Classifying learners based on skills

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Questions are (a) Compute the set of all divisors of 230. (b) Compute the greatest common divisor of 275 and 385. (c) Compute the least common multiple of 275 and 385. (d) Compute all common divisors of 172 and 258.

	a	b	c	d
Huey		×	×	
Dewey		×		
Louie	×			×
Doofus			×	

Data are formalized by Formal Concept Analysis [5] into a formal context $\langle L, Q, \square\rangle$, where L is a set of learners, Q is a set of questions and $l\square q$ expresses that the learner l masters the question q.

Skills: (x) computing the greatest common divisor by Euclidean algorithm, (y) knowing relationship between the greatest common divisor and the least common multiple, (z) computing all divisors of a given number, and a skill function is given in Figure **??**.

Skill function:

q	a	b	c	d
$\sigma(q)$	$\{\{z\}\}$	$\{\{z\},\{x\}\}$	$\{\{x,y\}\}$	$\{\{x,z\}\}$

Let S be a set of skills. A skill function [2] is a mapping $\sigma : Q \to \mathbf{2}^{\mathbf{2}^S}$ such that $\sigma(q)$ is an antichain for each $q \in Q$.

Theoretical competence model assigning to a learning state mastered skills:

	x	y	z
s_1			
s_2			×
s_3	×		×
s_4		×	×
s_5	×	×	×

A theoretical competence model is given by a formal context $\langle T, S, *\rangle$ where T is a set of learning states and $t * s$ expresses that in state t skill s is mastered. The concept lattice $\mathcal{B}(T, S, *)$ plays the role of a knowledge space [3].

Our goal is to assign to each learner an appropriate concept of $\mathcal{B}(T, S, *)$ via a mapping α.

Dependencies between skills:

R	x	y	z
x	1	0	0.3
y	0.4	1	0.5
z	0	0	1

For expressing dependencies between skills we use fuzzy logic [4] where a statement is true in the certain degree which is taken from a residuated lattice. Logical connectives are interpreted by operations of a residuated lattice. Let $\mathbf{L}$ be a residuated lattice and R be a binary $\mathbf{L}$-relation on the set S. The value $R(s_1, s_2)$ expresses the degree to which the following proposition is true: If a learner has the skill s_1, then the learner also has the skill s_2. Following natural properties has to be satisfied: for each $x, y, z \in S$ it holds $R(x, x) = 1$ (reflexivity) and $R(x, y) \otimes R(y, z) \leq R(x, z)$ (transitivity). An example of an $\mathbf{L}$-relation R is given in the figure above. Łukasiewicz chain $\mathbf{L} = \{0, 0.1, 0.2, \ldots, 0.9, 1\}$ is used as the structure of truth degrees in the example.

The $\mathbf{L}$-relation R can be extended to subsets of S by the following [1]. For each $A, B \in \mathbf{2}^S$ we set

$$R^{\#}(A, B) = \left(\bigwedge_{x\in A} \bigvee_{y\in B} R(x, y)\right) \wedge \left(\bigwedge_{y\in B} \bigvee_{x\in A} R(x, y)\right)$$

Possibilities of mastered skill by learners:

l	l_1	l_2	l_3	l_4
$\beta(l)$	$\{\{x,y,z\},\{x,y\}\}$	$\{\{z\},\{x\}\}$	$\{\{x,z\}\}$	$\{\{x,y\}\}$

For a learner we compute possibilities of mastered skills by $\beta(l) = \dot{\bigcup}\sigma(\{l\}^{\uparrow\square})$, where $\dot{\bigcup}\{A_1, \ldots A_n\} = \{\bigcup\{S_1, \ldots, S_n\} \mid S_1 \in A_1, \ldots, S_n \in A_n\}$.

Suitability of concepts of the knowledge space for describing learners:

γ	l_1	l_2	l_3	l_4
$\emptyset$	0	0	0	0
$\{z\}$	0.3	1	0.3	0.3
$\{x,z\}$	0.5	0.3	1	0.5
$\{y,z\}$	0.3	0	0	0.3
$\{x,y,z\}$	1	0	0	0.5

The degree to which a concept $\langle A, B\rangle \in \mathcal{B}(T, S, *)$ is suitable to describe the learning state of a learner l is given by $\gamma(\langle A, B\rangle, l) = \bigvee_{S\in\beta(l)} R^{\#}(S, B)$.

A function α for a learner l chooses a concept $\langle A, B\rangle$ which maximizes the value $\gamma(\langle A, B\rangle, l)$. From ambiguity of choice several possible mappings α_i arise. For the learners from the example we obtain two possible mappings α_1, α_2 as shown in Figure **??**.

Two possibilities of classifying learners in the knowledge space:

l	l_1	l_2	l_3	l_4
$\alpha_1(l)$	$\{x,y,z\}$	$\{z\}$	$\{x,z\}$	$\{x,z\}$
$\alpha_2(l)$	$\{x,y,z\}$	$\{z\}$	$\{x,z\}$	$\{x,y,z\}$

Using mapping α we can make interesting observations about learners. For example, the learner l_1 solved only difficult problems. From the fact that intent of $\alpha_1(l_1)$ and also $\alpha_2(l_1)$ is $\{x, y, z\}$ we can conclude that the learner has all the skills needed for solving all problems. On the other hand, the learner l_4 solved only the most difficult problem c. Mappings α_1 and α_2 differ in classifying this learner. Since according α_1 the learner has skills $\{x, z\}$, but for solving problem c one needs skills $\{x, y\}$, so we may be suspicious that the learner has been cheating.

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