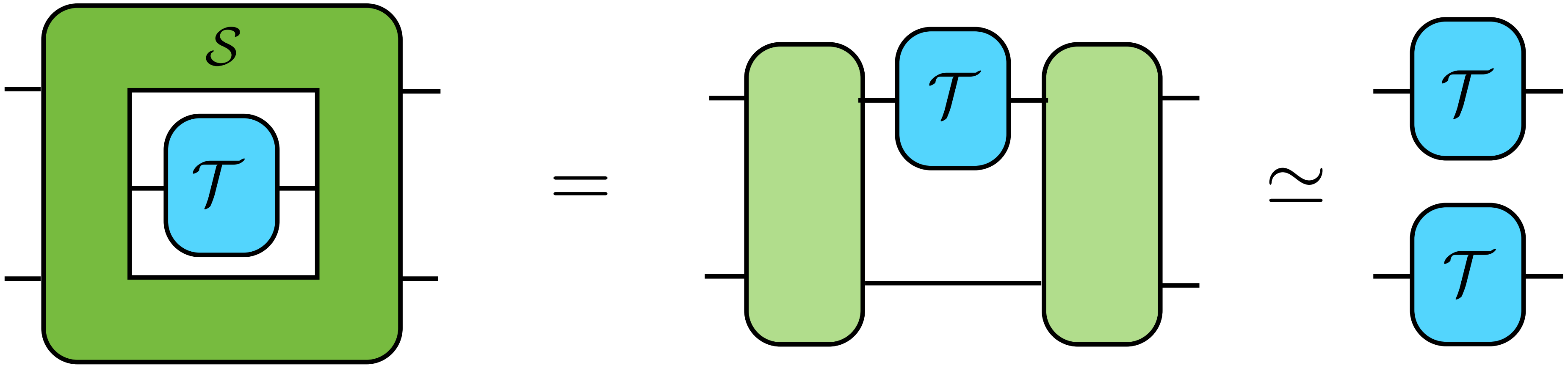


Realization theorem

channel
estimation

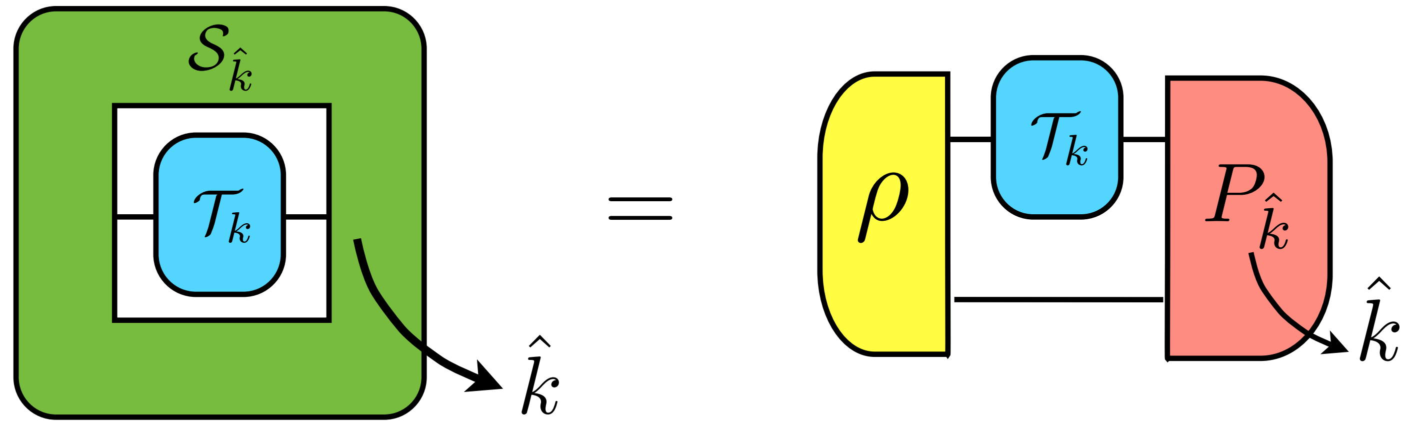


cloning



Realization theorem

channel estimation



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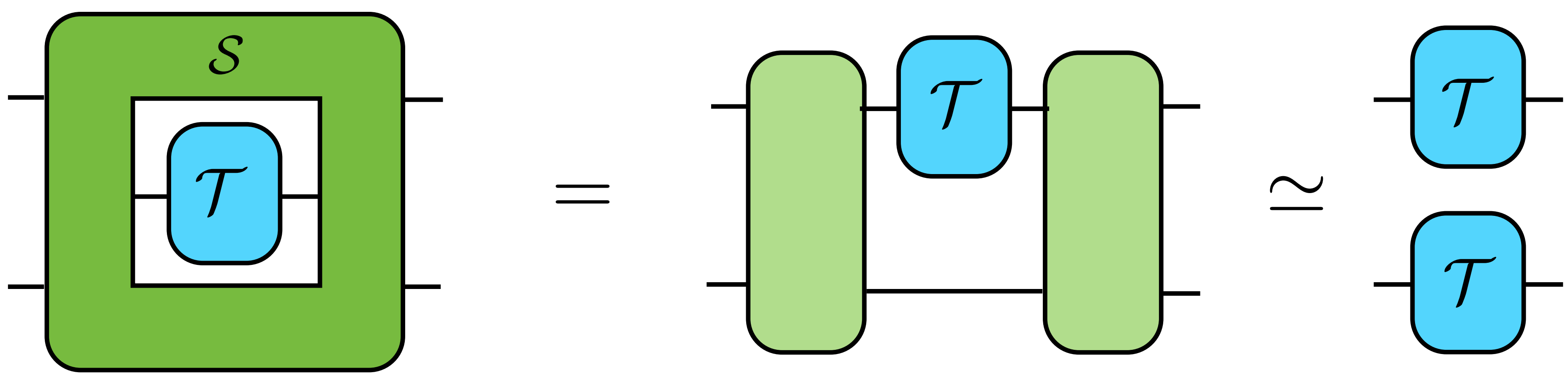

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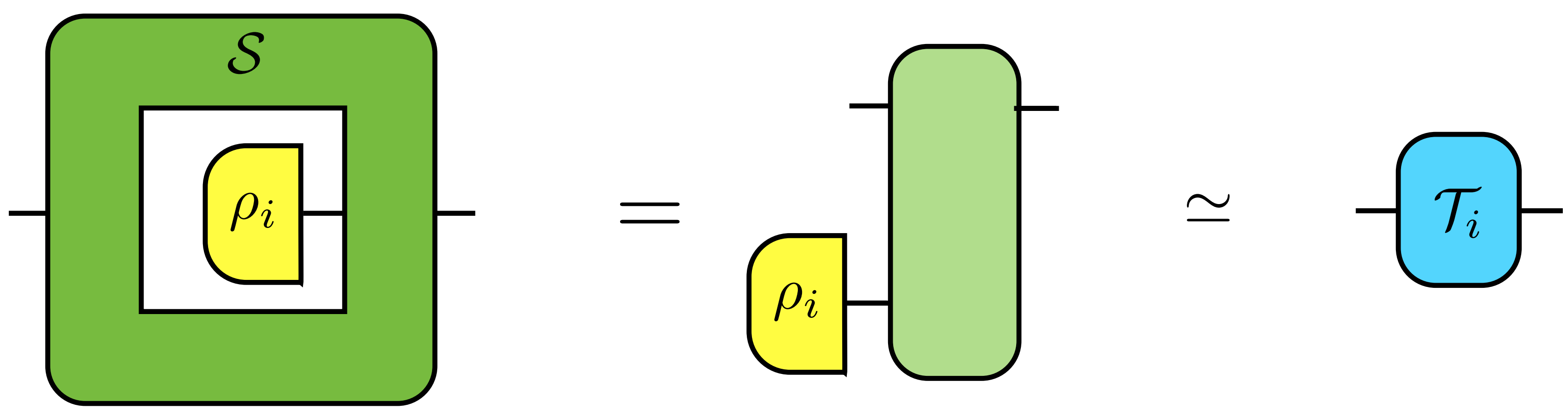
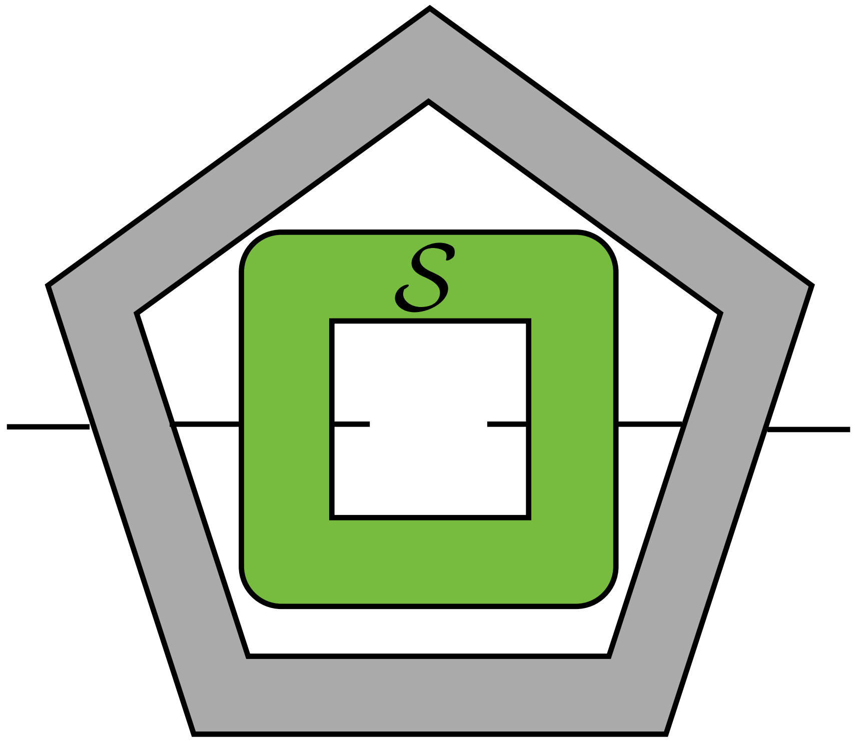
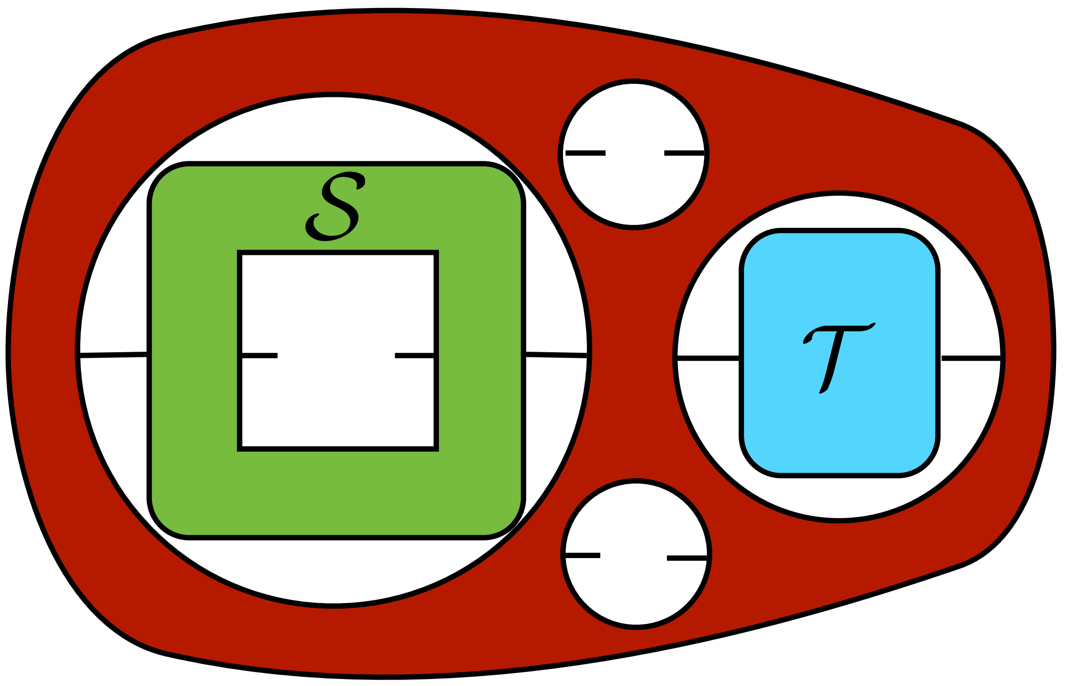


Transforming supermaps?

Transforming supermaps?

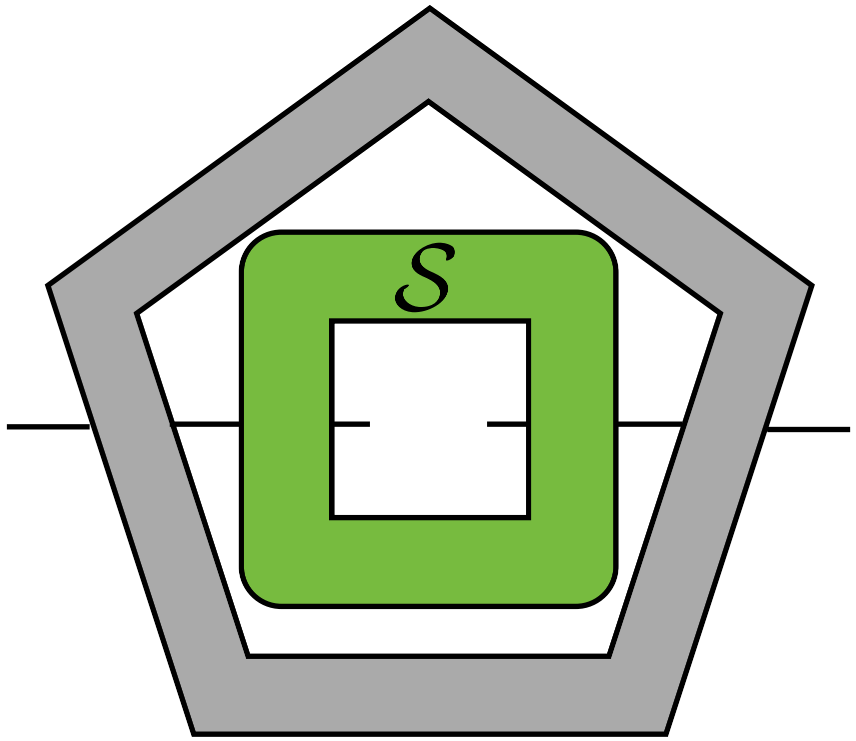
Higher Order Quantum Maps

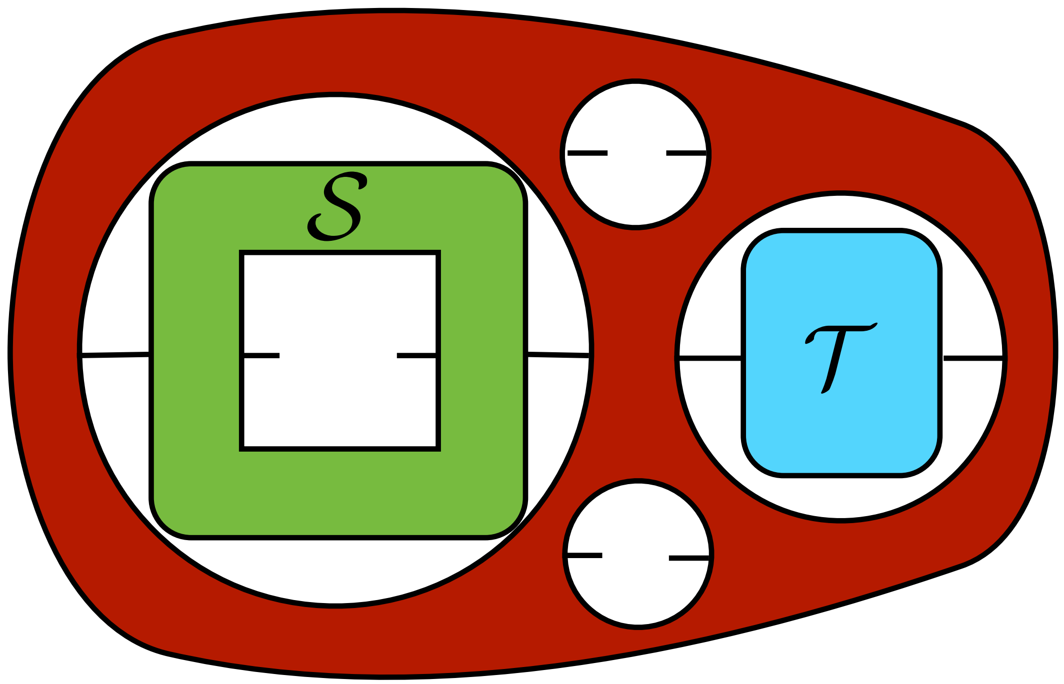




Transforming supermaps?

Higher Order Quantum Maps



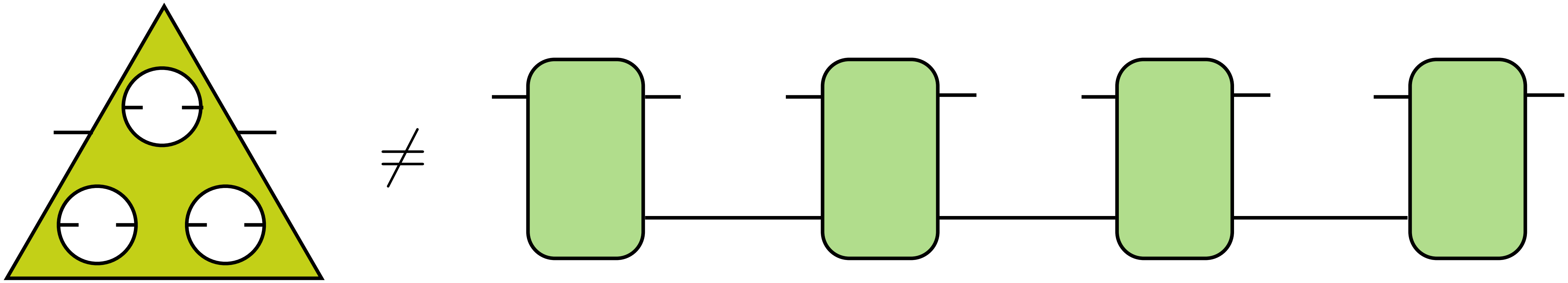


road to quantum lambda calculus...

Realization theorem?

Realization theorem?

Not every higher order map is realizable as a quantum circuit



Realization theorem?

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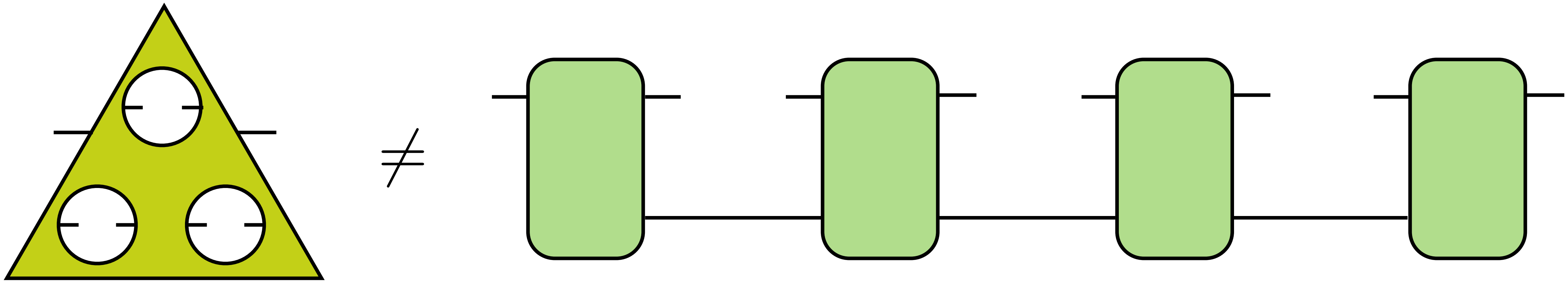
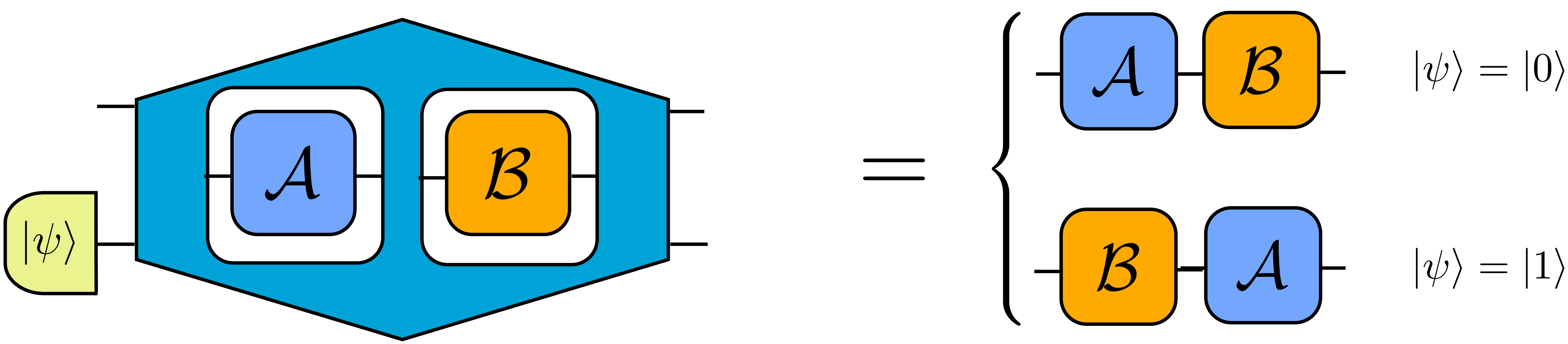
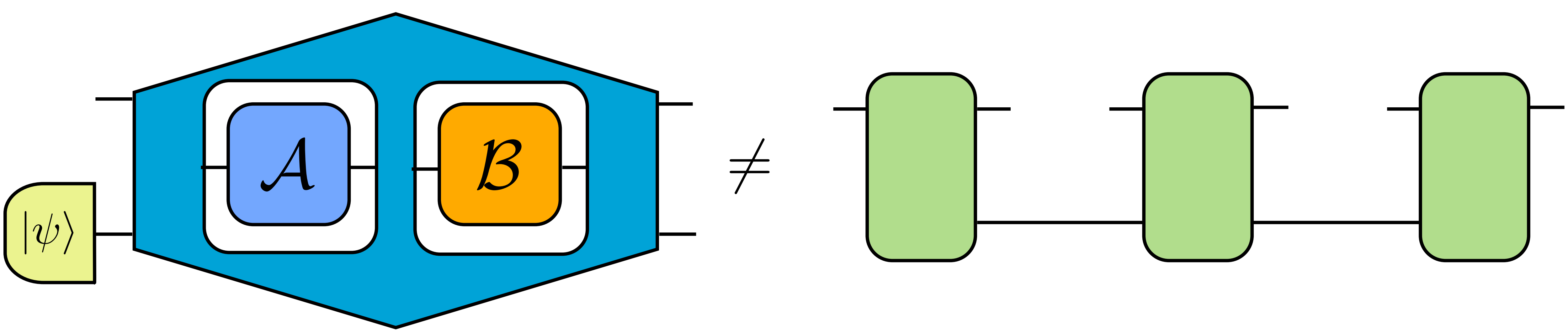


Example: Quantum Switch

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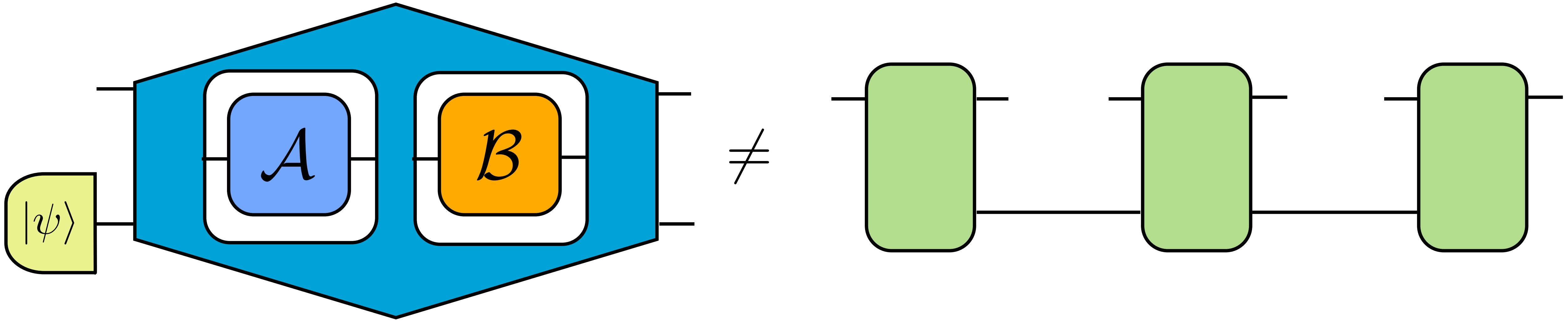
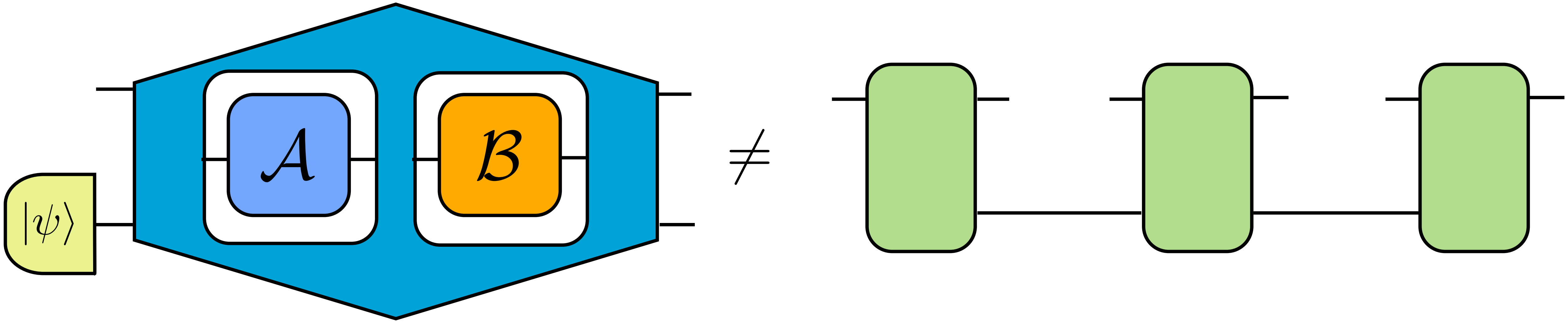
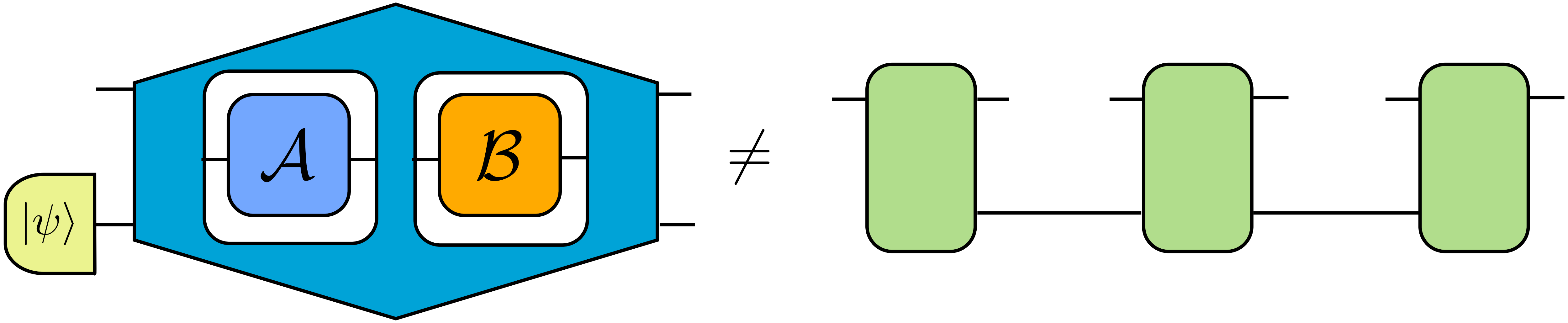


$|\psi\rangle$
$\mathcal{A}$
$\mathcal{B}$
$\neq$



Quantum circuit = probabilistic structure + causal order



Quantum circuit = probabilistic structure + causal order

Admissibility conditions $\Rightarrow$ probabilistic structure

Admissibility conditions $\not\Rightarrow$ causal order

Many open problems

Many open problems

Application?

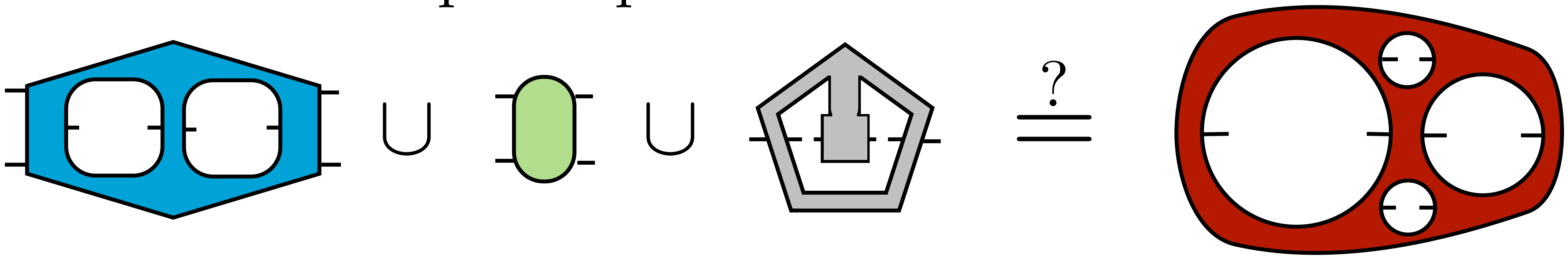
Discrimination of no-signalling channels, non local games...

Many open problems

Application?

Discrimination of no-signalling channels, non local games...

Universal set of supermaps?

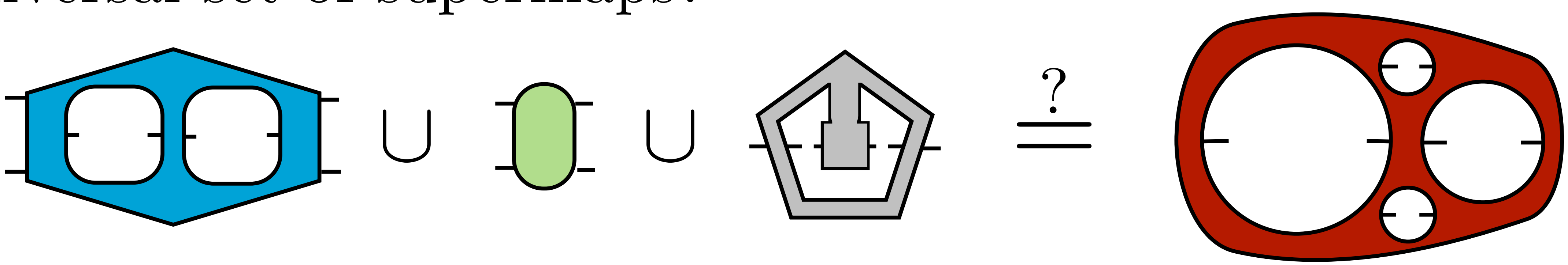


Many open problems

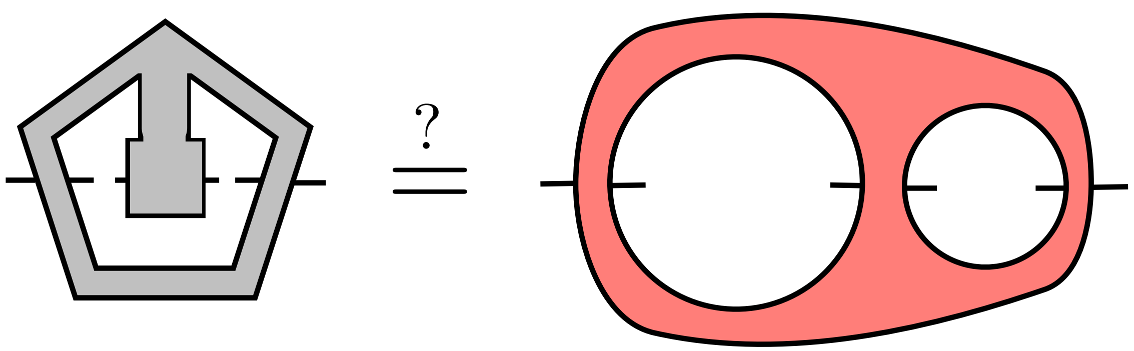
Application?

Discrimination of no-signalling channels, non local games...

Universal set of supermaps?



Equivalence of supermaps?



Many open problems

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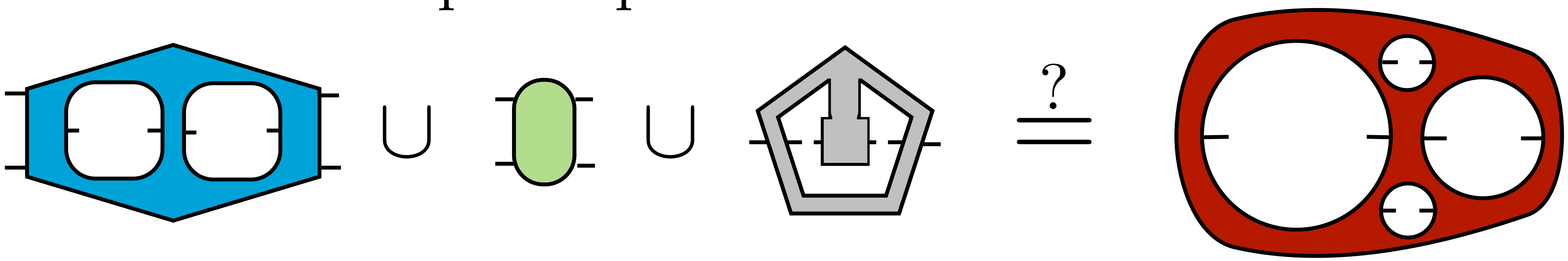
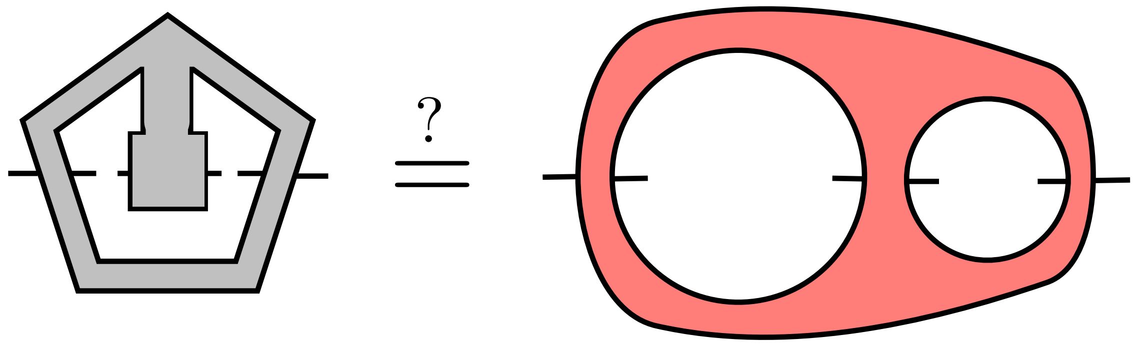
Universal set of supermaps?

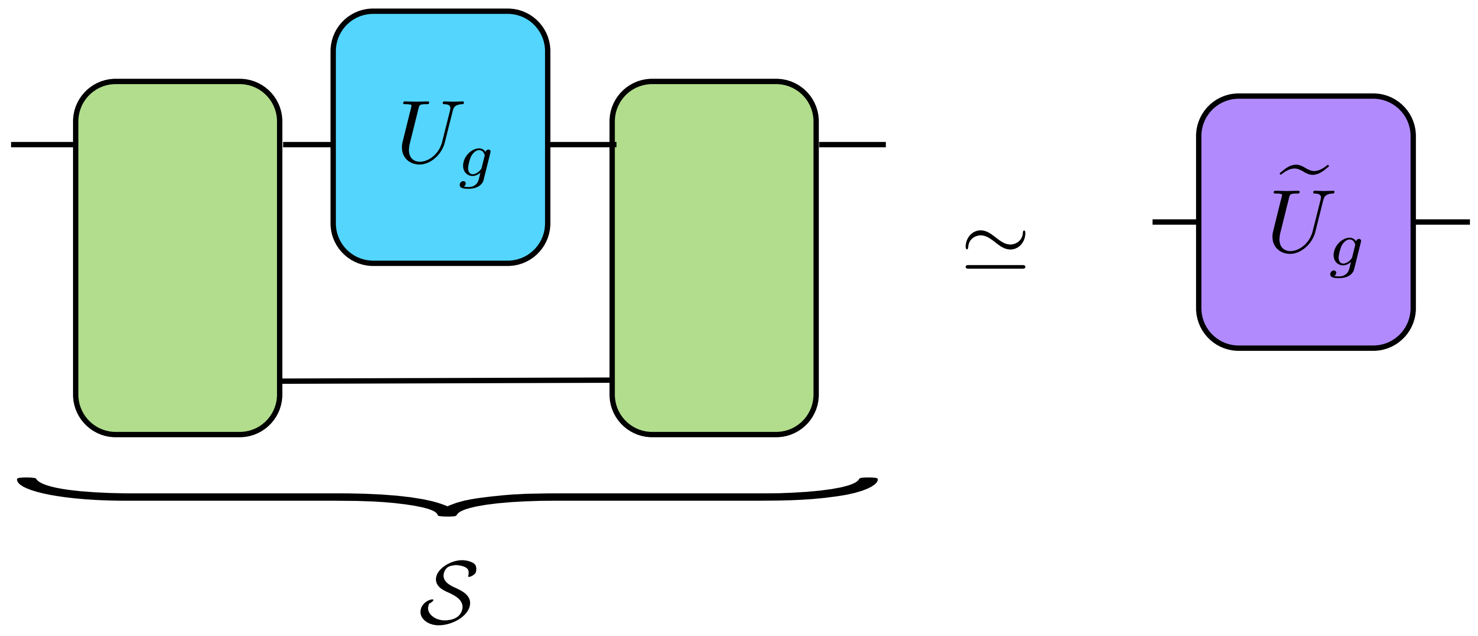


Equivalence of supermaps?



Physically realizable supermaps?

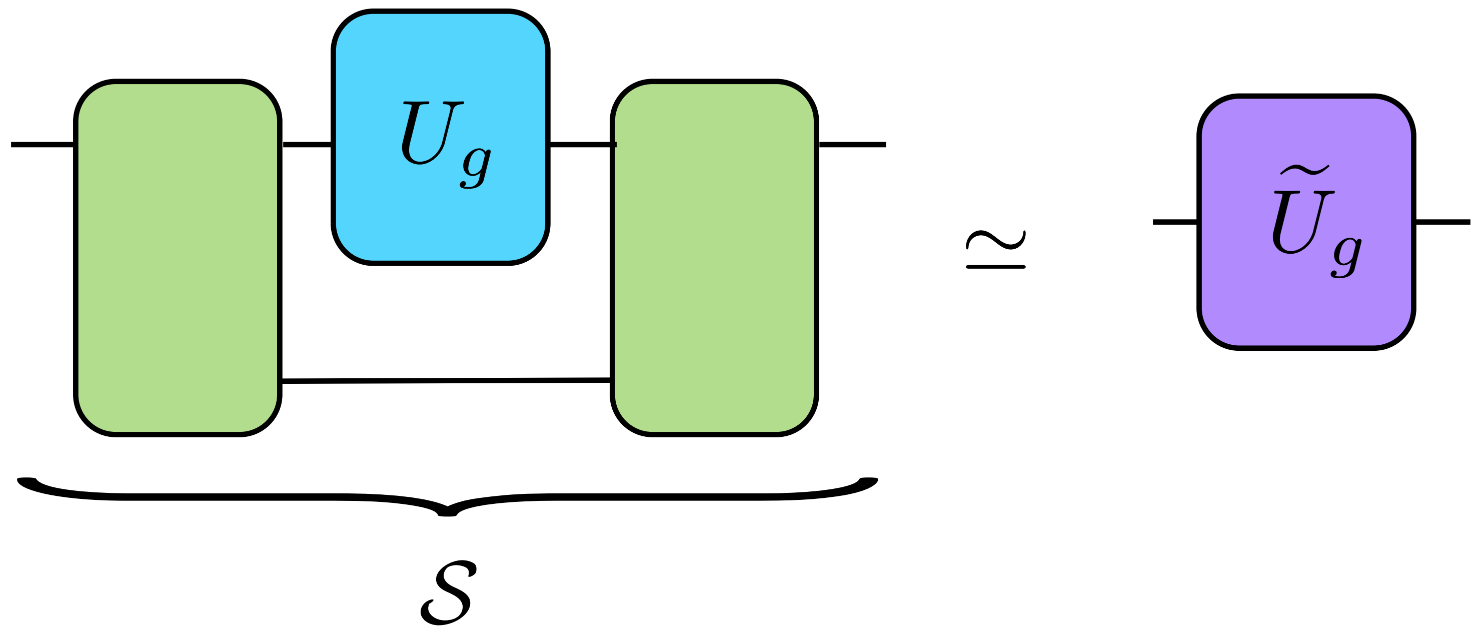
Processing unitary channels



U_g, $\widetilde{U}_g$ two different unitary representations of the group G

Which is the $\mathcal{S}$ that best achieves this task?

Processing unitary channels



$U_g\,,\ \widetilde{U}_g$ two different unitary representations of the group G

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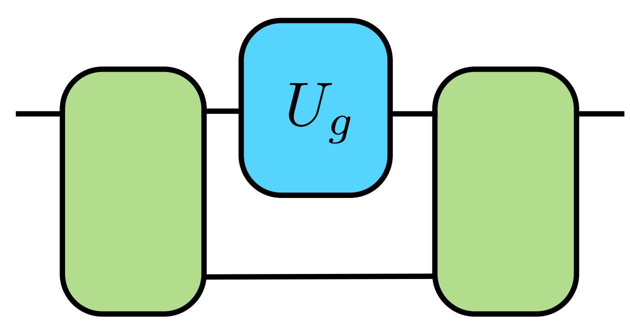
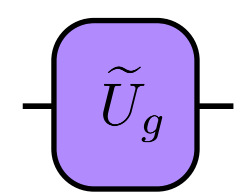
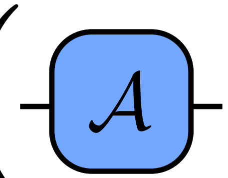
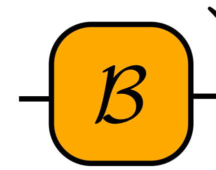
criterion: $F = \int \mathrm{d}g\ \mathcal{F}\Bigg($ 

$,$ 

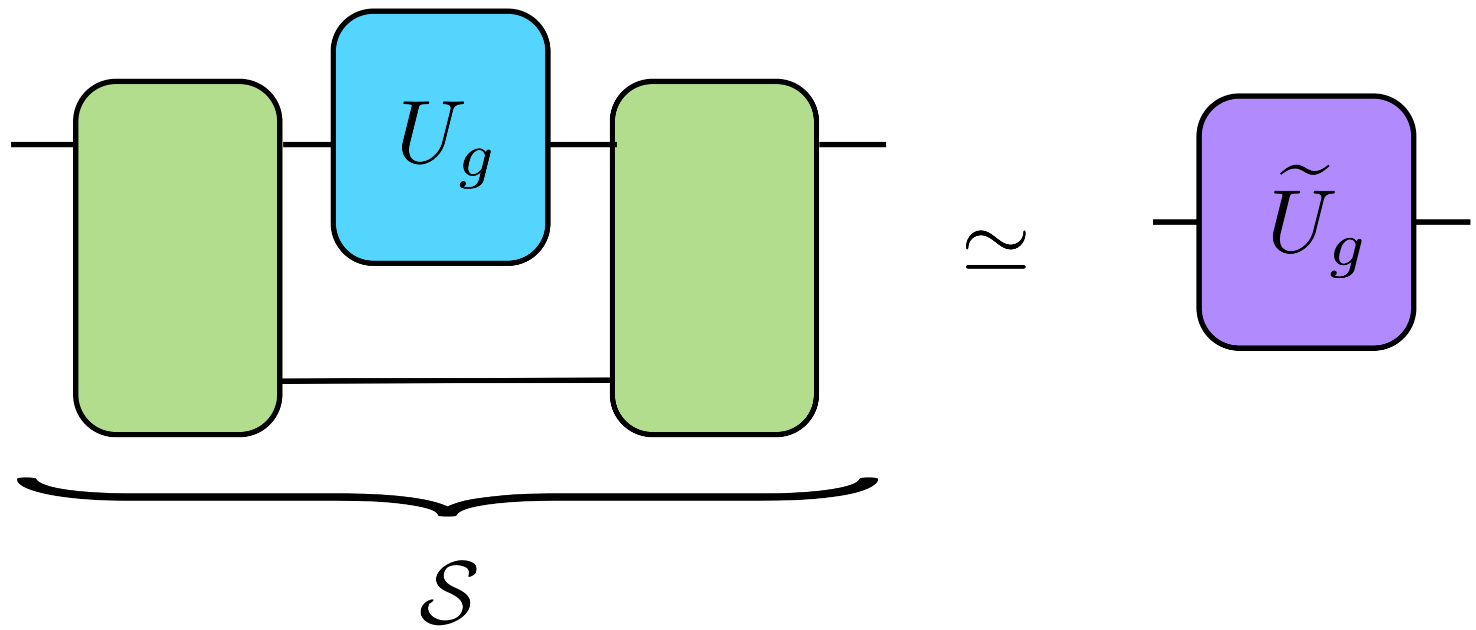
$\Bigg)$

where $\mathcal{F}\Big($ 

$,$ 

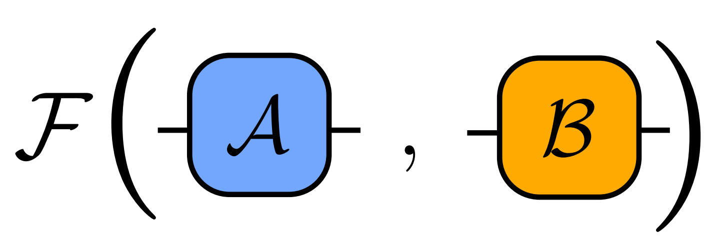
$\Big)$ is the channel fidelity

Processing unitary channels



$U_g\,,\ \widetilde{U}_g$ two different unitary representations of the group G

Which is the $\mathcal{S}$ that best achieves this task?



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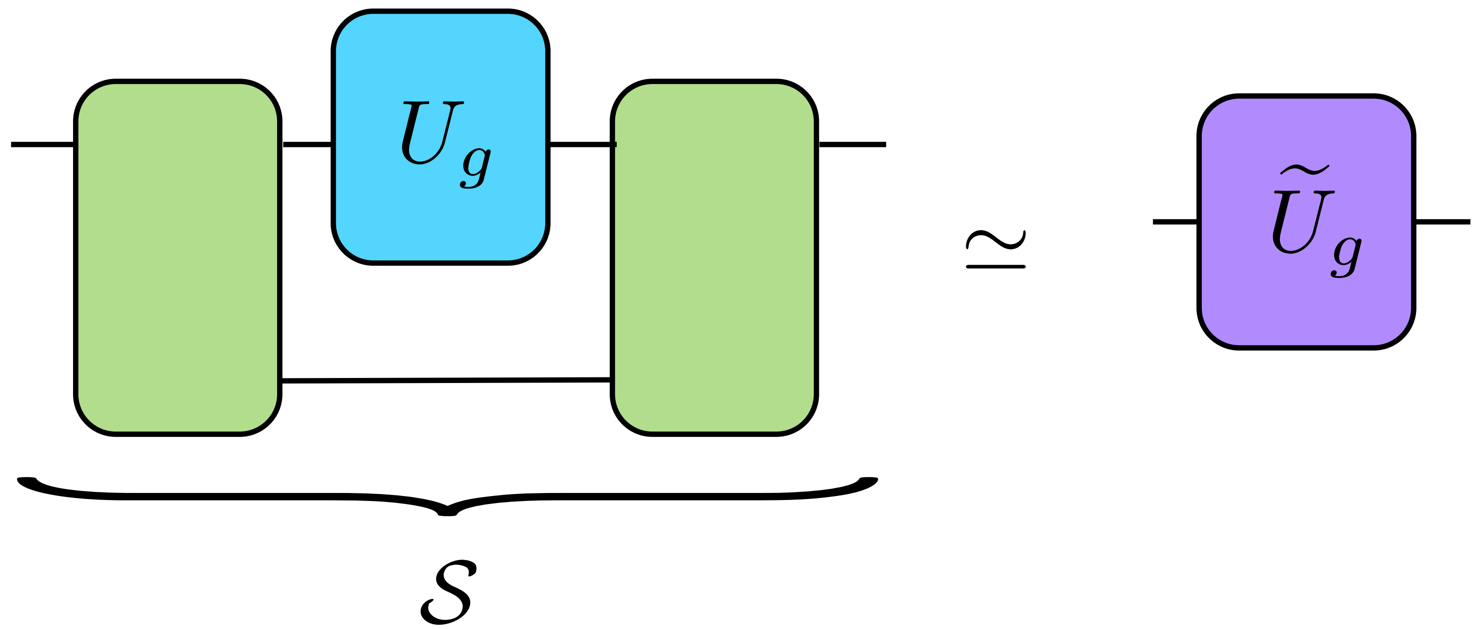

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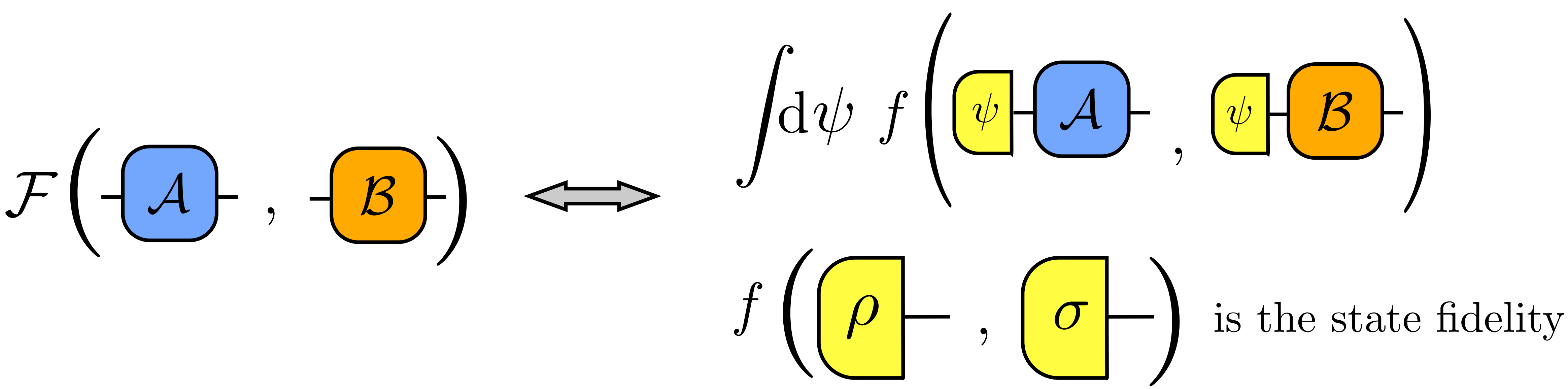

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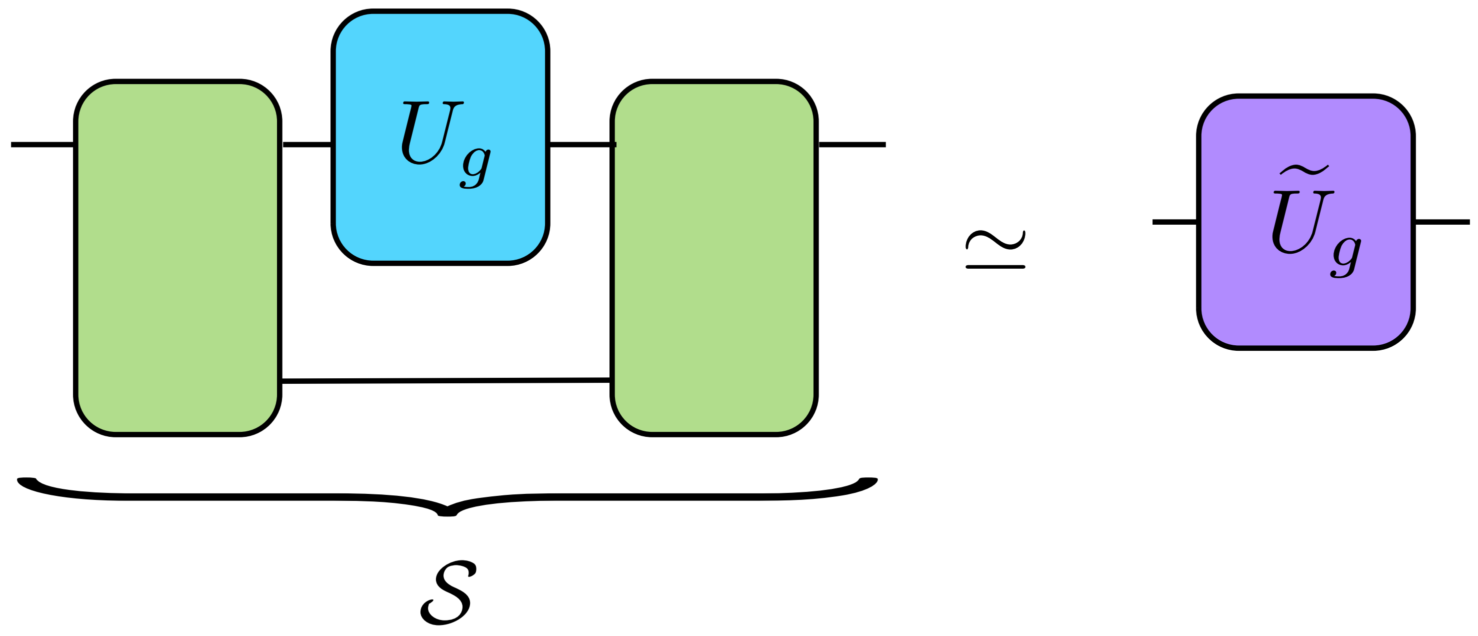
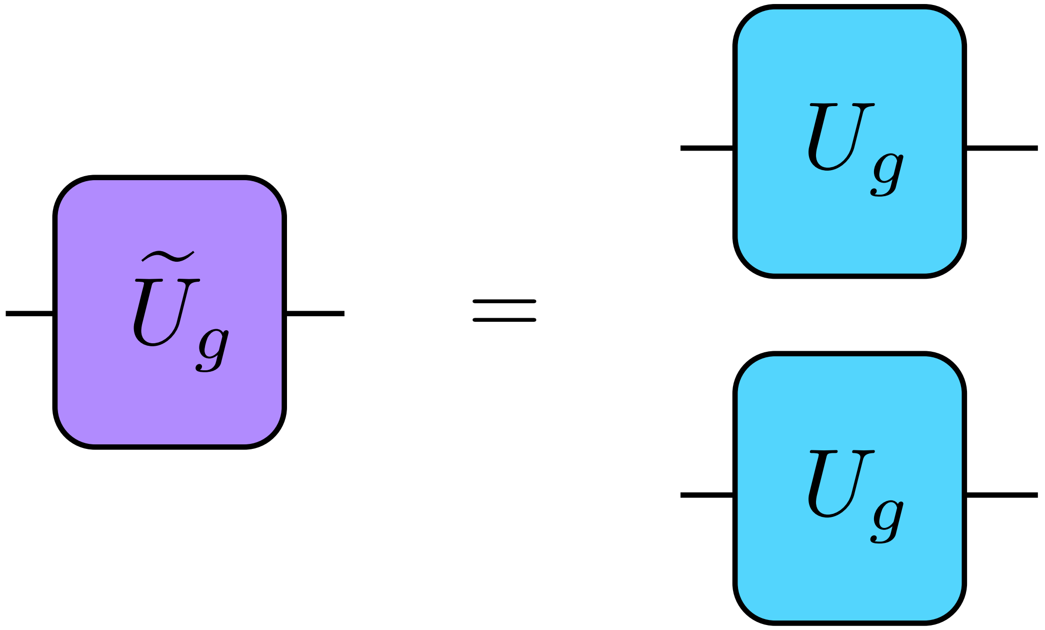
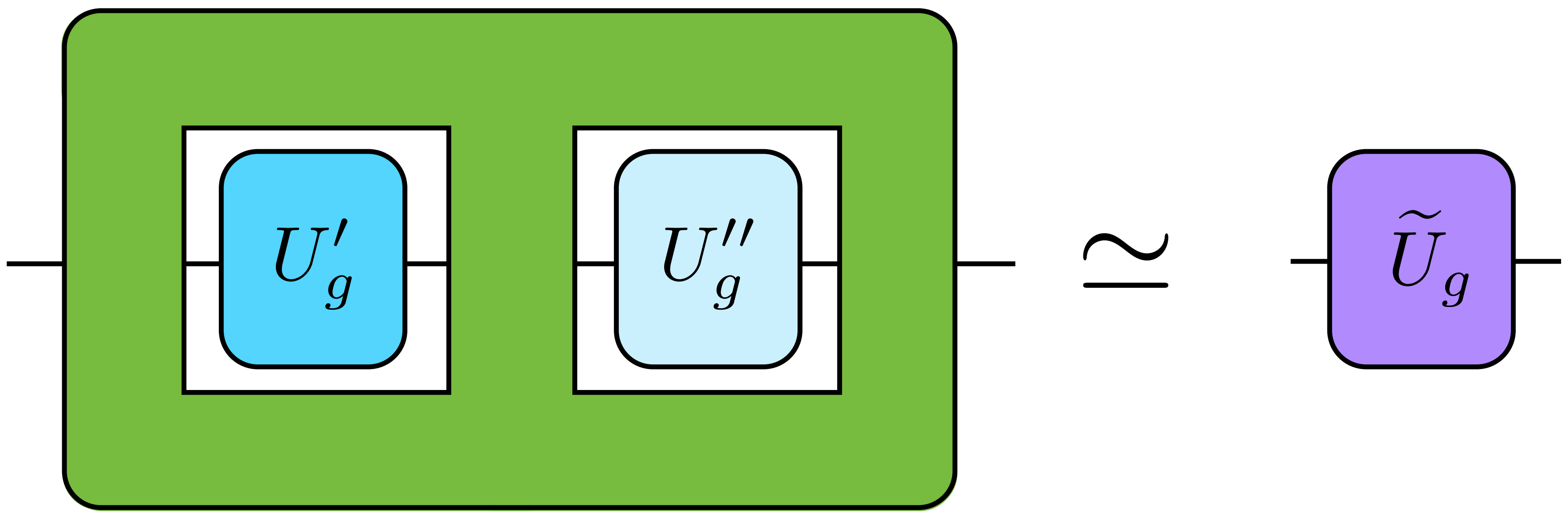


The task is very general, cloning is the case $\widetilde{U}_g = U_g \otimes U_g$



Many uses:



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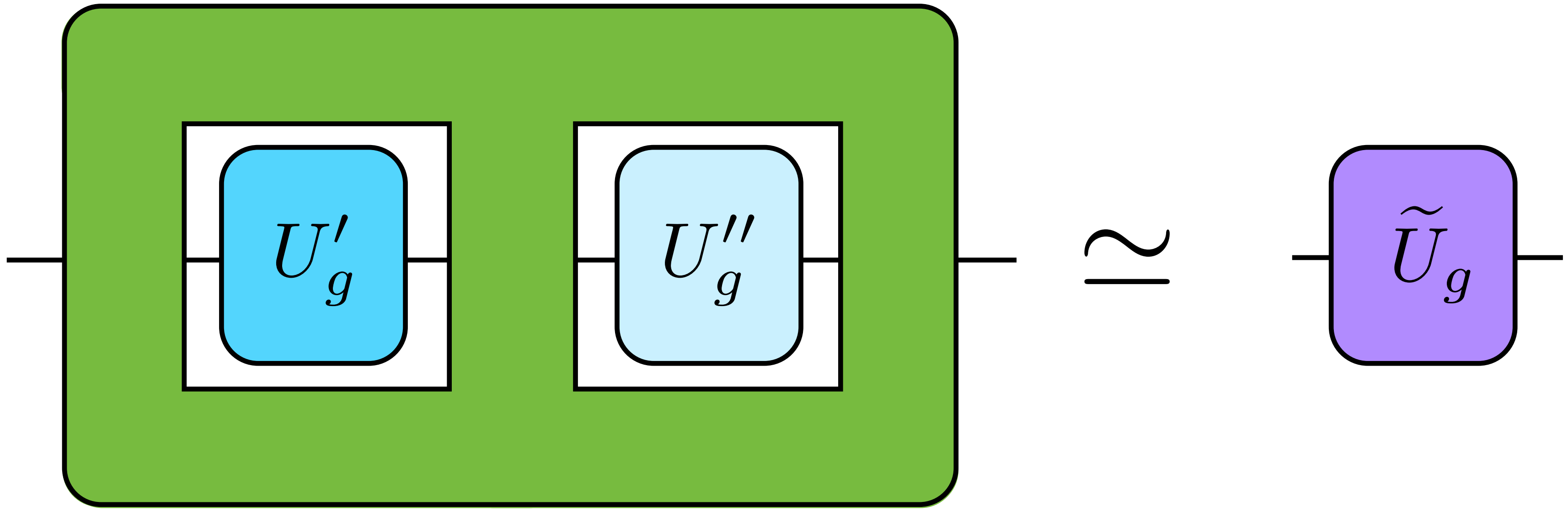

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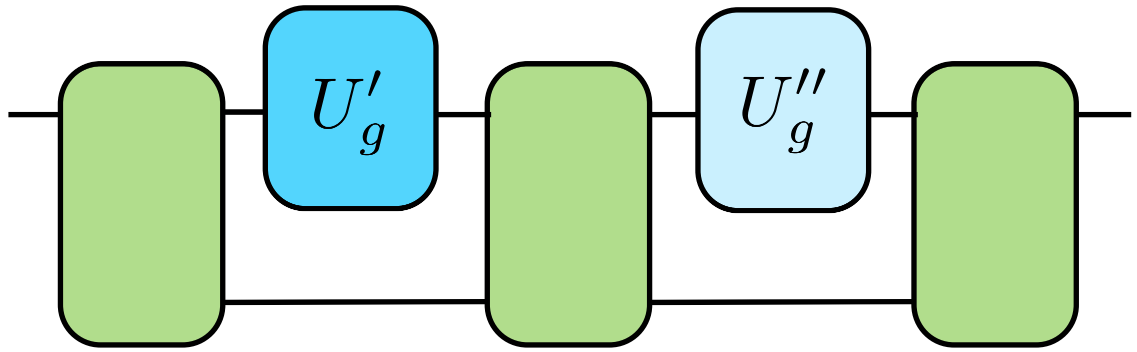

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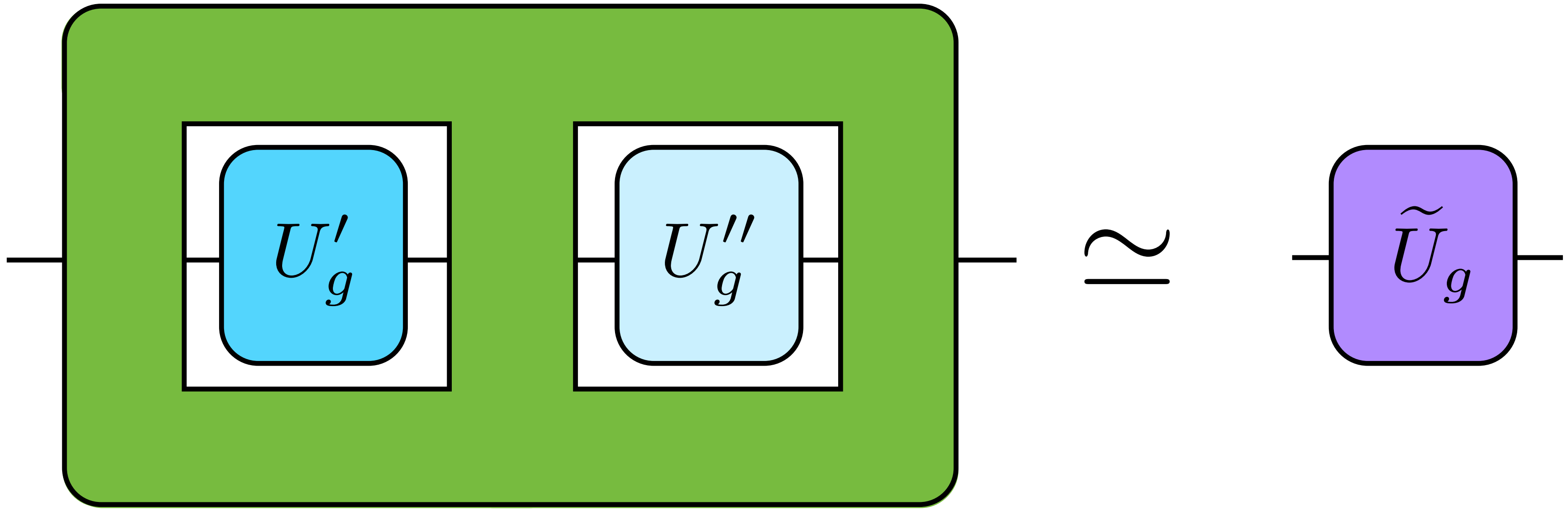

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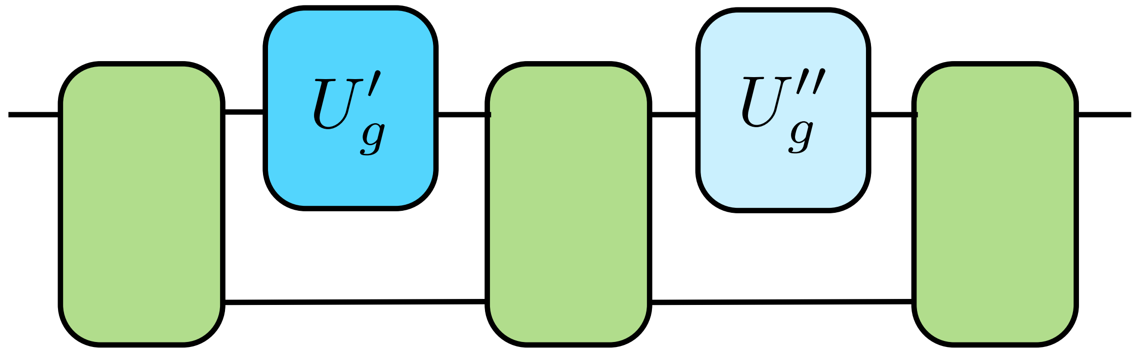

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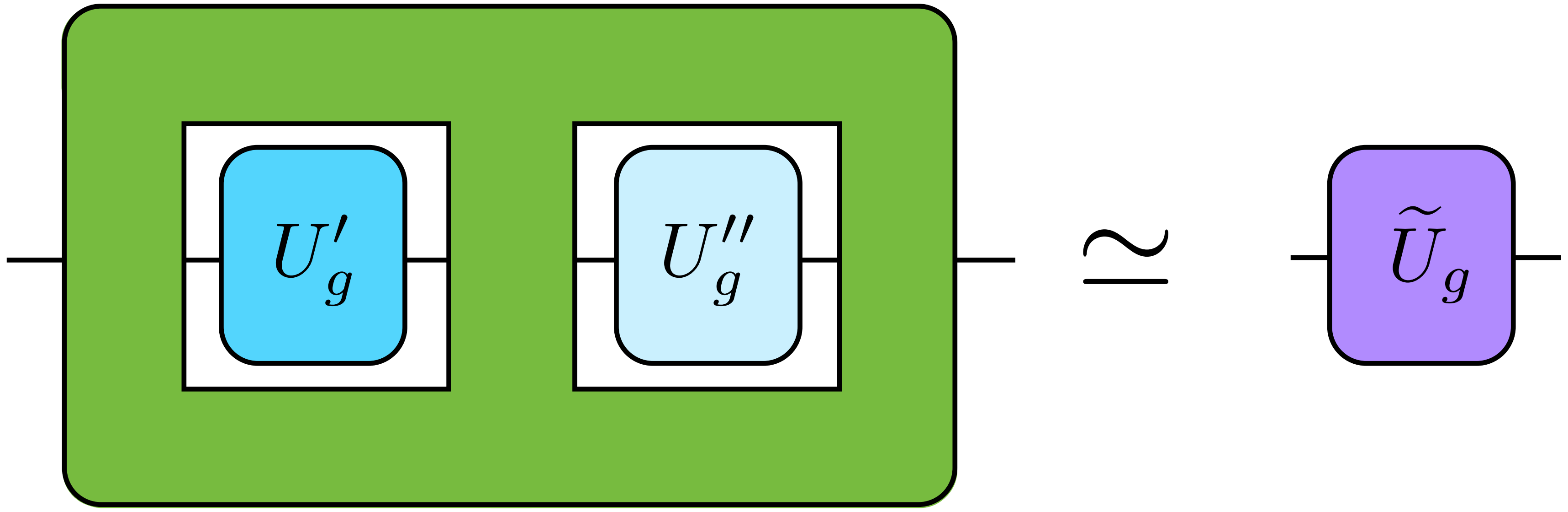

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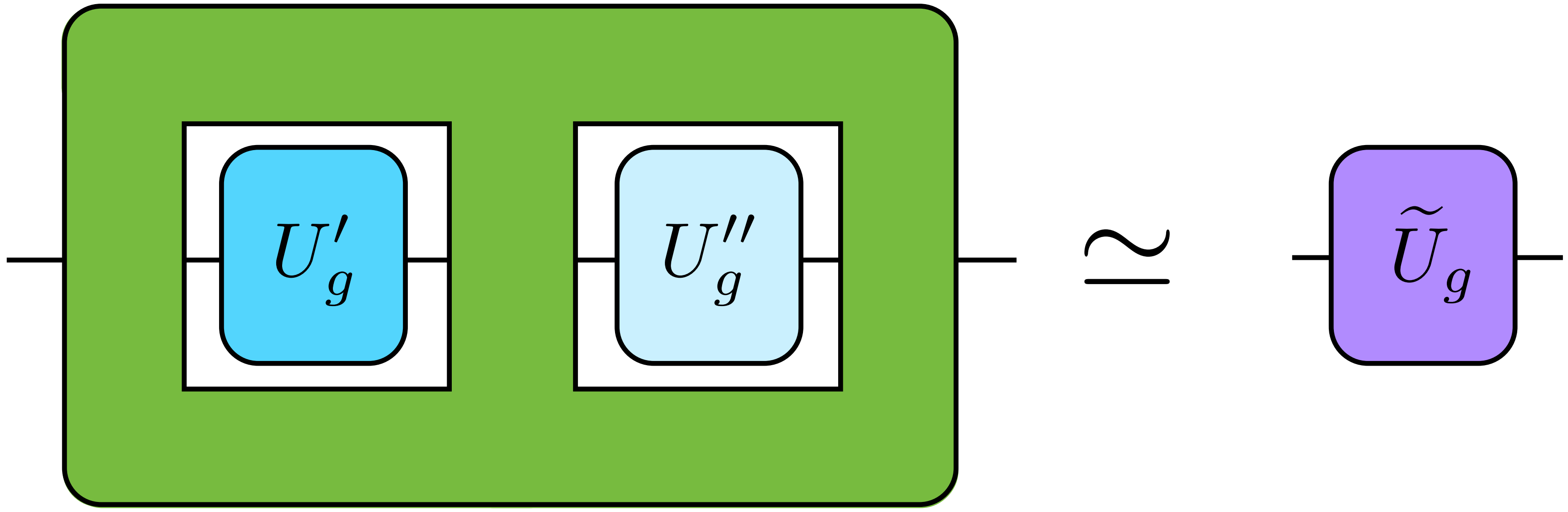

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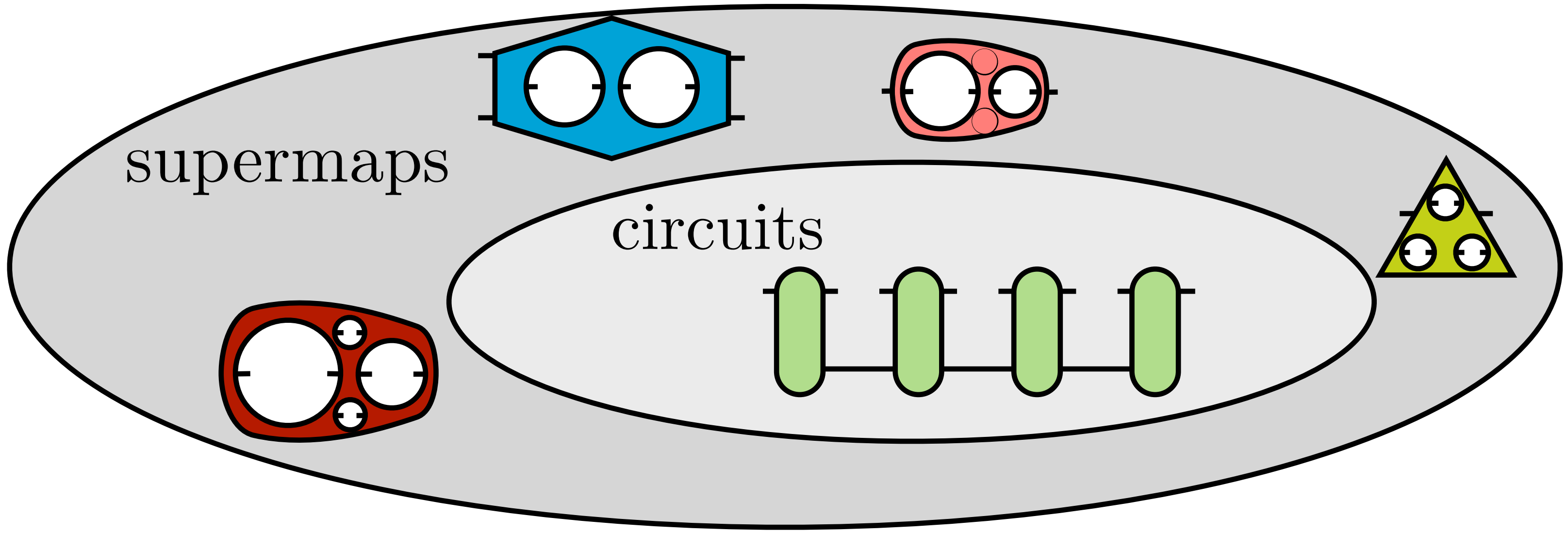
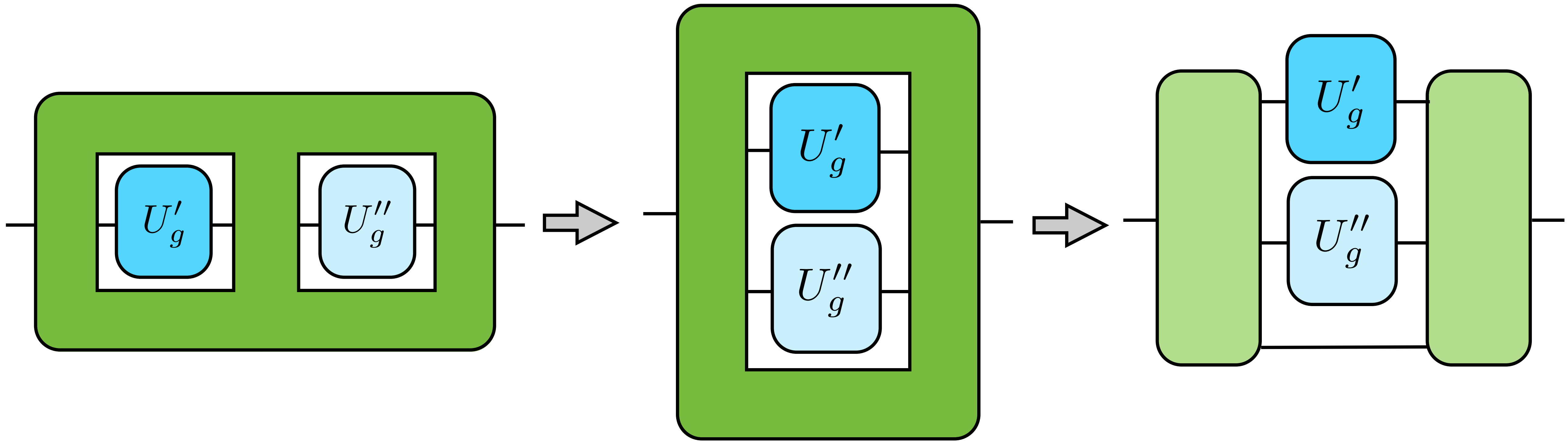


Where do we find the optimal supermap?

Processing of
unitary transformations
picked from a group G

$\Rightarrow$

parallel use
is optimal



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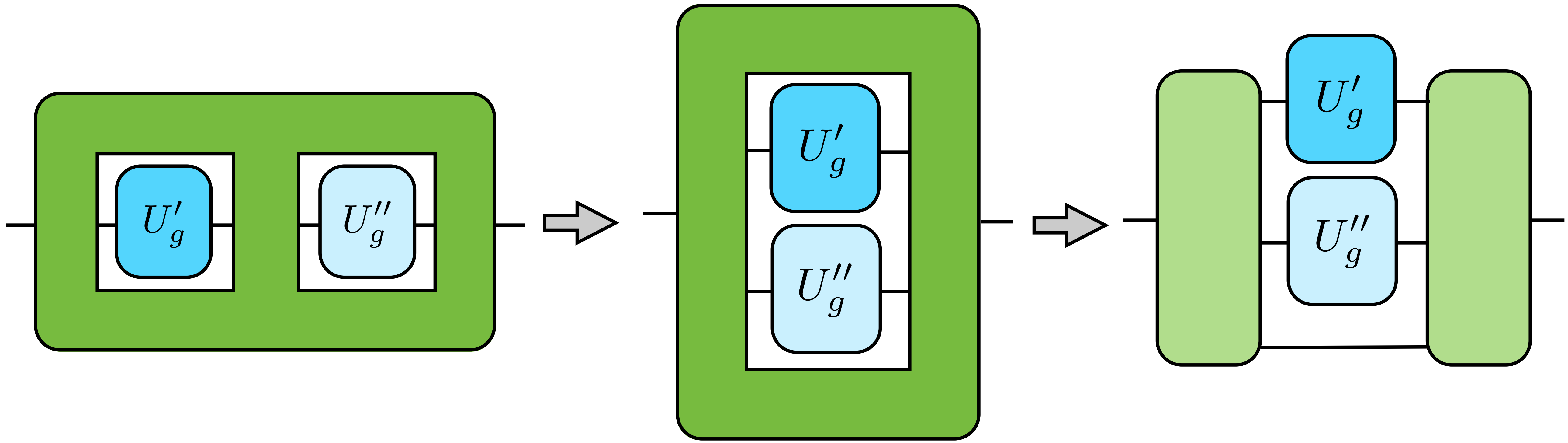

The optimal supermap is realizable as a quantum circuit

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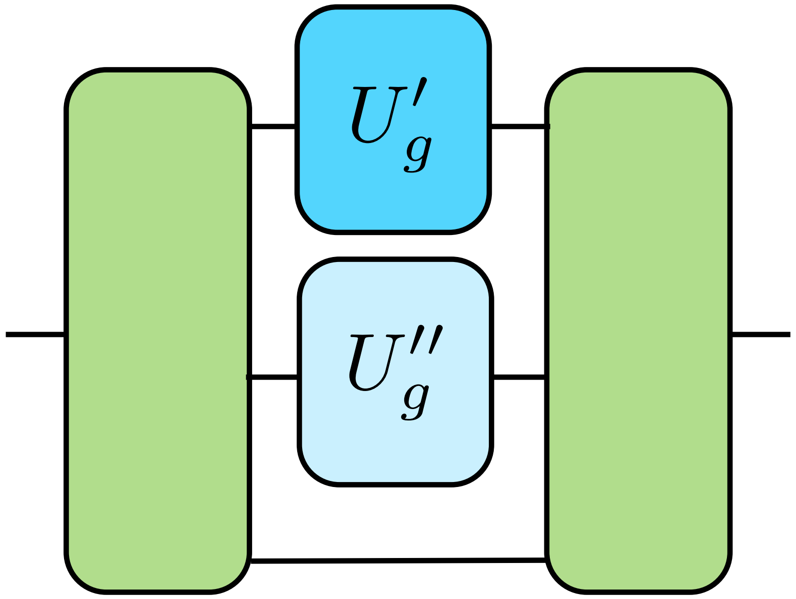

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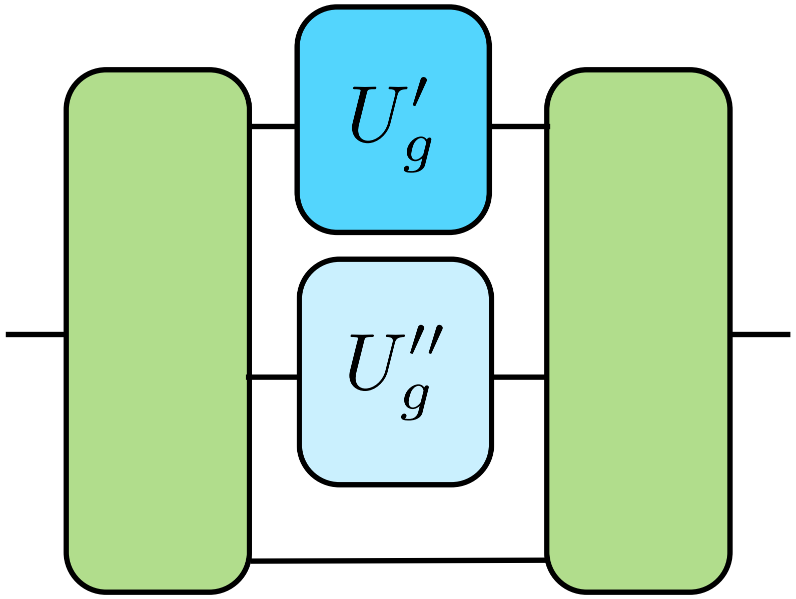

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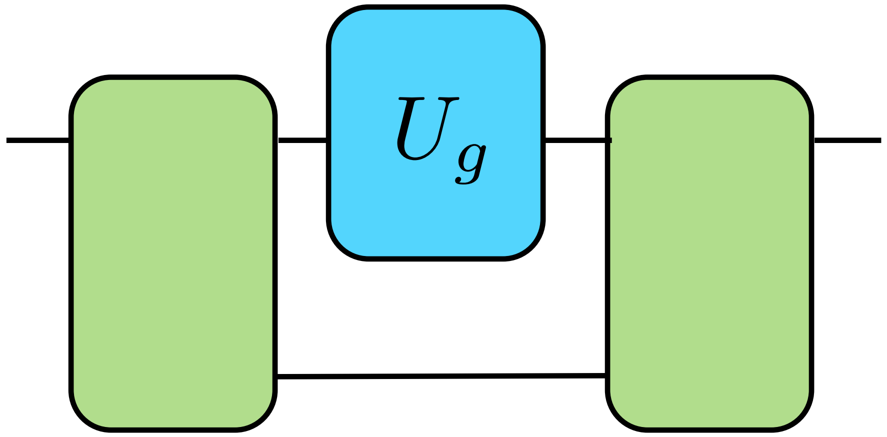
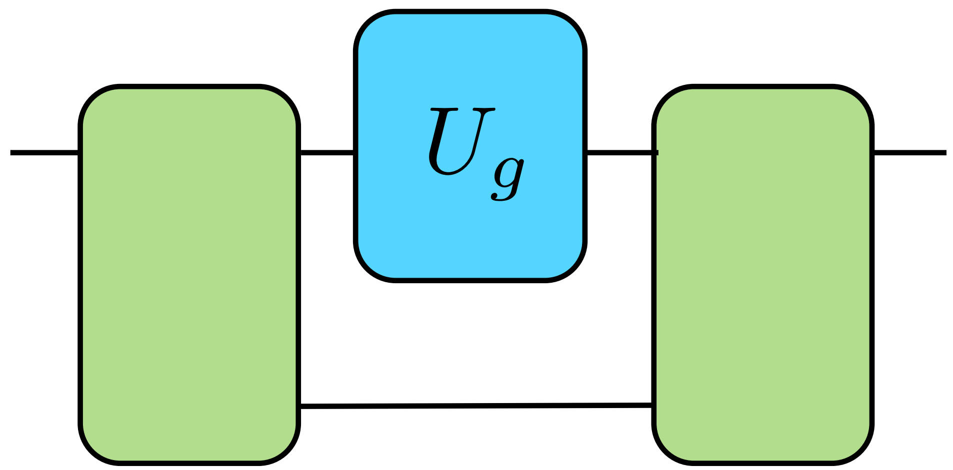


Optimal processing of unitary channels



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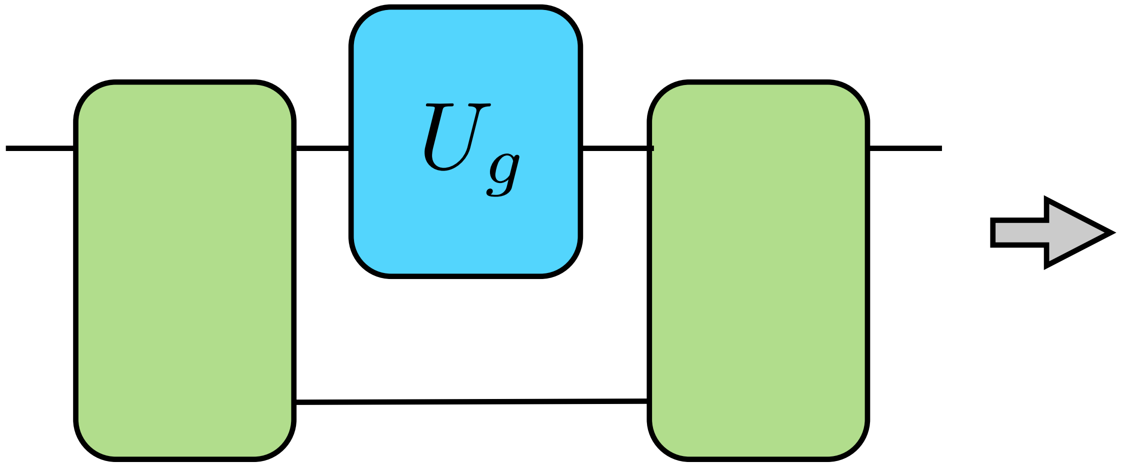

we can reduce the problem to a set of equations:

$$F = \max_{P_{K,a}} \sum_K \left(\sum_a \sqrt{Q_{K,a}\ P_{K,a}} \right)^2 \qquad \sum_K P_{K,a} = 1$$

where:

$$Q_{K,a} = \frac{m_a d_a}{d_K} \sum_j m_K^{j,a} d_j$$

$$U_g = \bigoplus_j U_g^{(j)} \qquad \widetilde{U}_g = \bigoplus_a U_g^{(a)} \otimes I_{m_a}$$

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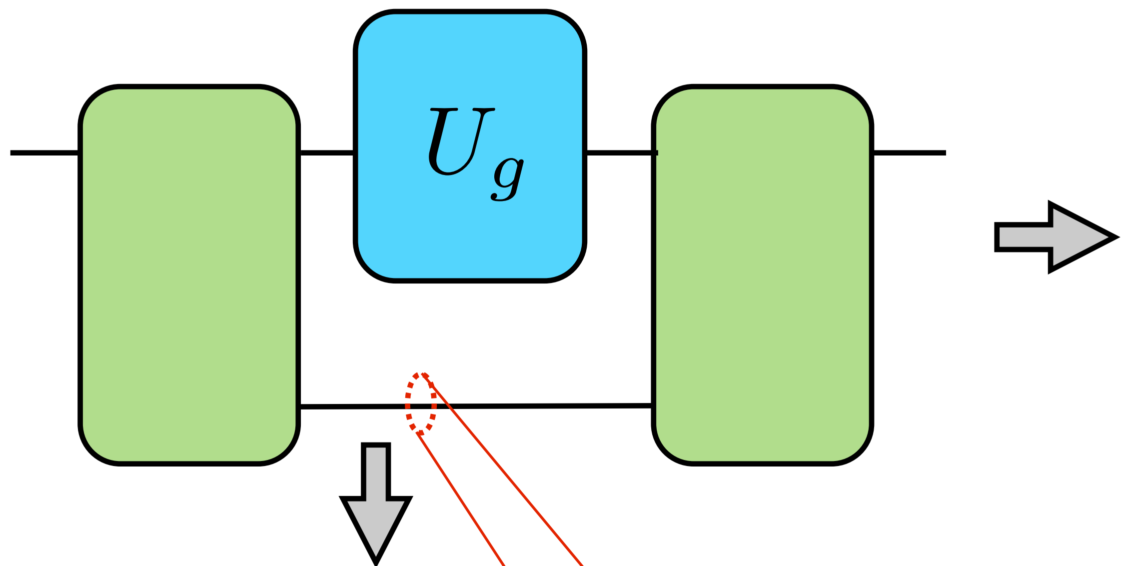

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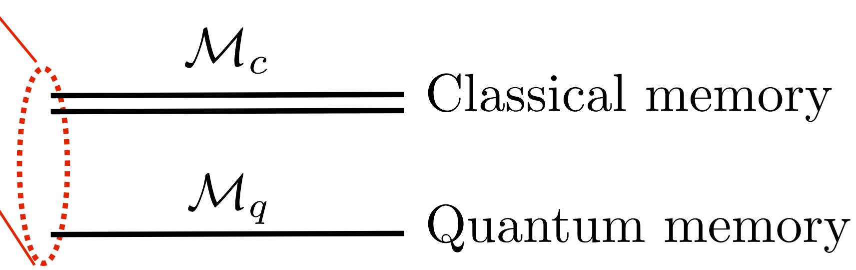
we can upper bound the amount of quantum memory:

$$\dim \mathcal{M}_q \leq \max_K m_k$$

where: $m_K = \sum_{a,j} m_K^{j,a}\, m_a$



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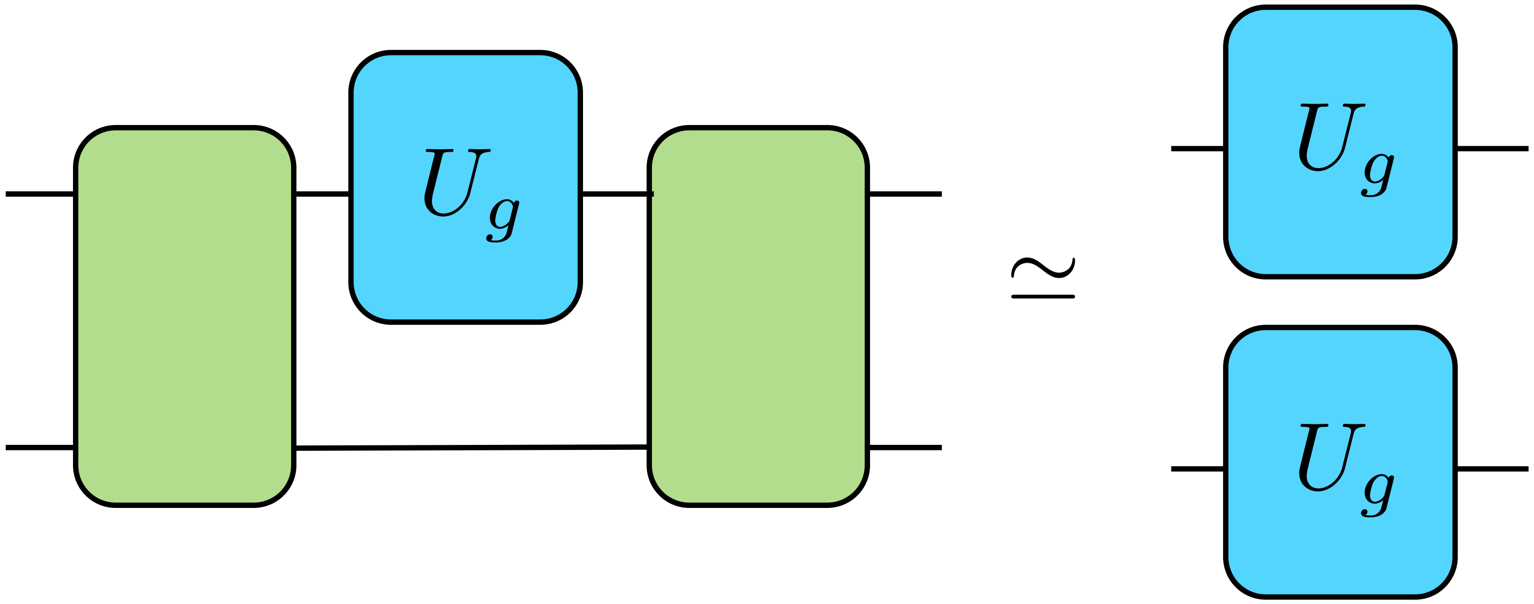
Cloning of a phase gate:

$$U_g|0\rangle = |0\rangle \qquad U_g|1\rangle = e^{ig}|1\rangle \qquad 0 \leq g < 2\pi$$



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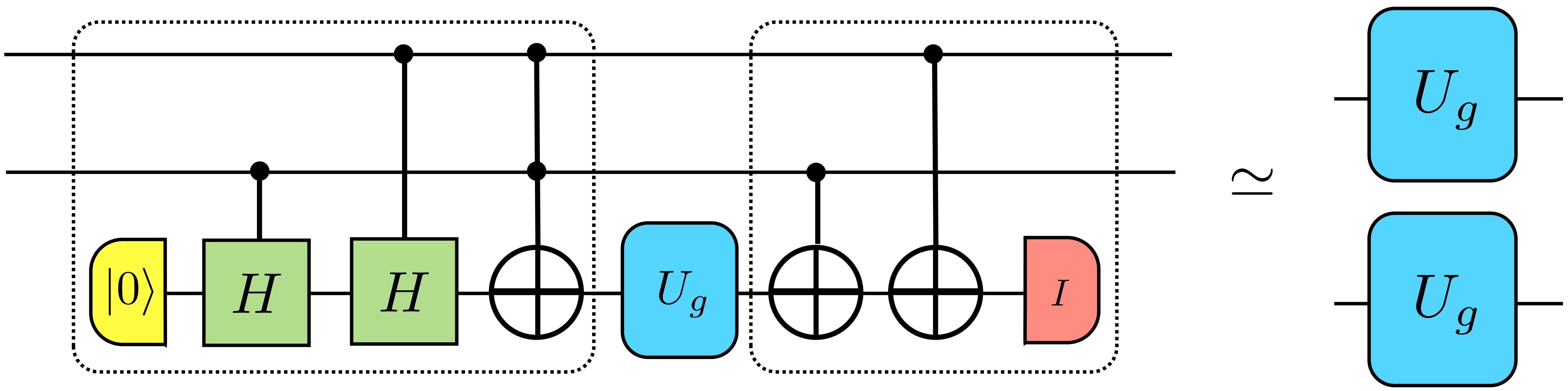

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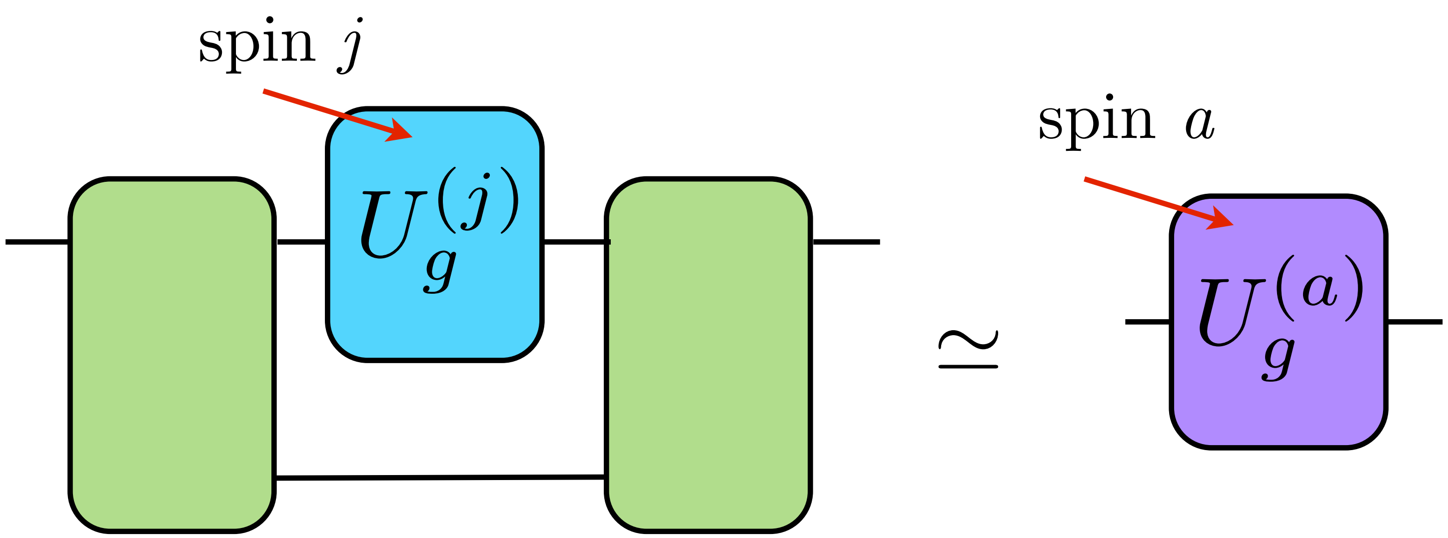
$U_g, \widetilde{U}_g$ are $SU(2)$ irreducible representations



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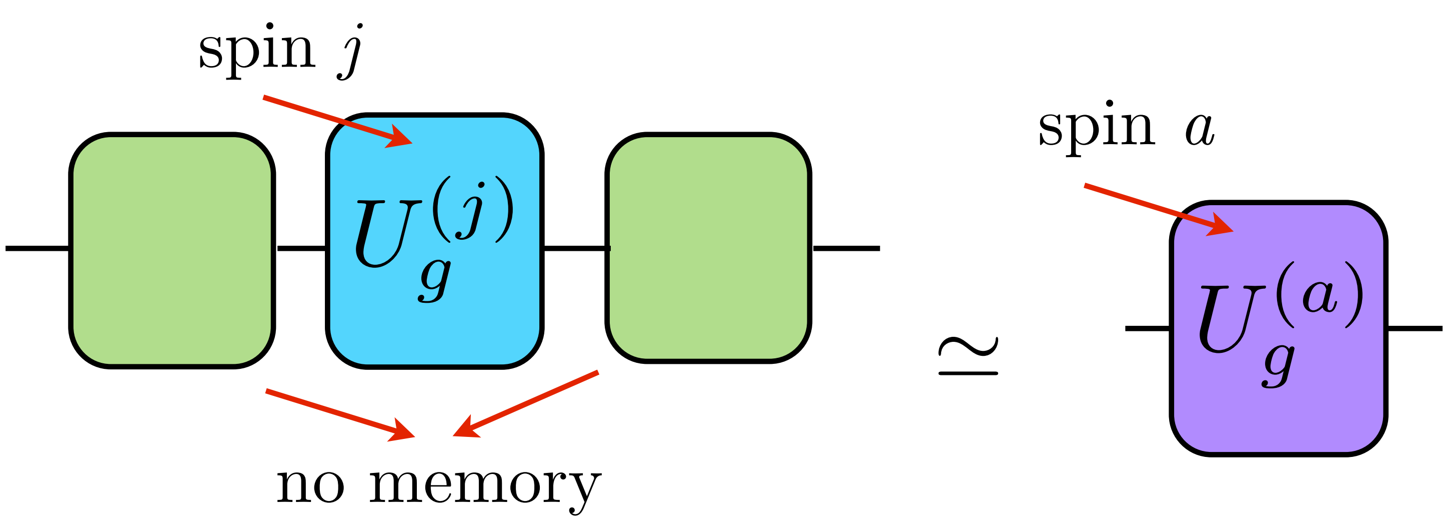
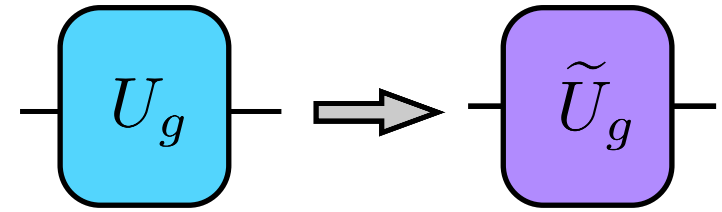


$$F = \frac{d_j}{d_a d_{|a-j|}}$$

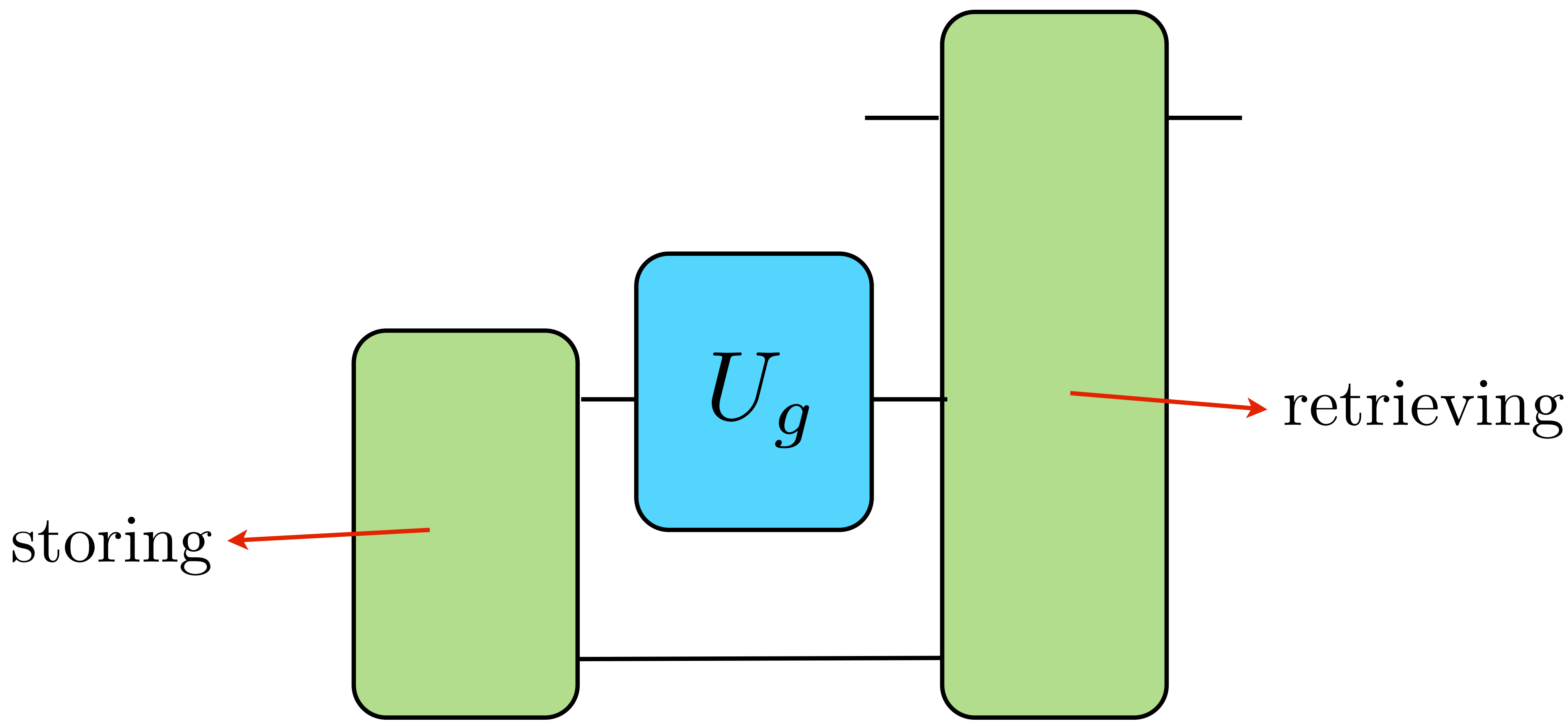
$$d_n = 2n + 1$$

A different scenario

Another possible way to achieve the mapping 

is provided by the following scheme:

Storing and retrieving



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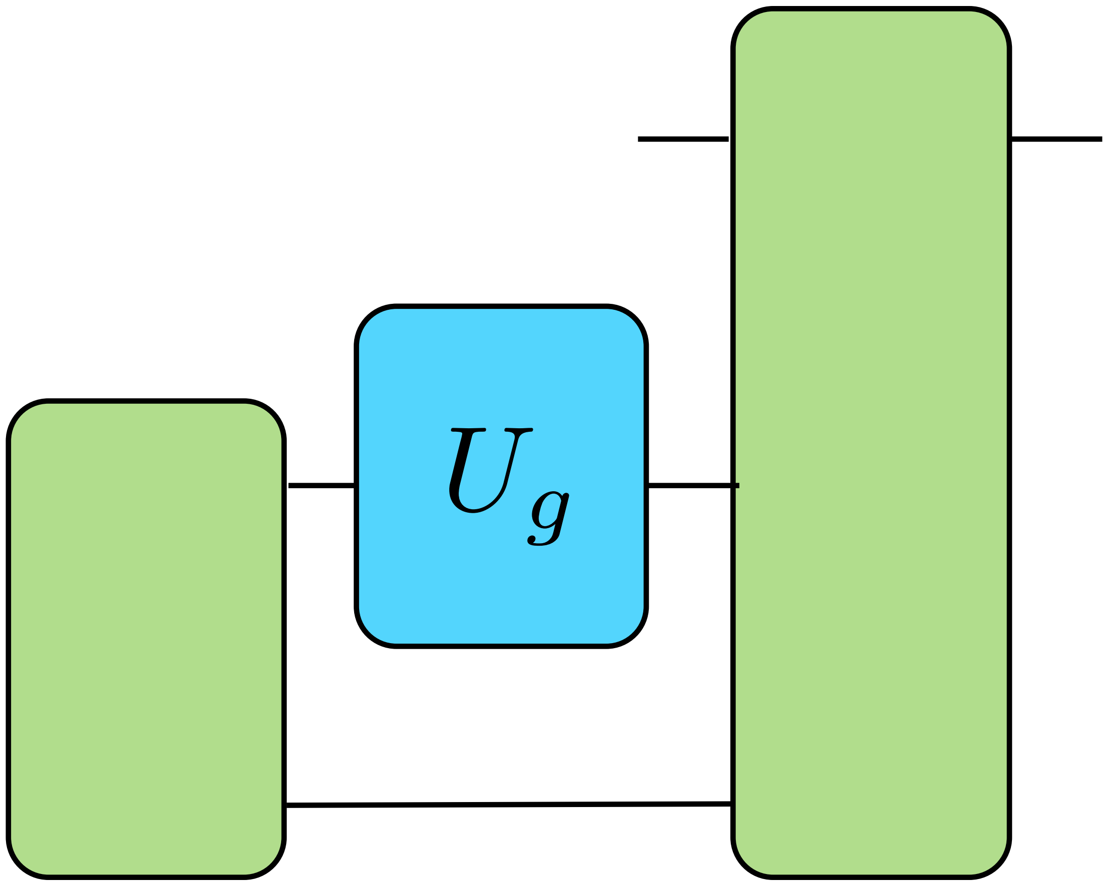

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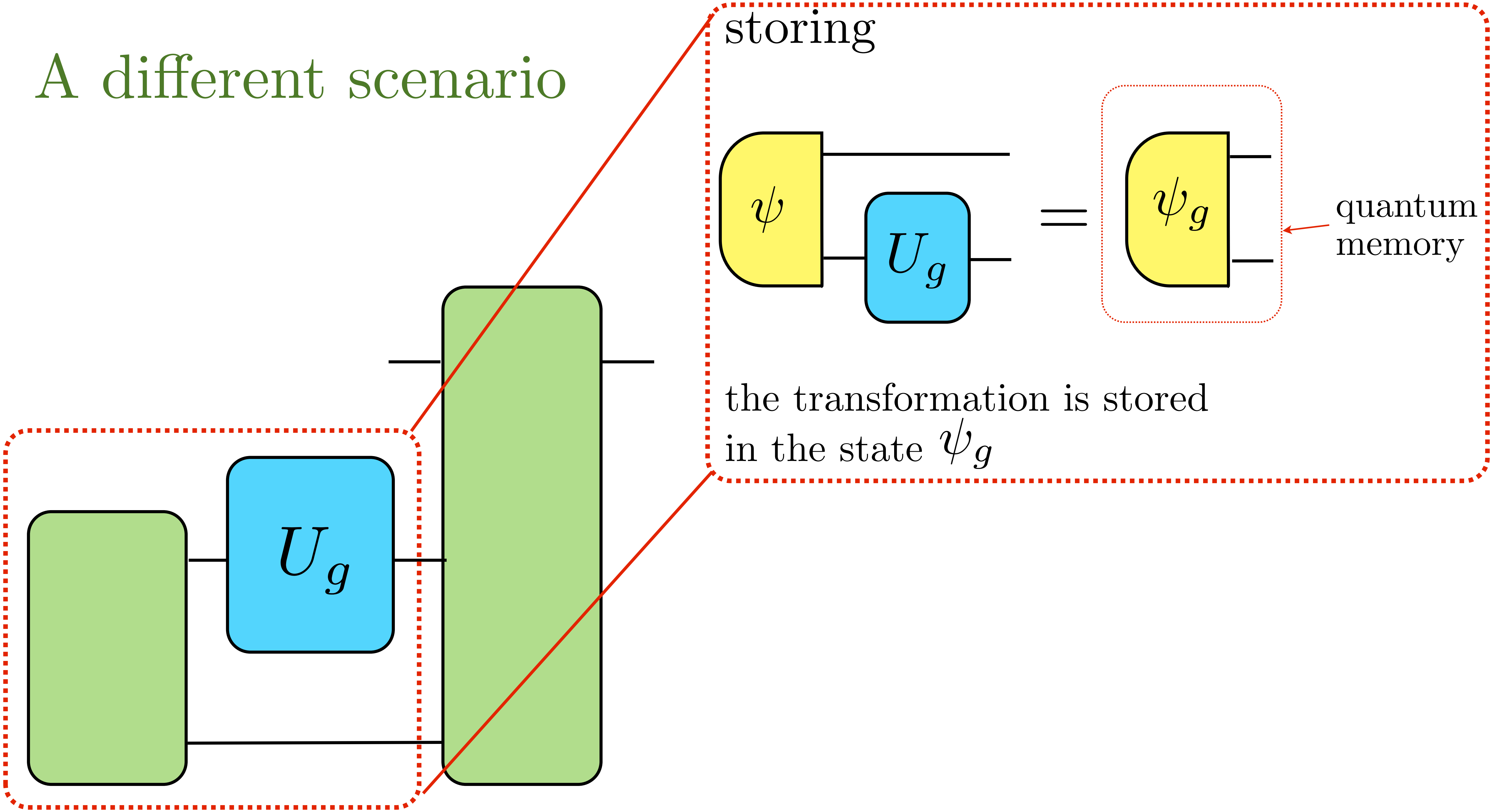

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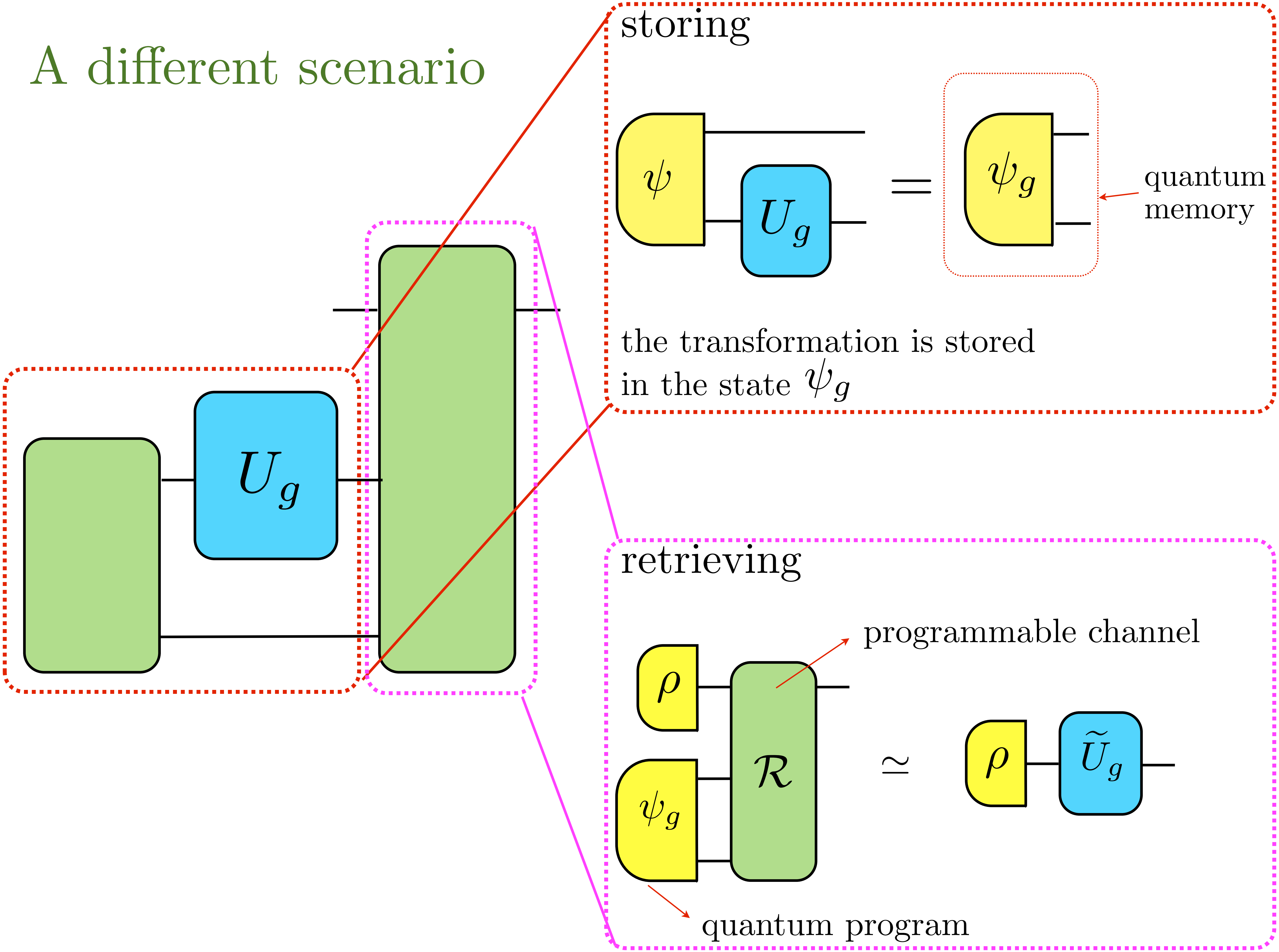

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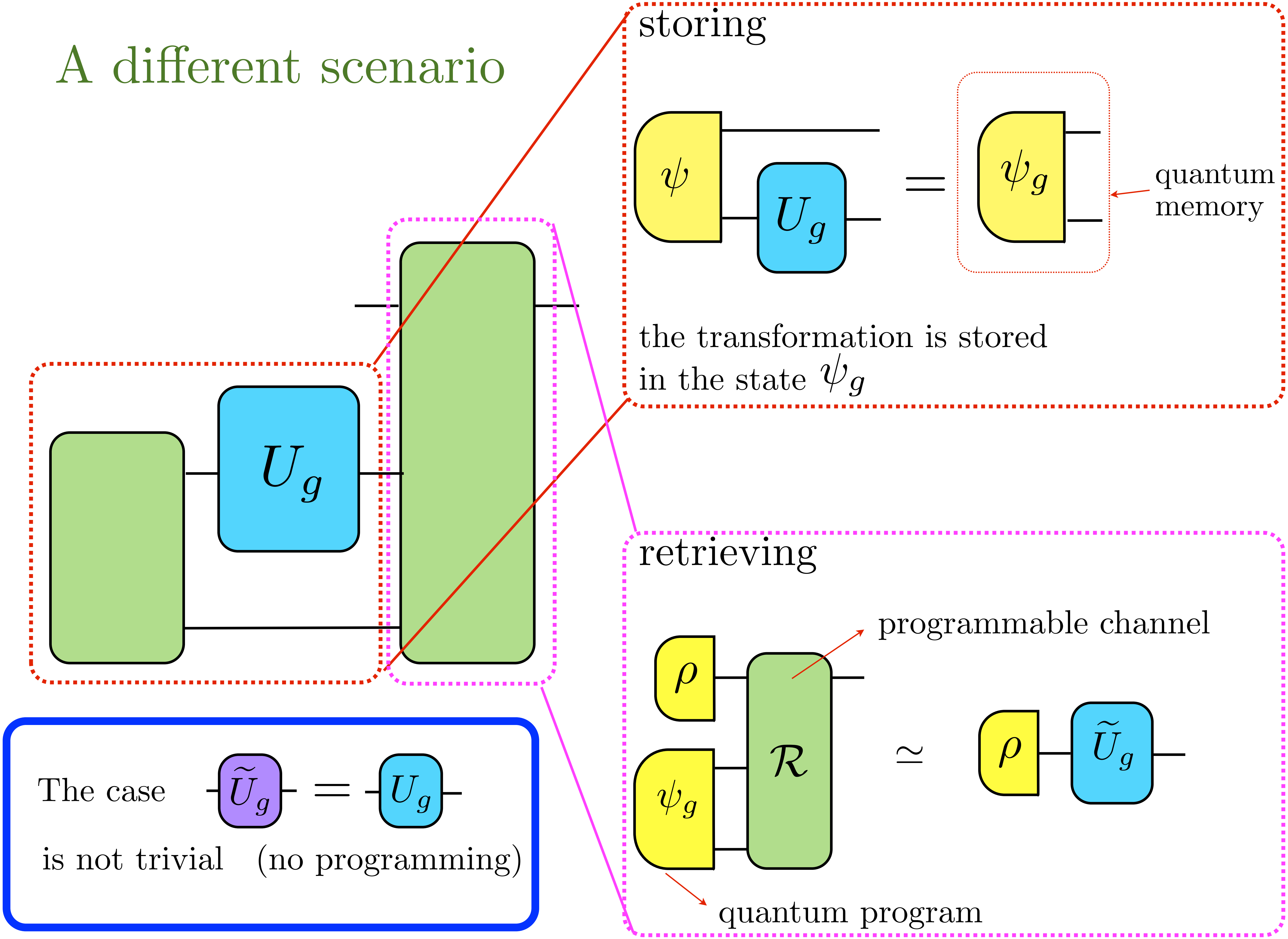
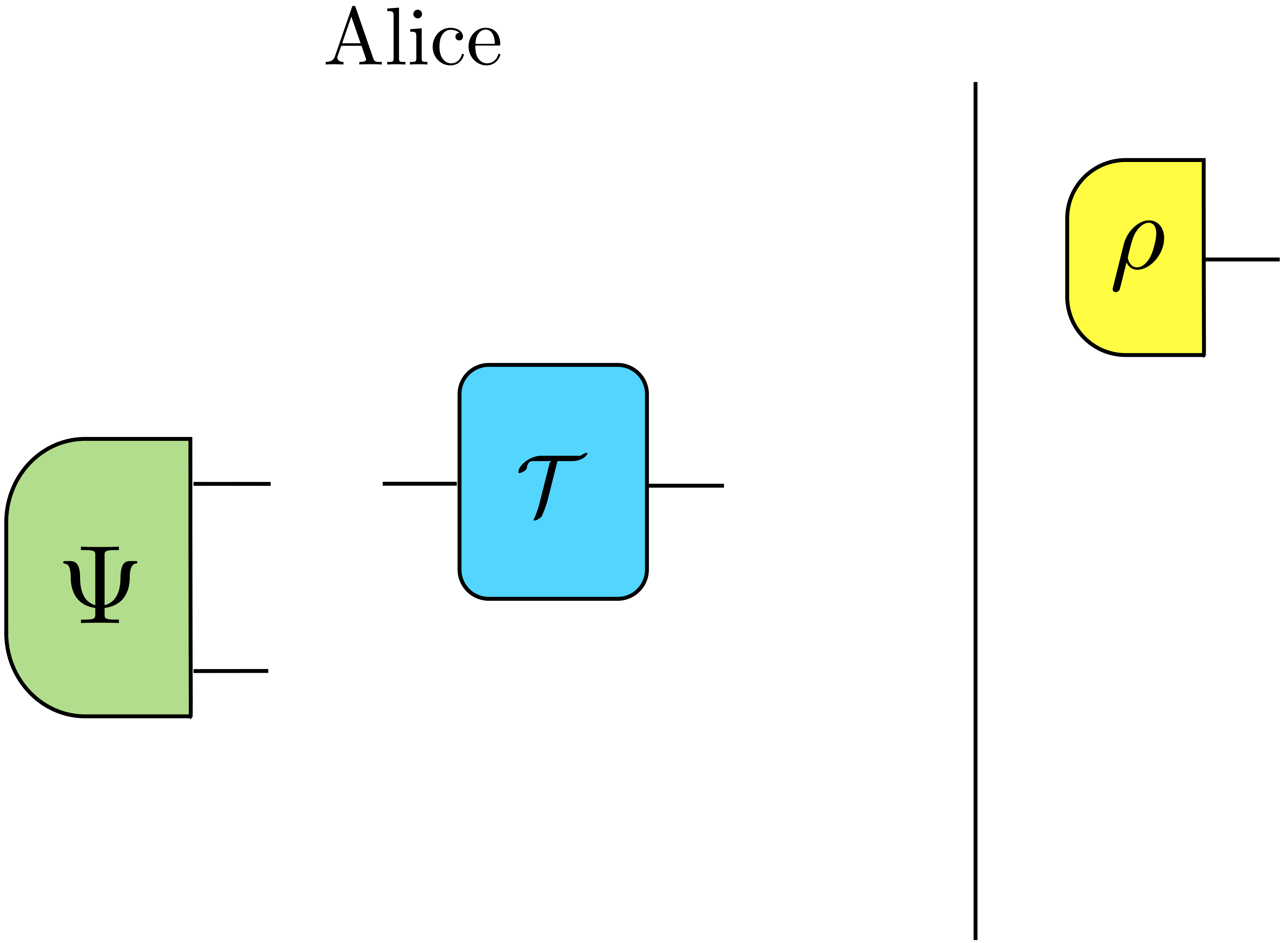


Possible application: sending a transformation without sending the device



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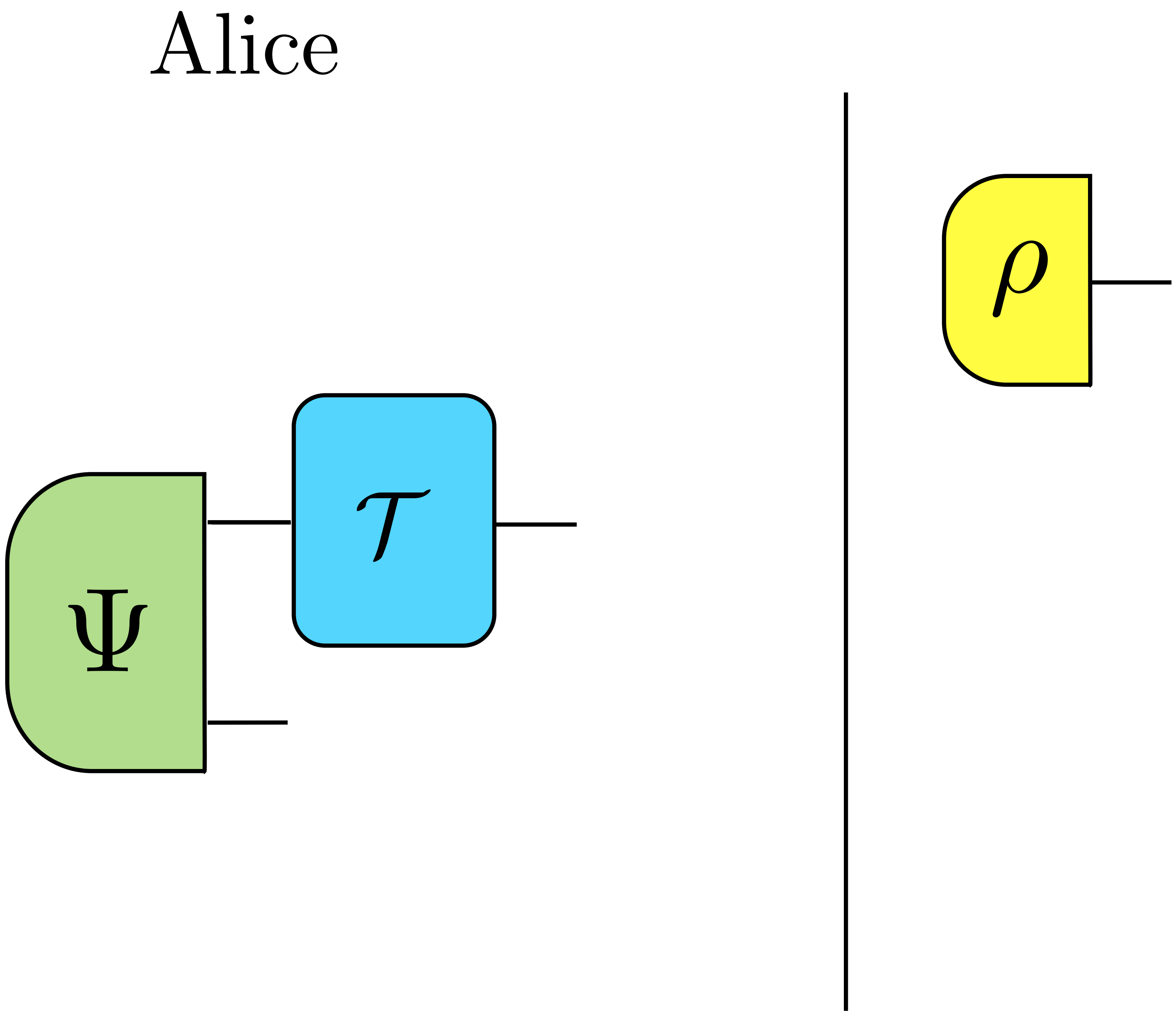

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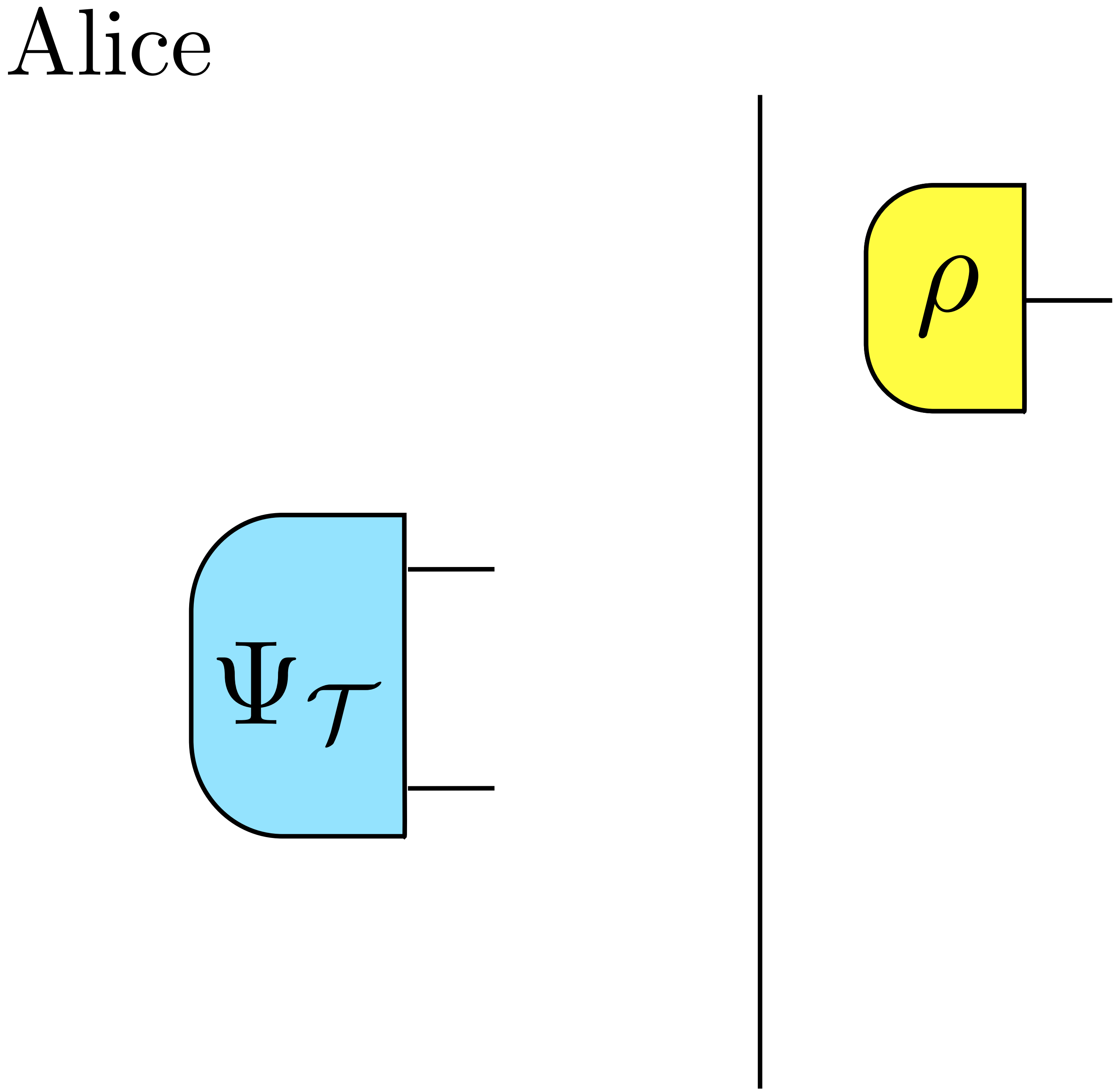

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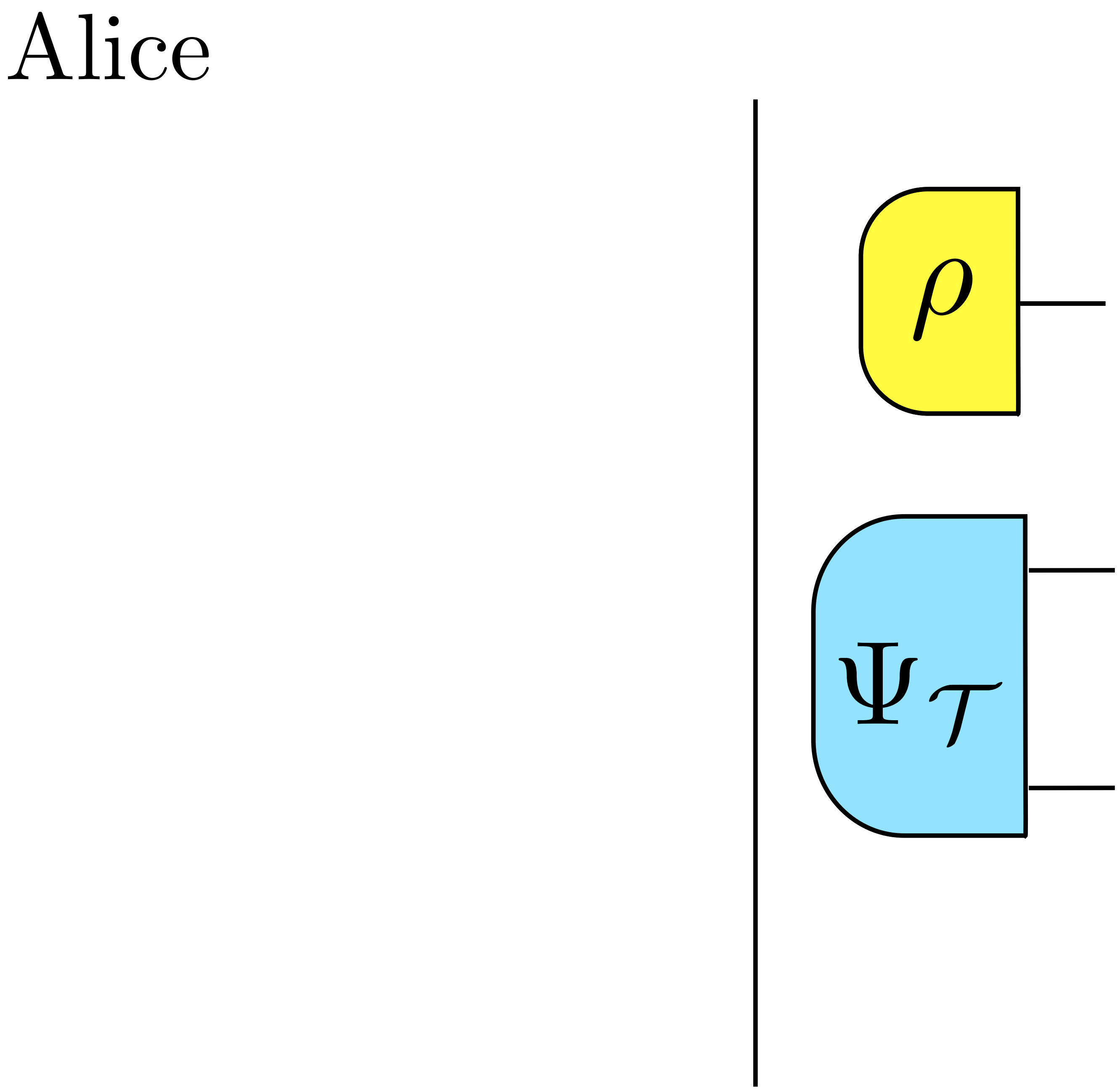

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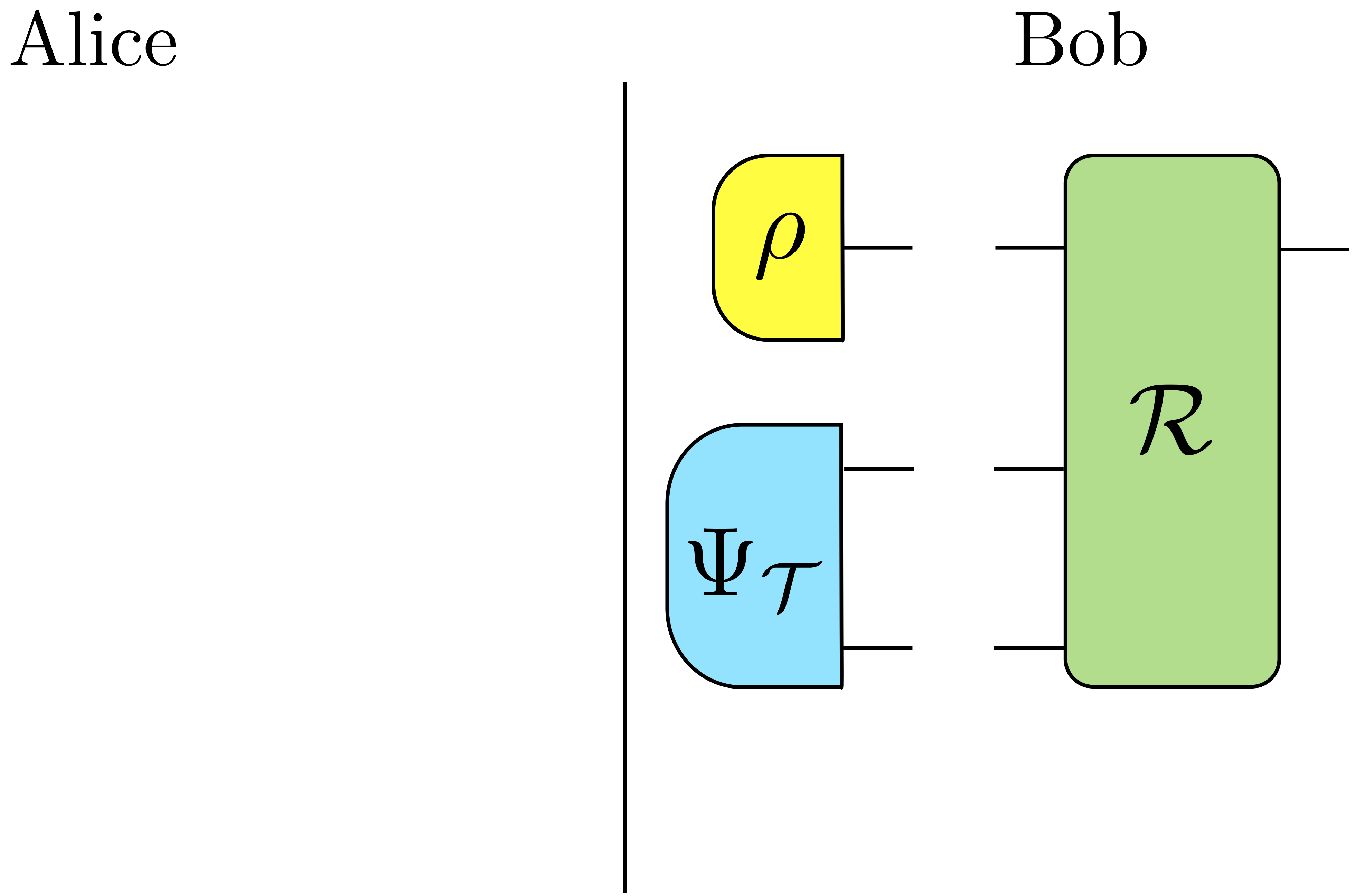

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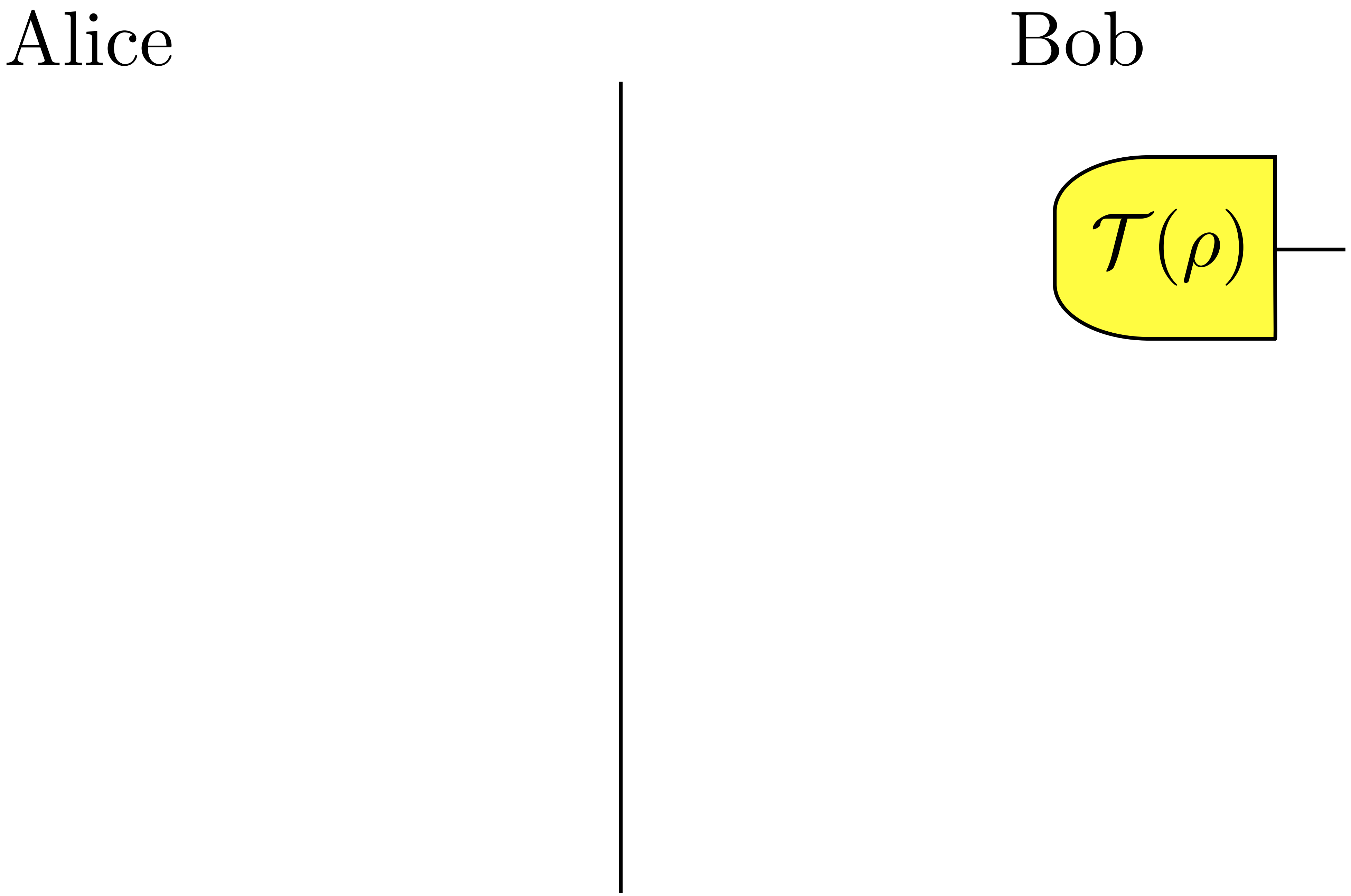
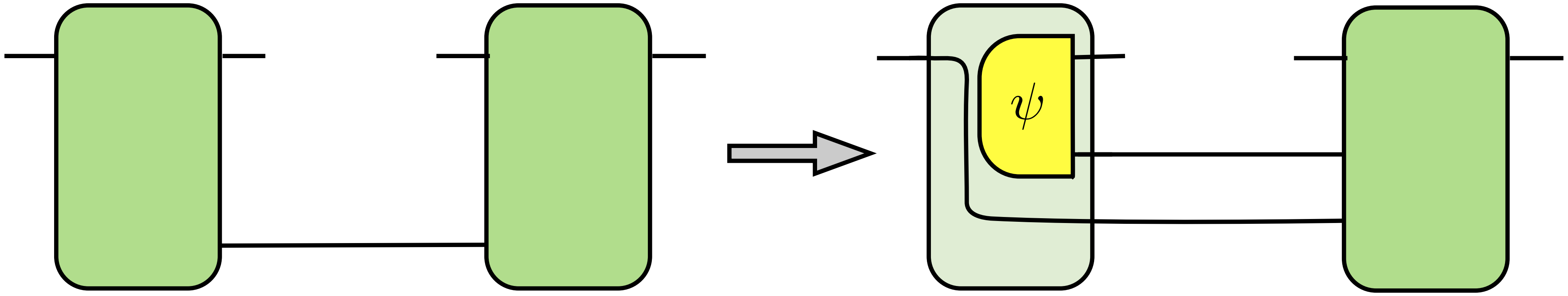


Storing and retrieving is a special case of pre- and postprocessing:



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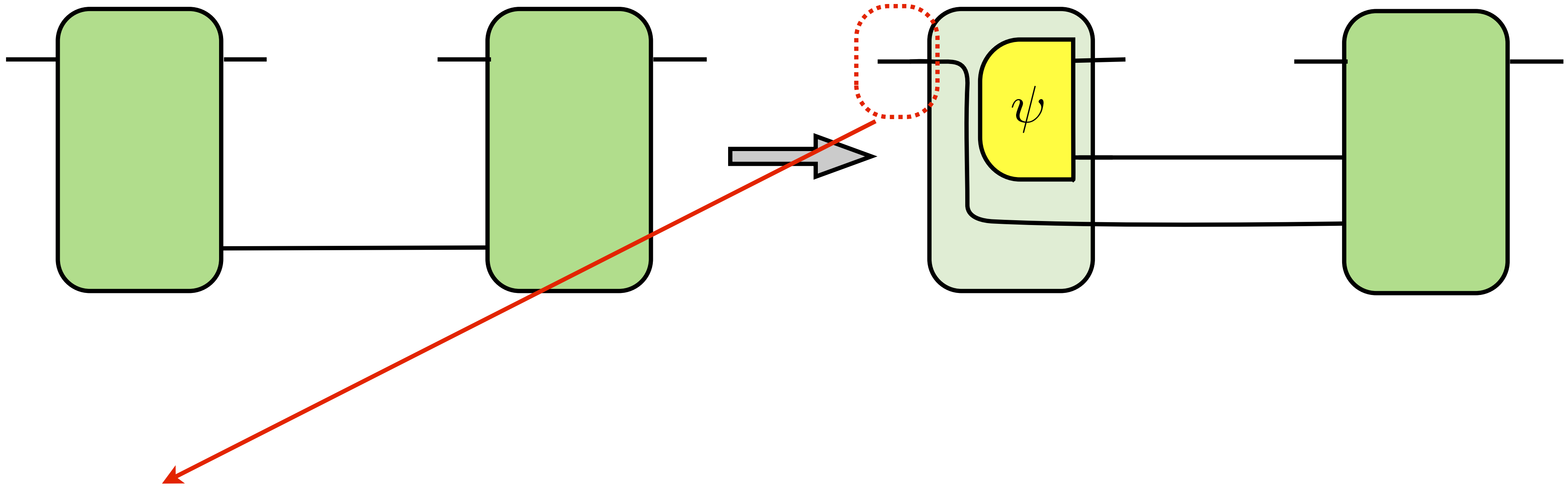

In the s&r scenario we cannot process the input state before using the unknown unitary

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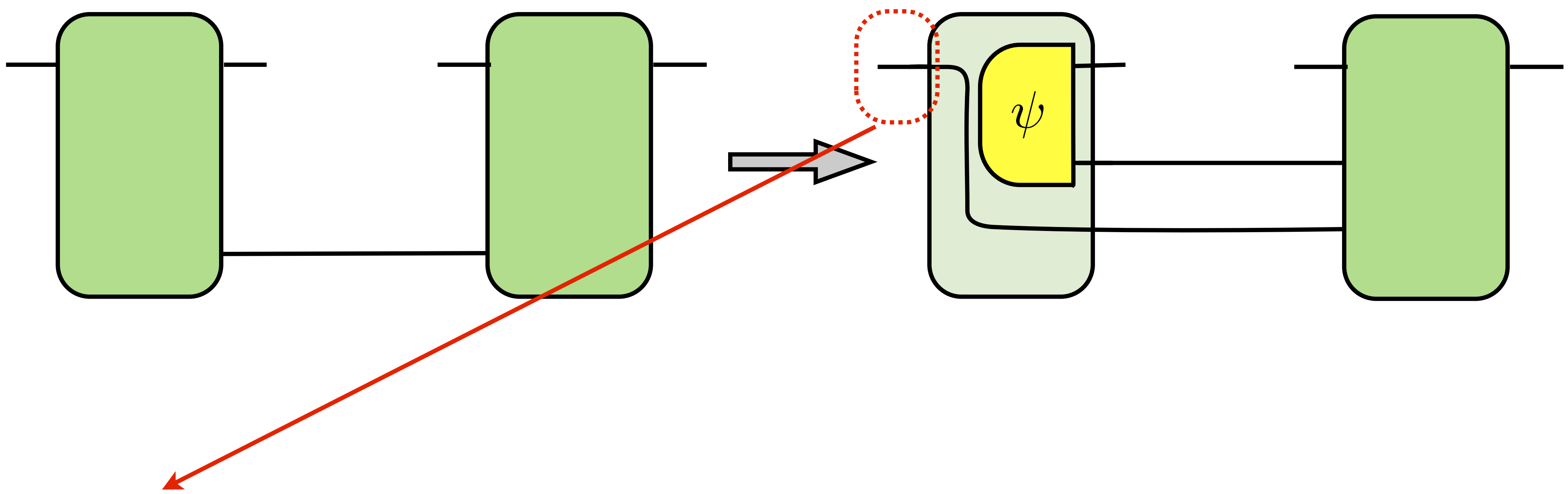

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The input state and the use of the unitary are not available at the same time

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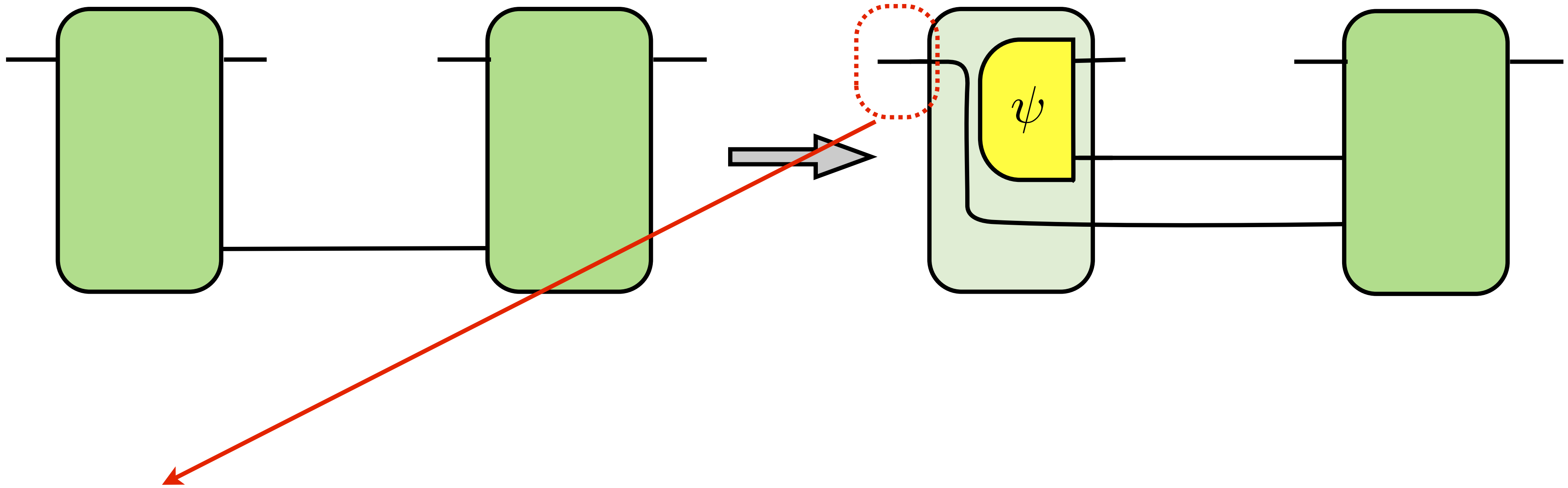

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More restrictive causal structure

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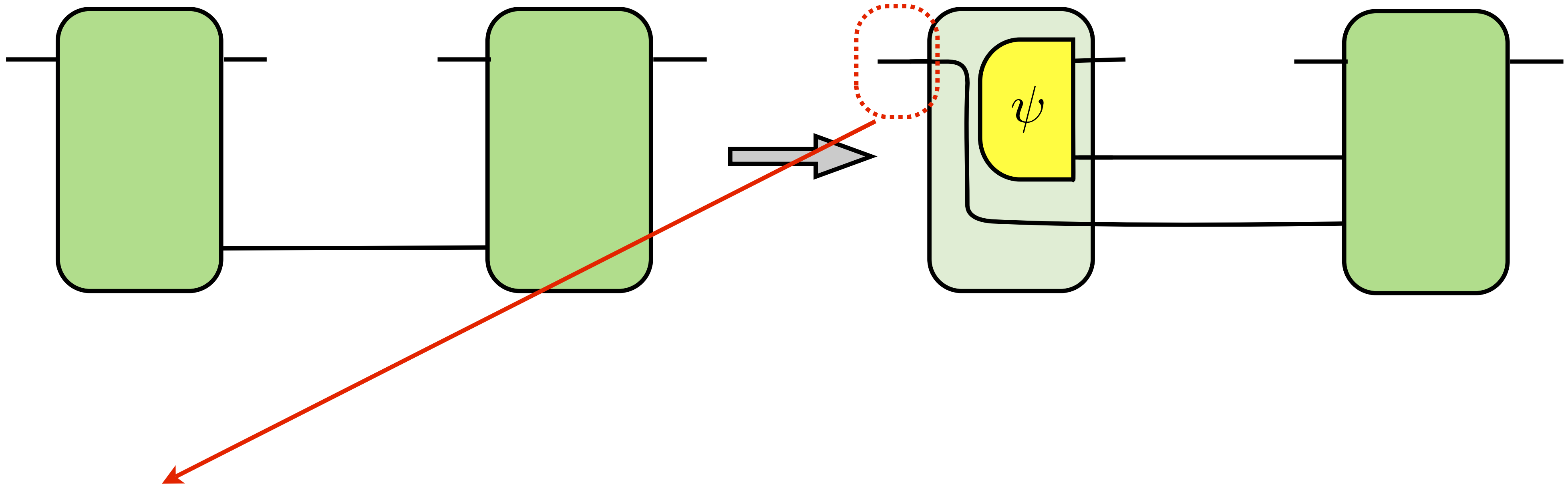

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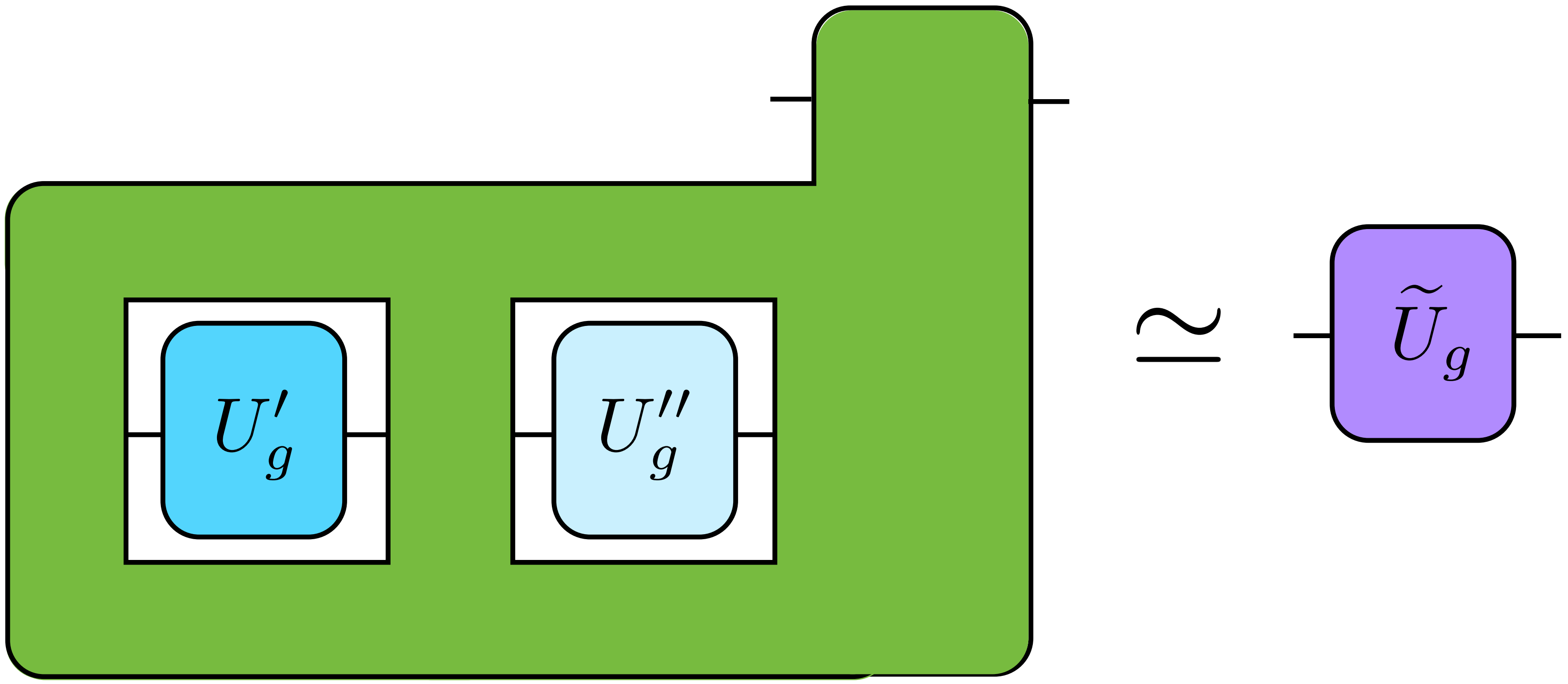
More restrictive causal structure

Lower performances

Many uses:



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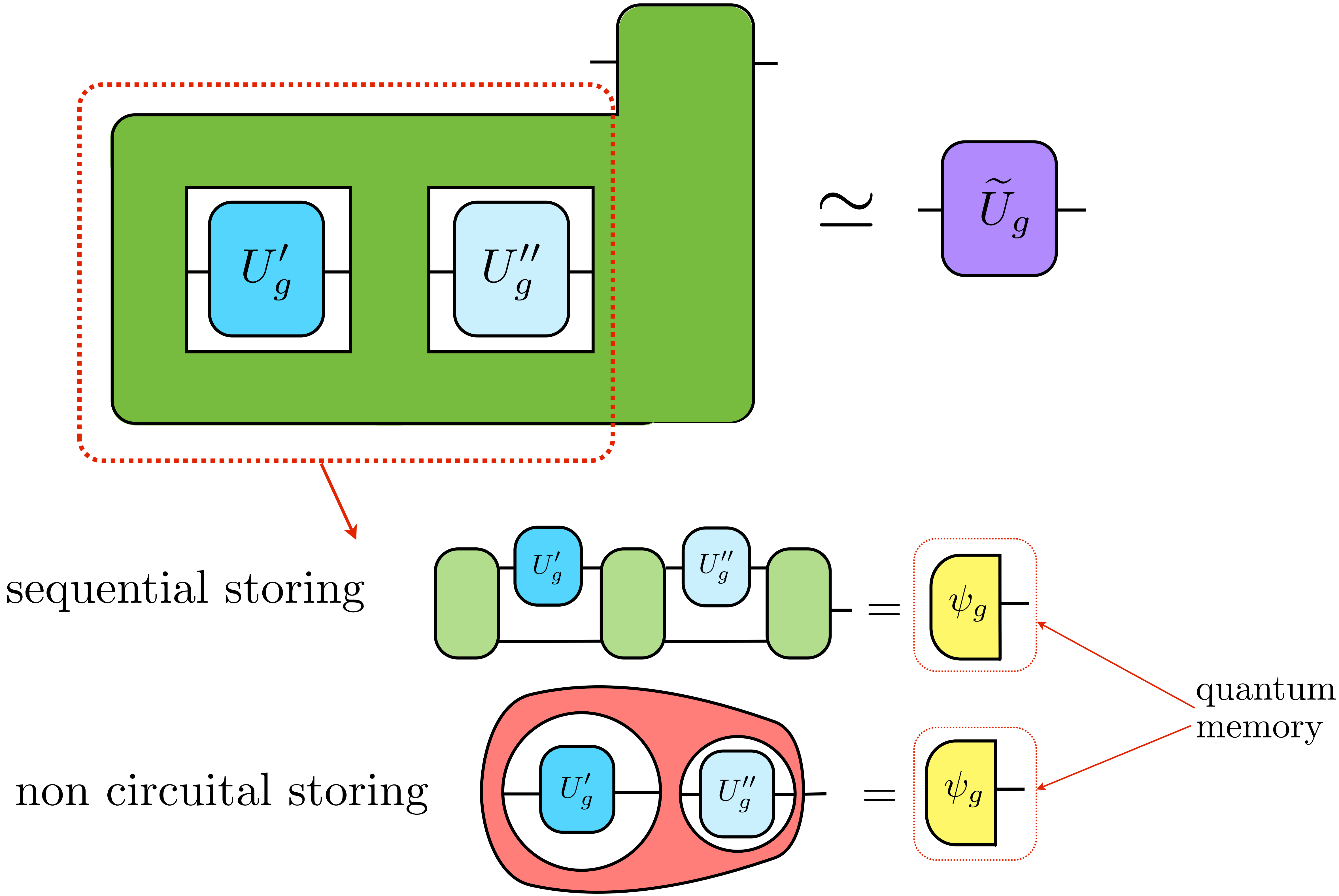

U'_g
U''_g
$\simeq$
$\widetilde{U}_g$
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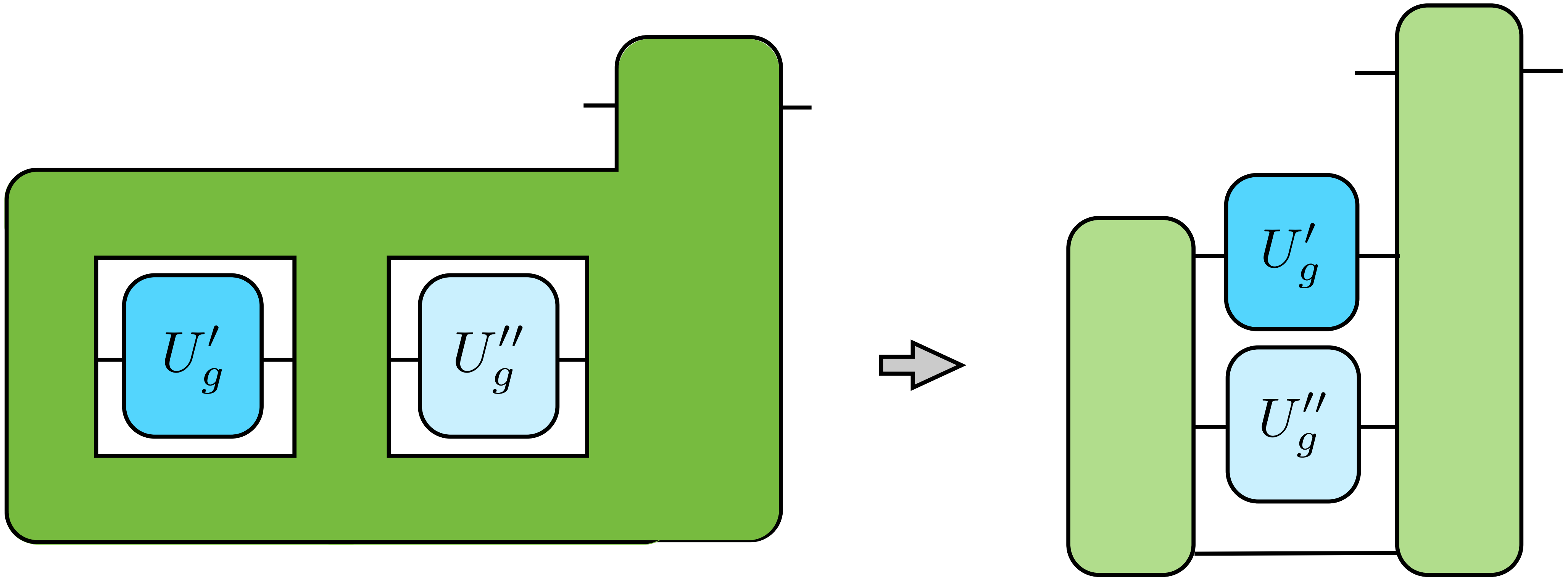
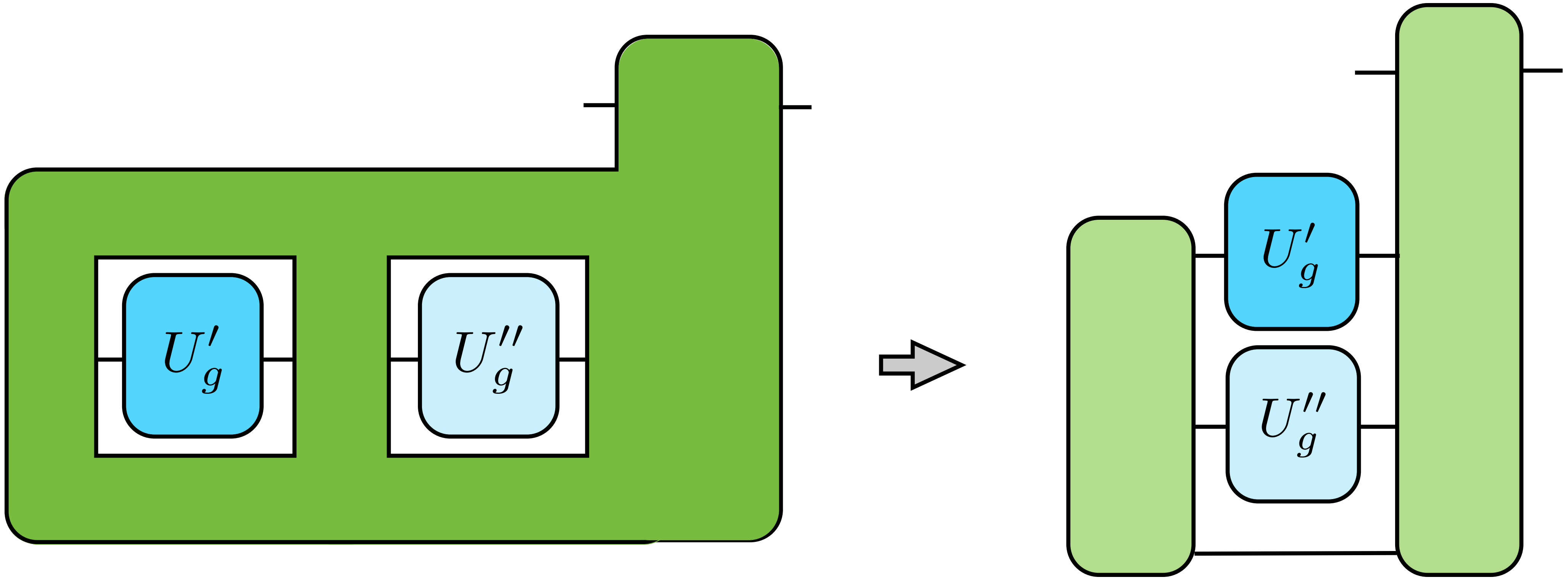
Optimal storing



The optimal storing is parallel

Optimal storing

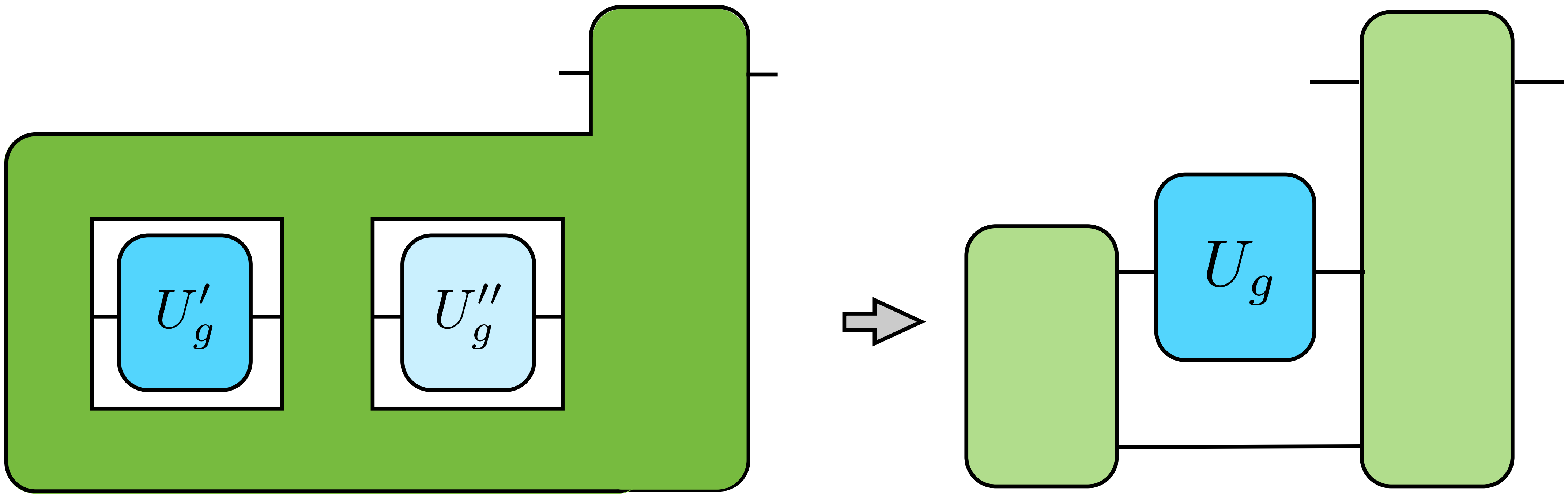


The optimal storing is parallel

⇓

it is a quantum circuit

Optimal storing



$U_g = U'_g \otimes U''_g$

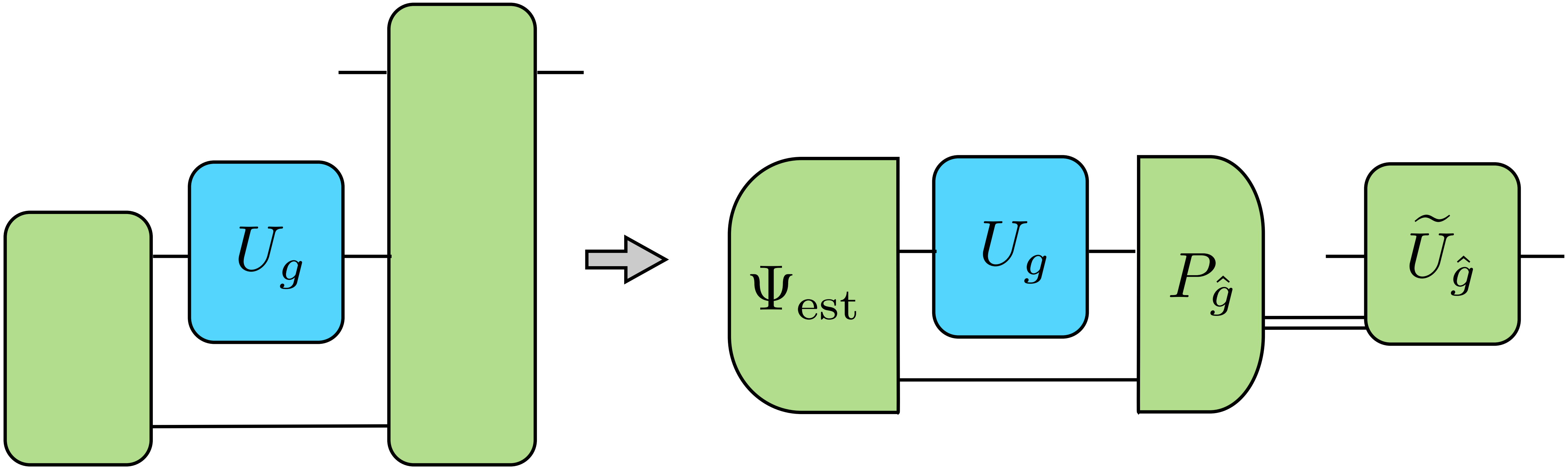
The optimal storing is parallel

it is a quantum circuit

single use case

Optimal storing and retrieving

Measure and prepare



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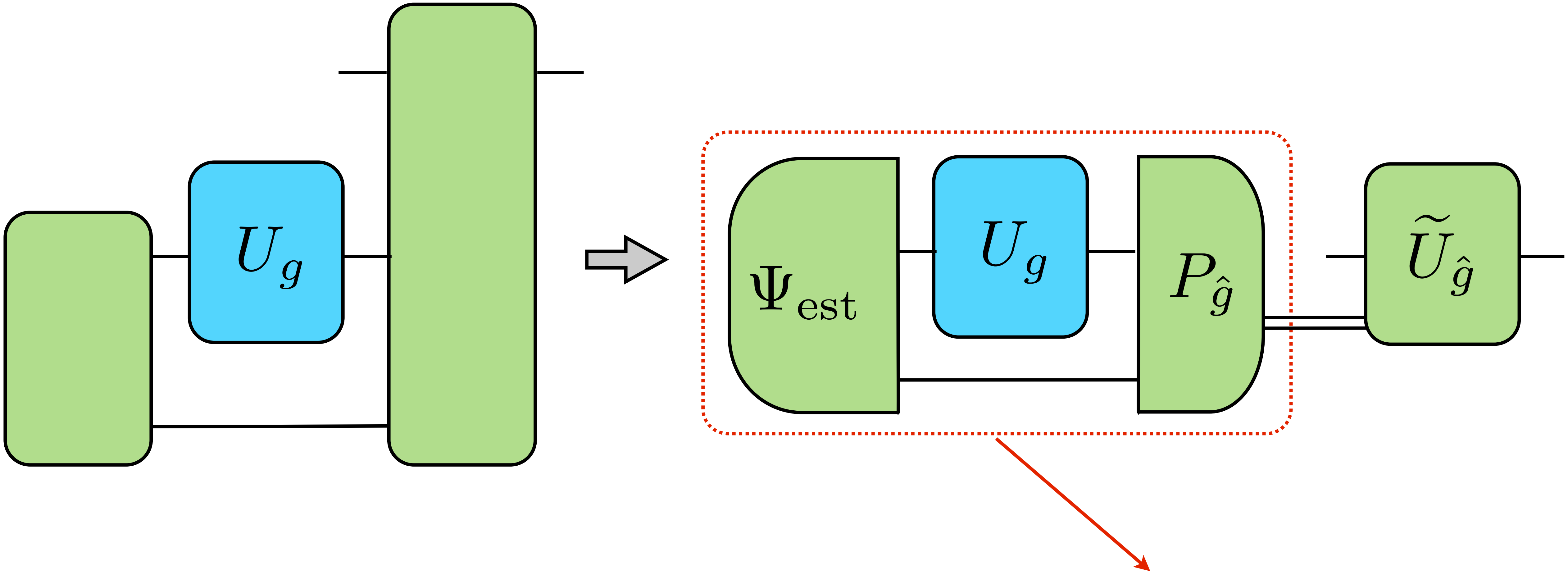
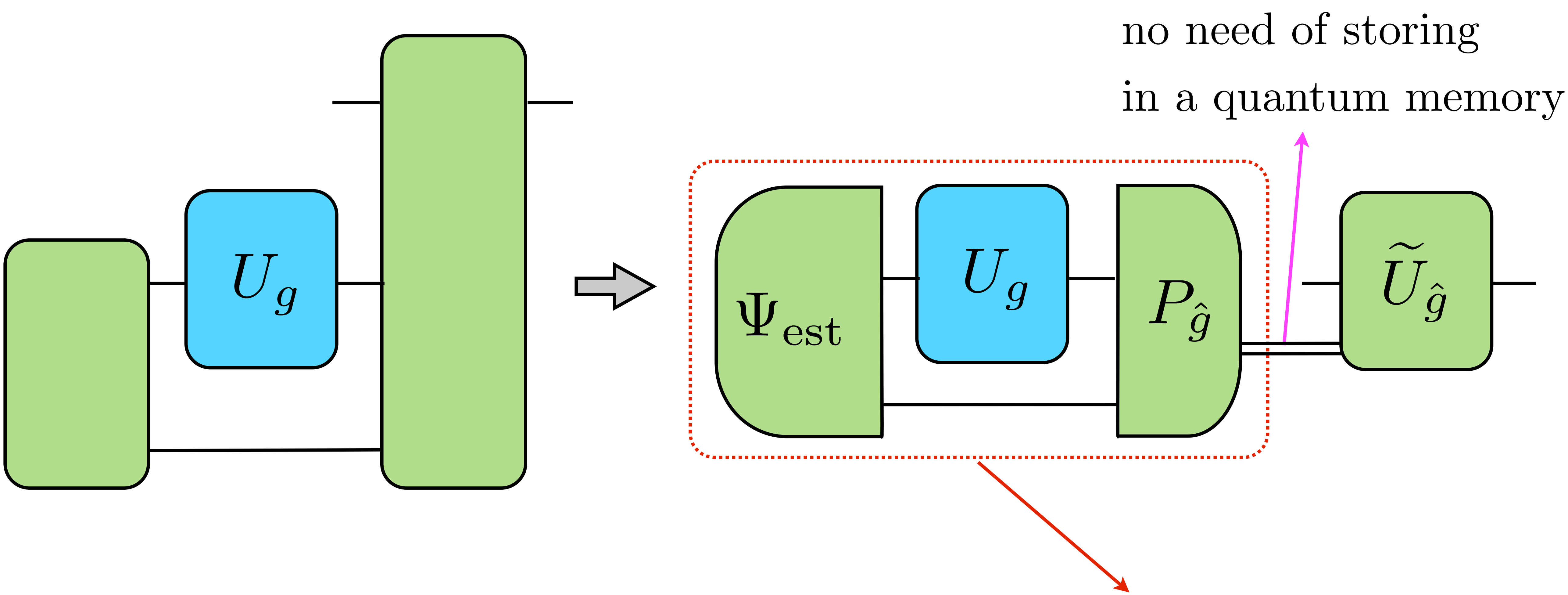


Optimal storing and retrieving ⟺ Optimal estimation

Optimal storing and retrieving

Measure and prepare



Optimal storing and retrieving ⟺ Optimal estimation

Comparing the two scenarios

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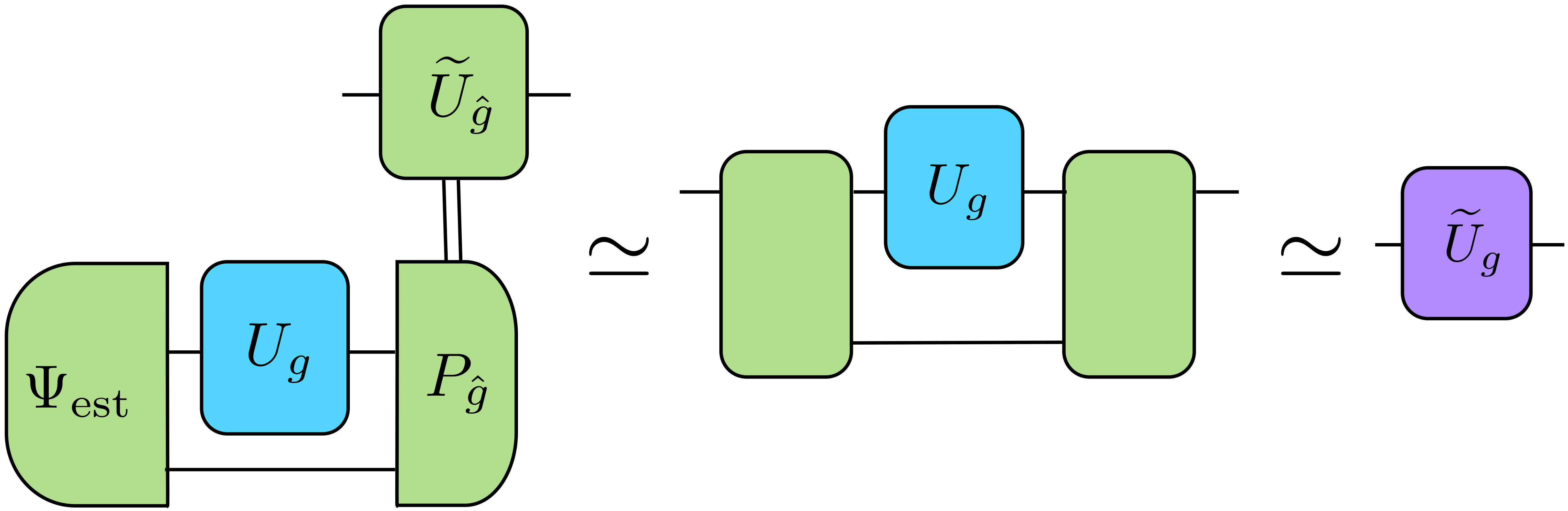
When are the performances of the optimal measure and prepare close to the performances of the optimal pre- and postprocessing?



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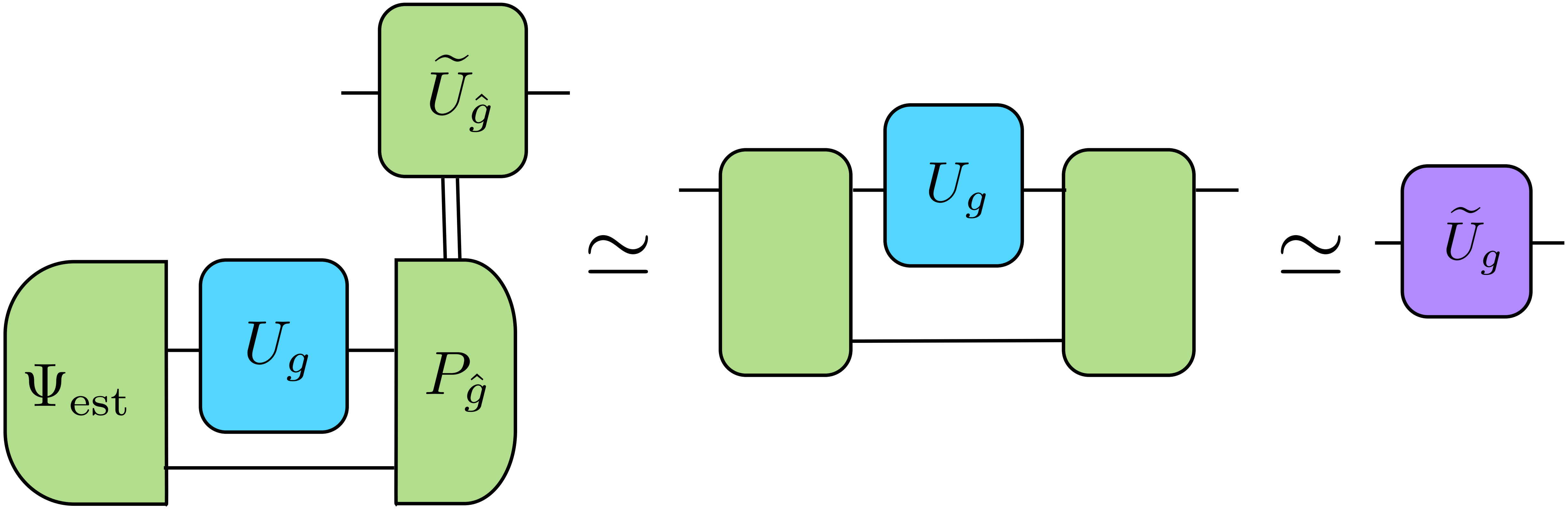

Intuitive answer: when U_g and $\widetilde{U}_g$ are "far apart"

U_g and $\widetilde{U}_g$ are "far apart"

U_g and $\widetilde{U}_g$ are "far apart" → very vague statement

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Let us consider a more specific context:

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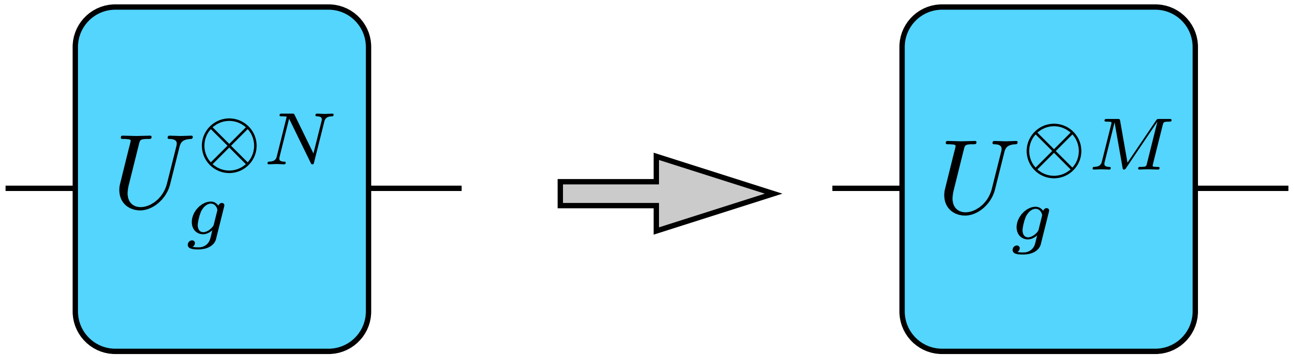

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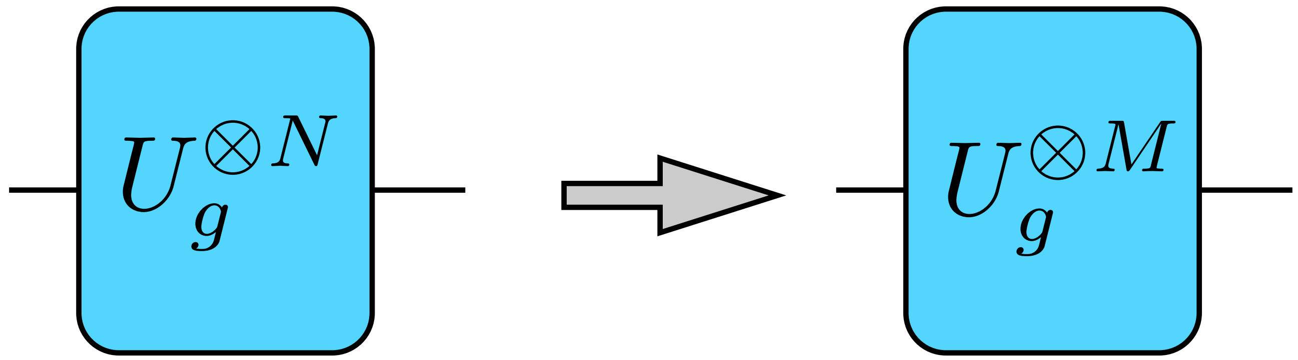

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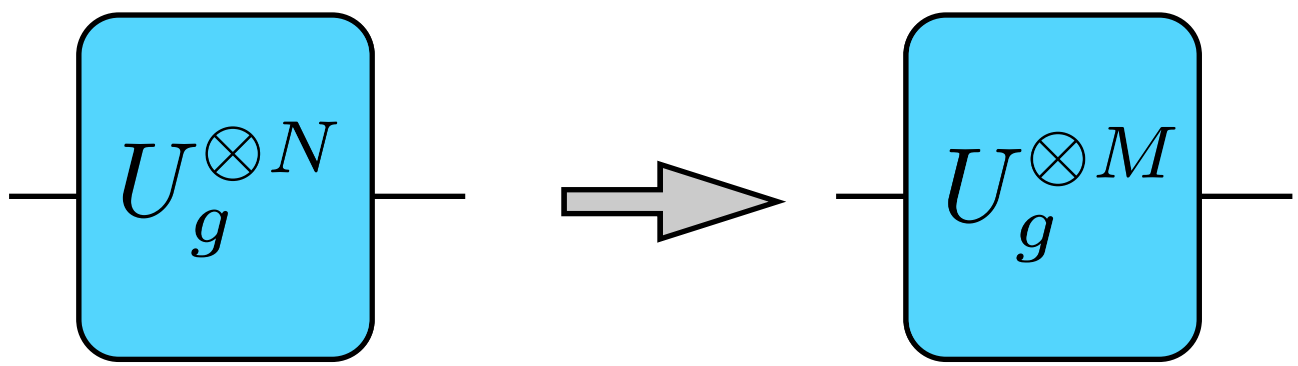

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Can we apply what we learned from states?

Thank you