Quantum limits to the detection of weak signals?

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Quantum limits to the detection of weak signals?

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Quantum limits to the detection of weak signals?

Motivation:

Deals with fundamental quantum concepts: quantum statistics, uncertainty relations, nonclassical light….

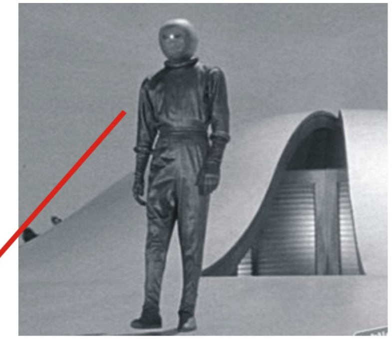
Has practical applications: accuracy or real experiments

Easy to be introduced: basic quantum physics/optics

Richer than it seems at first sight

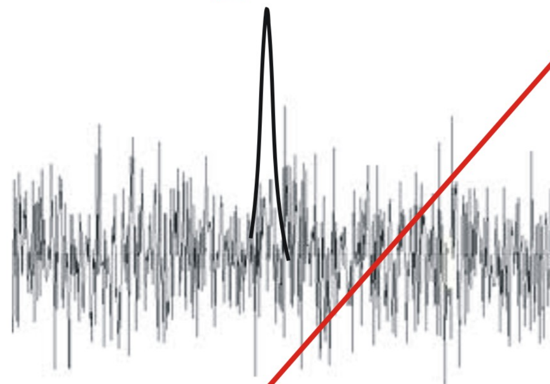
Task: detecting a weak signal χ over background noise

Practical example: an alien send us a message…….



signal χ

Detection if signal > noise



quantum noise

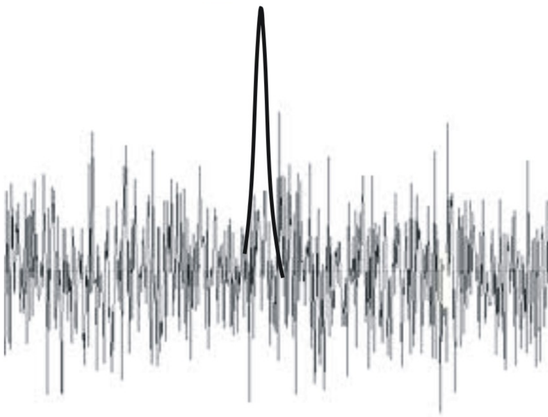
Even if all technical noise is suppressed…. quantum noise will remain

Since quantum noise cannot be removed, seemingly quantum fluctuations will impose a threshold to the minimum signal that can be detected

$$\chi \geq \chi_{\min}$$

or equivalently a minimum uncertainty.

$$\Delta\chi \geq \Delta\chi_{\min} = \chi_{\min}$$



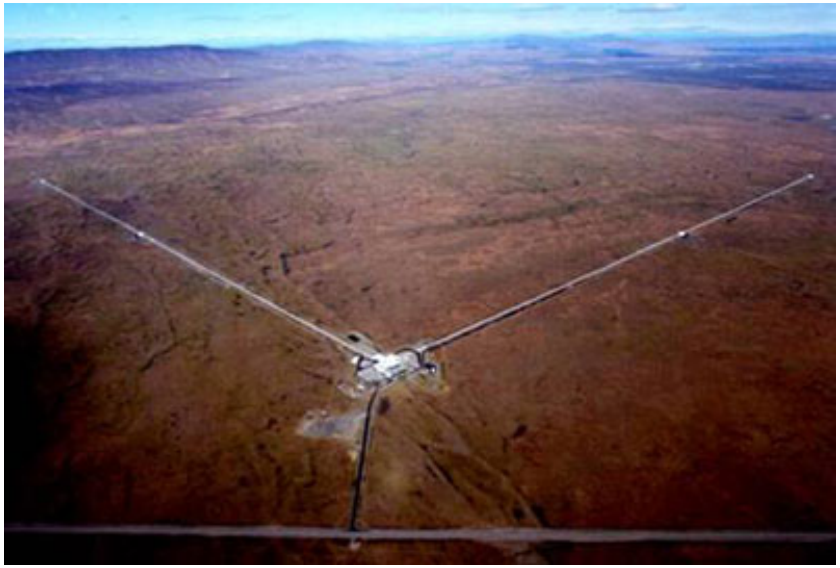
In this talk we address the following points:

✓ Are there quantum limits to the detection of weak signals?

✓ How can they be reached?

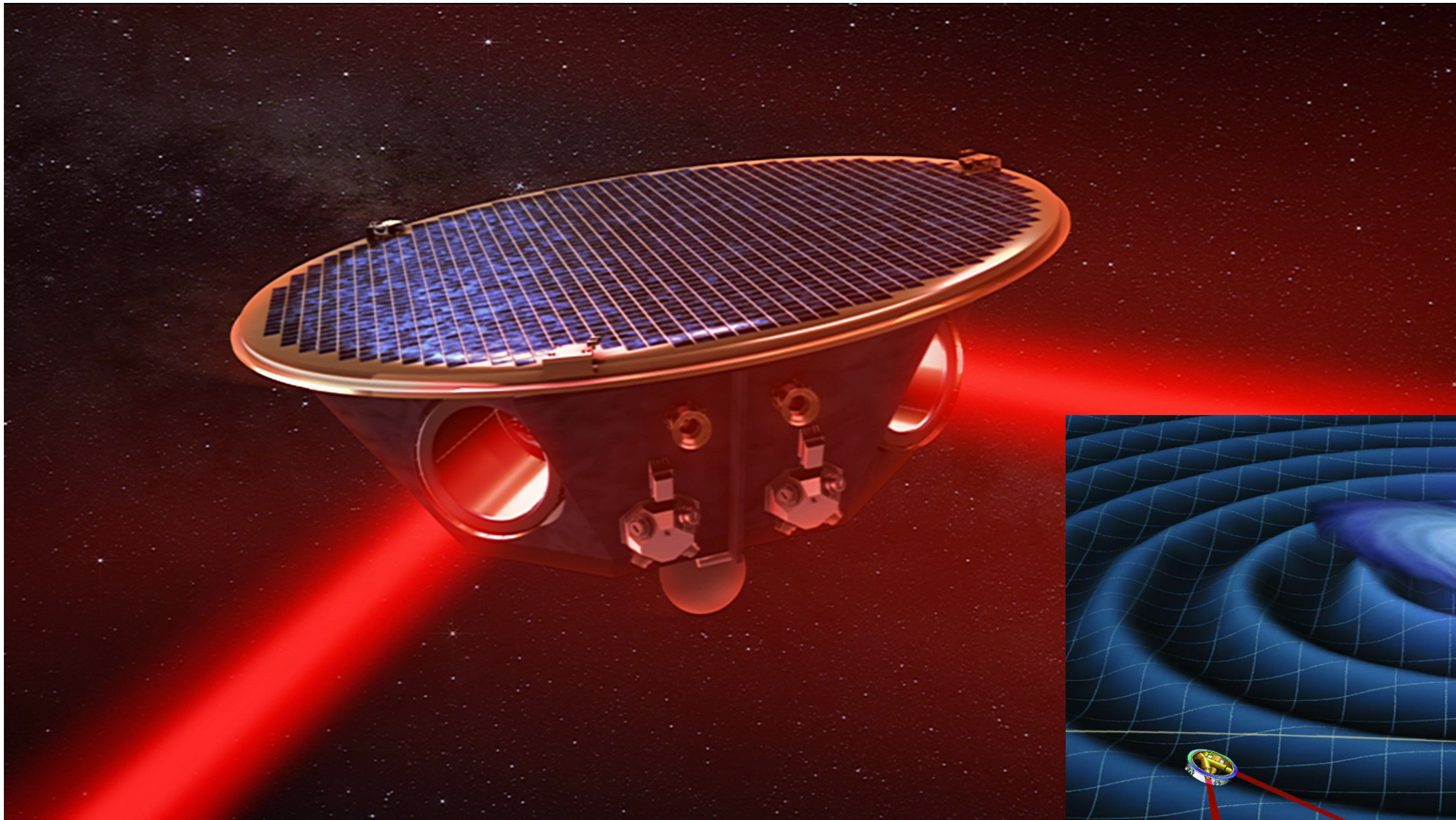
✓ Might be eventualy beaten?

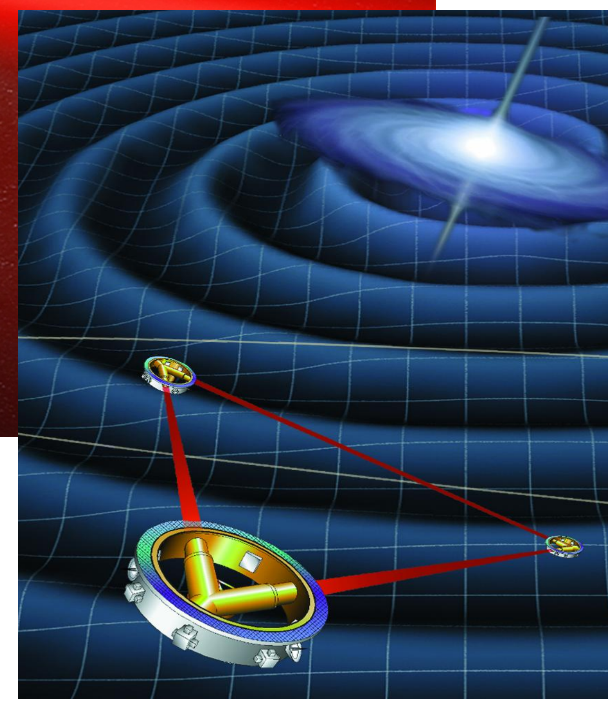
LIGO = Laser Interferometer Gravitational-Wave Observatory



Interferometers with 4 km arms lenght to detect the pass of gravitaional waves. The task is to detect lenght changes of one-thounsand of the proton diameter (10-18 m), so quantum noise is a practical matter.

LISA = Laser Interferometer Space Antenna





Interfermeter made of three satellites separated **5 million kilometers**.

Quantum Limits to Precision for Space Based Devices: Developing the Next Generation of Sensor, Detector and Gyroscope

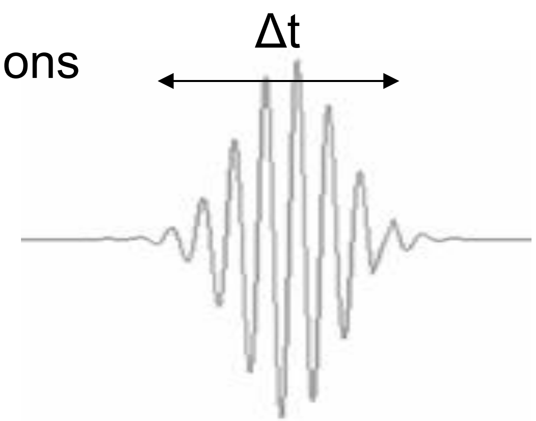
Our research and development focuses on exploring the limits of physical measurement in space based devices The quantum gravitational gradiometer based on an atom interferometer could potentially be used to discover unseen features such as **caves below the surface of Mars**, or lava tubes on the moon in a totally non-invasive fashion, without the need for drilling or impactors. Quantum limited interferometers for gravity wave sensing can operate at the same precision as LIGO but at dramatically reduced laser power.

Very simple illustration: one-photon wave packet

Measuring time/distances with a pulse of duration Δt made of frequencies in an spectral interval Δω

$$\Delta t \approx \frac{1}{\Delta\omega}$$

Quantum domain, pulse of a single photon and Einstein relations



$$E = \hbar\omega \qquad \Delta E = \hbar\Delta\omega \qquad \Delta t \approx \frac{\hbar}{\Delta E}$$

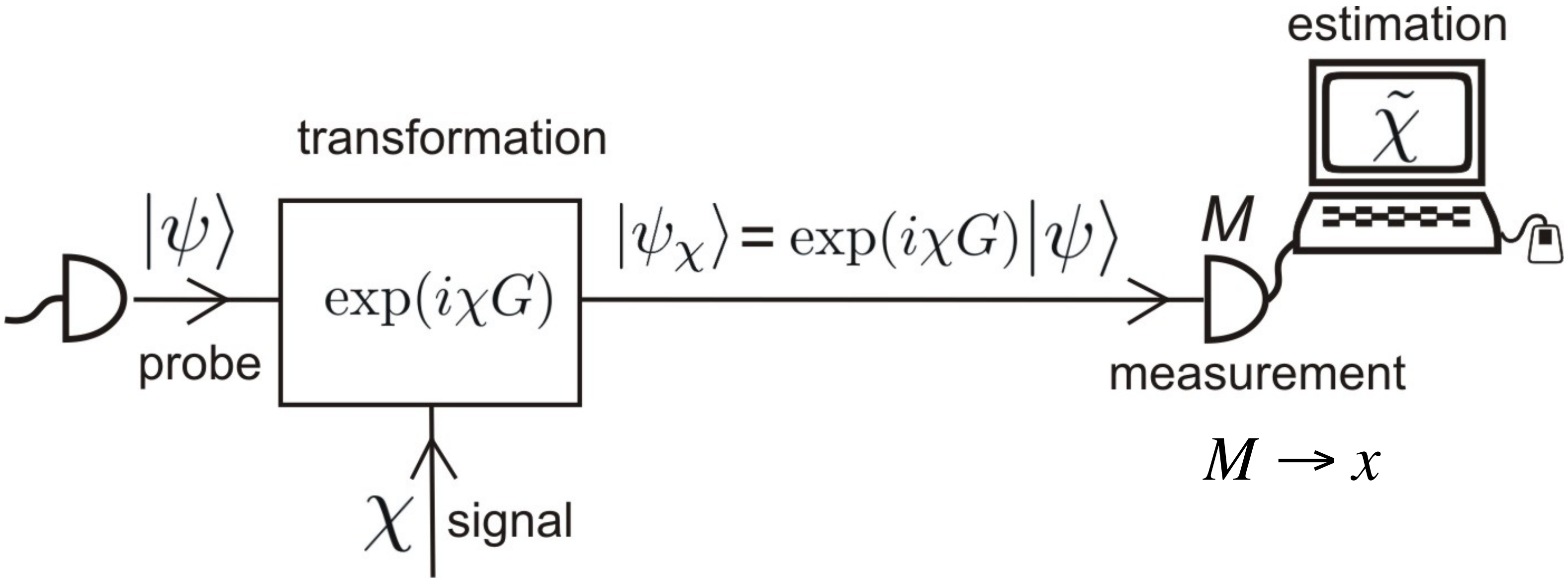
Time-energy uncertainty relation

If moreover $\Delta\omega < \langle\omega\rangle \qquad \Delta E < \langle E\rangle \qquad \Delta t > \frac{\hbar}{\langle E\rangle}$

Key ingredients to be repeated over an over again:

- In general signals are encoded as time or phase
- Quantum limits have to do with some uncertainty relation
- Minimum uncertainty is inversely proportional to energy uncertainty
- Maybe minimum uncertainty inversely proportional to mean energy.

Universal signal detection scheme



$M \rightarrow x$

A probe is prepared in a known state experiencing a signal-dependent transformation.

The state change is monitored measuring some observable M
whose outputs x allow to estimate the value of the signal.

The signal X takes a deterministic nonrandom value (but unknown)

G = generator of the transformation
Typically a time/phase change caused by free propagation
G = n = number of photons = energy

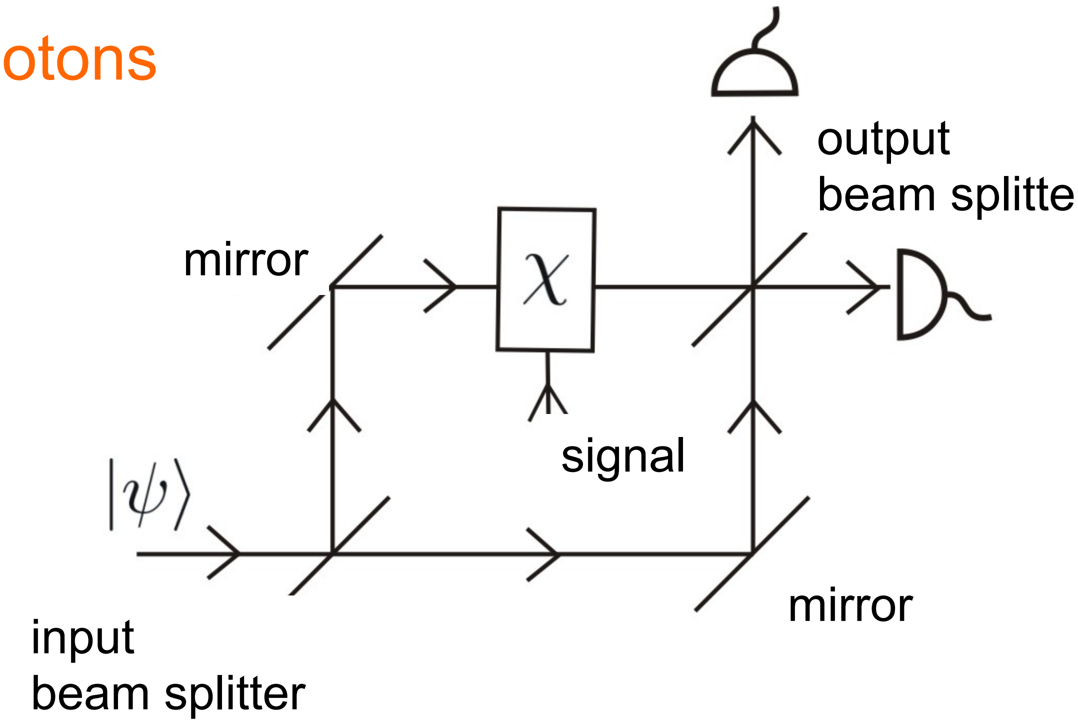
Practical implementation: Mach-Zehnder interferometer

Probe = two-mode input state

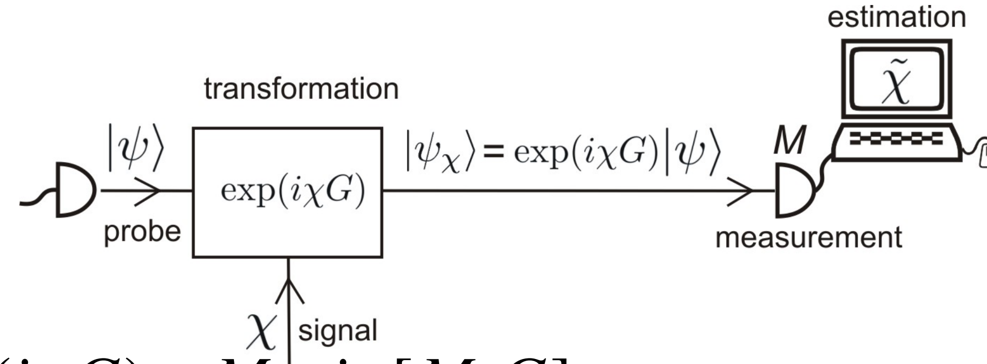
Signal = phase change   X=ωt

G ≈ n = number of photons

M = output intensities



Weak signal X<<1

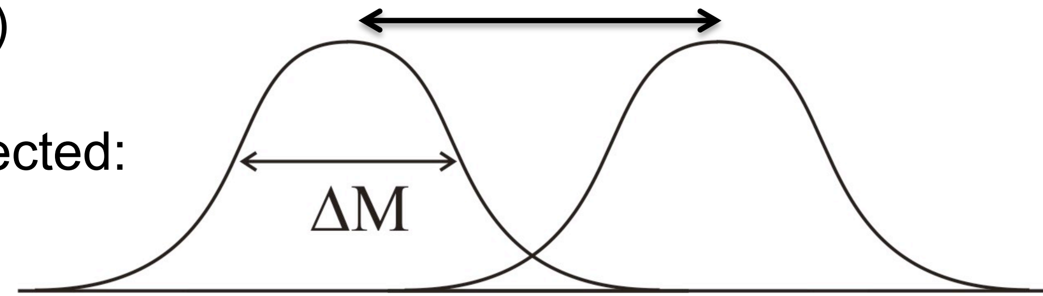


$$M(\chi) = \exp(-i\chi G) M \exp(i\chi G) \approx M + i\chi[M,G]$$

Signal = shift of the measured observable M

The shift must be larger than the noise (uncertainty of M)

$$\left|\langle [G,M] \rangle\right| \chi_{\min} = \left|M(\chi_{\min}) - M(0)\right| > \Delta M$$



Minimum signal that can be detected:

$$\Delta M \approx \left|\langle [M,G] \rangle\right| \chi_{\min}$$

$$\chi_{\min} \approx \frac{\Delta M}{\left|\langle [M,G] \rangle\right|}$$

$$\Delta M \Delta G \geq \frac{1}{2}\left|\langle [M,G] \rangle\right|$$

$$\chi_{\min} = \Delta\chi_{\min} = \frac{1}{2\Delta G}$$

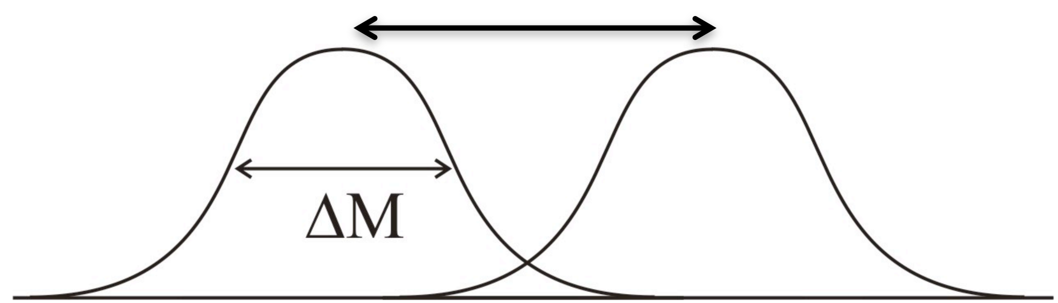
Quantum limit as consequence of uncertainty relations

$M(\chi) \approx M(0) + i\chi[M(0), G]$ Necessary condition $[G, M] \neq 0$

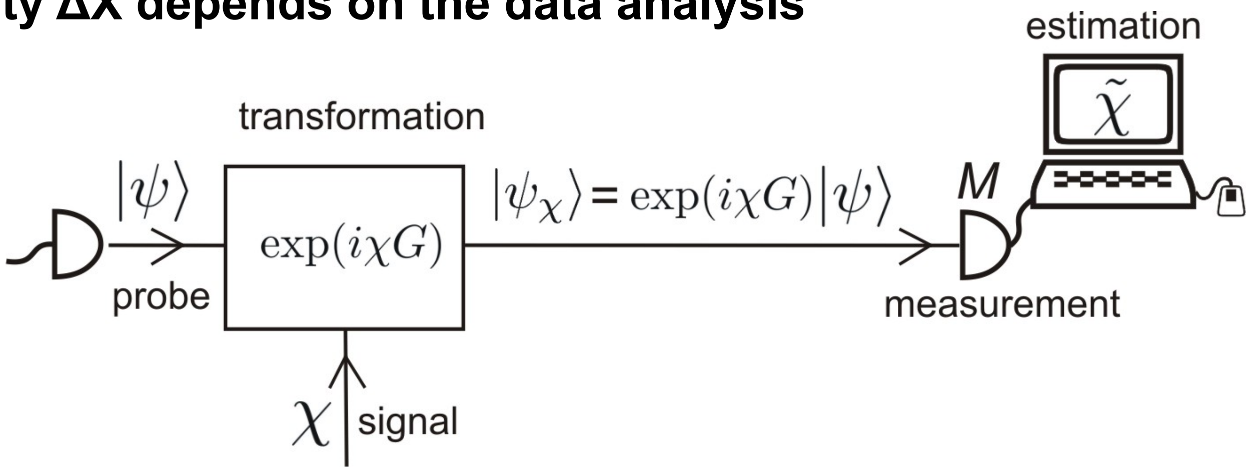
Thus $[M(X), M(0)] \neq 0$, unless $[G, M] \propto I$

$\Delta M(X)\Delta M(0) > 0$ and $X_{\min} = \Delta X_{min} \neq 0$

$$\left|\left\langle [G, M] \right\rangle\right| \chi_{\min} = \left|M(\chi_{\min}) - M(0)\right| > \Delta M$$



A more rigorous approach:
The final uncertainty ΔX depends on the data analysis



The measurement is repeated *m* times $\vec{x}_m = (x_1, \ldots, x_m)$

The total mean number of photons used *N*= *m*<*n*>
<*n*> = mean number of photons of the probe

Inference: estimating the signal and its uncertainty after the results obtained

$$\tilde{\chi}(\vec{x}_m) \qquad \Delta\tilde{\chi}$$

For example estimating the signal via **maximum likelihood**:
$\tilde{\chi}$ = the signal that maximizes the probability of the outcomes actually obtained

Typical estimation of uncertainty = **mean squared error**

$$\Delta_\chi^2 \tilde{\chi} = \sum_{\vec{x}_m} \left[\tilde{\chi}(\vec{x}_m) - \chi \right]^2 P(\vec{x}_m \mid \chi)$$

$P(\vec{x}_m \mid \chi)$ probability of outcomes x_m when the signal is X

Bounded from below by the **Cramér-Rao lower bound**

For unbiased estimators $\langle \tilde{\chi} \rangle = \chi$

$$\Delta_\chi^2 \tilde{\chi} \geq \frac{1}{mF(\chi)} \geq \frac{1}{mF_Q(\chi)} = \frac{1}{4m\Delta^2 G}$$

← for probes in pure states

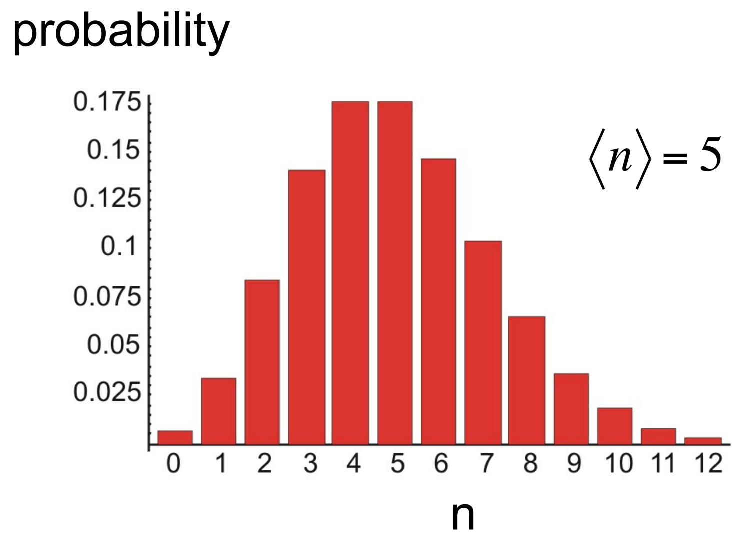
F= Fisher information F_Q = quantum Fisher information

$$F(\chi) = \sum_m \frac{1}{P(\vec{x}_m \mid \chi)} \left[\frac{\partial P(\vec{x}_m \mid \chi)}{\partial \chi} \right]^2 \leq F_Q(\chi) = 4m\Delta^2 G$$

$$\Delta_\chi \tilde{\chi} \geq \frac{1}{2\sqrt{m}\Delta G}$$

The same result we obtained before

Typical case: $G = n, \quad \Delta\chi \geq \dfrac{1}{2\sqrt{m}\Delta n}$



Classical light: **standard quantum limit**

Coherent state = Poissonian statistics

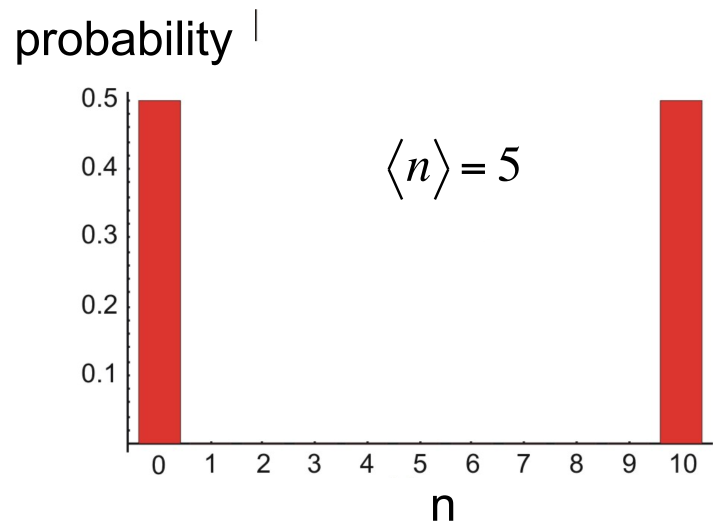
$$\Delta n = \sqrt{\langle n \rangle} \qquad \Delta\chi \geq \frac{1}{2\sqrt{m\langle n \rangle}}$$

Nonclassical light: Heisenberg limit

Coherent superposition vacuum n=0 and number n=2<n> $\quad |\psi\rangle \propto |n=0\rangle + |n=2\langle n \rangle\rangle$

$$\Delta n = \langle n \rangle >> \sqrt{\langle n \rangle} >> 1$$

$$\Delta\chi \geq \frac{1}{2\sqrt{m}\langle n \rangle} << \frac{1}{2\sqrt{m\langle n \rangle}}$$



Heisenberg limit (weak form)

Heisenberg limit (strong form)
Ultimate quantum limit?

Minimum uncertainty given by the inverse of the total number of photons (or total energy) used in the detection process

$$\Delta\chi_{\min} \approx \frac{1}{2m\langle n\rangle}$$

Universally adopted as quantum limit, the best that can be done in any case

Necessary conditions in typical approaches: after $\Delta\chi \geq \frac{1}{2\sqrt{m}\Delta n}$

i) All photons must be used in a single realization of the measurement m=1
ii) The probe must be in a nonclassical state with Δn ~<n>

Surprisingly without universally valid demonstration !!!???

On what follows: Discussion of four alternatives to the Heisenberg limit

ALTERNATIVE TO HEISENBERG LIMIT NUMBER 1

Typically uncertainty Δ is assessed via variance. But this is not the only possibility nor the better behaved one, specially for non Gaussian statistics (all the previous examples)

Other estimators of uncertainty

TSALLIS ENTROPIES

$$S_q = \frac{1 - \sum_j p_j^q}{q - 1}$$

EXPONENTIAL OF RENYI ENTROPIES

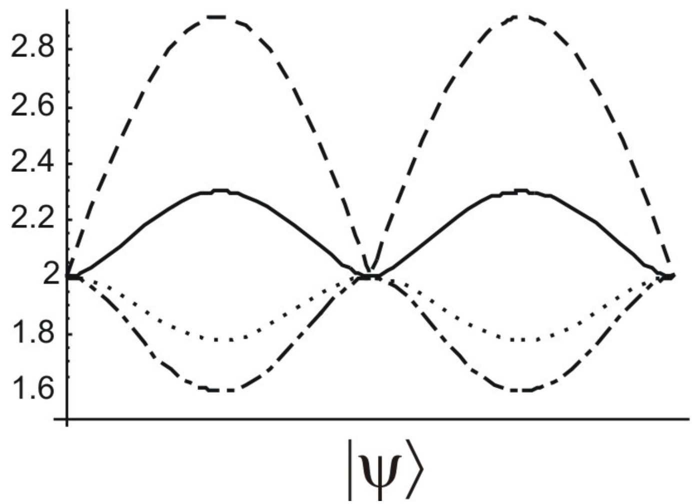
$$R_q = \left(\sum_j p_j^q \right)^{1/(1-q)}$$

p_j = probability distribution q = parameter (arbitrary) $q \to 1$ Shannon entropy

Generalized entropy as a measure of quantum uncertainty
M Portesi, A Plastino, Physica A **225**, 412 (1996)

AMBIGUITY!

$R_q(\sigma_x)R_q(\sigma_y)$ Joint uncertainty for two complementary observables for a qubit



The same state can be of maximum or minimum uncertainty depending on the q value.

In the figure q=0.5,1,2,3

Effect of fluctuation measures on the uncertainty relations between two observables: Different measures lead to opposite conclusions
A. Luis, Phys. Rev. A **84**, 034101 (2011)

On the connection between complementarity and uncertainty principles in the Mach-Zehnder interferometric setting
G. M. Bosyk, M. Portesi, F. Holik, and A. Plastino Phys. Scr. **87**, 065002 (2013)

Contradictory entropic joint uncertainty relations for complementary observables in two-level systems, Alfredo Luis arXiv:1306.5211

NO UNCERTAINTY RELATIONS ??!!!

For q=2 there is no position-momentum uncertainty relation

$$R_2(x)R_2(p) \to 0$$

Quantum limits arise from uncertainty relations?

Some extensions of the uncertainty principle

Steeve Zozor[a,*], Mariela Portesi[b], Christophe Vignat[c]

[a] GIPSA-Lab, Département Images et Signal, 961 Rue de la Houille Blanche, B.P. 46, 38420 Saint Martin d'Hères Cedex, France

[b] Instituto de Física La Plata (IFLP, CCT-La Plata, CONICET), and Departamento de Física, Facultad de Ciencias Exactas, Universidad Nacional de La Plata, C.C. 67, 1900 La Plata, Argentina

[c] Laboratoire d'Informatique de l'Institut Gaspard Monge, Equipe SYSCOM, Université de Marne-la-Vallée, 77 454 Marne-la-Vallée Cedex 2, France

Physica A **387**, 4800 (2008)

An attempt of application of Renyi measures in metrology

Comparing the observed statistics before and after the signal.
Establishing a threshold we obtain a minimum detectable signal.

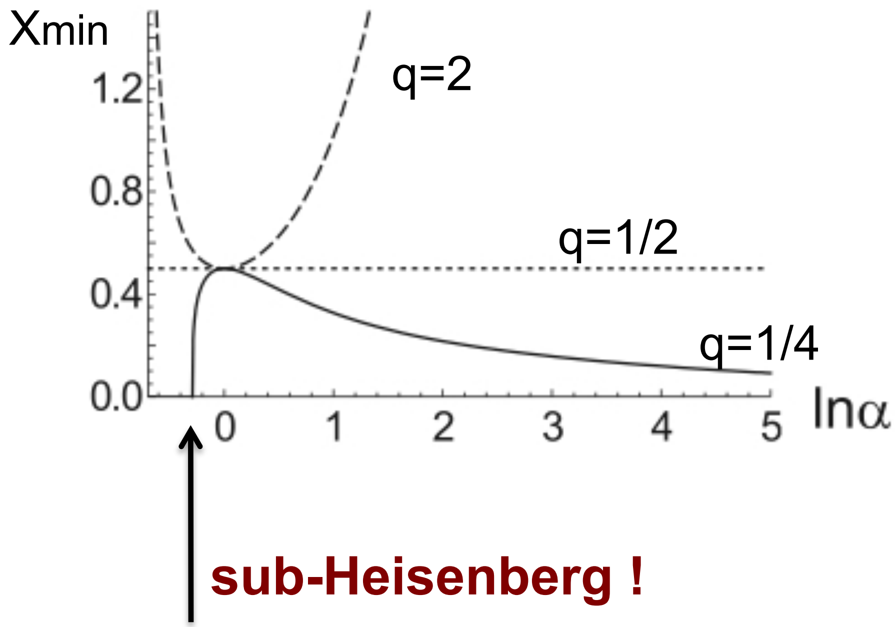
$$\int_{-\infty}^{\infty} dx \left([P(x-\chi)]^q - [P(x)]^q \right)^{1/q} > \text{threshold} \leftrightarrow \chi > \chi_{\min} \qquad P(x) = \frac{\alpha 2^{1/\alpha}}{2\gamma\Gamma(1/\alpha)} \exp\left(-2|x/\gamma|^\alpha\right)$$

P(x), statistics
α, γ = parameters

Same mean energy in all cases

Contradictions between different q values

No minimum signal for q<1/2 and α≈1-q



Alternative measures of uncertainty in quantum metrology: Contradictions and limits
Alfredo Luis and Alfonso Rodil, Phys. Rev. A **87**, 034101 (2013)

ALTERNATIVE TO HEISENBERG LIMIT NUMBER 2

NONLINEAR OPTICS

$$G \propto n^2 \to \Delta G \propto 2\langle n \rangle \Delta n \qquad \Delta\chi \propto \frac{1}{2\sqrt{m}\Delta G} \propto \frac{1}{4\langle n \rangle \sqrt{m}\Delta n}$$

For example, if classical probe $\Delta n = \sqrt{\langle n \rangle}$

$$\Delta\chi \propto \frac{1}{4\langle n \rangle \sqrt{m\langle n \rangle}} << \frac{1}{2m\langle n \rangle} \quad \text{if } \langle n \rangle >> m$$

Classical light can do much better than the Heisenberg limit

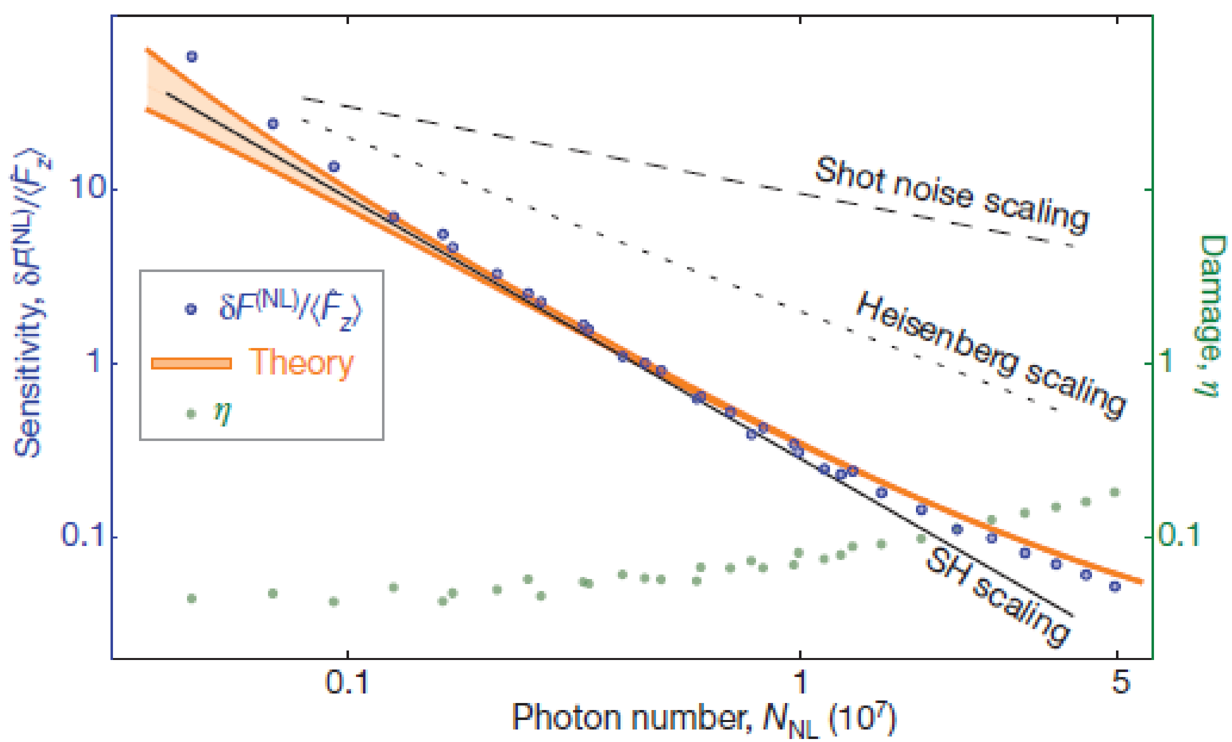
Nonlinear transformations and the Heisenberg limit
A. Luis, Phys. Lett. A **329**, 8 (2004)

Experimental confirmation ICFO:

Interaction-based quantum metrology showing scaling beyond the Heisenberg limit

M. Napolitano[1], M. Koschorreck[1], B. Dubost[1,2], N. Behbood[1], R. J. Sewell[1] & M. W. Mitchell[1]

486 | NATURE | VOL 471 | 24 MARCH 2011



Trick? We still have $\Delta\chi \geq \frac{1}{2\sqrt{m\Delta G}}$ for different G. **But what matters** $\Delta\chi << \frac{1}{2m\langle n \rangle}$

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Batido el límite de precisión de medida que impone la mecánica cuántica

Un experimento realizado en Barcelona muestra cómo soslayar el principio de incertidumbre de Heisenberg, pero no lo niega

EL PAÍS - Madrid - 25/03/2011

Press release:

Heisenberg uncertainty principle *is avoided….*

….. without being denied

http://sociedad.elpais.com/sociedad/2011/03/25/actualidad/1301007607_850215.html

More striking features of nonlinear detection schemes:

Mixed classical probes can do much better than classical pure probes with the same <n>

Mixed states have larger uncertainty than pure states?

Precision Quantum Metrology and Nonclassicality in Linear and Nonlinear Detection Schemes
Angel Rivas and Alfredo, Luis Phys. Rev. Lett. **105**, 010403 (2010)

ALTERNATIVE TO HEISENBERG LIMIT NUMBER 3

Generator different from energy beyond nonlinear optics

Signal not encoded as phase shift, but momentum shift of a free particle

$$p \rightarrow p + \chi$$

Generator = position operator

$$G = x \neq \text{energy} \propto p^2, \quad M = p$$

$$\text{if } \langle p \rangle = 0, \quad \Delta\chi \approx \Delta p \approx \sqrt{\langle \text{energy} \rangle}$$

The lesser the energy the lesser the uncertainty
Exactly the opposite of the Heisenberg limit

Note that *[M(X),M(0)]=0* since *[G,M(0)]~I* ***Quantum Non-demolition detection***

Signal detection without finite-energy limits to quantum resolution
A. Luis, Ann. Phys. 331, 1-8 (2013)

ALTERNATIVE TO HEISENBERG LIMIT NUMBER 4

Back to the usual case of signal= phase and G = n

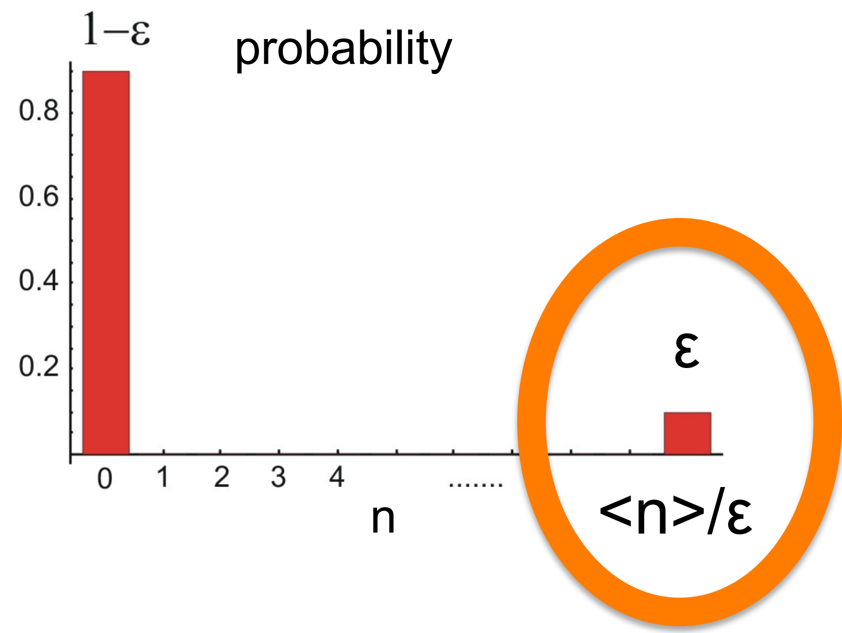
Uncertainty depends on Δn, not on <n> $\Delta\chi \geq \dfrac{1}{2\sqrt{m}\Delta n}$ While Heisenberg limit goes as 1/<n>

We can have Δn -->∞ with finite <n>

$$|\psi\rangle = \sqrt{1-\varepsilon}\,|n=0\rangle + \sqrt{\varepsilon}\,|n=\langle n\rangle/\varepsilon\rangle$$

Same <n> for all ε

$$\Delta n = \langle n\rangle\sqrt{\frac{1-\varepsilon}{\varepsilon}} \rightarrow \infty \quad \text{si} \quad \varepsilon \rightarrow 0$$

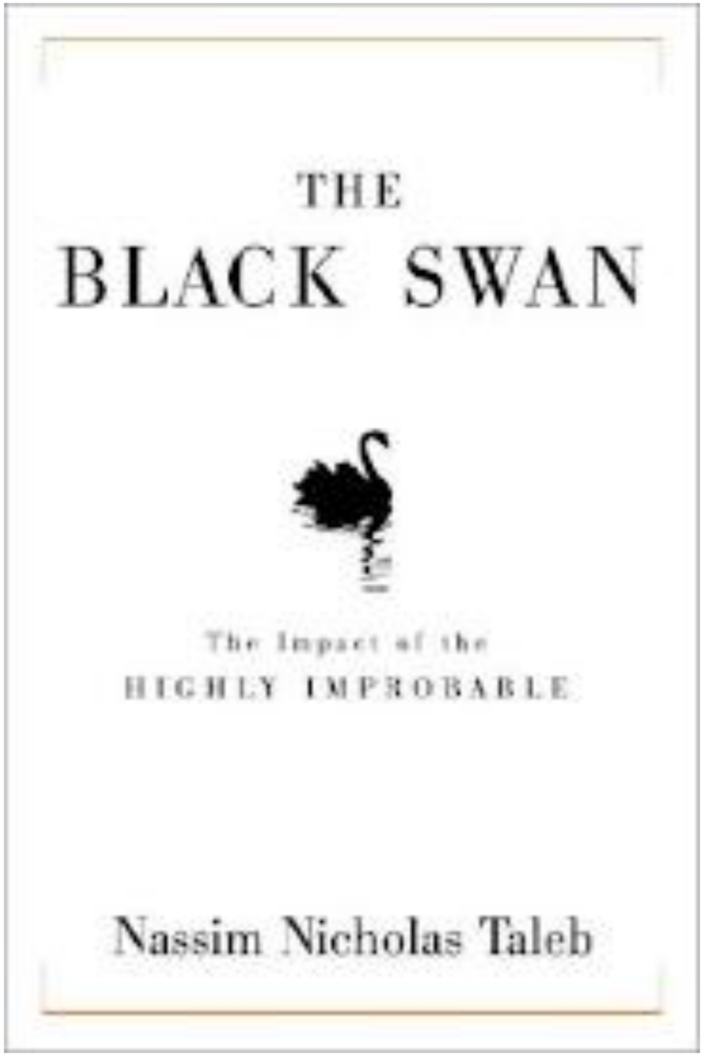


Paradoxical?

When ε→0 the probe tends to be the vacuum insensitive to phase shifts

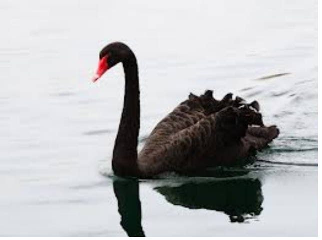
Nonlinearity of variance amplifies the small probability of large numbers

Importance of unlikely events in social sciences



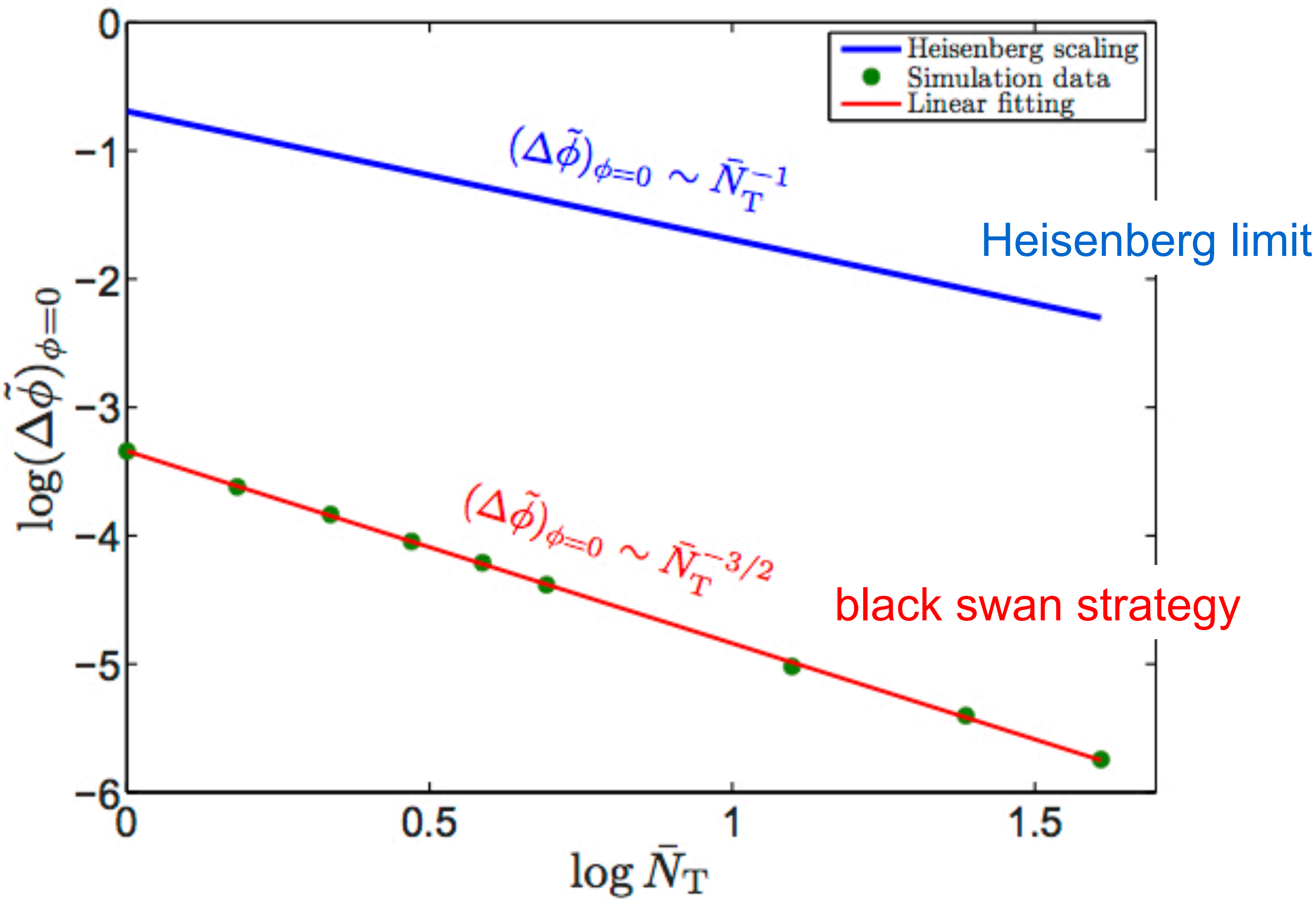
http://www.fooledbyrandomness.com/

The case $\Delta n \to \infty$ with finite $\langle n \rangle$
might be called *black swan* strategy



Many repetitions are necessary to find a black swan but this does not spoil subHeisenberg uncertainty

Numerical demonstration of the beating of the strong form of Heisenberg limit via *black swan* strategy for zero signal



Sub-Heisenberg estimation of non-random phase shifts
A. Rivas and A. Luis, New Journal of Physics 14, 093052 (2012)

Simple extreme demonstration that the Heisenberg limit can be violated

Vanishing signal $\chi = 0$ and probe prepared in an eigenstate of M

$\Delta M = 0$ and $\Delta\chi = 0$ for all $\langle n \rangle$

No Heisenberg limit at all

But only works for definite known values of the signal

Key point: $\Delta\tilde{\chi}$ **depends on the signal value** χ

Mean square error depends on the unknown value of the signal X

$$\Delta_{\chi}^{2}\tilde{\chi} = \sum_{\vec{x}_m} \left[\tilde{\chi}\left(\vec{x}_m\right) - \chi \right]^2 P\left(\vec{x}_m \mid \chi\right)$$

The most popular strategy averages the uncertainty over some prior range

$$\Delta^2 \tilde{\chi} = \int d\chi P(\chi) \Delta_{\chi}^{2} \tilde{\chi}$$

P(X) = some prior distribution for the signal value

Generator = number n
The average obeys a true strong-form Heisenberg limit

$$\Delta\tilde{\chi} \geq \frac{1}{2m\langle n \rangle}$$

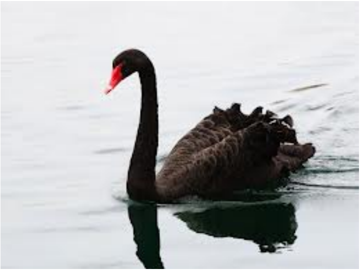
V. Giovannetti, L. Maccone, Phys. Rev. Lett. 108, 210404 (2012)
M. Tsang, Phys. Rev. Lett. 108, 230401 (2012)
V. Giovannetti, S. Lloyd, L. Maccone, Phys. Rev. Lett. 108, 260405 (2012)
M. Hall, D. W. Berry, M. Zwierz, H. M. Wiseman, Phys. Rev. A 85, 041802 (2012)
M. J. W. Hall, H. M. Wiseman, New J. Phys. 14, 033040 (2012)
R. Nair, arXiv:1204.3761 [quant-ph].

$$\Delta\tilde{\chi} \geq \frac{1}{2m\langle n\rangle}$$

This average reveals that sub-Heisenberg for some X implies super-Heisenberg for other X .

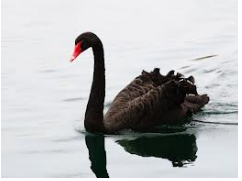
Typically transition from sub to super holds for X = Heisenberg limit

Sub-Heisenberg would require sub-Heisenberg prior knowledge



black swan = ugly duck?

Nevertheless, black swan strategies might still be useful?



They may provide better experimental bounds for quantities with strong theoretical support, such as the photon mass?

Currently $m < 1.2 \times 10^{-54}\,\mathrm{kg}$

New Experimental Limit on the Photon Rest Mass with a Rotating Torsion Balance
J. Luo, L.-Ch. Tu, Z.-K. Hu, and E.-J. Luan, Phys. Rev. Lett. 90, 081801 (2003)

Two forms of Heisenberg limits depending on m

Weak form $$\Delta\tilde{\chi} \propto \frac{1}{\sqrt{m\langle n\rangle}}$$ Repetitions are statistically independent

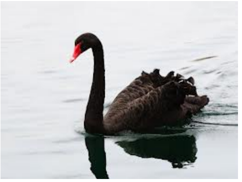
Strong form $$\Delta\tilde{\chi} \propto \frac{1}{m\langle n\rangle}$$ Theoretical limit

Coincide only for *m* = 1 (all photons used in a single realization of the experiment)

Heisenberg limit only reached for nonclassical states, small photon numbers say $\langle n \rangle \approx 10$

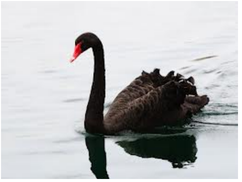
Therefore practical detection requires high *m* and thus far from the strong limit.

First results suggest that black swans might allow to reach the strong limit for large m!!!



Previous results assume unbiased estimators $\langle \tilde{\chi} \rangle = \chi$

Sub-Heisenberg = bias



On sub-Heisenberg phase uncertainties
Luca Pezzé, Phys. Rev. A. 88, 060101 (2014)

Biased estimators can do better than unbiased ones

Rethinking Biased Estimation
Steven Kay and Yonina C. Eldar
[lecture NOTES] IEEE SIGNAL PROCESSING MAGAZINE [133] MAY 2008

Rethinking Biased Estimation: Improving Maximum Likelihood and the Cramér–Rao Bound
Yonina C. Eldar
Foundations and Trends in Signal Processing
Vol. 1, No. 4 (2007) 305–449 2008 DOI: 10.1561/2000000008

Averaging over signal values that will never arise?

we are assuming deterministic non-random signal

$$\Delta^2\tilde{\chi} = \int d\chi P(\chi)\Delta_\chi^2\tilde{\chi} \geq \frac{1}{4m^2\langle n\rangle^2}$$

P(X) = prior distribution for the signal value

This suggest addressing a Bayesian picture

Bayesian alternative. Probability = degree of belief

Bayes theorem: $P\left(\tilde{\chi} \mid \vec{x}_m\right) \propto P\left(\vec{x}_m \mid \tilde{\chi}\right) P\left(\tilde{\chi}\right)$

$P\left(\tilde{\chi}\right) =$ what we know about $\tilde{\chi}$ before measurement

$P\left(\tilde{\chi} \mid \vec{x}_m\right) =$ probability a posteriori for $\tilde{\chi}$ given the outcomes $\vec{x}_m$

$P\left(\vec{x}_m \mid \tilde{\chi}\right) =$ probability of outcomes $\vec{x}_m$ when the signal is $\tilde{\chi}$

Averaging over all possible outcomes gives a posterior distribution

$$P\left(\tilde{\chi} \mid \chi\right) \propto \sum_{\vec{x}_m} P\left(\tilde{\chi} \mid \vec{x}_m\right) P\left(\vec{x}_m \mid \chi\right)$$

Uncertainty = width of $P\left(\tilde{\chi} \mid \chi\right)$ ≠ mean square error

(there is no estimator $\tilde{\chi}\left(\vec{x}_m\right)$ *at work)*

$$\Delta_\chi^2 \tilde{\chi} = \sum_{\vec{x}_m} \left[\tilde{\chi}\left(\vec{x}_m\right) - \chi\right]^2 P\left(\vec{x}_m \mid \chi\right)$$

Quantum Phase in Interferometry
Z. Hradil, R. Myška, J. Peřina, M. Zawisky, Y. Hasegawa, and H. Rauch
Phys. Rev. Lett. **76**, 4295 (1996)

Estimation of counted quantum phase
Zdeněk Hradil
Phys. Rev. A **51**, 1870 (1995)

Entropy of phase measurement: Quantum phase via quadrature measurement
Zdeněk Hradil, Robert Myška, Tomáš Opatrný, and Jiří Bajer
Phys. Rev. A **53**, 3738 (1996)

Summarizing: Potential open questions regarding quantum limits

Uncertainty measures different from variance

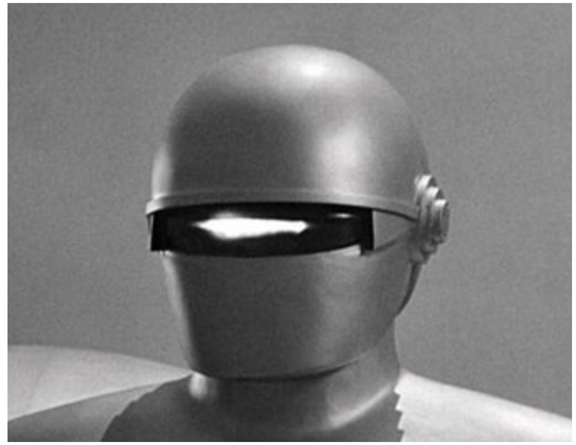
Transformations different from phase/time shifts

Breaking weak Heisenberg for multiple repetitions

Biased estimators

Bayesian picture

THANKS



….for there are some who weary themselves out in learning and proving things that, after they are known and proved, are not worth a farthing to the understanding or memory…

Miguel de Cervantes, *Don Quixote* Chapter XXII

To give an accurate description of what has never occurred is not merely the proper occupation of the historian, but the inalienable privilege of any man of parts and culture.

Oscar Wilde, *The Critic as Artist*

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