CENTRE FOR QUANTUM COMPUTATION & COMMUNICATION TECHNOLOGY
AUSTRALIAN RESEARCH COUNCIL CENTRE OF EXCELLENCE

Enhancing Optical Quantum Channels

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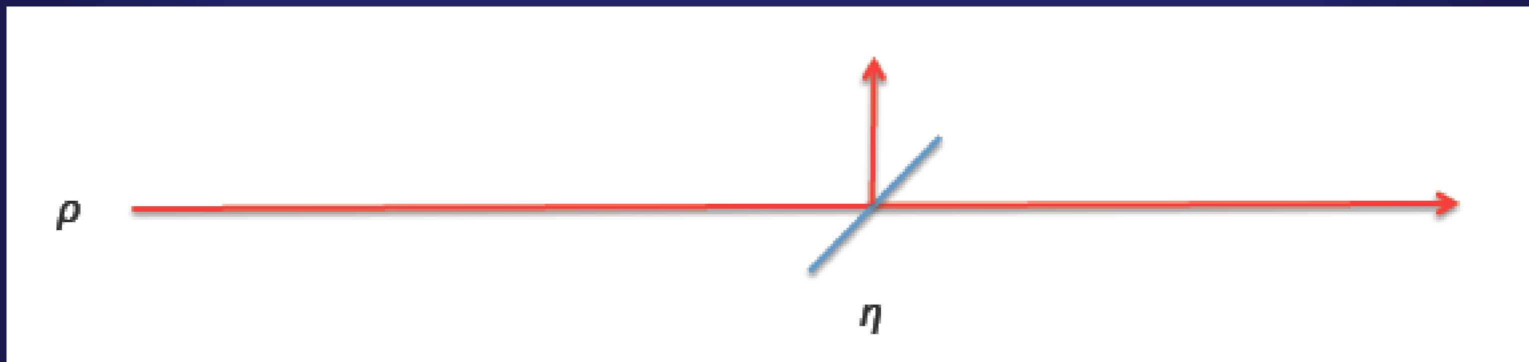
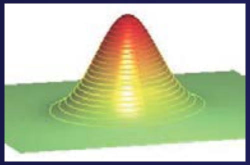
The University of Queensland Australia

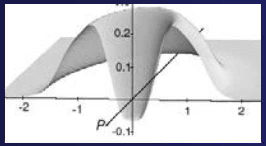
Quantum Optics Quantum Information Theory Group

Motivation – What if you could error correct the optical channel itself!

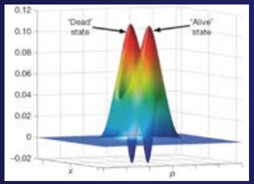




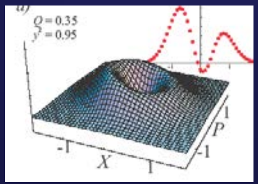
Coherent states → reduced signal to noise (more errors)



Single photon polarization qubits → erasure errors



Cat states → phase flip errors



Field qubits → amplitude damping errors

Overview

* Noiseless Linear Amplification

* Channel enhancement via NLA

* NLA via Post-selection
  - experiment
  - teleportation

* Quantum Repeater for continuous variables

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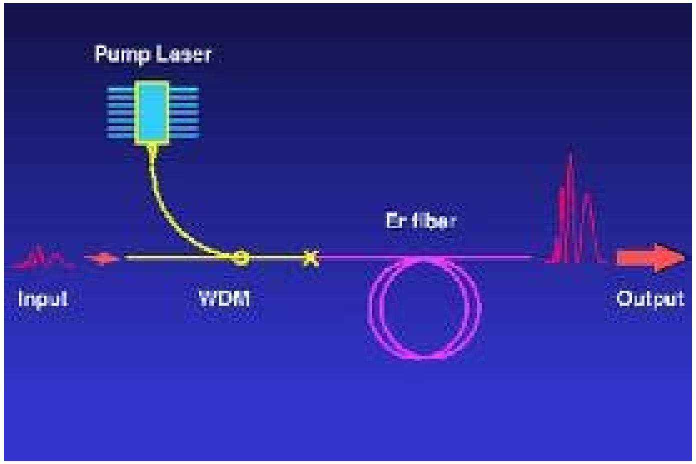
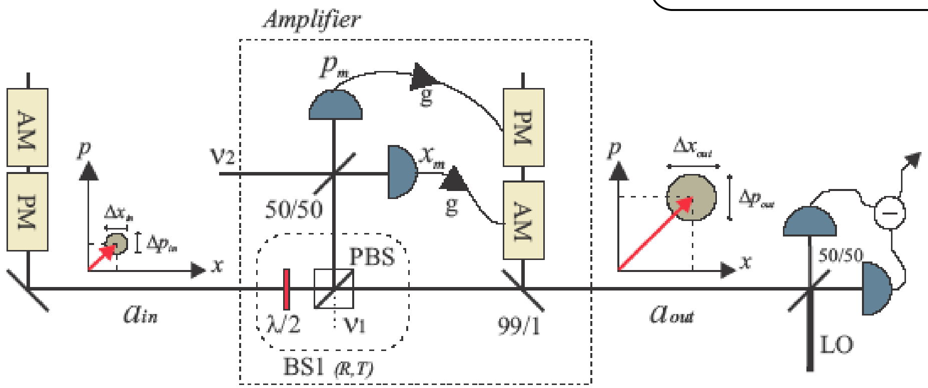
Amplifiers in Quantum Communication



$$\hat{a}_{out} = \sqrt{G}\hat{a}_{in} + \sqrt{G-1}\hat{a}^{\dagger}_{int}$$



V. Josse, M. Sabuncu, N. J. Cerf, G. Leuchs,
U. L. Andersen, Phys. Rev. Lett. 96, 163602 (2006)

C.M.Caves, Phys.Rev.D **23**, 1693 (1981).

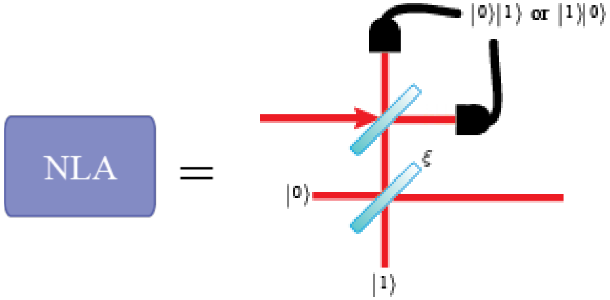
Heralded Noiseless Linear Amplifiers

$|g| > 1$

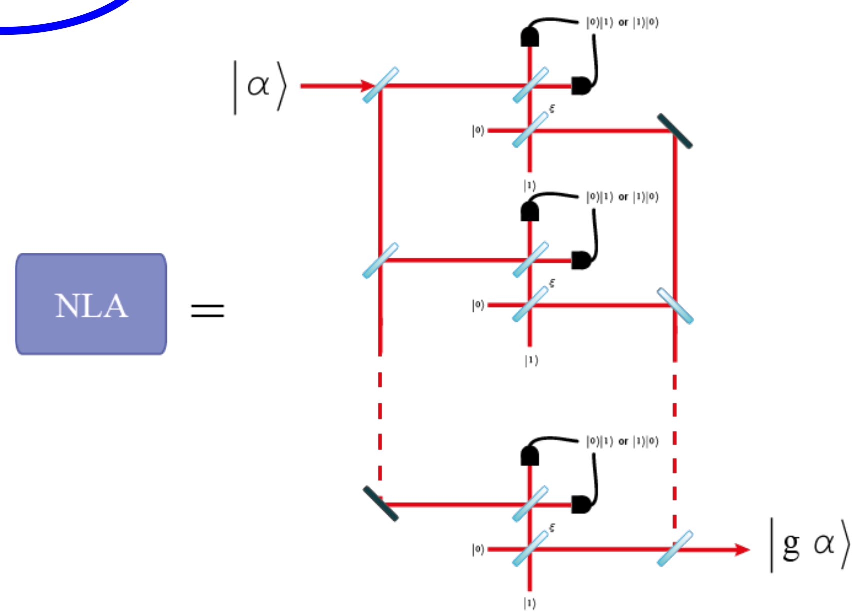
$|\alpha\rangle$ → noiseless amplifier → $e^{-(1-g^2)|\alpha|^2}\,|g\alpha\rangle$

$|n\rangle$ → $g^n|n\rangle$

$$|\alpha\rangle\langle\alpha| \rightarrow \rho(\alpha) = P|g\alpha\rangle\langle g\alpha| + (1-P)|0\rangle\langle 0|.$$



low photon number



G.Y.Xiang, T.C.Ralph, A.P.Lund, N.Walk and G.J.Pryde, *Nature Photonics* **4**, 316 (2010).

Experiments

* G.Y.Xiang, T.C.Ralph, A.P.Lund, N.Walk and G.J.Pryde,
  Nature Photonics **4**, 316 (2010)

* F. Ferreyrol et al.,
  Phys Rev Lett **104**, 123603 (2010)

* A. Zavatta, J. Fiurásek, M. Bellini,
  Nature Photonics **5**, 52 (2011)

* N.Bruno, et al, New Journal of Physics **15**, 093002 (2013)

* S. Kocsis, G. Y. Xiang, T. C. Ralph and G. J. Pryde,
  Nature Physics **9**, 23 (2013).

*A.E.Ulanov, I.A.Fedorov, A.A.Pushkina, Y.V.Kurochkin,
T.C.Ralph and A. I. Lvovsky, arXiv:1504.00886

Overview

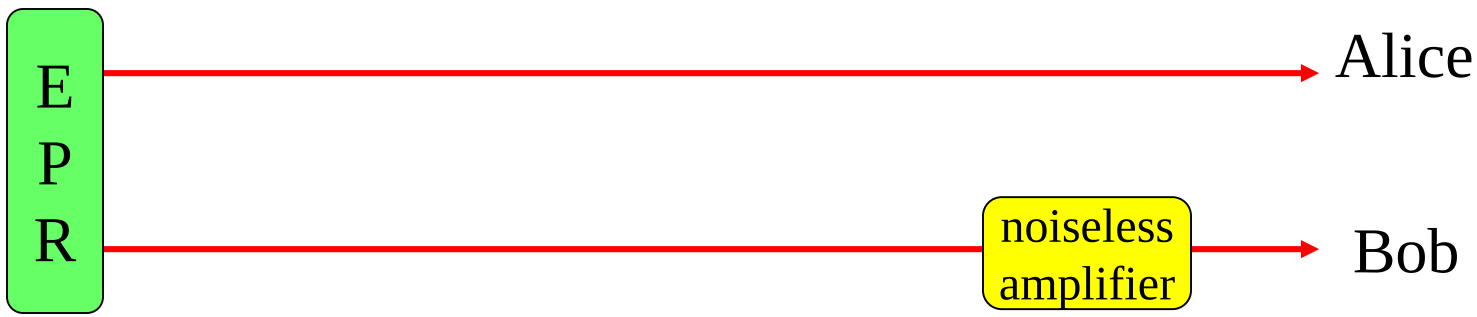
* Noiseless Linear Amplification

* Channel enhancement via NLA

* NLA via Post-selection
    -experiment
    - teleportation

* Quantum Repeater for continuous variables

Distillation via Noiseless Amplification



$$|EPR\rangle = \sqrt{1-\chi^2}\;\Sigma_{n=0}^{\infty}\chi^n|n\rangle|n\rangle$$

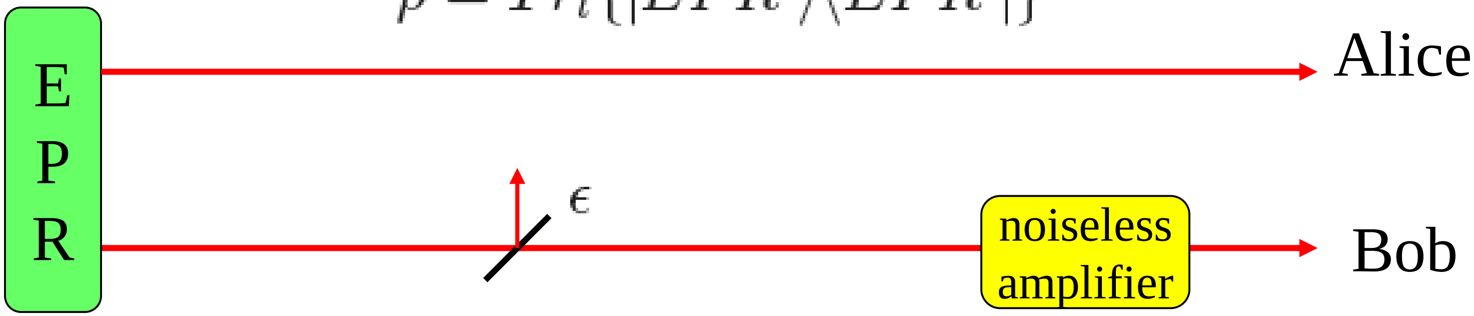
$$|EPR\rangle \rightarrow \sqrt{1-\chi^2}\;\Sigma_{n=0}^{\infty}\chi^n g^n|n\rangle|n\rangle$$

$|g|>1$ entanglement increased $\chi' = g\chi$

T.C.Ralph and A.P. Lund, QCMC 9th Proceedings,155 (2009).

Distillation via Noiseless Amplification

$$\rho = Tr_l\{|EPR'\rangle\langle EPR'|\}$$



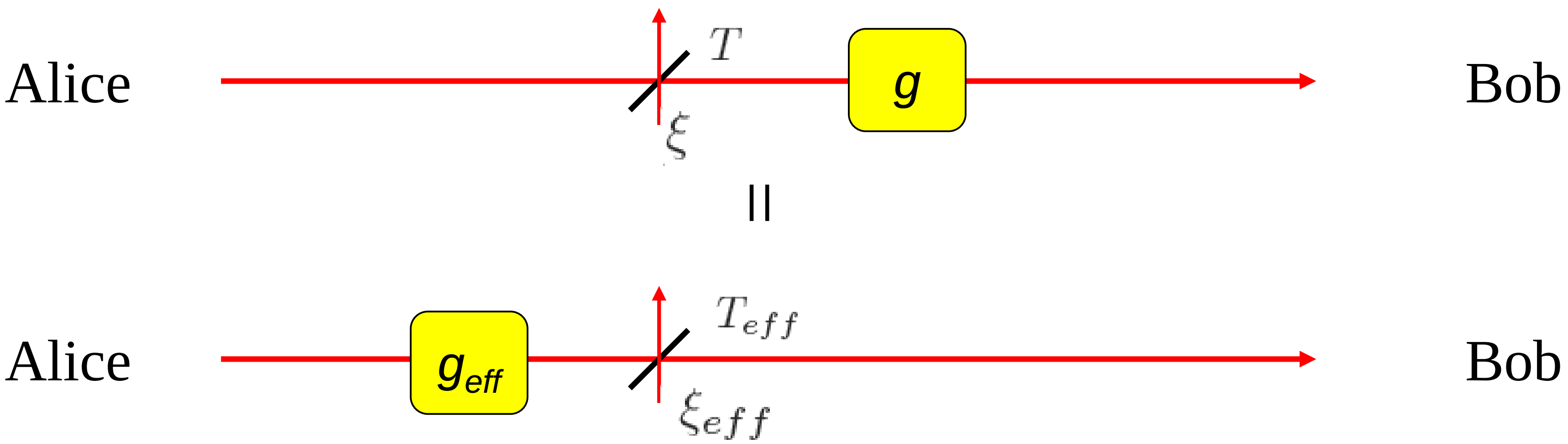
$$|EPR'\rangle = \sqrt{1-\chi^2}\,\Sigma_{n=0}^{\infty}\chi^n \times \Sigma_{k=0}^{n}\sqrt{\binom{n}{k}}(1-\epsilon)^{k/2}\epsilon^{(n-k)/2}|n-k\rangle|n\rangle|k\rangle_l$$

$$|EPR'\rangle \rightarrow \sqrt{1-\chi^2}\,\Sigma_{n=0}^{\infty}\chi'^n \times \Sigma_{k=0}^{n}\sqrt{\binom{n}{k}}(1-\epsilon')^{k/2}\epsilon'^{(n-k)/2}|n-k\rangle|n\rangle|k\rangle_l$$

$$\chi' = \chi\sqrt{1+(g^2-1)\epsilon} \qquad \epsilon' = \frac{g^2\epsilon}{1+(g^2-1)\epsilon}$$

Entanglement and transmission increased

Effective Channel with Noiseless Amplification



$$g_{eff} = \sqrt{1 + \frac{2T(1-g))}{gT\xi - T\xi - 2)}}$$

$$T_{eff} = \frac{4gT}{(-2+(-1+g)T(-2+\xi))(-2+(-1+g)T\xi)}$$

$$\xi_{eff} = \xi - \xi T(\xi - 2)(g-1)/2$$

R. Blandino, M. Barbieri, P. Grangier, R. Tualle-Brouri, arXiv:1407.6798 (2014)

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NLA via Post-selection

$$\rho \rightarrow g^{\hat{n}} \rho g^{\hat{n}} \quad g^{\hat{n}}|\alpha\rangle = e^{\frac{1}{2}(g^2-1)|\alpha|^2}|g\alpha\rangle \quad Q_\rho(\alpha) = \langle\alpha|\rho|\alpha\rangle$$

$$Q_{\rho'}(\alpha) = \langle\alpha|g^{\hat{n}}\rho g^{\hat{n}}|\alpha\rangle$$

heterodyne detection after NLA

$$= e^{(g^2-1)|\alpha|^2}\langle g\alpha|\rho|g\alpha\rangle$$

$$= e^{(1-1/g^2)|\beta|^2}\langle\beta|\rho|\beta\rangle$$

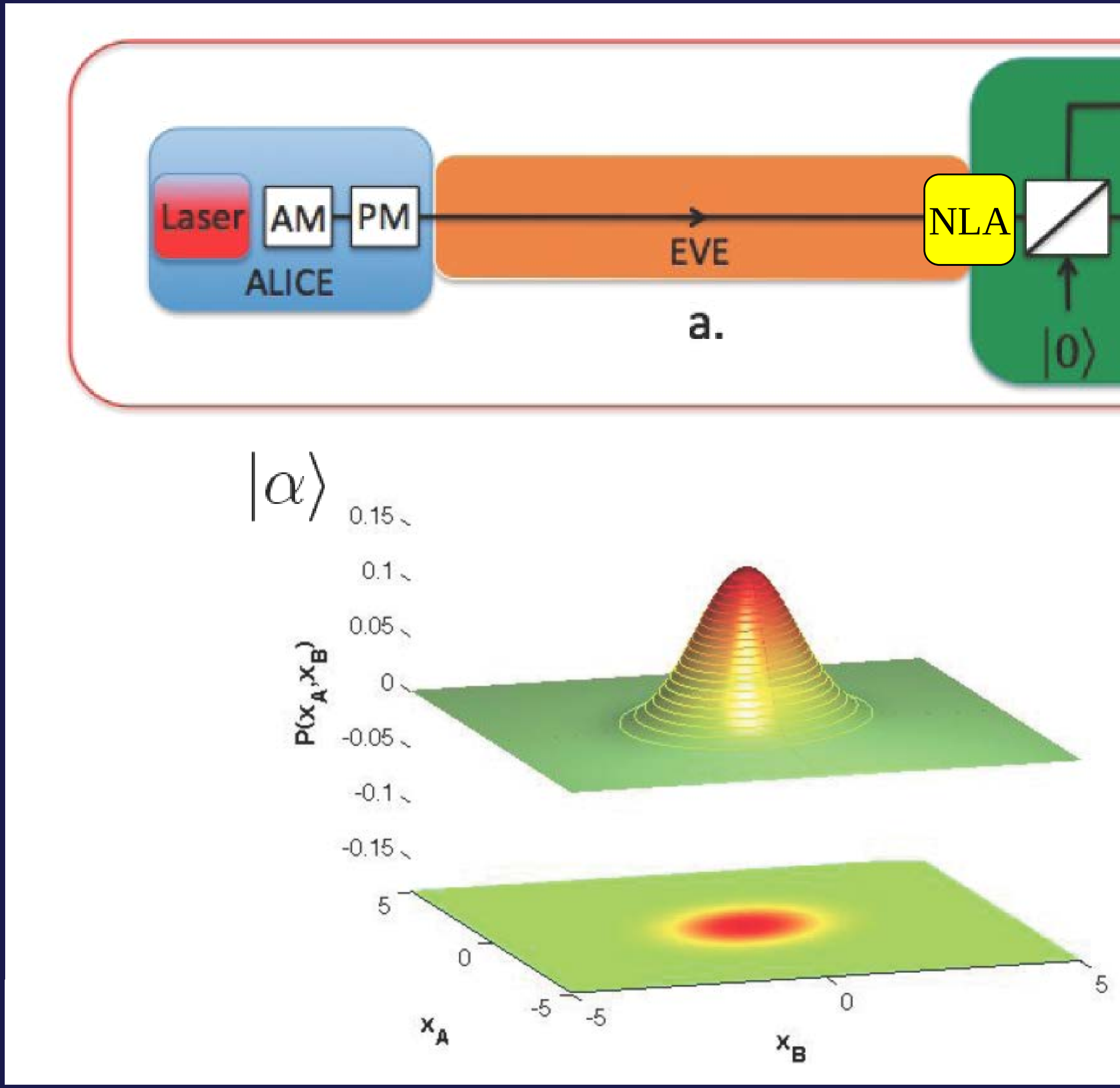
heterodyne detection with post-selection

$$P(\alpha) = \begin{cases} e^{\frac{1}{2}(|\alpha|^2-|\alpha_C|^2)(1-g^{-2})}, & \alpha < \alpha_C \\ 1, & \alpha \geq \alpha_C \end{cases}$$

post-selection with a cut-off

Fiurasek and Cerf, Physical Review A **86**, 060302(R) (2012).

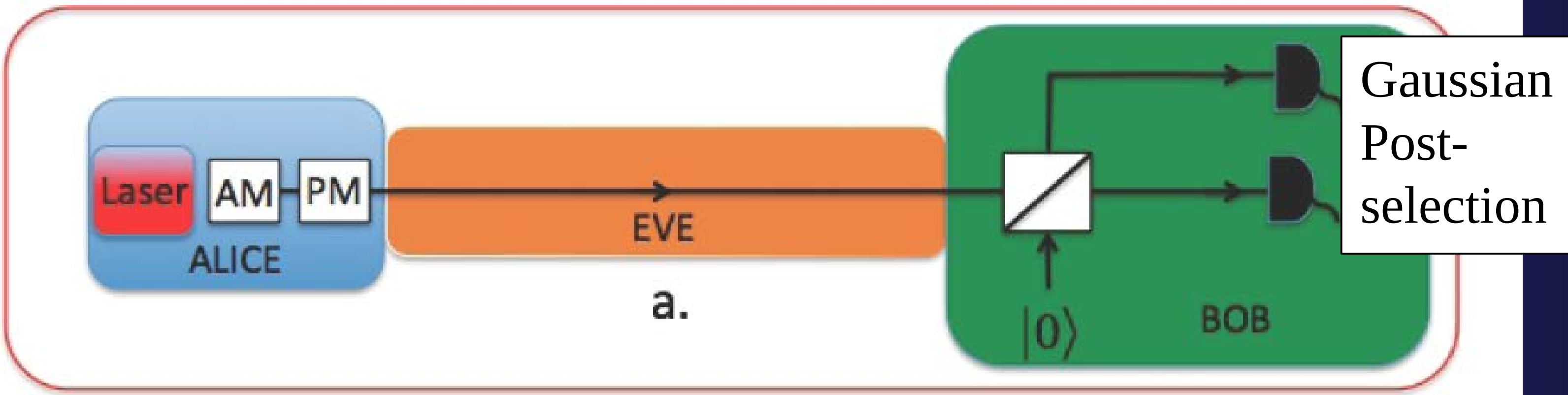
Continuous variable quantum key distribution

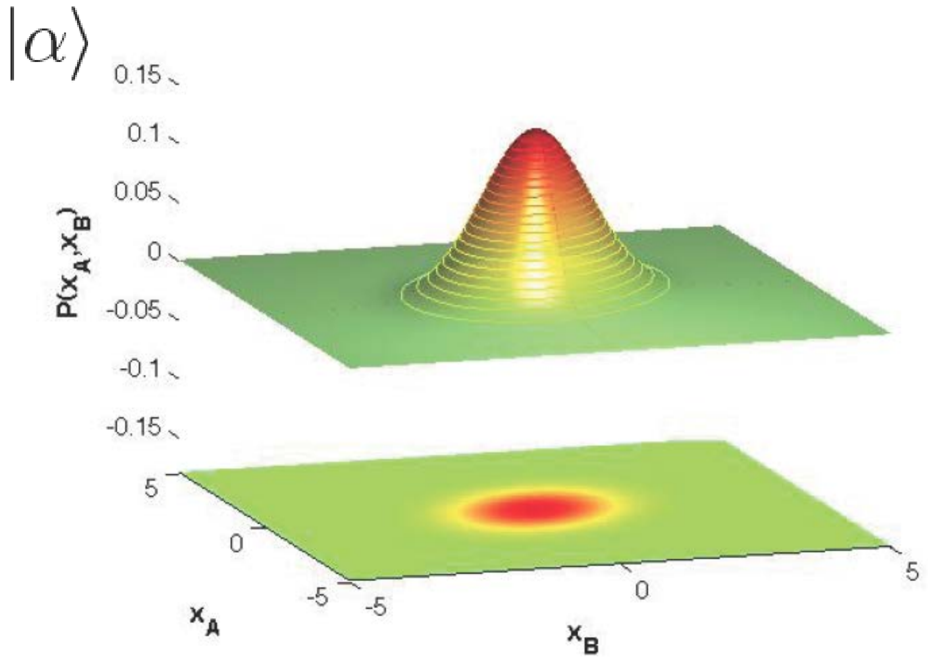


homodyne
or
heterodyne
detection

R.Blandino, et al, Phys. Rev. A **86**, 012327 (2012).

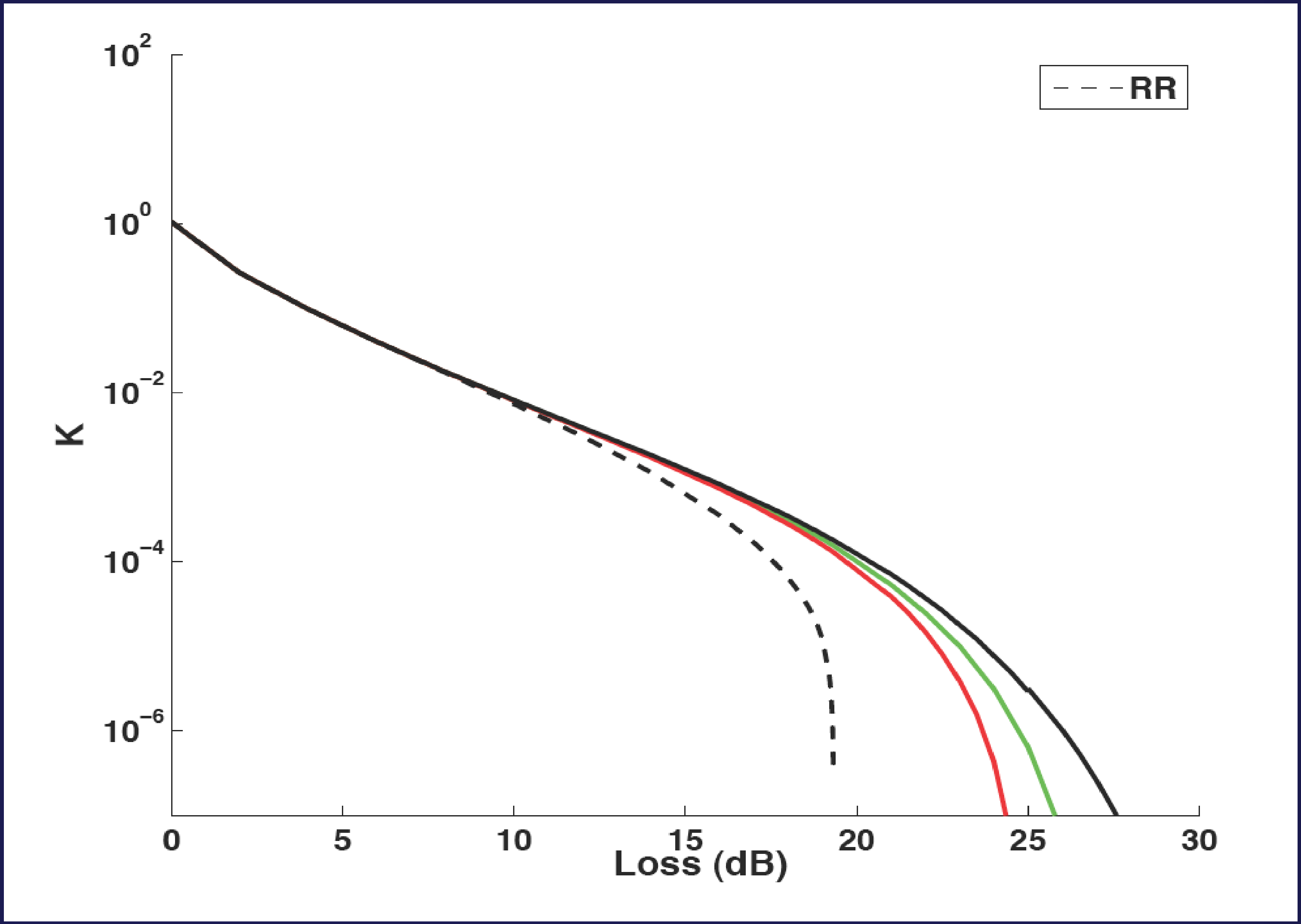
Continuous variable quantum key distribution





homodyne
or
heterodyne
detection

Improved CV QKD



Walk, Ralph, Symul, Lam, Phys Rev A **87**, 020303(R) (2013).

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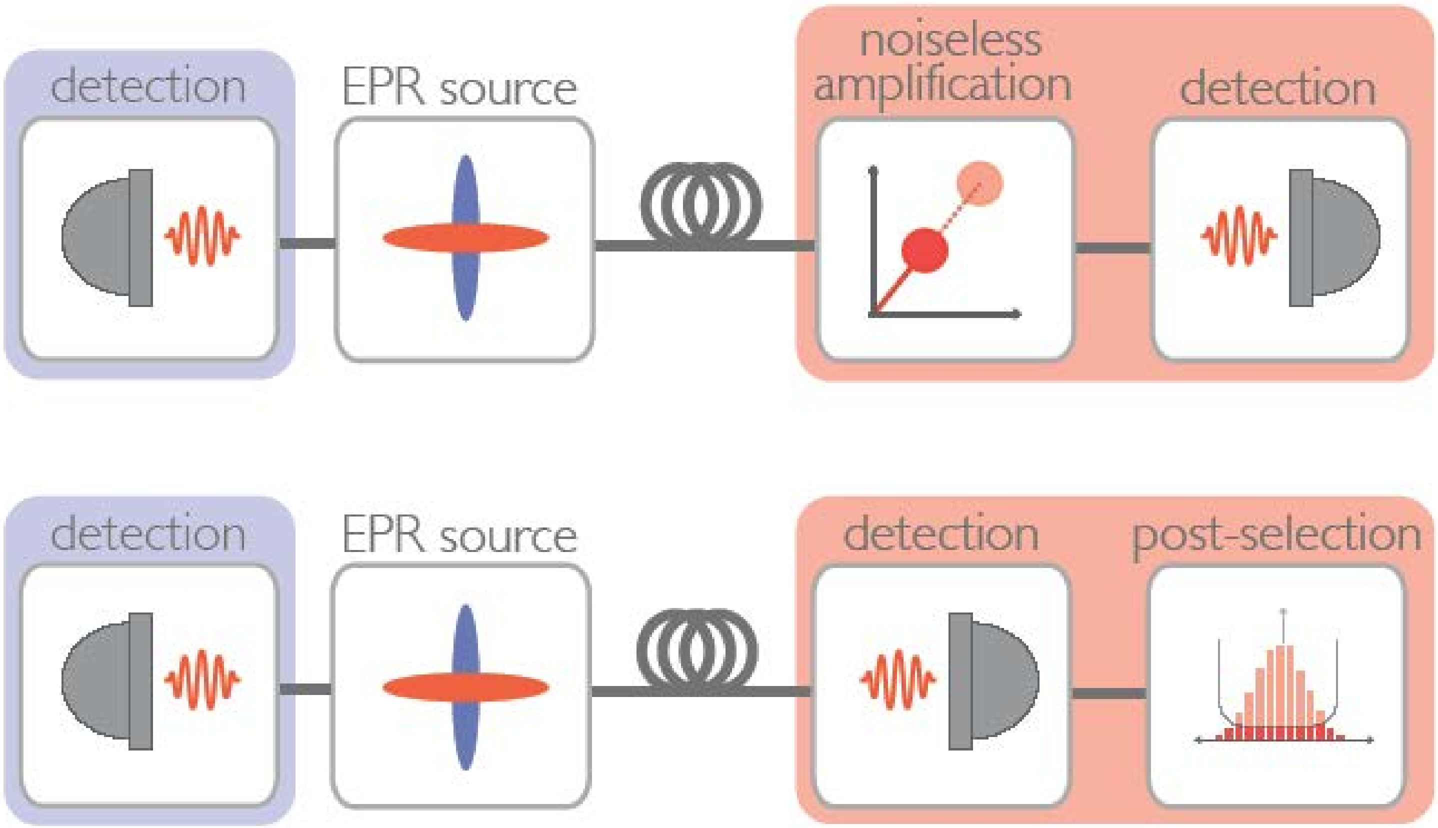
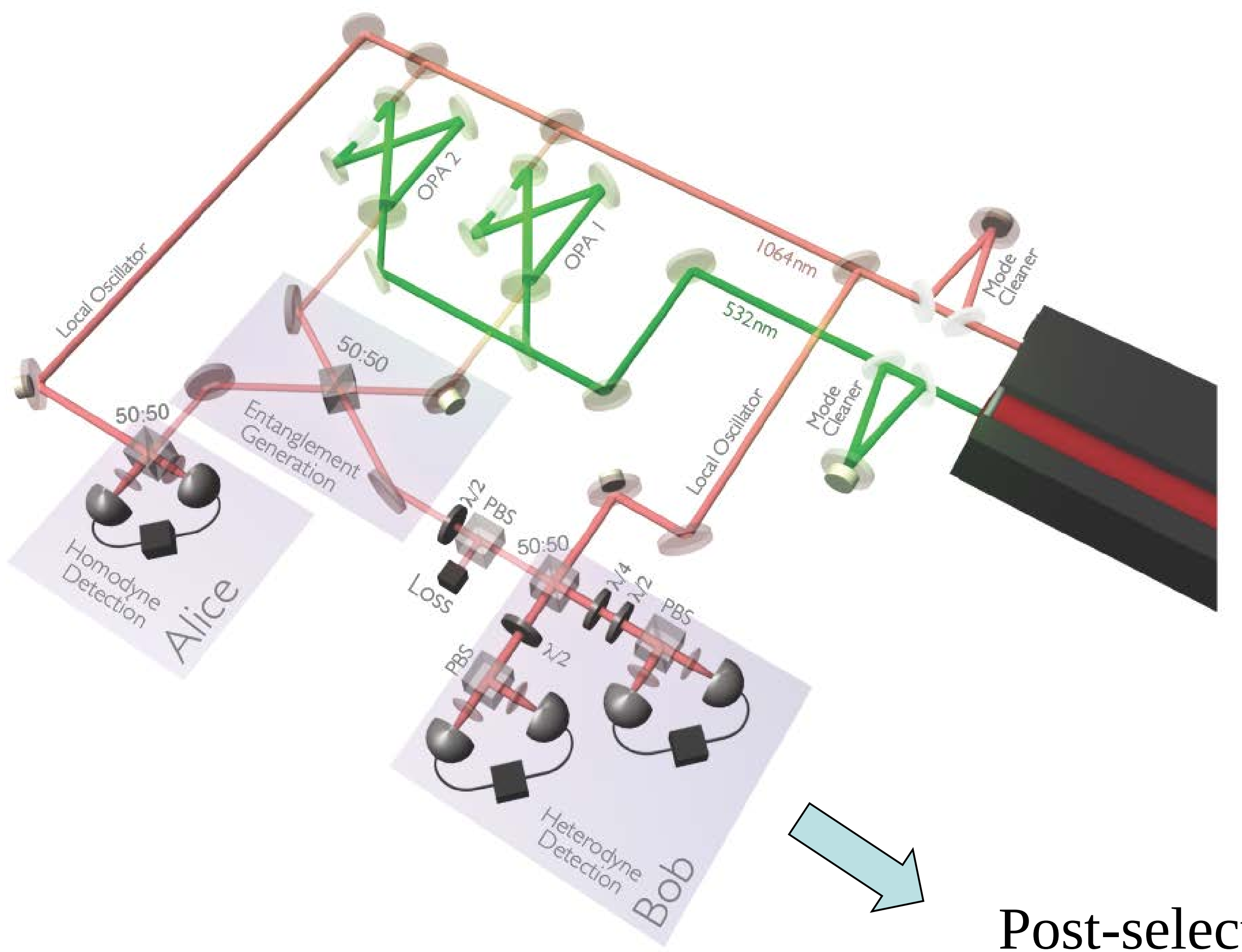
Measurement based entanglement distillation



Helen Chrzanowski, et al, *Nature Photonics*, **8**, 333 (2014)

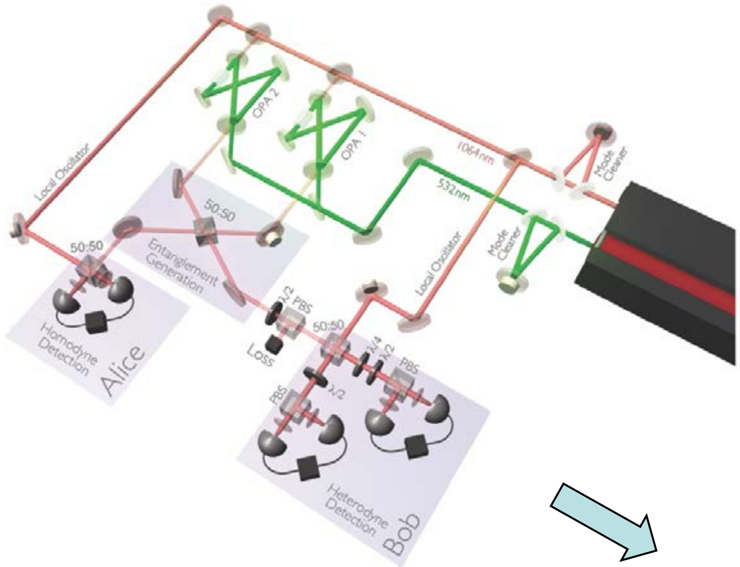
Measurement based entanglement distillation



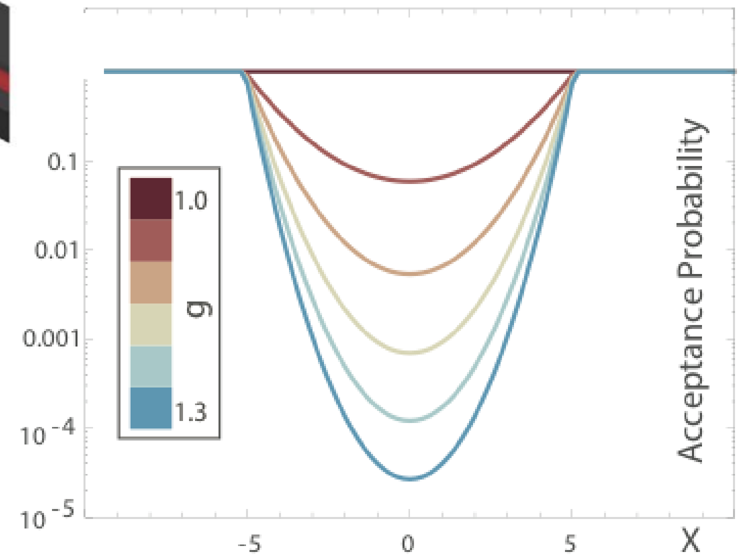
Post-select

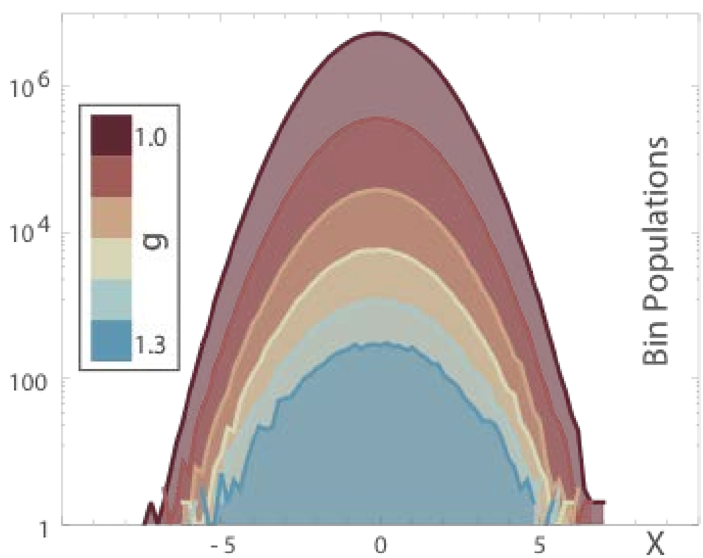
Helen Chrzanowski, et al, *Nature Photonics*, **8**, 333 (2014)

Measurement based entanglement distillation



Gaussian Post-selection





New distribution
~ Gaussian with
larger variance

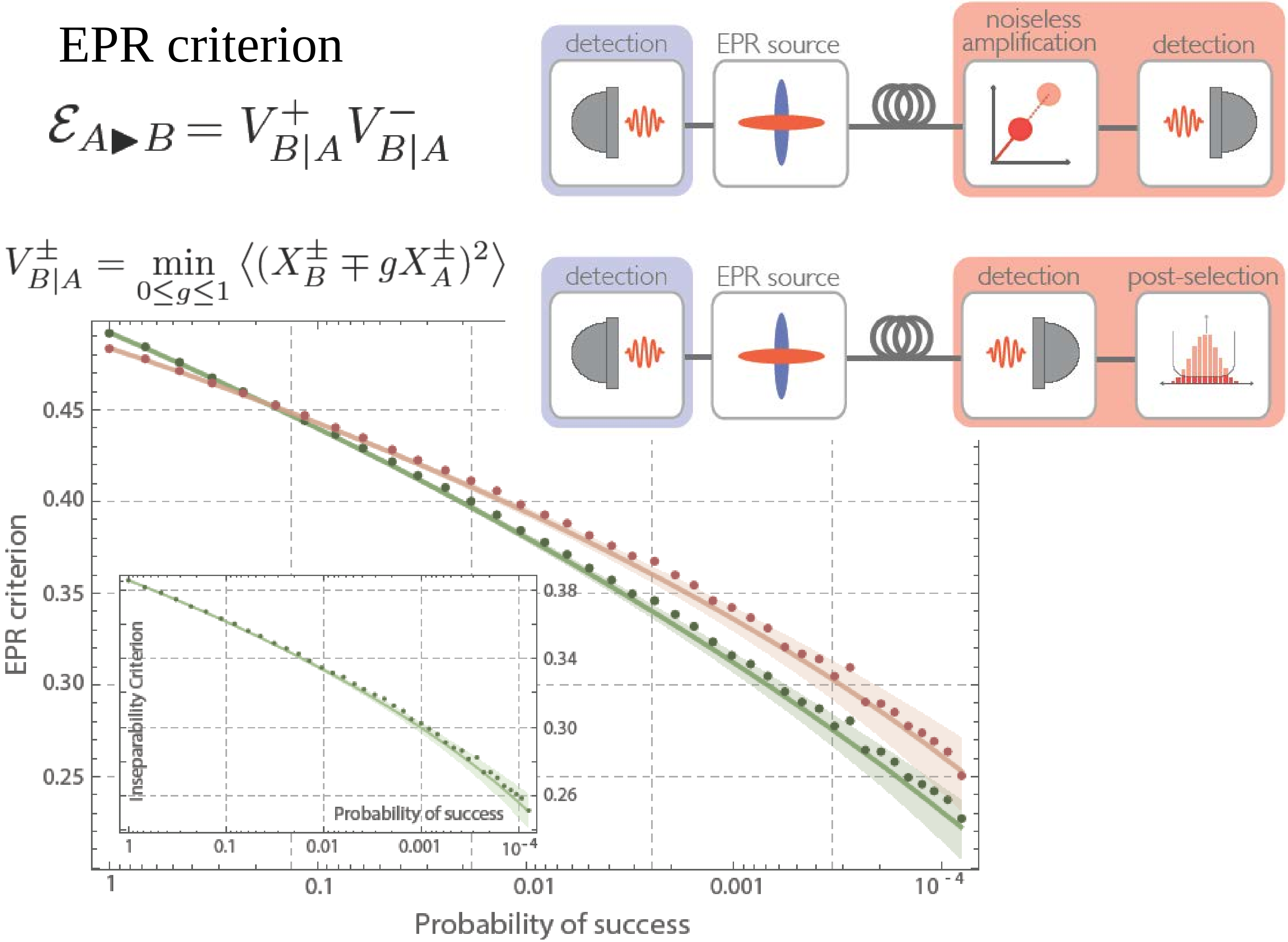
Helen Chrzanowski, et al, *Nature Photonics*, **8**, 333 (2014)

Measurement based entanglement distillation

EPR criterion

$$\mathcal{E}_{A\blacktriangleright B} = V^{+}_{B|A} V^{-}_{B|A}$$

$$V^{\pm}_{B|A} = \min_{0\le g\le 1} \langle (X^{\pm}_{B} \mp g X^{\pm}_{A})^2 \rangle$$



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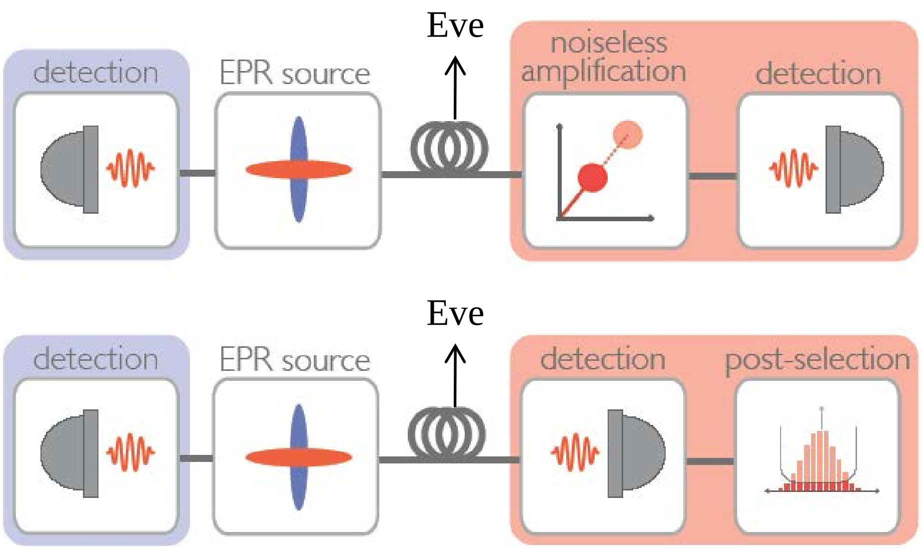
Measurement based entanglement distillation



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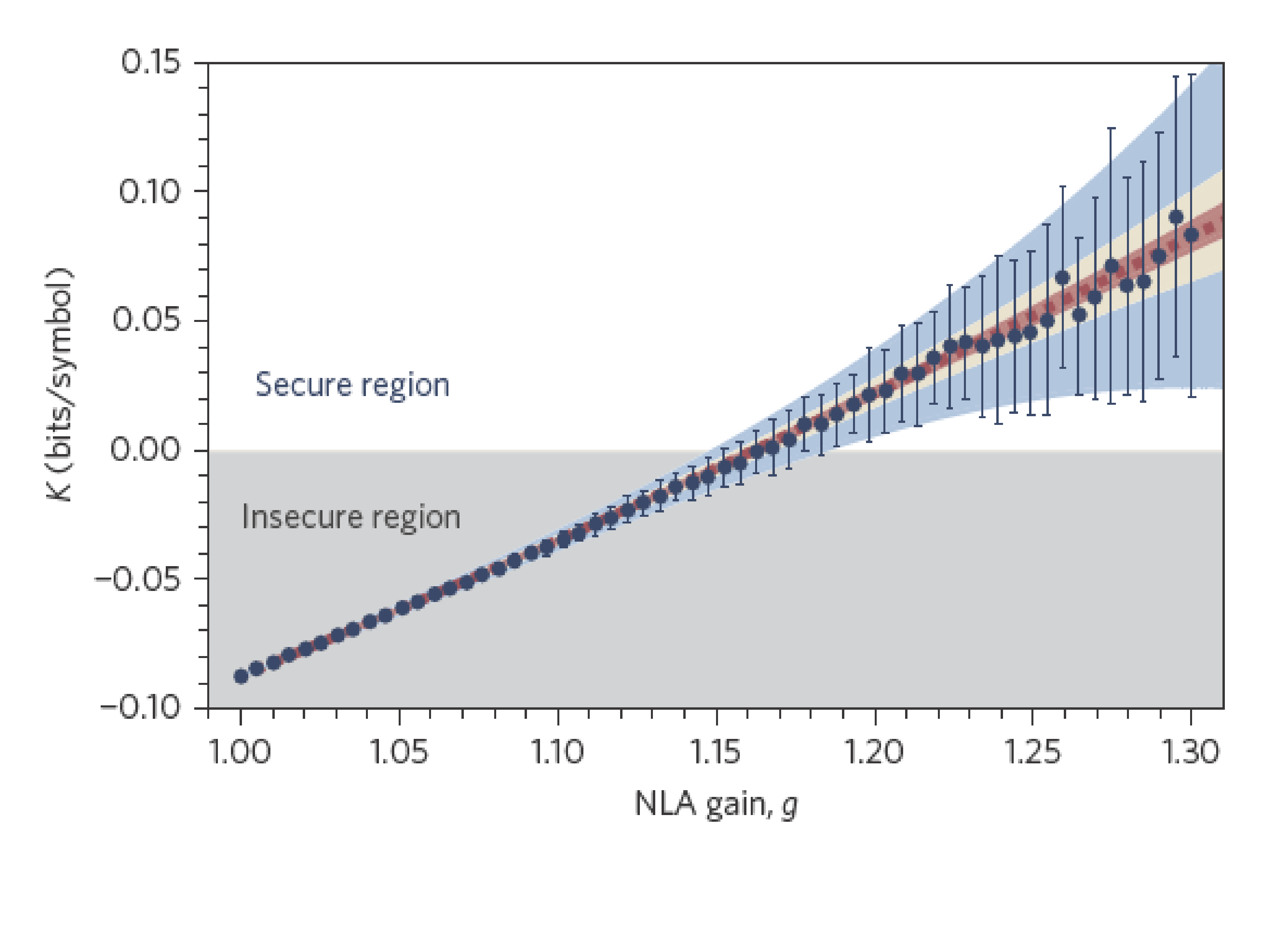

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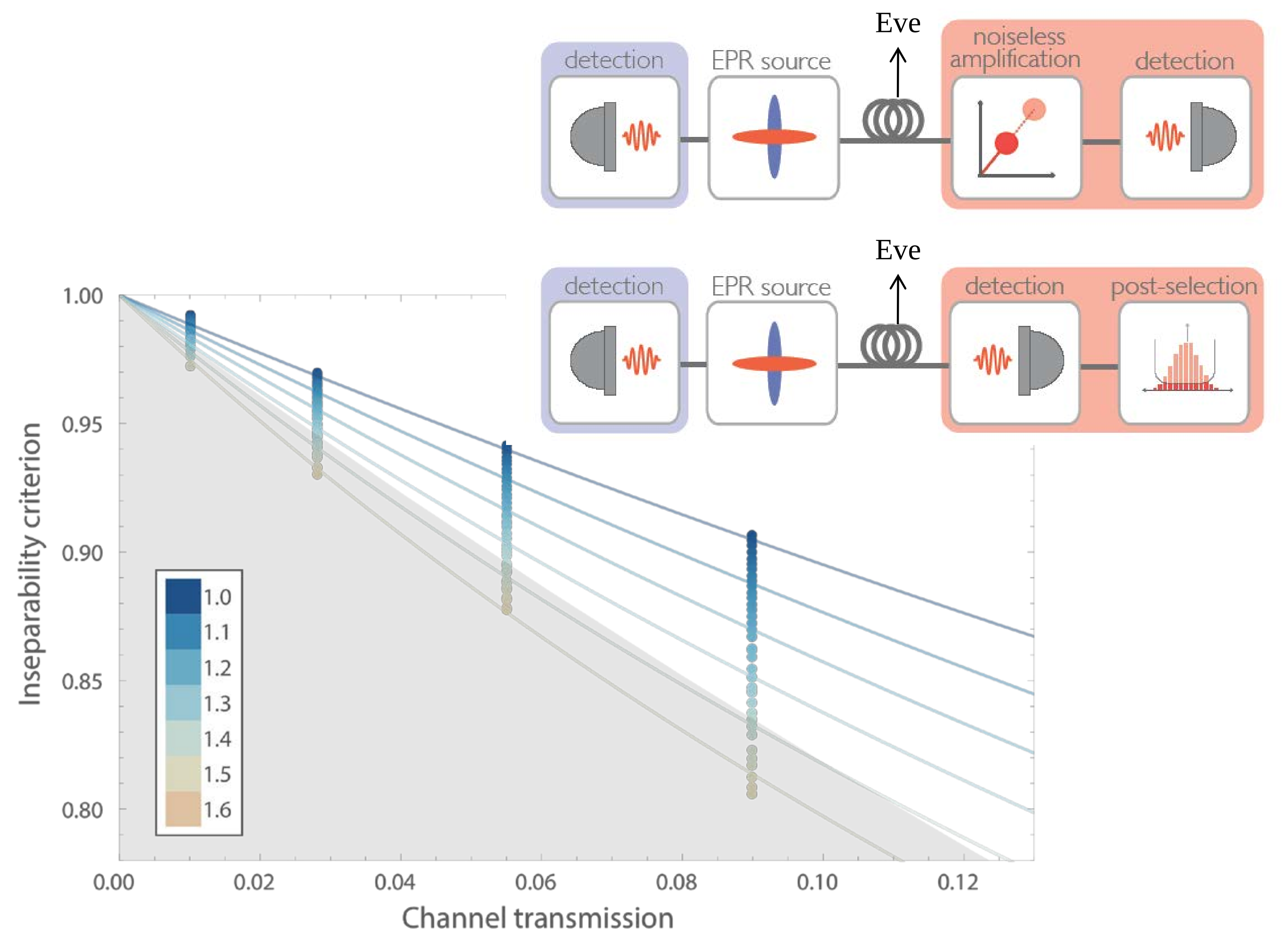

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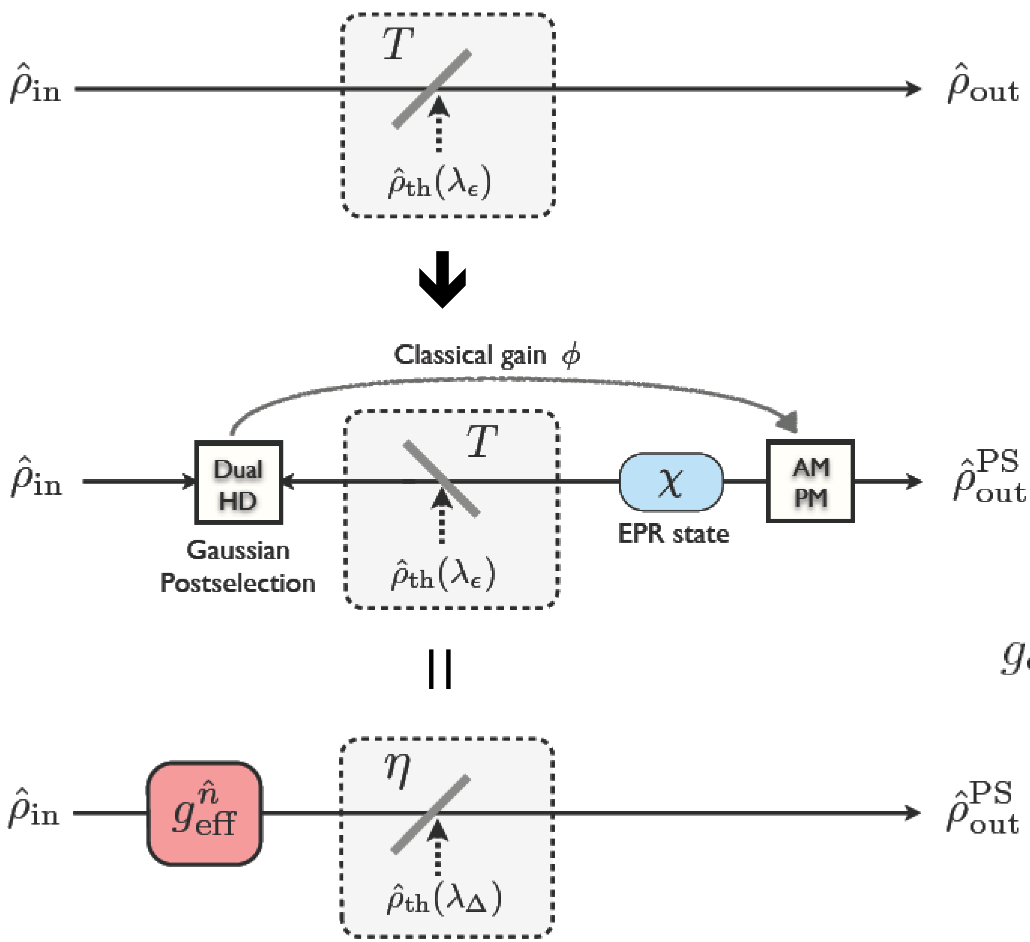
Continuous variable teleportation with Post-selection



$$\eta = 1$$

$$\Delta_{g=1} = \frac{2(1-\sqrt{T}\chi)^2}{1-\chi^2}$$

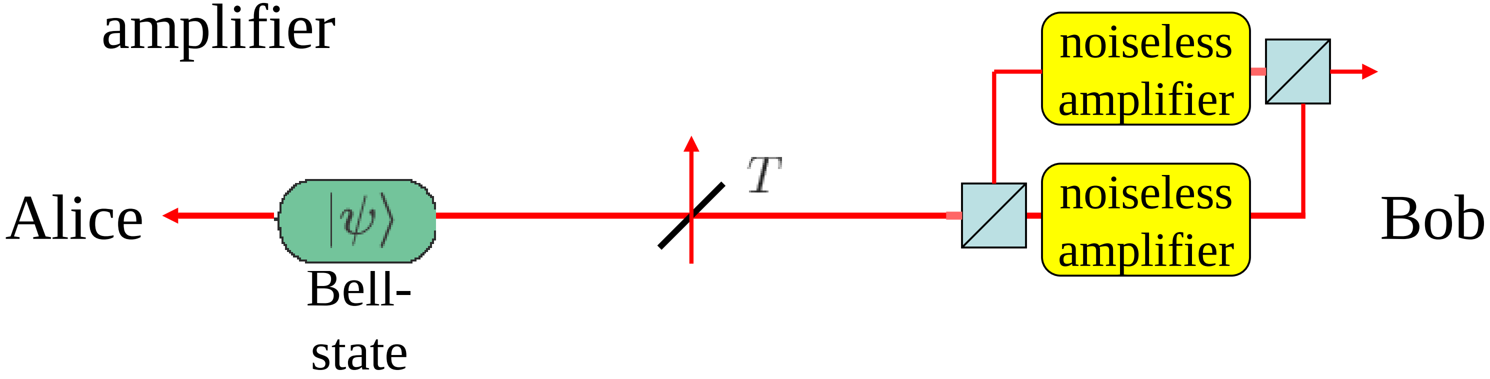
$$g_{\text{eff}} = \frac{\sqrt{T}\chi(1-\sqrt{T}\chi)}{1-\sqrt{T}\chi+(T-1)\chi^2}$$

R.Blandino, A.Lund, N.Walk, T.C.Ralph, arXiv:1408.6018 (2014)

heralded correction of qubit loss

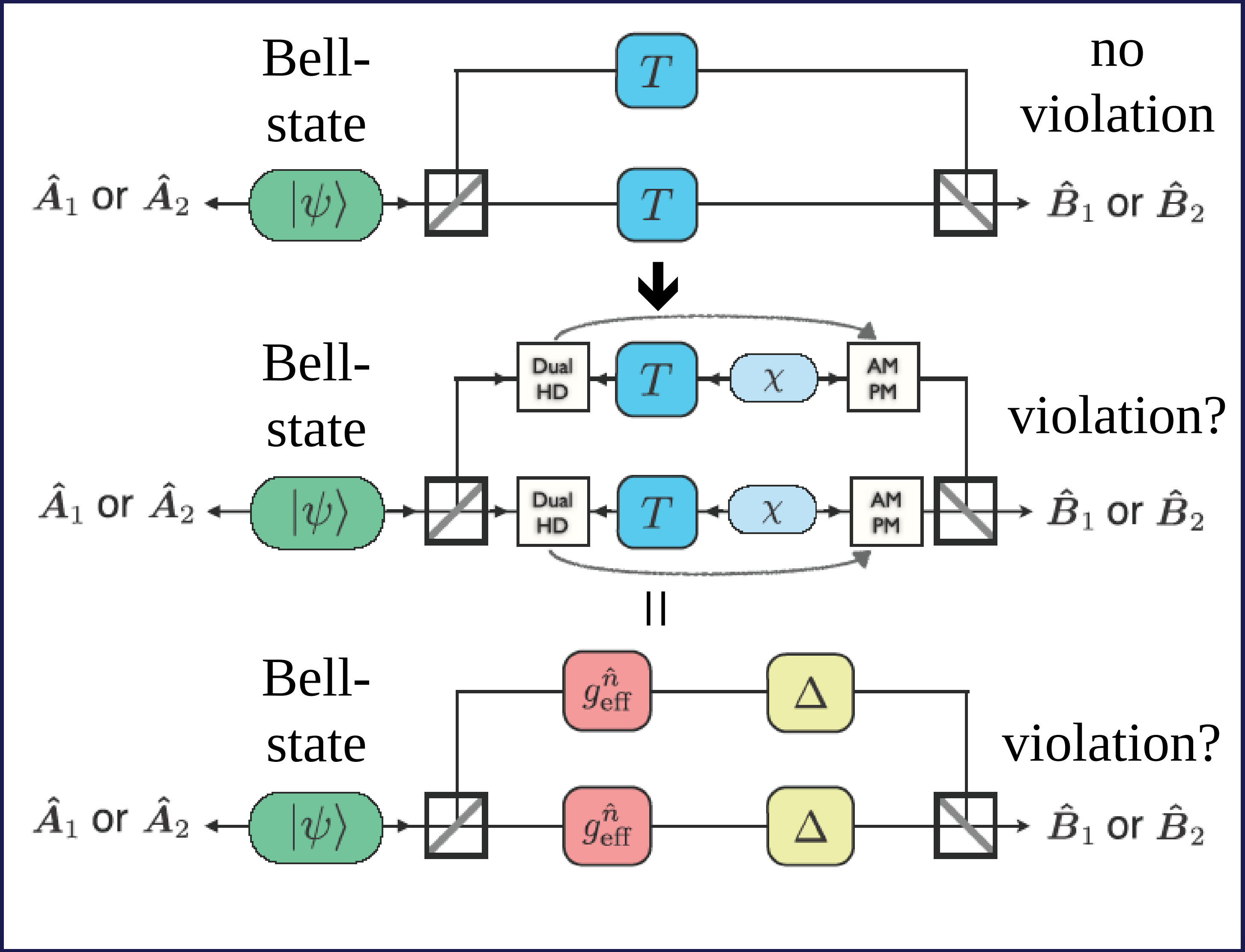
N.Gisin et al, PRL **105**, 070501 (2010)

qubit
amplifier



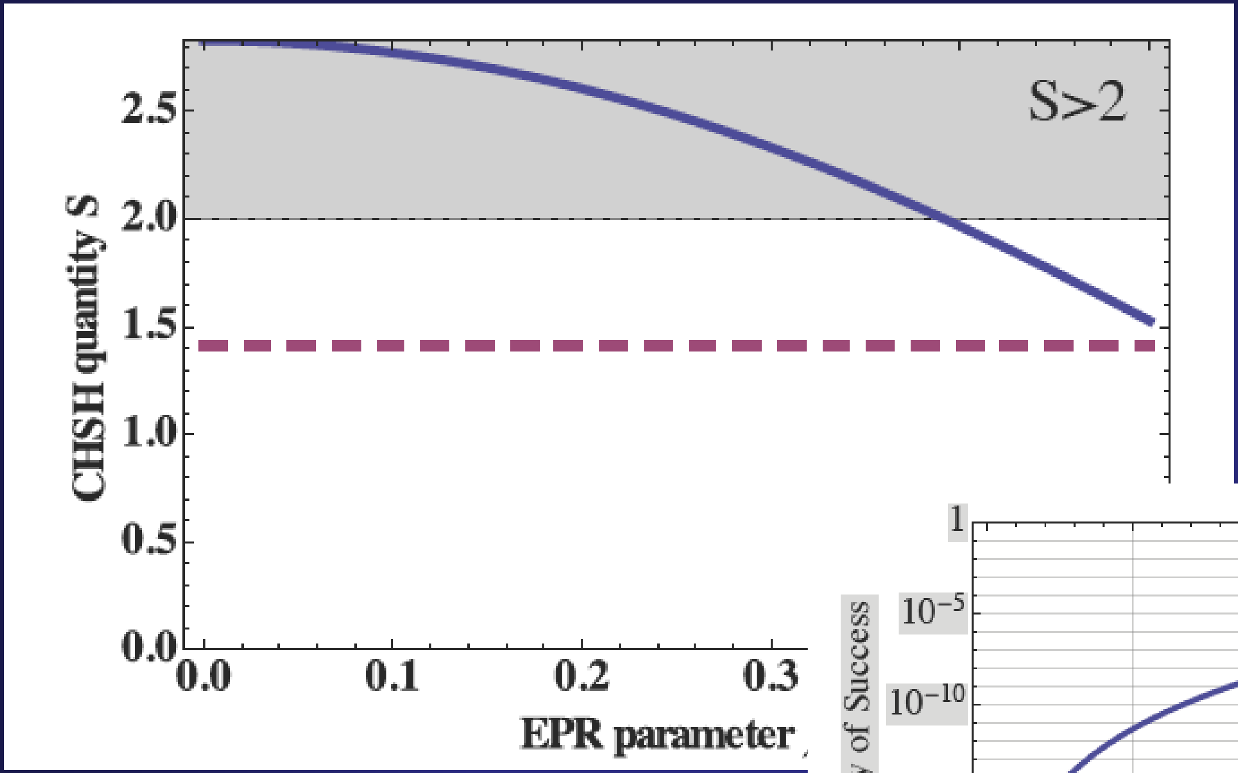
* Loop-hole free Bell inequalities
* Devise independent QKD
* Repeaters
* discrete variable error correction of loss

Violating a loop-hole free Bell inequality

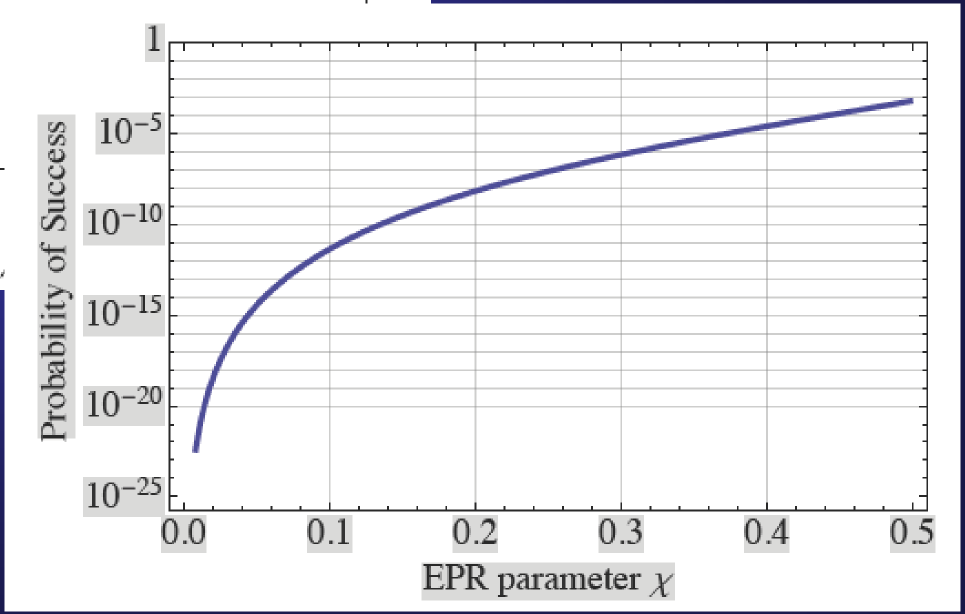


R.Blandino, A.Lund, N.Walk, T.C.Ralph, arXiv:1408.6018 (2014)

Violating a loop-hole free Bell inequality



T = 0.5



R.Blandino, A.Lund, N.Walk, T.C.Ralph, arXiv:1408.6018 (2014)

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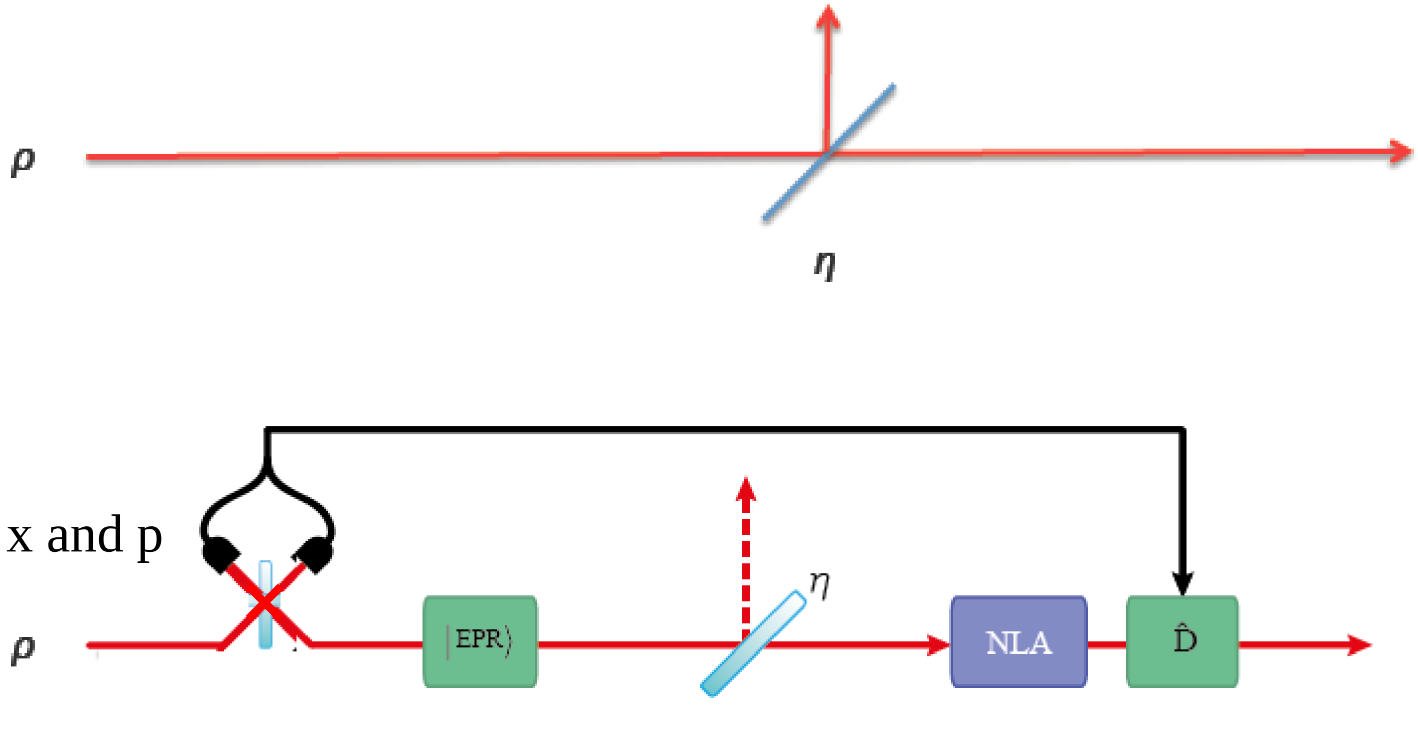
CV error correction of loss -

T.C.Ralph, Phys. Rev. A **84**, 022339 (2011).

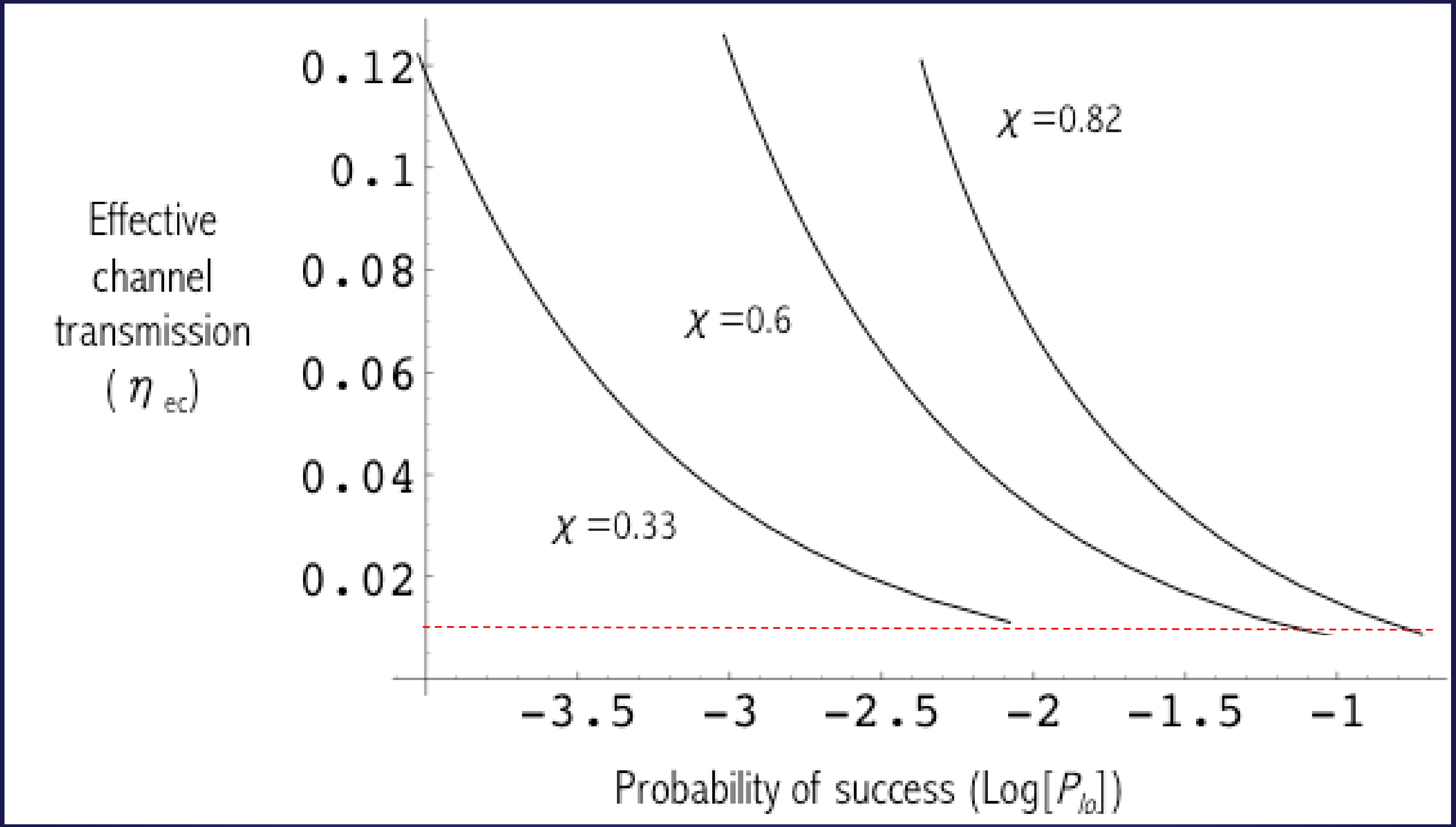


dual homodyne detection

$$\eta_{ec} = G\eta(\frac{V-1}{V+1})^2 = G\eta\chi^2$$

amplitude and phase modulation

Probability vs transmission for linear optics implementation



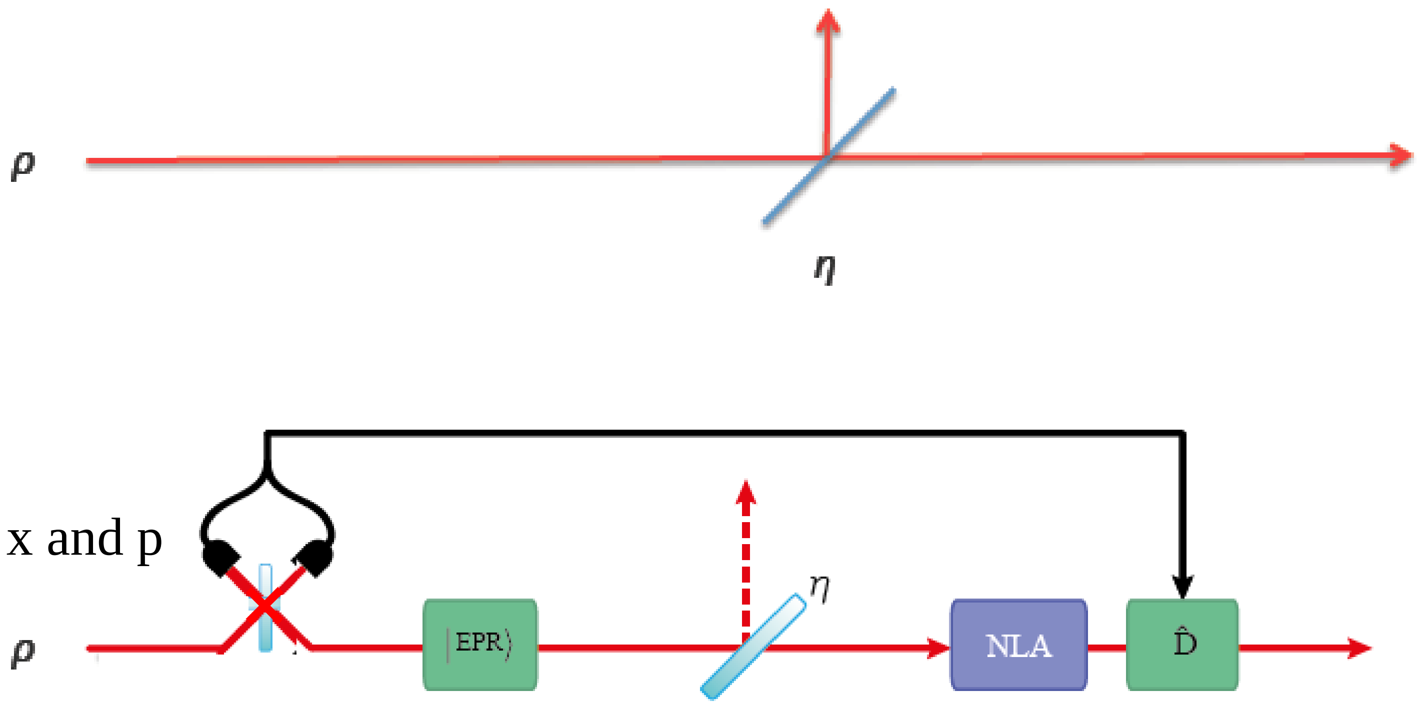
$$\eta_{ec} = G\eta(\frac{V-1}{V+1})^2 = G\eta\chi^2$$

$$P_{lo} = \frac{\xi}{(1+G)^N}$$

T.C.Ralph, Phys. Rev. A **84**, 022339 (2011).

CV error correction of loss -

T.C.Ralph, Phys. Rev. A **84**, 022339 (2011).



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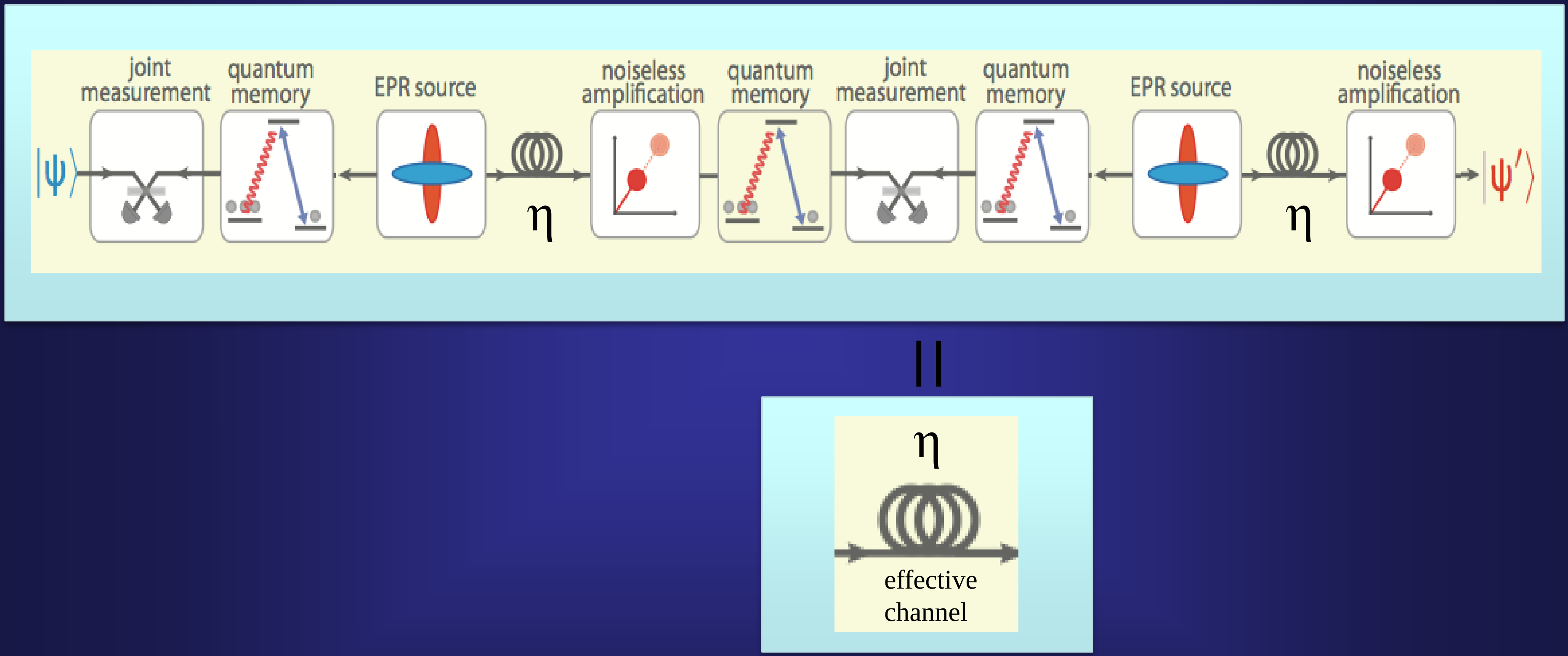
The Quantum Repeater Model

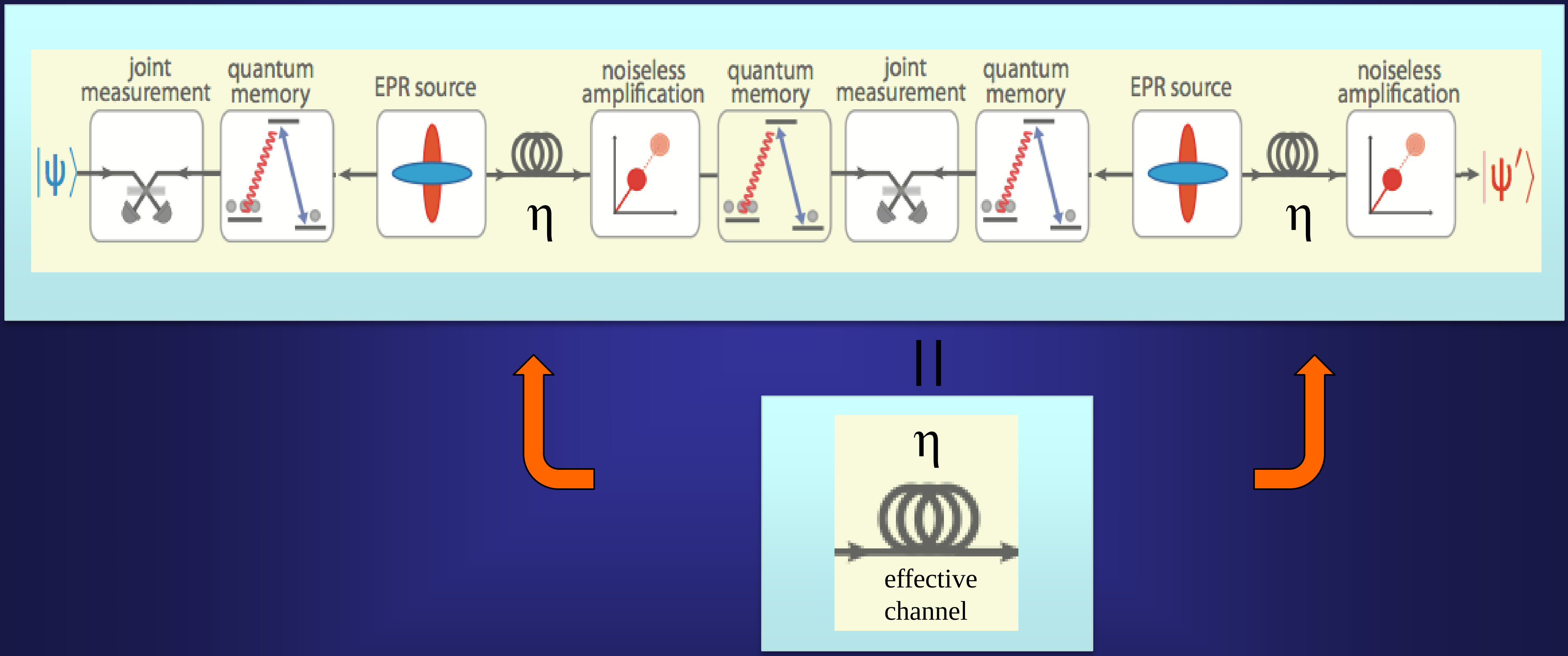


If we choose $g = \frac{1}{\eta^{1/4}\chi}$, then $|g\sqrt{\eta}\chi\alpha\rangle = |\eta^{1/4}\alpha\rangle$ and we have

$$\eta \rightarrow \sqrt{\eta}$$

T. Ralph, Physical Review. A 84 (2011).

The Quantum Repeater Model



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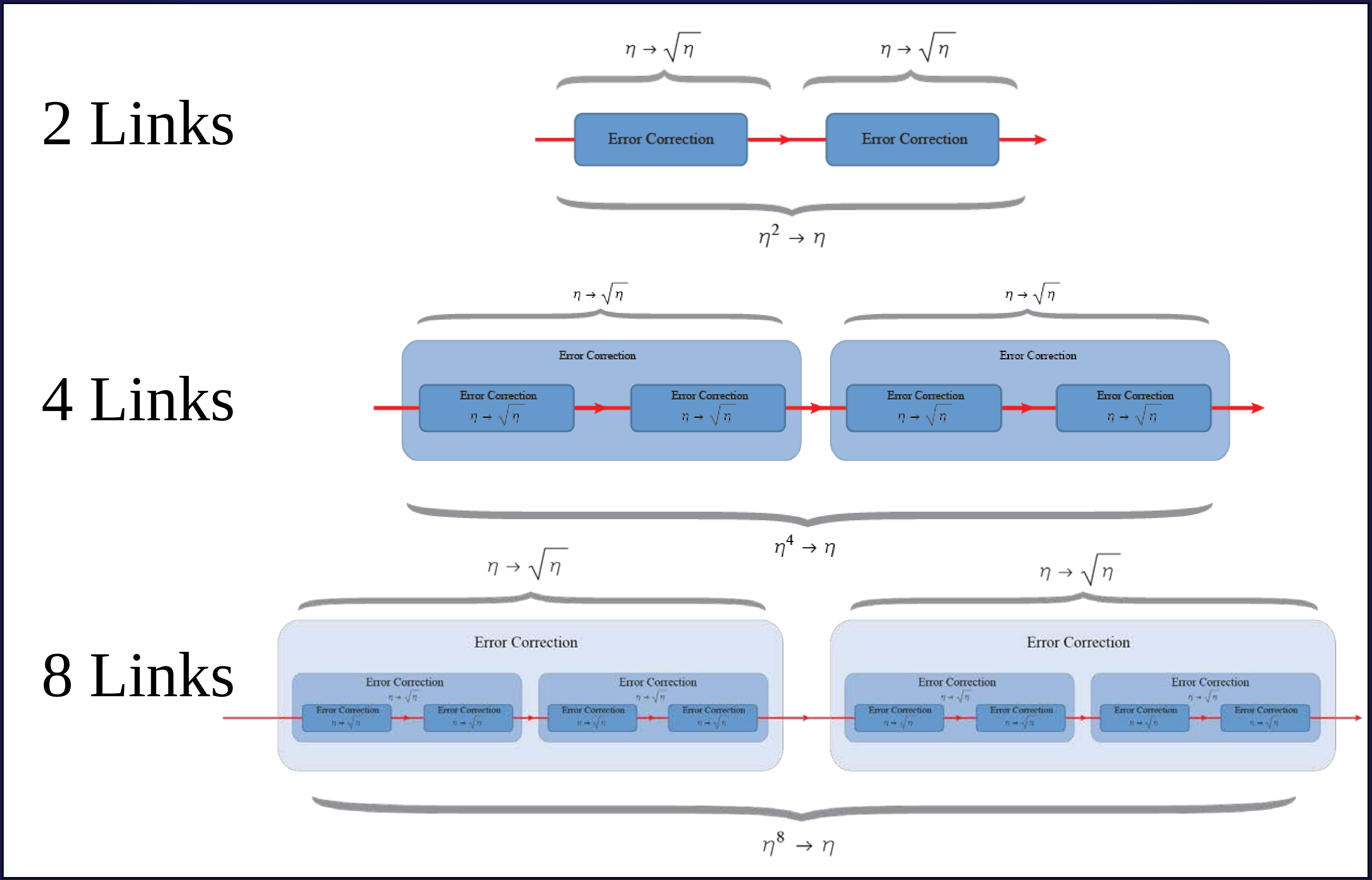
T. Ralph, Physical Review. A 84 (2011).

Concatenation of the error correction protocol: $\eta^M \rightarrow \eta$



Concatenation of the error correction protocol: $\eta^M \rightarrow \eta$

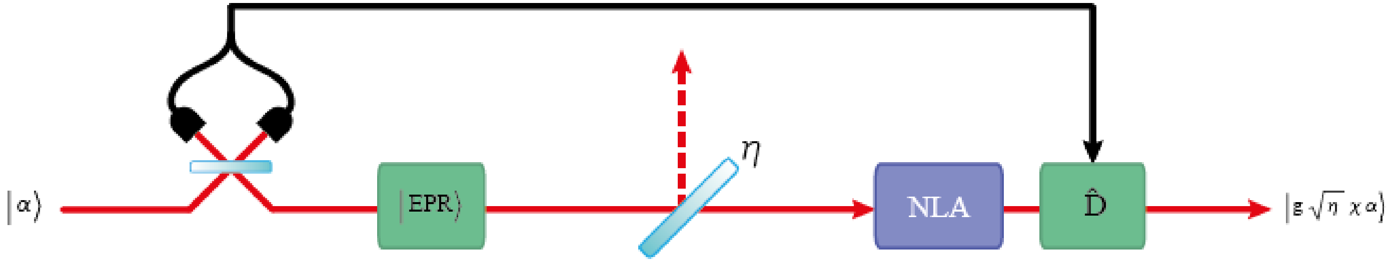
Fidelity for M links:

$$Fidelity \geq F^{(M-1)/2}$$

Probability of Success:

$$P_{success} = \mathrm{M}^{\log_2 P}$$

Where F and P are the fidelity and probability of success for one iteration of the error correction protocol.



Concatenation of the error correction protocol: $\eta^M \longrightarrow \eta$

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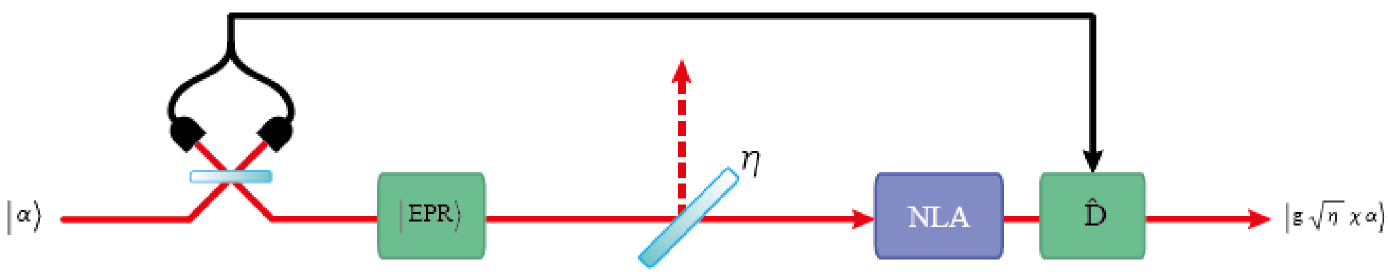
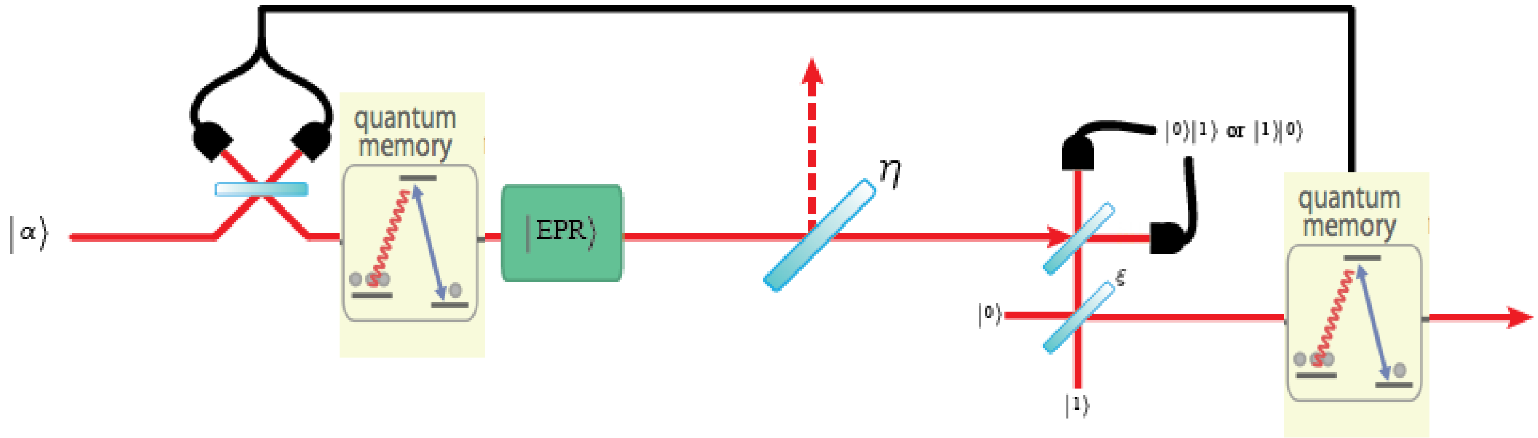


Quantum Scissors Implementation of NLA

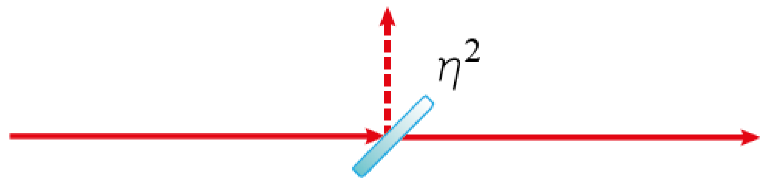
Consider first an NLA implemented with a single quantum scissor



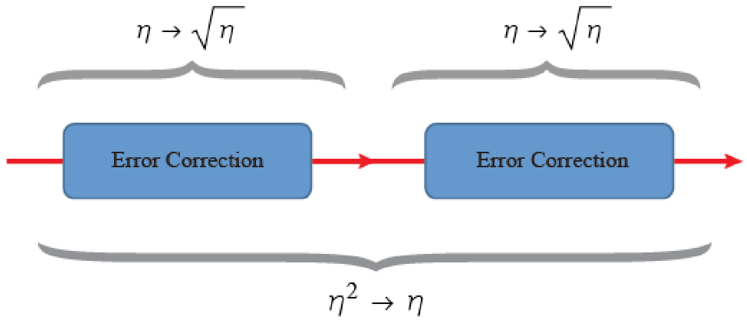
Josephine Dias, R.Blandino and T.C.Ralph, in preparation (2014).

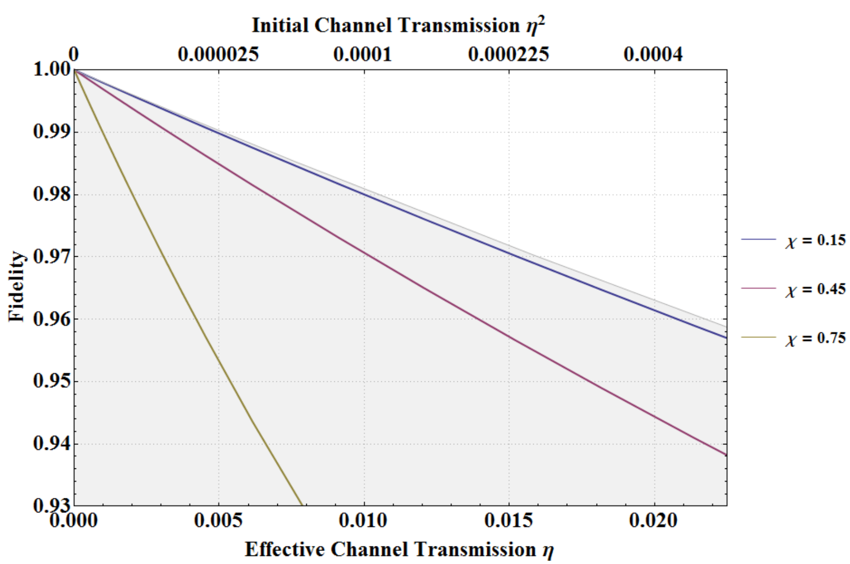
2 Links

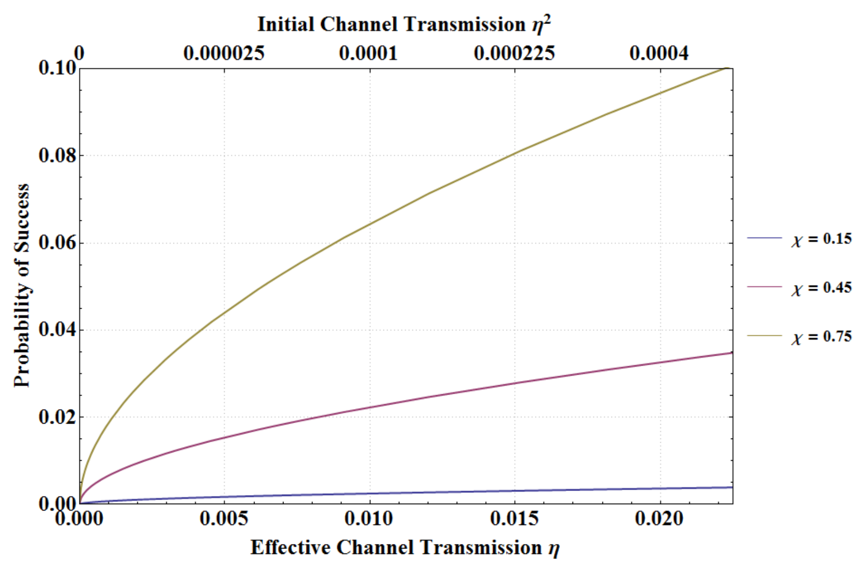
A quantum channel of transmission η^2:



becomes a channel of effective transmission η:

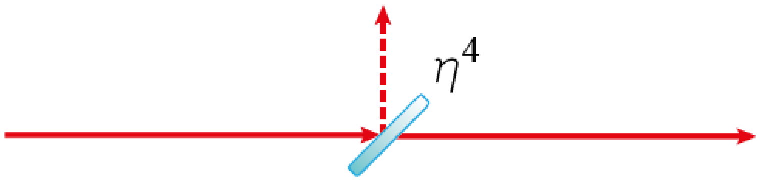




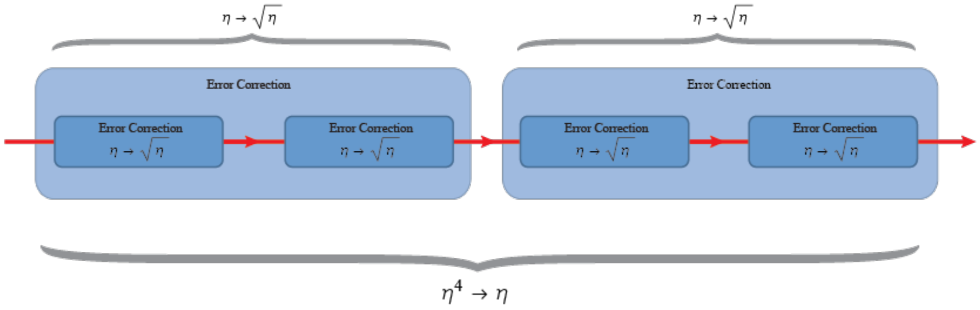


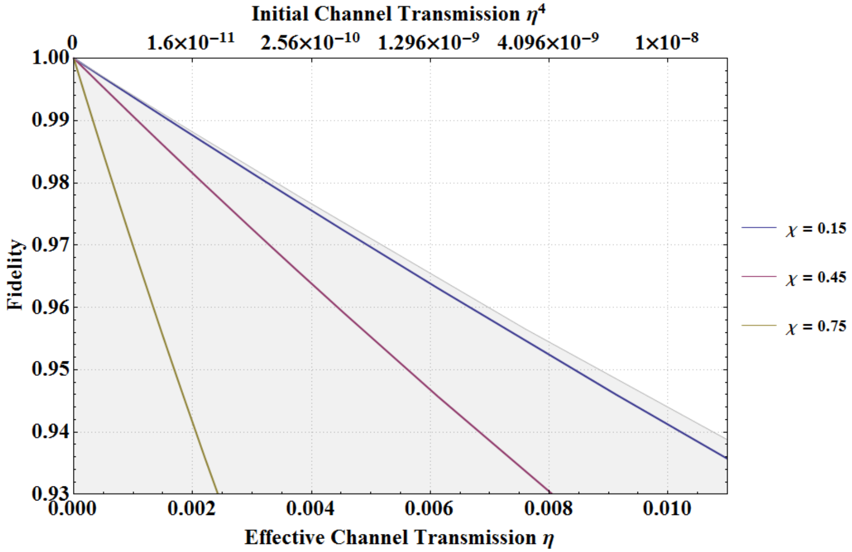
4 Links

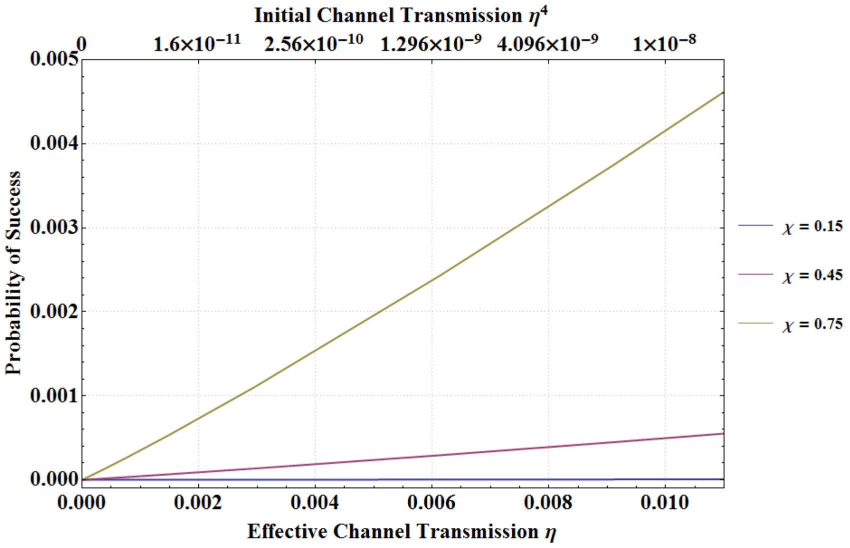
A quantum channel of transmission η^4:



becomes a channel of effective transmission η:

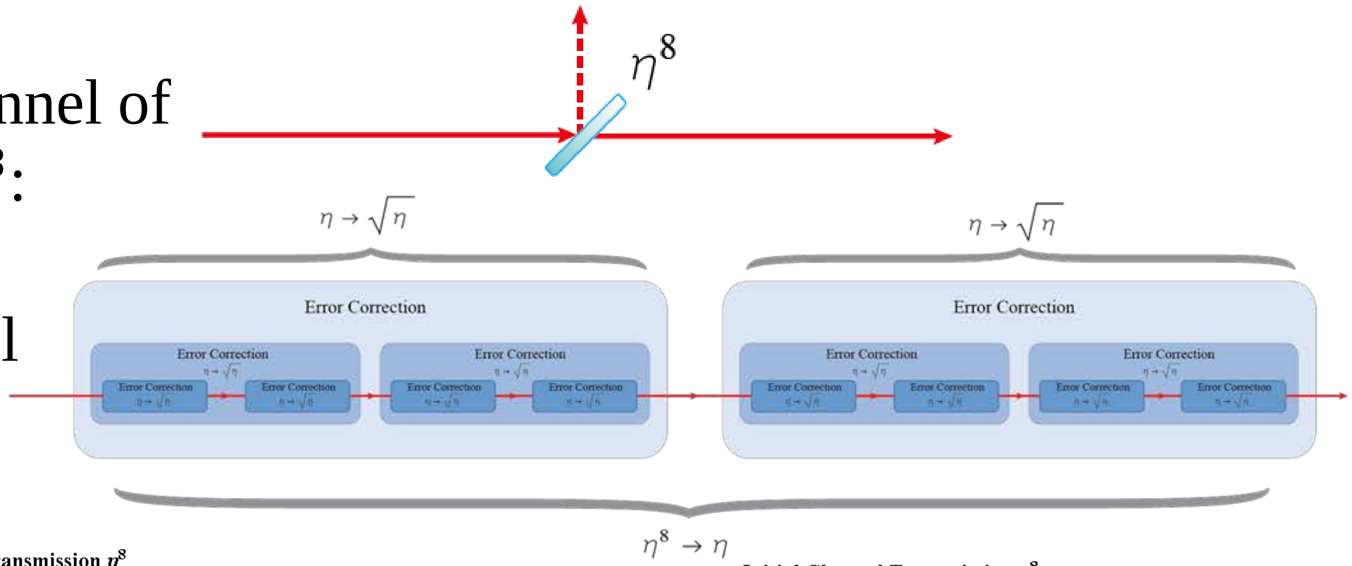




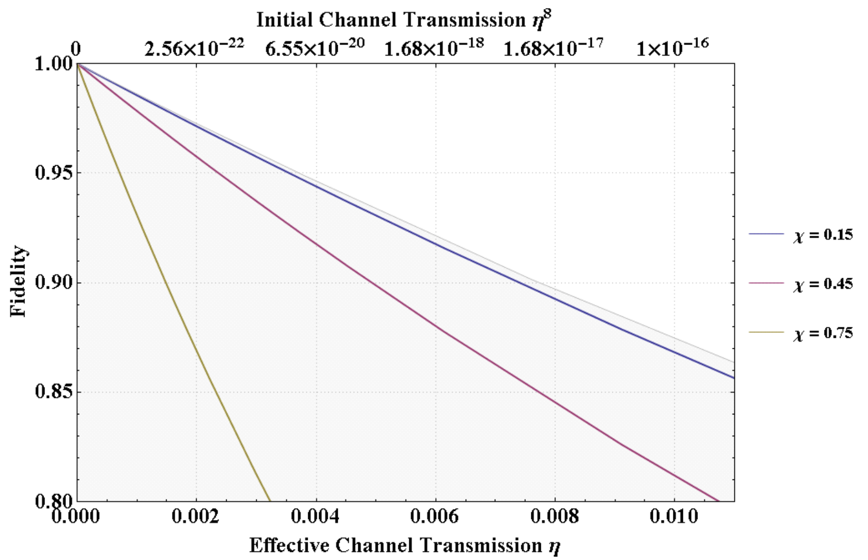


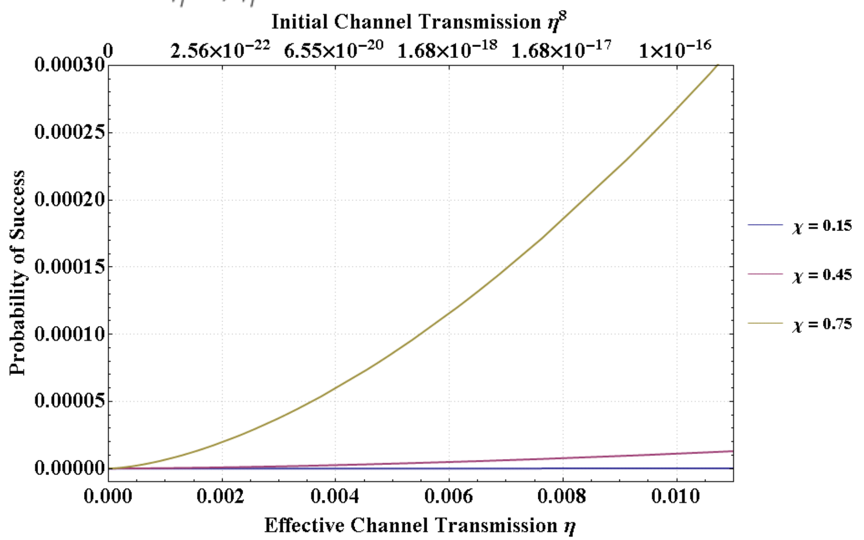
8 Links

A quantum channel of transmission η^8:



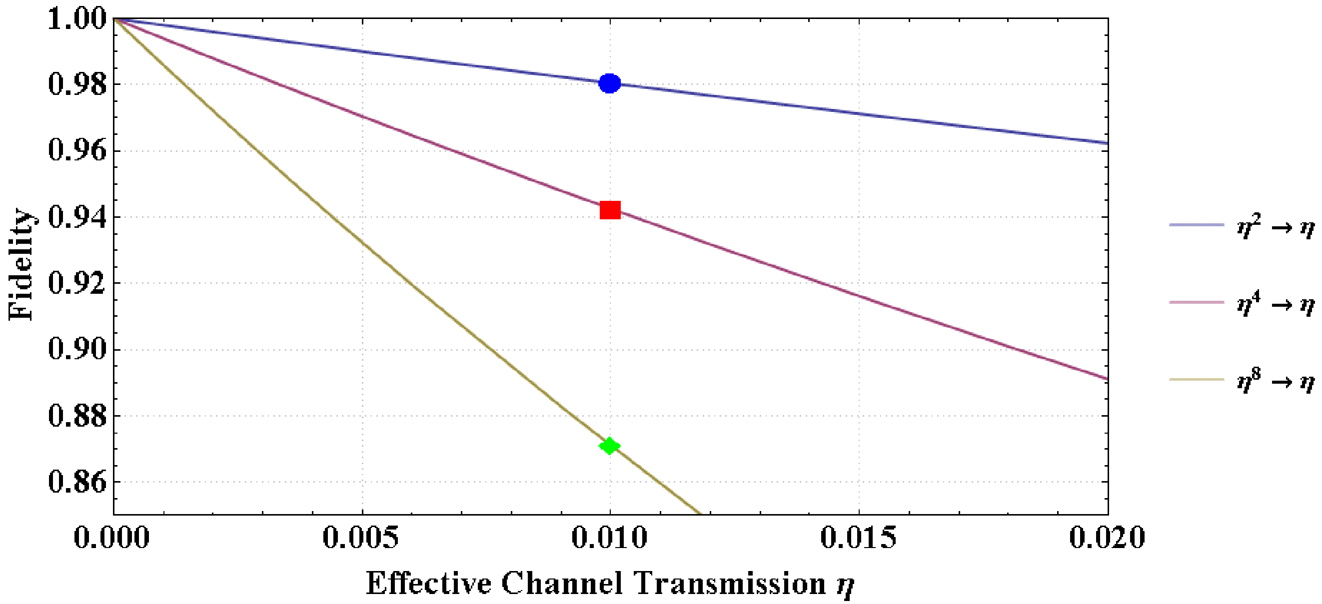
becomes a channel of effective transmission η:





Just How Far Can We Get?

As an example, using $\chi = 0.1$:



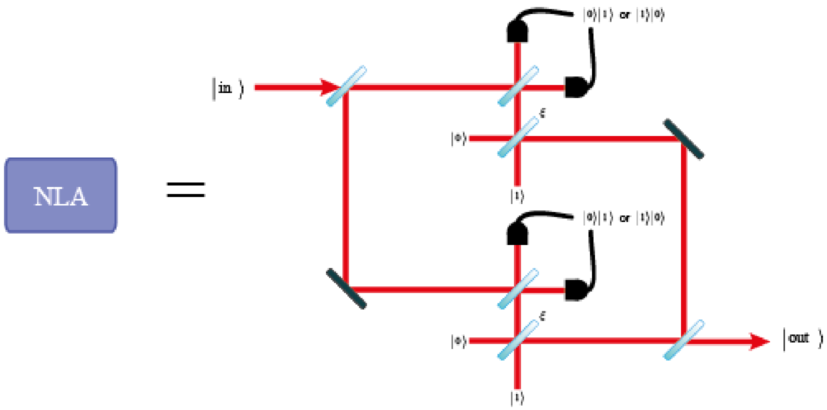
A channel transmission of $\eta = 0.01$ corresponds to the loss after $\sim 100km$ of optic fibre.

	Distance	**Fidelity**	**Success Probability**
● (blue circle)	$\sim 200km$	0.98	0.001
■ (red square)	$\sim 400km$	0.94	1.2×10^{-6}
◆ (green diamond)	$\sim 800km$	0.87	1.3×10^{-9}

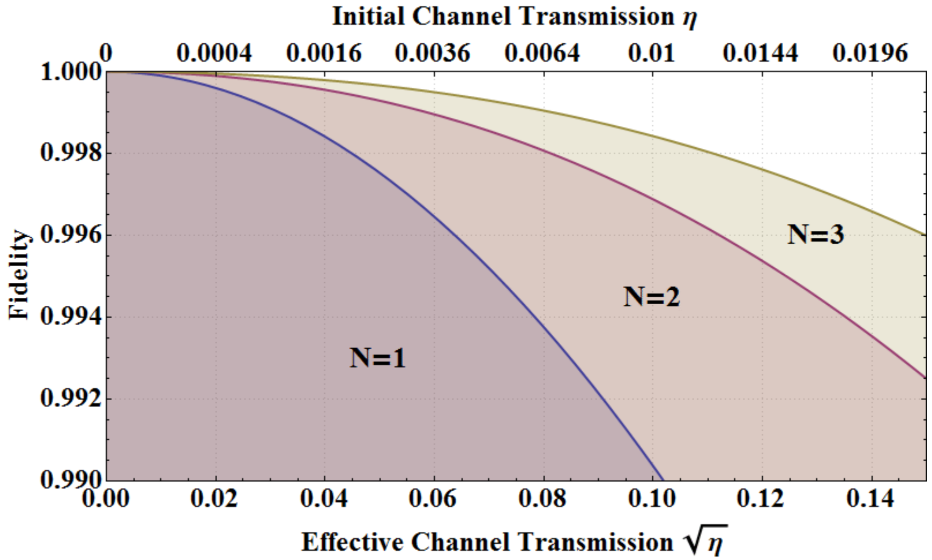
Increasing the Fidelity

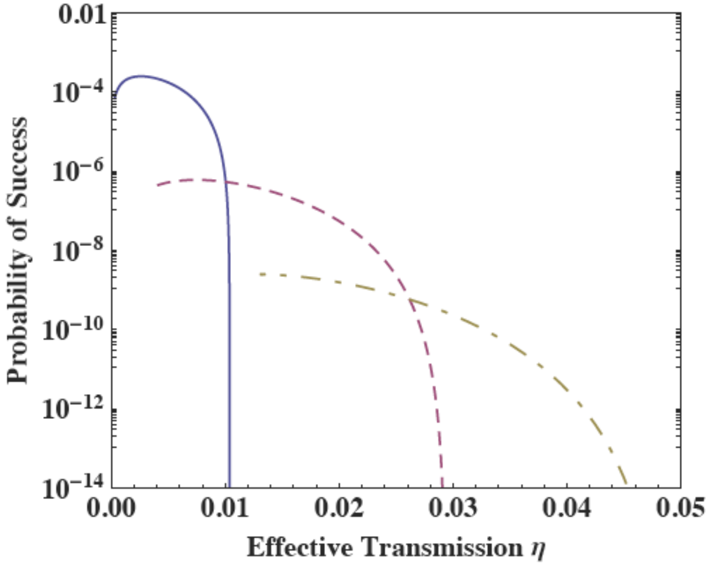
While the single quantum scissor produces good success probabilities, it limits the achievable fidelity.

Using more quantum scissors devices for purification will improve fidelity but decrease success probability.



NLA with two quantum scissors (N=2)





Advantages

By concatenating the error correction protocol, we are able to preserve channel transmission for increasing distance with success probability that decreases polynomially with distance.

The use of continuous variable teleportation means this repeater protocol will work for any optical encoding of quantum information.

Summary

* Just as linear amplification can enhance classical channels so noiseless linear amplification can enhance quantum channels

* Most general implementation via photon post-selection

* For some applications can use homodyne post-selection

* Quantum Repeater Protocol for continuous variables

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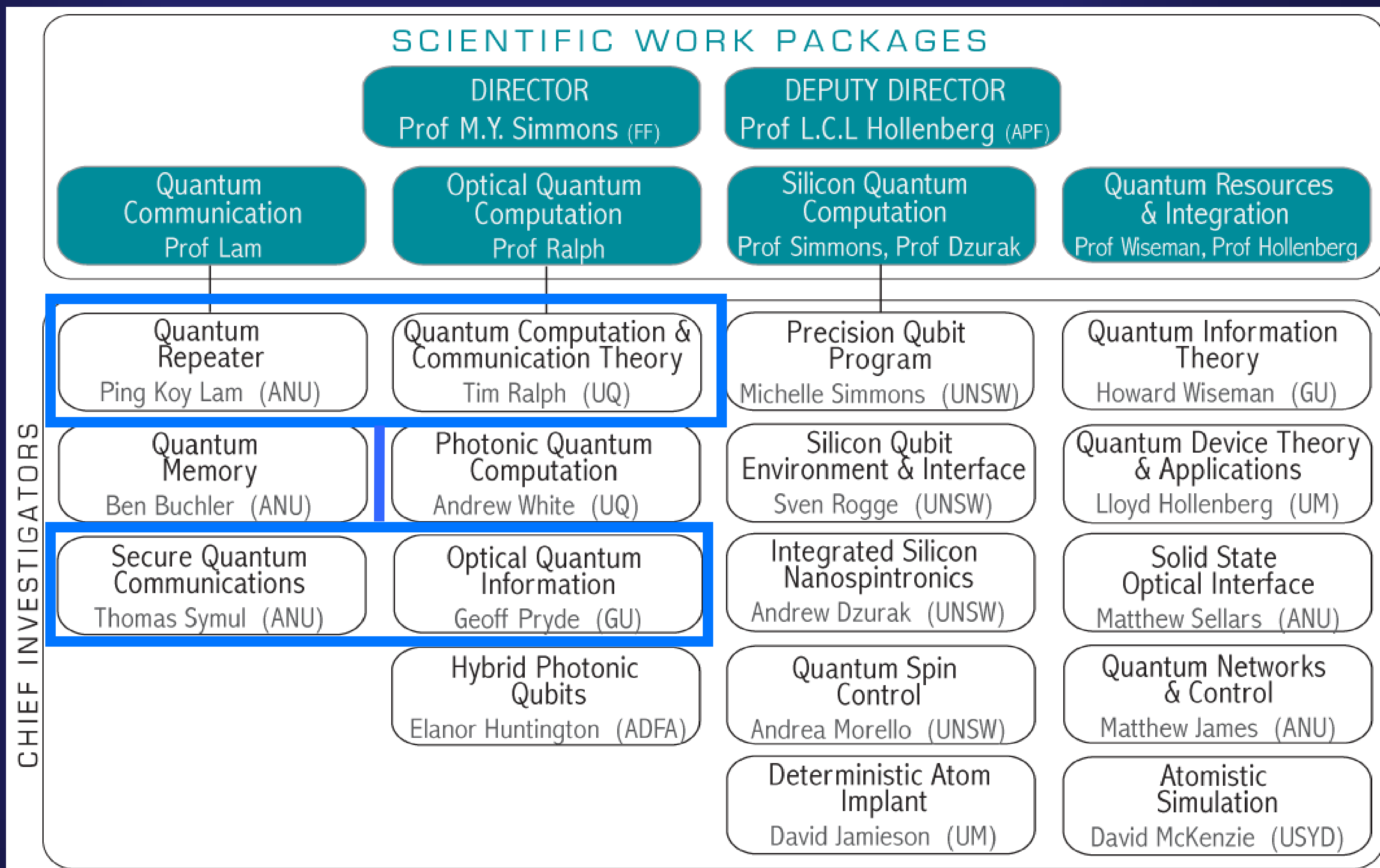

thanks