Quantum tomography, uncertainties and information

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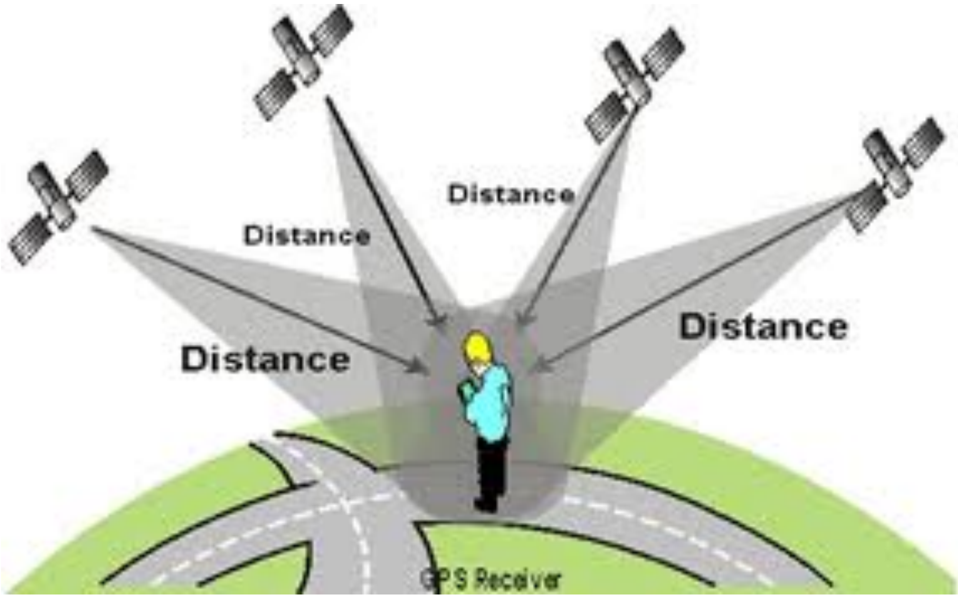
How to use uncertainties for improved tomography

Outline

- Brief Review of MaxLik Tomography
- Borders in Quantum World
- Moments and Uncertainties
- Basic and Advanced Uncertainties
- Brief Review of Information Concepts
- Summary

Tomography as GPS navigation



Quantum mechanics

Probability in Quantum Mechanics:

$p_j = Tr(\rho A_j)$

Measurement: Elements of positive-valued operator measure (POVM) $A_j \geq 0$

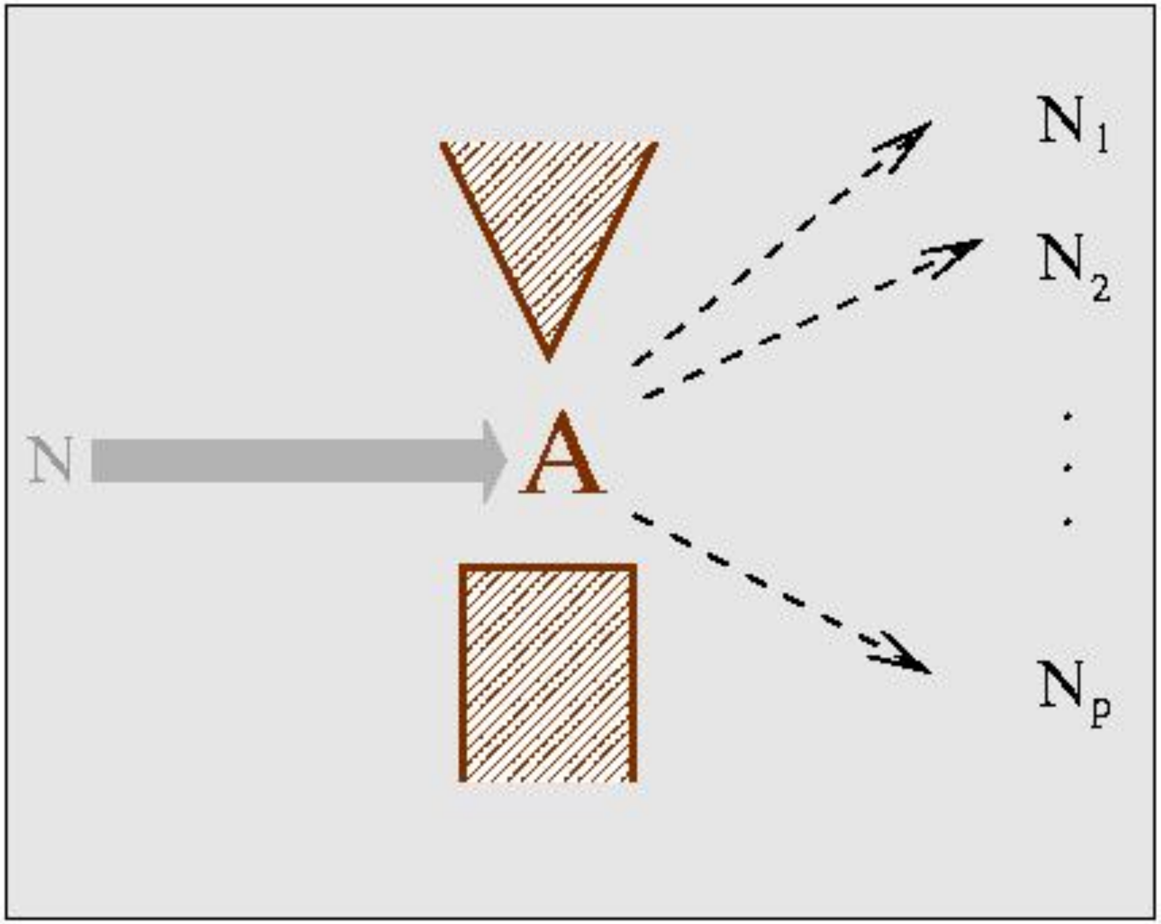
Relation of completeness: $\Sigma_j A_j = 1$

Signal: density matrix $\rho \geq 0$

"Coordinates" : $d^2 - 1$ parameters in d dimensional Hilbert space

"Favorite QM Navigation": Mutually Unbiased Basis (MUB) corresponding to (d+1) observables

Von Neumann Measurement



MaxLik tomography

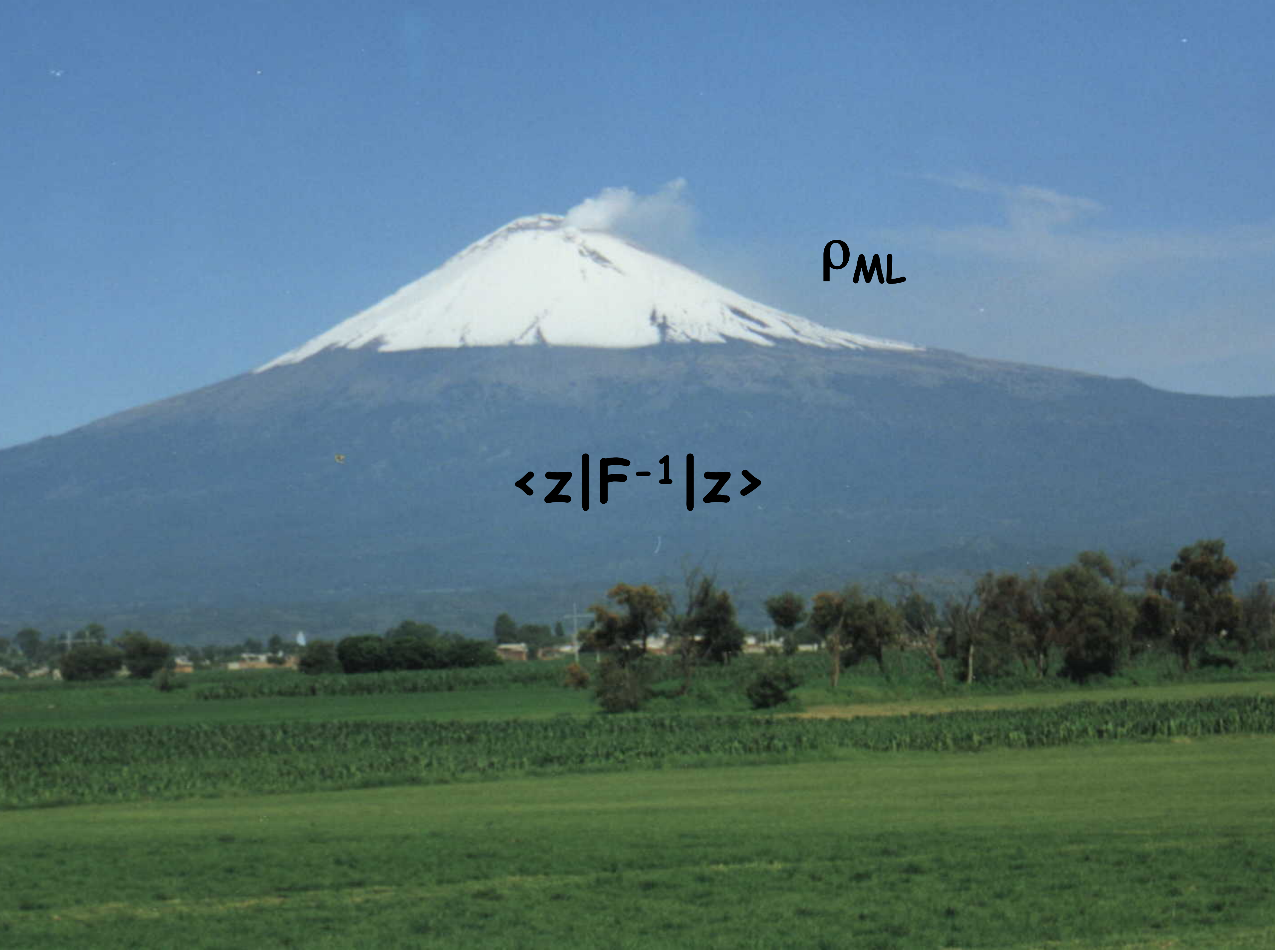
Maximum Likelihood (MaxLik) principle: Bet Always On the Highest Chance!

log-Likelihood: $\log L = \sum_i N_j \log p_j / (\sum_k p_k)$
(logarithm of the product of normalized probabilities)

Uncertianties of inferred variable $\mathbf{z} = \mathbf{Tr(Z\rho)}$

$$(\Delta z)^2 = \langle z|F^{-1}|z\rangle$$

$$\mathbf{z} = \mathbf{Tr(Z\rho_{ML})} \pm \{\langle z|F^{-1}|z\rangle\}^{1/2}$$


ρ_{ML}
$\langle z|F^{-1}|z\rangle$

and its problems …

- Dimensionality: There is a big difference between parameter estimated as 0 and the parameter set to 0 !

- States on the border: $\rho \geq 0$

$$\begin{pmatrix} \rho_{11} & \dots & \dots \\ \dots & 0 & \dots \\ \dots & \dots & \rho_{dd} \end{pmatrix}$$

Borders in Hilbert space

Moments and their hierarchy

- Signal: Complex amplitude $\langle a \rangle$
- Noise: Second order moments $\langle n \rangle$ (energy), $\langle a^2 \rangle$ (phase sensitive noise) = squeezing

$$\lambda^2_{1,2} = \tfrac{1}{2} + \langle n \rangle \pm |\langle a^2 \rangle| \quad \text{(noise ellipse)}$$

- Energy fluctuations $(\Delta n)^2 = \langle n^2 \rangle - \langle n \rangle^2$

… and many other moments with unclear meaning …

Uncertianties:borders in configuration space

Cauchy-Schwartz inequalities

$$\langle\alpha|\alpha\rangle\langle\beta|\beta\rangle \geq |\langle\alpha|\beta\rangle|^2$$

Well-known inequalities:

saturated by coherent states

$$\langle \Delta a^\dagger \Delta a\rangle \geq 0$$

saturated by squeezed states

$$|\langle \Delta a^2\rangle|^2 \leq \langle \Delta a^\dagger \Delta a\rangle[\langle \Delta a^\dagger \Delta a\rangle + 1]$$

(equivalent to $\lambda^2{}_1\,\lambda^2{}_2 \geq \frac{1}{4}$)

“New uncertainties”

- Heisenberg uncertainties for the commutator

$$[n, X_1(\theta)] = -iX_2(\theta)$$

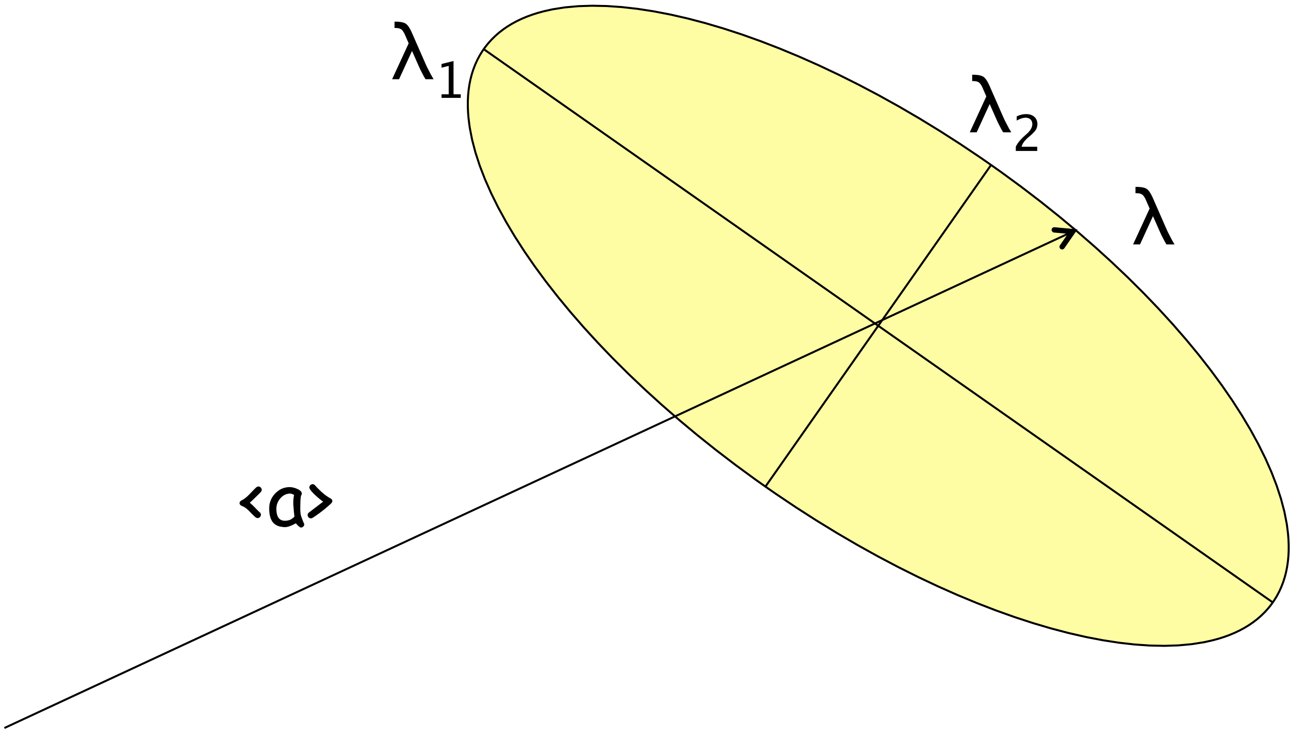
$$(\Delta n)^2 \geq \frac{|\langle a \rangle|^2}{2} \frac{\lambda^2}{\lambda_1^2 \lambda_2^2}$$

$$\lambda^2 = \lambda_1^2 \sin^2 \Phi + \lambda_2^2 \cos^2 \Phi,$$

$$\Phi = \frac{1}{2} \arg[\langle (\Delta a)^2 \rangle].$$

Z.H, Phys. Rev. A41, 400 (1990); Phys. Rev. A 44, 793 (1991), PhD thesis 1991

Noise ellipse



$$(\Delta n)^2 \geq \frac{|\langle a \rangle|^2}{2} \frac{\lambda^2}{\lambda_1^2 \lambda_2^2}$$

Extremal states

- Crescent states

$$|\psi\rangle_{crescent} \propto (a^\dagger + \xi^*)^M |\eta\rangle_{coh}$$

- Well approximated by superposition of coherent and photon added coherent states

$$|\psi\rangle \approx (1 + \gamma a^\dagger)|\eta\rangle_{coh}$$

Phase insensitive inequalities

$$(\Delta n)^2 \geq \frac{|\langle a\rangle|^2}{2(1+2\langle \Delta a^\dagger \Delta a\rangle)} = \frac{|\langle a\rangle|^2}{2(\lambda_1^2+\lambda_2^2)}$$

I. Urizar-Lanz, G. Toth, Phys. Rev. A 81, 052108 (2010)

Another inequality…

CS inequality $$|\langle a^2\rangle|^2 \leq \langle a^{\dagger 2} a^2|\rangle$$

$$(\Delta n)^2 \geq |\langle a^2\rangle|^2 - \langle a^\dagger a\rangle^2 + \langle a^\dagger a\rangle$$

can be saturated by the symmetric-cat like states

$$|-\alpha\rangle + |\alpha\rangle$$

This shows nontrivial structure of the quantum border !

$$[n^2 + \alpha a^2 + \alpha^* a^{\dagger 2} + \gamma n + \delta a + \delta^* a^\dagger]|\psi\rangle = \omega|\psi\rangle$$

Some boarders for 2nd moments…

In[15]:= `Show[y1, y2]`

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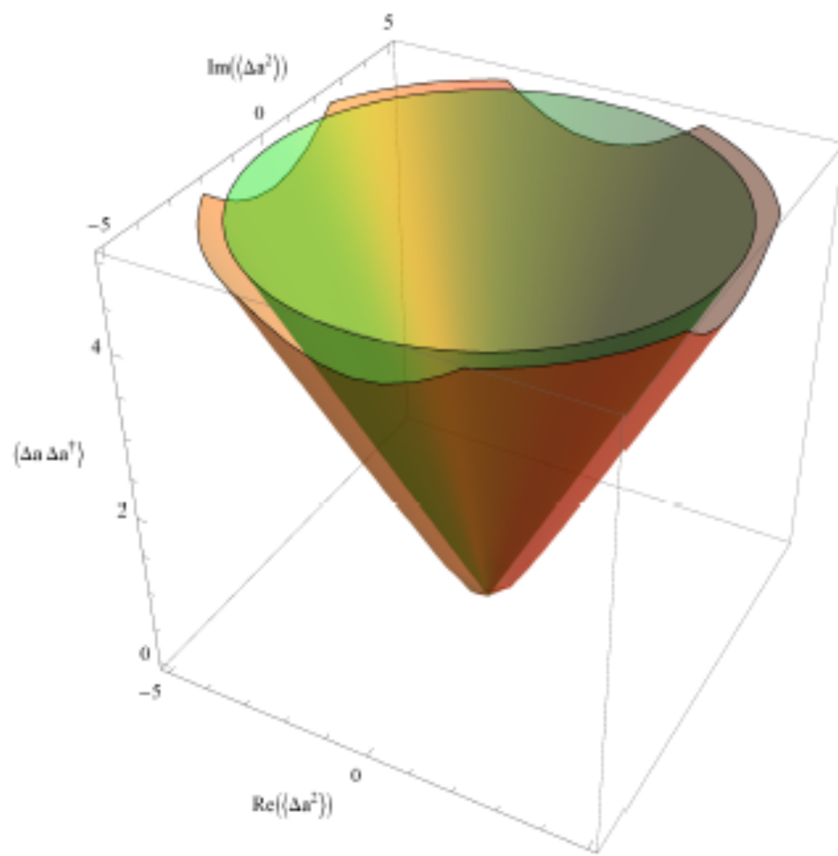

Statistics, models, information,...

Moments and uncertainties provide an alternative formulation to “quantum information” …

Three fundamental attributes of statistical models:

1. Parsimony: model simplicity
2. Goodness-of-fit: conformity of the fitted model to the data at hand
3. Generalizability: applicability of the fitted model to describe or predict new data

Information concepts follows: Relative information, Akaike informaion, Schwartz information, Entropy

By-product of the tomo

Performance measures for diagnostics of photon-added states

$$G_1 \equiv (\Delta n)^2 - \frac{|\langle a \rangle|^2}{2} \frac{\lambda^2}{\lambda_1^2 \lambda_2^2} \geq 0$$

$$G_2 \equiv (\Delta n)^2 - |\langle a^2 \rangle|^2 - \langle a^\dagger a \rangle^2 + \langle a^\dagger a \rangle \geq 0$$

Summary

- Quantum tomography is not omnipotent!!!!

(To optimize Wigner function at the origin or fidelity is not the best option…)

- Uncertainties can define performance measures and distances
- Borders should be respected and sometimes used advantageously
- “Any” state is extremal with respect to “some” measure…
- Resources are ALWAYS limited
- Non-Gaussianity and non-linearity included for testing cat-like and photon added coherent states.

Thanks for your attention!

The parable of the fishing net (Eddington 1939):

If an ichthyologist casts a net with meshes two inches wide for exploring the life on the ocean, he must not be surprised if he finds that "no sea-creature is less than two inches long".

Please, don't confuse with Parable of the Fishing Net
(Matthew 13:47-50)....