Optimal entanglement-assisted discrimination of projective single-qubit measurements

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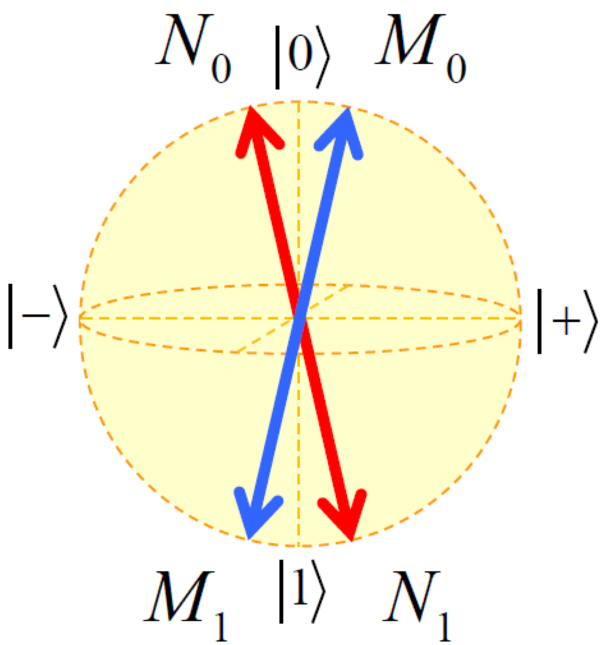
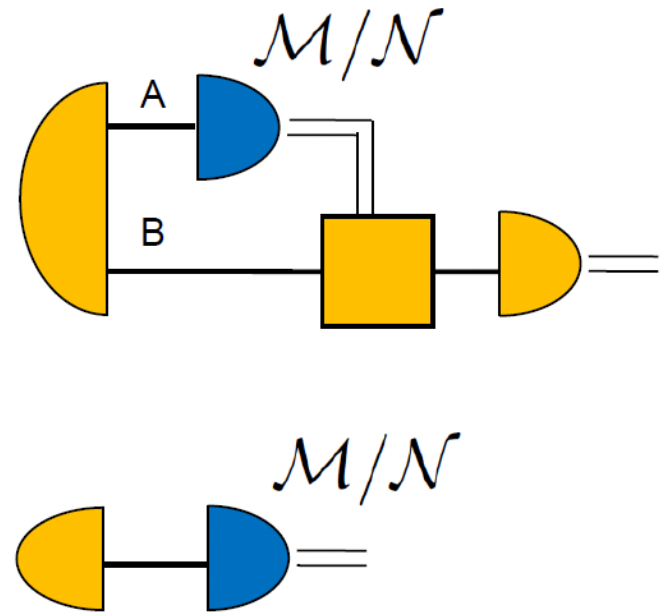
Discrimination of quantum measurements



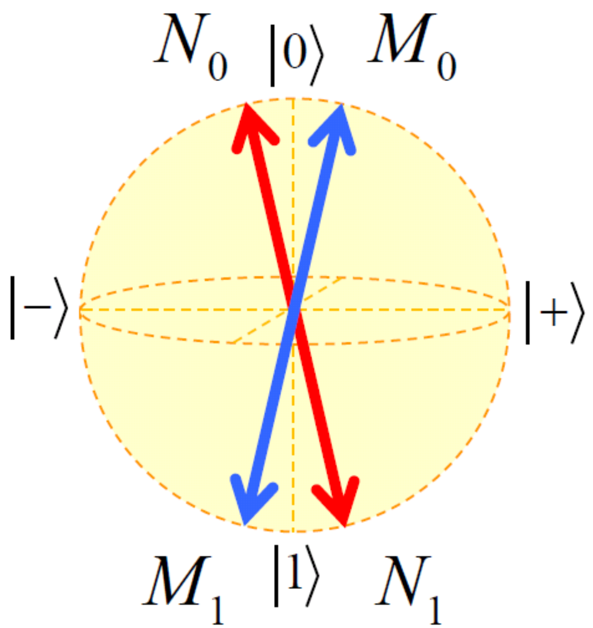


The task is to discriminate between two single-qubit projective measurements M and N when the measurement can be performed only once.

We consider a general discrimination strategy that can involve certain fraction of inconclusive outcomes.

M. Miková, M. Sedlák, I. Straka, M. Mičuda, M. Ziman, M. Ježek, M. Dušek, and J. Fiurášek , Phys. Rev. A **90**, 022317 (2014).

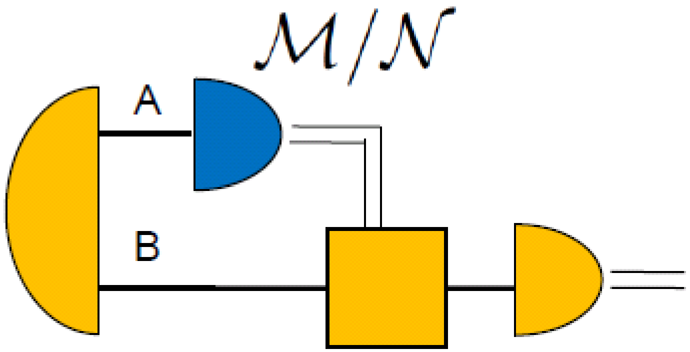
Measurement bases



$$M_0 = |\phi\rangle\langle\phi|, \qquad M_1 = |\phi^\perp\rangle\langle\phi^\perp|,$$
$$N_0 = |\psi\rangle\langle\psi|, \qquad N_1 = |\psi^\perp\rangle\langle\psi^\perp|,$$

$$|\phi\rangle = \cos\theta|0\rangle + \sin\theta|1\rangle, \quad |\phi^\perp\rangle = \sin\theta|0\rangle - \cos\theta|1\rangle,$$
$$|\psi\rangle = \cos\theta|0\rangle - \sin\theta|1\rangle, \quad |\psi^\perp\rangle = \sin\theta|0\rangle + \cos\theta|1\rangle,$$

Entanglement-assisted discrimination procedure



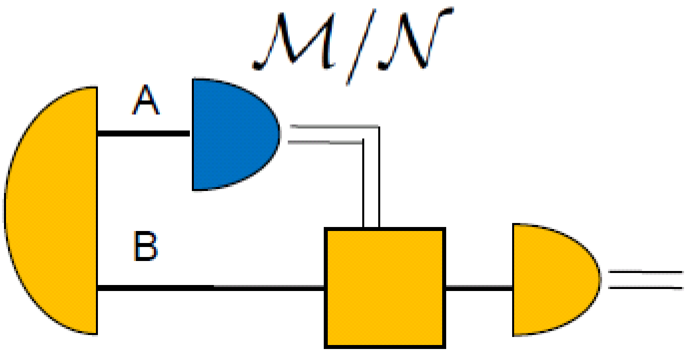
Prepare an entangled state of qubits A and B.

Perform the measurement M/N on qubit A.

Measure qubit B in a basis determined by the outcome of measurement on qubit A.

Guess M, N, or declare an inconclusive outcome depending on the measurement outcomes.

Entanglement-assisted discrimination procedure



Prepare an entangled state of qubits A and B.

Perform the measurement M/N on qubit A.

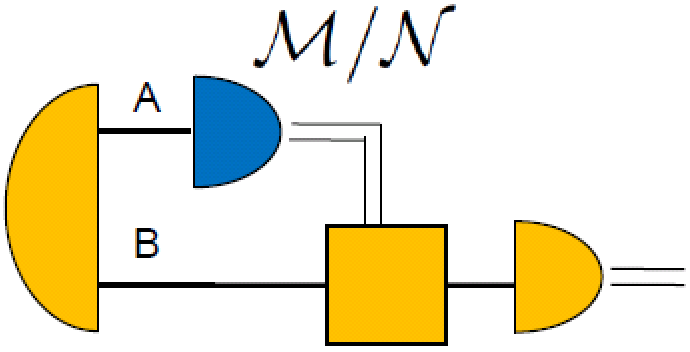
Measure qubit B in a basis determined by the outcome of measurement on qubit A.

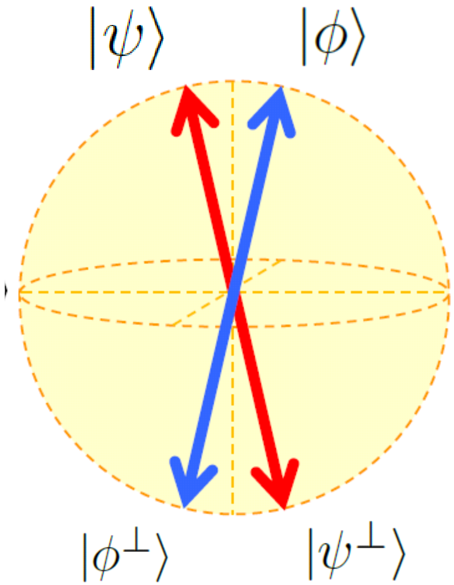
Guess M, N, or declare an inconclusive outcome depending on the measurement outcomes.

We assume equal a-priori probabilities of M and N. In this case it is optimal to employ a maximally entangled probe state:

$$|\Psi^-\rangle = \frac{1}{\sqrt{2}}(|01\rangle - |10\rangle)$$

Entanglement-assisted discrimination procedure

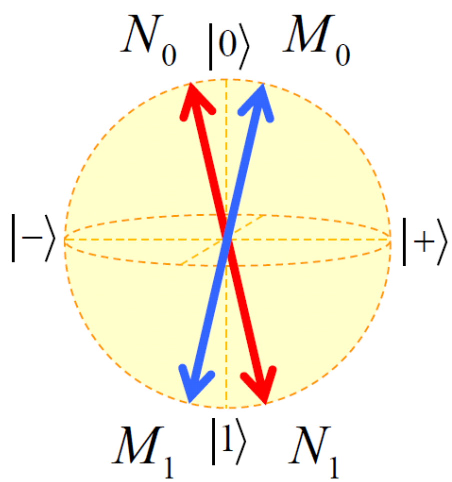




$$|\Psi^-\rangle = \frac{1}{\sqrt{2}}(|01\rangle - |10\rangle)$$

Outcome of measurement on A	State of qubit B if the measurement was M	State of qubit B if the measurement was N
0	$\lvert\phi^\perp\rangle$	$\lvert\psi^\perp\rangle$
1	$\lvert\phi\rangle$	$\lvert\psi\rangle$

Entanglement-assisted discrimination procedure



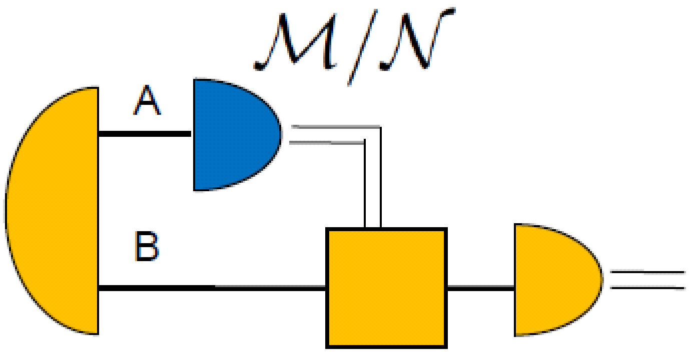
$$|\phi\rangle = \cos\theta|0\rangle + \sin\theta|1\rangle, \quad |\phi^\perp\rangle = \sin\theta|0\rangle - \cos\theta|1\rangle,$$
$$|\psi\rangle = \cos\theta|0\rangle - \sin\theta|1\rangle, \quad |\psi^\perp\rangle = \sin\theta|0\rangle + \cos\theta|1\rangle,$$

$$|\phi\rangle = \sigma_Y|\phi^\perp\rangle, \quad |\psi\rangle = -\sigma_Y|\psi^\perp\rangle,$$

Outcome of measurement on A	State of qubit B if the measurement was M	State of qubit B if the measurement was N
0	$\vert\phi^\perp\rangle$	$\vert\psi^\perp\rangle$
1	$\vert\phi\rangle$	$\vert\psi\rangle$

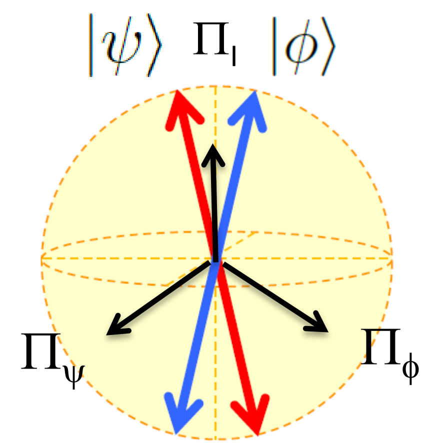
We apply unitary σ_Y operation if the measurement outcome is 0. Discrimination of quantum measurements is thus reduced to discrimination of quantum states ϕ and ψ.

Entanglement-assisted discrimination procedure



$$|\phi\rangle = \cos\theta|0\rangle + \sin\theta|1\rangle,$$
$$|\psi\rangle = \cos\theta|0\rangle - \sin\theta|1\rangle,$$

General discrimination strategy with a three-component POVM – we allow for a tunable probability of inconclusive outcomes P_I.



P_S – probability of success

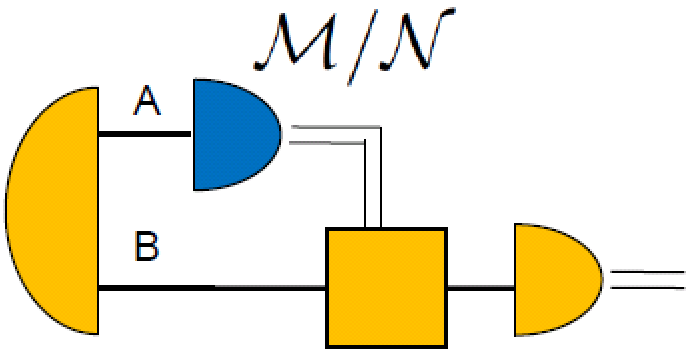
P_I – probability of inconclusive outcomes

P_E – probability of error

$$P_S + P_E + P_I = 1$$

A. Chefles and S.M. Barnett, J. Mod. Opt. 45, 1295 (1998).
C.W. Zhang, C.F. Li, and G.C. Guo, Phys. Lett. A 261, 25 (1999).
M. Miková, M. Sedlák, I. Straka, M. Mičuda, M. Ziman, M. Ježek, M. Dušek, and J. Fiurášek (2014).

Entanglement-assisted discrimination procedure



$$|\phi\rangle = \cos\theta|0\rangle + \sin\theta|1\rangle,$$
$$|\psi\rangle = \cos\theta|0\rangle - \sin\theta|1\rangle,$$

General discrimination strategy with a three-component POVM – we allow for a tunable probability of inconclusive outcomes P_I.

Maximum probability of a successful guess for a fixed P_I:

$$P_S = \frac{1}{2}\left(1 - P_I + \sin(2\theta)\sqrt{1 - \frac{P_I}{\cos^2\theta}}\right)$$

Optimality of this procedure can be proved using the formalism of process POVM.

A. Chefles and S.M. Barnett, J. Mod. Opt. 45, 1295 (1998).
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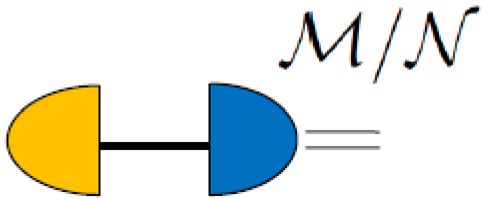
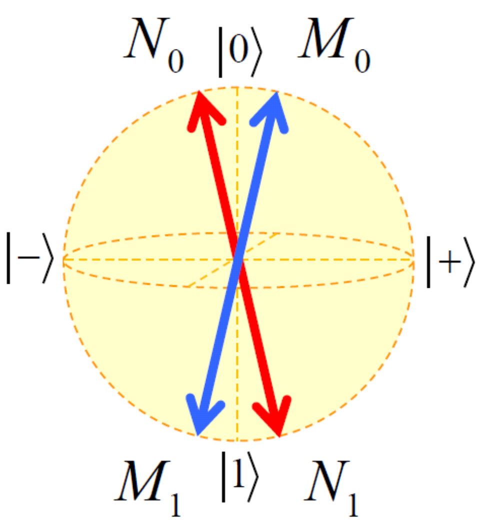
M. Miková, M. Sedlák, I. Straka, M. Mičuda, M. Ziman, M. Ježek, M. Dušek, and J. Fiurášek (2014).

Single-qubit probe



Pure probe state

$$|\vartheta\rangle = \cos\vartheta|0\rangle + \sin\vartheta|1\rangle$$



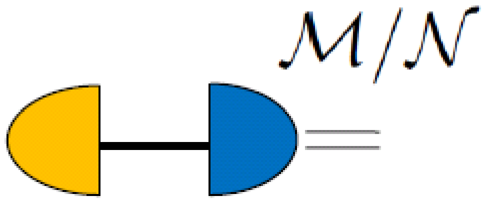
Minimum error discrimination:

$$|\vartheta\rangle = |+\rangle = \frac{1}{\sqrt{2}}(|0\rangle + |1\rangle)$$

$$P_S = \frac{1}{2}[1 + \sin(2\theta)] \qquad P_I = 0$$

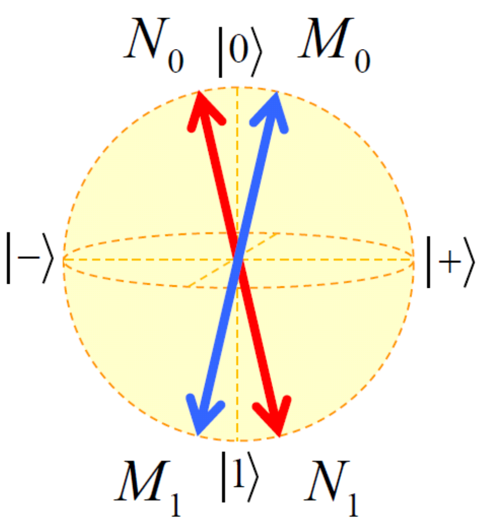
Globally optimal strategy. Entanglement is not needed for minimum error discrimination.

Single-qubit probe



Pure probe state

$$|\vartheta\rangle = \cos\vartheta|0\rangle + \sin\vartheta|1\rangle$$



Unambiguous discrimination:

$$|\vartheta\rangle = |\psi^{\perp}\rangle$$

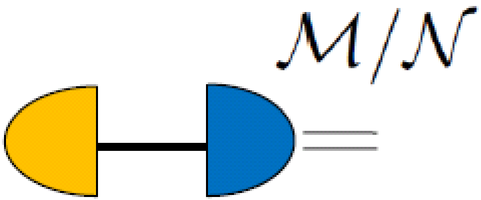
$$P_S = \frac{1}{2}\sin^2(2\theta) \qquad P_I = 1 - P_S$$

The optimal entanglement-assisted unambiguous discrimination achieves:

$$P_{S,\mathrm{ent}} = 1 - \cos(2\theta) > P_S$$

In fact, one can prove that entanglement helps for any $P_I>0$.

Single-qubit probe – general strategy



It is optimal to use a pure probe state:

$$|\vartheta\rangle = \cos\vartheta|0\rangle + \sin\vartheta|1\rangle$$

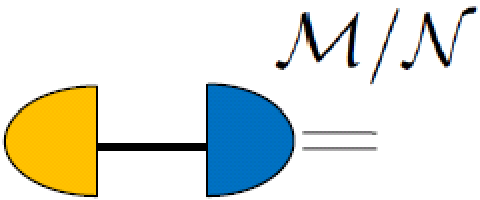
Without loss of generality, the following strategy can be proved to be optimal:

Outcome 0 -> always guess M

Outcome 1 -> with probability q guess N, with probability 1-q announce an inconclusive outcome

One can construct symmetric protocol by randomly replacing the roles of M and N.

Single-qubit probe – general strategy



Pure probe state:

$$|\vartheta\rangle = \cos\vartheta|0\rangle + \sin\vartheta|1\rangle$$

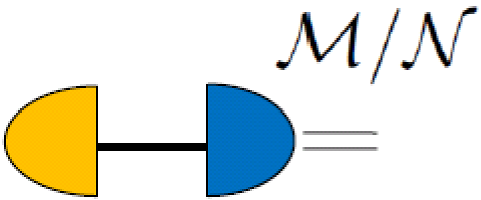
Define a threshold value on P_I:

$$P_{I,B} \; = \; \left[3 + (1 + 8c^2)^{1/2}\right]/8 \qquad\qquad c \; = \; \cos(2\theta)$$

When $P_I < P_{I,B}$, the optimal probe state can be determined by solving a cubic equation:

$$c^2x^3 - 2cx^2 + (1 - P_I)x + P_I c = 0 \qquad\qquad x \; = \; \cos(2\vartheta)$$

Single-qubit probe – general strategy



Pure probe state:

$$|\vartheta\rangle = \cos\vartheta|0\rangle + \sin\vartheta|1\rangle$$

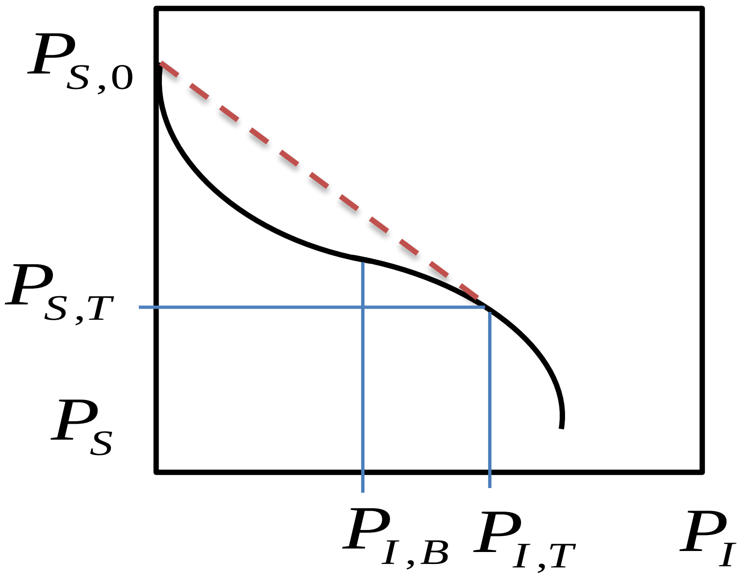
Define a threshold value on P_I:

$$P_{I,B} \;=\; \left[3 + (1 + 8c^2)^{1/2}\right]/8 \qquad\qquad c \;=\; \cos(2\theta)$$

When $P_I > P_{I,B}$, we have q=0 and

$$\cos(2\vartheta) = (1 - 2P_I)/c \qquad\qquad P_S = \frac{1}{2}(1 - P_I) + \frac{1}{4}\sin(2\theta)\sqrt{1 - \frac{(1 - 2P_I)^2}{\cos^2(2\theta)}}$$

Single-qubit probe – general strategy



The dependence of P_S on P_I is a convex function for $P_I<P_{I,B}$

We need to construct a convex hull of the single-qubit strategies.

$$P_{I,T} = \frac{1+3c^2+2c^2\sqrt{1+3c^2}}{2(1+4c^2)}$$

$$P_I < P_{I,T}$$

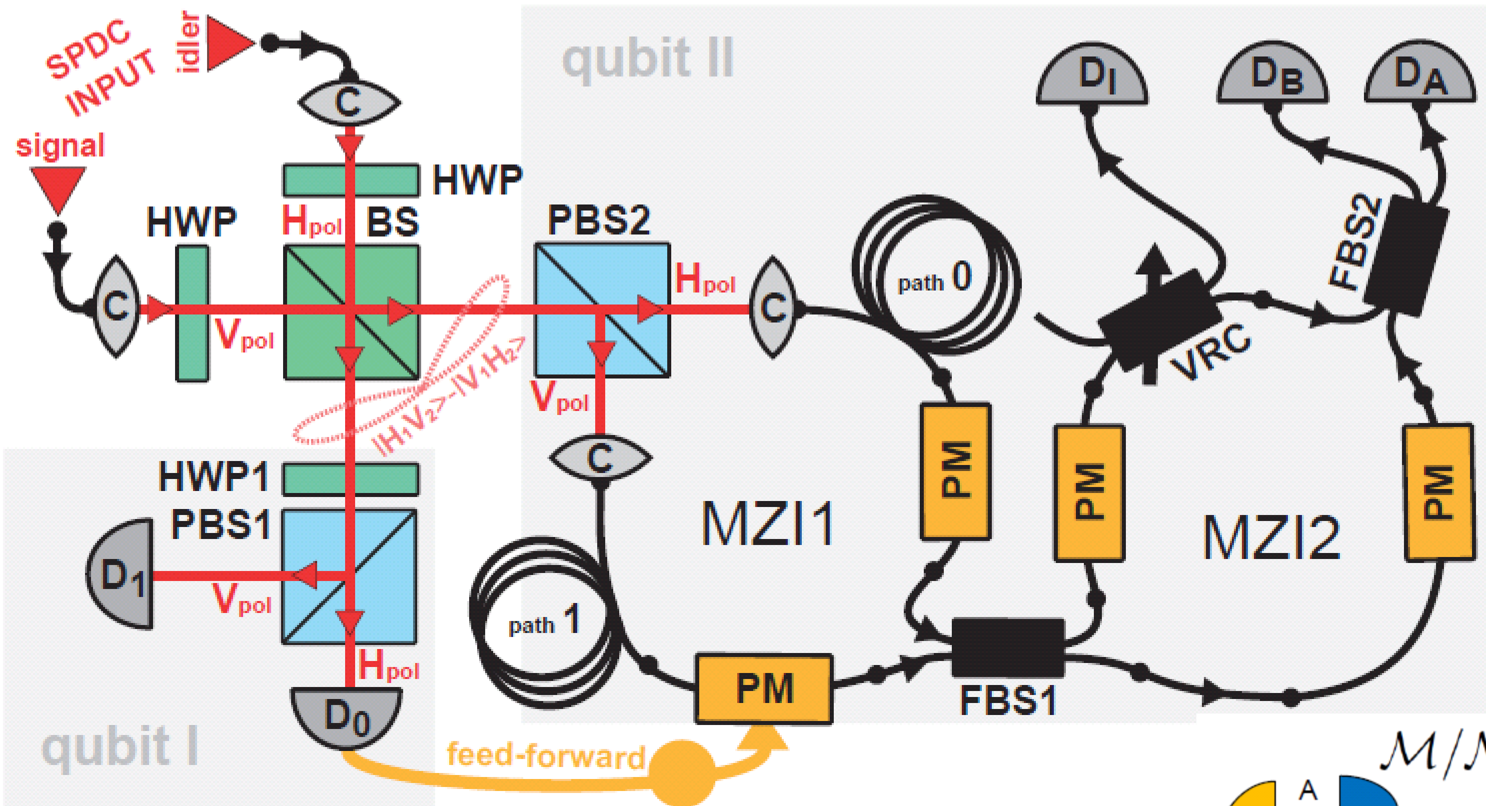
$$P_S = \left(1-\frac{P_I}{P_{I,T}}\right)P_{S,0} + \frac{P_I}{P_{I,T}}P_{S,T}$$

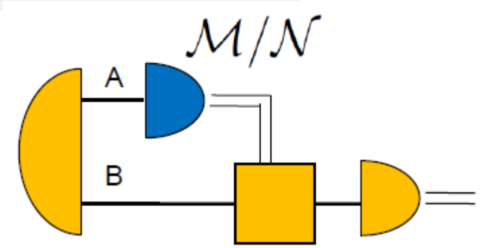
$$P_{S,0} = \frac{1}{2}[1+\sin(2\theta)]$$

$$P_I \geq P_{I,T}$$

$$P_S = \frac{1}{2}(1-P_I) + \frac{1}{4}\sin(2\theta)\sqrt{1-\frac{(1-2P_I)^2}{\cos^2(2\theta)}}$$

Experimental setup





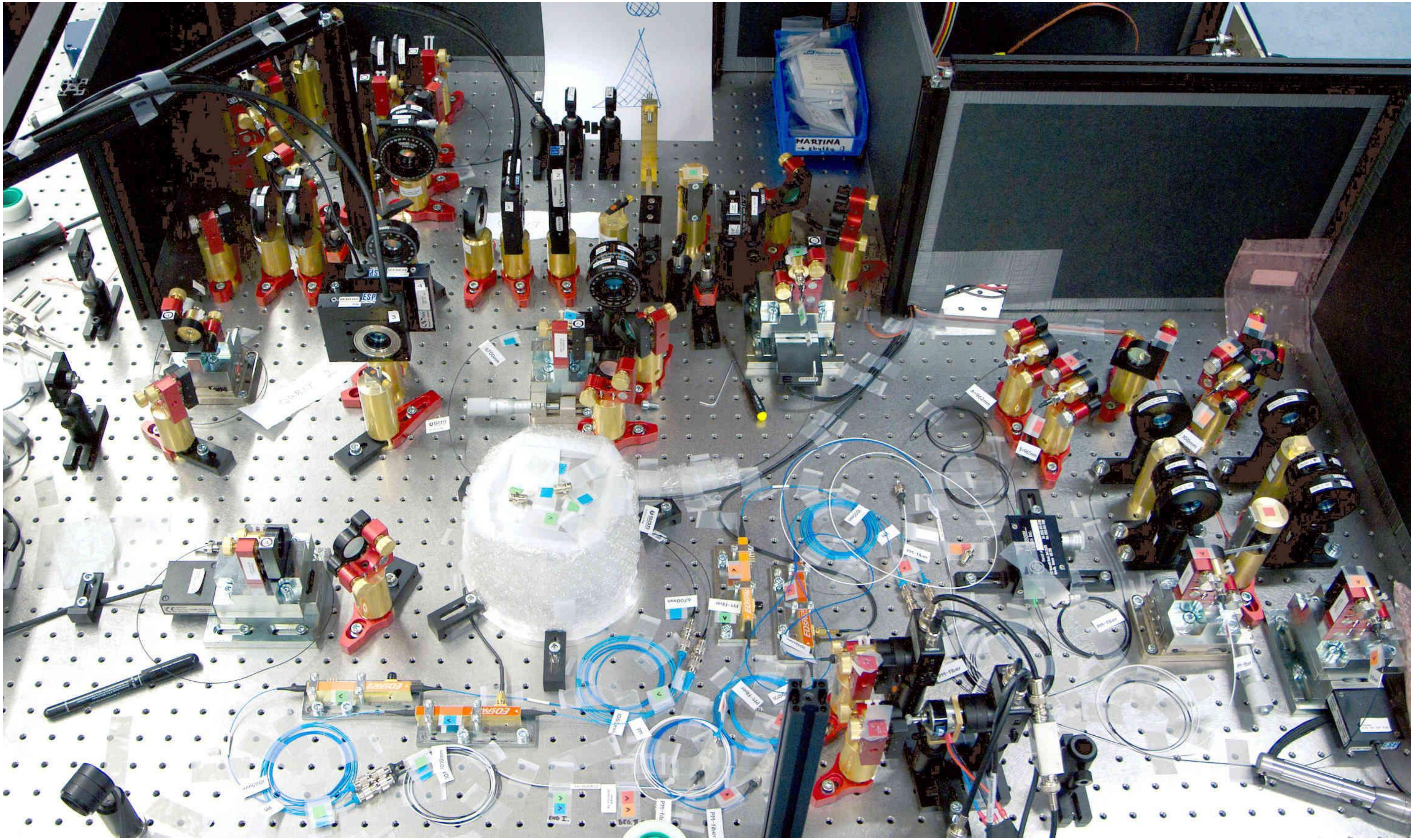
Qubits encoded into polarization states of single photons.

Two-qubit entangled state is conditionally generated by interference on a BS.

The conditional unitary on qubit B is applied using a real-time electronic feed-forward loop.

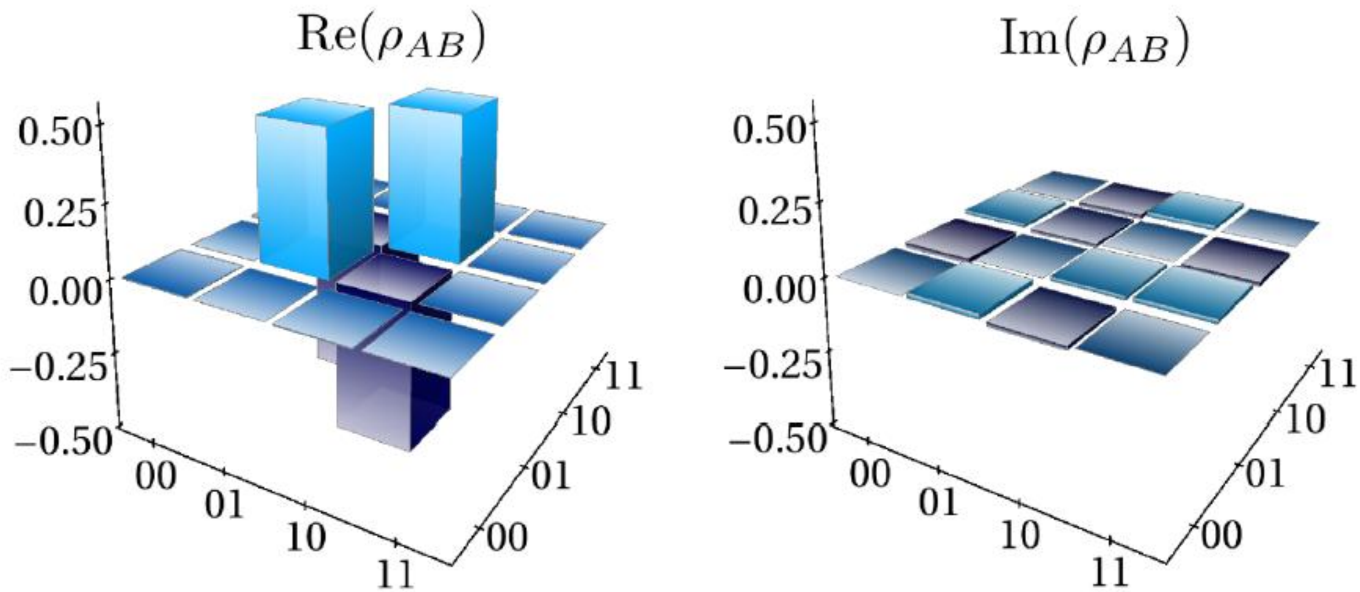
The POVM on qubit B is determined by the transmittance of VRC.

Experimental setup



Characterization of entangled probe state

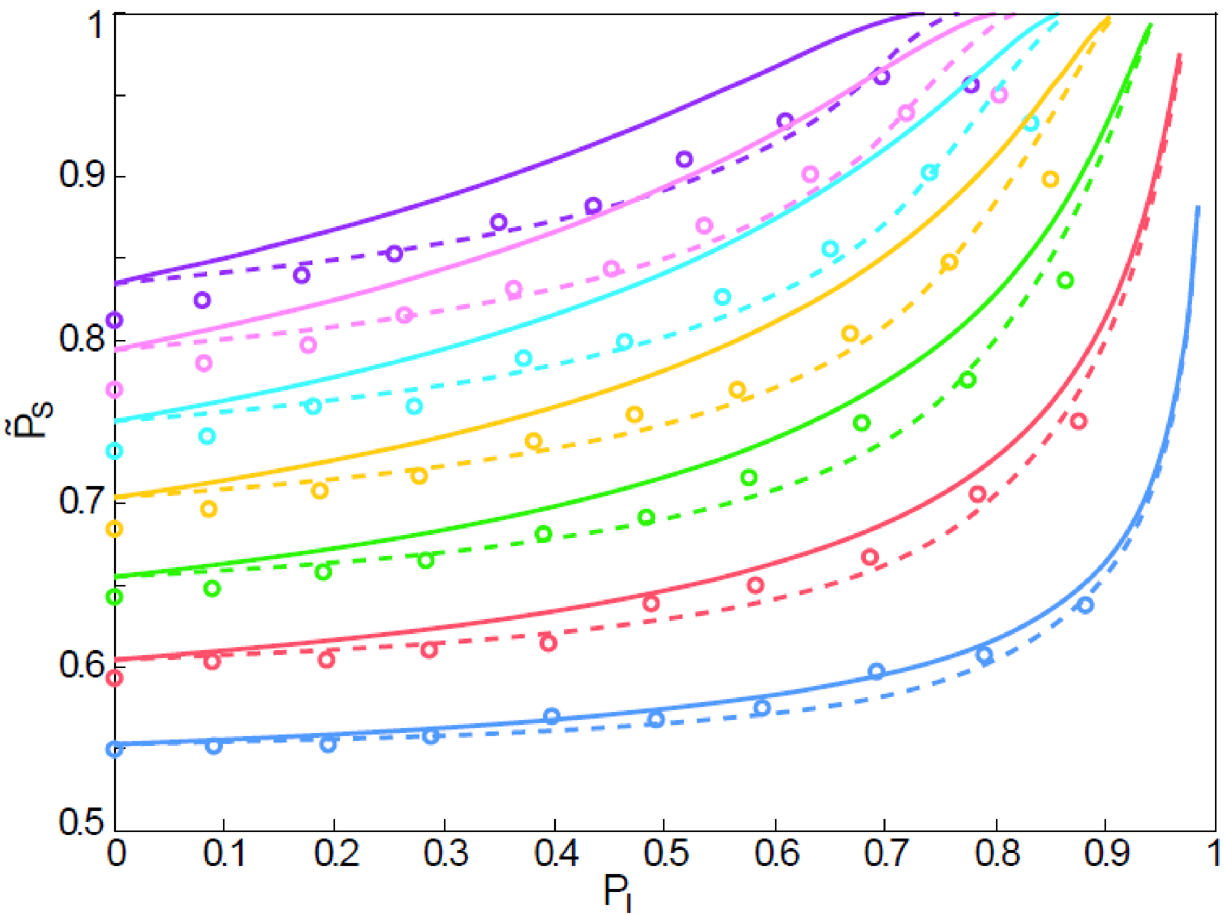
Target singlet state: $$|\Psi^-\rangle = \frac{1}{\sqrt{2}}(|01\rangle - |10\rangle)$$



purity 98 %, fidelity 99 %, concurrence 98%, ent. of formation 97%

Experimental results I

Dependence of relative success probability $\tilde{P}_S$ on probability of inconclusive results P_I for 7 values of $\theta_j = j\pi/30, \quad j = 1, \ldots, 7$



$$\tilde{P}_S = \frac{P_S}{1 - P_I}$$

Circles – experiment | Solid lines – theory entangled probe | Dashed lines – theory single-qubit probe

Experimental results II

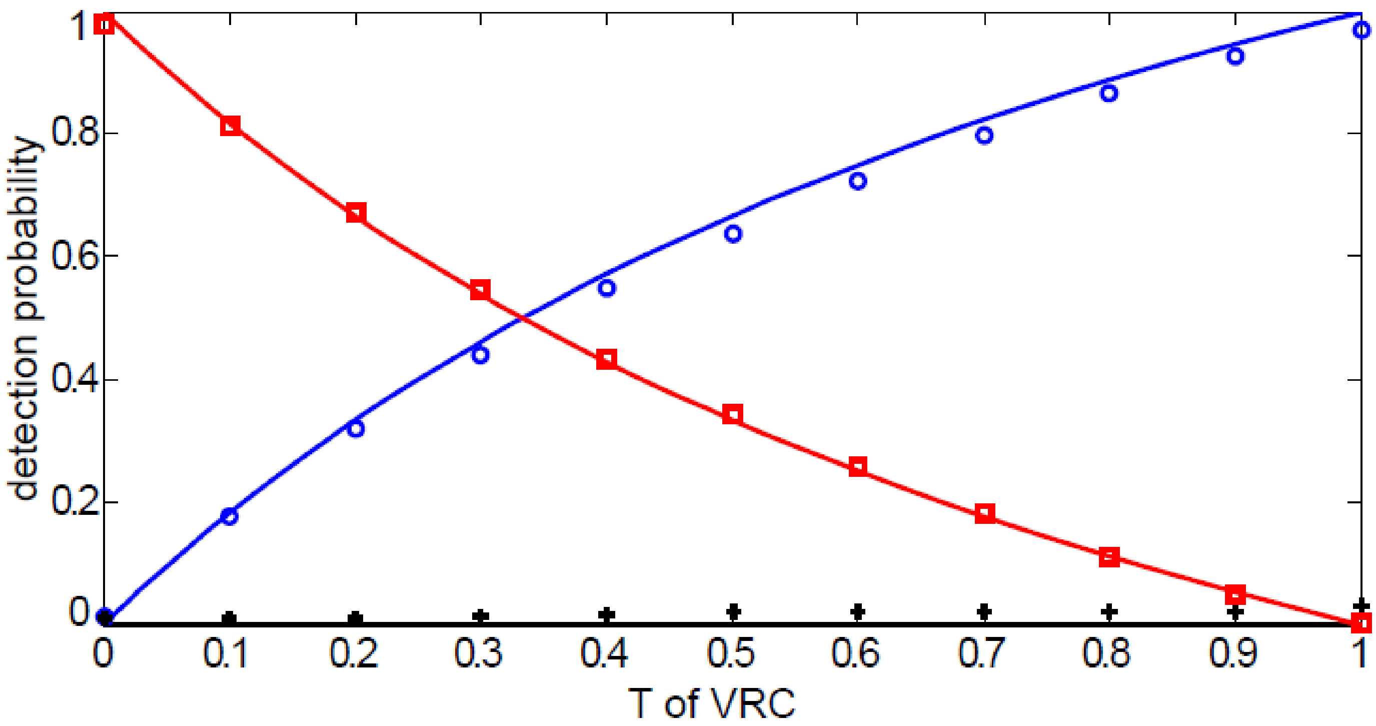
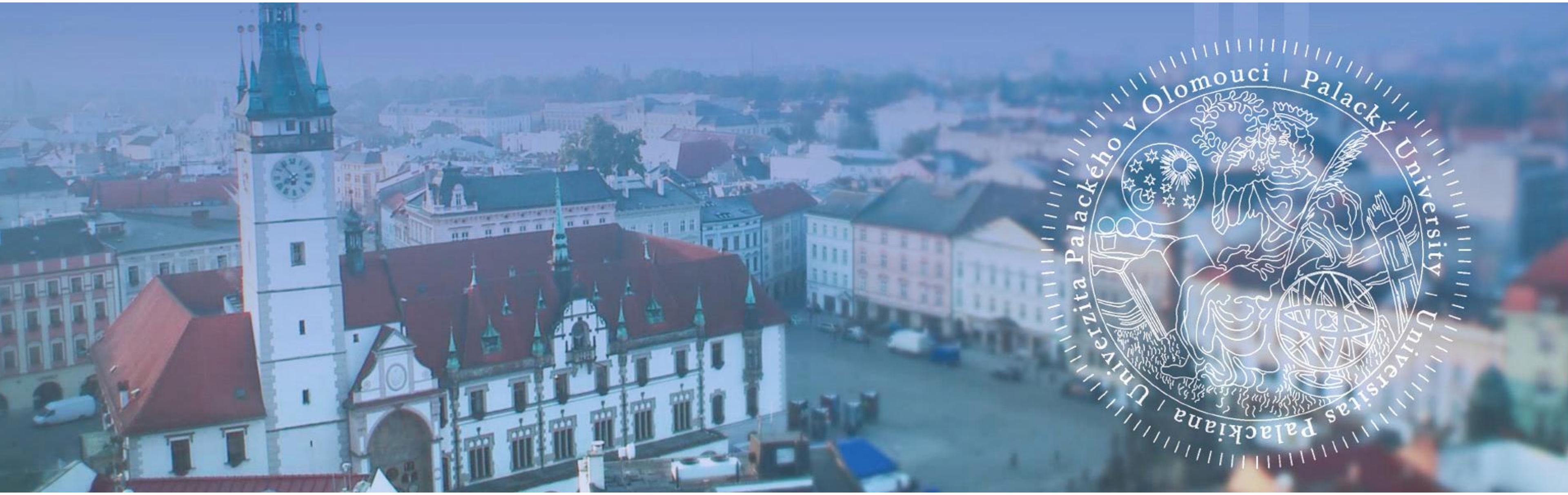


FIG. 4: Unambiguous discrimination of quantum measurements. The probabilities P_S (blue circles), P_I (red squares), and P_E (black crosses) are plotted as functions of the VRC splitting ratio T. The lines represent theoretical predictions.

Thank you for your attention!



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