Low-power optical communication: Approaching the quantum limit

M. Jarzyna, P. Kuszaj, A. Klimek, W. Lipińska, K. Banaszek

Wydział Fizyki, Uniwersytet Warszawski

M. G. A. Paris

Dipartimento di Fisica, Università degli studi di Milano

UNIVERSITY OF WARSAW

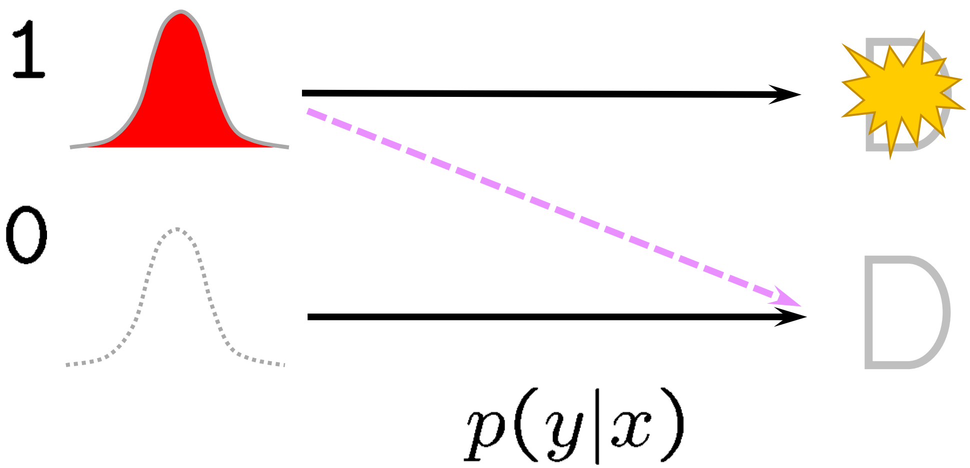
siqs

SEVENTH FRAMEWORK PROGRAMME

PhoQuS@UW

OOK: On-off keying

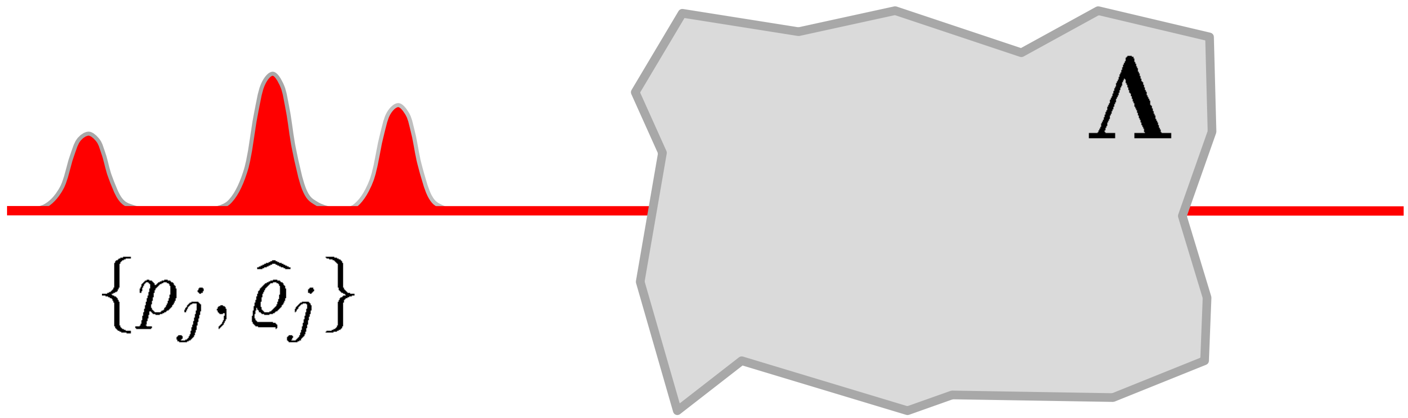
0 1 1 0 0 1 0 1 1 0 1 1 0 0 1 0 1 0 1 1



Mutual information:

$$I(X,Y) = \sum_{x,y} p(y|x)p(x) \log_2 \frac{p(y|x)}{\sum_{x'} p(y|x')p(x')}$$

Channel capacity



Holevo quantity

$$\chi\Big(\{p_j, \hat{\varrho}_j\}\Big) = \mathsf{S}\left(\sum_j p_j \Lambda(\hat{\varrho}_j)\right) - \sum_j p_j \mathsf{S}\Big(\Lambda(\hat{\varrho}_j)\Big)$$

Classical channel capacity

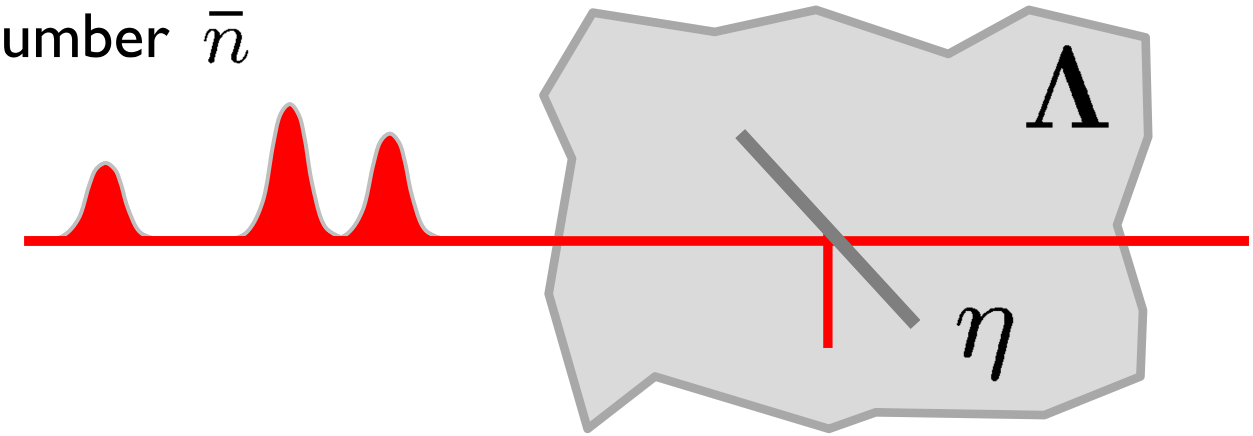
$$\mathsf{C} = \max_{\{p_j, \hat{\varrho}_j\}} \chi\Big(\{p_j, \hat{\varrho}_j\}\Big)$$

UNIVERSITY
OF WARSAW

Lossy bosonic channel

V. Giovannetti, R. García-Patrón, N. J. Cerf, A. S. Holevo, Nature Photon. **8**, 796 (2014)

Average input photon number $\bar{n}$



Channel capacity

$$C = (\eta\bar{n}+1)\log_2(\eta\bar{n}+1) - \eta\bar{n}\log_2 \eta\bar{n}$$

$\equiv$ entropy of a thermal state with the mean output photon number $\eta\bar{n}$

$$\approx \eta\bar{n}\log_2\frac{1}{\eta\bar{n}} + O(\eta\bar{n}) \qquad \text{for} \quad \eta\bar{n} \ll 1$$

Optimal ensemble: lossless case

Fock states $(n = 0, 1, 2, \ldots)$:

$$p_n = \frac{1}{1+\bar{n}} \left(\frac{\bar{n}}{1+\bar{n}} \right)^n, \qquad \hat{\varrho}_n = |n\rangle\langle n|$$

Coherent state ensemble (α arbitrary complex):

$$p_\alpha = \frac{1}{\pi \bar{n}} e^{-|\alpha|^2/\bar{n}}, \qquad \hat{\varrho}_\alpha = |\alpha\rangle\langle\alpha|$$

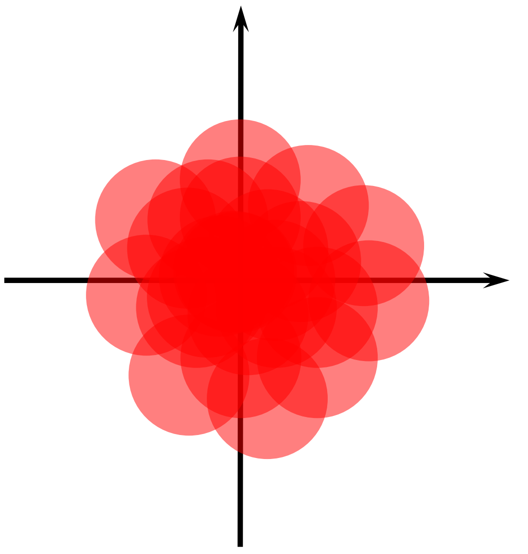
Average state:

$$\hat{\varrho}_{\mathrm{av}} = \sum_{n=0}^{\infty} p_n \hat{\varrho}_n = \int_{\mathbb{C}} \mathrm{d}^2\alpha \, p_\alpha \hat{\varrho}_\alpha$$

Losses: Coherent state ensemble

Coherent states remain *pure:*

$$\boldsymbol{\Lambda}\left(\hat{\varrho}_{\alpha}\right) = |\sqrt{\eta}\alpha\rangle\langle\sqrt{\eta}\alpha|$$



Average output state

$$\int \mathrm{d}^2\alpha \, p_\alpha \boldsymbol{\Lambda}\left(\hat{\varrho}_{\alpha}\right)$$

Thermal state with $\eta\bar{n}$ photons on average

- Continuum of input states
- Phase reference needed

Losses: Fock state ensemble

$$\boldsymbol{\Lambda}(\hat{\varrho}_n) = \sum_{k=0}^{n} \binom{n}{k} \eta^k (1-\eta)^{n-k} |k\rangle \langle k|$$

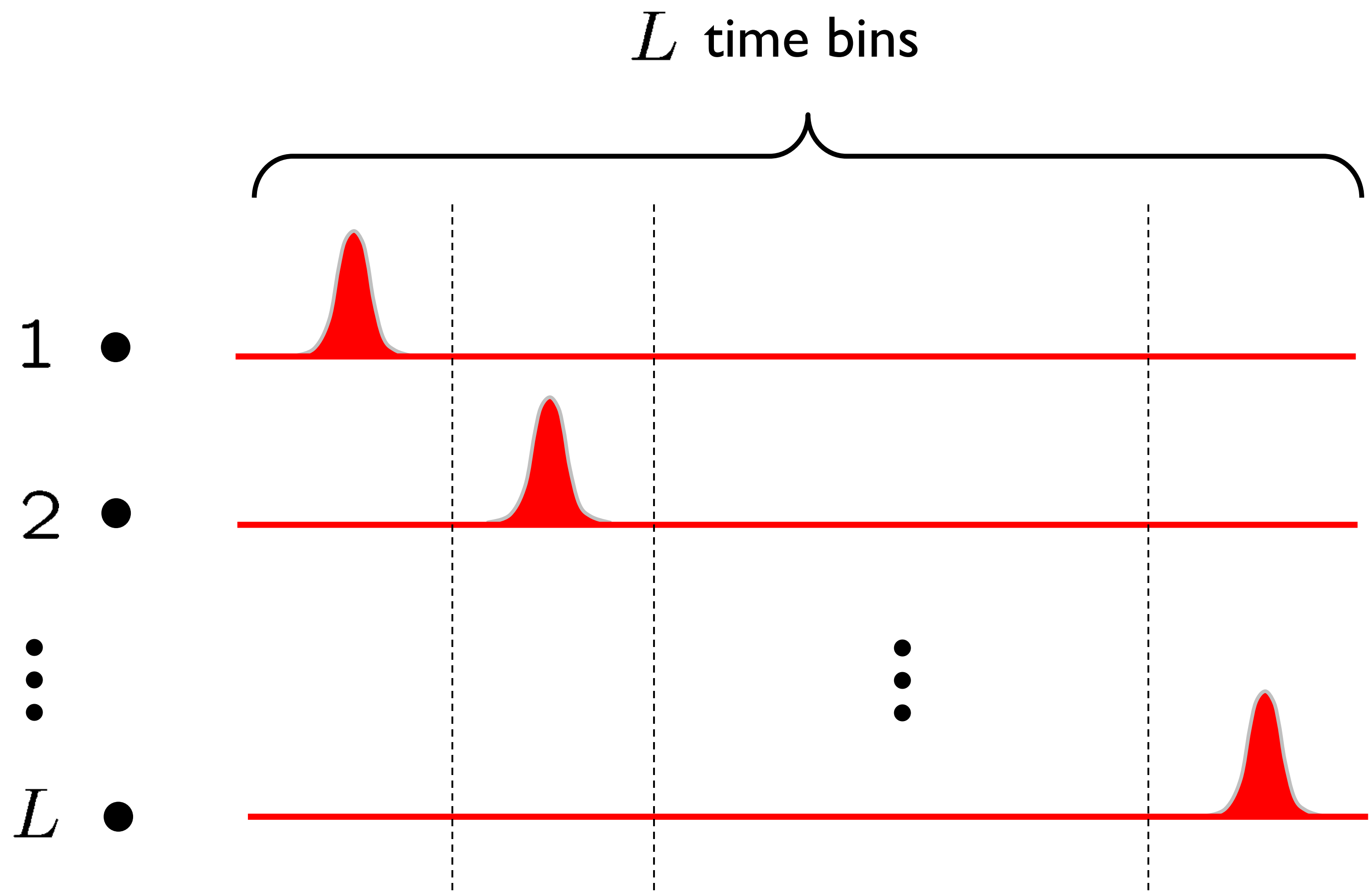
Entropy of individual states:

$$\mathsf{S}\big(\boldsymbol{\Lambda}(\hat{\varrho}_n)\big) > 0$$

Holevo quantity:

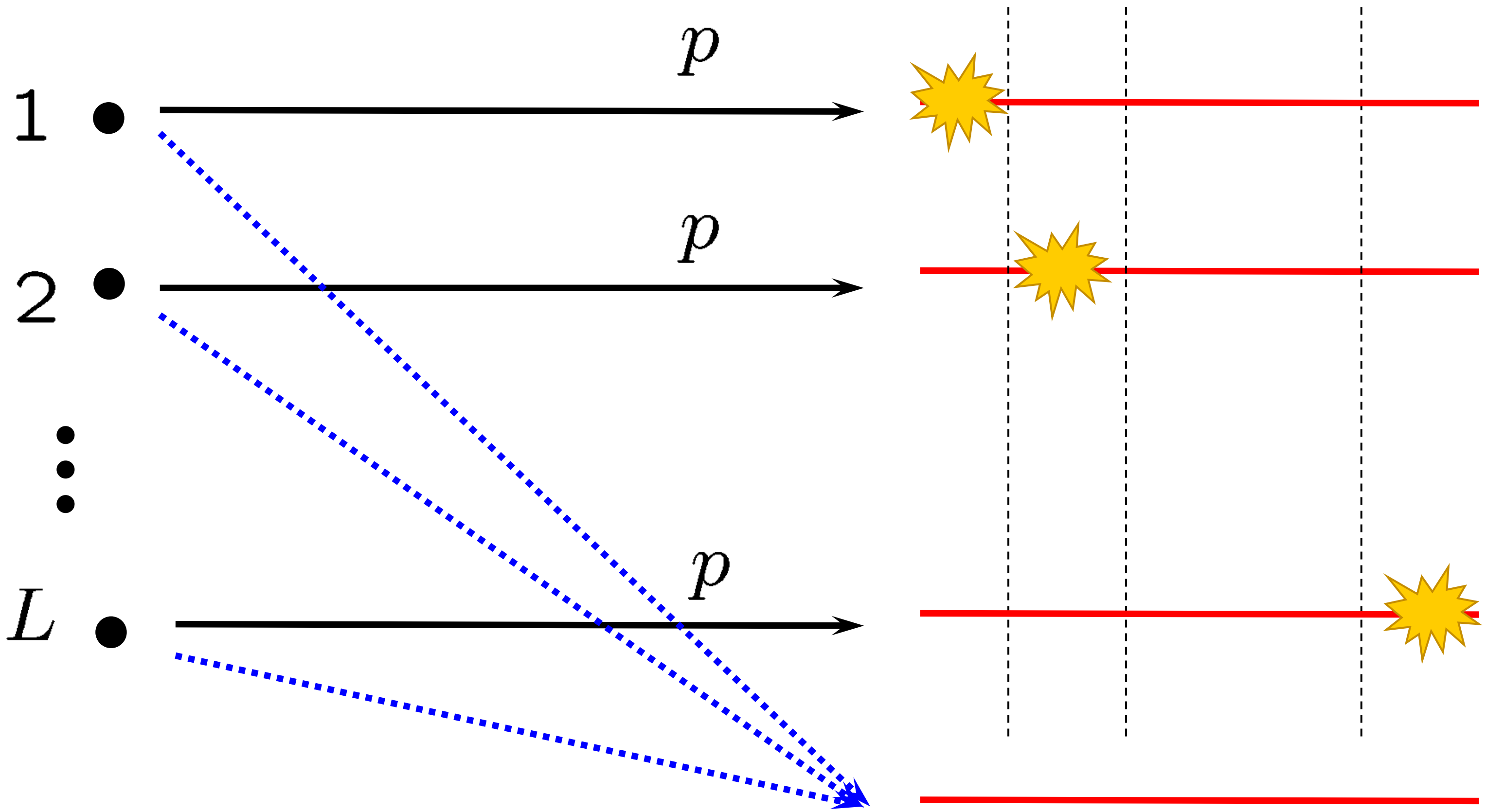
$$\chi\big(\{p_n, \hat{\varrho}_n\}\big) \approx \eta\bar{n}\log_2\frac{1}{\bar{n}} + O(\bar{n}^2)$$

PPM: Pulse Position Modulation



Uniform input probabilities $\frac{1}{L}$

Direct detection



Erasure probability

$$1 - p = \langle : \exp(-\eta \hat{N}) : \rangle$$

UNIVERSITY
OF WARSAW

Maximum transmission rate

Mutual information per time bin

$$\mathsf{R} = \frac{p}{L}\log_2 L$$

Energy constraint

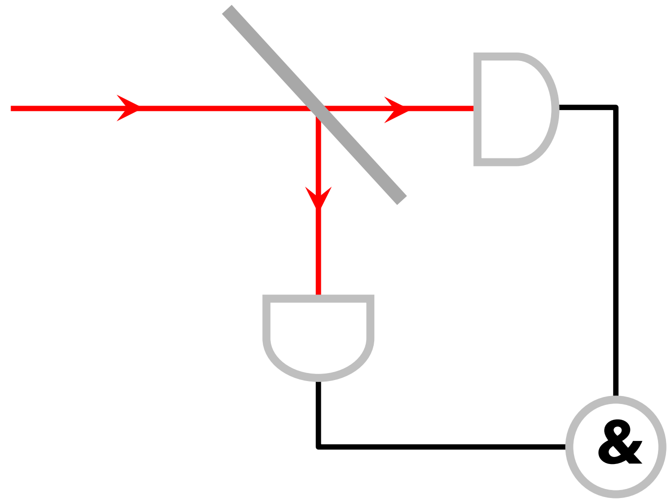
$$\langle \hat{N} \rangle = L\bar{n}$$

Weak output pulse regime, $\eta\langle \hat{N} \rangle \ll 1$

$$\begin{aligned} p &\approx \eta\langle \hat{N} \rangle - \frac{1}{2}\eta^2\langle : \hat{N}^2 : \rangle \\ &= L\eta\bar{n} - \frac{g^{(2)}}{2}(L\eta\bar{n})^2 \end{aligned}$$

UNIVERSITY
OF WARSAW

Intensity correlations



$$g^{(2)} = \frac{\langle : \hat{N}^2 : \rangle}{\langle \hat{N} \rangle^2}$$

Non-classical light $g^{(2)} < 1$

Stochastic classical light $g^{(2)} > 1$

Poissonian statistics $g^{(2)} = 1$

UNIVERSITY OF WARSAW

Optimisation

$$\mathrm{R} = \eta\bar{n}\left(1 - \frac{g^{(2)}}{2}L\eta\bar{n}\right)\log_2 L$$

For sequence length $L \gg 1$ treated as a continuous and unconstrained parameter:

$$L_{\mathrm{opt}} = \frac{2}{g^{(2)}\eta\bar{n}}\left[W\left(\frac{2e}{g^{(2)}\eta\bar{n}}\right)\right]^{-1}$$

Lambert function $W(x) \approx \ln x$ for $x \gg 1$

Hierarchy in the low-power regime:

$$\eta\bar{n} \ll L_{\mathrm{opt}}\eta\bar{n} \approx \frac{2}{g^{(2)}}\left(\ln\frac{2e}{g^{(2)}\eta\bar{n}}\right)^{-1} \ll 1$$

UNIVERSITY OF WARSAW

Efficiency

M. Jarzyna, P. Kuszaj, K. Banaszek, Opt. Express **23**, 3170 (2015)

Optimal transmission rate

$$\mathsf{R} = \eta\bar{n}\Pi\left(g^{(2)}\eta\bar{n}\right)$$

Photon information efficiency (PIE) for Poissonian statistics:

$$\Pi(\eta\bar{n}) = \frac{1}{\ln 2}\left\{W\left(\frac{2\mathrm{e}}{\eta\bar{n}}\right) - 2 + \left[W\left(\frac{2\mathrm{e}}{\eta\bar{n}}\right)\right]^{-1}\right\}$$

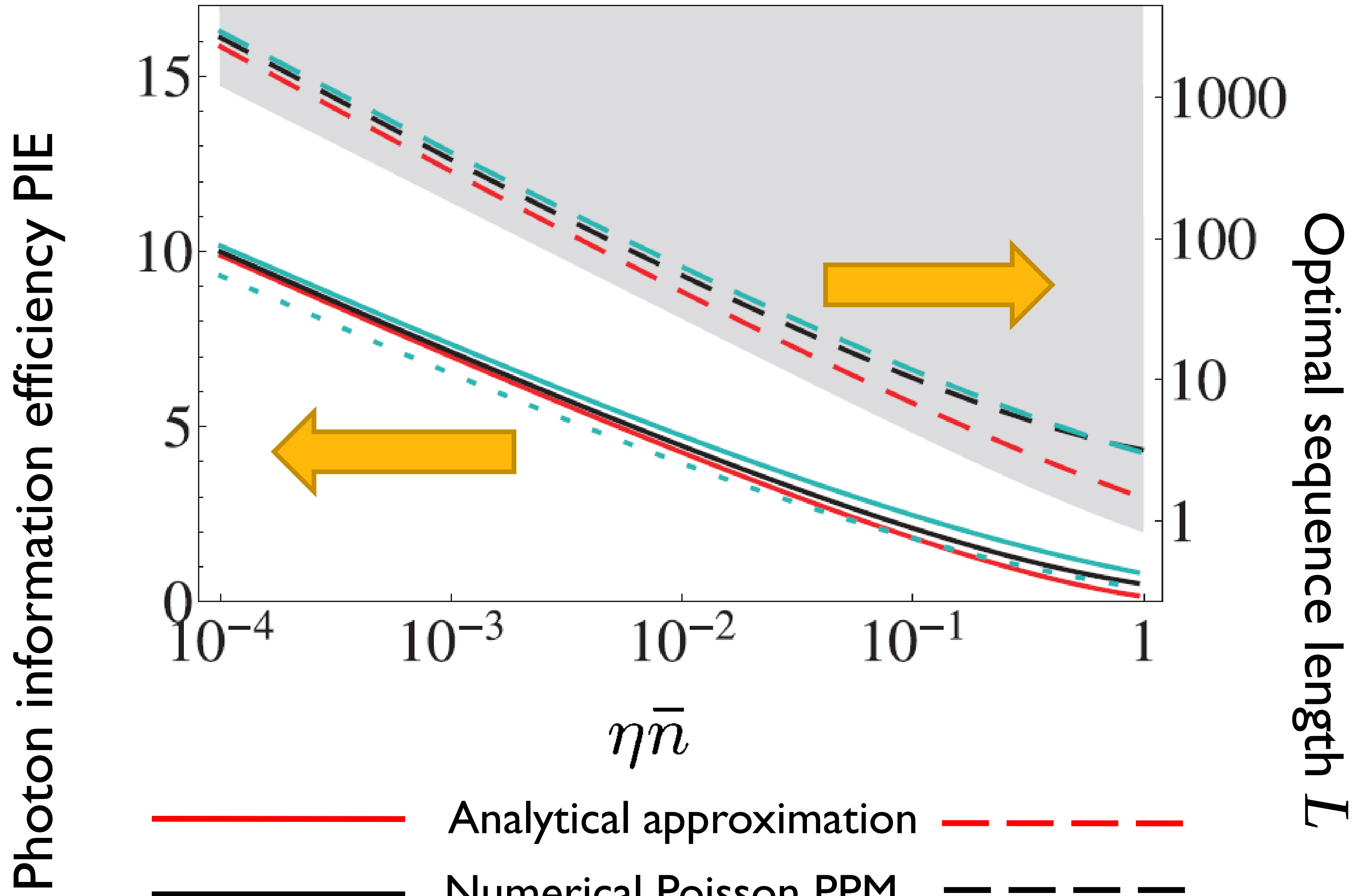
Asymptotics

$$\Pi(\eta\bar{n}) = \log_2\frac{1}{\eta\bar{n}} - \log_2\ln\frac{1}{\eta\bar{n}} + O(1)$$

Channel capacity:

$$\frac{1}{\eta\bar{n}}\mathsf{C} = \log_2\frac{1}{\eta\bar{n}} + \frac{1}{\ln 2} + O(\eta\bar{n})$$

Comparison



M. Jarzyna, P. Kuszaj, K. Banaszek, Opt. Express **23**, 3170 (2015)

Nonclassical light

$$\mathsf{R} = \eta\bar{n}\Pi\left(g^{(2)}\eta\bar{n}\right)$$

For nonclassical light optimization over the pulse photon number is *constrained:*

$$\langle \hat{N}^2 \rangle \geq \langle \hat{N} \rangle^2 \Rightarrow \langle \hat{N} \rangle \leq \frac{1}{1 - g^{(2)}}$$

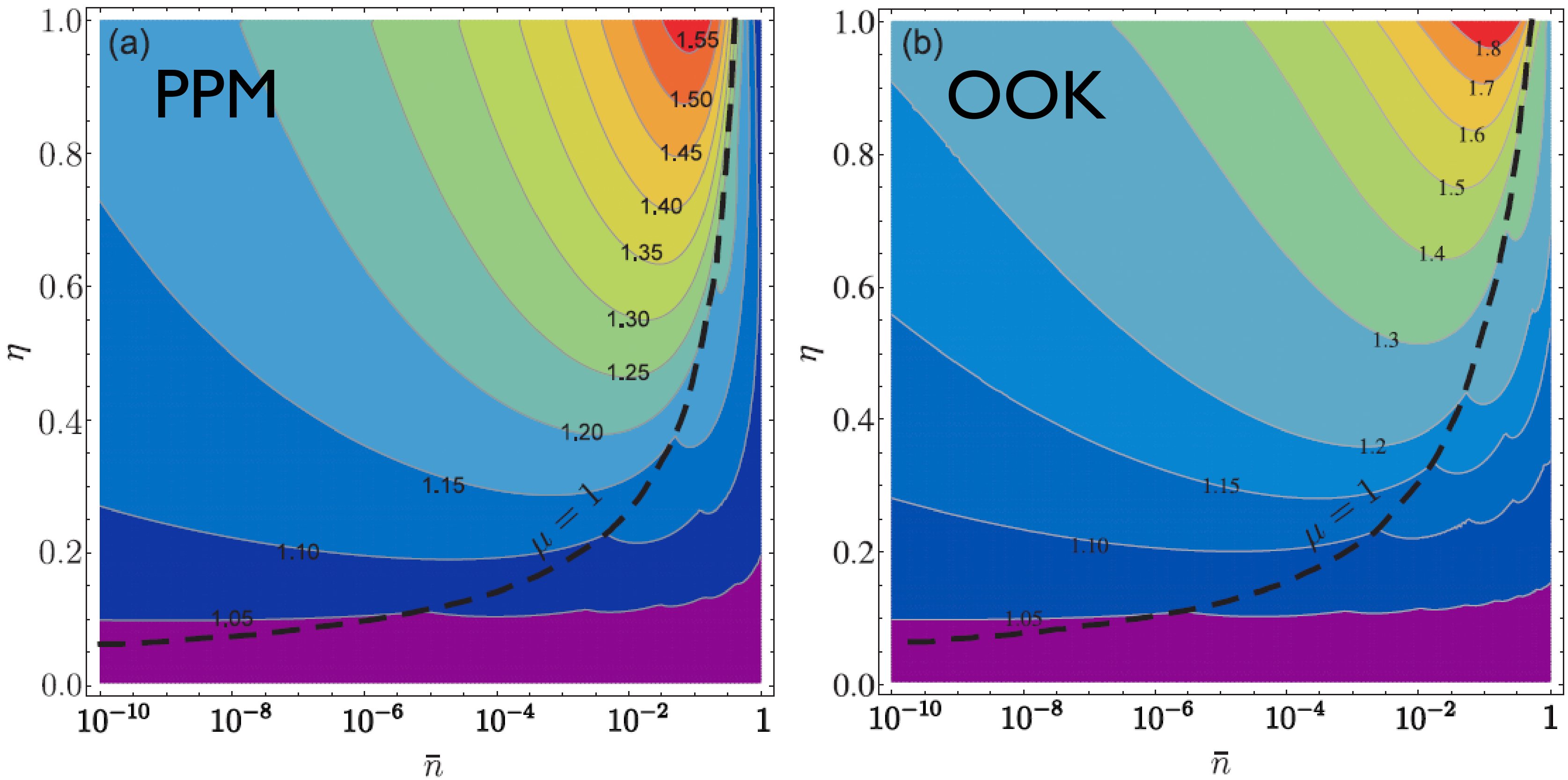
When $\eta > 2/\ln\frac{1}{\bar{n}}$, single photons are optimal

$$\mathsf{R}_{\mathsf{PPM}} = \eta\bar{n}\log_2\frac{1}{\bar{n}}$$

UNIVERSITY OF WARSAW

Nonclassical enhancement

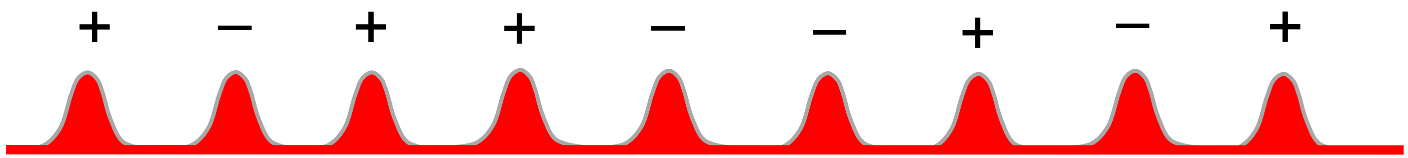
$R_{Fock}/R_{Poisson}$:



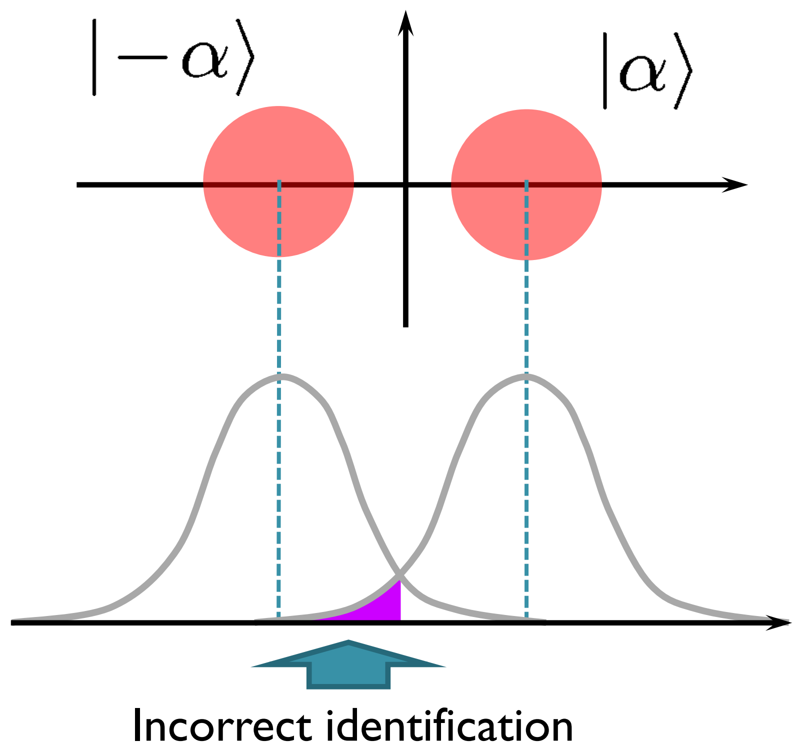
M. Jarzyna, P. Kuszaj, K. Banaszek, Opt. Express **23**, 3170 (2015)

Binary phase shift keying (BPSK)

$$\alpha = \sqrt{\bar{n}}$$

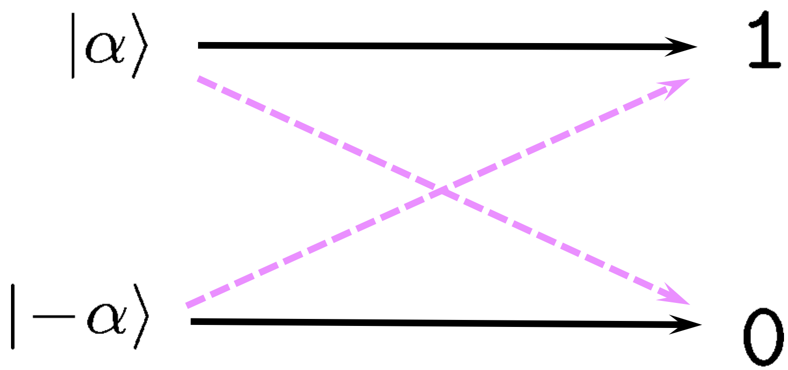


Homodyne detection



UNIVERSITY OF WARSAW

Binary symmetric channel



Capacity: $\mathrm{C} = 1 - \mathrm{H}(e)$

$$\mathrm{H}(e) = -e\log_2 e - (1-e)\log_2(1-e)$$

Error probability for homodyne detection

$$e_{\mathrm{hom}} = \tfrac{1}{2}[1 - \mathrm{erf}(\sqrt{2}\alpha)]$$

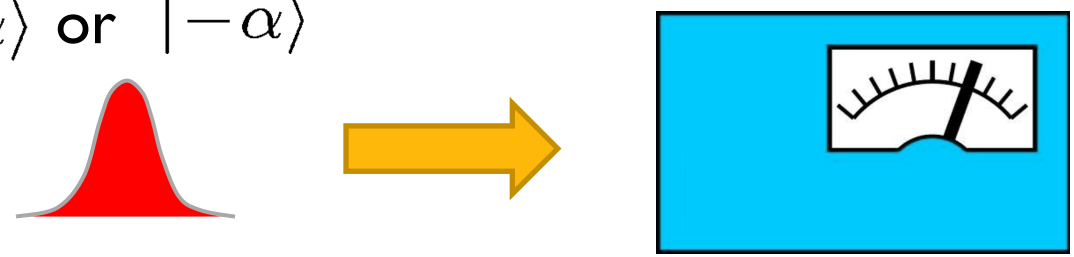
Transmission rate

$$\mathrm{R}_{\mathrm{hom}} = 1 - \mathrm{H}(e_{\mathrm{hom}}) \approx \frac{4\bar{n}}{\pi \ln 2} \qquad \text{for } \bar{n} \ll 1$$

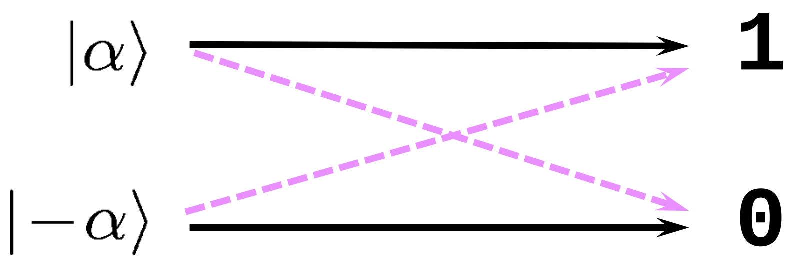
UNIVERSITY
OF WARSAW

Minimum-error measurement

$|\alpha\rangle$ or $|-\alpha\rangle$



$$\langle -\alpha|\alpha\rangle = \mathrm{e}^{-2\bar{n}}$$



Helstrom limit

$$e_{\mathrm{Hel}} = \frac{1}{2}\left(1 - \sqrt{1 - |\langle -\alpha|\alpha\rangle|^2}\right)$$

Maximum transmission rate *for individual detection:*

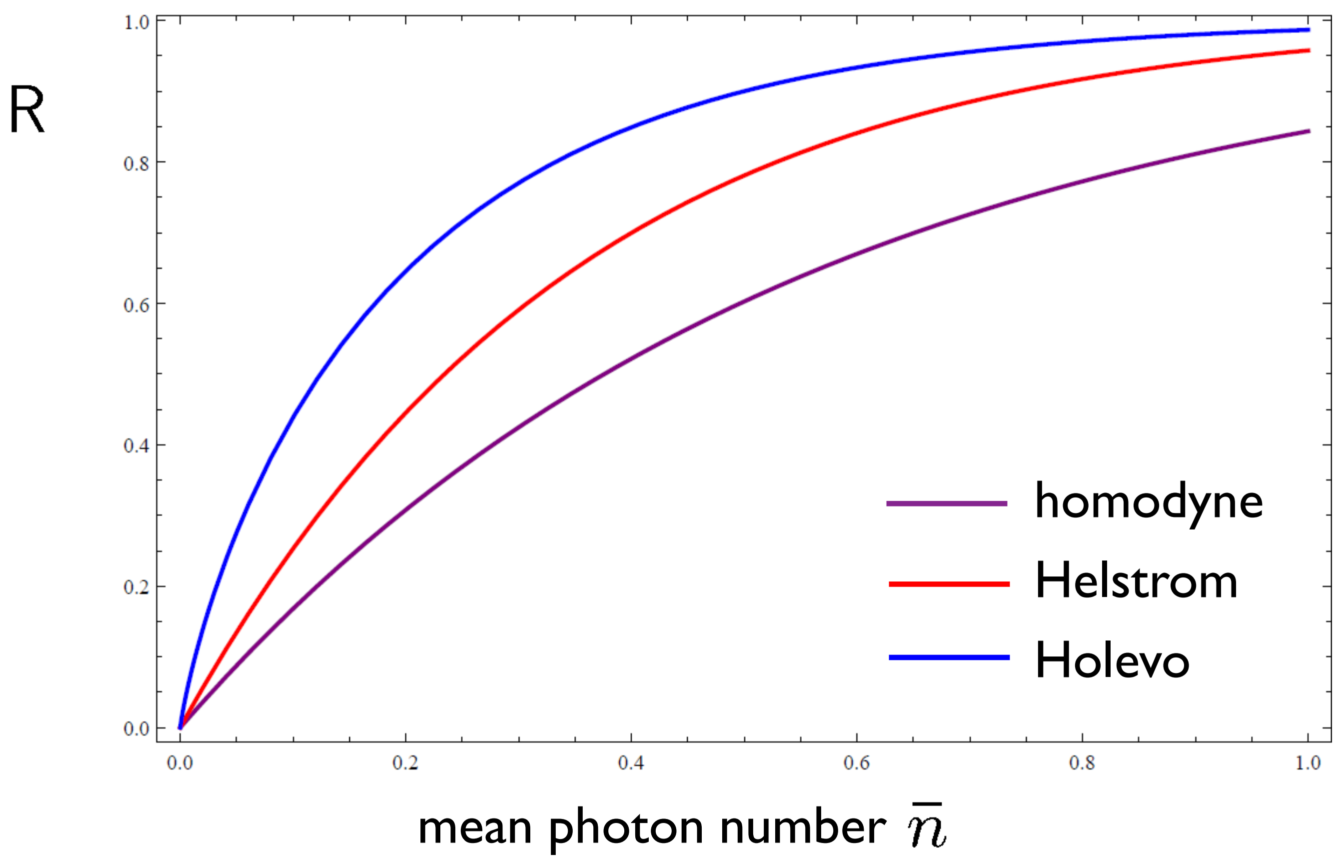
$$\mathrm{R}_{\mathrm{Hel}} = 1 - \mathrm{H}(e_{\mathrm{Hel}}) \approx \frac{2\bar{n}}{\ln 2}$$

UNIVERSITY
OF WARSAW

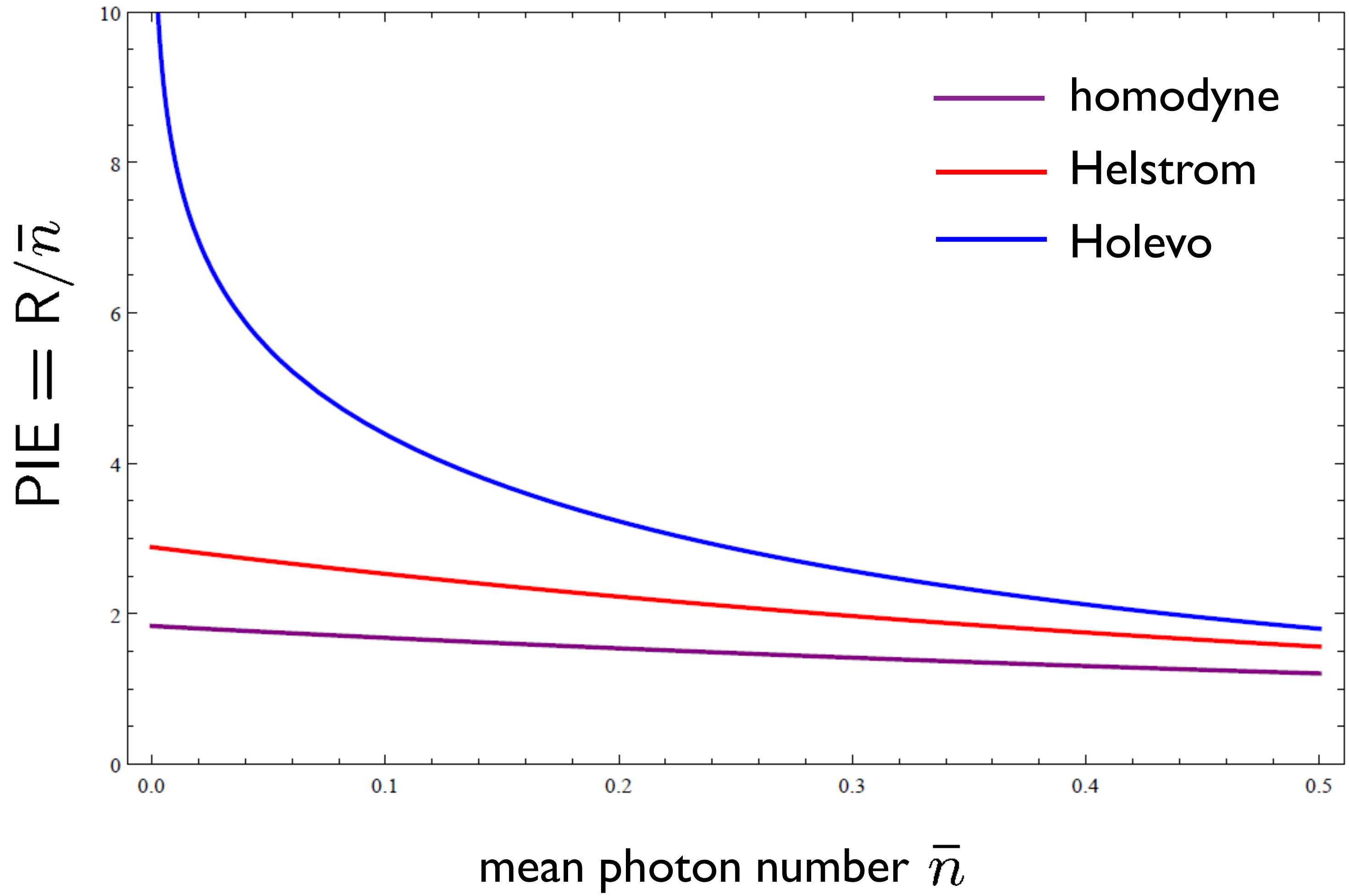
Holevo quantity

$$\chi = S\left(\tfrac{1}{2}|\alpha\rangle\langle\alpha| + \tfrac{1}{2}|-\alpha\rangle\langle-\alpha|\right)$$

$$\approx H(\bar{n}) = \bar{n}\log_2\frac{1}{\bar{n}} + \frac{\bar{n}}{\ln 2} + O(\bar{n}^2)$$

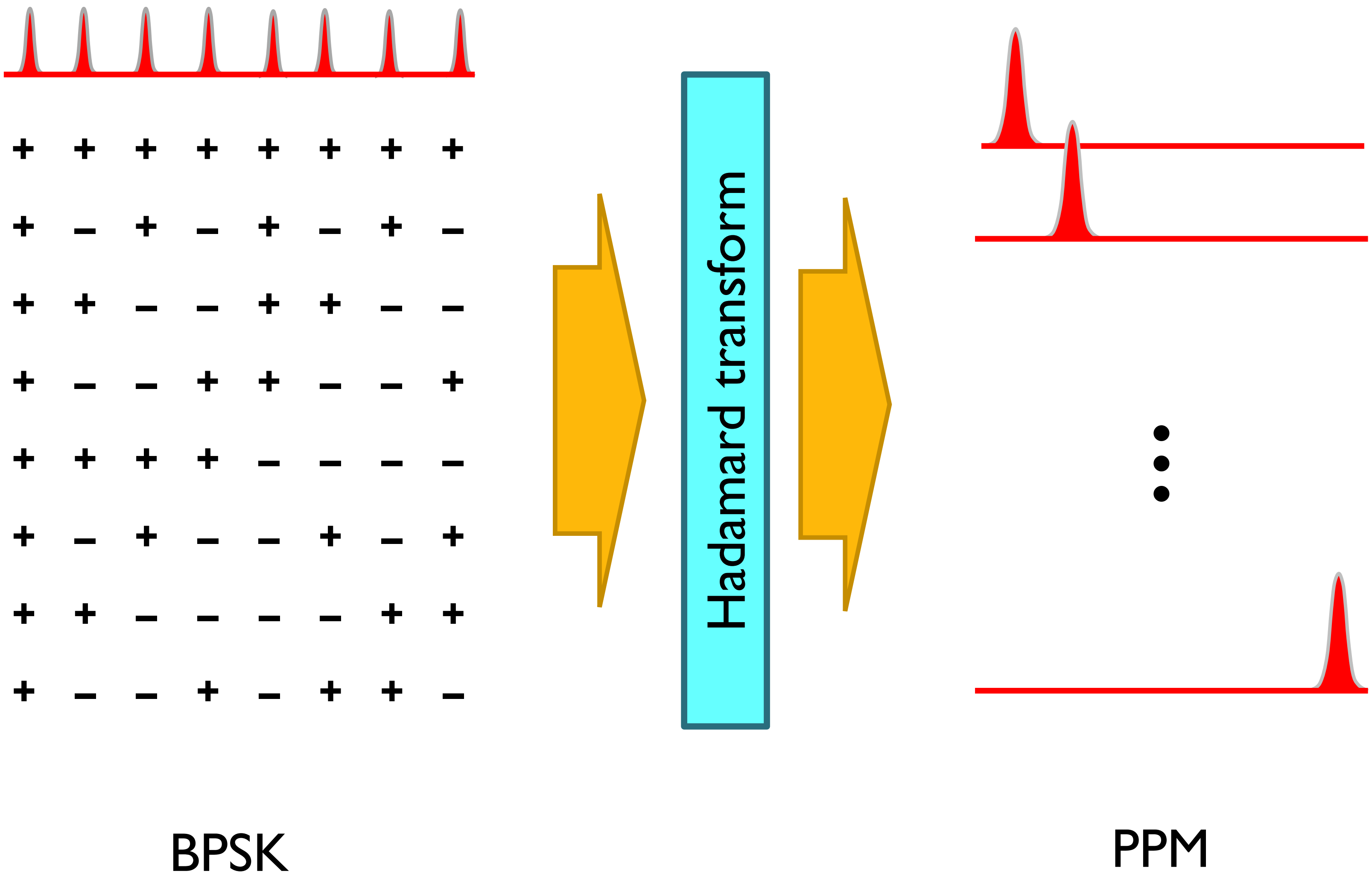


Photon information efficiency



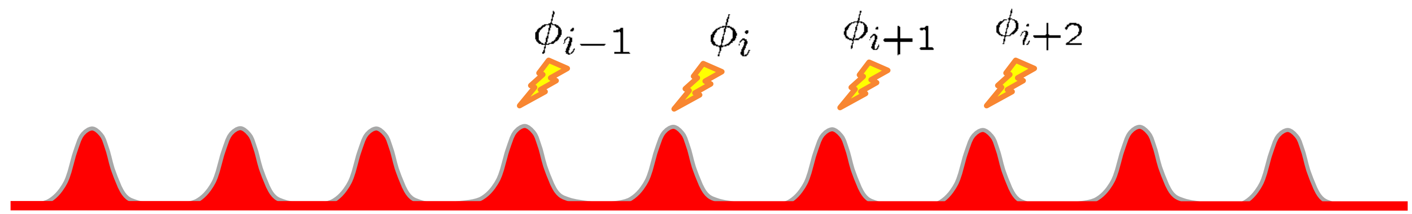
Hadamard words

S. Guha, Phys. Rev. Lett. **106**, 240502 (2011)



+	+	+	+	+	+	+	+
+	−	+	−	+	−	+	−
+	+	−	−	+	+	−	−
+	−	−	+	+	−	−	+
+	+	+	+	−	−	−	−
+	−	+	−	−	+	−	+
+	+	−	−	−	−	+	+
+	−	−	+	−	+	+	−

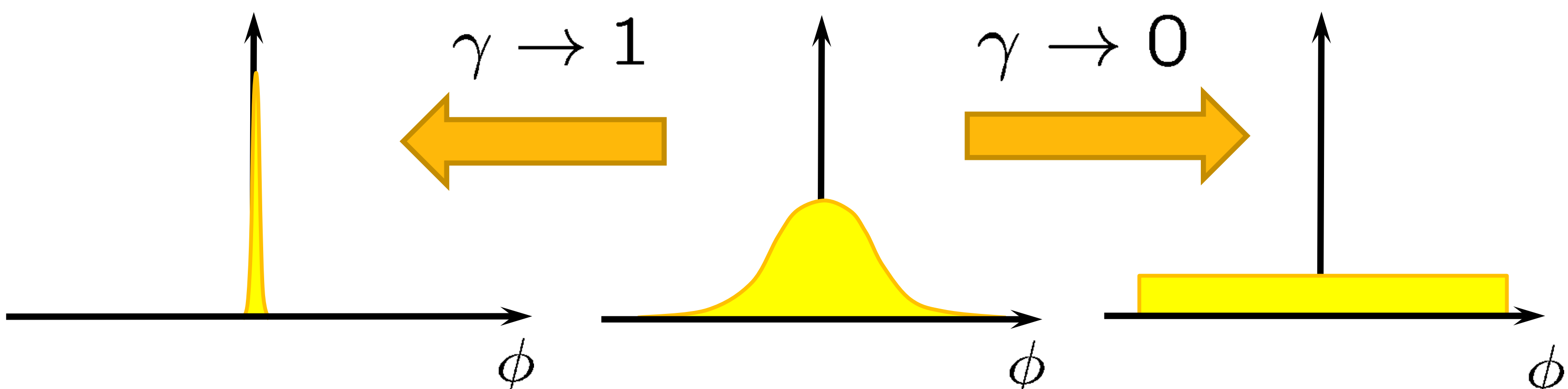
Dephasing



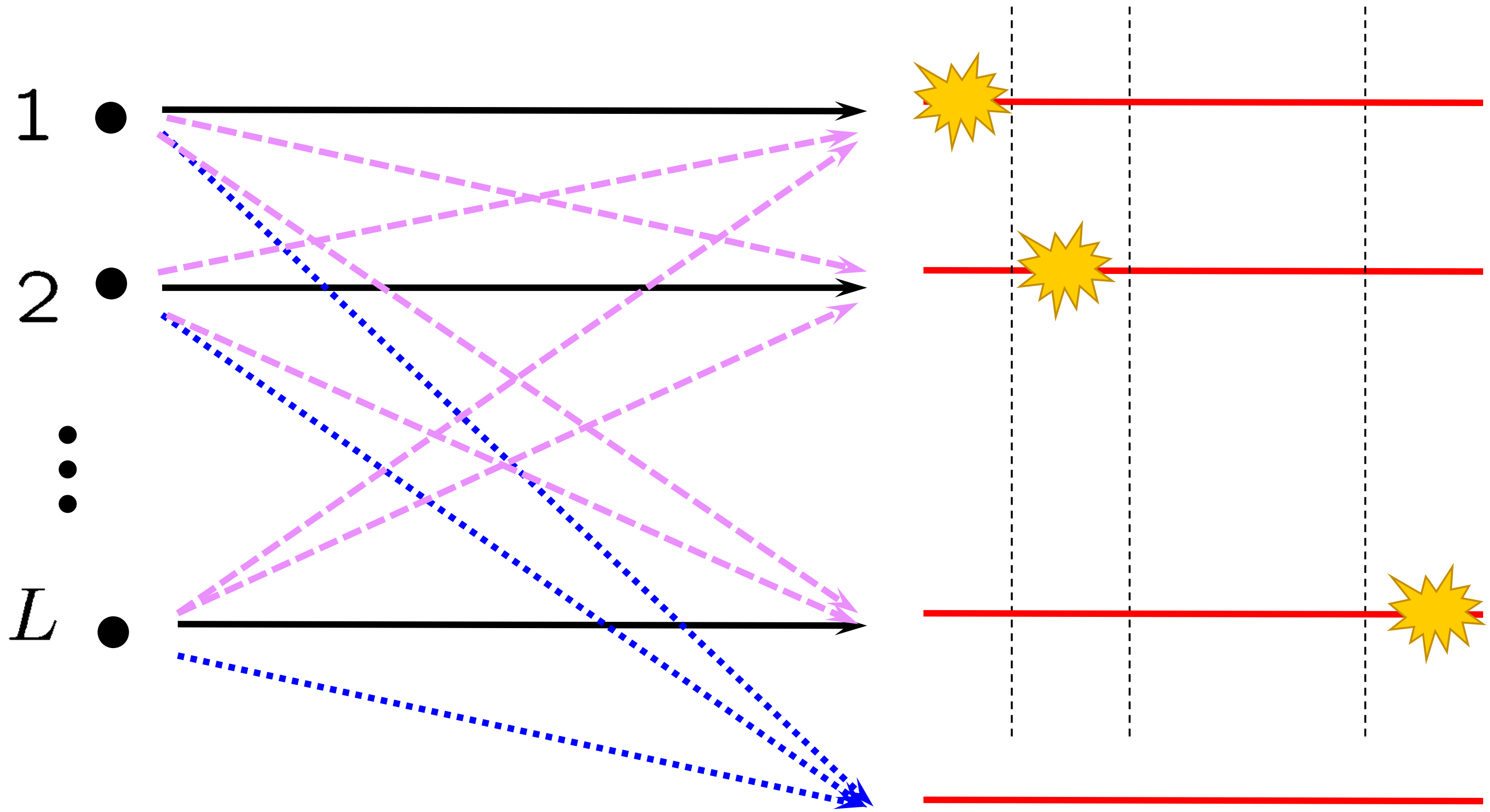
Phase distribution

$$p(\phi) = \frac{1}{\sqrt{2\pi\sigma^2}} \mathrm{e}^{-\phi^2/2\sigma^2}$$

Dephasing parameter $\gamma = \mathrm{e}^{-\sigma^2}$



PPM with uniform noise

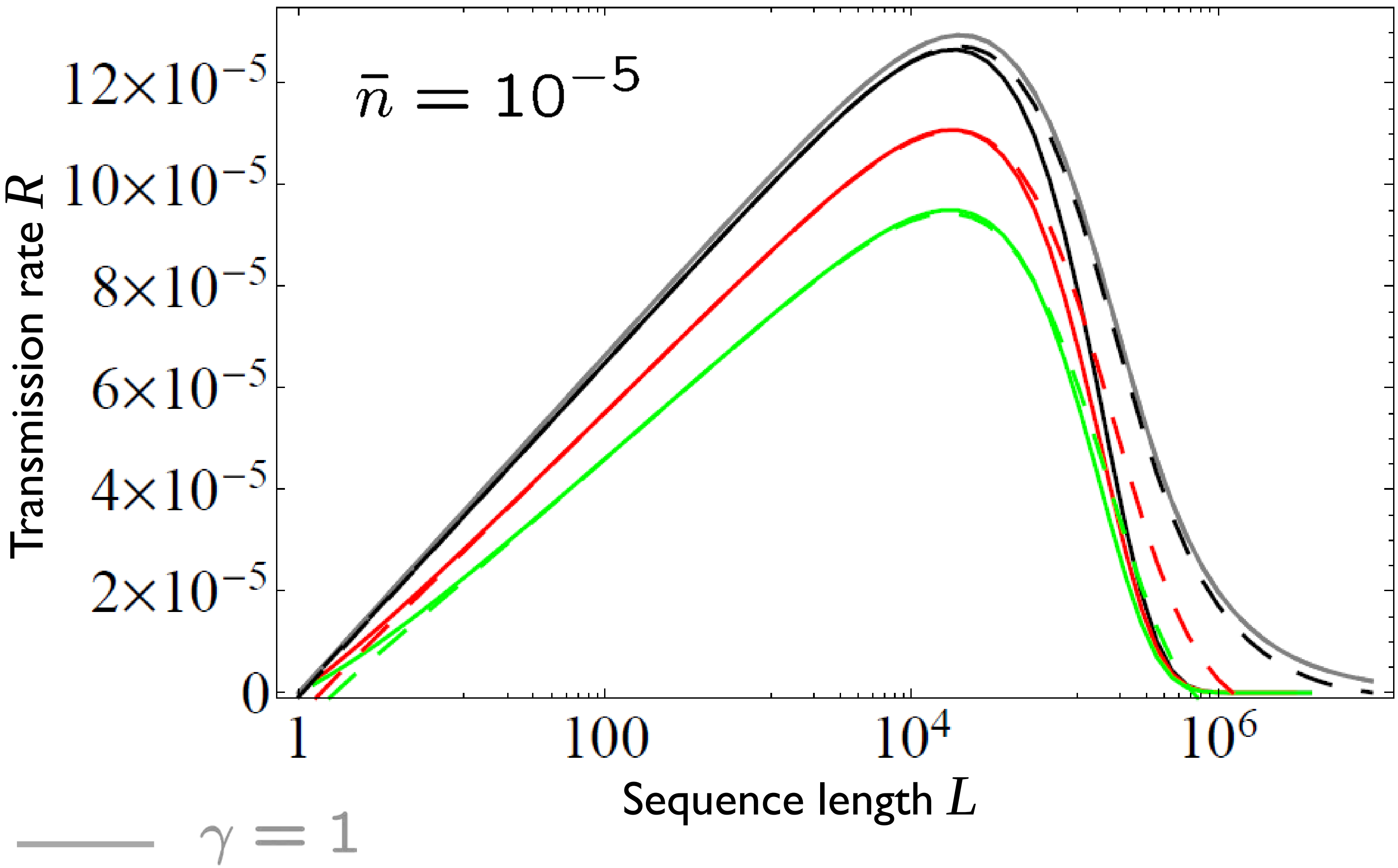


$$p \geq \tilde{p} = \mathrm{e}^{-(L-1)\bar{n}(1-\gamma)} - \mathrm{e}^{-L\bar{n}}$$

$$q \leq \tilde{q} = \bar{n}(1-\gamma)$$

UNIVERSITY
OF WARSAW

Optimal sequence length



Numerical with p, q expanded up to $\bar{n}^2$:

$\gamma = 0.99$ $\gamma = 0.9$ $\gamma = 0.8$

Transmission rate

$$\mathsf{R} \approx \frac{p}{L}\log_2 L - \bar{n}\mathsf{H}(\gamma)$$

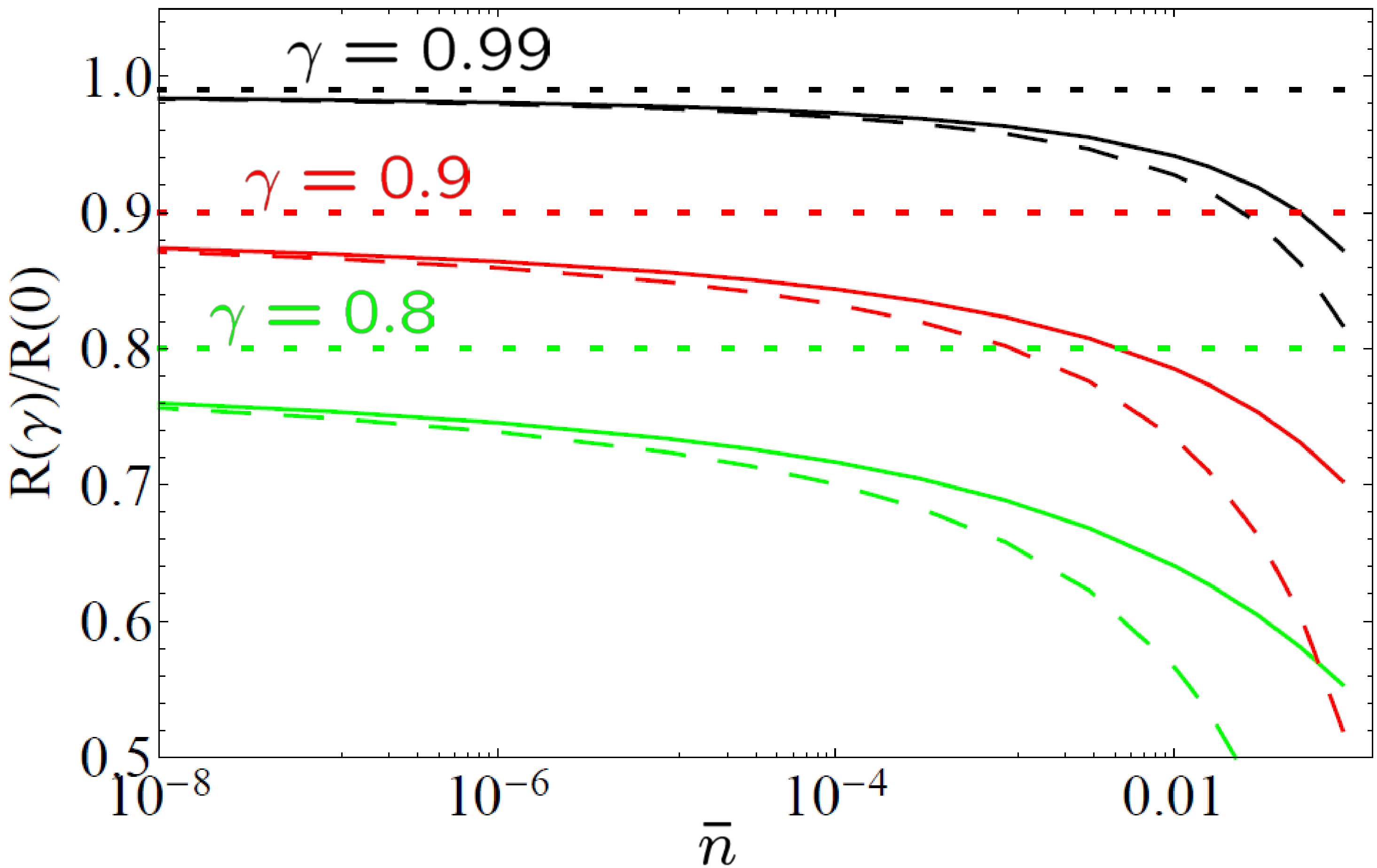
Expanding p up to $(L\bar{n})^2$:

$$p \approx \underbrace{\gamma L\bar{n}}_{\textit{Effective } \langle \hat{N} \rangle} - \frac{1}{2}\underbrace{(2\gamma^{-1} - 1)}_{\textit{Effective } g^{(2)} > 1}(\gamma L\bar{n})^2$$

Closed analytical expression:

$$\tilde{\mathsf{R}} = \gamma\bar{n}\Pi\Big((2-\gamma)\bar{n}\Big) - \bar{n}\mathsf{H}(\gamma).$$

Approximation

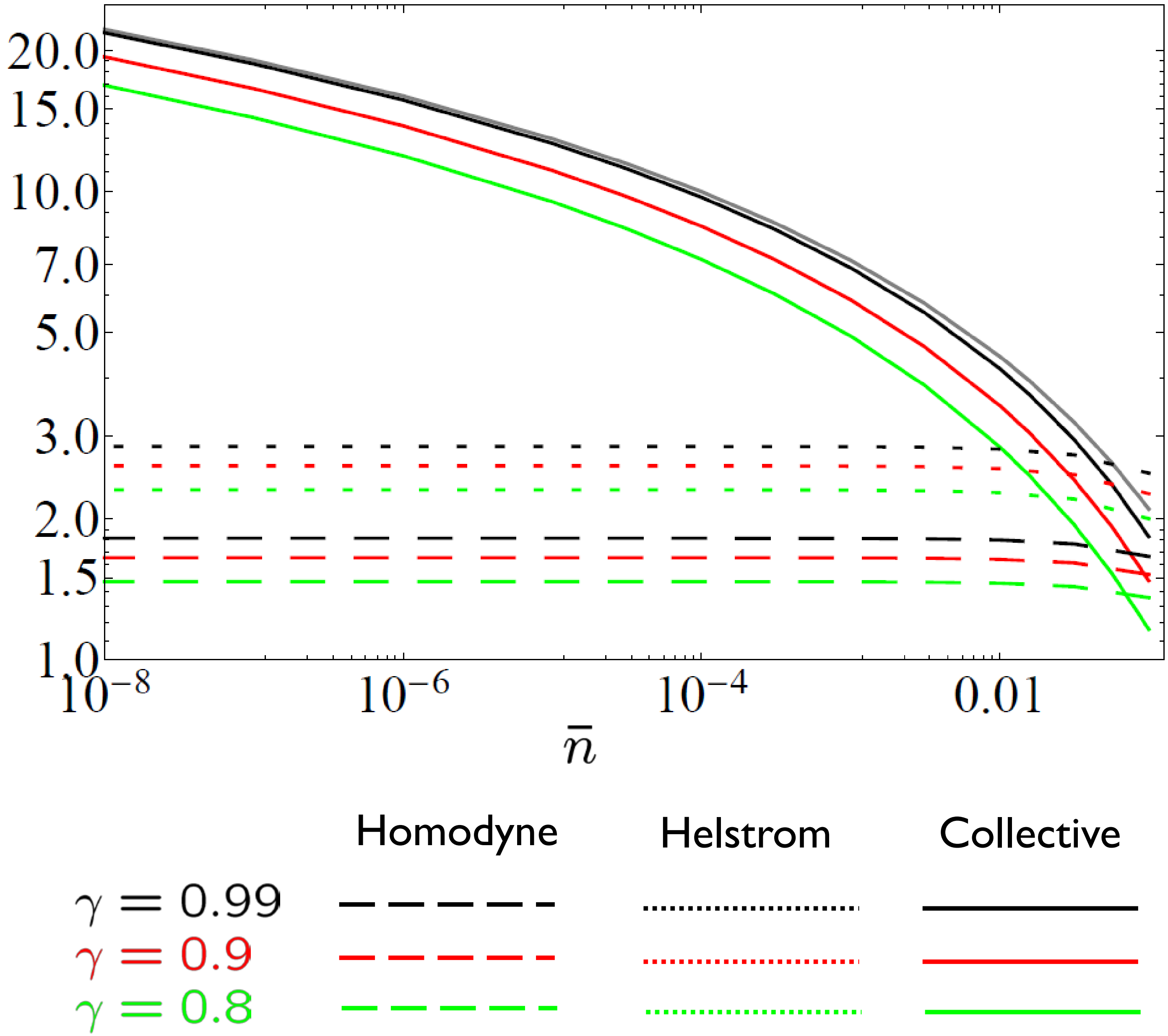


Numerical with p, q expanded up to $\bar{n}^2$

Closed analytical expression for $\tilde{R}$

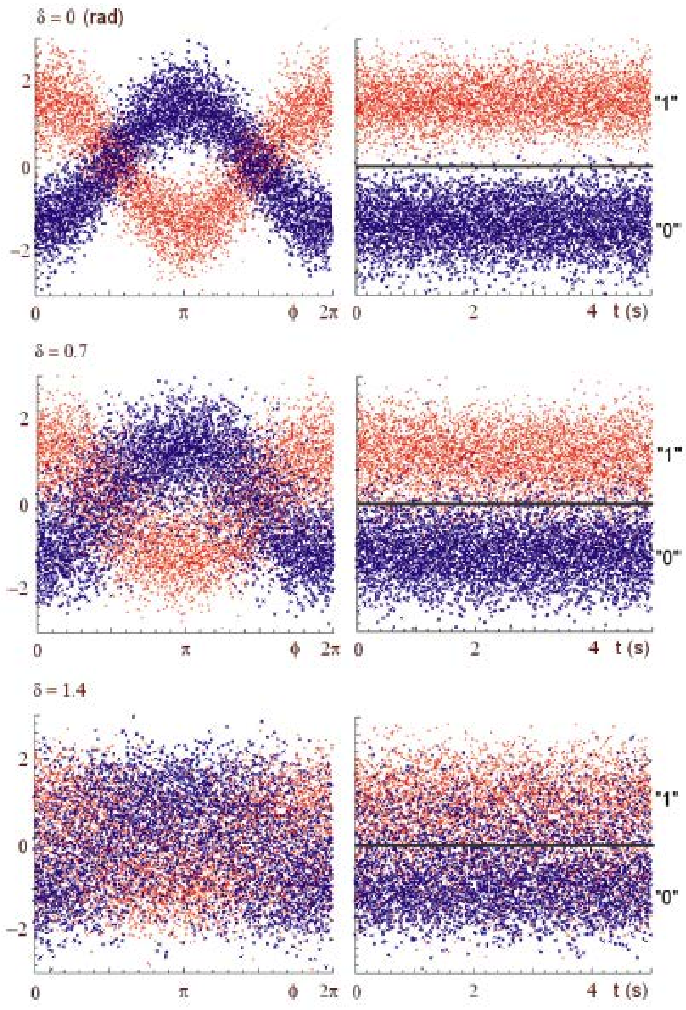
Asymptotics: $R \approx \gamma \bar{n} \log_2 \frac{1}{\bar{n}}$

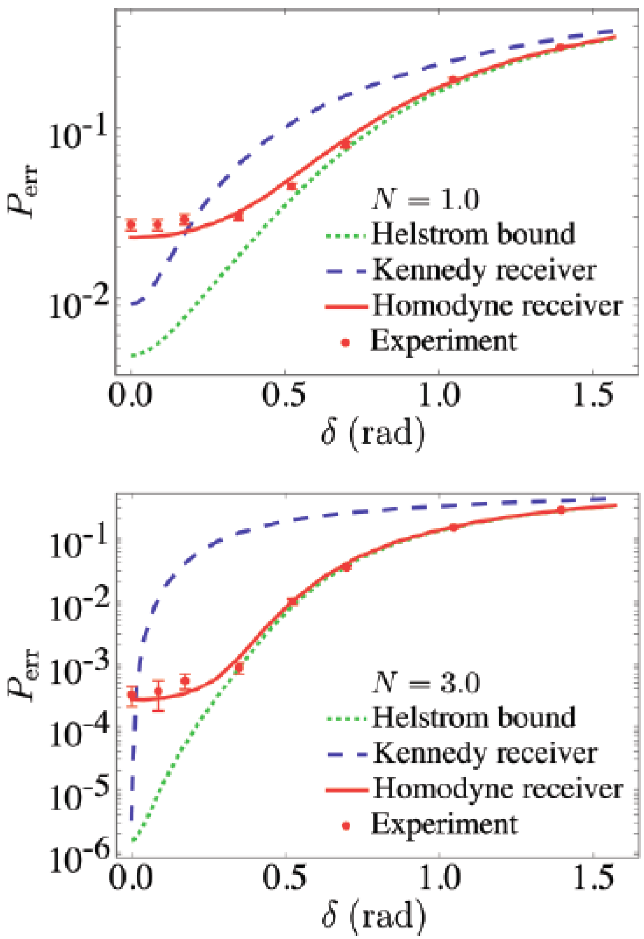
Photon information efficiency



Homodyne vs Kennedy receiver

S. Olivares, S. Cialdi, F. Castelli, and M. G. A. Paris,
Phys. Rev. A **87**, 050303(R) (2013)





UNIVERSITY OF WARSAW

Conclusions

- Approximate analytical expression for maximum PPM / OOK transmission rates in the low-power regime
- Approaching channel capacity in the leading order of the average power
- Quantified effects of excess noise in pulse photon statistics
- Non-classical enhancement in the low-loss regime
- Phase noise in collective BPSK
- Open questions:
  - Robustness to dark counts, amplification noise, etc.
  - Resources needed to saturate channel capacity (coherent / collective detection)

UNIVERSITY OF WARSAW