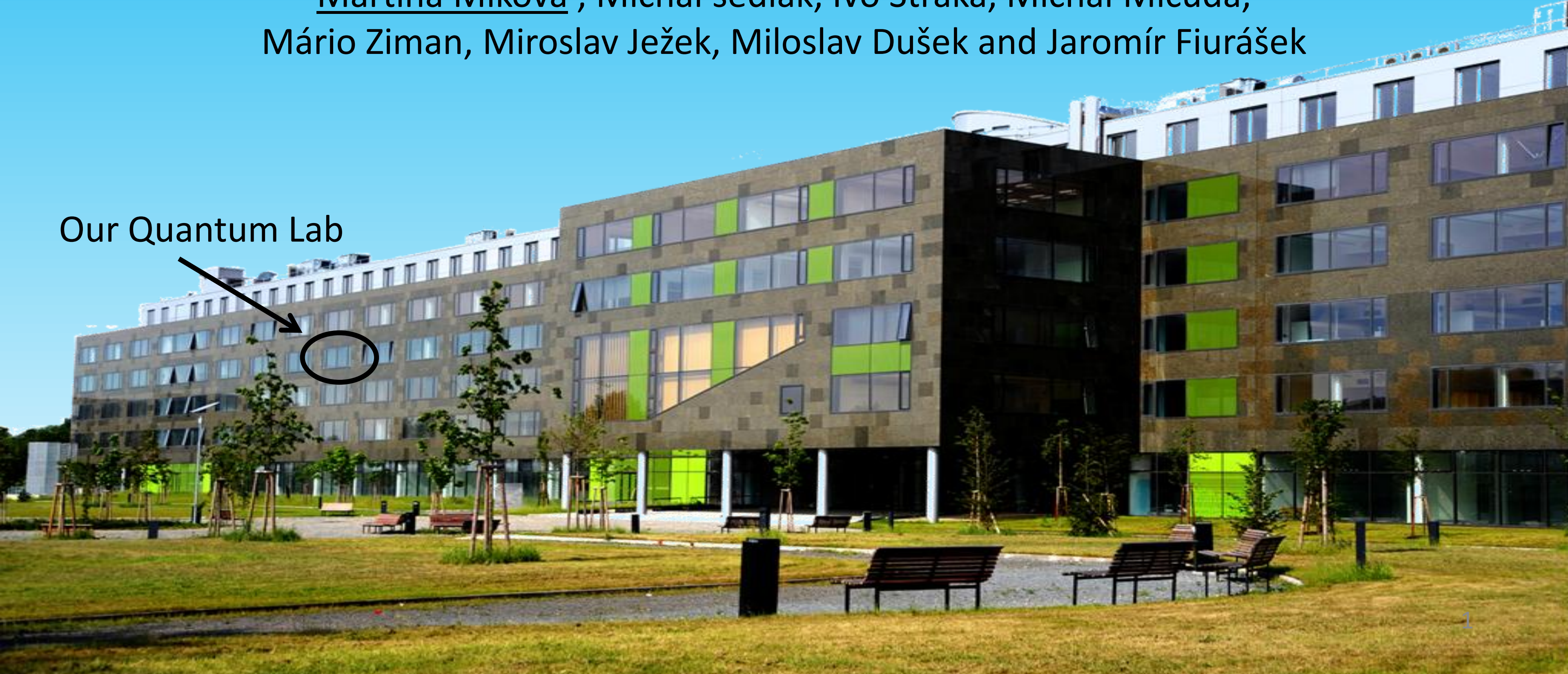
Department of Optics, Faculty of Science, Palacký University in Olomouc

Optimal entanglement-assisted discrimination of quantum measurements

<u>Martina Miková</u> , Michal sedlák, Ivo Straka, Michal Mičuda,
Mário Ziman, Miroslav Ježek, Miloslav Dušek and Jaromír Fiurášek



Our Group

THEORY

EXPERIMENT

Jaromr Fiurášek
assoc. professor

Miloslav Dušek
professor

Michal Mičuda
postdoc

Ivo Straka
PhD student

Michal Sedlák
postdoc

Mário Ziman
assoc. professor

Miroslav Ježek
postdoc

Martina Miková
PhD student

Our Lab

Outline

- **Motivation**
- **Methods** - Feed-forward
- **Experiment**
    - Theory
    - Experiment
    - Results
- **Conclusion**

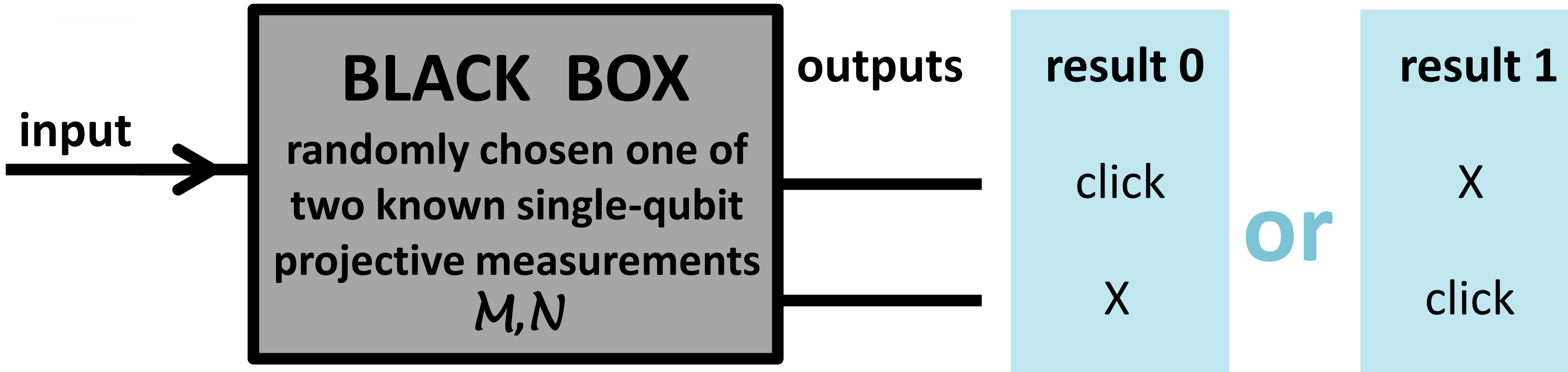
Motivation

- One of the characteristic traits of quantum mechanics is the impossibility to perfectly discriminate two nonorthogonatl quantum states.

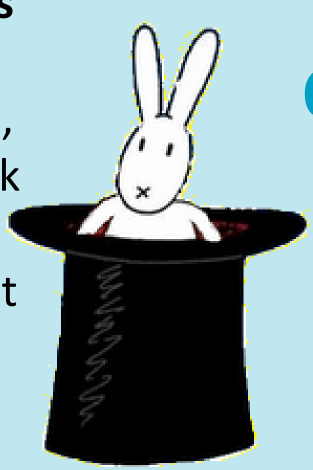
- It triggers the question what is the optimal approximate or probabilistic discrimination strategy.

- Recently the discrimination strategies has been extended to discrimination of quantum operations and **measurements**.

Motivation



We can **guess** from the results **0** or **1**, whether Black box applied measurement $\mathcal{M}$ or $\mathcal{N}$.



or

We can find something better than simple guess: **optimal single-qubit strategy**. Where we can also say **I do not know** whether $\mathcal{M}$ or $\mathcal{N}$.

or

We can also find some sophisticated strategy: **optima entanglement-assisted strategy**. In the cases when we can sometime say **I do not know** it beats the previous ones.

Motivation

- BUT here is a QUESTION:

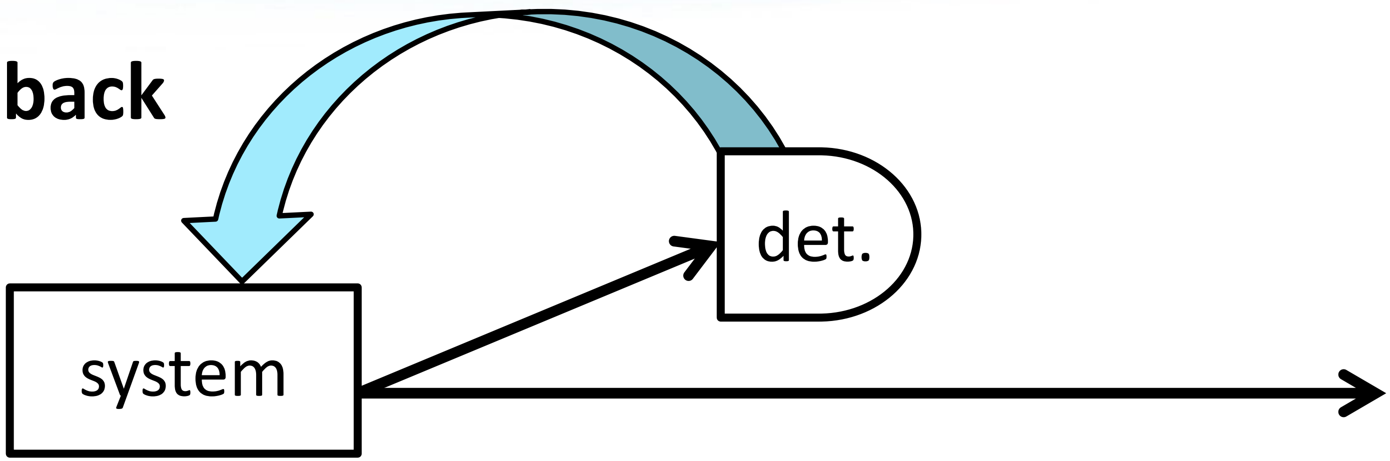
Does the optimal entanglement-assisted strategy beats the single-qubit one **NOT only at the paper but in a real experiment?**

- qubits $\longrightarrow$ photons
- manage their interaction
- linear optics + quantum measurement

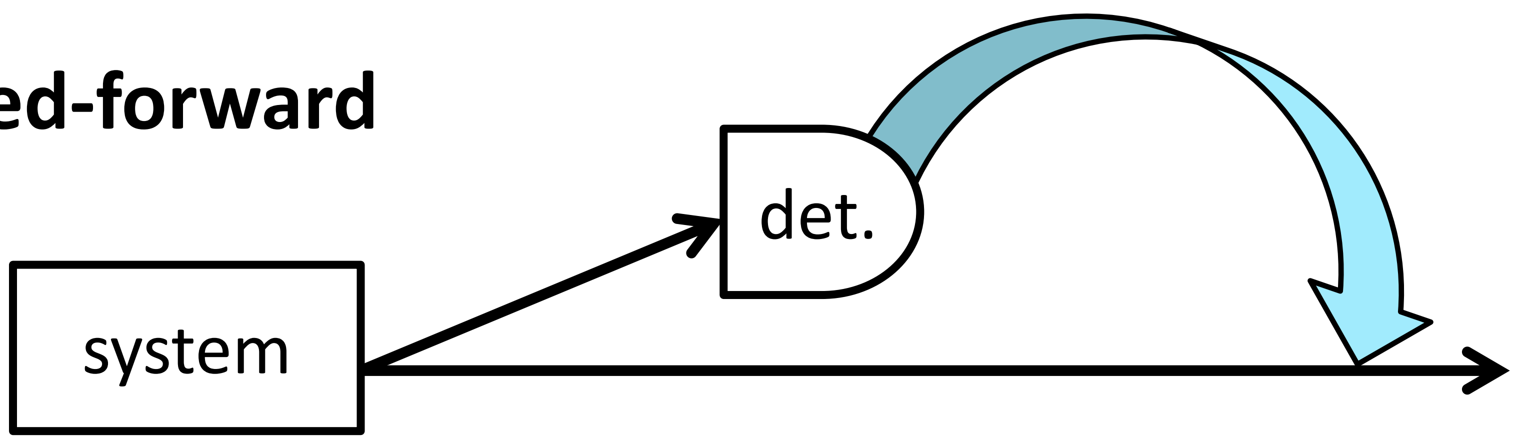
- experimental imperfections

Methods feed-back / feed-forward

- **feed-back**

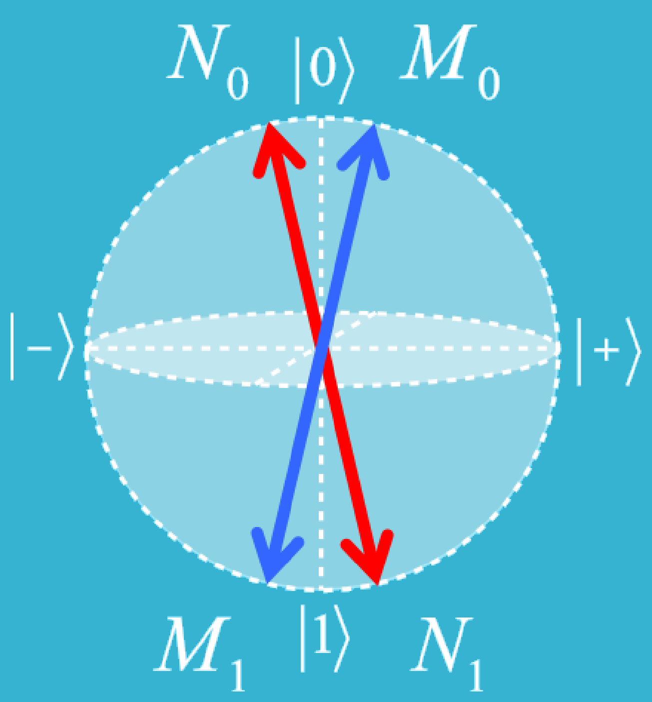


- **feed-forward**



Optima discrimination of two know single-qubit quantum measurements $\mathcal{M},\mathcal{N}$

in scenario where the measurement can be performed only once



$\mathcal{M}$: $M_0 = |\phi\rangle\langle\phi|, \quad M_1 = |\phi^{\perp}\rangle\langle\phi^{\perp}|$

$\mathcal{N}$: $N_0 = |\psi\rangle\langle\psi|, \quad N_1 = |\psi^{\perp}\rangle\langle\psi^{\perp}|$

The projectors of the measurement bases $\mathcal{M}$ and $\mathcal{N}$ can be parameterized by single angle θ. Where θ denotes half of the angle between the states $|\psi\rangle$ and $|\phi\rangle$, $0 \leq \theta \leq \pi/4$.

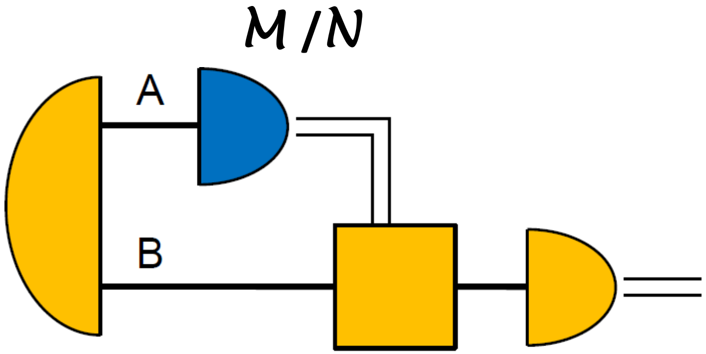
$$
\begin{aligned}
|\phi\rangle &= \cos\theta|0\rangle + \sin\theta|1\rangle \\
|\phi^{\perp}\rangle &= \sin\theta|0\rangle - \cos\theta|1\rangle \\
|\psi\rangle &= \cos\theta|0\rangle - \sin\theta|1\rangle \\
|\psi^{\perp}\rangle &= \sin\theta|0\rangle + \cos\theta|1\rangle
\end{aligned}
$$

Theory optimal discrimination with **entangled** probe state

It was shown in such case it is optimal to employ maximally entangled singlet Bell state $|\Psi^-\rangle = (|01\rangle - |10\rangle)/\sqrt{2}$.

- The measurement that should be discriminated is performe on **qubit A**.
- The measurement outcomes (0 and 1) specifies, which measurement is then performend on the **qubit B**.

outcome 0 at A heralds B in state $|\psi^\perp\rangle$ or $|\phi^\perp\rangle$
outcome 1 at A heralds B in state $|\psi\rangle$ or $|\phi\rangle$



- When outcome on **A** reads **0** ⟶ we apply a suitable unitary operation on qubit **B**.

- The suitable unitary operation rotates states $|\psi^\perp\rangle, |\phi^\perp\rangle$ in such way that we end up with the task to discriminate between two fixed non-orthogonal states $|\psi\rangle$ and $|\phi\rangle$.

Theory

optimal discrimination with **entangled** probe state

- Perfect error-free discrimination between nonorthogonal states $|\psi\rangle$ and $|\phi\rangle$ is possible if we allow for a certain probability of inconclusive outcomes. It was shown by Ivanovic, Dieks, and Peres (IDP, three-component POVM).
- $P_I = |\langle\psi|\phi\rangle|$ explicitly, we have $P_I = cos(2\theta)$, $P_S = 2\sin^2\theta$.

prob. of conclusive results + prob. of inconclusive results = 1

- success prob. + prob. of inconclusive results = 1 $$P_S + P_I = 1$$
- Due to the various experimental imperfections
  → **erroneous conclusive results P_E** $$P_S + P_I + P_E = 1$$
- Thus we consider a general discrimination scheme where we maximize P_S hence we minimize P_E for a fixed fraction of P_I. This intermediate strategy optimally interpolates between IDP and Helstrom approaches.

Theory optimal discrimination with **entangled** probe state

Intermediate strategy
maximize P_S
minimize P_E
for a fixed fraction of P_I.

$$P_S + P_I + P_E = 1$$

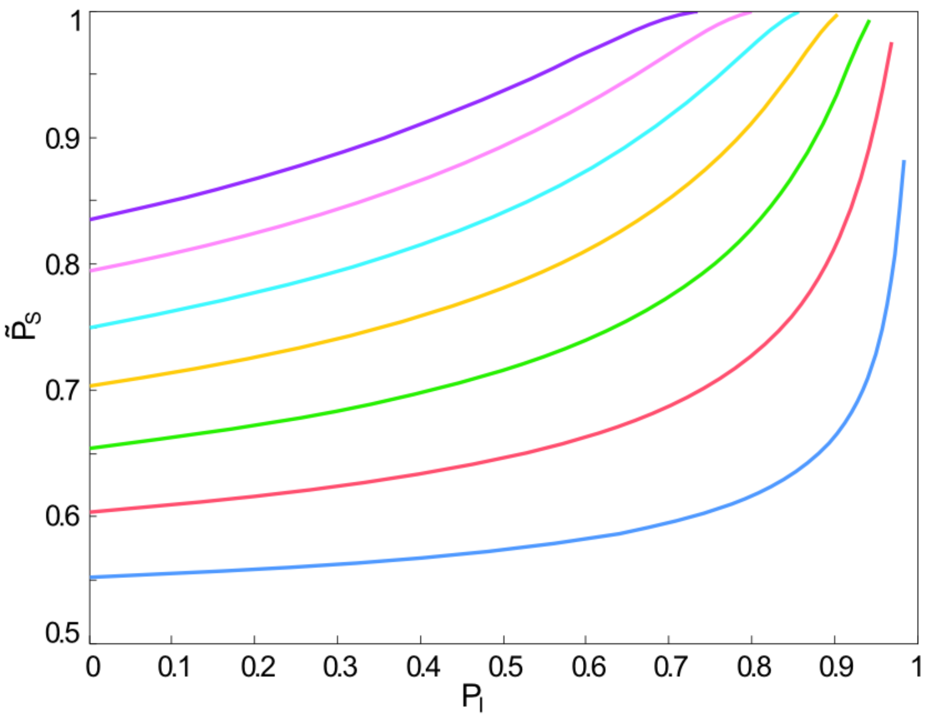
relative probability of successful discrimination

$$\tilde{P}_S = \frac{P_S}{1 - P_I}$$

where P_S is given as

$$P_S = \frac{1}{2}\left(1 - P_I + \sin(2\theta)\sqrt{1 - \frac{P_I}{\cos^2\theta}}\right)$$

Dependence of relative success probability on probability of inconclusive results.



It is plotted for 7 values of $\theta_j = j\pi/30,\ j = 1,2,3,4,5,6,7$. The value of *j* increases from the bottom to top.

Theoretical curves of maximum $\tilde{P}_S$ achievable by the optimal scheme using entangled state (solid line).

Theory optimal discrimination with **unentangled** single-qubit probes

as a BENCHMARK for the experiment

relative probability of successful discrimination $\tilde{P}_S = \frac{P_S}{1 - P_I}$

where:

$$P_S = \left(1 - \frac{P_I}{P_{I,T}}\right) P_{S,0} + \frac{P_I}{P_{I,T}} P_{S,T}$$

and:

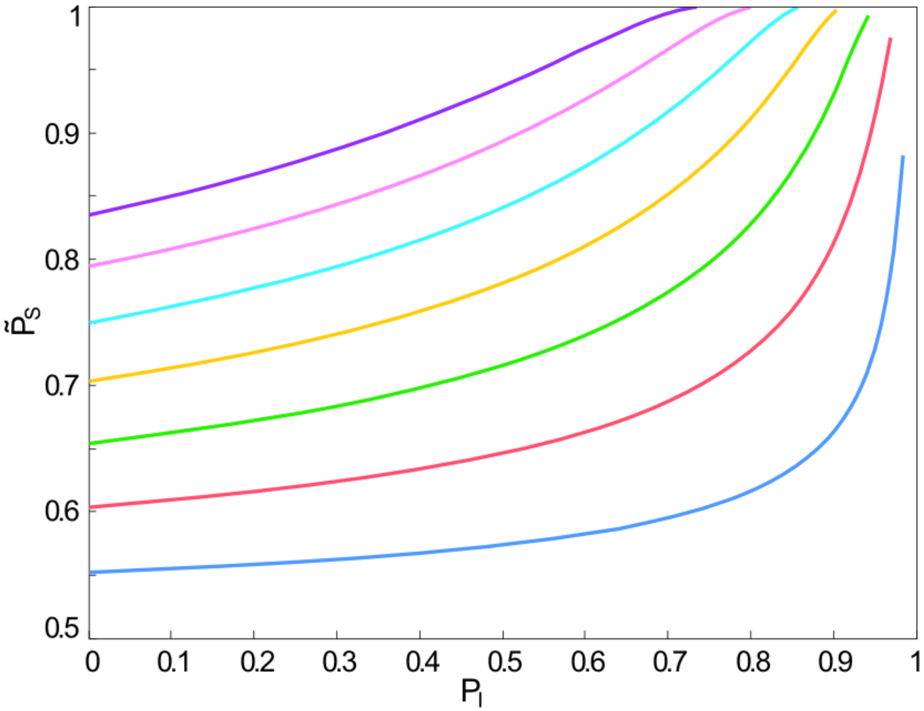
$$P_{S,T} = \frac{1}{2}(1 - P_{I,T}) + \frac{1}{4}\sin(2\theta)\sqrt{1 - \frac{(1 - 2P_{I,T})^2}{\cos^2(2\theta)}}$$

$$P_{S,0} = \frac{1 + \sin(2\theta)}{2}$$

$$P_{I,T} = \frac{1 + 3c^2 + 2c^2\sqrt{1 + 3c^2}}{2(1 + 4c^2)}$$

$$c = \cos(2\theta)$$

Dependence of relative success probability on probability of inconclusive results.



It is plotted for 7 values of $\theta_j = j\pi/30,\ j = 1, 2, 3, 4, 5, 6, 7$. The value of *j* increases from the bottom to top.

Theoretical curves of maximum $\tilde{P}_S$ achievable by the optimal scheme using entangled state (solid line).

Theory — optimal discrimination with **unentangled** single-qubit probes

as a BENCHMARK for the experiment

relative probability of successful discrimination $\tilde{P}_S = \frac{P_S}{1-P_I}$

where:

$$P_S = \left(1 - \frac{P_I}{P_{I,T}}\right) P_{S,0} + \frac{P_I}{P_{I,T}} P_{S,T}$$

and:

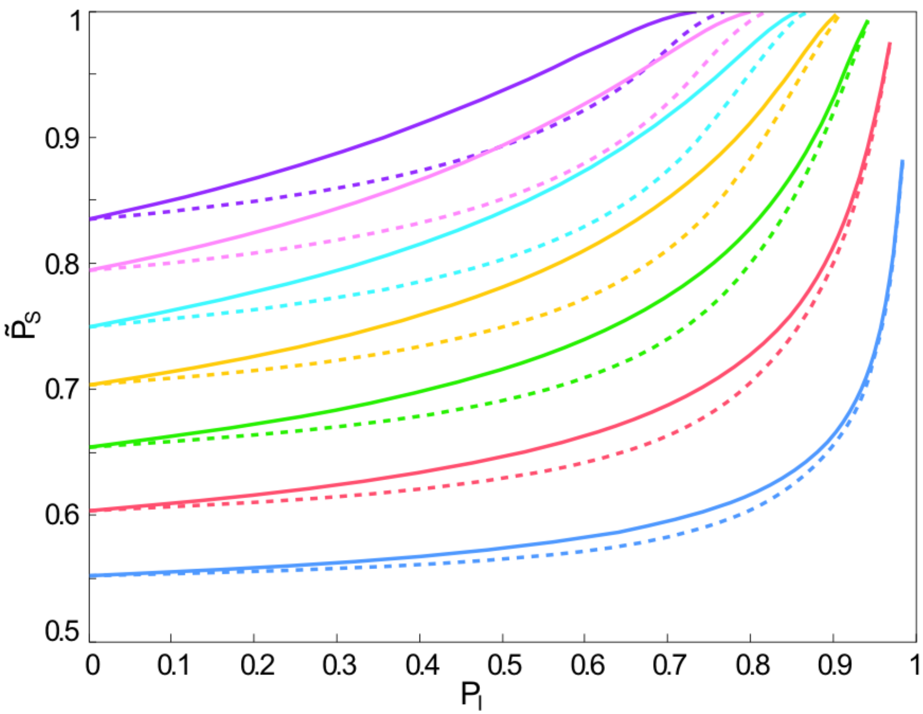
$$P_{S,T} = \frac{1}{2}(1-P_{I,T}) + \frac{1}{4}\sin(2\theta)\sqrt{1 - \frac{(1-2P_{I,T})^2}{\cos^2(2\theta)}}$$

$$P_{S,0} = \frac{1+\sin(2\theta)}{2}$$

$$P_{I,T} = \frac{1+3c^2+2c^2\sqrt{1+3c^2}}{2(1+4c^2)}$$

$$c = \cos(2\theta)$$

Dependence of relative success probability on probability of inconclusive results.



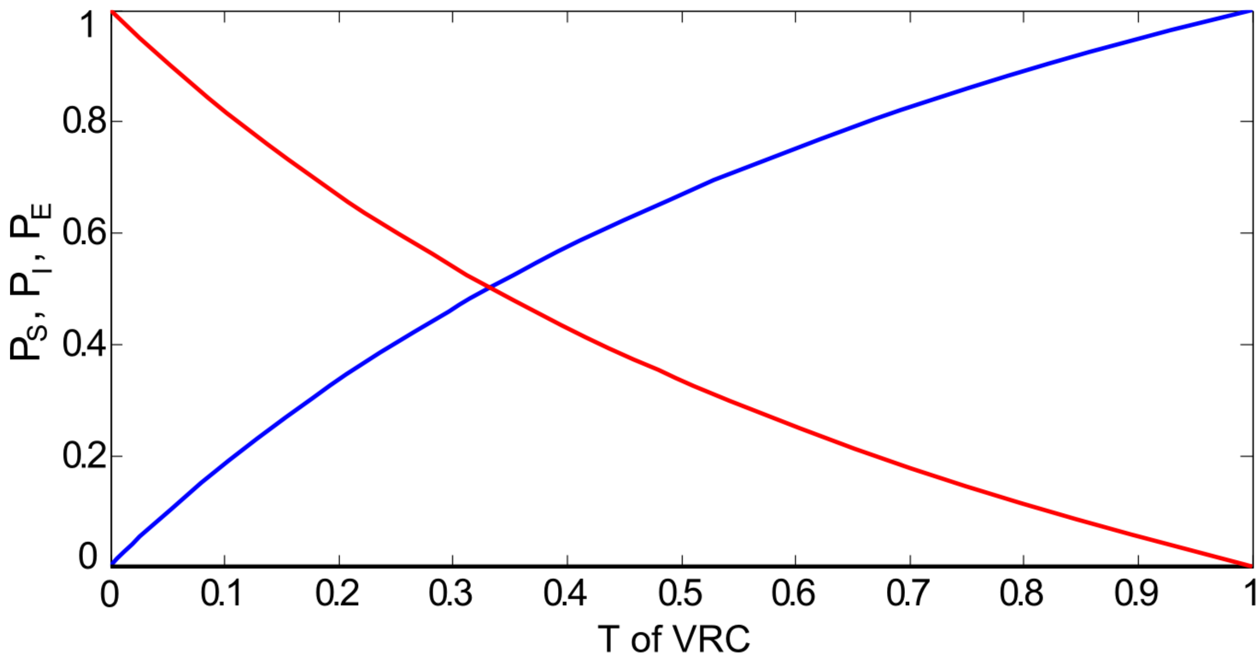
It is plotted for 7 values of $\theta_j = j\pi/30,\ j = 1,2,3,4,5,6,7$. The value of *j* increases from the bottom to top.

—— Theoretical curves of maximum $\tilde{P}_S$ achievable by the optimal scheme using entangled state (solid line).

- - - - Single-qubit probes only (dash line)

Theory optimal **unambiguous** discrimination with **entangled** probe state

- One certain angle between basis $\mathcal{M},\mathcal{N}$ corresponds with just one certain amount of inconclusive results. When the erroneous results should be in ideal case zero in real case as low as possible.



P_E - Probability of error detection

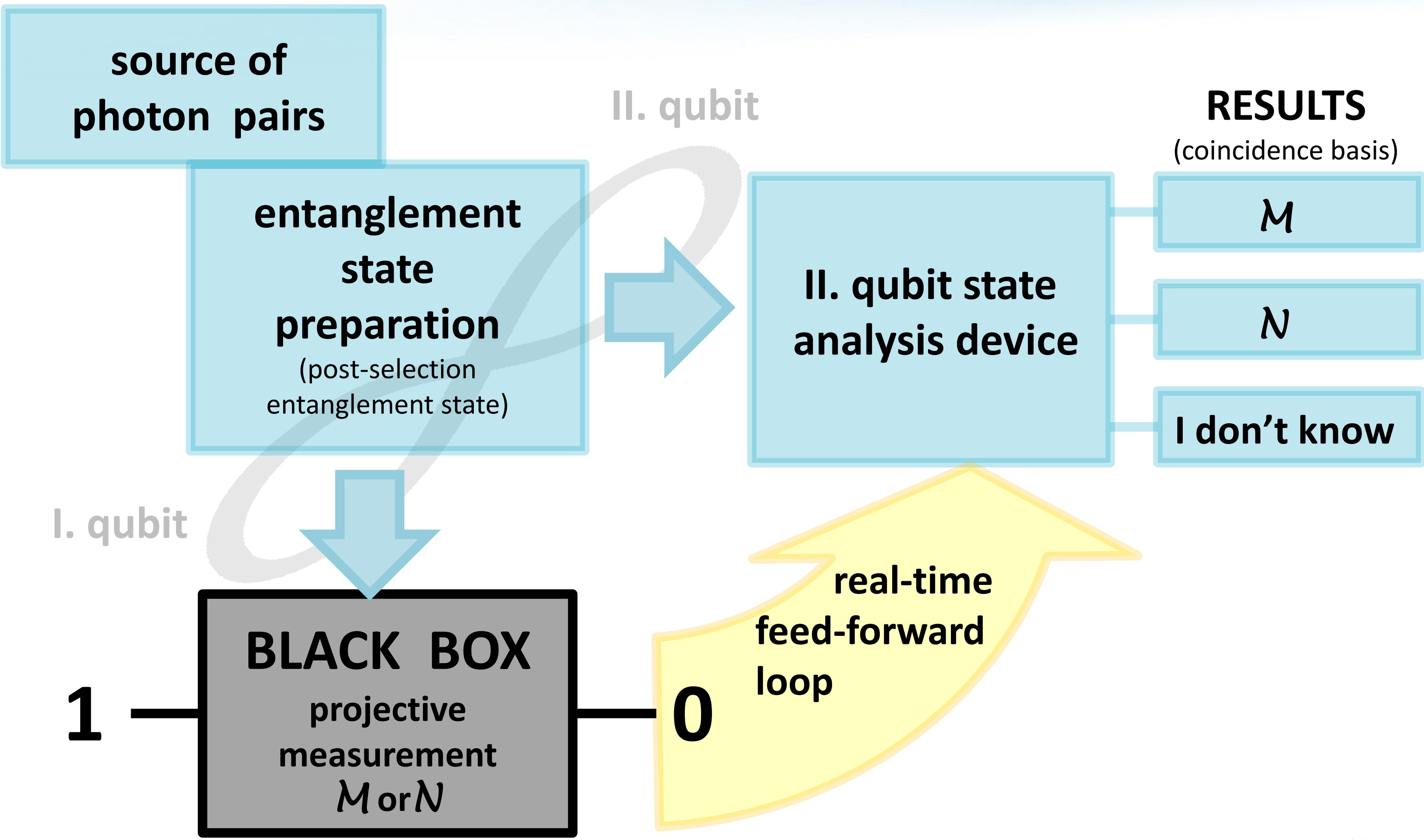
P_S - Probability of successful unambiguous detection

P_I - Probability of incocnclusive results

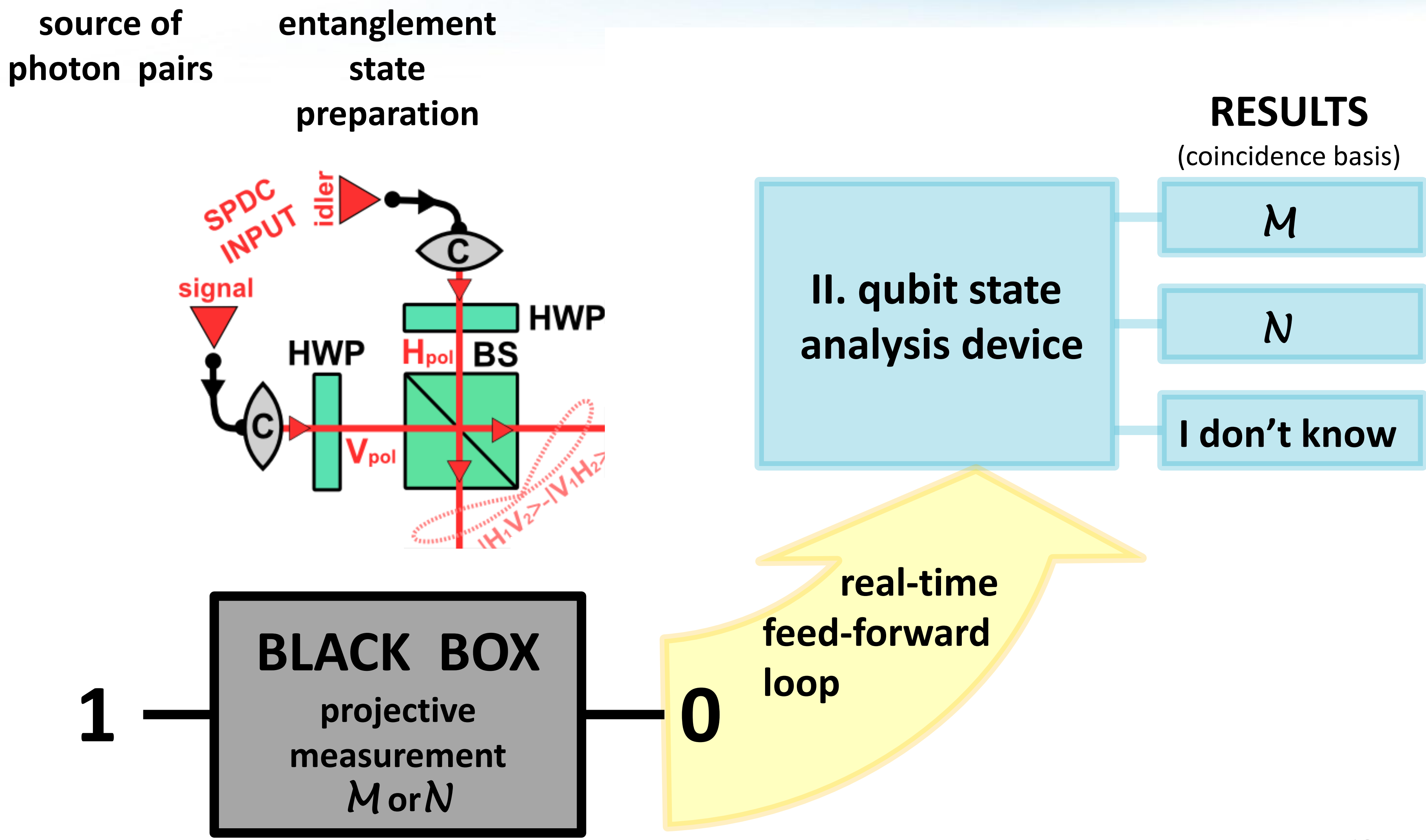
Experiment

**We experimentally implemented
the optimal entanglement-assisted discrimination
for projective measurements
on polarization states of single photons.**

Experiment - block idea



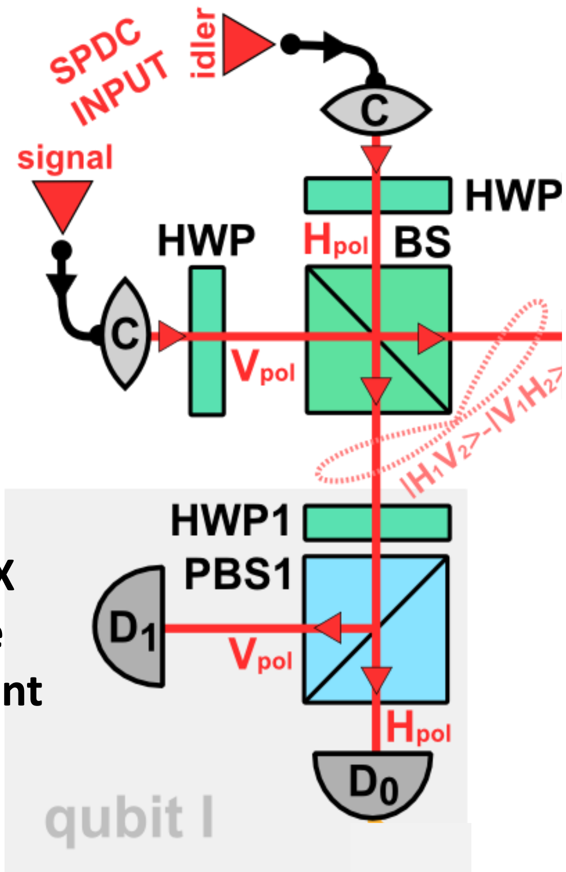
Experiment - simple scheme



Experiment - simple scheme

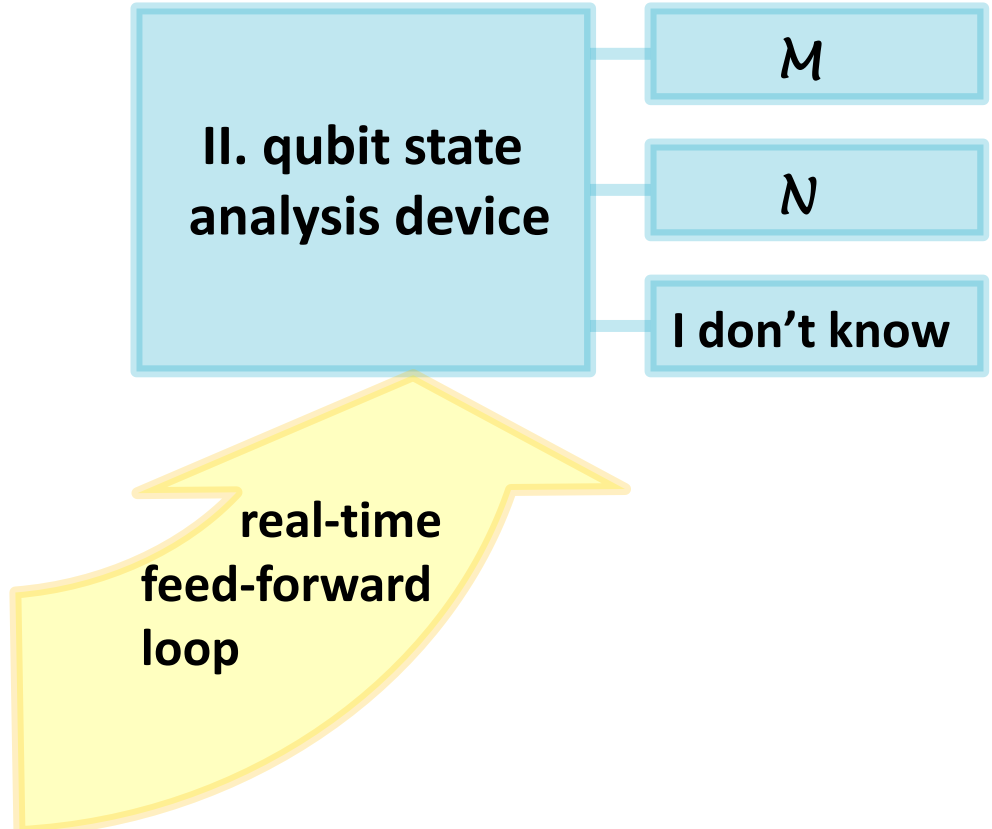
source of photon pairs

entanglement state preparation



BLACK BOX
projective measurement
$\mathcal{M}$ or $\mathcal{N}$

RESULTS
(coincidence basis)



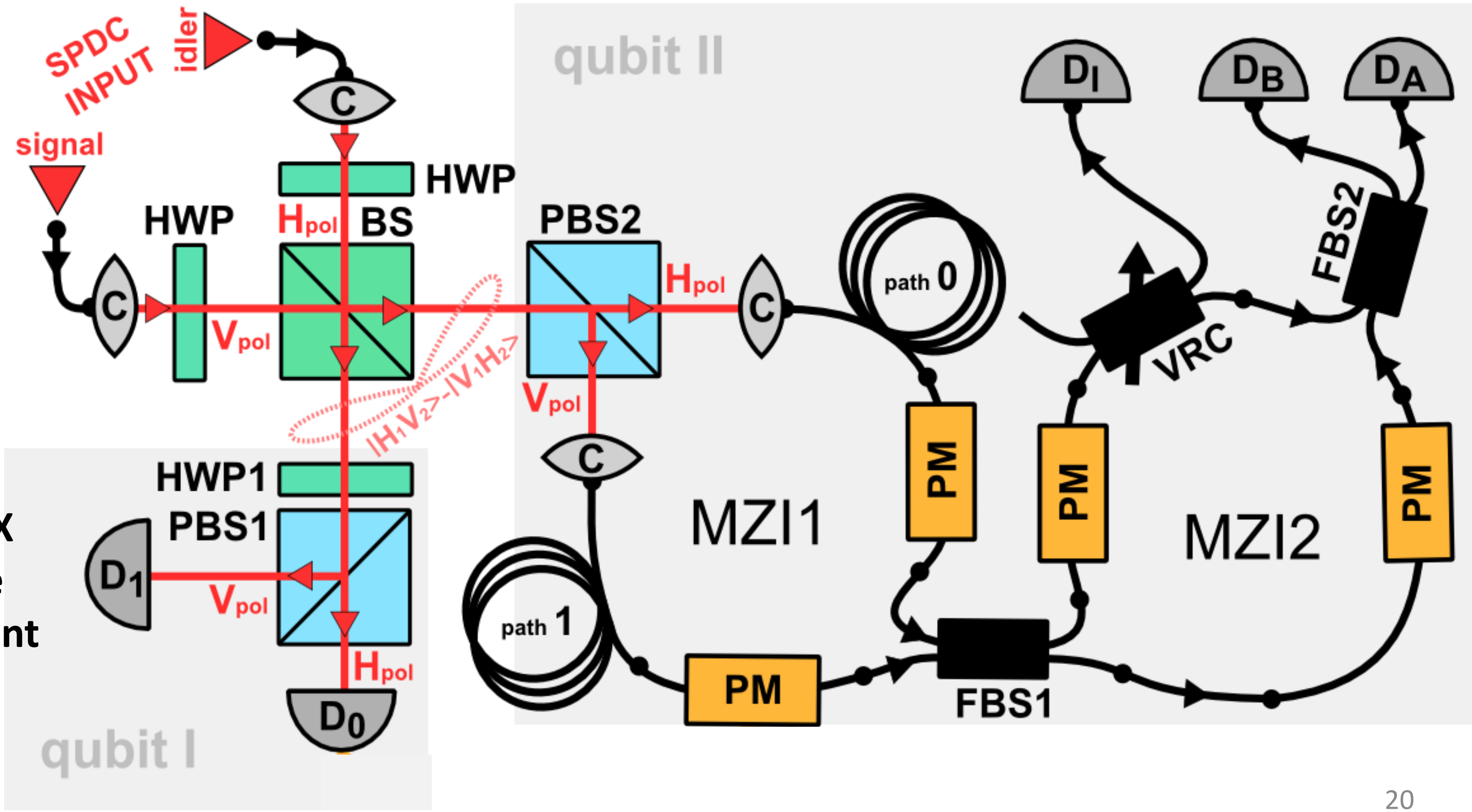
Experiment - simple scheme

source of photon pairs

entanglement state preparation

II. qubit state analysis device

BLACK BOX projective measurement $\mathcal{M}$ or $\mathcal{N}$



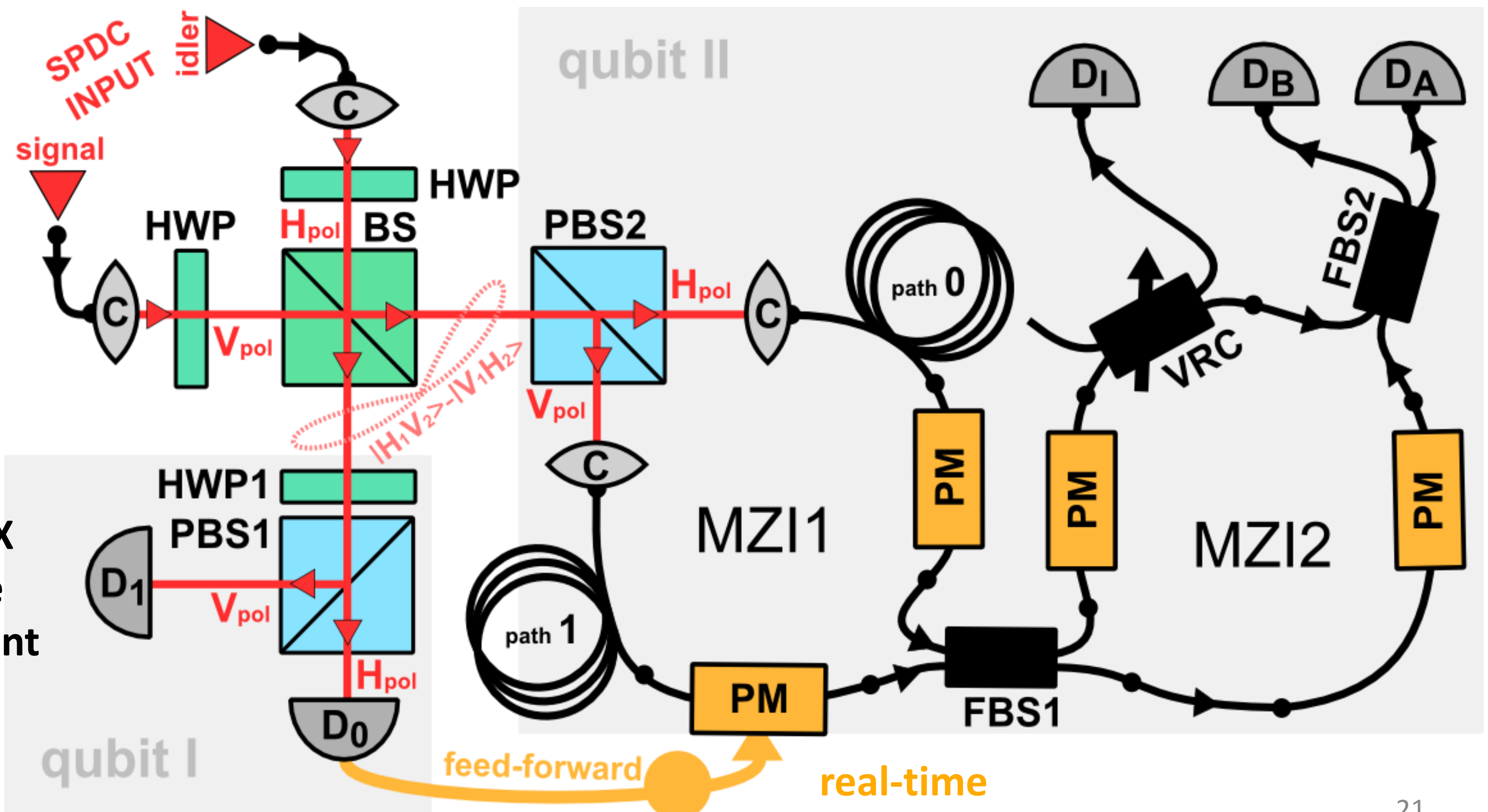
Experiment - simple scheme

source of photon pairs

entanglement state preparation

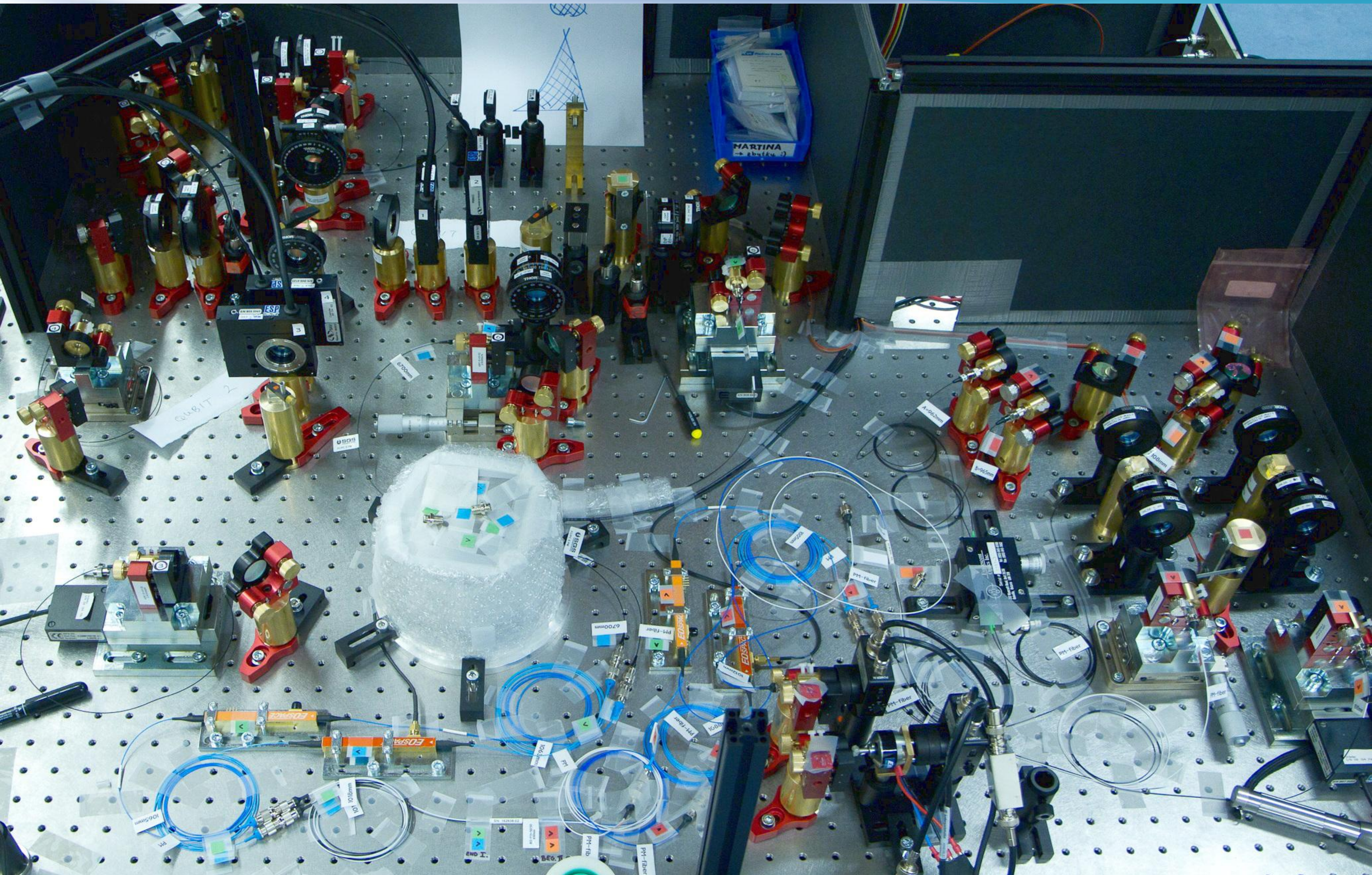
II. qubit state analysis device

BLACK BOX projective measurement $\mathcal{M}$ or $\mathcal{N}$

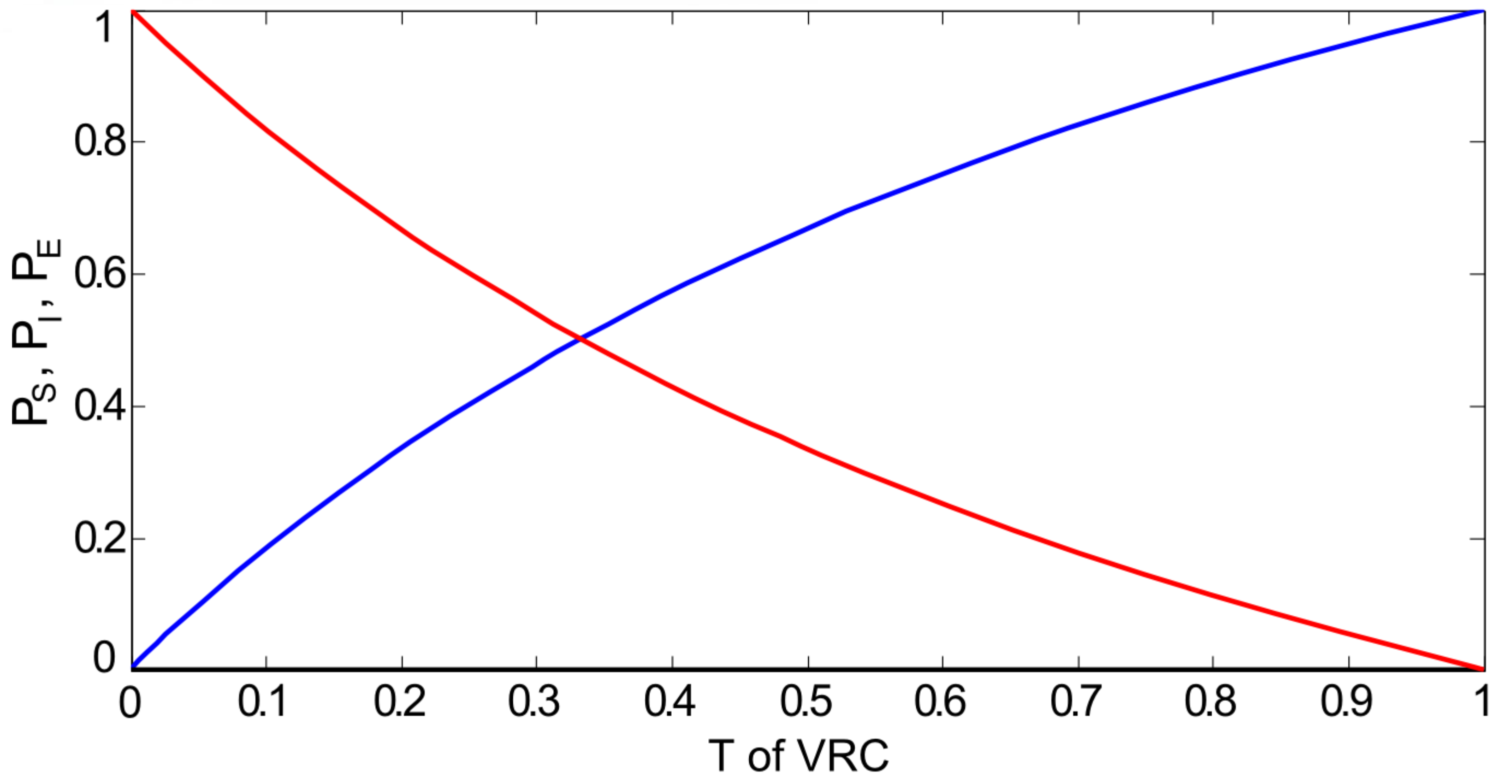


real-time feed-forward loop

Experiment - photo



Results unambiguous discrimination

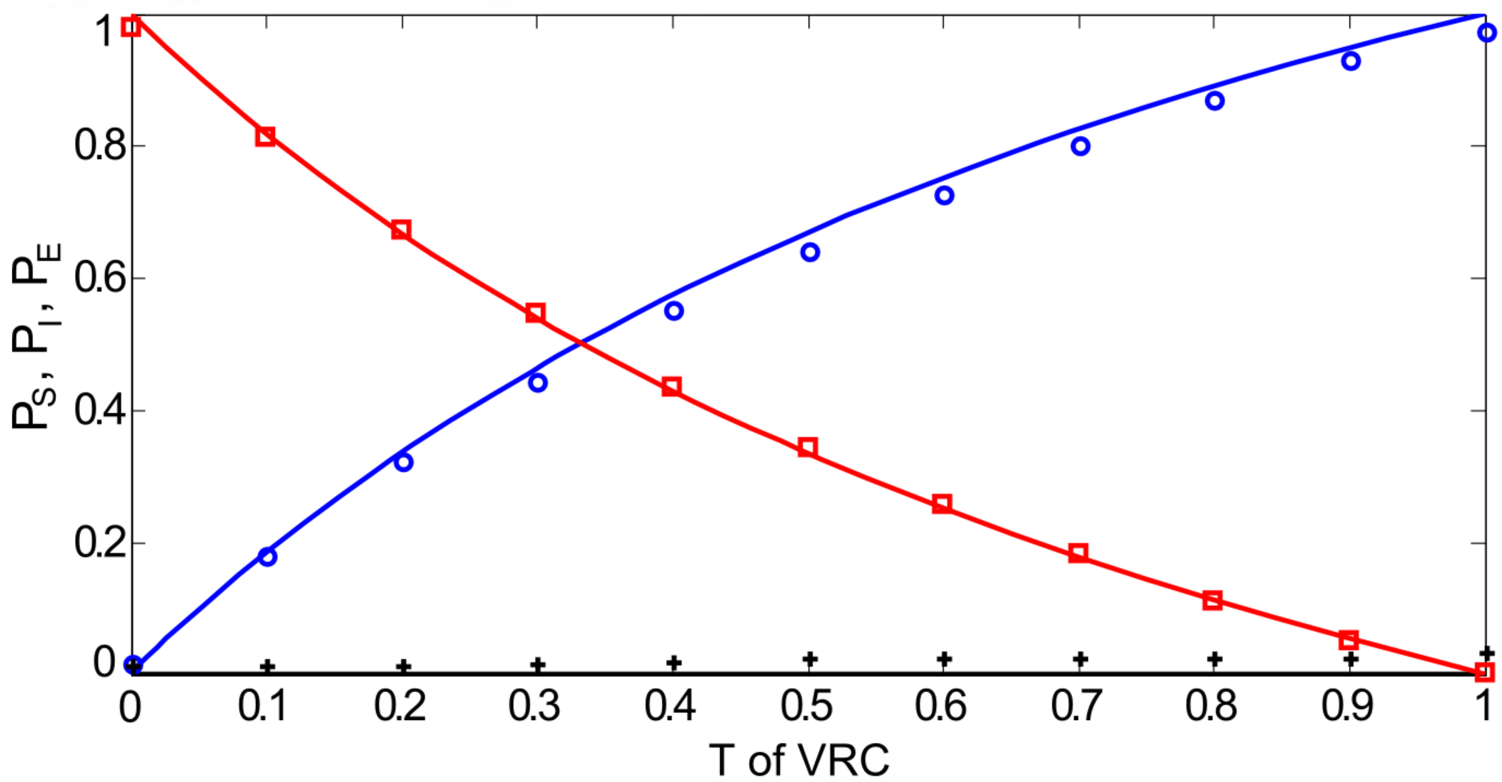


P_E - probability of error detection

P_S - probability of successful unambiguous detection

P_I - probability of incocnclusive results

Results unambiguous discrimination



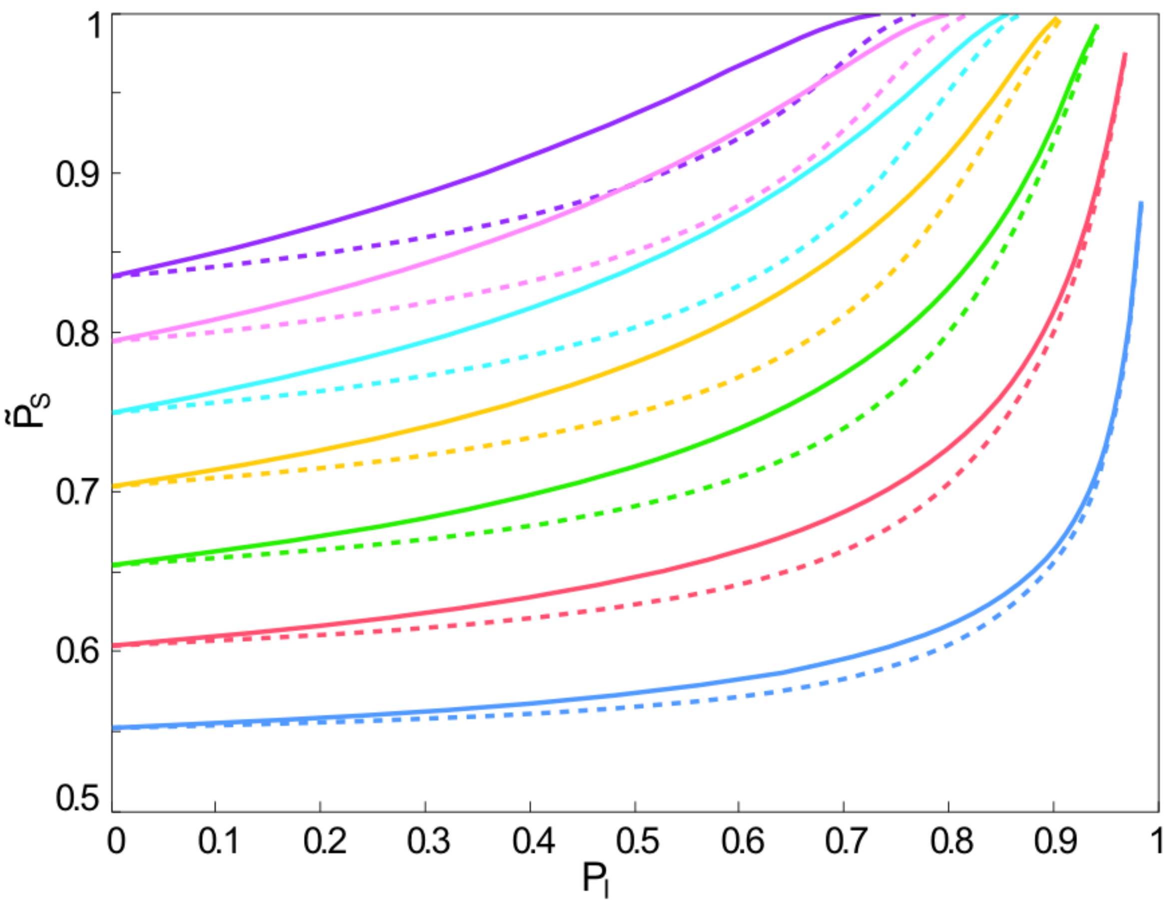
P_E - probability of error detection

P_S - probability of successful unambiguous detection

P_I - probability of incocnclusive results

Results for certain ratio of inconclusive results

Dependence of relative success probability on probability of inconclusive results.



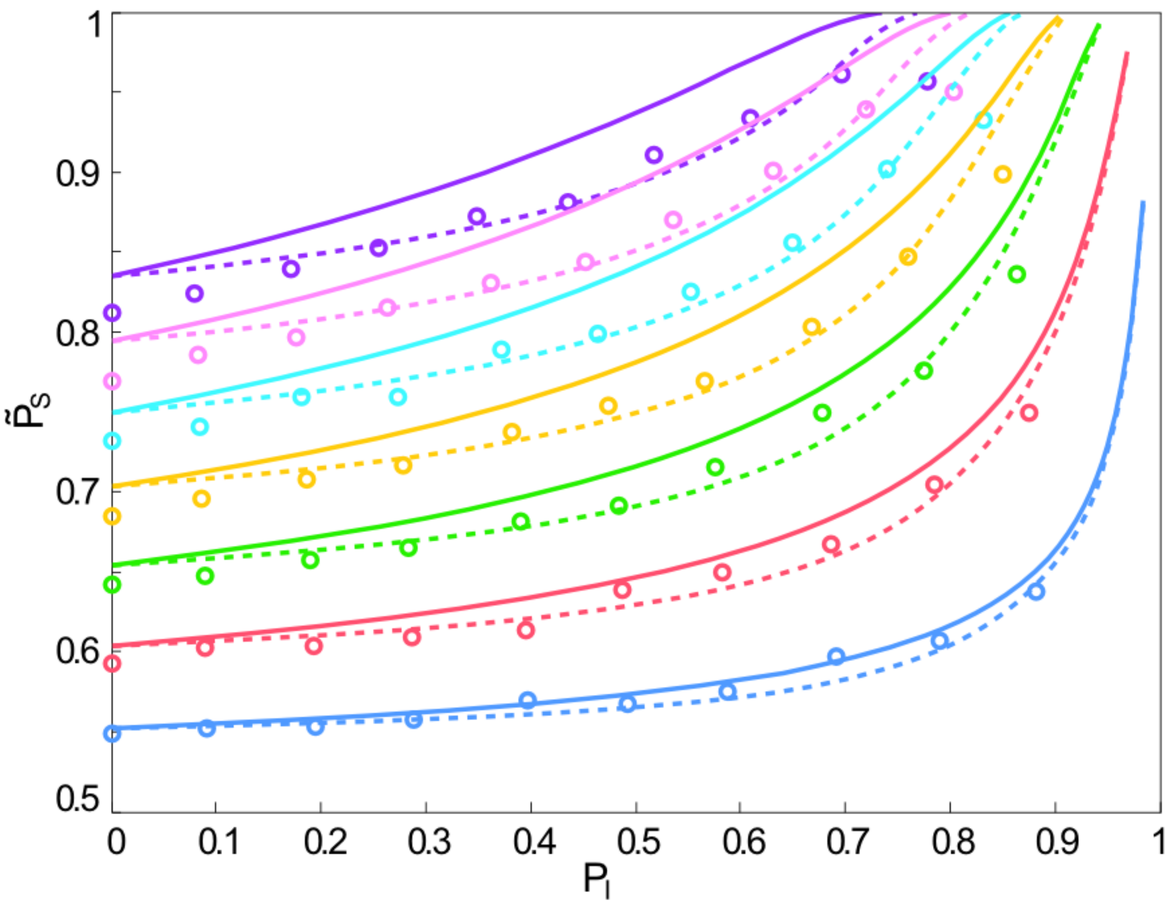
It is plotted for 7 values of $\theta_j = j\pi/30,\ j = 1,2,3,4,5,6,7$
The value of j increases from the bottom to top.

Theoretical curves of maximum $\tilde{P}_S$ achievable by the optimal scheme using entangled state (solid line).

Single-qubit probes only (dash line).

Results for certain ratio of inconclusive results

Dependence of relative success probability on probability of inconclusive results.



It is plotted for 7 values of $\theta_j = j\pi/30,\ j = 1, 2, 3, 4, 5, 6, 7$

The value of j increases from the bottom to top.

Theoretical curves of maximum $\tilde{P}_S$ achievable by the optimal scheme using entangled state (solid line).

Single-qubit probes only (dash line).

Experimental data (circles), errorbars are smaller that the symbols.

Conclusion

- We determined theoretically and implemented experimentaly the optimal strategies for discrimination between two projective single-qubit quantum measurements.

- The experimental data clearly demonstrate the advantage of entanglement-based discrimination strategy (compared to unentangled single-qubit probes).

- Results was published in Physical Review A **90**, 022317 (2014)

  M. Miková, M. Sedlák, I. Straka, M. Mičuda, M. Ziman, M. Ježek, M. Dušek, and J. Fiurášek, Optima entanglement-assisted discrimination of quantum measurements

Acknowledgement

- **To ALL my colleagues**
- **This work was supported by**

  The Czech Science Foundation (GAČR): GA13-20319S
  Internal Grant of Palacky University: PrF-2013-008, and IGA-PrF-2014008
  European Social Fund (ESF): CZ.1.07/2.3.00/20.0060

european social fund in the czech republic
EUROPEAN UNION
MINISTRY OF EDUCATION, YOUTH AND SPORTS
OP Education for Competitiveness
INVESTMENTS IN EDUCATION DEVELOPMENT

Thank you for your attention