Stable optical spring in aLIGO detector and in dual-recycling Michelson-Sagnac interferometer

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Olomouc, 2014

Gravitational waves

Detectors

Standard Quantum Limit

Optical spring effect

Stable optical spring

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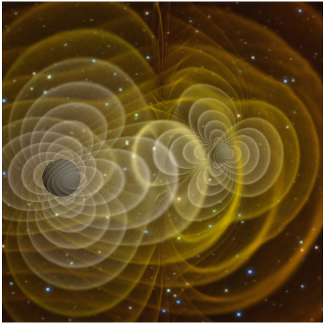
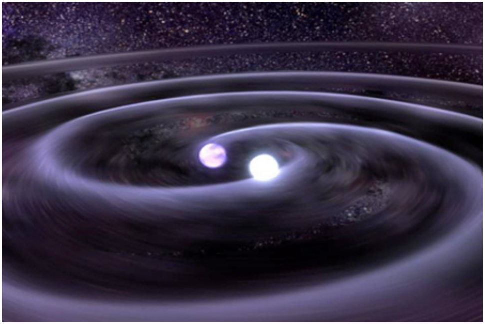
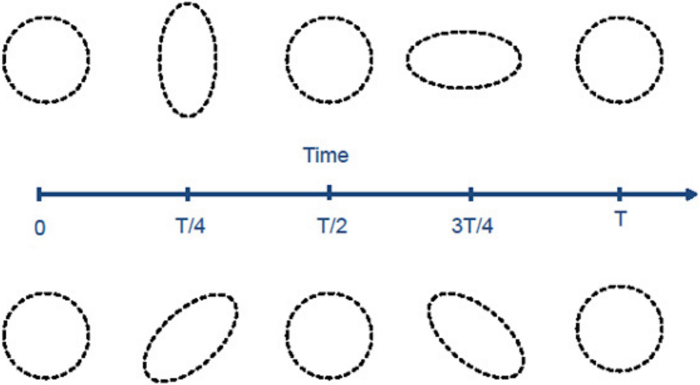




1) The new window to explore the Universe
2) The confirmation of the General Theory of Relativity
[1]

[1] www.ligo.org, www.nasa.gov

How do they interact with matter?



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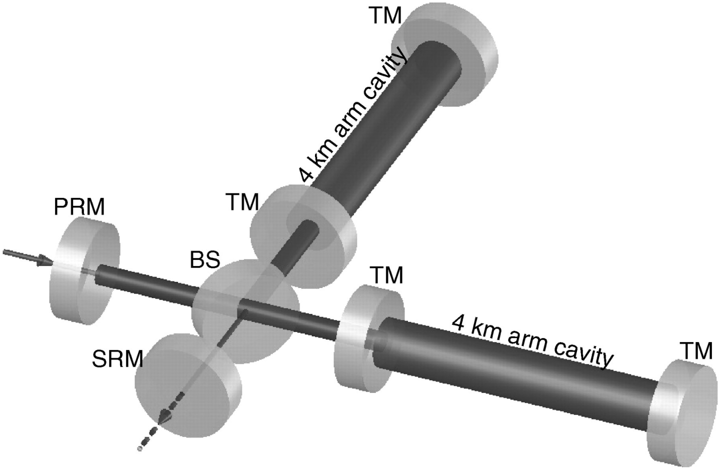
[2] source: Blayne Heckel

How can we catch them? Advanced LIGO



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[3] http://www.bssaonline.org/

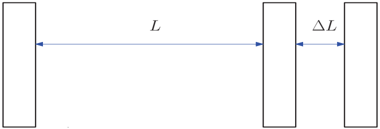
How can we catch them?

LIGO interferometers

GEO 600

VIRGO

Order of effect



$$\Delta L = \frac{1}{2} h L, \quad L = 4 km$$

- Initial LIGO:

$$h \sim 10^{-21} \Rightarrow \Delta L \sim 10^{-18} cm \quad \sim 0.5 events/year$$

- Advanced LIGO:

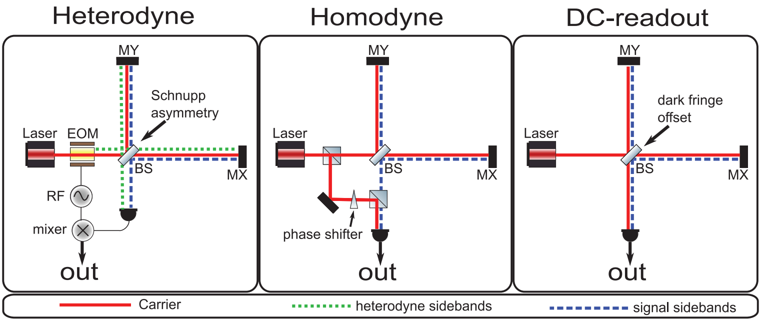
$$h \sim 10^{-22} \Rightarrow \Delta L \sim 10^{-19} cm \quad \sim 10^{3} events/year$$

Dark port

Possible detection regimes:

- Dark port regime
  Arms of interferometer are balanced $\rightarrow$ no mean power in the signal port
- Operating off-dark port
  Arms of interferometer are unbalanced $\rightarrow$ mean power in the signal port appears

DC readout

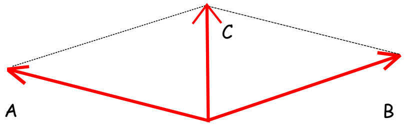


[4] S Hild et al 2009 Class. Quantum Grav. 26 055012

DC readout

Operating off-dark port

a) Balanced scheme



b) Unbalanced scheme

Gravitational waves

Detectors

Standard Quantum Limit

Optical spring effect

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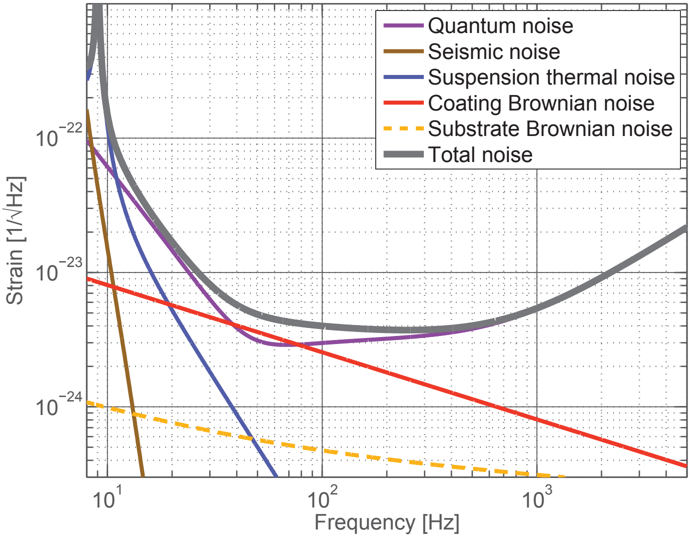
Noise budget of Advanced LIGO detector



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[5] arXiv:1103.2728

Standard quantum limit

Quantum noise

The light produces two types of noise:

- Photon shot-noise
- Radiation pressure noise

$$S_h^{shot} \sim \frac{1}{\mathcal{I}_0}, \quad S_h^{rad} \sim \mathcal{I}_0$$

The optimal case:

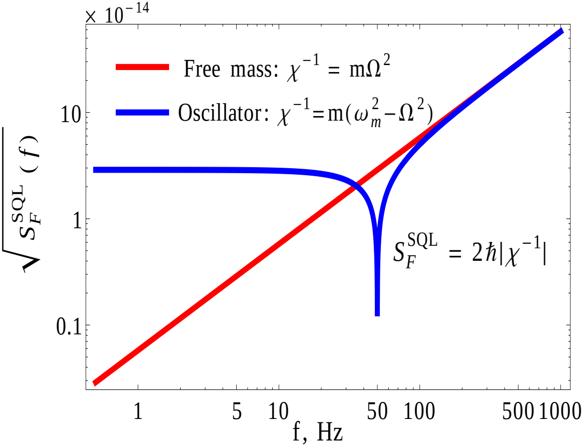
$$S_h^{shot} = S_h^{rad}$$

The resulting net noise:

$$S_h = S_h^{shot} + S_h^{rad} = 2S_h^{shot}$$

Standard quantum limit

This minimum achievable noise is called the “standard quantum limit” (SQL) and is denoted S_F^{SQL}



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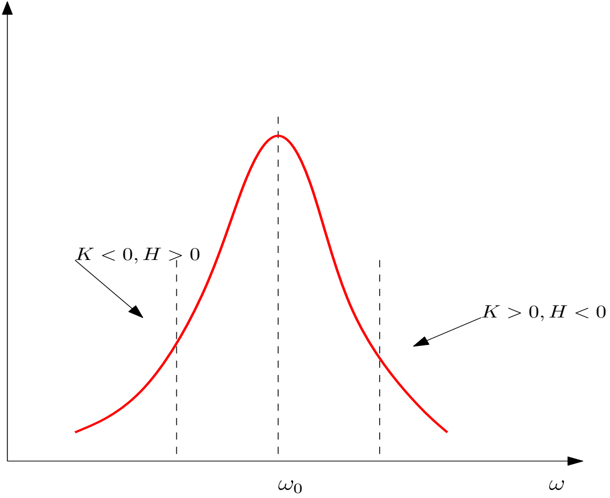
Optical spring effect

$$\omega_p - \omega_0 = \delta < 0 \Rightarrow \boxed{K = -\frac{dF}{dx} < 0}, \qquad \omega_p - \omega_0 = \delta > 0 \Rightarrow \boxed{K = -\frac{dF}{dx} > 0}$$



Optical spring effect

- Circumvent of the SQL: The optical spring turns free mass into the oscillator and that allows to beat the SQL in some frequency band [6].
- Instability:

$$m_{eq}\ddot{y} + h\dot{y} + (K + \Delta K)y = 0,$$

$$y(t - \tau^*) \simeq y(t) - \tau^* \dot{y}(t)$$

$$m_{eq}\ddot{y}(t) + (h \mp \tau^* \Delta K)\dot{y}(t) + (K \pm \Delta K)y(t) = 0.$$

[6] V.B. Braginsky, F.Ya. Khalili, Physics Letters A, **257**, 241 (1999).
F.Ya. Khalili, PLA, **288**, 251-256 (2001);
A. Buonanno, Y. Chen, PRD, **64**, 042006 (2001);
V. I. Lazebny, S. P. Vyatchanin, PLA, **344**, 7 (2005);
F. Ya. Khalili, V. I. Lazebny, S. P. Vyatchanin, PRD **73**, 062002 (2006)

Instability

Ways to avoid instability

1) Feedback [7]
2) Double optical spring [8]

[7] A. Buonanno, Y. Chen, PRD, **65**, 042001 (2002)

[8] H. Rehbein, H. Müller-Ebhardt, S.L. Danilishin, C. Li, R. Schnabel, K. Danzmann, and Y. Chen, Phys. Rev. **D78**,062003 (2008),
A.A. Rakhubovsky, S. Hild, S.P. Vyatchanin, Phys. Rev. **D84**,062002 (2011)

Gravitational waves

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Standard Quantum Limit

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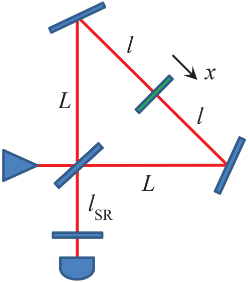
Stable optical spring

The Idea

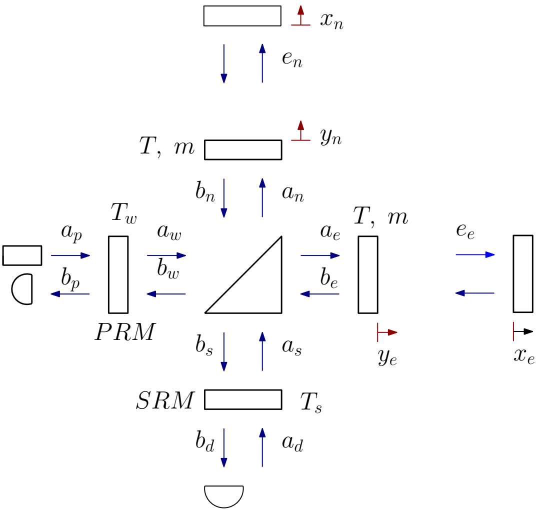
It was shown that it is possible to achieve stable optical rigidity in interferometer operated off-dark port [9]

- It was shown on the example of the Michelson-Sagnac interferometer without power-recycling mirror.
- Relatively big arms detuning (operating far from dark port).



[9] S. P. Tarabrin, H. Kaufer, F. Ya. Khalili, R. Schnabel, and K. Hammerer, Physical Review A 88, 023809 (2013)

Advanced LIGO scheme



Some basic points

Dynamics of interferometer is described by two mechanical and two optical modes.

$$z_{e,n} = x_{e,n} - y_{e,n}$$

- Mechanical modes:
  1) Differential mode (antisymmetric): $z_- = \frac{z_e - z_n}{2}$
  2) Common mode (symmetric): $z_+ = \frac{z_e + z_n}{2}$
- Optical modes:
  1) Differential mode
  This mode is characterized by optical detuning δ_s and decay rate γ_s
  2) Common mode
  This mode is characterized by optical detuning δ_w and decay rate γ_w

Our aim

- To get the stability with single pump
- To find a minimal value of arms detuning necessary to obtain stable optical spring

Consideration and assumptions:

- Common and differential modes became weakly coupled
- Small decay rates and detuning in arms.

$$\gamma_w \ll \delta_w, \quad \gamma_s \ll \delta_s, \quad \delta \ll \delta_s,\ \delta_w$$

- We exclude symmetric mechanical mode

$$z_+ \equiv \frac{(x_e - y_e) + (x_n - y_n)}{2} = 0$$

Initial equations

$$
\begin{cases}
(\gamma_w - i\delta_w - i\Omega)\, e_+(\Omega) - i\delta\, e_-(\Omega) - \dfrac{ik}{\tau}\, E_- z_-(\Omega) = \dfrac{\sqrt{\gamma_w}\, g_p(\Omega)}{\sqrt{\tau}}, \\
-i\delta\, e_+(\Omega) + (\gamma_s - i\delta_s - i\Omega)\, e_-(\Omega) - \dfrac{ik}{\tau}\, E_+ z_-(\Omega) = \dfrac{\sqrt{\gamma_s}\, g_d(\Omega)}{\sqrt{\tau}}, \\
(\gamma_w + i\delta_w - i\Omega)\, e_+^\dagger(-\Omega) + i\delta\, e_-^\dagger(-\Omega) + \dfrac{ik}{\tau}\, E_-^* z_-(\Omega) = \dfrac{\sqrt{\gamma_w}\, g_p^\dagger(-\Omega)}{\sqrt{\tau}}, \\
i\delta e_+^\dagger(-\Omega) + (\gamma_s + i\delta_s - i\Omega)\, e_-^\dagger(-\Omega) + \dfrac{ik}{\tau}\, E_+^* z_-(\Omega) = \dfrac{\sqrt{\gamma_s}\, g_d^\dagger(-\Omega)}{\sqrt{\tau}}, \\
\hbar k \left\{ E_+^* e_-(\Omega) + E_-^* e_+(\Omega) + E_+ e_-^\dagger(-\Omega) + E_- e_+^\dagger(-\Omega) \right\} + \mu\Omega^2 z_-(\Omega) = 0.
\end{cases}
$$

Notations

Normal coordinates

- Balanced aLIGO interferometer:

Common (e_+) and differential (e_-) modes:

$$e_+ = \frac{e_e + e_n}{\sqrt{2}}, \quad e_- = \frac{e_e - e_n}{\sqrt{2}}$$

- Unbalanced aLIGO interferometer:

Normal modes ($b_\pm$):

$$b_+ = e_+ - \varkappa e_-, \quad b_- = \varkappa e_+ + e_-, \quad \varkappa \sim \delta$$

Characteristic equation:

$$\boxed{\lambda^2 + \frac{\mathcal{I}_1\left[1 + \alpha_1(\lambda + \tilde{\gamma}_s)\right]}{(\lambda + \tilde{\gamma}_s)^2 + \tilde{\delta}_s^2} + \frac{\mathcal{I}_2\left[1 + \alpha_2(\lambda + \tilde{\gamma}_w)\right]}{(\lambda + \tilde{\gamma}_w)^2 + \tilde{\delta}_w^2} = 0}$$

Notations

Analysis

We can rewrite characteristic equation in the next form:

$$\underbrace{D_1^{(0)} D_2^{(0)}}_{\text{zero order}} + \underbrace{D^{(1)}}_{\text{first order}\sim\delta^2} = 0,$$

$$D_1^{(0)} = \lambda^2\left[(\lambda+\tilde{\gamma}_s)^2+\tilde{\delta}_s^2\right] + \mathcal{I}_1\left[1+\alpha_1(\lambda+\tilde{\gamma}_s)\right],$$

$$D_2^{(0)} = \left(\lambda+\tilde{\gamma}_w\right)^2+\tilde{\delta}_w^2,$$

$$D^{(1)} = \left[(\lambda+\tilde{\gamma}_s)^2+\tilde{\delta}_s^2\right]\mathcal{I}_2\left(1+\alpha_2(\lambda+\tilde{\gamma}_w)\right).$$

<u>Zero order</u>:

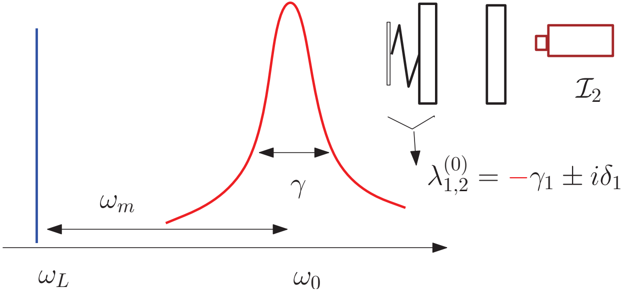
$$\underbrace{D_1^{(0)}}_{\lambda_{1,2}^{(0)},\ \lambda_{3,4}^{(0)}} \times \underbrace{D_2^{(0)}}_{\lambda_{5,6}^{(0)}} = 0, \quad \lambda_{1,2}^{(0)} = \gamma_1 \pm i\delta_1, \quad \gamma_1 > 0 \ \text{— instability,}$$

$$\lambda_{3,4}^{(0)} = -\gamma_3 \pm i\delta_3, \quad \lambda_{5,6}^{(0)} = -\tilde{\gamma}_w \pm i\tilde{\delta}_w, \quad \gamma_3,\ \tilde{\gamma}_w > 0$$

First order iteration

Problem: To introduce maximal relaxation into unstable mode by using “additional” pump $\mathcal{I}_2$ as small as possible.

Solution: We use an analogy of laser-cooling : $\boxed{\tilde{\delta}_w = -\delta_1}$



First order iteration

We choose $\tilde{\delta}_w = -\delta_1$

$$\left((\lambda - \gamma_1)^2 + \delta_1^2\right)\left((\gamma + \tilde{\gamma}_w)^2 + \delta_w^2\right) - b = 0,$$

where

$$b \equiv - \left. \frac{D^{(1)}}{(\lambda + \gamma_3)^2 + \delta_3^2} \right|_{\lambda = \lambda_{1,2}^{(0)}}$$

We consider b as a constant of first order of smallness

$$\lambda = \frac{\gamma_1 - \tilde{\gamma}_w}{2} \pm i \sqrt{\delta_1^2 - \left[\frac{\gamma_1 + \tilde{\gamma}_w}{2}\right]^2 \pm \sqrt{b - \delta_1^2 \left[\gamma_1 + \tilde{\gamma}_w\right]^2}}$$

Stability conditions

$$\lambda = \frac{\gamma_1 - \tilde{\gamma}_w}{2} \pm i\sqrt{\delta_1^2 - \left[\frac{\gamma_1 + \tilde{\gamma}_w}{2}\right]^2 \pm \sqrt{b - \delta_1^2\left[\gamma_1 + \tilde{\gamma}_w\right]^2}}$$

Stability conditions:

$$\Re b = \delta_1^2\left[\gamma_1 + \tilde{\gamma}_w\right]^2, \quad \gamma_1 < \tilde{\gamma}_w$$

Estimation of minimal value of δ:

$$\boxed{\delta \sim \tilde{\gamma}_w}$$

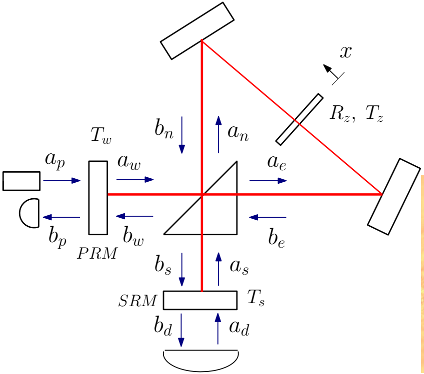
$\tilde{\gamma}_w$ - decay rate of common optical mode.

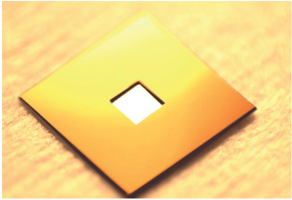
Michelson-Sagnac interferometer

We show that after substitution

$$\delta^2 \to R_z^2\delta^2, \quad J_+ \to R_z^2 J_+, \quad \mu \to m,$$

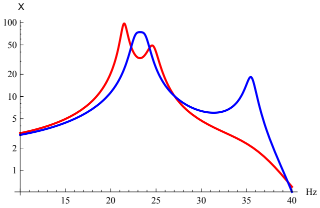
the characteristic equation is the same that for MSI.





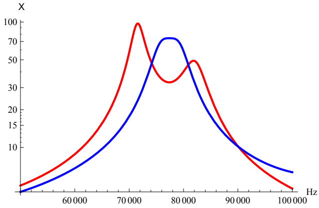
Numerical estimations

aLIGO



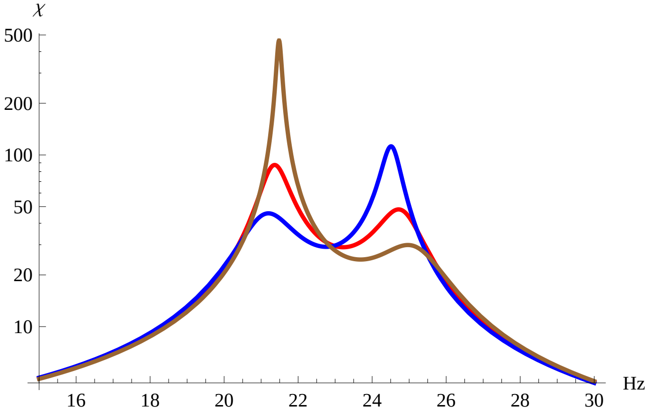
Common mode detuning (δ_w)	-23.0 Hz
Differential mode detuning (δ_s)	42.4 Hz
Decay rate of common mode (γ_w)	3.0 Hz (1.5)
Decay rate of differential mode (γ_s)	3.0 Hz (0.3)
Test mass (m)	40 kg
Arm length (L)	4 km
Circulating power (I_{circ})	24 kW
Arms detuning (δ)	4.6 Hz (1.5)
Output power (I_{out})	0.03 W

MSI



Common mode detuning (δ_w)	-77.2 kHz
Differential mode detuning (δ_s)	141.0 kHz
Decay rate of common mode (γ_w)	10 kHz (5)
Decay rate of differential mode (γ_s)	10 kHz (1)
Mass of membrane (m)	10^{-10} kg
Arm length (L)	8.7 cm
Circulating power (I_{circ})	318 mW
Arms detuning (δ)	15 kHz (5)
Membrane's reflectivity (R_z^2)	0.17

Shape of susceptibility curve



Susceptibility χ of aLIGO with $\tilde{\delta}_w = -\delta_1 + \Delta$. Red curve – $\Delta = 0$. Blue curve – $\Delta = 0.5\ Hz$. Brown curve – $\Delta = -0.5\ Hz$

Summary

- Very high-sensitive detectors are required to detect GW.
- Quantum effects come into play. The SQL.
- Optical spring is one of the possible ways to beat SQL. Optical spring is unstable.
- It was shown that it is possible to achieve stable optical spring in the UNBALANCED detector with SINGLE PUMP.
- The stable optical spring may be created with small arm detuning comparable with optical bandwidths:
$$\delta \sim \tilde{\gamma}_{w,s}$$
- These results can be applied to the table top Michelson-Sagnac interferometer with membrane inside to create stable optical spring (and to another optomechanical systems)

Thank you!

Routh-Hurwitz stability criterion

$$D = \sum_{n=0}^{6} a_n \lambda^n$$

Hurwitz matrices $H_1,\ H_2,\ H_3,\ H_4,\ H_5,\ H_6$.

Determinants of all matrices must be positive: $Det(H_i) > 0$.

Our analysis shows:

$$\mathrm{Det}(H_1) = 3.6, \quad \mathrm{Det}(H_2) = 2721.22, \quad \mathrm{Det}(H_3) = 1.4 * 10^6,$$
$$\mathrm{Det}(H_4) = 9.7 * 10^{11}, \quad \mathrm{Det}(H_5) = 2.9 * 10^{15}, \quad \mathrm{Det}(H_6) = 1.0 * 10^{24}.$$

Hurwitz matrices

Notations

Differential and common optical modes:

$$e_\pm = \frac{e_e \pm e_n}{\sqrt{2}}, \quad E_\pm = \frac{E_e \pm E_n}{\sqrt{2}}.$$

We introduce a new basis:

$$g_p = e^{i\alpha_w} a_p, \quad g_d = e^{i\alpha_s} a_d.$$

so that

$$[g_d(\Omega),\, g_d^\dagger(\Omega')] = 2\pi\,\delta(\Omega - \Omega'),$$
$$[g_p(\Omega),\, g_p^\dagger(\Omega')] = 2\pi\,\delta(\Omega - \Omega')\,.$$

Back

Notations

Initial equations in normal coordinates:

$$
\begin{cases}
\big(-i\Omega-\lambda_+\big)b_+(\Omega)-iz_-\left[\xi-\varkappa\right]=0,\\
\big(-i\Omega-\lambda_-\big)b_-(\Omega)-iz_-\Big[1+\varkappa\xi\Big]=0,\\
\big(-i\Omega-\lambda_+^*\big)b_+^\dagger(-\Omega)+iz_-\left[\xi^*-\varkappa^*\right]=0,\\
\big(-i\Omega-\lambda_-^*\big)b_-^\dagger(-\Omega)-iz_-\Big[1+\varkappa^*\xi^*\Big]=0,\\
\dfrac{b_+(\Omega)\Big[\xi^*-\varkappa\Big]+b_-(\Omega)\Big[1+\xi^*\varkappa\Big]}{d}+\\
+\dfrac{b_+^\dagger(-\Omega)\Big[\xi+\varkappa^*\Big]++b_-^\dagger(-\Omega)\Big[1+\xi\varkappa^*\Big]}{d^*}+\dfrac{\Omega^2}{J_+}z_-=0\,,
\end{cases}
$$

Notations

$$\lambda_\pm = -\left(\Gamma_+ \pm \Gamma_- \sqrt{1+\Delta^2}\right), \quad J_+ \equiv \frac{kI_+}{L\mu},$$

$$\Gamma_\pm \equiv \frac{\gamma_w - i\delta_w \pm (\gamma_s - i\delta_s)}{2}, \quad d \equiv 1 + \varkappa^2,$$

$$\varkappa \equiv \frac{i\delta}{\Gamma_w + \lambda_-} = \frac{\Delta}{1+\sqrt{1+\Delta^2}}, \quad \xi \equiv \frac{i\delta}{\gamma_s - i\delta_s}, \quad \Delta \equiv \frac{i\delta}{\Gamma_-}.$$

We introduce

$$\lambda_+ \equiv -\tilde{\gamma}_w + \tilde{\delta}_w, \quad \lambda_- \equiv -\tilde{\gamma}_s + \tilde{\delta}_s,$$

$$\mathcal{I}_1 \equiv \frac{2J_+\tilde{\delta}_s\,\Re\phi}{|d|^2}, \quad \alpha_1 \equiv \frac{\Im\,\phi}{\tilde{\delta}_s\,\Re\phi}, \quad \mathcal{I}_2 \equiv \frac{2J_+\tilde{\delta}_w\,\Re\psi}{|d|^2}, \quad \alpha_2 \equiv \frac{\Im\,\psi}{\tilde{\delta}_w\,\Re\psi},$$

$$\phi \equiv (1+\xi^*\varkappa)(1+\varkappa\xi)d^*, \quad \psi \equiv (\xi^* - \varkappa)(\xi - \varkappa)d^*.$$

Back

Hurwitz matrices

$$H_1 = a_1,$$

$$H_2 = \begin{pmatrix} a_1 & 1 \\ a_3 & a_2 \end{pmatrix}, \quad H_3 = \begin{pmatrix} a_1 & 1 & 0 \\ a_3 & a_2 & a_1 \\ a_5 & a_4 & a_3 \end{pmatrix}, \quad H_4 = \begin{pmatrix} a_1 & 1 & 0 & 0 \\ a_3 & a_2 & a_1 & 1 \\ a_5 & a_4 & a_3 & a_2 \\ a_7 & a_6 & a_5 & a_4 \end{pmatrix},$$

$$H_5 = \begin{pmatrix} a_1 & 1 & 0 & 0 & 0 \\ a_3 & a_2 & a_1 & 1 & 0 \\ a_5 & a_4 & a_3 & a_2 & a_1 \\ a_7 & a_6 & a_5 & a_4 & a_3 \\ 0 & a_8 & a_7 & a_6 & a_5 \end{pmatrix}, \quad H_6 = \begin{pmatrix} a_1 & 1 & 0 & 0 & 0 & 0 \\ a_3 & a_2 & a_1 & 1 & 0 & 0 \\ a_5 & a_4 & a_3 & a_2 & a_1 & 1 \\ a_7 & a_6 & a_5 & a_4 & a_3 & a_2 \\ 0 & a_8 & a_7 & a_6 & a_5 & a_4 \\ 0 & 0 & 0 & a_8 & a_7 & a_6 \end{pmatrix}$$

Back