Ordinal factor analysis of graded data

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Boolean Factor Analysis

Boolean factor analysis concerns with reduction of space dimension of binary data. Its goal can be formalized as follows: decompose $I = A \circ B$ where

- A ...object $\times$ factors matrix, B ...factors $\times$ attributes matrix
- aim: no. factors as small as possible

Given an object $\times$ attribute Boolean matrix I like

$$I = \begin{pmatrix} 1 & 1 & 0 & 0 & 0 \\ 1 & 1 & 0 & 0 & 0 \\ 1 & 1 & 1 & 1 & 0 \\ 1 & 0 & 0 & 0 & 1 \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 \\ 1 & 0 & 0 \\ 1 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} \circ \begin{pmatrix} 1 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 1 & 0 \\ 1 & 0 & 0 & 0 & 1 \end{pmatrix}$$

Using terms of formal concept analysis:

- the goal: express attributes of objects using factors
- no. factors $\ll$ no. attributes.

Boolean Factor Analysis

R. Belohlavek, V. Vychodil
Discovery of optimal factors in binary data via novel method of matrix decomposition
J. Computer and System Sci.

Formal concepts are universal and optimal factors.

$$\mathcal{F} \subseteq \mathcal{B}(X, Y, I)$$

extents in $\mathcal{F}$ intents in $\mathcal{F}$

$$I = \begin{pmatrix} 1 & 1 & 0 & 0 & 0 \\ 1 & 1 & 0 & 0 & 0 \\ 1 & 1 & 1 & 1 & 0 \\ 1 & 0 & 0 & 0 & 1 \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 \\ 1 & 0 & 0 \\ 1 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} \circ \begin{pmatrix} 1 & 1 & 0 & 0 & 0 \\ 1 & 1 & 1 & 1 & 0 \\ 1 & 0 & 0 & 0 & 1 \end{pmatrix}$$

Graded Factor Analysis

R. Belohlavek
Optimal decompositions of matrices with entries from residuated lattices.
Journal of Logic and Computation 22(6)(2012), pp 1405–1425.

Formal fuzzy concepts are universal and optimal factors.

$$I = \begin{pmatrix} 0.5 & 0.5 & 0.0 & 0.0 & 0.0 \\ 1.0 & 1.0 & 0.0 & 0.0 & 0.0 \\ 1.0 & 1.0 & 0.5 & 0.5 & 0.0 \\ 1.0 & 0.0 & 0.0 & 0.0 & 1.0 \end{pmatrix} = \begin{pmatrix} 0.5 & 0.0 & 0.0 \\ 1.0 & 0.0 & 0.0 \\ 1.0 & 0.5 & 0.0 \\ 0.0 & 0.0 & 1.0 \end{pmatrix} \circ \begin{pmatrix} 1.0 & 1.0 & 0.0 & 0.0 & 0.0 \\ 0.0 & 0.0 & 1.0 & 1.0 & 0.5 \\ 1.0 & 0.0 & 0.0 & 0.0 & 1.0 \end{pmatrix}$$

Ordinal Factor Analysis

The factors can be divided into conceptually meaningful subsets;

They can be interpreted as many-valued factors.

ordinal factor – a chain of conceptual factors.

B. Ganter, C. Glodeanu
Ordinal Factor Analysis.
ICFCA'12, LNCS 7278 (2012) 128–139

$$I = \begin{pmatrix} 1 & 1 & 0 & 0 & 0 \\ 1 & 1 & 0 & 0 & 0 \\ 1 & 1 & 1 & 1 & 0 \\ 1 & 0 & 0 & 0 & 1 \end{pmatrix} = \left(\begin{array}{cc:c} 1 & 0 & 0 \\ 1 & 0 & 0 \\ 1 & 1 & 0 \\ 0 & 0 & 1 \end{array}\right) \circ \left(\begin{array}{ccccc} 1 & 1 & 0 & 0 & 0 \\ 1 & 1 & 1 & 1 & 0 \\ \hdashline 1 & 0 & 0 & 0 & 1 \end{array}\right)$$

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$$I = \begin{pmatrix} 1 & 1 & 0 & 0 & 0 \\ 1 & 1 & 0 & 0 & 0 \\ 1 & 1 & 1 & 1 & 0 \\ 1 & 0 & 0 & 0 & 1 \end{pmatrix} = \left(\begin{array}{c:c} 1 & 0 \\ 1 & 0 \\ 2 & 0 \\ 0 & 1 \end{array}\right) \circ \left(\begin{array}{ccccc} 2 & 2 & 1 & 1 & 0 \\ \hdashline 1 & 0 & 0 & 0 & 1 \end{array}\right)$$

R. Belohlavek
Optimal decompositions of matrices with entries from residuated lattices.
Journal of Logic and Computation 22(6)(2012), pp 1405–1425.

B. Ganter, C. Glodeanu
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?

What is in the paper.

- study of ordinal factors for graded data
  structure of degrees $\mathbf{L}$= residuated lattice
  Context $\langle X, Y, I\rangle$; X, Y – ordinary sets, I – $\mathbf{L}$-relation. Concept-forming operators:

$$A^{\uparrow}(y) = \bigwedge_{x \in X} A(x) \to I(x, y)$$
$$B^{\downarrow}(x) = \bigwedge_{y \in Y} B(y) \to I(x, y)$$

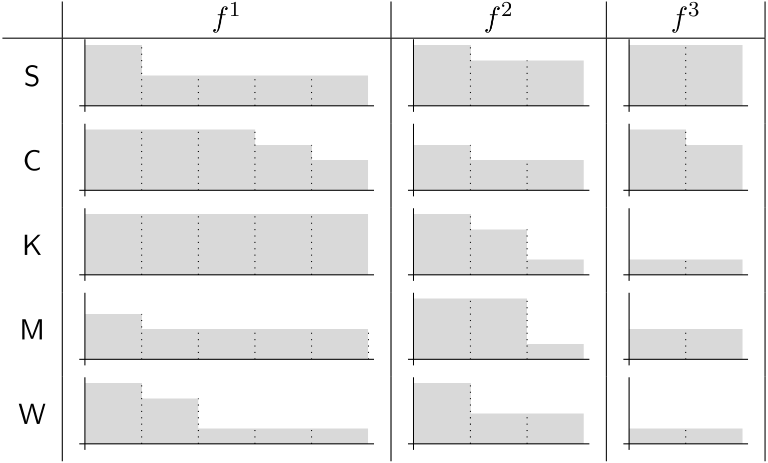
- illustration of factorization of a decathlon data set

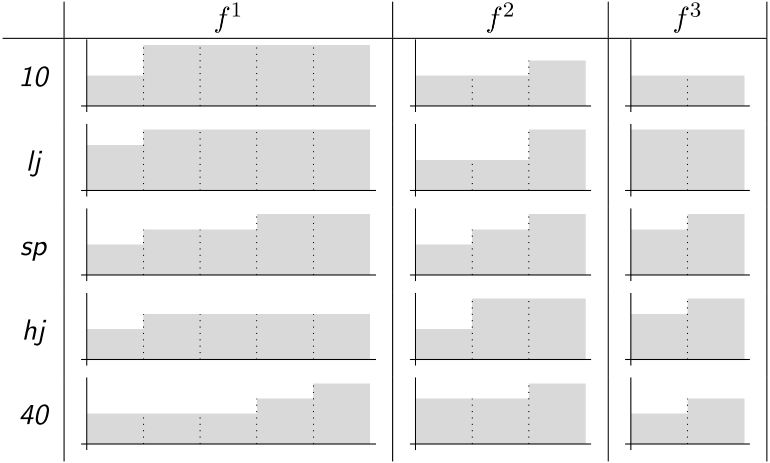
Remarks

- analogous results as in the crisp case
- nontrivial extension

	10	*lj*	*sp*	*hj*	*40*	*hu*	*di*	*pv*	*ja*	*15*
S: Sebrle	0.50	1.00	1.00	1.00	0.75	1.00	0.75	0.75	1.00	0.75
C: Clay	1.00	1.00	0.75	0.75	0.50	1.00	1.00	0.50	1.00	0.50
K: Karpov	1.00	1.00	1.00	0.75	1.00	1.00	1.00	0.25	0.25	0.75
M: Macey	0.50	0.50	0.75	1.00	0.75	0.75	0.75	0.25	0.50	1.00
W: Warners	0.75	0.75	0.50	0.50	0.75	1.00	0.25	0.50	0.25	0.75

Figure : Scores of top 5 athletes in the 2004 Olympic Decathlon scaled into 5-element chain. The abbreviations of the attributes have the following meaning: *10* – 100 meters sprint race; *lj* – long jump; *sp* – shot put; *hj* – high jump; *40* – 400 meters sprint race; *hu* – 110 meters hurdles; *di* – discus throw; *pv* – pole vault; *ja* – javelin throw; *15* – 1500 meters run.

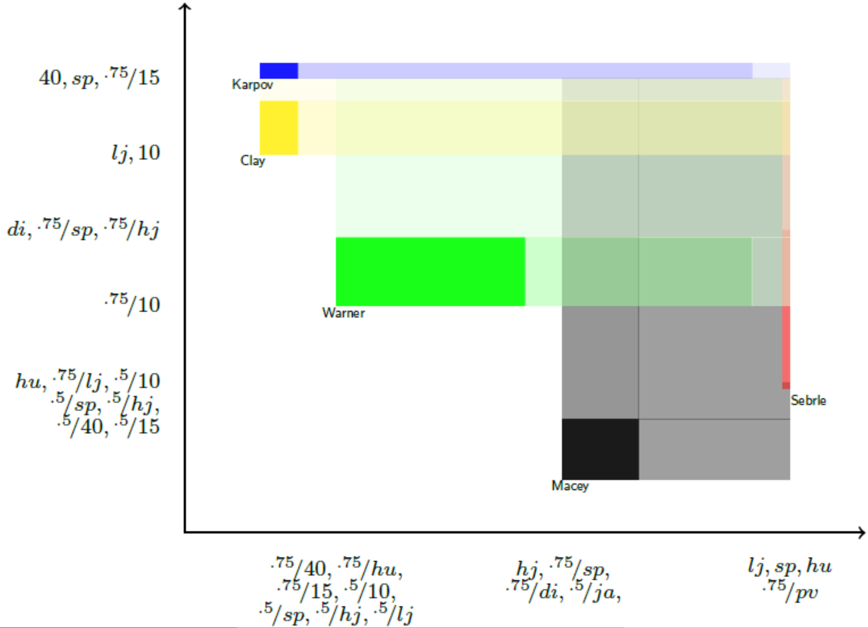

f^1
f^2
f^3
S
C
K
M
W



. . .

interpretation?

Some attempt...



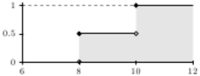
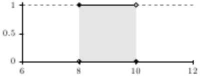
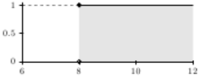
Does this bring better understanding of the data?

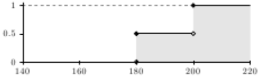
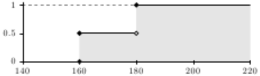
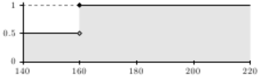
Does this bring better understanding of the data?

NO

On the other hand, it resembles this ...

	consumption	speed
F_1	quite high	fast
F_2	8-10l/100 km	quite fast
F_3	at least 8l/100 km	not so fast as F_2
F_4	at least 8l/100 km	fast

consumption	
quite high	
8-10l/100 km	
at least 8l/100 km	

speed (km/h)	
fast	
quite fast	
not so fast as F_2	

. . . can be interpreted as many-valued factors.

Ganter, B., Glodeanu, C.:
Ordinal Factor Analysis.
ICFCA'12, LNCS 7278 (2012) 128–139

Present approach . . . fuzzy-valued factors

as in

Silke Pollandt.
Fuzzy Begriffe.
Springer Verlag, 1997.

. . . almost . . . isotone concept-forming operators

Isotone concept-forming operators ${}^{\cap}, {}^{\cup}$

For $\mathbf{L}$-context $\langle X, Y, I\rangle$

$$A^{\cap}(y) = \bigvee_{x \in X} A(x) \otimes I(x, y)$$
$$B^{\cup}(x) = \bigwedge_{y \in Y} I(x, y) \to B(y)$$

A. Popescu
A general approach to fuzzy concepts.
Math. Log. Quart. 50 (2004), 1-17

G. Georgescu, A. Popescu
Non-dual fuzzy connections.
Archive for Mathematical Logic 43 (2004)

subconcept–superconcept hierarchy $\leqslant$

$$\langle A_1, B_1\rangle \leqslant \langle A_2, B_2\rangle \text{ iff } A_1 \subseteq A_2 (\text{iff } B_1 \subseteq B_2)$$

Fuzzy concept lattice ($\mathbf{L}$-concept lattice) of $\langle X, Y, I\rangle$ w.r.t. $\langle {}^{\cap}, {}^{\cup}\rangle$

$$\mathcal{B}^{\cap\cup}(X, Y, I) = \{\langle A, B\rangle \mid A^{\cap} = B, B^{\cup} = A\} + \leqslant$$

R. Belohlavek
Optimal triangular decompositions of matrices with entries from residuated lattices.
Int. Journal of Approximate Reasoning 50(8)(2009), 1250-1258

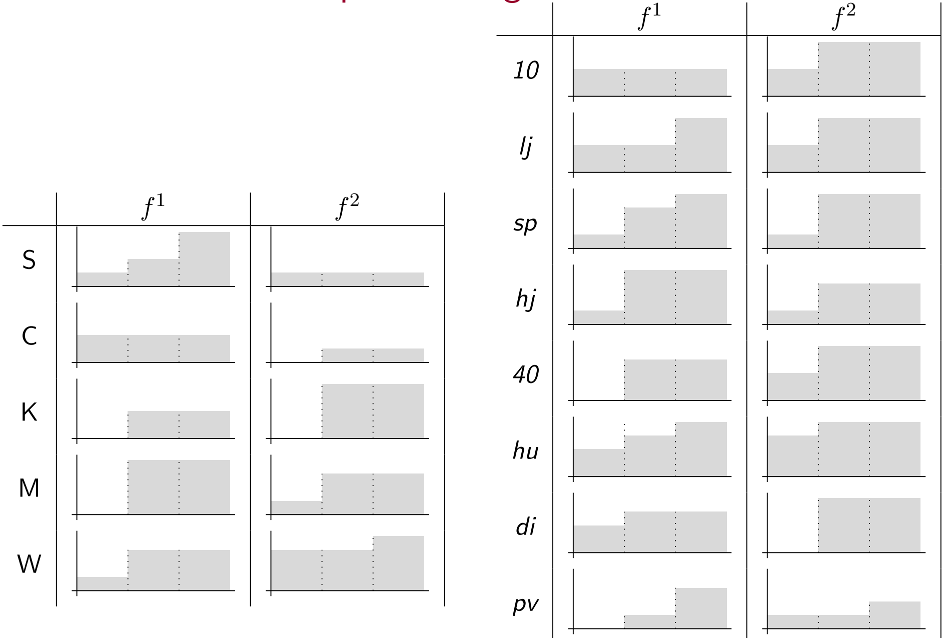
Formal concepts from $\mathcal{B}^{\cap\cup}(X, Y, I)$ are universal and optimal factors for

$$I = A \triangleleft B.$$

$$(A \triangleleft B)(x, y) = \bigwedge_{f \in F} A(x, f) \rightarrow B(f, y)$$

L. J. Kohout, W. Bandler.
Relational-product architectures for information processing.
Information Sciences, 37(1-3):25–37, 1985.

Decathlon data decomposed using 2 factors



For better interpretation...crisply generated ordinal factors

Fuzzy concepts generated by crisp sets

Denote

$$\mathcal{B}_{\mathrm{c}}(X,Y,I) = \{\langle A^{\uparrow\downarrow}, A^{\uparrow}\rangle \mid A \in \mathbf{2}^X\}$$

and similarly

$$\mathcal{B}_{\mathrm{c}}^{\cap\cup}(X,Y,I) = \{\langle A^{\cap\cup}, A^{\cap}\rangle \mid A \in \mathbf{2}^X\}$$

We look for factors...

$\mathcal{F} \subseteq \mathcal{B}_{\mathrm{c}}(X,Y,I)$

$$I = A_{\mathcal{F}}^{*} \circ B_{\mathcal{F}}$$

or $\mathcal{F} \subseteq \mathcal{B}_{\mathrm{c}}^{\cap\cup}(X,Y,I)$

$$I = A_{\mathcal{F}}^{*} \triangleleft B_{\mathcal{F}}$$

with $\mathcal{F}$ containing as small number of incomparable elements as possible.

$*$ denotes the globalization

Two main ways how to achieve it (first one)

R. Belohlavek
Optimal decompositions of matrices with entries from residuated lattices.
Journal of Logic and Computation 22(6)(2012), pp 1405–1425.

R. Belohlavek
Sup-t-norm and inf-residuum are one type of relational product: unifying framework and consequences.
Fuzzy Sets and Systems 197(2012), 45-58.

Two main ways how to achieve it (second one)

E. Bartl, R. Belohlavek, J. Konecny
Optimal decompositions of matrices with grades into binary and graded matrices.
Annals of Mathematics and Artificial Intelligence 59(2)(2010), 151-167.

Formal fuzzy concepts in $\mathcal{B}_{\mathrm{c}}(X, Y, I)$ are universal and optimal factors for

$$I = A^{*}_{\mathcal{F}} \circ B_{\mathcal{F}}.$$

Approximation algorithms for finding optimal $\mathcal{F}$

Summary

- Properties fuzzy ordinal factors (for both, $\circ, \triangleleft$),
- Properties of crisply generated fuzzy ordinal factors (for both, $\circ, \triangleleft$),
- Relationship to the general framework,
- Algorithms for finding crisply generated fuzzy ordinal factors (for both, $\circ, \triangleleft$).

Summary

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- Relationship to the general framework,
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THANK YOU