Nonlinear Losses

or

How to make friends out of worst enemies

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Work done in cooperation with

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Motivation: turning the worst enemy into the best friend

Purpose: generating nice non-classicality
in a robust deterministic way

Outline:

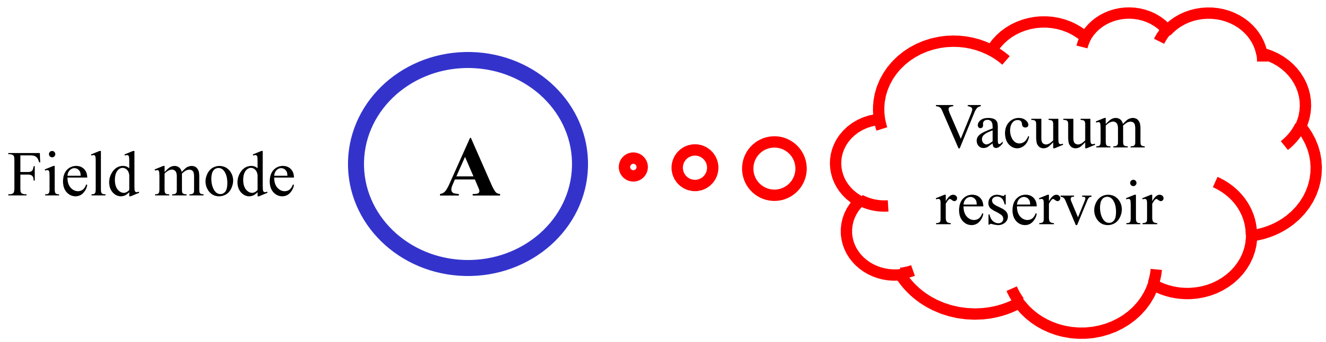
1. Are losses always bad for non-classicality?

2. What can nonlinear losses do?

3. How can one produce these losses?

4. How can one avoid linear losses?

What common linear loss is commonly doing?



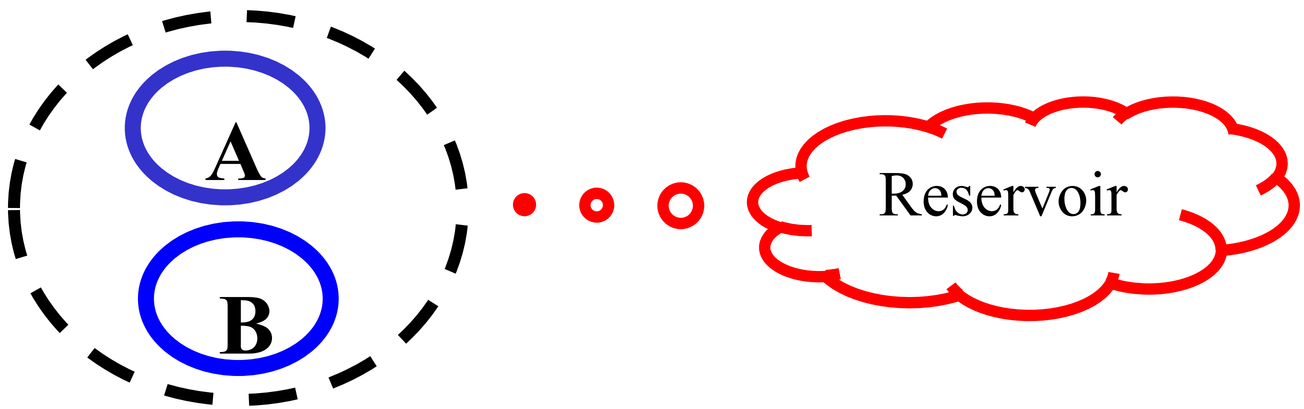
For usual unstructured "Markovian" reservoir

$$\frac{d}{dt}\rho_a = L\left\{\sqrt{\gamma}a\right\}\rho_a = \gamma\left(2a\rho_a a^+ - a^+ a\rho_a - \rho_a a^+ a\right)$$

Any initial state just turns into the vacuum eventually ….

But what will happen if we take two modes coupled linearly to the same reservoir?

Correlated (collective) loss



An example of interaction Hamiltonian: $V_{ab} = (a^+ + b^+)R + h.c.$

Markovian master equation $\frac{d}{dt}\rho_{ab} = L\{\sqrt{\gamma}(a+b)\}\rho_{ab}$

Decoherent-free subspace appears!

These states are not affected by dissipation: $(a+b)|\Psi\rangle = 0$

G.M. Palma, K.-A. Suominen and A.K. Ekert. Quantum Computers and Dissipation. Proc. Roy. Soc. London Ser. A, 452:567, 1996.

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Consequences of having correlated loss:

The initial state is projected on the decoherence-free subspace

↓

One can preserve entanglement. One can create entanglement.

Example for the initial single photon ("dissipative beam-splitter"):

$$\frac{d}{dt}\rho_{ab} = L\left\{\sqrt{\gamma}(a+b)\right\}\rho_{ab}$$

$$(a+b)|\Psi\rangle = 0 \quad \Rightarrow \quad |\Psi\rangle = \frac{1}{\sqrt{2}}\left(|1\rangle_a|0\rangle_b - |0\rangle_a|1\rangle_b\right)$$

Initial state: $\rho_{ab}(t=0) = |\varphi\rangle\langle\varphi|, \qquad |\varphi\rangle = |1\rangle_a|0\rangle_b$

Final state: $\rho_{ab}(t\to\infty) = \frac{1}{2}|\Psi\rangle\langle\Psi| + \frac{1}{2}|0,0\rangle\langle 0,0|.$

Daniel A. Lidar, K. Birgitta Whaley, Decoherence-Free Subspaces and Subsystems, in "Irreversible Quantum Dynamics"

Lecture Notes in Physics Volume 622, 2003, pp 83-120

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Correlated loss is not a theoretical fiction

Paul G. Kwiat, Andrew J. Berglund, Joseph B. Altepeter, Andrew G. White, "Experimental Verification of Decoherence-Free Subspaces", Science 290, no. 5491 pp. 498-501 (2000)

Correlated loss can indeed produce and preserve non-classical states.

BUT …

Correlated loss is rather rather fragile tool …

- *Non-correlated loss destroys effects of correlated loss.*
- *Interaction between quantum objects destroys effects of correlated loss.*
- *Even self-interaction of a quantum object can destroy effects of correlated loss (for example, even Lamb shifts can do that eventually …)*

Well, there are other kinds of useful losses …

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The general scheme

$$\frac{d}{dt}\rho_a = L\{A\}\rho_a = 2A\rho_a A^+ - A^+ A\rho_a - \rho_a A^+ A$$

Let us look for such Lindblad operators, *A*, that $A\rho_{needed} = 0$

The general idea

Start with the simple-to-prepare initial state and let loss drive it toward the required state.

http://thesteamworks.blogspot.com/

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Nonlinear loss can indeed drive deterministically a wide class of initial states into a predefined stationary one.

Let us demonstrate it with a single mode: $$\frac{d}{dt}\rho_a = L\left\{\sqrt{\Gamma}\, a f\left(a^{+}, a\right)\right\}\rho_a$$

We want to generate the state such as $$a f\left(a^{+}, a\right)\rho_{needed} = 0$$

The way to proceed:

We create the initial state with sufficiently large number of photons $$Tr\left\{a^{+} a \rho(t=0)\right\} > Tr\left\{a^{+} a \rho_{needed}\right\}$$

We switch on the loss.

R. R. Carvalho, P. Milman, R. L. de Matos Filho, and L. Davidovich, PRL ***86****, 4988 (2001);*

Z. Kis, W. Vogel, and L. Davidovich, PRA ***64****, 033401 (2001);*

B. Kraus, H. P. Buchler, S. Diehl, A. Kantian, A. Micheli, and P. Zoller, PRA ***78****, 042307 (2008);*

R. Wu, A. Pechen, C. Brif, and H. Rabitz, J. Phys. A ***40****, 5681 (2007).*

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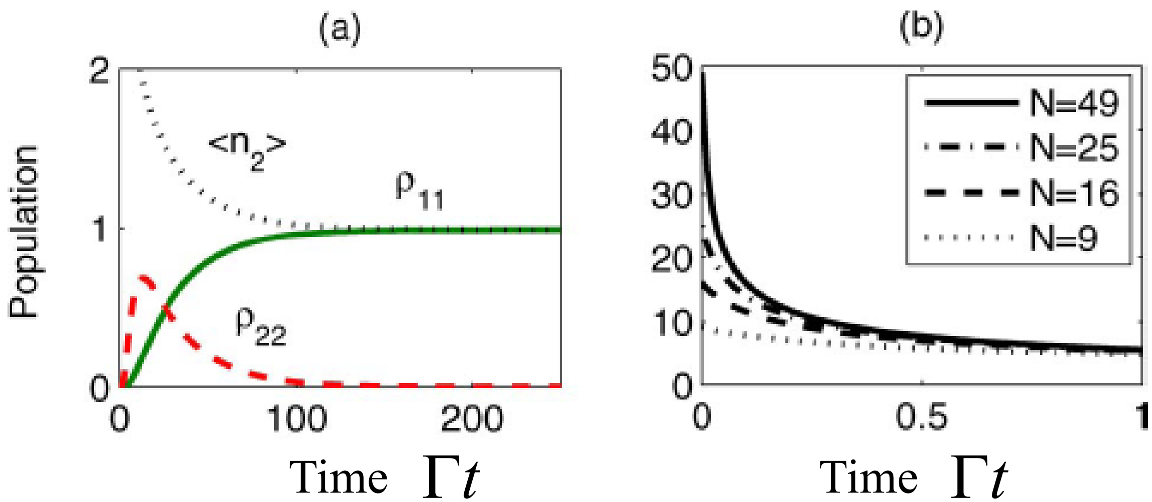
Example: producing single photons from initial coherent states

$$\frac{d}{dt}\rho_a \approx L\left\{\sqrt{\Gamma}a\left(a^{+}a-1\right)\right\}\rho_a$$



Decay is non-exponential.
Decay time does not depend on the initial state:
the larger it is, the faster it goes down …

D. Mogilevtsev, V. S. Shchesnovich, Opt. Lett. ***35****, 3375 (2010).*

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Example: Nonlinear coherent loss

$$\frac{d}{dt}\rho_a \approx \Gamma L\{af(a^+a)\}\rho_a$$

Annihilation operator for *f*-deformed harmonic oscillator $F = af(a^+a)$

Commutation relations: $[F,F^+] = (a^+a+1)f^2(a^+a+1) - a^+af^2(a^+a)$

Eigenstates of F are nonlinear coherent states.

Any pure state non-orthogonal to an arbitrary Fock state can be exactly represented as a nonlinear coherent state. Other states can be well approximated by them.

Nonlinear coherent loss can serve as a tool for producing Fock states.

*R. L. de Matos Filho and W. Vogel, PRA **54**, 4560 (1996).*

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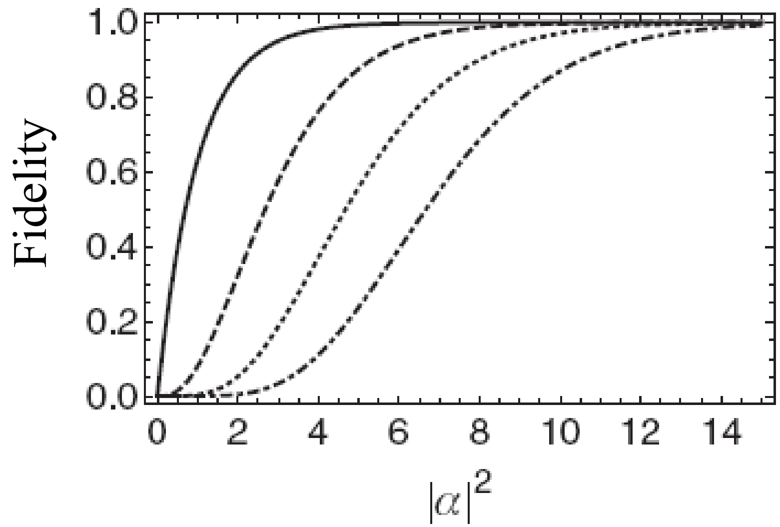
Good example: Fock state generation from coherent states

$$\frac{d}{dt}\rho_a \approx \Gamma L\{af(a^+a)\}\rho_a, \qquad f(n_1)=0, \qquad f(n)\neq 0, \qquad \forall n\neq n_1.$$



Notice: provided that *f* is non-negative, its form is not of any importance for the stationary state

Figure 2. Fidelity of generation of the Fock state $|n_1\rangle$ by NCL starting from the coherent state $|\alpha\rangle$: the solid, dashed, dotted, dash-dotted lines correspond to $n_1 = 1, 3, 5, 7$, respectively.

A Mikhalychev, D Mogilevtsev and S Kilin, J. Phys. A: Math. Theor. ***44*** *325307 (2011).*

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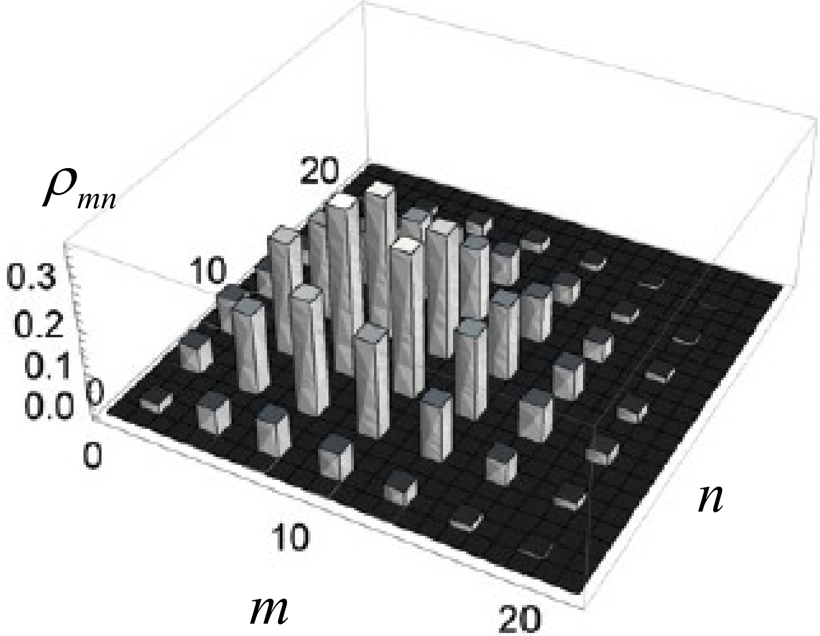
Good but exotic example: "combing" the state

$$f(jN + n_0) = 0, \qquad j = 0,1,2\ldots$$

Just getting rid of unnecessary terms in the density matrix!



"Combed" coherent state with *N=3*.

A Mikhalychev, D Mogilevtsev and S Kilin, J. Phys. A: Math. Theor. ***44*** *325307 (2011).*

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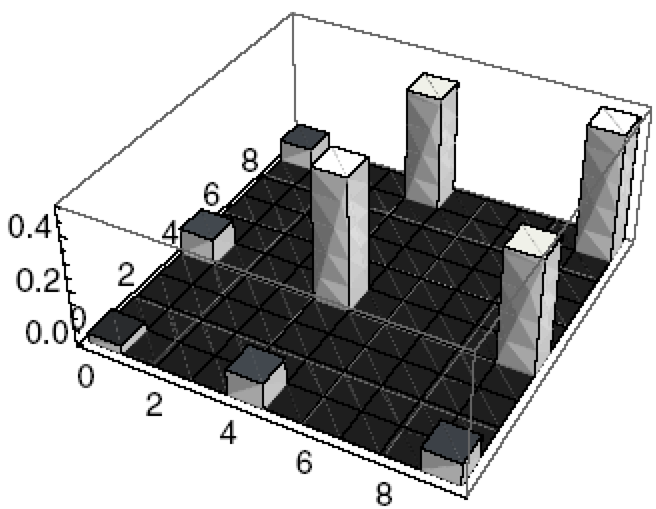
No so good example: generating finite superpositions of Fock states

$$f(n_1) = f(n_2) = 0, \qquad f(n) \neq 0, \qquad \forall n \neq n_1, n_2.$$

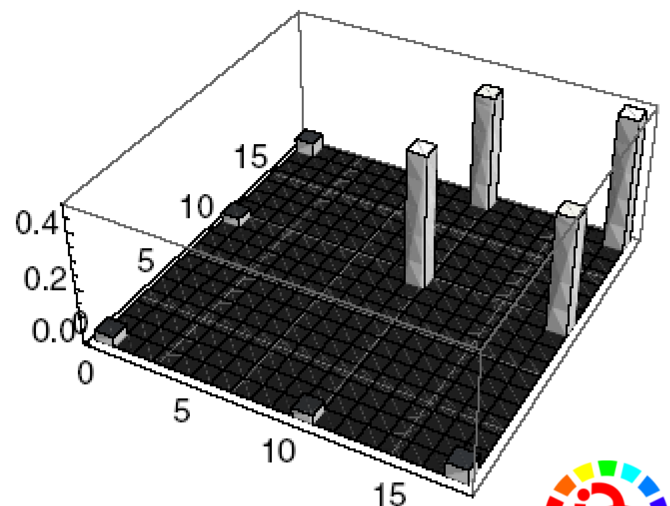
One aims for $$|\varphi(n_1, n_2)\rangle = \frac{1}{\sqrt{2}}\left(|n_1\rangle + |n_2\rangle\right).$$

Alas, one cannot generate pure superpositions this way ...

$\rho_{kl}(n_1, n_2), \qquad n_1 = 4, n_2 = 9$



$\rho_{kl}(n_1, n_2), \qquad n_1 = 10, n_2 = 17$



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Producing nonlinear loss: examples

1. Bose-Einstein condensates

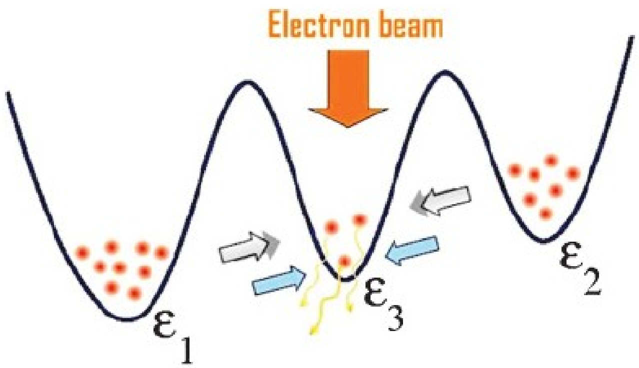
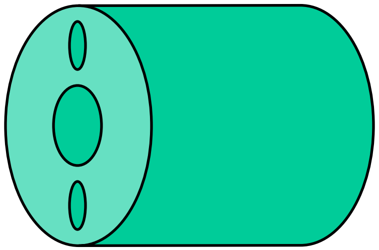
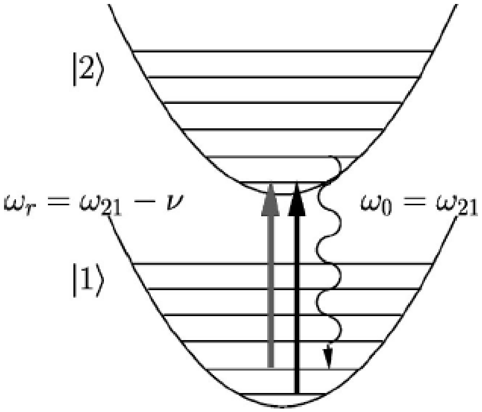


FIG. 1. (Color online) The three-site Bose-Hubbard model setup.

2. Multi-core nonlinear fibers



3. Motional states of a trapped ion



4. Combination of high-order nonlinearities and multi-photon absorption

V. S. Shchesnovich, D. S. Mogilevtsev, PRA ***82****, 043621 (2010);*
D. Mogilevtsev, V. S. Shchesnovich, Opt. Lett. ***35****, 3375 (2010);*
J. F. Poyatos, J. I. Cirac, and P. Zoller PRL 77, 4728 (1996);
*Tao Hong,Michael W. Jack,and Makoto Yamashita, PRA 7****0****, 013814 (2004).*

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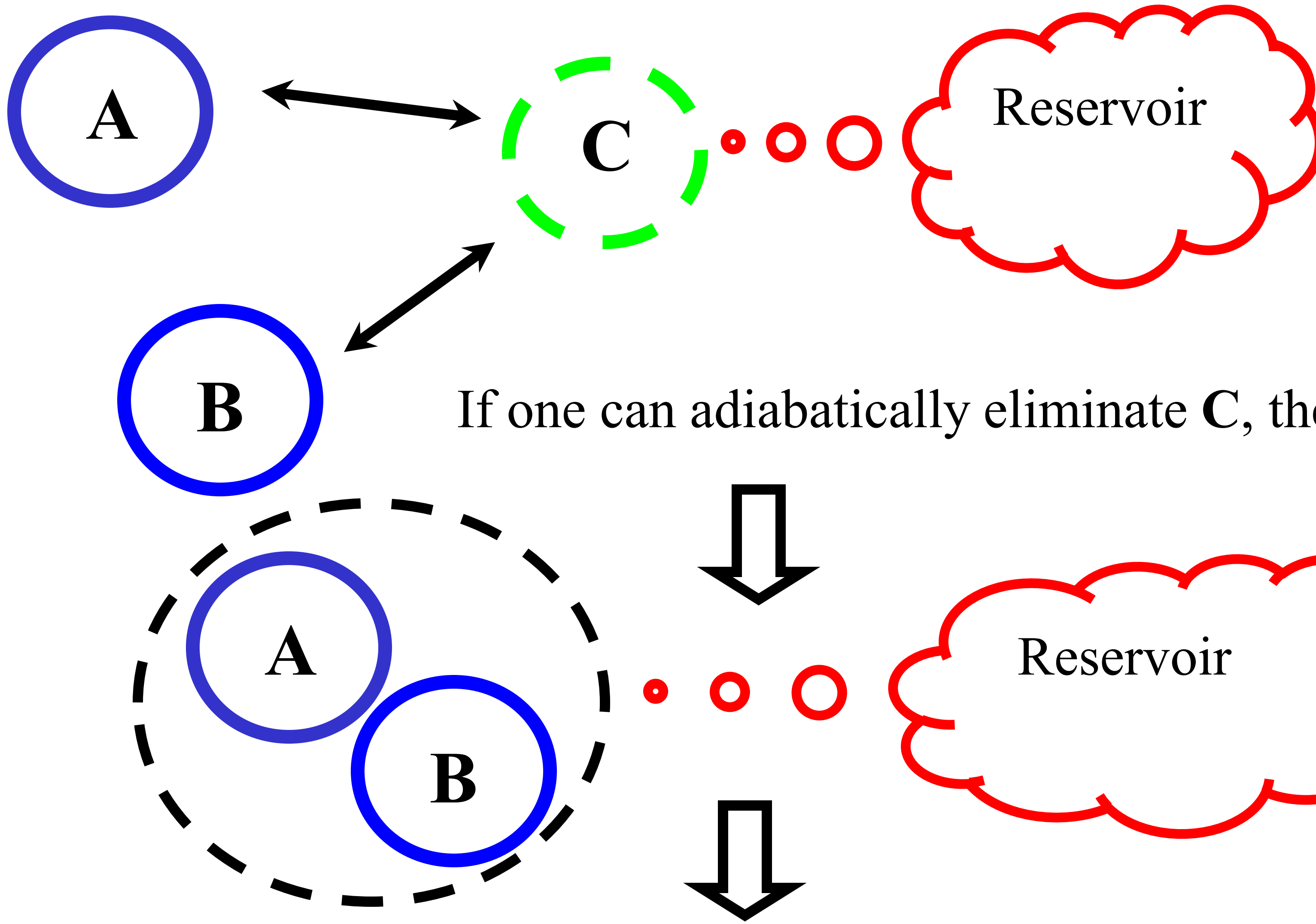
The way to make nonlinear loss, stage I: creating correlated loss



If one can adiabatically eliminate **C**, then

Correlated loss

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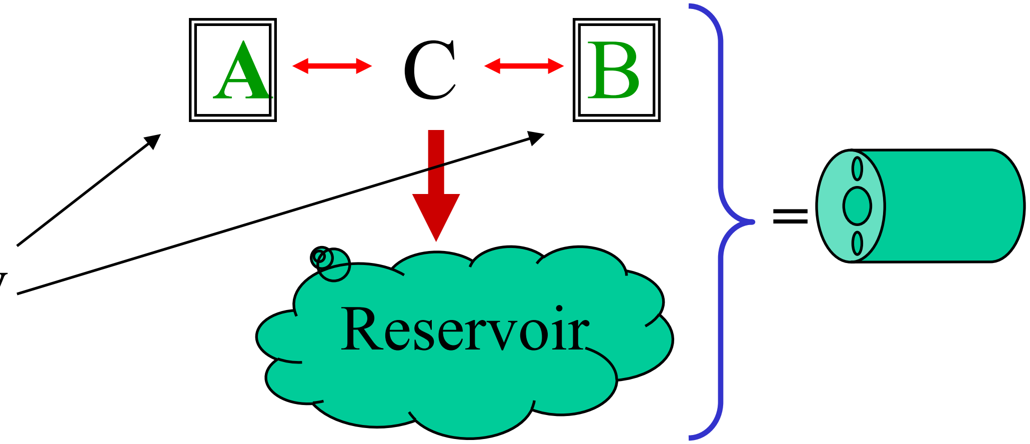
Example: creating correlated loss in a three-mode system

Three linearly interacting modes with self-Kerr nonlinearity

Nonlinearity



$$\frac{d}{dt}\rho_{abc} = -i\left[\left(g_a a^+ + g_b b^+\right)c + U\left(\left(a^+\right)^2 a^2 + \left(b^+\right)^2 b^2\right) + h.c., \rho_{abc}\right] + \gamma L\{c\}\rho_{abc}.$$

For $\gamma >> \left|g_{a,b}\right|$ one eliminates C adiabatically.

$$\frac{d}{dt}\rho_{ab} \approx -i\left[\widetilde{H}, \rho_{ab}\right] + \frac{G^2}{\gamma} L\{g_a a + g_b b\}\rho_{ab}, \qquad G = g_a^2 + g_b^2.$$

The way to make nonlinear loss, stage II

Nonlinearity + correlated loss can give rise to nonlinear loss!

$$\frac{d}{dt}\rho_{ab} \approx -i\left[\widetilde{H}, \rho_{ab}\right] + \Gamma L\{g_a a + g_b b\}\rho_{ab}.$$

For $\Gamma \equiv \frac{G^2}{\gamma} >> U$ one eliminates $d_+ = \frac{1}{G}\left(g_a a + g_b b\right)$

↓

$$\frac{d}{dt}\rho_{d-} \approx -i\left[x d_-^+ d_- + y\left(d_-^+ d_-\right)^2, \rho_{d-}\right] + \Gamma_1 L\left\{d_-^2\right\}\rho_{d-} + \Gamma_2 L\left\{d_-^+ d_-^2\right\}\rho_{d-}.$$

$$d_- = \frac{1}{G}\left(g_a b - g_b a\right)$$

Nonlinear loss!!

*V. S. Shchesnovich, D. S. Mogilevtsev, PRA **82**, 043621 (2010);*
*D. Mogilevtsev, V. S. Shchesnovich, Opt. Lett. **35**, 3375 (2010).*

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So, we have seen that nonlinear loss is good.
One puts in a coherent state, switches the interaction on and gets identical non-classical output states deterministically.

Nevertheless, the main enemy is very much alive: linear loss can easily destroy the result of nonlinear loss.

Indeed, the condition $A\rho_a = 0$ means than near the desired state an influence of nonlinear loss is very weak. So, linear loss is always dominating there.

So, what can one do to save the day?

The answer: be brief and have a strong drive!

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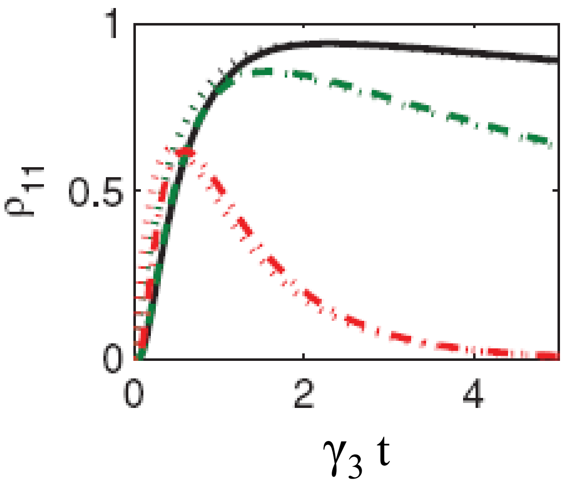
Example: linear loss breaking down effects of nonlinear loss for single-photon generation from the coherent state

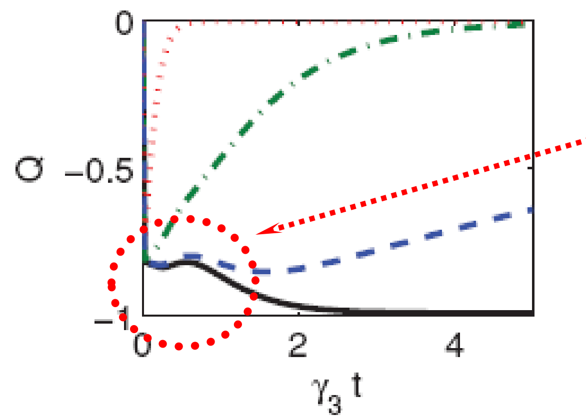
$$\frac{d}{dt}\rho_a = \left(\gamma_3 L\left\{a\left(a^+ a - 1\right)\right\} + \Gamma L\{a\}\right)\rho_a .$$

Linear loss term



Dynamics of single-photon component for the linear loss rate $\Gamma = 0.01\,\gamma_3$, $0.1\,\gamma_3$, γ_3 (solid, dashed, and dash-dotted lines)

Dynamics of Mandel parameter for the linear loss rate $\Gamma = 0.01\,\gamma_3$, $0.1\,\gamma_3$, γ_3 (solid, dashed, and dash-dotted lines)



A hint: should we aim for sufficiently small interaction times?

V. S. Shchesnovich, D. S. Mogilevtsev, PRA ***84****, 013805 (2011)*

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Overcoming linear loss, the way number 1: let's outrun them!

Let us demonstrate a possibility to generate an arbitrary state from the initial coherent state.

Lindblad operator: $A=\sqrt{\gamma}|\varphi\rangle\langle\alpha|a^{k}, \qquad k>1.$

Target state — Initial coherent state

Result: $\langle\varphi|\rho(t>>\gamma_{eff}^{-1})|\varphi\rangle\approx 1-O(|\langle\varphi|\Psi\rangle|^{2}), \qquad |\Psi\rangle=\frac{(a^{+})^{k}}{\mathrm{N}}|\alpha\rangle,$

Time-scale: $\gamma_{eff}=\gamma\mathrm{N}, \qquad \mathrm{N}=\langle\alpha|a^{k}(a^{+})^{k}|\alpha\rangle.$

So, taking sufficiently intensive initial coherent state, one can generate the target state until the linear loss takes any effect.

D. Mogilevtsev, A. Mikhalychev, V. S. Shchesnovich, and N. Korolkova, PRA 87, 063847 (2013).

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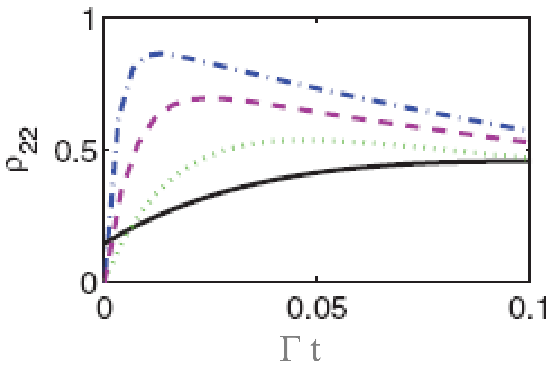
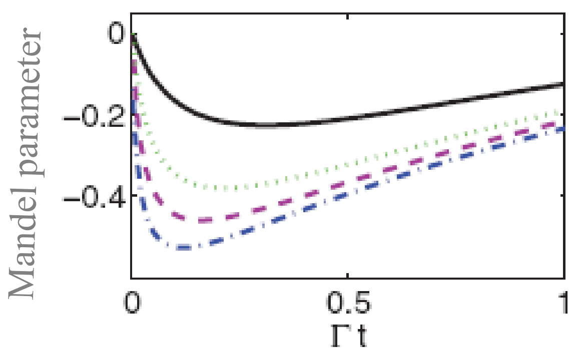
Example: generation of two-photon state



$A = \sqrt{\gamma}|2\rangle\langle\alpha|a^2$ Linear loss rate Γ = 5 γ

Solid, dotted, dashed, and dash-dotted lines correspond to α = 2,3,4,5.

Notice: to have a non-classical state for small times, one needs not constructing complicated loss schemes.



$A = \sqrt{\gamma}\, a\left(a^{+}a - 1\right)$ Linear loss rate Γ = 5 γ

Solid, dotted, dashed, and dash-dotted lines correspond to α = 2,4,6,8.

D. Mogilevtsev, A. Mikhalychev, V. S. Shchesnovich, and N. Korolkova, PRA 87, 063847 (2013).

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Overcoming linear loss, the way number 2: let’s drive them out!

$$\frac{d}{dt}\rho_a = -i\left[\Omega\left(a^+ + a\right), \rho_a\right] + \left(\gamma_3 L\{A\} + \Gamma L\{a\}\right)\rho_a .$$

http://www.supercoloring.com
/pages/kicking-out-the-princess/

Let us add coherent driving and see what happens next …

www.sodahead.com

Notice #1: Alas, it is not possible to get pure stationary non-classical state by driving.

Notice #2: Fortunately, it is possible to get mixed strongly non-classical stationary state by driving.

Moreover: driving non-linear loss, it is possible to make a stationary non-classical state INDEPENDENT of the linear loss rate!

B. Kraus, H. P. Buchler, S. Diehl, A. Kantian, A. Micheli, and P. Zoller, Phys. Rev. A 78, 042307 (2008);

D. Mogilevtsev, A. Mikhalychev, V. S. Shchesnovich, and N. Korolkova, PRA 87, 063847 (2013).

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Driving the nonlinear coherent loss coherently

Lindblad operator: $A = \sqrt{\gamma} a f\left(a^{+} a\right)$

It is sub-Poissonian. Always.

Analytic estimation for strong driving: $$\rho_{n,n} \approx \rho_{n-1,n-1} \frac{\Omega^2}{n\left(f^2(n)\gamma + \Gamma\right)^2}$$

Maximum of the photon-number distribution $$n_0\left(f^2(n_0)\gamma + \Gamma\right)^2 \approx \Omega^2$$

Just by increasing the driving amplitude, one can always get the photon number-distribution practically independent of an arbitrarily large linear loss rate. $$f^2(n_0)\gamma >> \Gamma$$

D. Mogilevtsev, A. Mikhalychev, V. S. Shchesnovich, and N. Korolkova, PRA 87, 063847 (2013).

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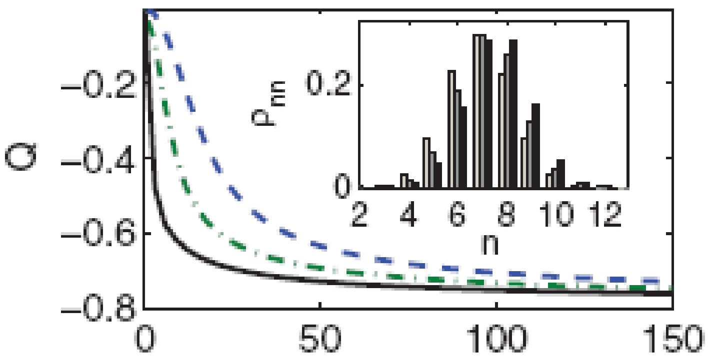
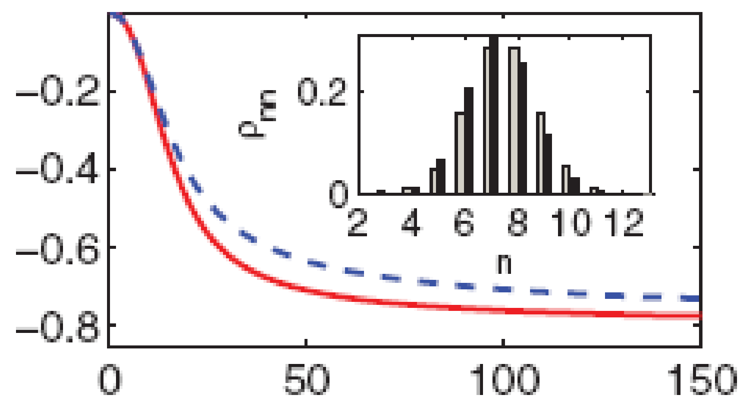
Example: nonlinear coherent loss + coherent driving

$$A = \sqrt{\gamma}\, a\left(a^{+} a - 1\right)$$



Exact solutions for Mandel parameter: solid, dash-dotted, and dashed curves correspond to the linear loss rate Γ= γ , 5γ, 10γ . Inset shows photon-number distribution of the stationary state for the driving amplitude α = 150; light-gray, gray, and black bars correspond to Γ= γ, 5γ, 10γ .

The exact solution (dashed line) and the approximate solution (red solid line) for Γ=10γ. Inset shows photon-number distributions for the solution of the exact (gray bars) and of the approximate (black bars) equations.



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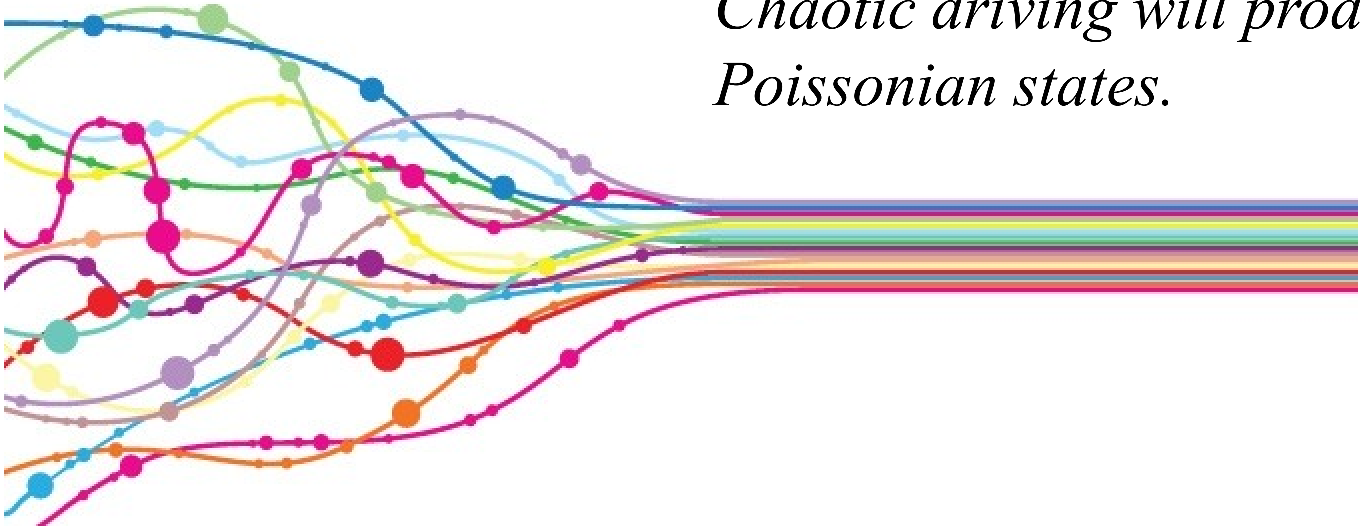
Well, nice coherent driving produces nice non-classical results.

What happens if our system is noised thermally?

Answer: it will create non-classicality!



Chaotic driving will produce sub-Poissonian states.

http://www.research-live.com/magazine/order-from-chaos/4004808.article

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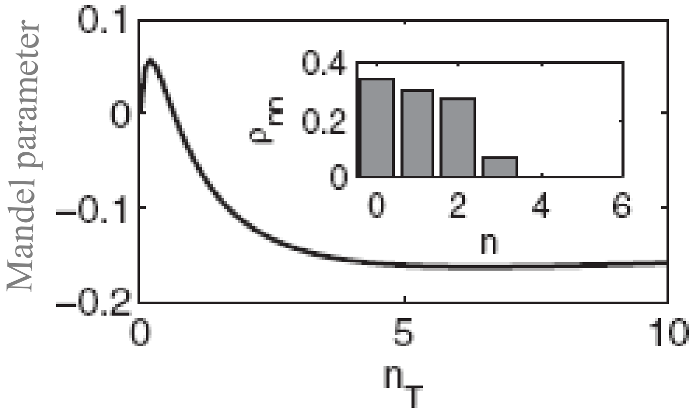
Driving the nonlinear coherent loss incoherently

$$A = \sqrt{\gamma}\, a \left(a^{+} a - 1\right)^{3}$$

$$\frac{d}{dt}\rho_a = \left(\gamma L\{A\} + \Gamma(n_T + 1) L\{a\} + \Gamma n_T L\{a^{+}\}\right)\rho_a .$$



Exact solutions for Mandel parameter: the curve corresponds to the linear loss rate $\Gamma = 10\gamma$. Inset shows photon-number distribution of the stationary state for $n_T = 5$.

Alas, strong thermal driving washes non-classicality away.

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A final touch of quirkiness: creating unit states

$$A=\sqrt{\gamma}\,a f\left(a^{+}a\right), \qquad f\left(n\geq n_0\right)=0.$$

Limit of strong nonlinear loss and simultaneous strong thermal driving (*alas, it is rather impractical …*)

$$n_T\to\infty, \qquad f(n)\frac{\gamma}{\Gamma n_T}\to\infty$$

In this limit one can have truncated unit states

$$\rho_{stationary}=\frac{1}{n_0+1}\sum_{m=0}^{n_0}|m\rangle\langle m|.$$

http://www.clipartguide.com/

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Conclusions

1. Losses can help you to create non-classical states.
2. Nonlinear losses can generate any state deterministically.
3. Nonlinear losses can be made robust with respect to linear ones.
4. Classical driving and nonlinear loss can produce stationary states independent of an arbitrarily large linear loss.

THANKS!

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