# Non-Markovian reactivation of quantum repeaters

**Stefano Pirandola**

**Computer Science & York Centre for Quantum Techologies**

**University of York (UK)**

EPSRC Engineering and Physical Sciences Research Council

UK NATIONAL QUANTUM TECHNOLOGIES PROGRAMME

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The Leverhulme Trust

Department of Optics, Palacky University
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# In collaboration with:

Sam Braunstein

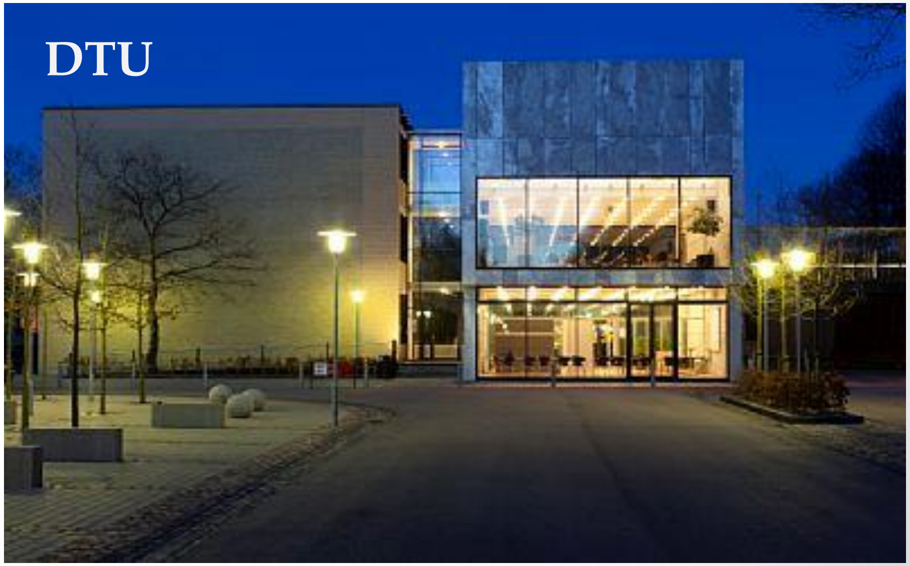
Carlo Ottaviani

Gaetana Spedalieri


Tobias Gehring

Christian S. Jacobsen

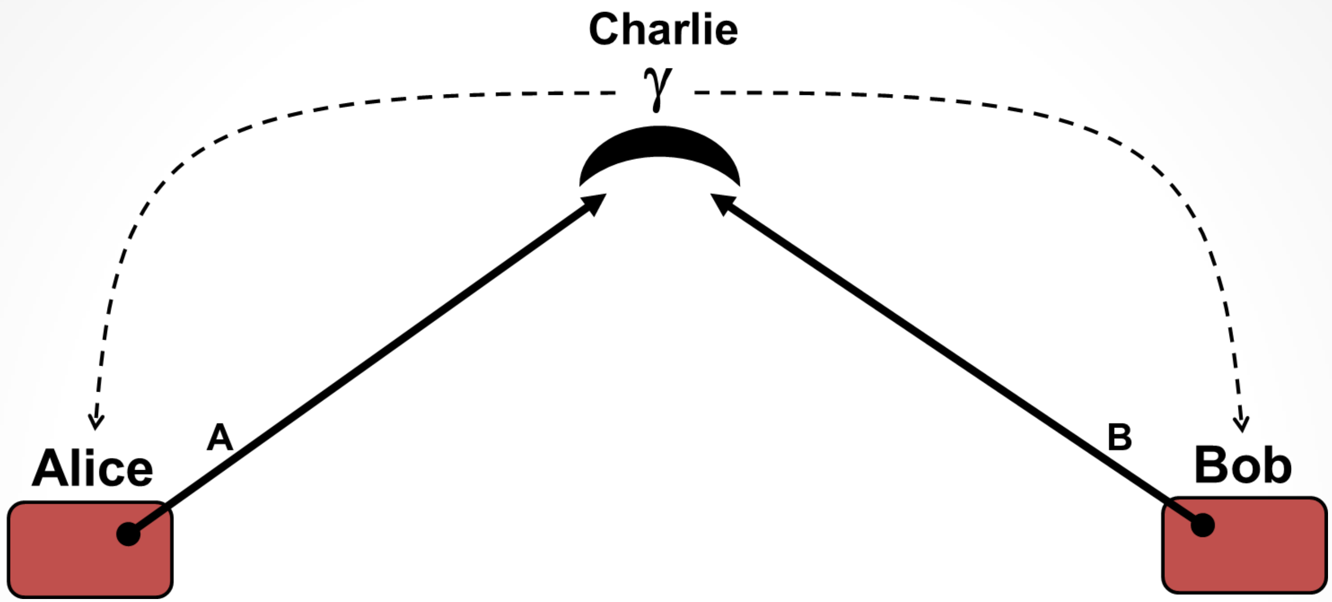
Ulrik L. Andersen



# Outline

- Repeater for CV protocols
- Markovian and non-Markovian noise
- Entanglement-breaking conditions
- Reactivation of multipartite entanglement
- Reactivation of repeater-based protocols
- Correlated and additive Gaussian noise
- Experimental demonstration

# Repeater-based protocols

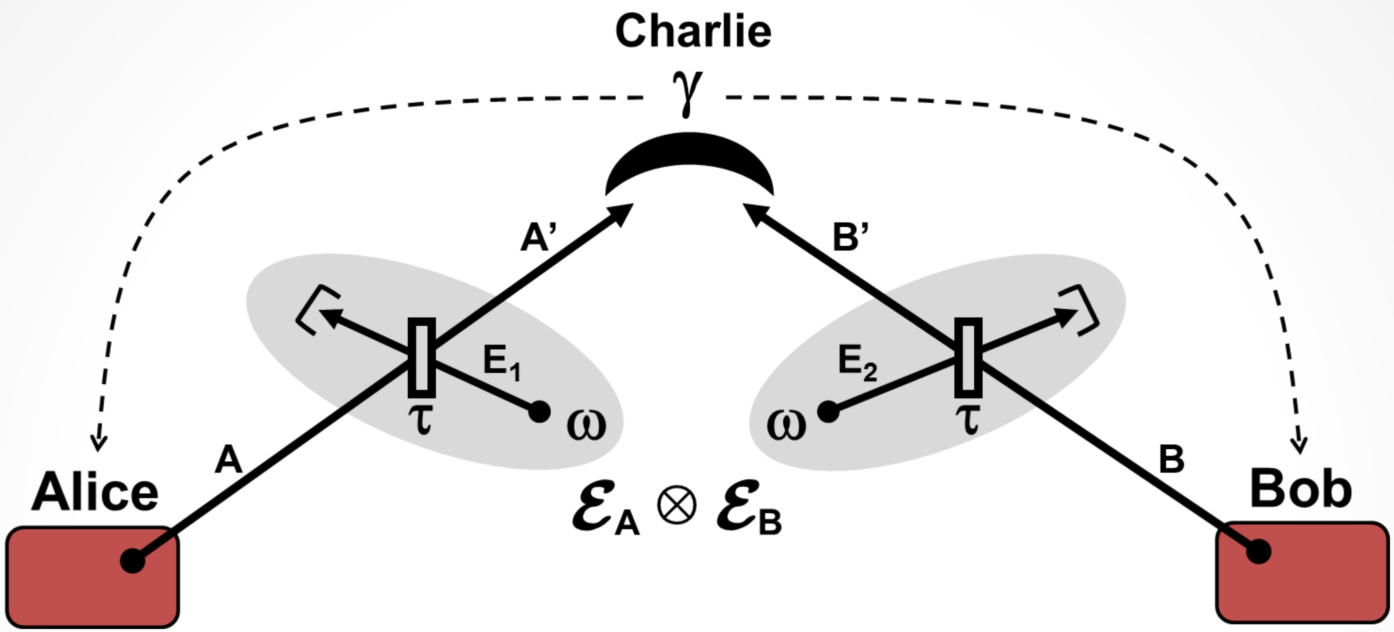



- Two parties send optical modes (A and B) to Charlie
- Charlie performs a CV-Bell detection and broadcast the outcome $\gamma$

In their labs, the parties can implement several protocols:

- **Entanglement Swapping (and Distillation)**
- **Quantum Teleportation**
- **Quantum Key Distribution**

# Markovian Environment



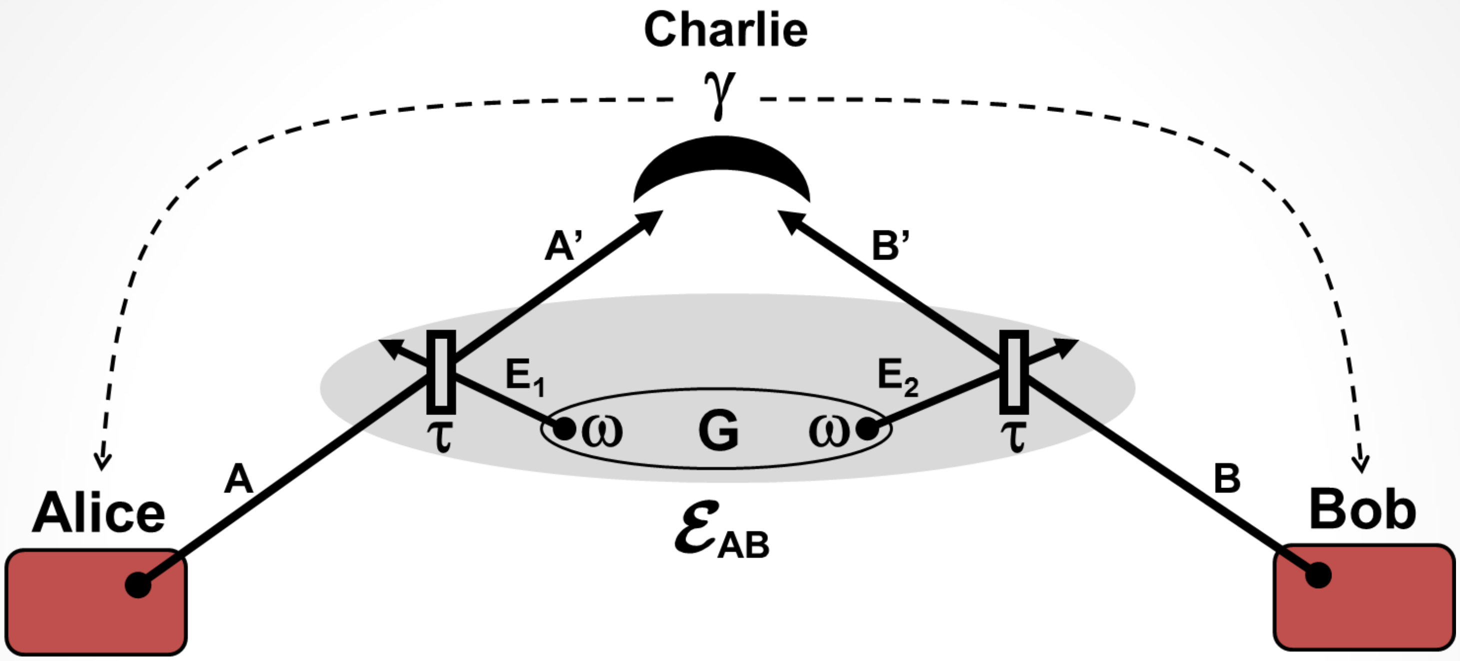

Markovian (memoryless) environment: Two independent lossy channels

Worst-case scenario: Entanglement Breaking (EB) channels

Thermal-noise $\boldsymbol{\omega \geq \omega_{EB} := \frac{1+\tau}{1-\tau}}$

- Repeater-based protocols can't work

# Non-Markovian Noise

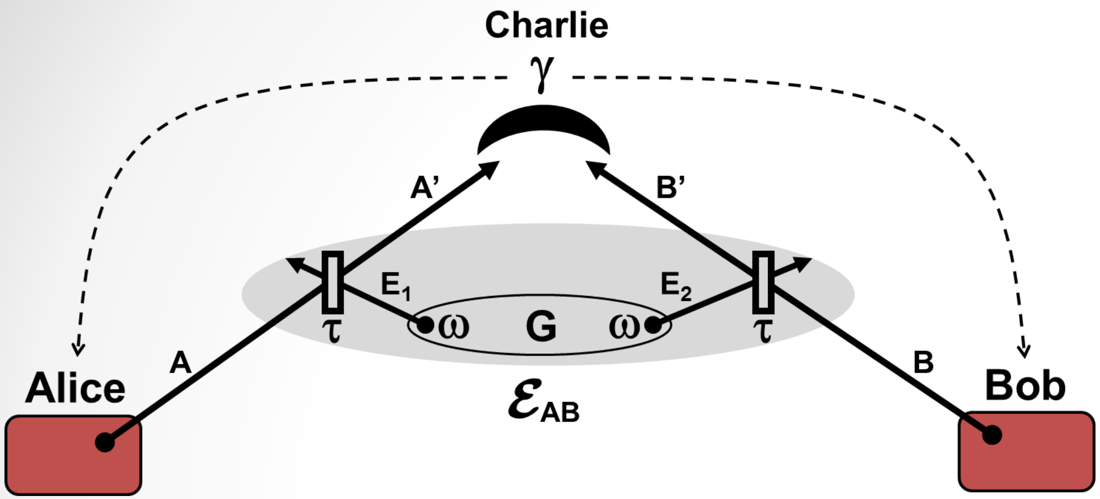



Non-Markovian Environment: Lossy channels with correlated thermal noise

$$\mathbf{V}_{E_1E_2} = \begin{pmatrix} \omega\mathbf{I} & \mathbf{G} \\ \mathbf{G} & \omega\mathbf{I} \end{pmatrix} \quad \mathbf{G} = \begin{pmatrix} g & \\ & g' \end{pmatrix}$$

Correlated (but separable) Gaussian state for the environment

# Non-Markovian Noise




**Why?**

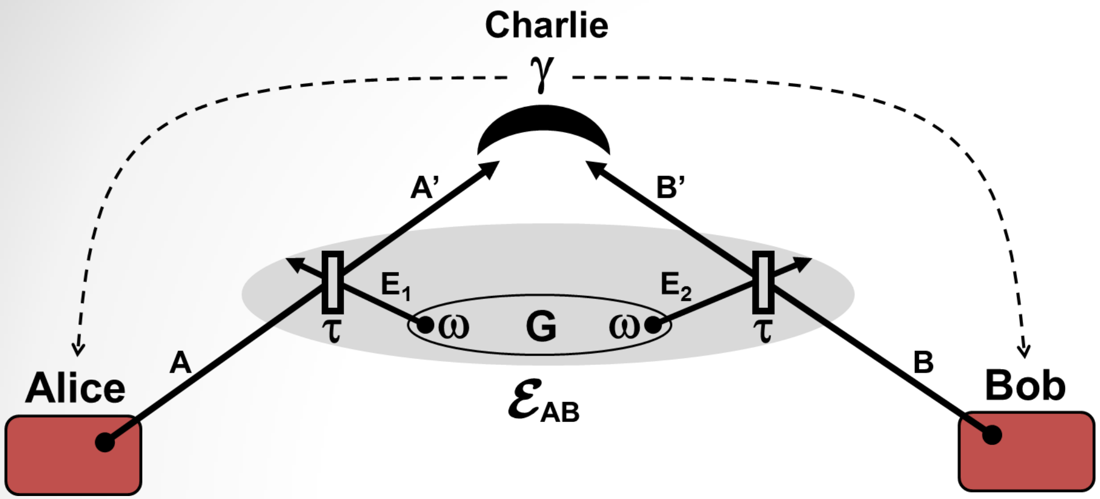
**Fundamental Reasons**

Markovianity is an approximation (Nature is non Markovian)

Quantum Information beyond i.i.d. formulation

Non-Markovianity may be a resource

# Non-Markovian Noise

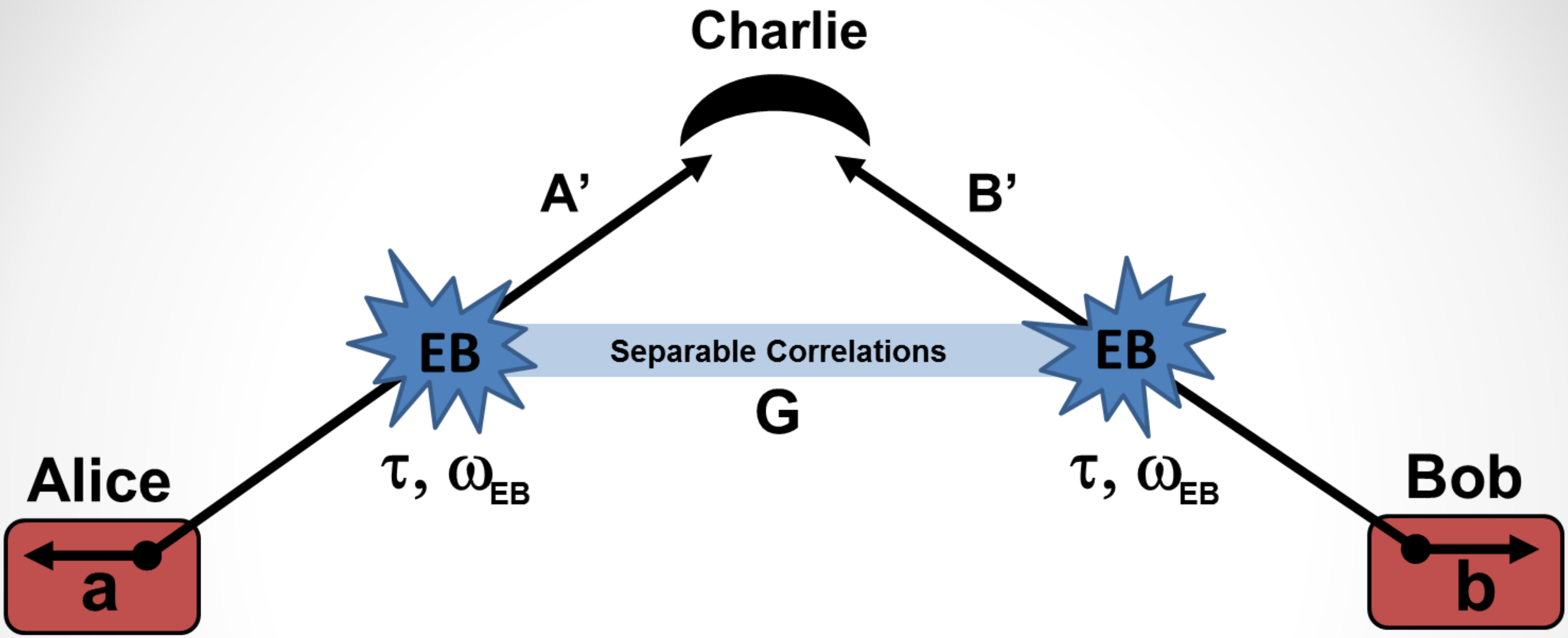



**Where?**

Short-distance high-rate implementations on photonic chips, where teleportation and swapping are primitive resources for quantum computing

Long distance QKD based on relays (e.g., MDI-QKD). In general, the attack is coherent between two links (2-mode coherent attacks after de Finetti). Correlated side-channel attacks of the relay are also possible.

# Entanglement-Breaking

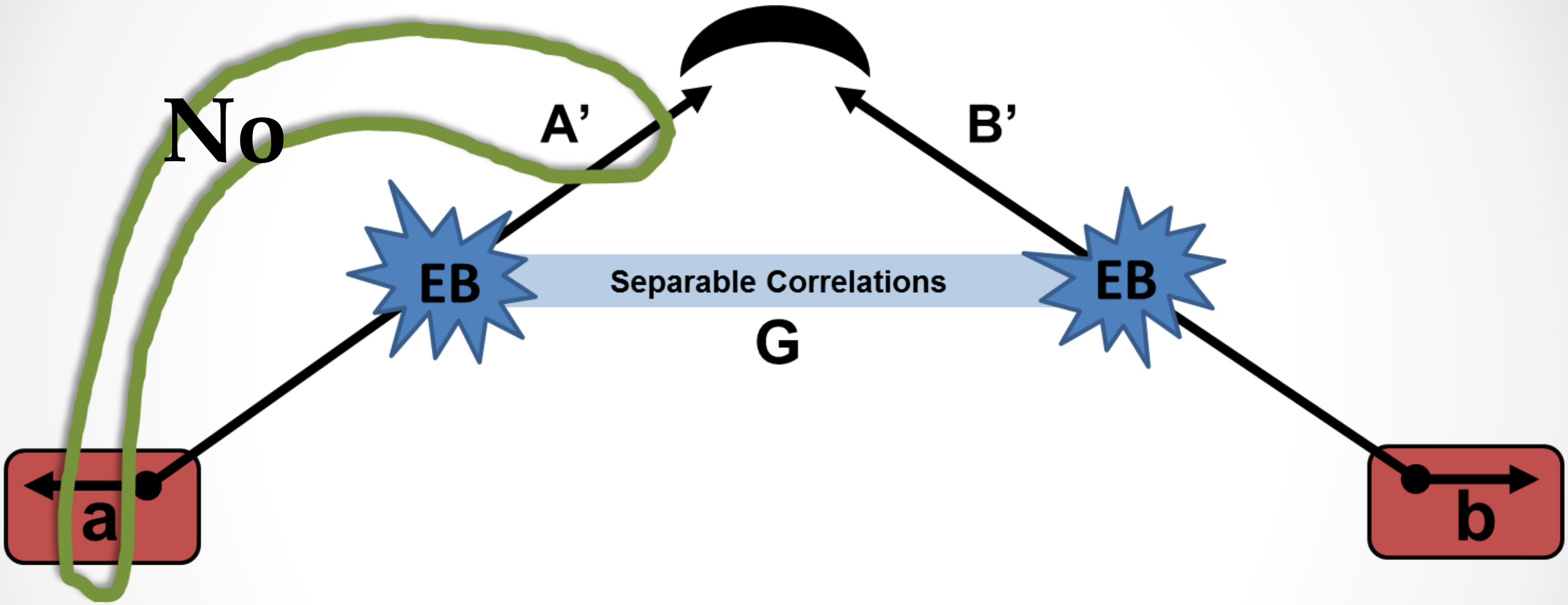



We consider entanglement-breaking thermal noise in each link

Even with correlations, entanglement is broken

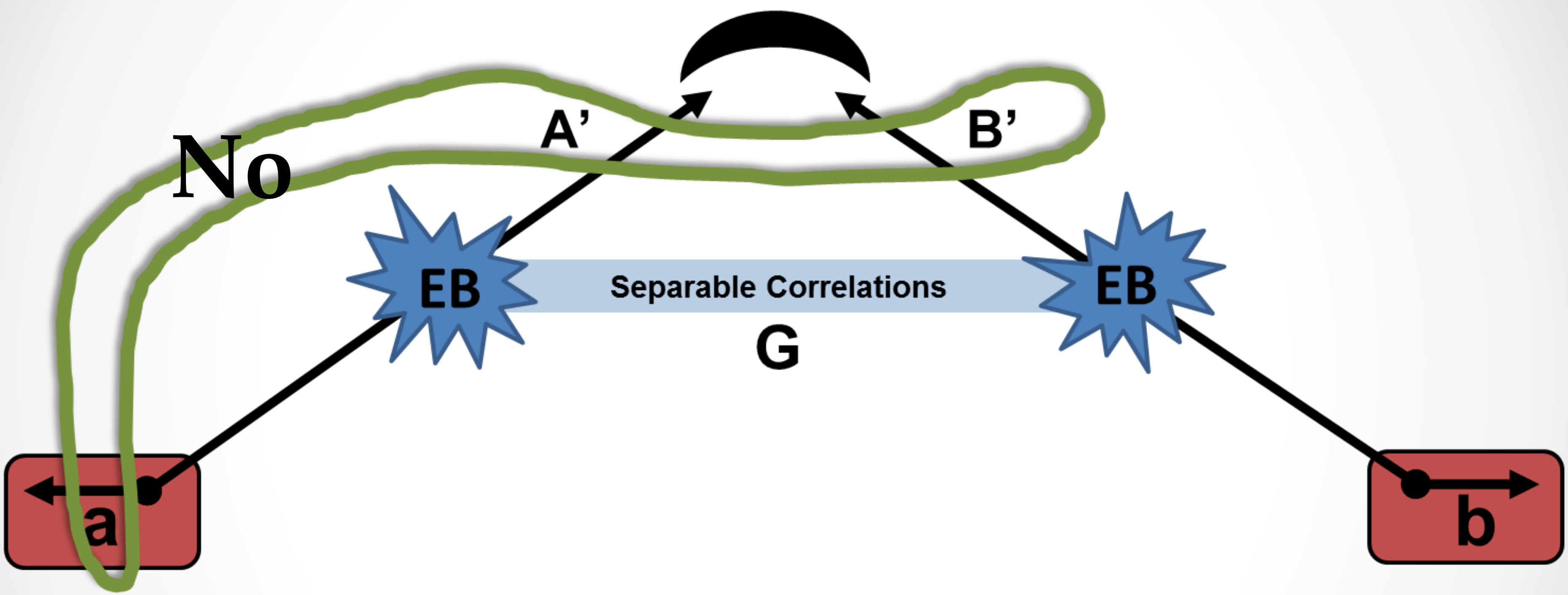
This is true for bipartite and tripartite entanglement

# Entanglement-Breaking




No bipartite entanglement survives before detection

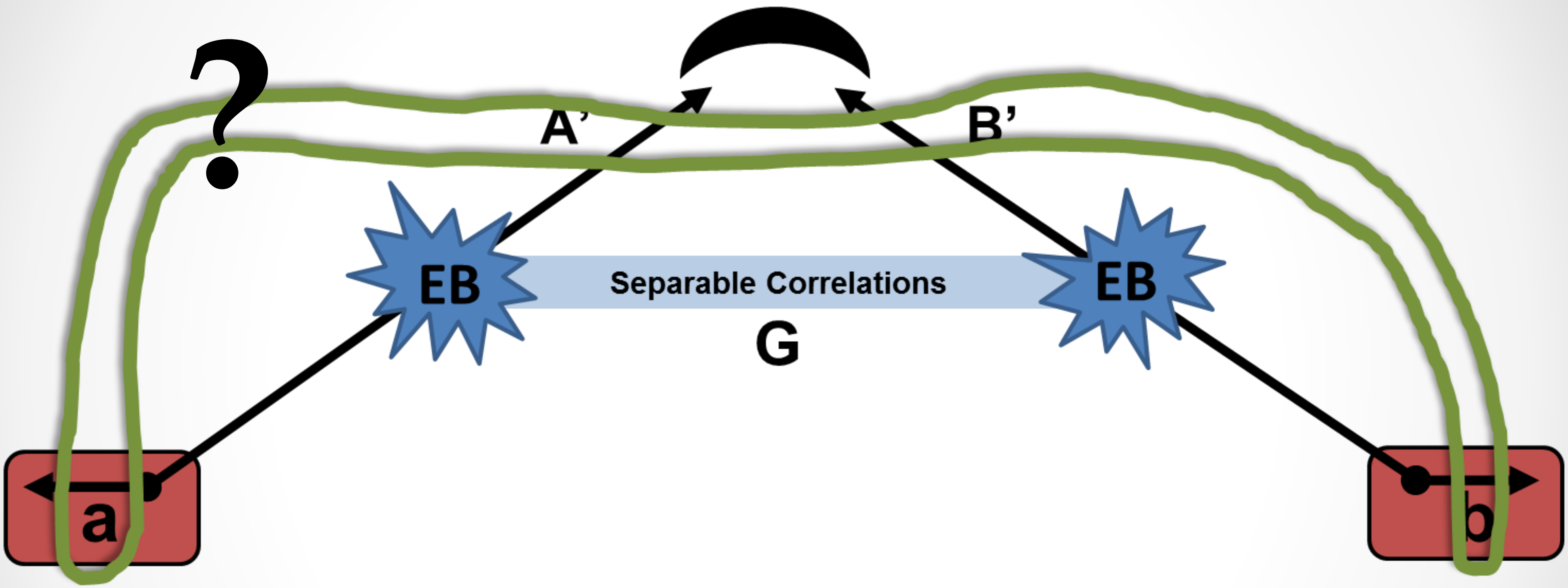
# Entanglement-Breaking




**No tripartite entanglement survives**

(Modes aA′B′ in a fully separable state)

# Entanglement-Breaking

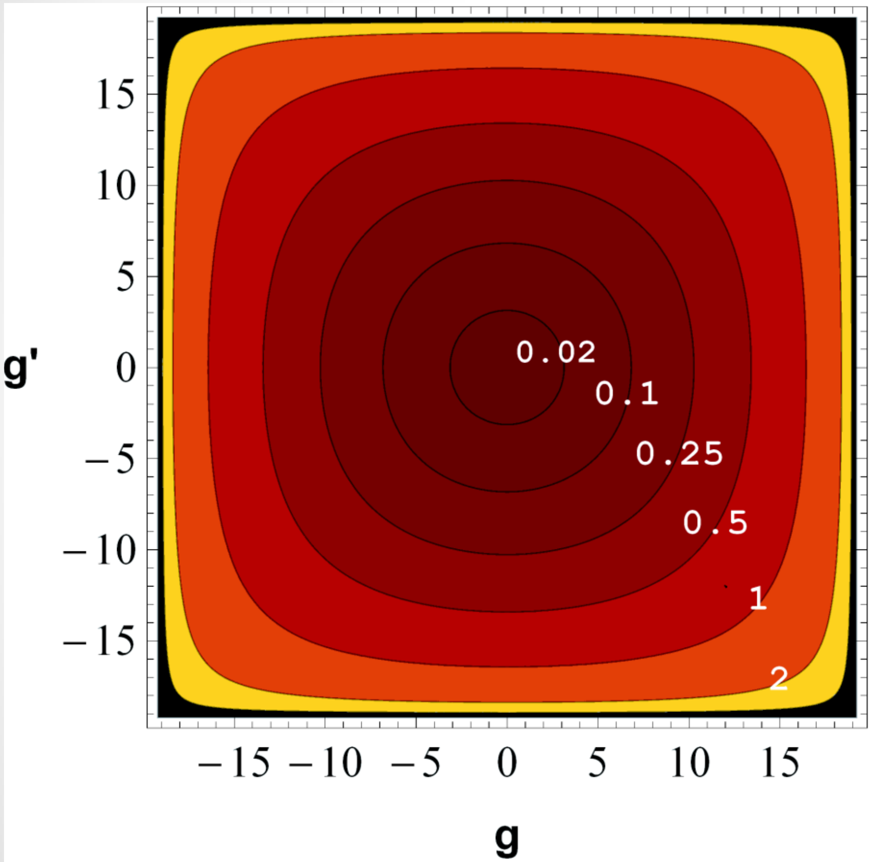



**What about 4-partite entanglement?**

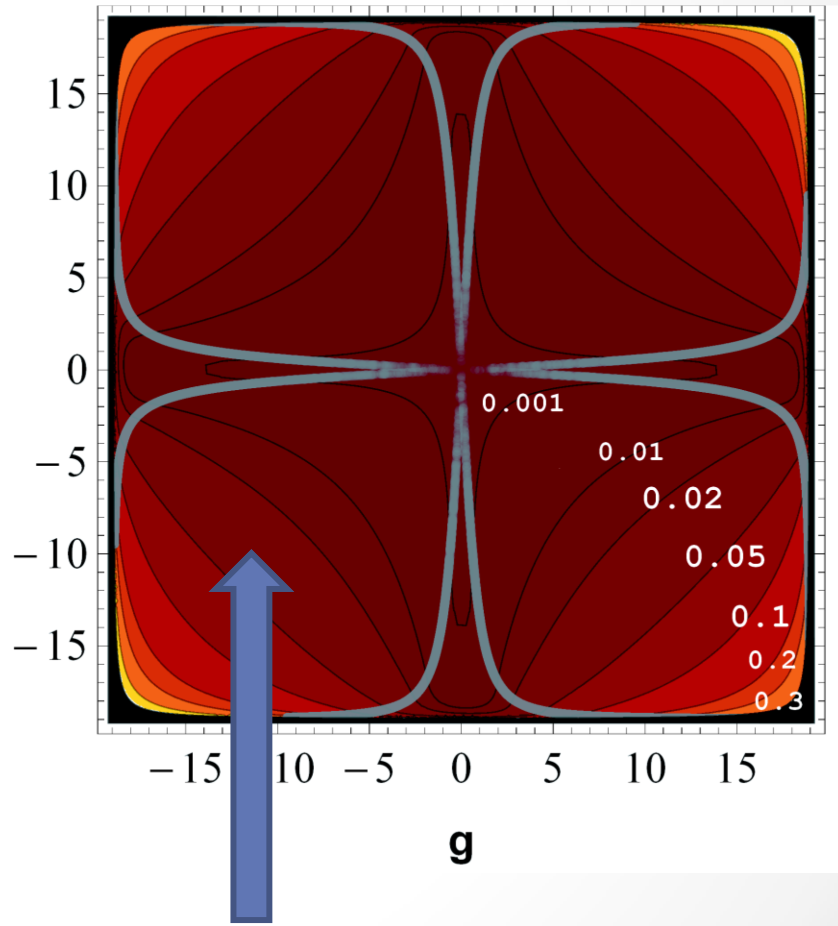
This is also broken unless the injected correlations surpass a critical threshold

# Environmental Correlations

Classical Correlations




Quantum Correlations

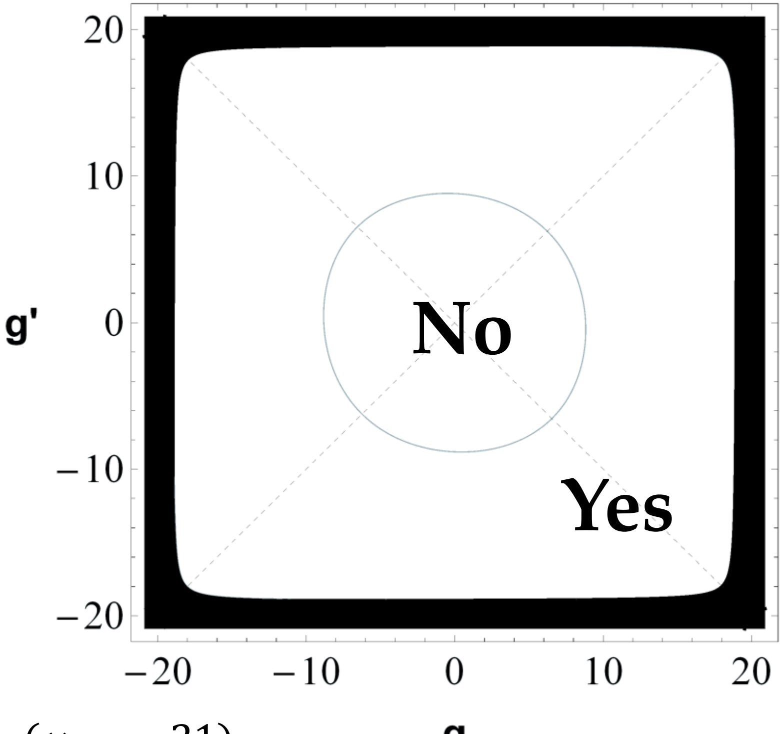



($\omega_{\mathrm{EB}} = 19$)

**Quantum Discord**

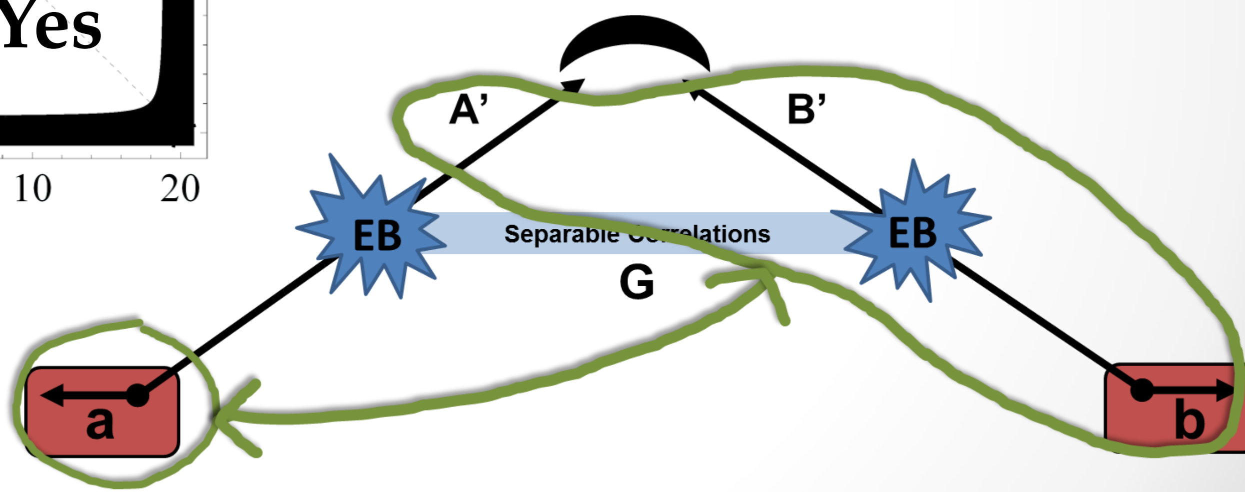
- [Pirandola, Spedalieri, Braunstein, Cerf & Lloyd, PRL 113, 140405 (2014)]

# Reactivation of 4-partite entanglement



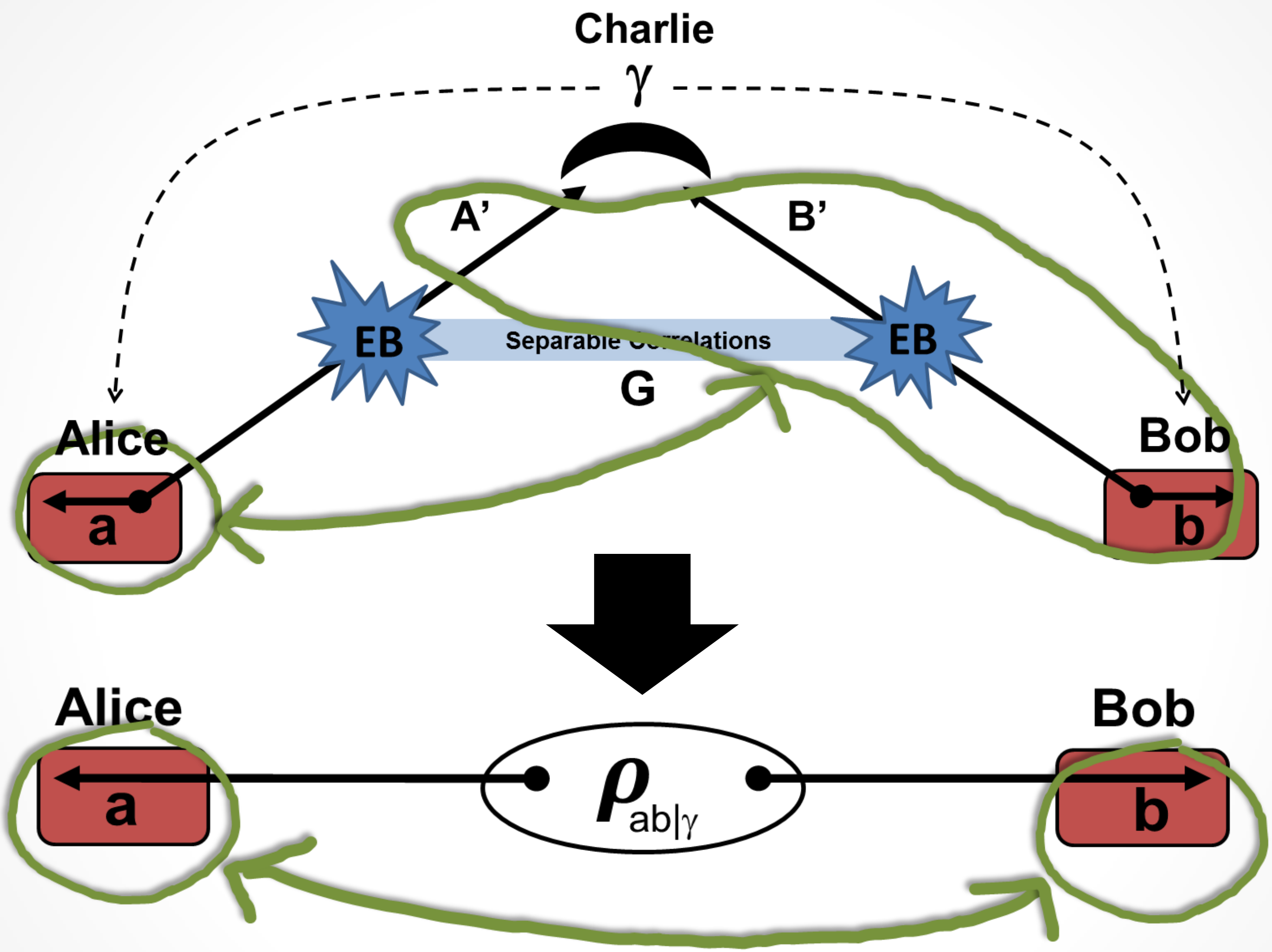

$(\omega_{\mathrm{EB}} = 21)$

Mode "a" becomes entangled with the 3-partite system A′B′b




Thx to the injected separable correlations, entanglement can survive in a weaker form

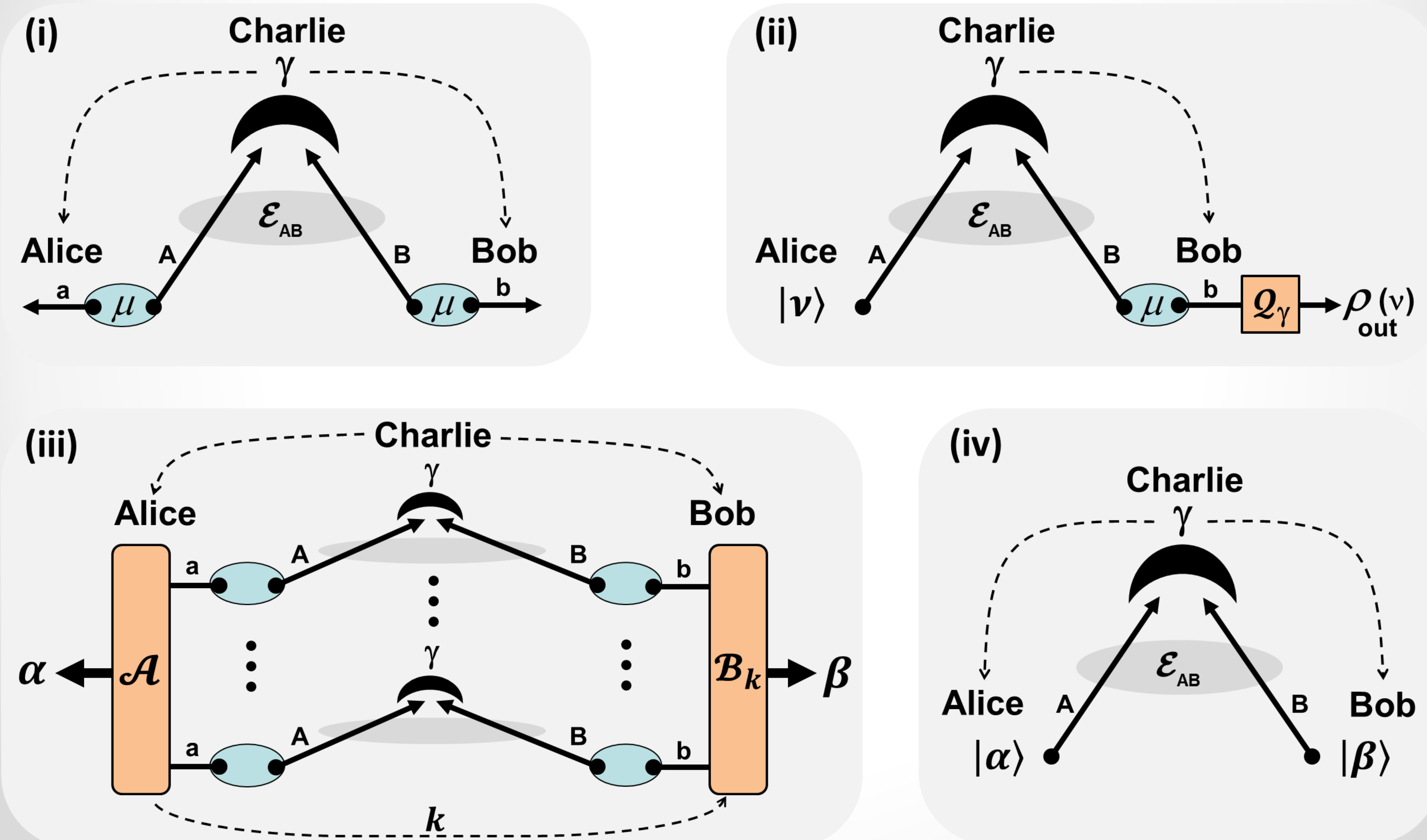
# Non-Markovian Reactivation



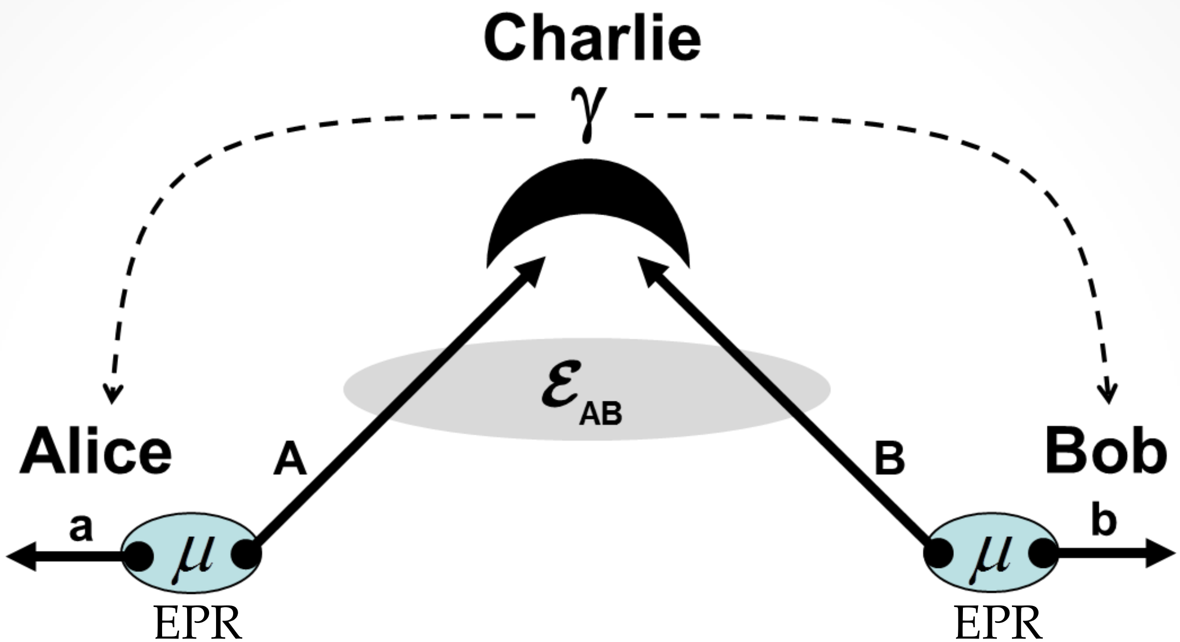

The projection of the Bell detection can now localize the entanglement into a bipartite form

# Non-Markovian Reactivation

Let us study the reactivation of all repeater-based protocols



# Entanglement Swapping

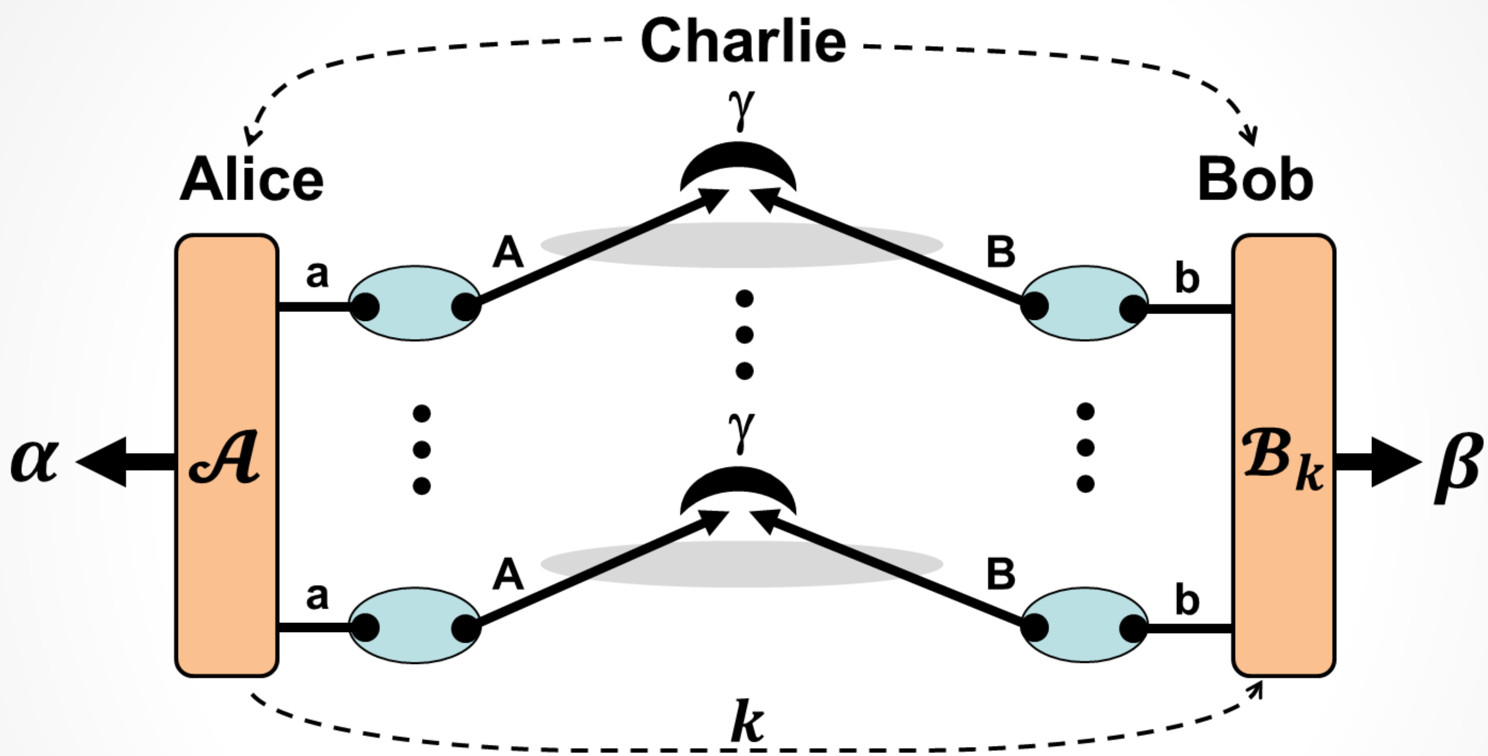



Remote log-neg $\mathcal{N} = \max\{0, -\log_2 \varepsilon\}$

$$\varepsilon = \left[\frac{(1+\mu\kappa)(1+\mu\kappa')}{(\mu+\kappa)(\mu+\kappa')}\right]^{1/2} \simeq \varepsilon_{\text{opt}} := \sqrt{\kappa\kappa'}$$ for large $\mu$

$$\kappa := (\tau^{-1}-1)(\omega - g), \;\; \kappa' := (\tau^{-1}-1)(\omega + g')$$

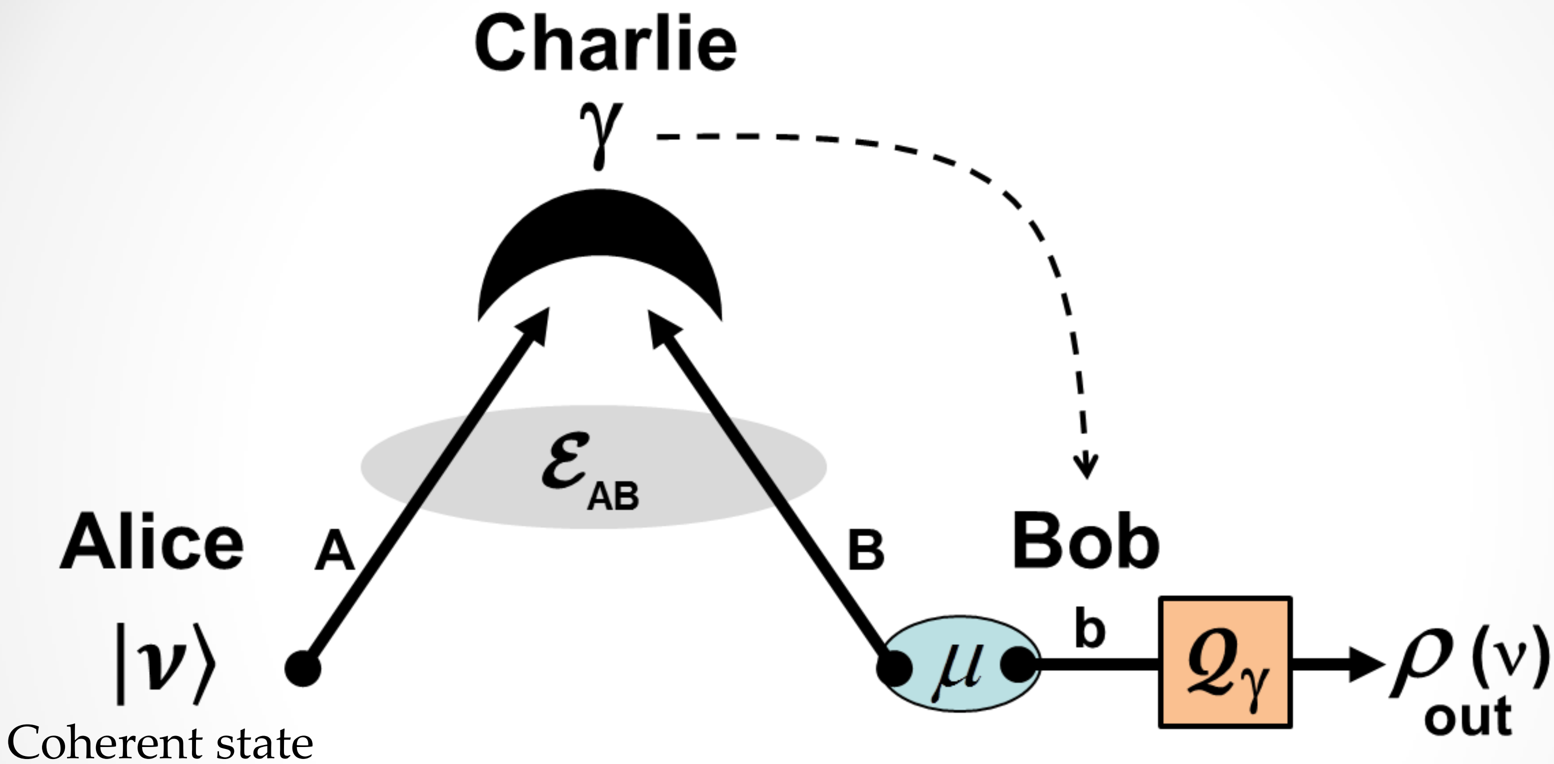
# Entanglement Distillation




Coherent Information

$$I_{\mathcal{C}} = I_{\mathcal{C}}(\mu, \kappa, \kappa') \simeq -\log_2(e\varepsilon_{\text{opt}})$$

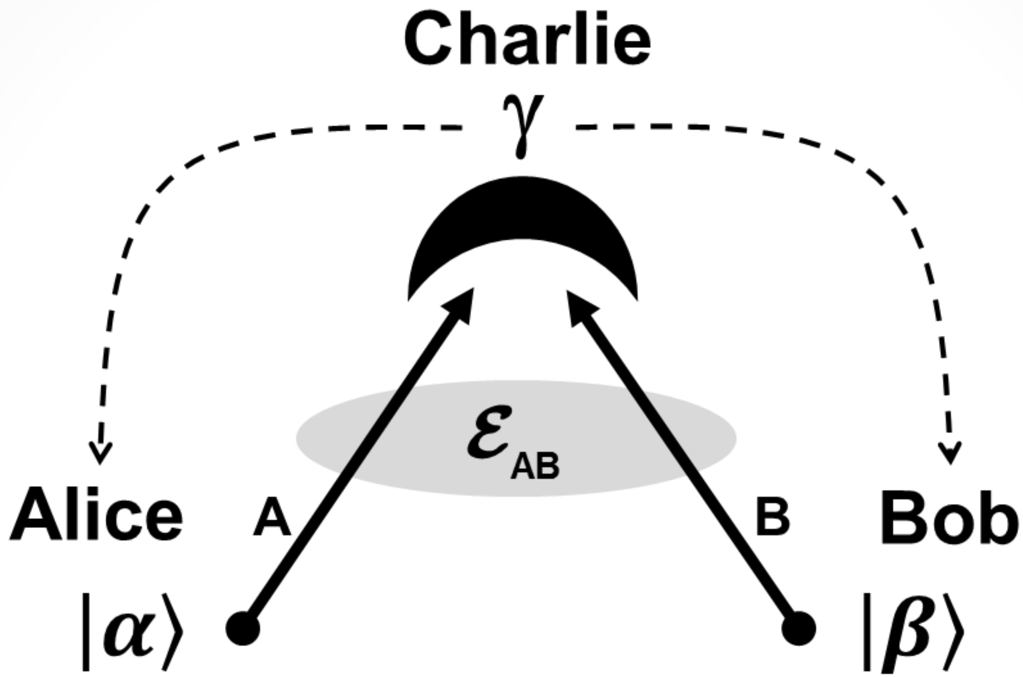
# Quantum Teleportation




Teleportation Fidelity

$$F \simeq F_{\text{opt}} := [(1+\kappa)(1+\kappa')]^{-1/2} \leq (1+\varepsilon_{\text{opt}})^{-1}$$

# Quantum Key Distribution




Secret Key Rate

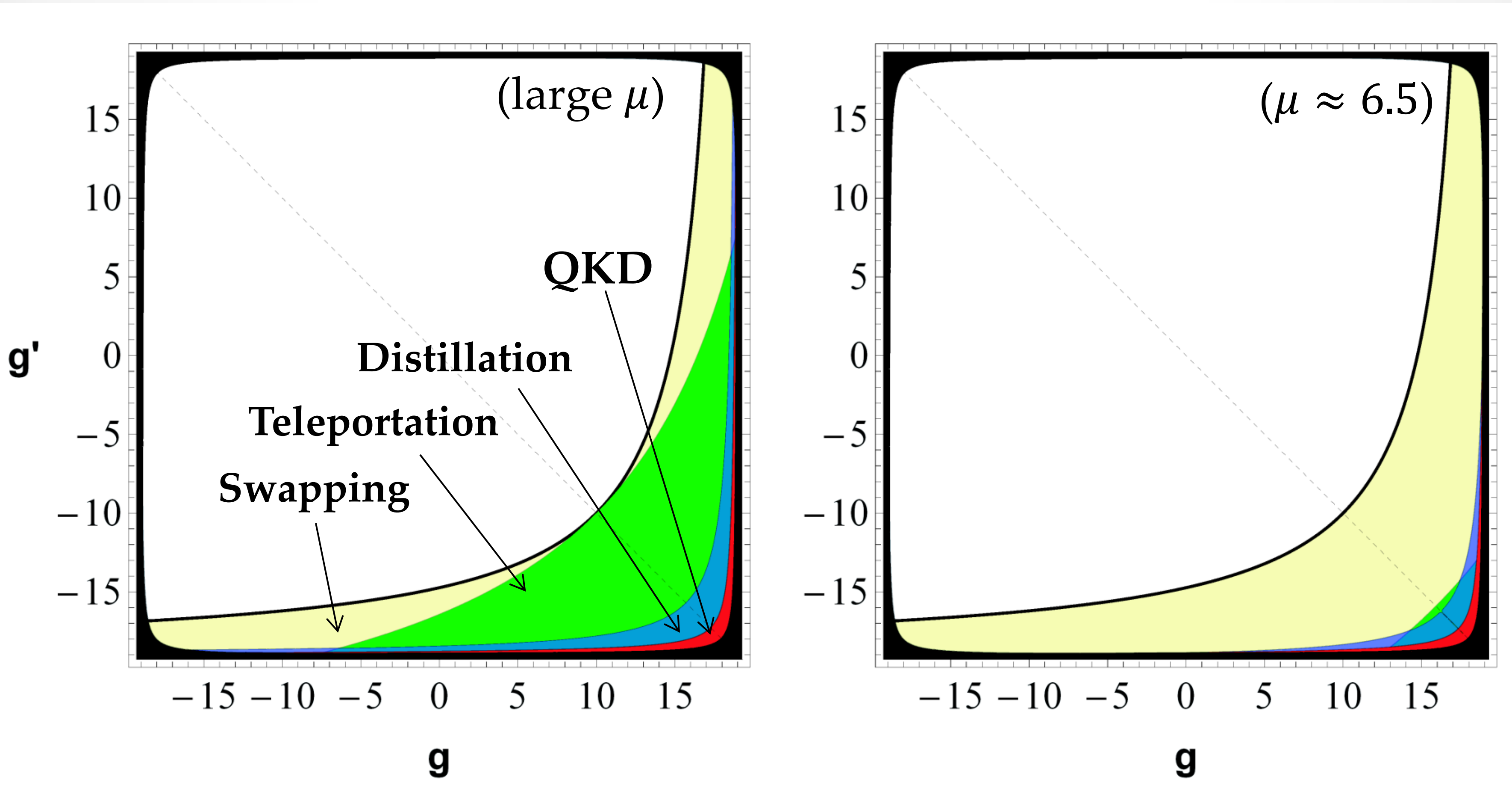
$$R_{\mathrm{opt}} \gtrsim \log_2\left(\frac{F_{\mathrm{opt}}}{e^2\varepsilon_{\mathrm{opt}}}\right) + h(1+2\varepsilon_{\mathrm{opt}})$$

See also MDI-QKD [Braunstein & Pirandola, PRL 108, 130502 (2012])

# Non-Markovian Reactivation

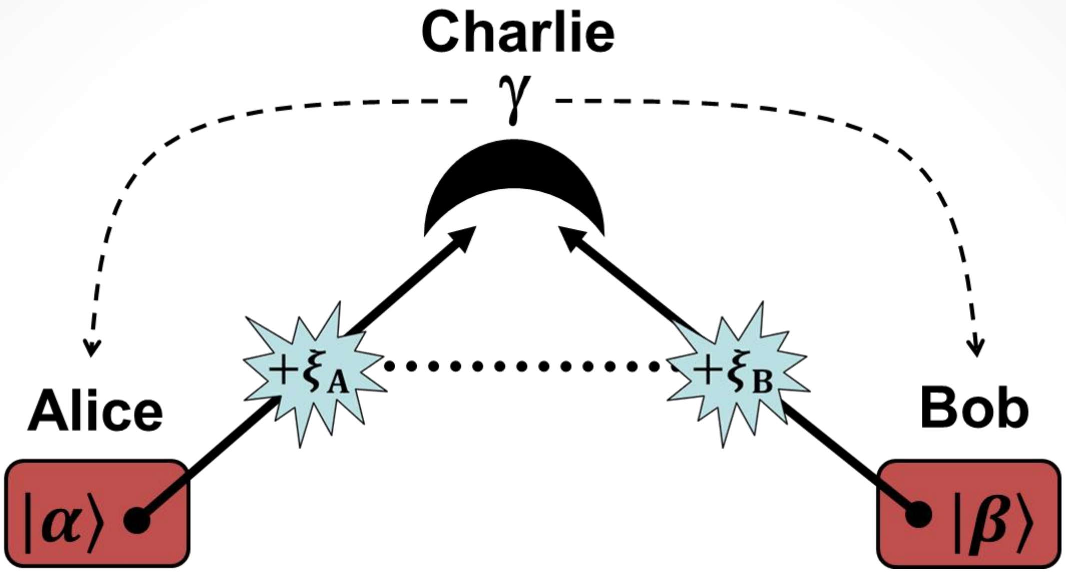
Assuming the EB condition in each link

**We reactivate all protocols by increasing the (separable) correlations of the environment**



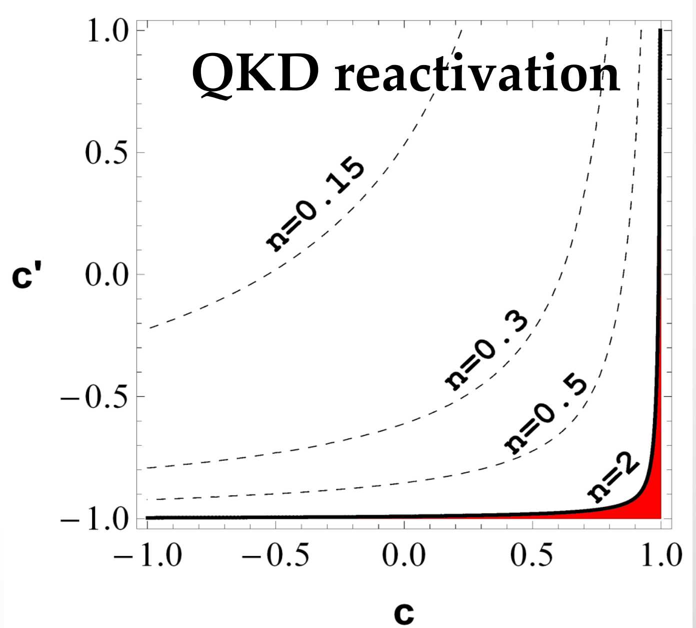

$\tau = 0.9 \quad \omega = \omega_{EB} = 19$

# Correlated Additive Noise

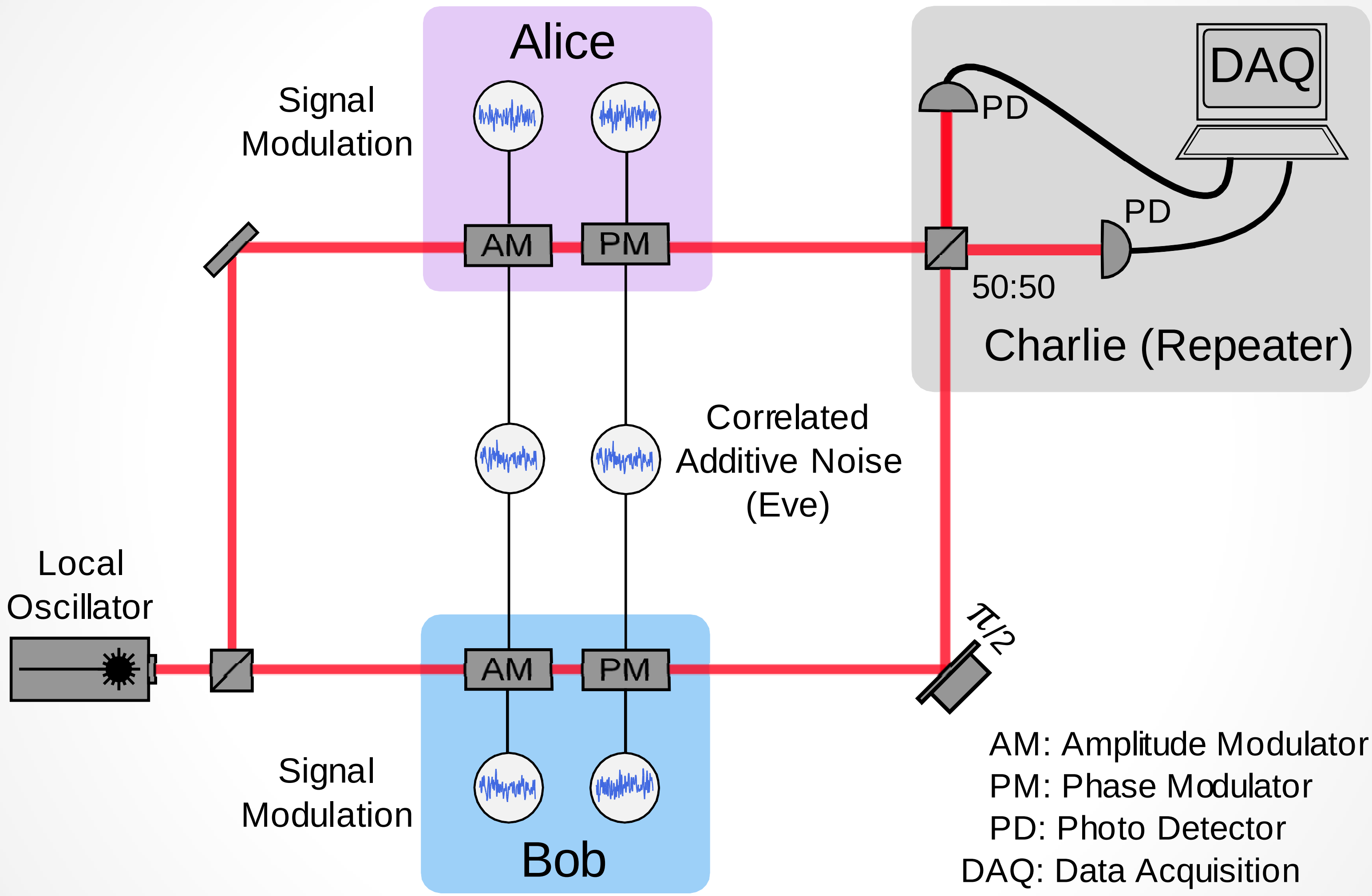



$$\mathbf{V}(n,c,c') = n \begin{pmatrix} 1 & 0 & c & 0 \\ 0 & 1 & 0 & c' \\ c & 0 & 1 & 0 \\ 0 & c' & 0 & 1 \end{pmatrix}$$

$n = 2$ corresponds to EB noise

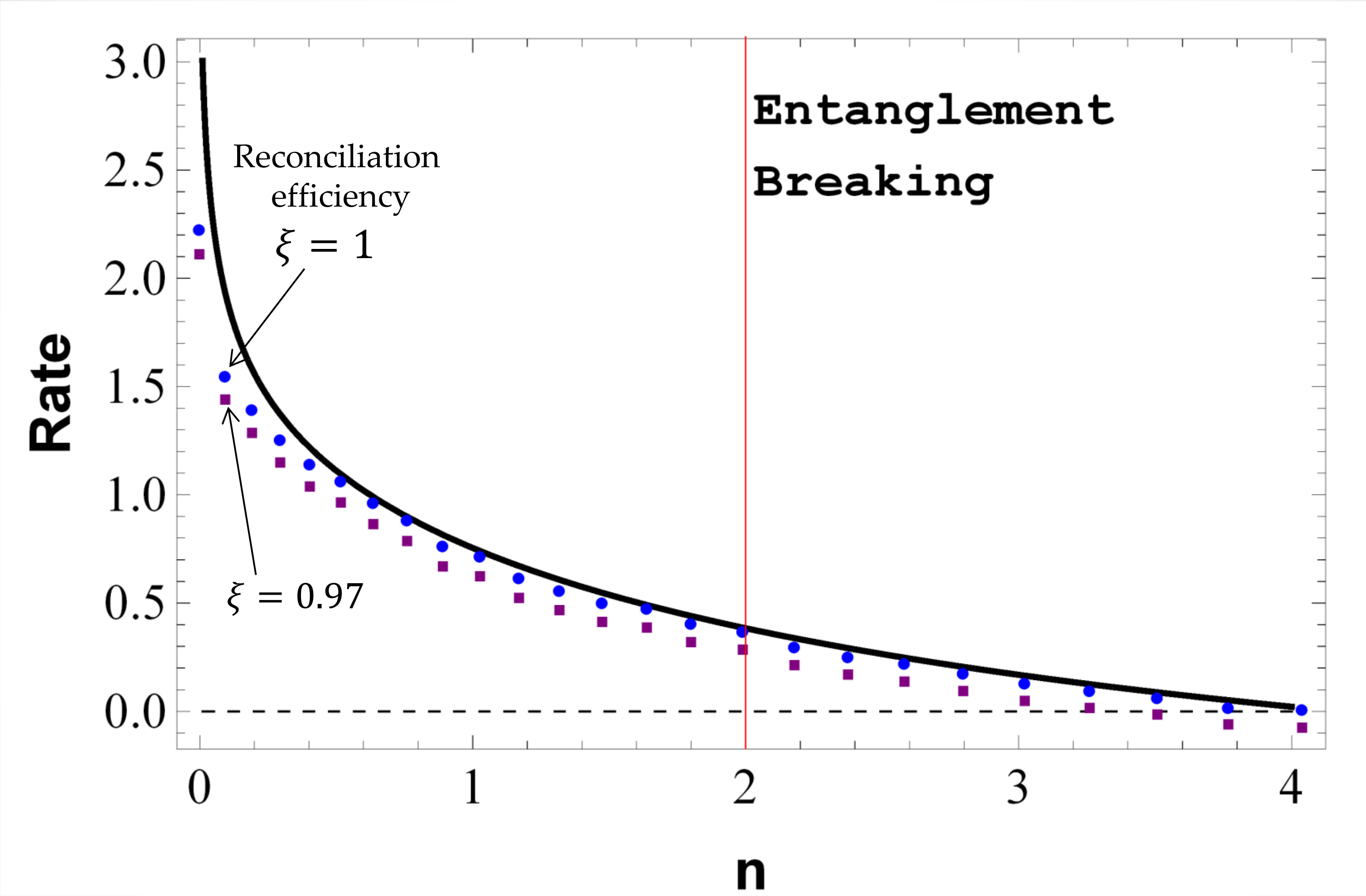


# Experimental Setup




**Two-mode (correlated) side-channel attack**

# Experimental Results




Environment with $c = c' = 1$

Signal modulation $\mu = 52$

# Conclusions

- Theoretical and experimental demonstration that repeater-based protocols can work in the presence of entanglement-breaking channels, as long as sufficient separable correlations are present in the non-Markovian environment.

- Protocols work thanks to a real (or virtual) 4-partite form of entanglement which is localized by the Bell detection into a bipartite form

- New perspectives for all quantum systems affected by correlated errors, memory effects, and non-Markovian dynamics

- This may involve both:
  - Short-distance implementations, such as chip-based quantum computing
  - Long-distance implementations (relay-based QKD, MDI-QKD)