Informace, neznalost a neurčitost v kvantové tomografii

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OP Vzdělávání pro konkurenceschopnost
INVESTICE DO ROZVOJE VZDĚLÁVÁNÍ

Work done in collaboration with J. Řeháček, D. Mogilevtsev. L. Sanchez-Soto, B. Englert, Y-S Teo, B. Stoklasa

Linear inverse problems

ML estimation is excellent tool for solving linear inverse problems with constraints (= tomography)

$$I_j = \sum_k c_{jk}\,\mu_k$$

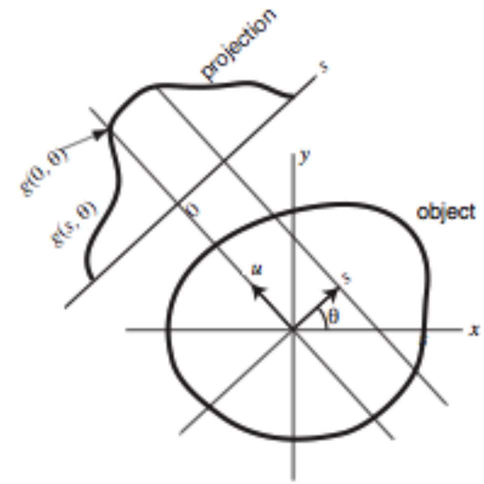
detected mean values I_j, $j = 1,2,\ldots M$
reconstructed signal μ_k $k = 1,2,\ldots N$

Over-determined problems **$M > N$**
Well defined problems **$M = N$**
Under-determined problems **$M < N$**

Tomography and Inverse Radon Transformation

Radon transformation

$$g(s,\theta)=\int dxdy f(x,y)\delta(x\cos\theta+y\sin\theta-s)$$



Projection theorem
(ray sum)

$$g(s,\theta)=\int_{-\infty}^{\infty} f(s\cos\theta-u\sin\theta, s\sin\theta+u\cos\theta)\,du$$

Inverse Radon transformation-
Fourier transformation method

$$G_\theta(\xi)=F(\xi\cos\theta,\xi\sin\theta)) \qquad f(x,y)=F^{-1}G_\theta$$

Elements of quantum theory

Probability in Quantum Mechanics:

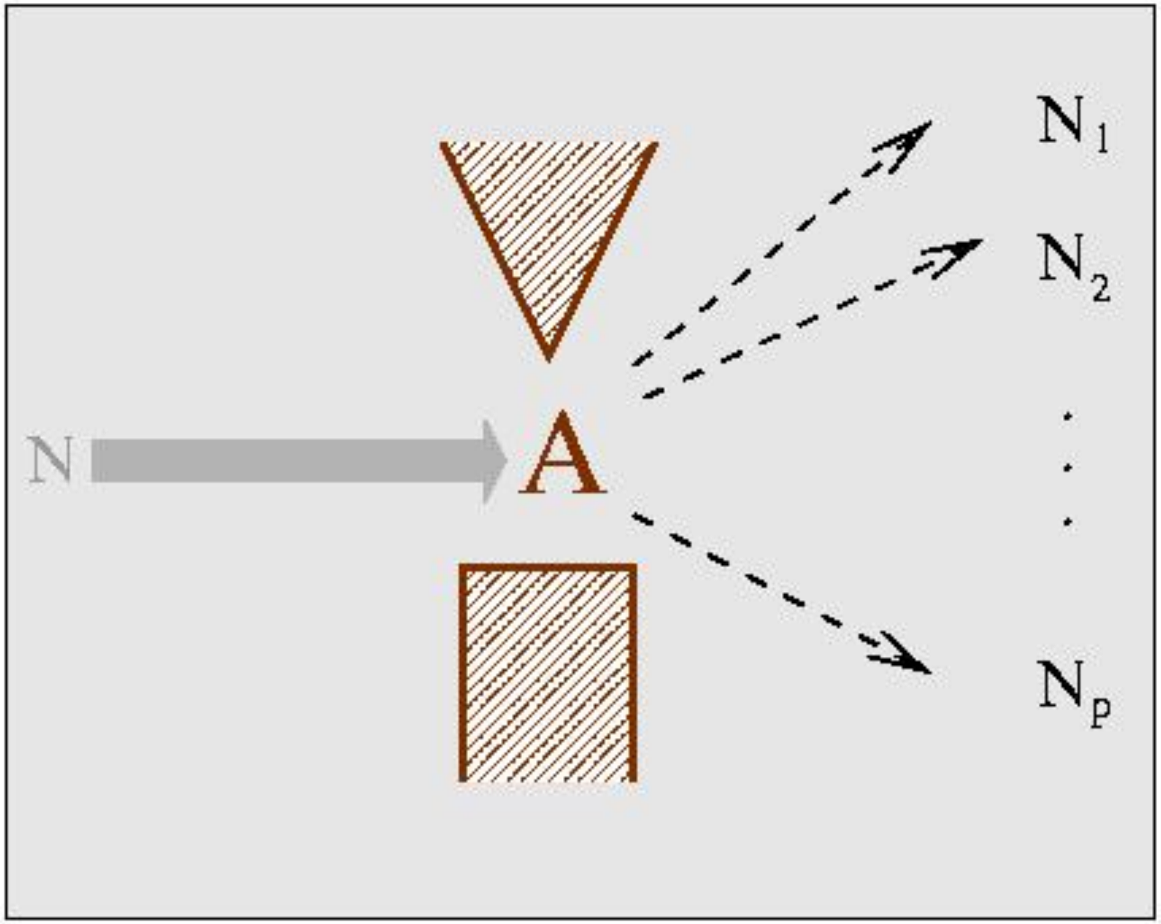
$$p_j = Tr(\rho A_j)$$

Measurement: elements of positive-valued operator measure (POVM) $A_j \geq 0$

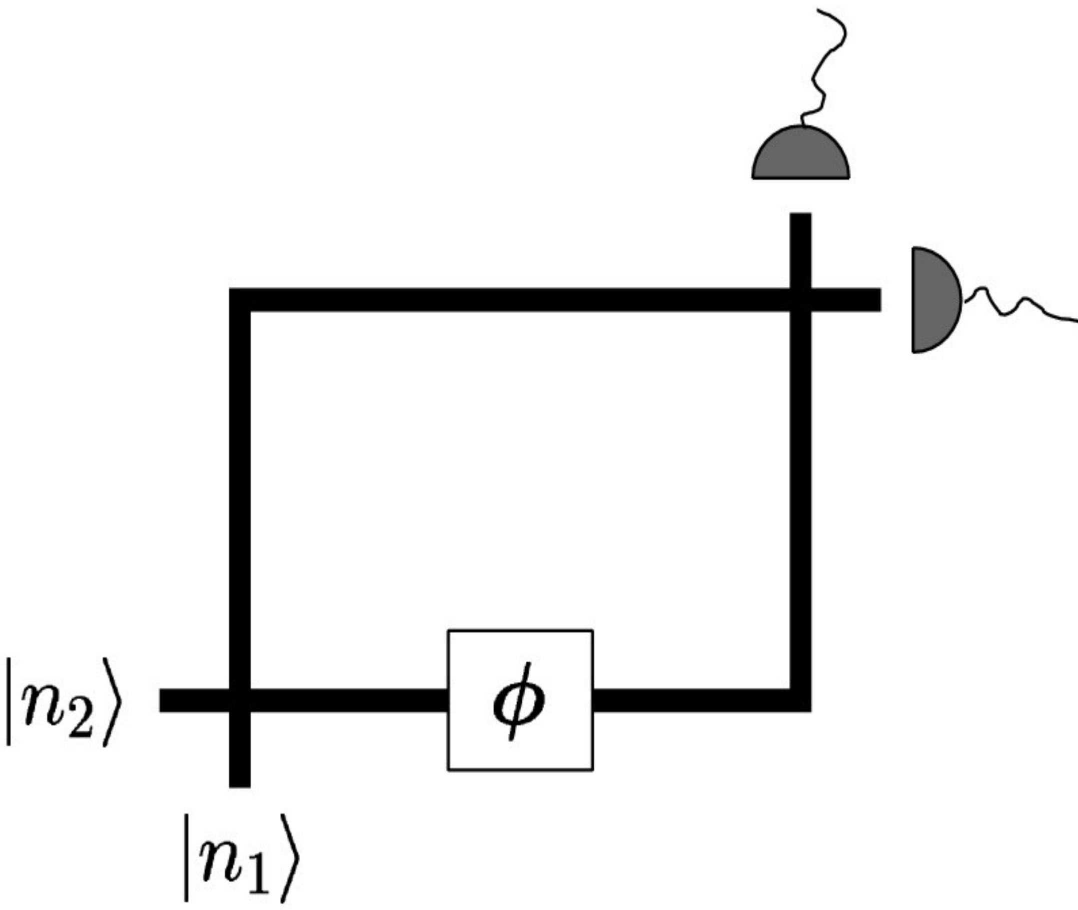
Relation of completeness $\Sigma_j A_j = 1$

Signal: density matrix $\rho \geq 0$

Von Neumann Measurement



Estimation Theory in Drawings



Necessary ingredients:

- Input signal
- Controllable transformation
- Feasible detection

Quantum Estimation Theory

Quantum Estimation Theory
= Quantum Theory + Estimation Theory

Some peculiarities:

- Quantum state ρ plays the role of c-number (matrix) with special constraints ($\rho \geq 0$)
- Quantum measurement must obey uncertainty principle

Maximum Likelihood Estimation (1922)

Sir Ronald Aylmer Fisher, FRS (17 February 1890 – 29 July 1962)
http://digital.library.adelaide.edu.au/coll/special/fisher/papers.html

- Maximum Likelihood (MaxLik) principle is not a rule that requires justification: Bet Always On the Highest Chance!
- Numerous applications in signal analysis, optics, geophysics, nuclear physics,…
- A. Witten, The application of ML estimator to tunnel detection, Inverse Problems 7(1991), 49.
- MaxLik analysis= pea plant experiment of G. Mendel was contrived (too good to be true, statistically ☺)

Maximum Likelihood Tomography

- Likelihood **L** quantifies the degree of belief in certain hypothesis under the condition of the given data.
- MaxLik principle selects the most likely configuration
- Information is updated according to the Bayes rule
prior probability ➔ posterior probability

$$P(\rho|D) = P(D|\rho)\, p(\rho)\, [p(D)]^{-1}$$

Generic reconstruction scheme

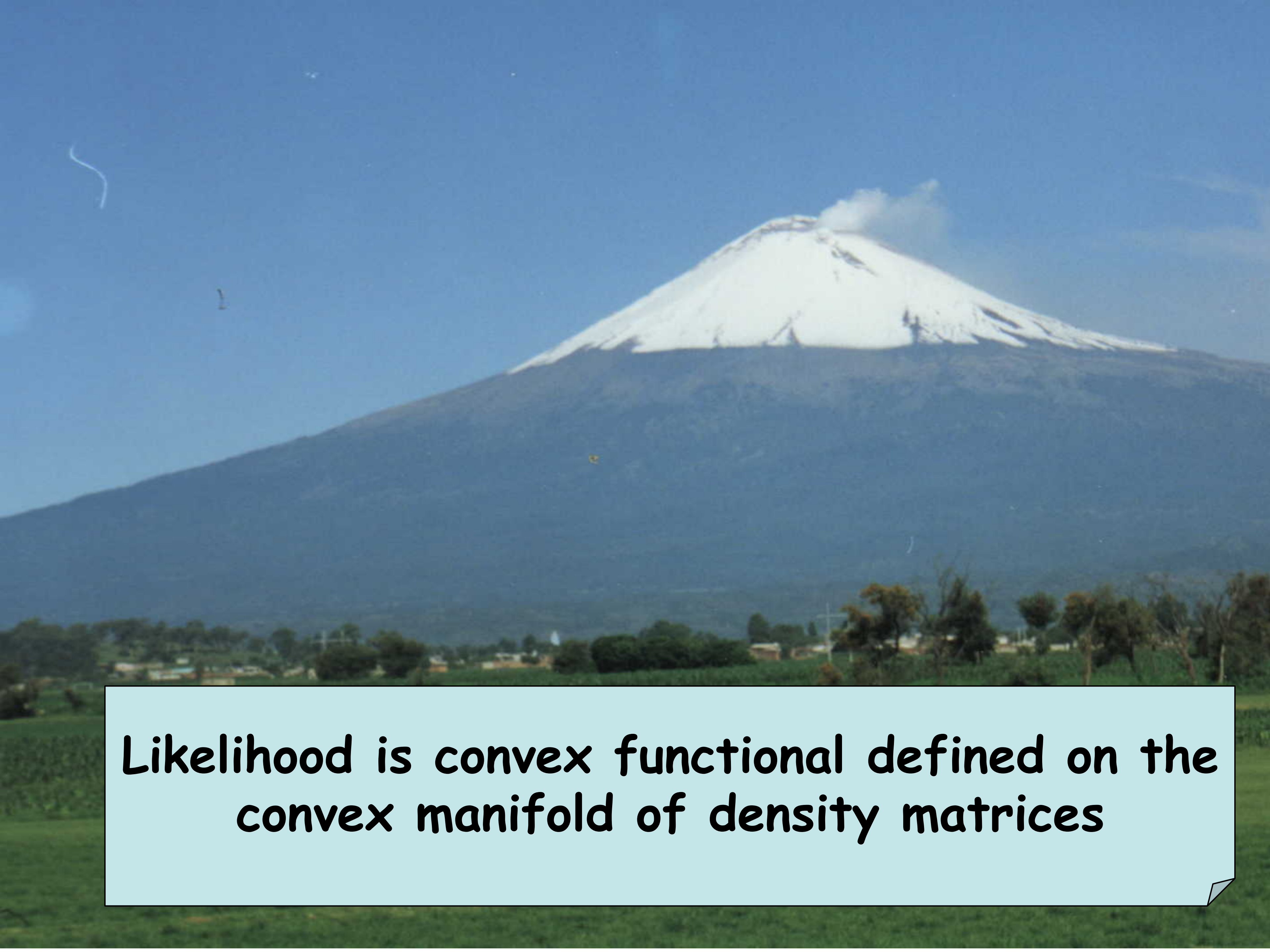
Log-likelihood for generic measurement

$\log L = \sum_i N_j \log p_j / (\sum_k p_k)$

(probabilities are mutually normalized)

Equivalent formulation: estimation of parameters with Poissonian probabilities and unknown mean λ (constrained MaxLik by Fermi)

$\log L = \sum_j N_j \log (\lambda\, p_j) - \lambda \sum_j p_j$



Likelihood is convex functional defined on the convex manifold of density matrices

Information criteria and MaxLik tomography

"The most valuable commodity I know of is information, wouldn't you agree?" (M. Douglas as tycoon Gordon Gekko in the movie Wall Street)

Good statistical models

Many random phenomena, such as those arising in biological and ecological applications, are extremely complex, potentially involving an endless assortment of variables and interactions, „good“ models are needed.An optimal statistical model is characterized by three fundamental attributes:

1. Parsimony (model simplicity)
2. Goodness-of-fit (conformity of the fitted model to the data at hand)
3. Generalizability (applicability of the fitted model to describe or predict new data)

Parsimony

- **Law of Parsimony: No more causes should be assumed than those that will account for the effect.**

More philosophy behind:

- Occam's Razor: "Plurality should not be posited without necessity." (Franciscan monk William of Ockham 1285–1349)
- "Everything should be made as simple as possible, but not simpler." (Albert Einstein, 1879–1955).
- "When you hear hoofbeats, think horses, not zebras." (popular adage from medical schools and residency programs)
- "Simplicity is the ultimate sophistication." (Leonardo da Vinci, 1452–1519).

- Laplace's Principle of Insufficient Reasoning: If there is no reason to prefer among several possibilities, than the best strategy is to consider them as equally likely and pick up the average.

All models are wrong, some are useful (George E. P. Box)

Akaike´s information criterion (AIC)

Akaike,IEEE Trans. Auto Control 19, 716 (1974)

Rationale behind AIC

- Could MaxLik be used for comparing various models?
- No, MaxLik favours overfitting: More complex model means better fit!!
- Akaike´s suggestion: Use Mean LogLik instead of LogLik itself !!!

- Akaike´s Information Criterion- remove the bias from MaxLik

$$AIC = \log L(x|\theta_{ML}) - M$$

M..... dimension of parameter space θ

Schwarz and Bayesian Information Criterion (BIC)

Schwarz, Annals of Stat. 6, 461 (1978)

Konishi, Ando, Imoto, Biometrica 91, 27 (2004)

Penalized MaxLik estimation

Hint: Consider the averaged Likelihood. The normalization term is state independent but dimension dependent!

Modified Schwarz information

$$I_{MS} = \log L(\rho) - \tfrac{1}{2} M \log N + \tfrac{1}{2} M \log(2\pi) - \tfrac{1}{2} \log \det F$$

M … dimension of estimated variable (density matrix)
N … dimension of data set

Entropy and quantification of ignorance

Yong Siah Teo, Huangjun Zhu, B-G Englert, J. Řeháček, Z. Hradil,
Quantum-State Reconstruction by Maximizing Likelihood and Entropy, **Phys. Rev. Lett. 107, 020404 (2011)**

MLME estimation

Likelihood $L(\rho)$ **quantifies the knowledge**

Entropy $S = -Tr(\rho \log \rho)$ **quantifies the ignorance**

$I(\lambda,\rho) = \lambda S(\rho) + 1/N \log L(\rho)$

In the limit $\lambda = 0$ we are searching for the most likely states with the highest entropy.

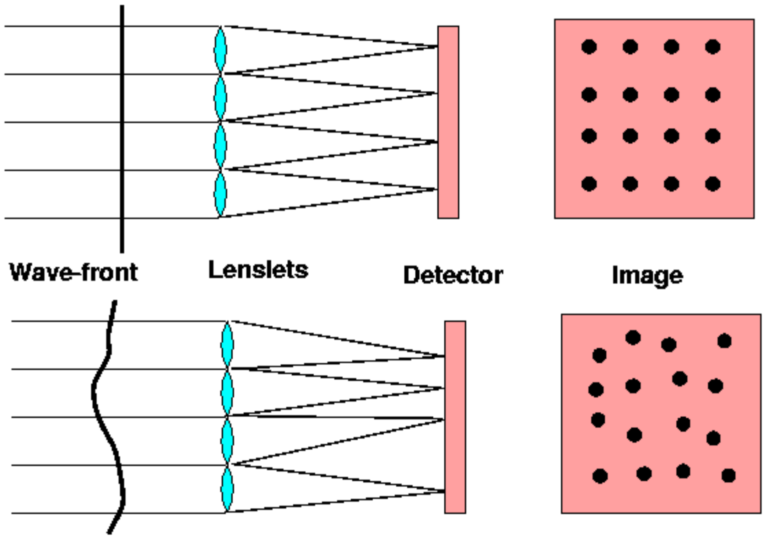
MLME is robust and always selects the single solution.

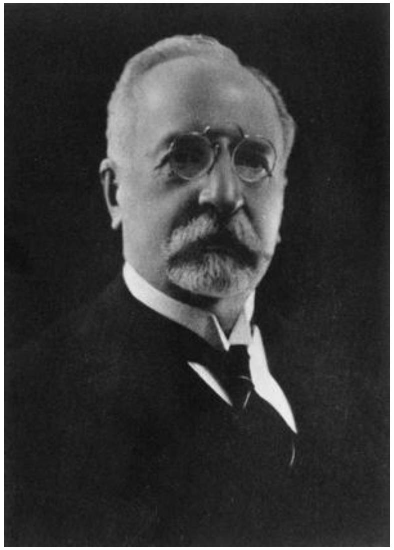
Examples…

Several examples

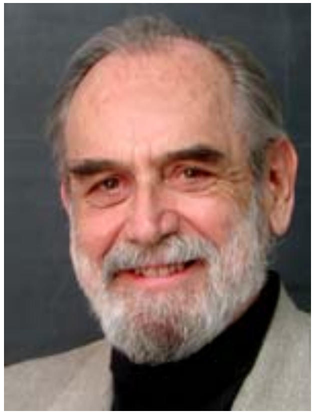
- Phase estimation
- Transmission tomography
- Tomography of CP maps
- Reconstruction of photocount statistics
- Image reconstruction
- Vortex beam analysis
- Quantification of entanglement
- Reconstruction of neutron wave packet
- Reconstruction based on homodyne detection
- Full reconstruction based on on/off detection
- Reconstruction of coherent matrix

Scanning of the optical field: Hartmann-Shack sensor



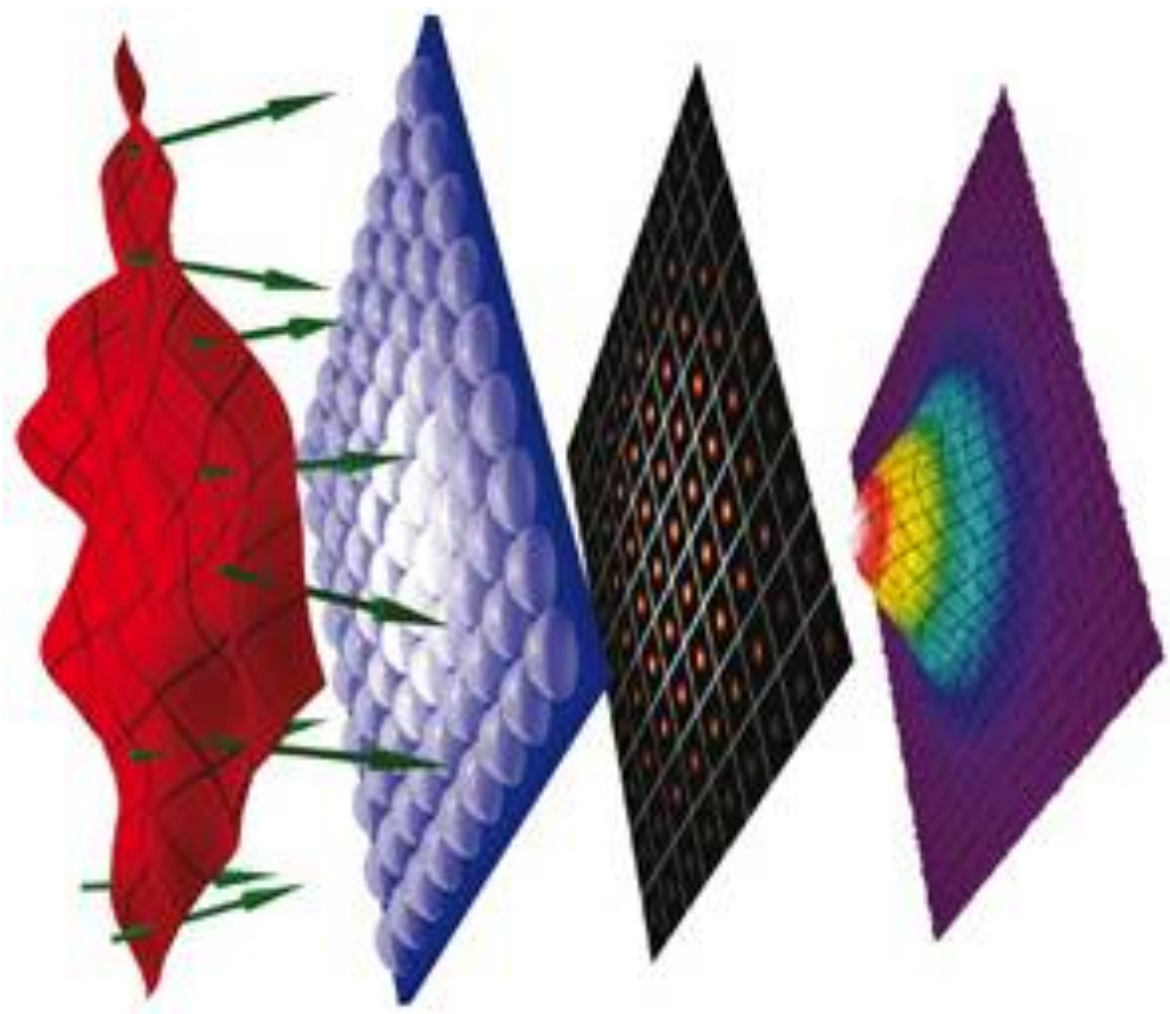


Johannes Hartmann
(1865-1936)



Roland Shack
(1970's)

Scheme of the wave-front reconstruction



Wave theory for HS sensor

- Detected amplitude:

$$\varphi_{det}(\xi)= \int dx'\, dq'\, \varphi(x')h(x'-q')A_i(q') \exp(i\, k\xi q'/f)$$

- Detected signal:

$$S_i(\xi) = \langle |\varphi_{det(}(\xi)|^2 \rangle_{average}$$

$$= \int dx'\, dx'' \int dq'\, dq''\, Q(x',x'')\, h(x'-q')\, a(q', \xi)\, h^*(x''-q'')\, a^*(q'', \xi)$$

where Q… function of mutual coherence

$$a_i(q',\xi) = A_i(q') \exp(i\, k\xi q'/f)$$

- Quantum formulation in x-representation

$$S_i(\xi) = \langle a_{i\xi} | U^\dagger\, Q\, U | a_{i\xi} \rangle$$

$Q(x',x'')= \langle x'|Q|x''\rangle$, $h(x'-q')= \langle q'|U|x'\rangle$, $\langle x'|a_{i\xi}\rangle = a_i(q',\xi)$

HS sensor: Quantum Consequences

- Smooth Gaussian approximation of aperture function:

$$A_i(q') \approx \exp[-(q'-x_i)^2/4(\Delta x)^2]$$

- Detection= Projection into the minimum uncertainty states

$$a_{i,\xi} = \exp[-(q'-x_i)^2/4(\Delta x)^2 + i\, k\xi q'/f\ ]$$

- Heisenberg uncertainty relations

$$\Delta x\ \Delta p \geq \hbar/2$$

- Generalized measurement of non-commuting variables x and p, (Arthurs, Kelly 1964)

$$\Delta X\ \Delta P \geq \hbar$$

See the excellent paper: S. Stenholm, *Simultaneous measurement of conjugate variables, Annals of Physics 218, 233-254 (1992).*

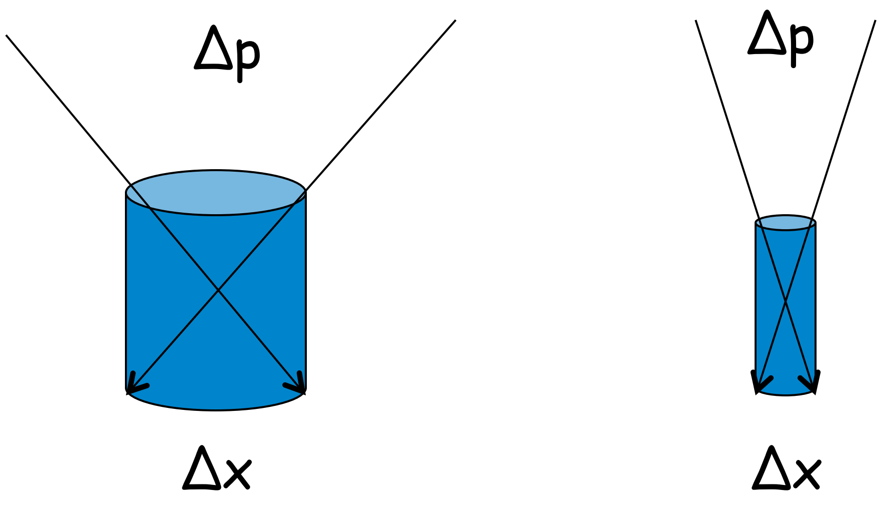
Further Quantum Consequences

- POVM corresponds to detection of annihilation operator

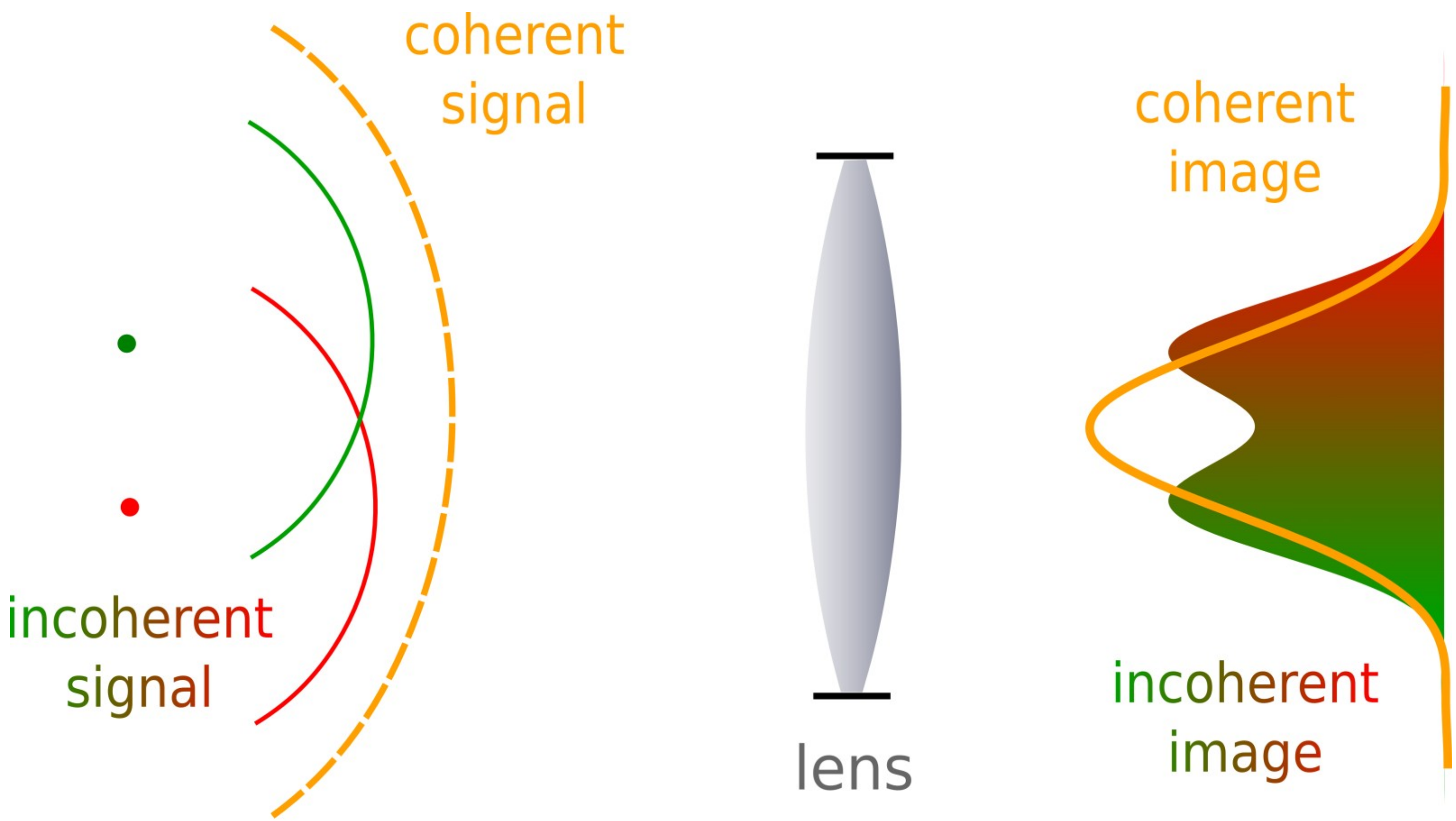
$$a = x + ip$$

$$1/\pi \int da^2 \; |a\rangle\langle a| = 1$$

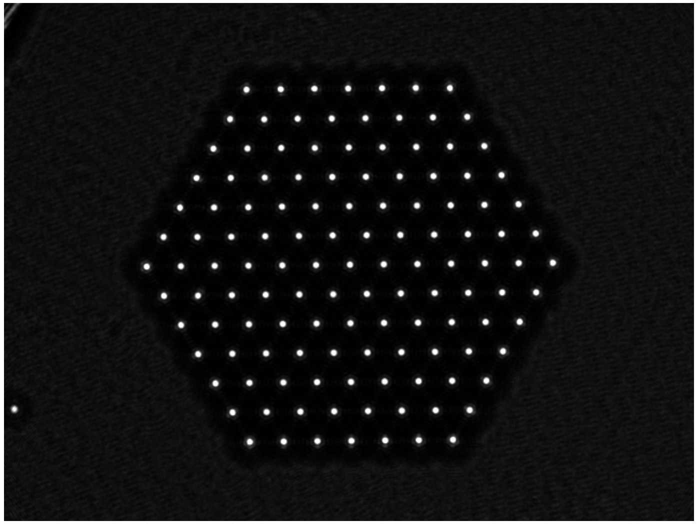
- Q-distribution (Husimi)

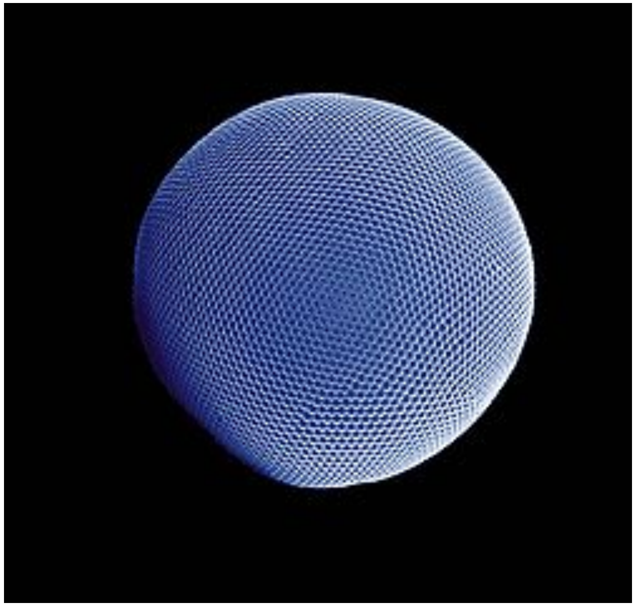


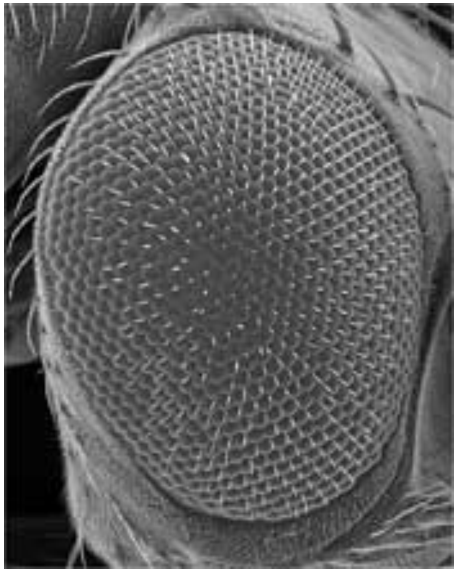
Detection of partially coherent signal



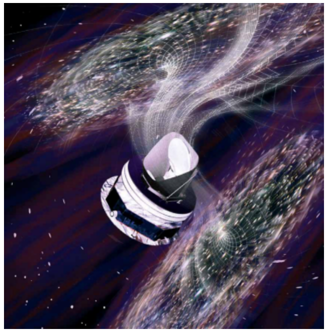
Hartmann-Shack sensor of the wavefront?

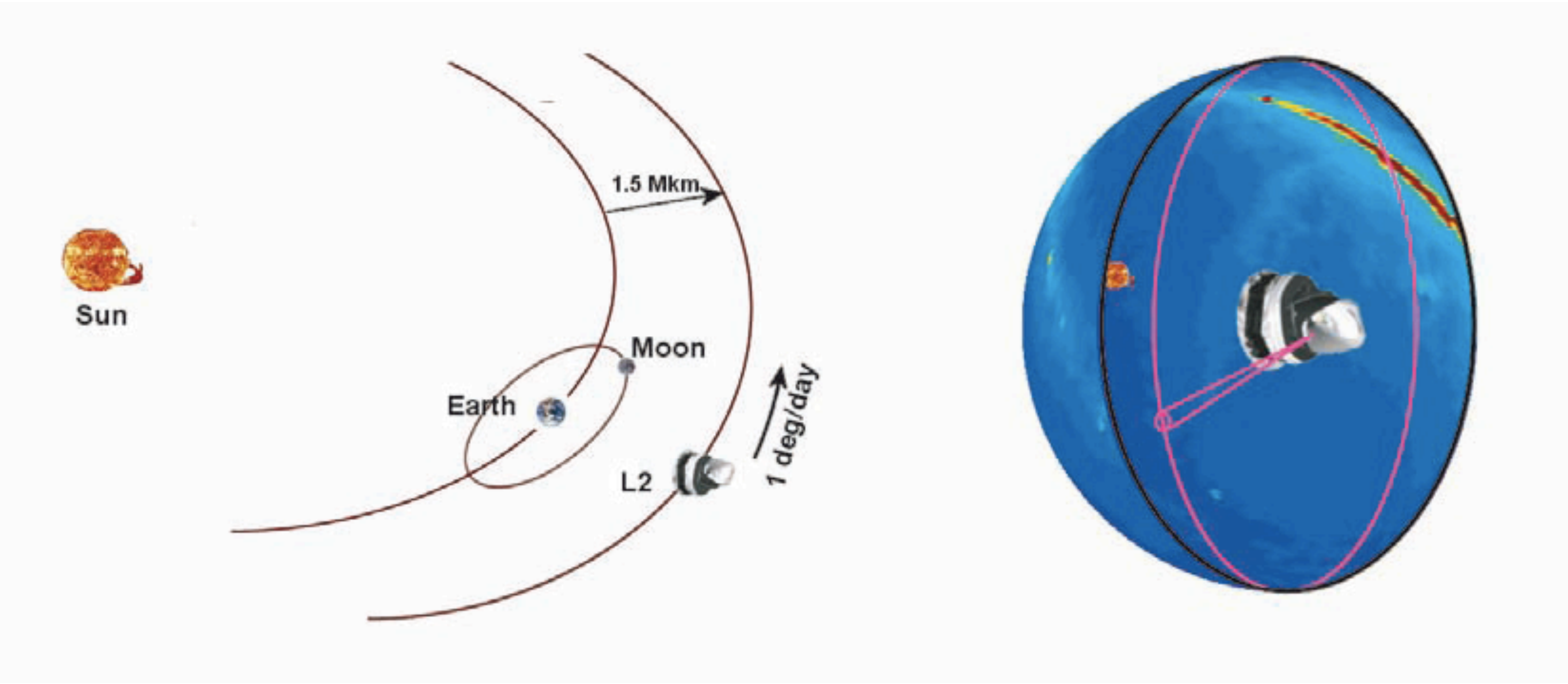




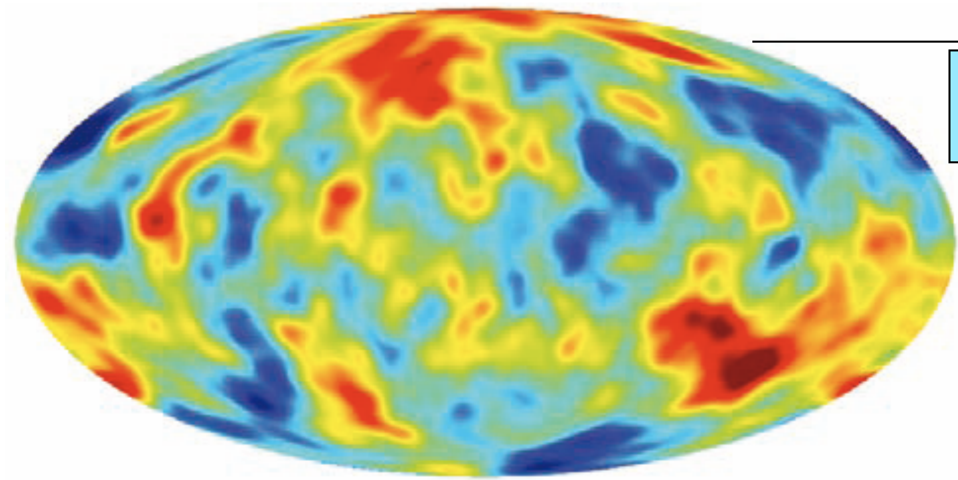


Planck mission of ESA: scanning of cosmic background radiation

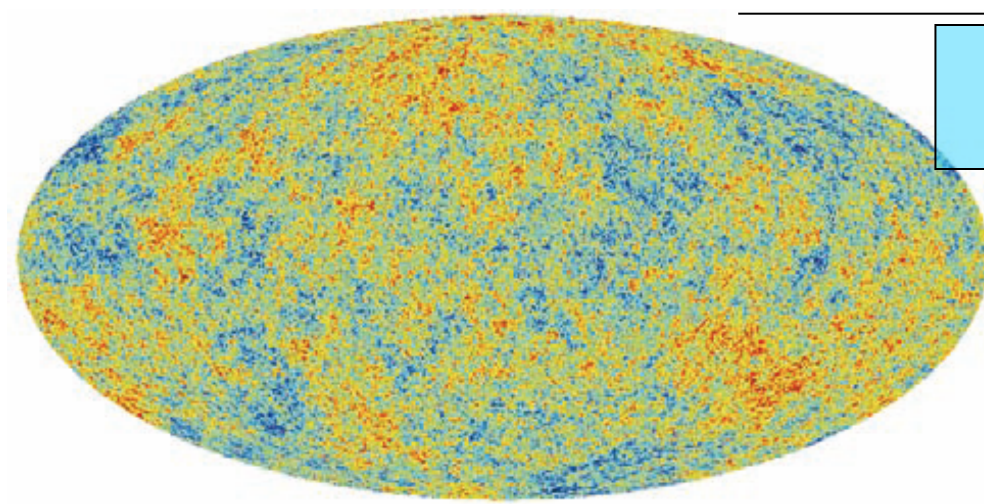




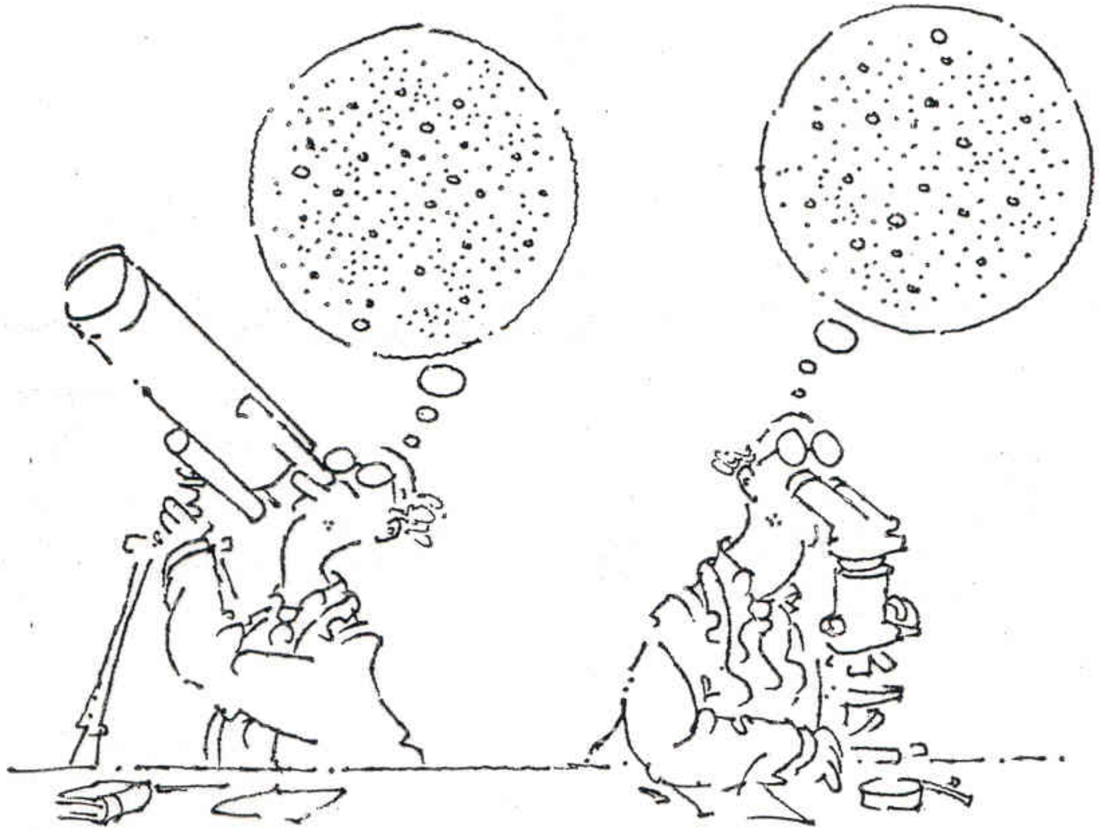
Temperature anisotropies

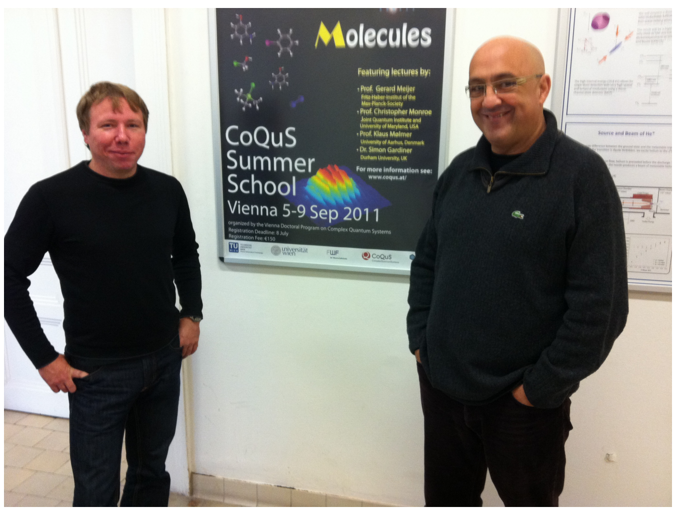


COBE-DMR resolution



Planck resolution

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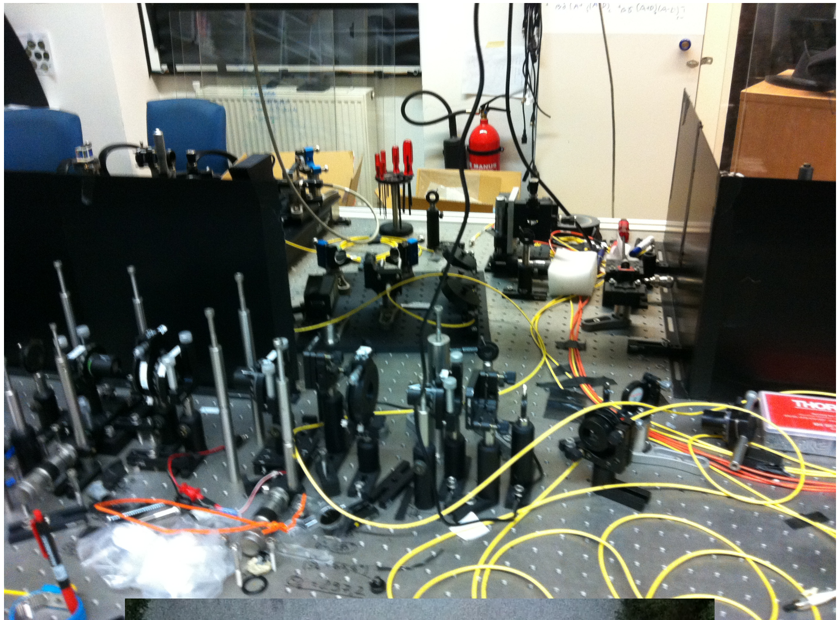
Atoms




S = k log W
LVDWIG
BOLTZMANN
1844 - 1906


STERN
BRÄU


CAFÉ
Joghurt Dreieck
Landtmann Kugel
Marzipankartoffel
Kunst! Stücke!
Kardinal Schnitte
Sacher Torte
Maroniblüte
Mohn Torte
Maroni Torte
Apfel-Nuss Torte
Gebackene Topfen Torte
Mozart Torte
Nussbeugerl
Rehrücken
Landtmann's feine Torte
Schokomousse Törtchen
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