Correlated&Nonlinear Losses, Dissipative Chains and Anomalous Heat

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Motivation: making losses do funny things

Purpose: to introduce and discuss a novel "dissipative gadget": a quantum chain with dissipative coupling

Outline:

1. Dissipative coupling and how to produce it
2. What can dissipative coupling do?
3. Dissipative chains.
4. Is it heat?

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What common linear loss is commonly doing?

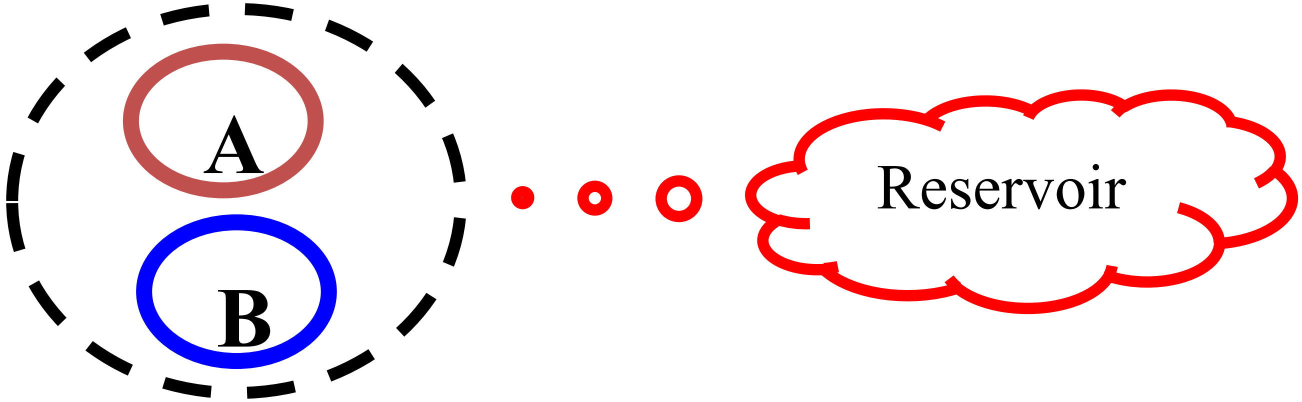
For usual unstructured “Markovian” reservoir

$$\frac{d}{dt}\rho_a = L\{\sqrt{\gamma}a\}\rho_a = \gamma\left(2a\rho_a a^+ - a^+ a\rho_a - \rho_a a^+ a\right)$$

Any initial state just turns into the vacuum eventually ….

But what will happen if we take two modes coupled linearly to the same reservoir?

Correlated (collective) loss



An example of interaction Hamiltonian: $V_{ab} = \left(a^{+} + b^{+}\right)R + h.c.$

Markovian master equation $\frac{d}{dt}\rho_{ab} = L\left\{\sqrt{\gamma}\left(a+b\right)\right\}\rho_{ab}$

Decoherent-free subspace appears!

These states are not affected by dissipation: $\left(a+b\right)\left|\Psi\right\rangle = 0$

G.M. Palma, K.-A. Suominen and A.K. Ekert. Quantum Computers and Dissipation. Proc. Roy. Soc. London Ser. A, 452:567, 1996.

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Consequences of having correlated loss:

The initial state is projected on the decoherence-free subspace

One can preserve entanglement. One can create entanglement.

Example for the initial single photon ("dissipative beam-splitter"):

$$\frac{d}{dt}\rho_{ab} = L\left\{\sqrt{\gamma}(a+b)\right\}\rho_{ab}$$

$$(a+b)|\Psi\rangle = 0 \quad \Rightarrow \quad |\Psi\rangle = \frac{1}{\sqrt{2}}\left(|1\rangle_a|0\rangle_b - |0\rangle_a|1\rangle_b\right)$$

Initial state:

$$\rho_{ab}(t=0) = |\varphi\rangle\langle\varphi|, \qquad |\varphi\rangle = |1\rangle_a|0\rangle_b$$

Final state:

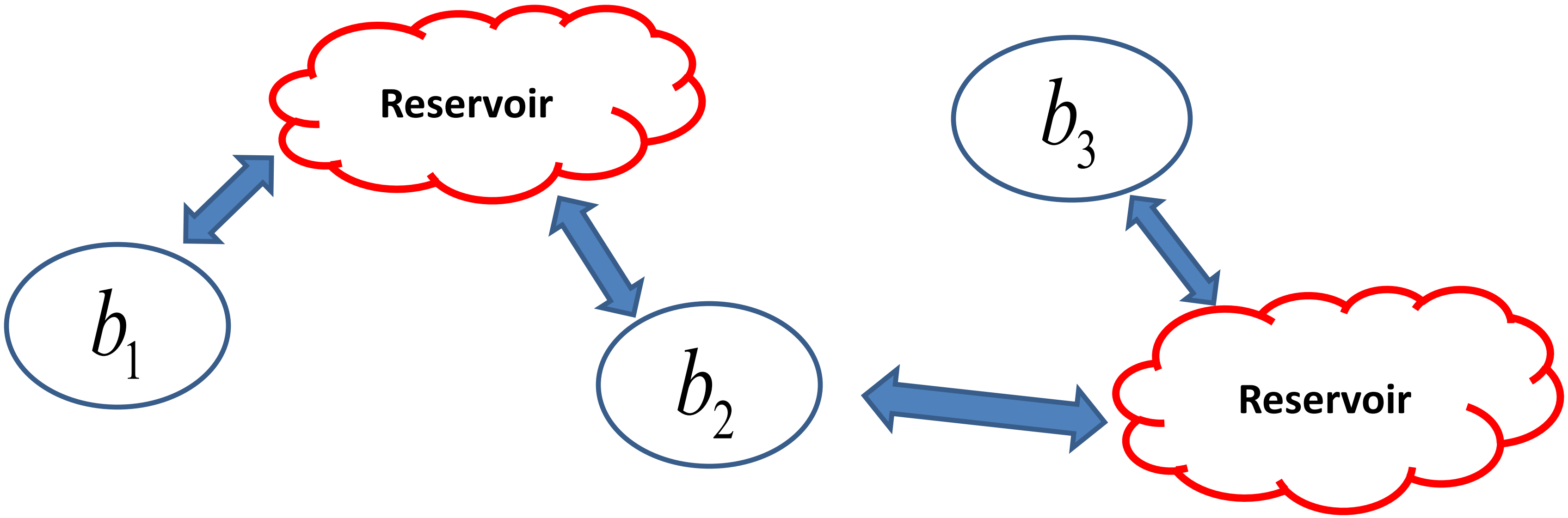
$$\rho_{ab}(t\to\infty) = \frac{1}{2}|\Psi\rangle\langle\Psi| + \frac{1}{2}|0,0\rangle\langle 0,0|.$$

Daniel A. Lidar, K. Birgitta Whaley, Decoherence-Free Subspaces and Subsystems, in "Irreversible Quantum Dynamics"

Lecture Notes in Physics Volume 622, 2003, pp 83-120

Dissipative coupling

or correlated loss into several reservoirs



$$\frac{d}{dt}\rho = \sum_{j\neq k} L\{g_{kj}b_j + g_{jk}b_k\}\rho$$

Objects are coupled through dissipative reservoirs

Consequences of dissipative coupling I

Possibility to produce nonlinear loss:

a. Stationary nonclassical states;

b. Possibility to generate them deterministically from a wide range of initial states;

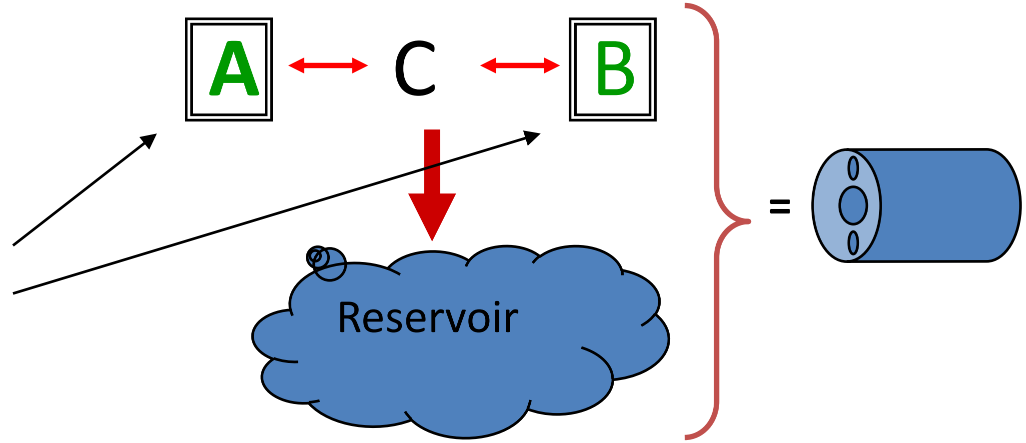
c. The generated stationary states can be made extremely robust with respect to linear loss by adding driving;

d. Nonlinear loss can produce nonclassical states on times when influence of linear loss is negligible.

Example: dissipative coupling in a three-mode system

Three linearly interacting modes with self-Kerr nonlinearity

Nonlinearity



$$\frac{d}{dt}\rho_{abc} = -i\left[\left(g_a a^+ + g_b b^+\right)c + U\left(\left(a^+\right)^2 a^2 + \left(b^+\right)^2 b^2\right) + h.c., \rho_{abc}\right] + \gamma L\{c\}\rho_{abc}.$$

For $\gamma >> \left|g_{a,b}\right|$ one eliminates C adiabatically.

↓

$$\frac{d}{dt}\rho_{ab} \approx -i\left[\widetilde{H}, \rho_{ab}\right] + \frac{G^2}{\gamma} L\{g_a a + g_b b\}\rho_{ab}, \qquad G = g_a^2 + g_b^2.$$

The way to make nonlinear loss, stage II

Nonlinearity + correlated loss can give rise to nonlinear loss!

$$\frac{d}{dt}\rho_{ab} \approx -i\left[\tilde{H}, \rho_{ab}\right] + \Gamma L\{g_a a + g_b b\}\rho_{ab}.$$

For $\Gamma \equiv \frac{G^2}{\gamma} >> U$ one eliminates $d_+ = \frac{1}{G}\left(g_a a + g_b b\right)$

$$\frac{d}{dt}\rho_{d-} \approx -i\left[x d_-^+ d_- + y\left(d_-^+ d_-\right)^2, \rho_{d-}\right] + \Gamma_1 L\left\{d_-^2\right\}\rho_{d-} + \Gamma_2 L\left\{d_-^+ d_-^2\right\}\rho_{d-}.$$

$$d_- = \frac{1}{G}\left(g_a b - g_b a\right)$$

Nonlinear loss!!

*V. S. Shchesnovich, D. S. Mogilevtsev, PRA **82**, 043621 (2010);*
*D. Mogilevtsev, V. S. Shchesnovich, Opt. Lett. **35**, 3375 (2010).*

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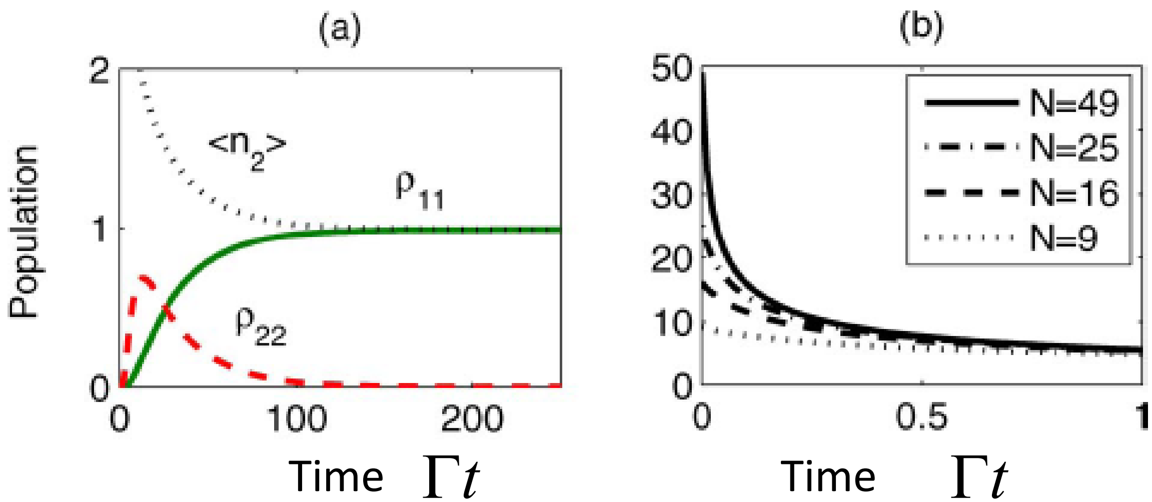
Example: producing single photons from initial coherent states

$$\frac{d}{dt}\rho_a \approx L\left\{\sqrt{\Gamma}a\left(a^{+}a-1\right)\right\}\rho_a$$



Decay is non-exponential.
Decay time does not depend on the initial state:
the larger it is, the faster it goes down …

D. Mogilevtsev, V. S. Shchesnovich, Opt. Lett. ***35****, 3375 (2010).*

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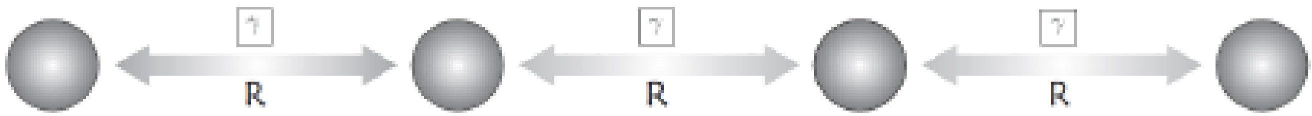
Consequences of dissipative coupling II

Possibility to build dissipatively coupled chains …



$$\frac{d}{dt}\rho = \sum_{j=1}^{N} \gamma_j \left\{ 2S_j^- \rho S_j^+ - S_j^+ S_j^- \rho - \rho S_j^+ S_j^- \right\},$$

$$S_j^- = g_j \sigma_j^- + f_j \sigma_{j+1}^-$$

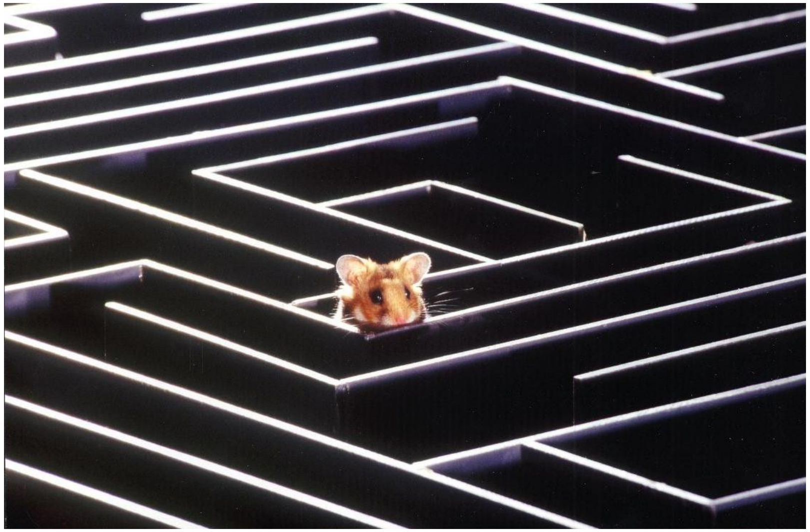
They can give us a number of surprises …

A simple example: homogeneous chain

$$\frac{d}{dt}\rho = \gamma \sum_{j=1}^{N} \left\{ 2S_j^- \rho S_j^+ - S_j^+ S_j^- \rho - \rho S_j^+ S_j^- \right\},$$

$$S_j^- = \sigma_j^- - \sigma_{j+1}^-$$

Let us put just one excitation inside it and see what happens …



$$\rho(0) = |1_k\rangle\langle 1_k|,$$

$$|1_k\rangle = |+\rangle_k \prod_{\forall j \neq k} |-\rangle_j$$

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Suddenly, we have a classical random walk!

$$\rho_{jk} = \langle 1_j | \rho | 1_k \rangle$$

$$\frac{d}{dt}\rho_{jk} = -4\gamma\rho_{jk} + \gamma\left(\rho_{j+1k} + \rho_{j-1k} + \rho_{jk-1} + \rho_{jk-1}\right).$$

Away from borders, of course …

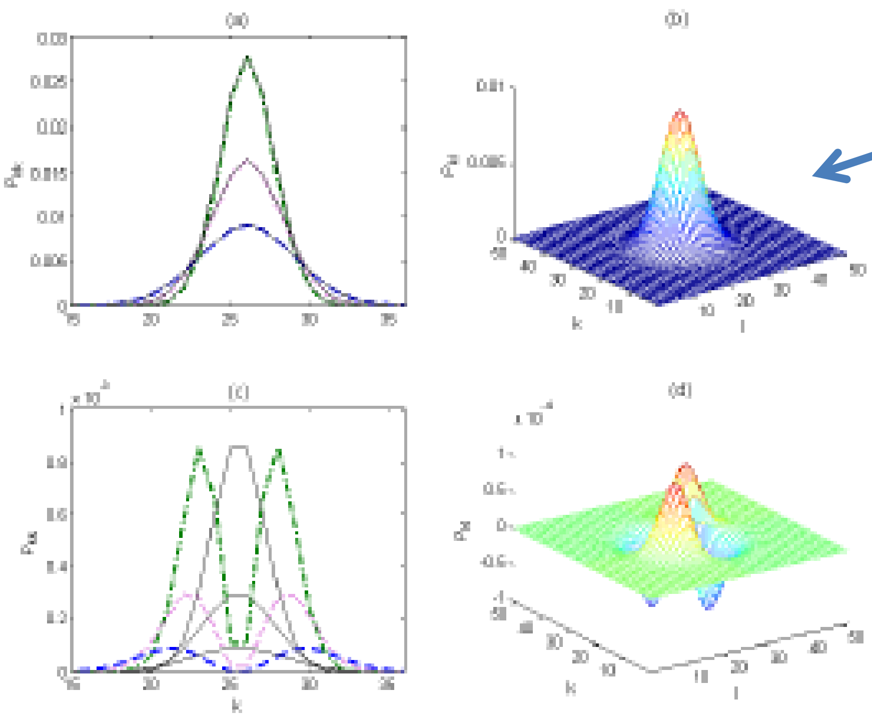
If the chain is long enough, we can try the continuous limit …

$$aj \to x, \qquad ak \to y$$

$$\frac{\partial}{\partial t}\rho(x,y,t) = \gamma a^2 \frac{\partial^2}{\partial x^2}\rho(x,y,t) + \gamma a^2 \frac{\partial^2}{\partial y^2}\rho(x,y,t).$$

Well, it is a heat equation …
What the hell is going on?

One can have polynomial decay of the degree dependent on the initial state



$$\rho(0) = |1_k\rangle\langle 1_k|,$$

$$\rho_{kk}(t) \propto 1/\gamma t.$$

$$\rho(0) = |\varphi\rangle\langle\varphi|,$$

$$|\varphi\rangle = \frac{1}{\sqrt{2}}\left(|1_k\rangle - |1_{k+1}\rangle\right),$$

$$\rho_{kk}(t) \propto 1/(\gamma t)^3.$$

But that is by far not the only exotic thing about it!

Putting in N initial excitations, we can model classical random walk in 2N dimensions.

We can model heat propagation from regions with positive, negative and even complex temperatures.

But it is not the strangest exotics that can be uncovered!

James Clerk Maxwell (1831–1879)

ALL HEAT IS OF THE SAME KIND

Well, eh … maybe not?

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First of all, it is not the energy that flows according to the heat law. Then, the stationary state of the system can be entangled.

$$\rho(t\to\infty)=\frac{W}{N}|\Psi\rangle\langle\Psi|+\left(1-\frac{W}{N}\right)|vac\rangle\langle vac|, \qquad |\Psi\rangle=\frac{1}{\sqrt{N}}\sum_{k=1}^{N+1}|1_k\rangle$$

The sum of all coherences is conserved!

$$W(t)=W(0)=\sum_{k,j=1}^{N+1}\rho_{jk}(0).$$

But the energy is not …

$$E(t)=\sum_{j=1}^{N+1}\rho_{jj}(t)\propto E(0)/\sqrt{\gamma t}.$$

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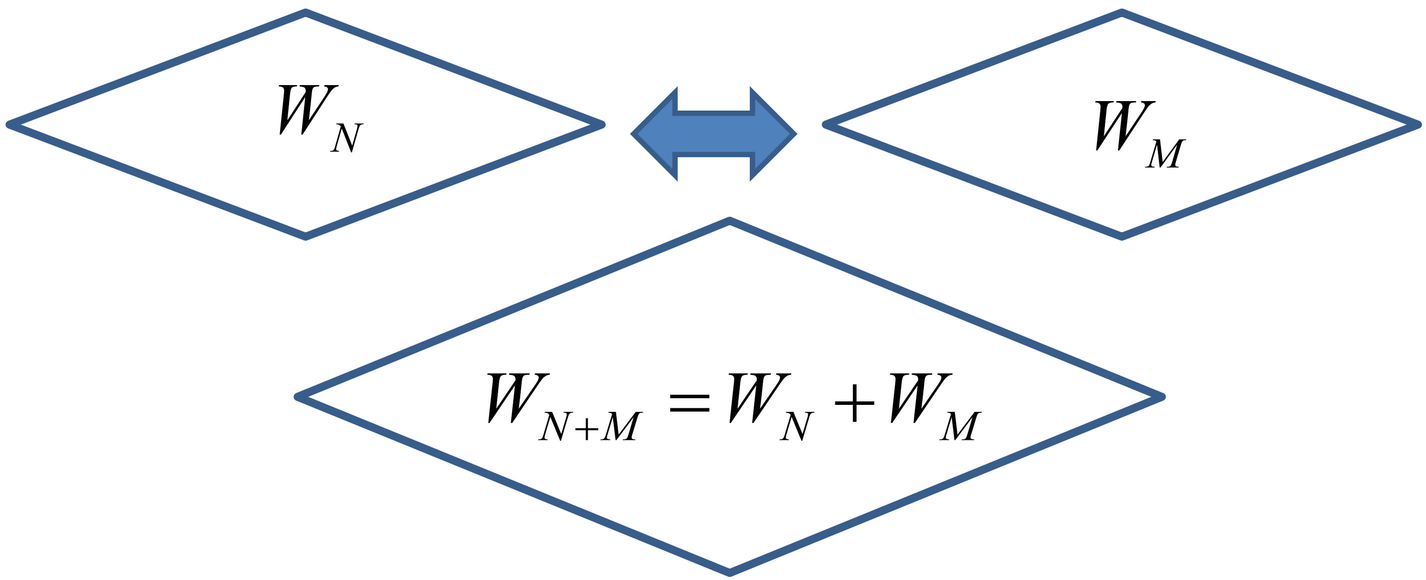
Well, one can build a sort of “thermodynamics” with it …

A coherence per element $\dfrac{W_N}{N^2}$ This is our “temperature”.



If we disconnect subchains, the “temperature” of them will remain the same:

$$W_{N+M} / (N+M)^2$$

Well, dissipatively coupled systems can really give us a lot of surprises!

THANKS!

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