Experimental quantum information processing exploiting combination of single-photon and two-photon interference

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Outline of the talk

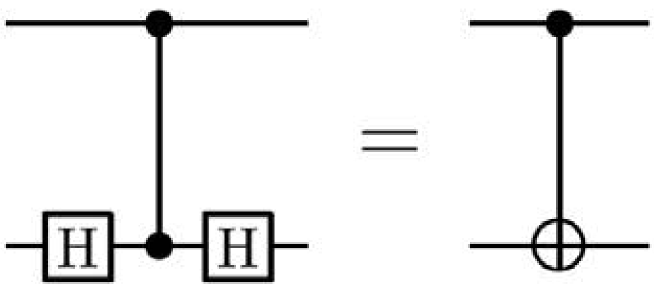
1. **Linear optical quantum CCZ/Toffoli gate**
2. **Perfect orthogonalization of partly unknown quantum states**
3. **Optimal entanglement assisted discrimination of projective quantum measurements**

All three experiments combine polarization and spatial encoding of quantum information into states of single photons and involve bulk-optics or fiber-based interferometers.

Linear optical quantum CZ/CNOT gate

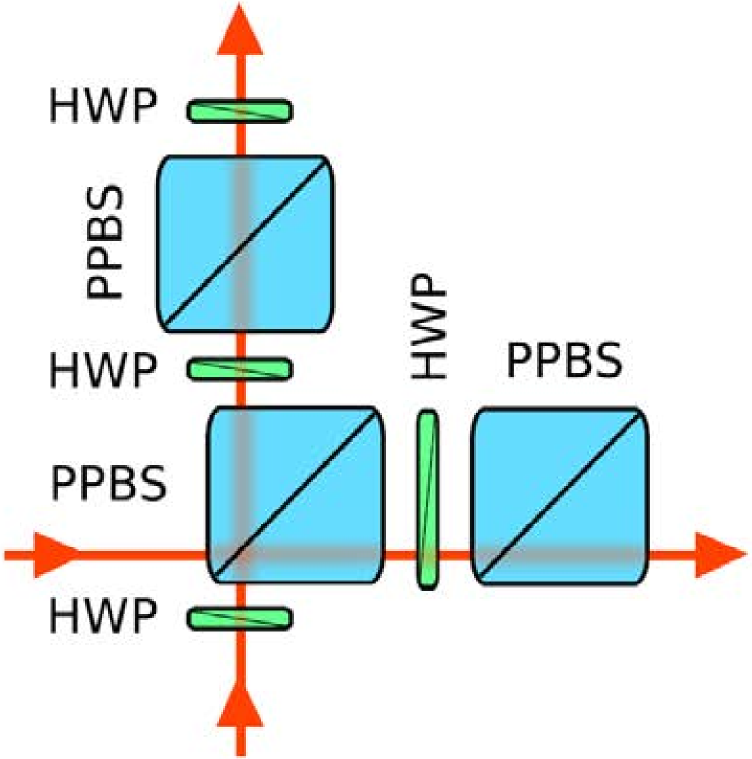
CZ$= I - 2\,|11\rangle\langle 11|$

CZ $|jk\rangle = (-1)^{jk}\,|jk\rangle$



$|0\rangle = |H\rangle,\ |1\rangle = |V\rangle$

PPBS: $T_H = 1,\ T_V = 1/3$



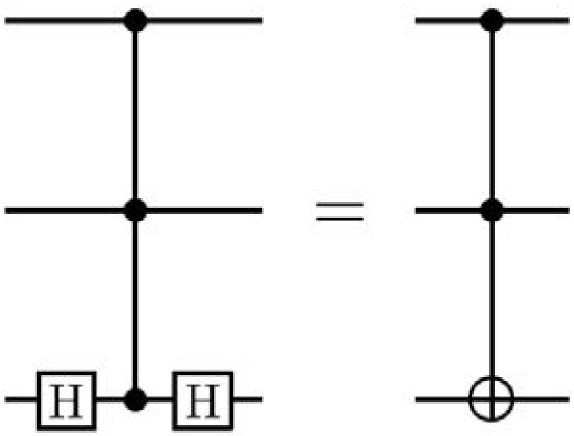
R. Okamoto, H.F. Hofmann, S. Takeuchi, and K. Sasaki, Phys. Rev. Lett. 95, 210506 (2005)

N. K. Langford, T.J. Weinhold, R. Prevedel, K. J. Resch, A. Gilchrist, J. L. OBrien, G. J. Pryde, and A. G. White, Phys. Rev. Lett. 95, 210504 (2005)

N. Kiesel, C. Schmid, U. Weber, R. Ursin, and H. Weinfurter, Phys. Rev. Lett. 95, 210505 (2005)

Linear optical quantum CCZ/Toffoli gate

$\text{CCZ} = I - 2\,|111\rangle\langle 111|$ $\quad\quad$ $\text{CCZ}\,|jkl\rangle = (-1)^{jkl}\,|jkl\rangle$



M. Mičuda, M. Sedlák, I. Straka, M. Miková, M. Dušek, M. Ježek, J. Fiurášek, Phys. Rev. Lett. 111, 160407 (2013)

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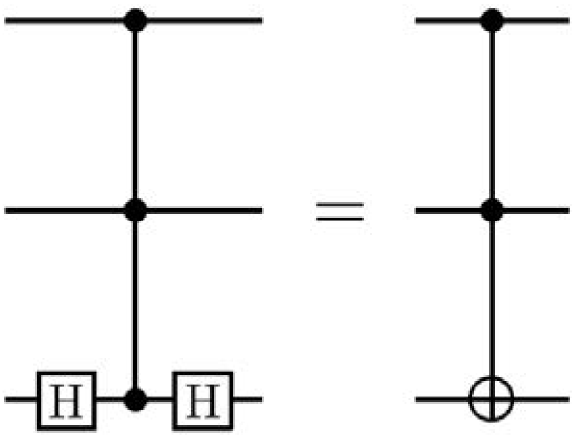

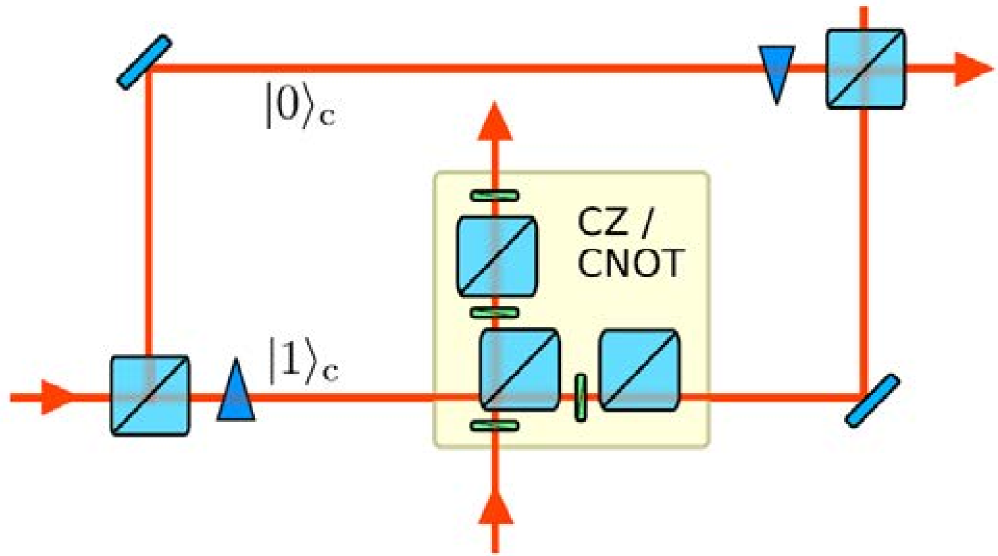

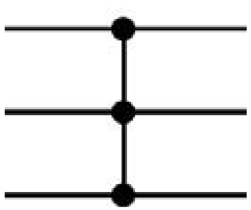

qubit 1: spatial degree of freedom of the first photon

qubit 2: polarization degree of freedom of the first photon

qubit 3: polarization degree of freedom of the second photon

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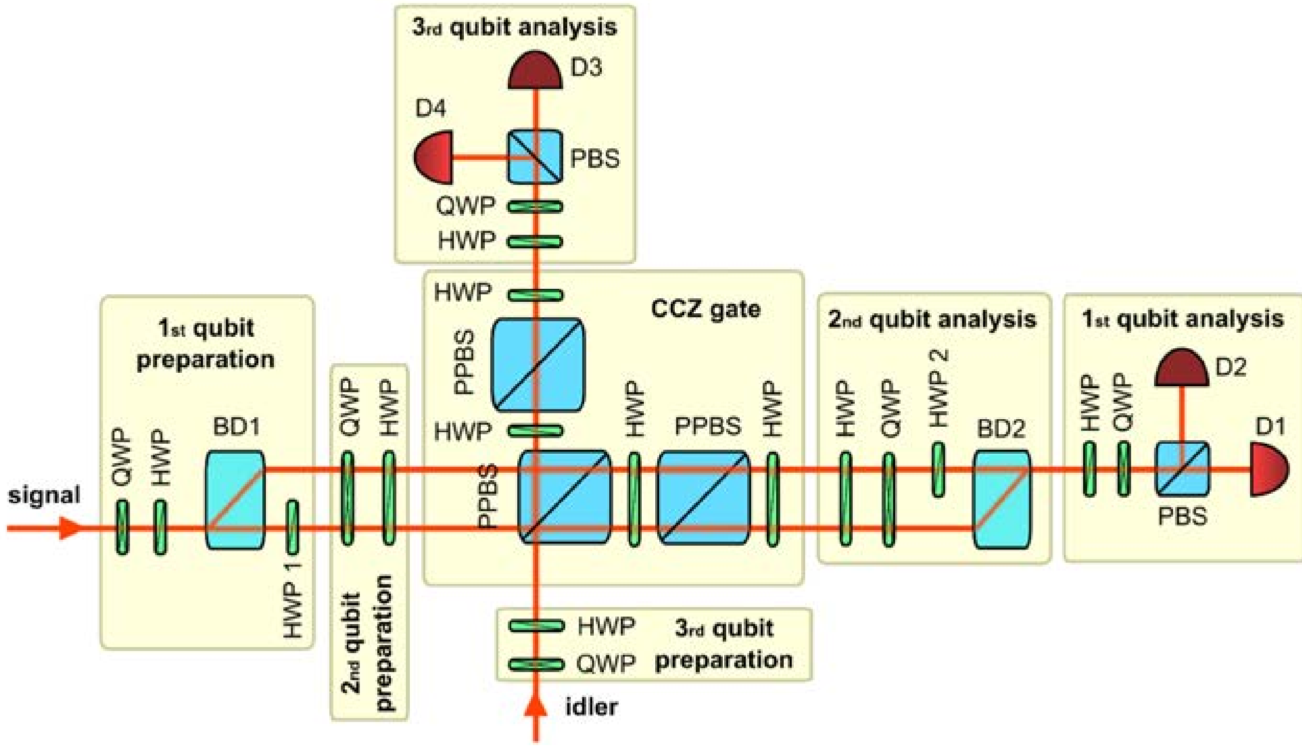
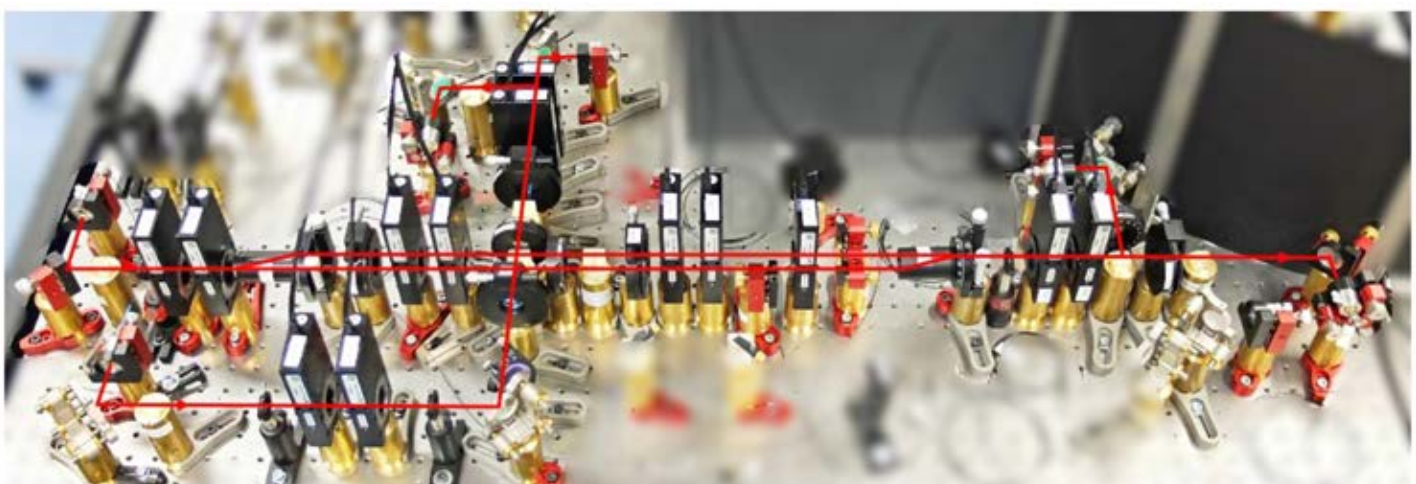
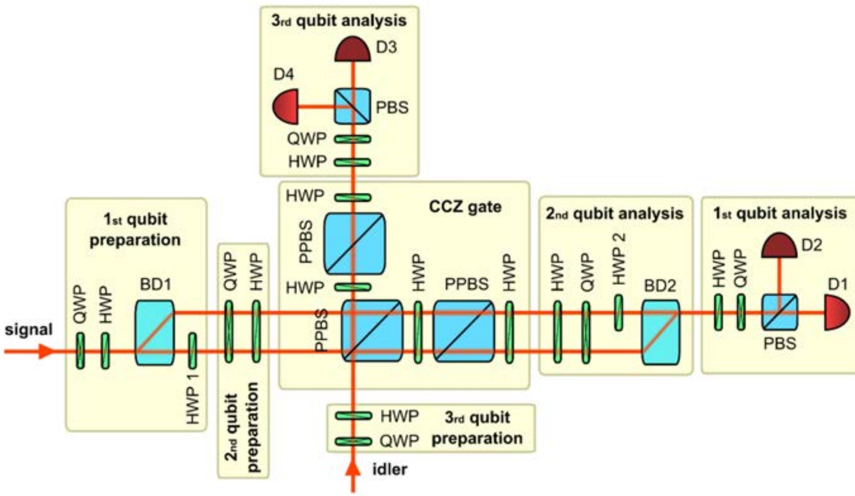
Linear optical CCZ gate – experimental setup





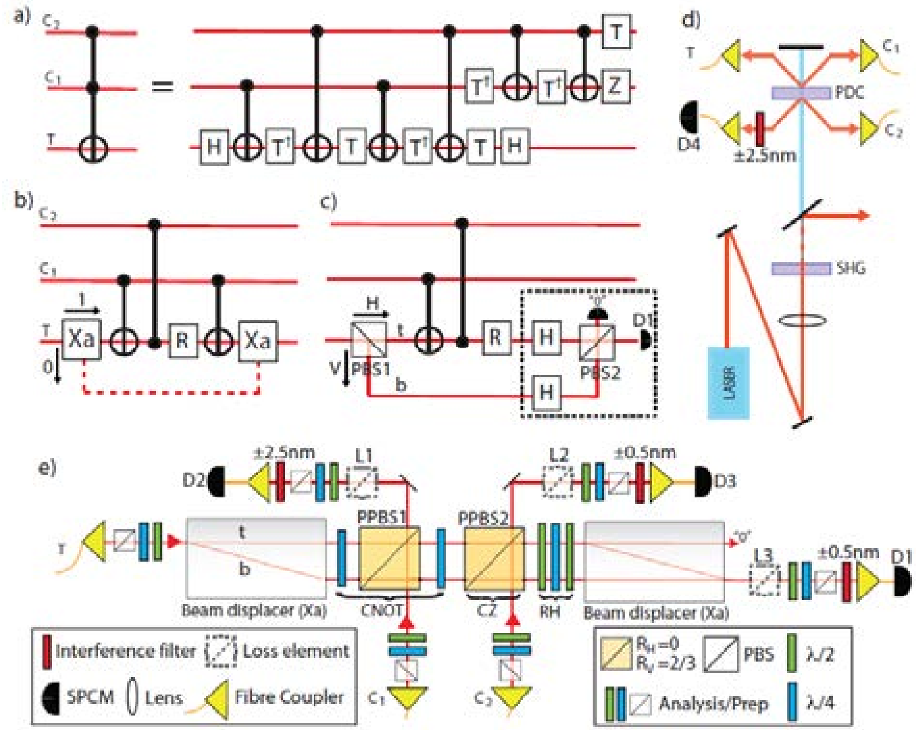
Comparison with Lanyon *et al.* scheme

$P_S=1/9$



M. Mičuda, M. Sedlák, I. Straka, M. Miková, M. Dušek, M. Ježek, J. Fiurášek, Phys. Rev. Lett. 111, 160407 (2013).

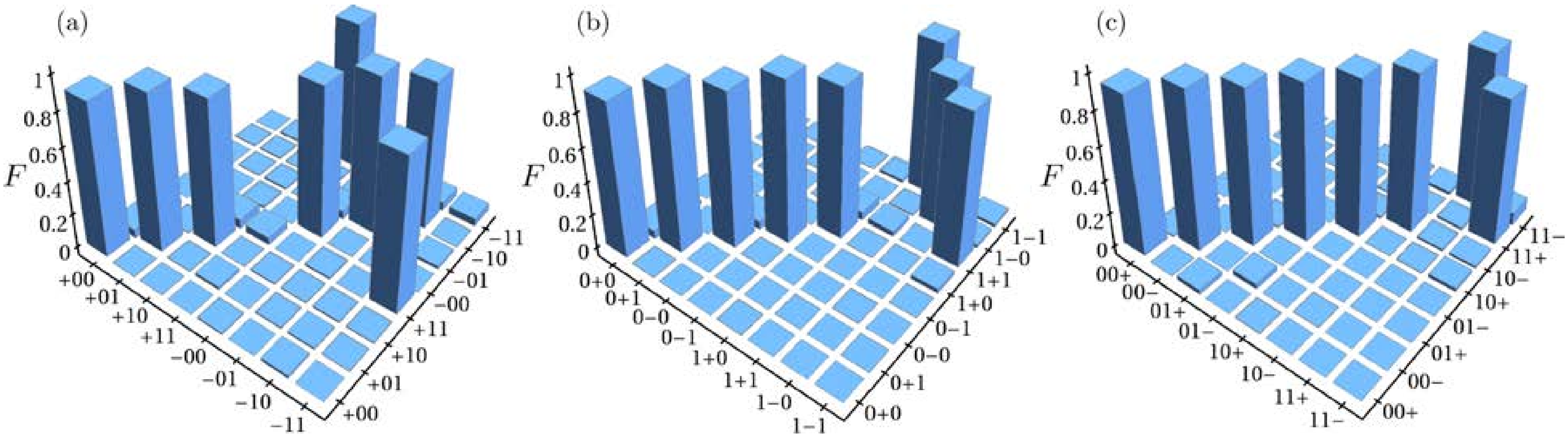
$P_S=1/72$

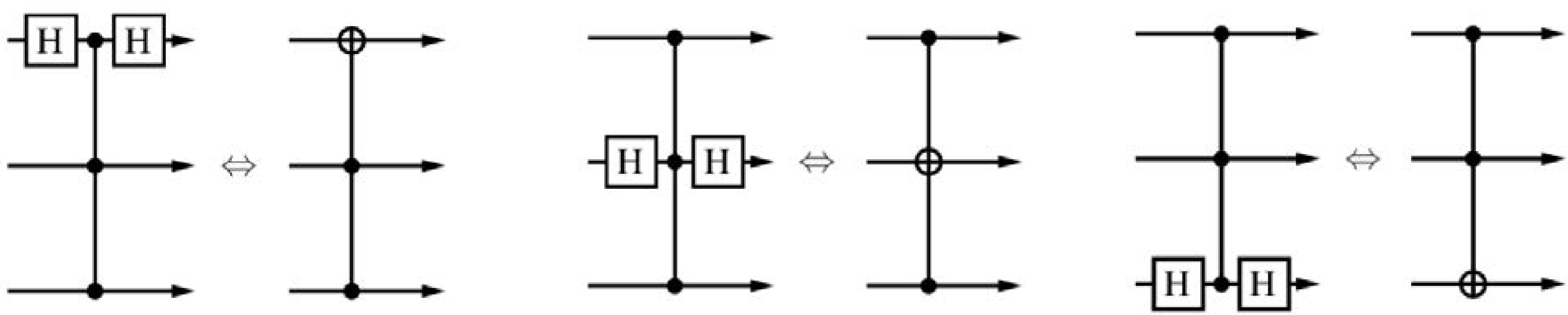


B.P. Lanyon, M. Barbieri, M.P. Almeida, T. Jennewein, T.C. Ralph, K.J. Resch, G.J. Pryde, J.L. O'Brien, A. Gilchrist, and A.G. White, Nature Phys. 5, 134 (2009).

Encoding each qubit in a seprate photon – four-photon coincident detection required which results in low coincidence rates.

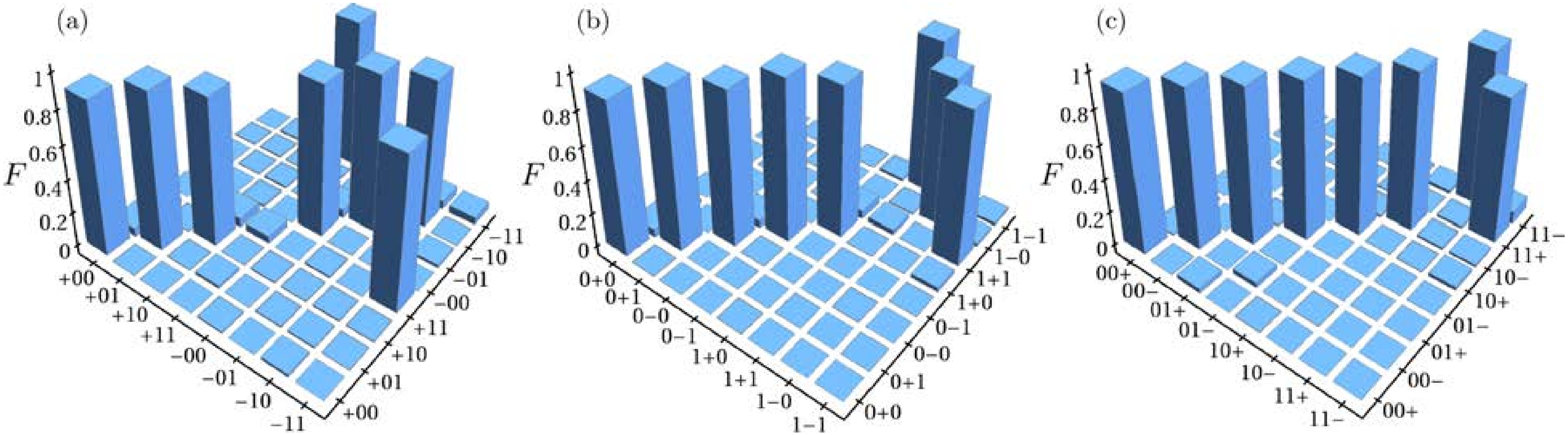
Measured truth tables for three product-state bases





Truth tables of three Toffoli gates, where the target qubit is the first, the second or the third qubit, respectively.

Generalized Hoffmann bound on gate fidelity



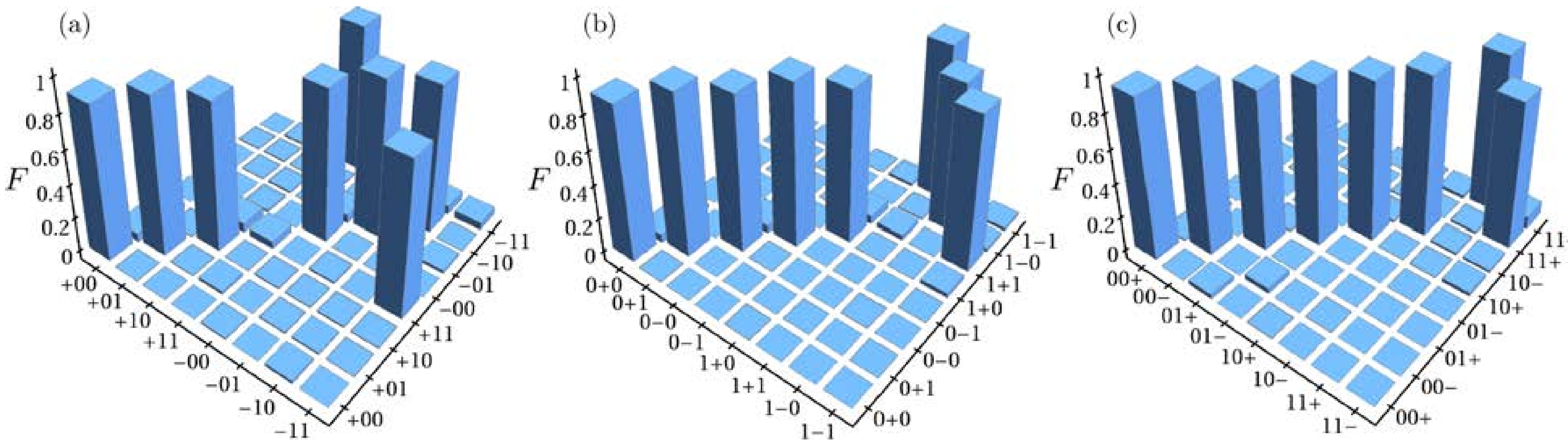
F_1=0.928(1) F_2=0.947(1) F_3=0.955(1)

Average state fidelities F_k – weighted averages with weights given by success probabilities of the gate for each input state.

H.F. Hofmann, Phys. Rev. Lett. 94,160504 (2005).

M. Mičuda, M. Sedlák, I. Straka, M. Miková, M. Dušek, M. Ježek, J. Fiurášek, Phys. Rev. Lett. 111, 160407 (2013).

Generalized Hoffmann bound on gate fidelity



F_1=0.928(1)

F_2=0.947(1)

F_3=0.955(1)

Average state fidelities F_k – weighted averages with weights given by success probabilities of the gate for each input state.

Lower bound on gate fidelity in terms of average state fidelities:

$$F_{CCZ} \geq F_1 + F_2 + F_3 - 2$$

F_{CCZ}>0.830(2)

H.F. Hofmann, Phys. Rev. Lett. 94,160504 (2005).

M. Mičuda, M. Sedlák, I. Straka, M. Miková, M. Dušek, M. Ježek, J. Fiurášek, Phys. Rev. Lett. 111, 160407 (2013).

Further characterization of gate fidelity

Original Hofmann bound

$$F_A + F_B - 1 \leq F_{CCZ} \leq \min(F_A, F_B)$$

F_A and F_B denote average state fidelities for two mutually unbiased bases

Requires measurement of fidelities of entangled output states – feasible with our setup due to encoding of two qubits into a single photon.

$0.876(2) < F_{CCZ} < 0.921(1)$

H.F. Hofmann, Phys. Rev. Lett. **94**,160504 (2005).
S. T. Flammia and Y.-K. Liu, Phys. Rev. Lett. **106**, 230501 (2011).
M. P. da Silva, O. Landon-Cardinal, and D. Poulin, Phys. Rev. Lett. **107**, 210404 (2011).

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$0.876(2) < F_{CCZ} < 0.921(1)$

Monte Carlo sampling of gate fidelity

Unbiased linear estimator of F_{CCZ}

Complete estimation of F_{CCZ} requires 4032 combinations of three-qubit preparations and measurements

$F_{CCZ} = 0.90$

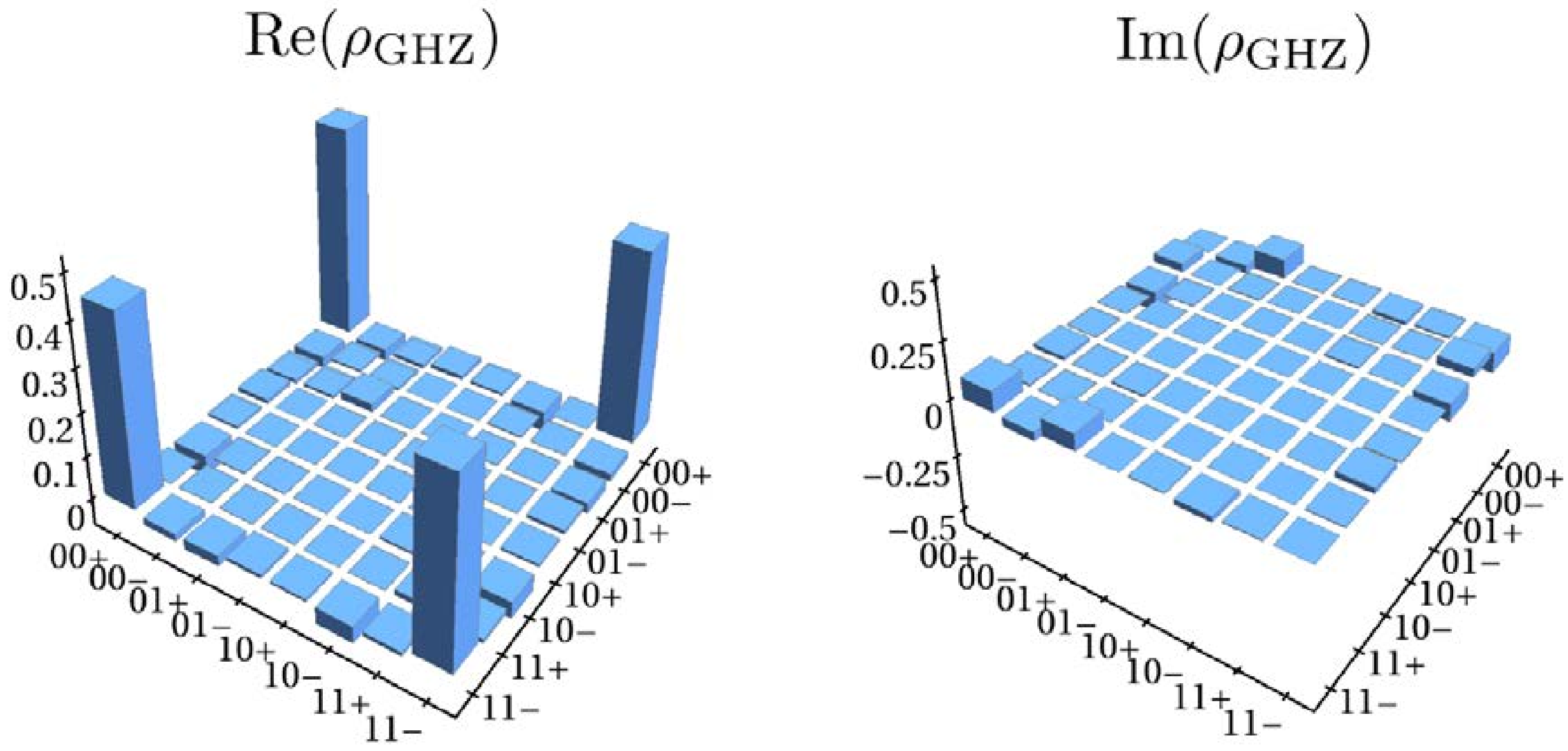
H.F. Hofmann, Phys. Rev. Lett. **94**,160504 (2005).

S. T. Flammia and Y.-K. Liu, Phys. Rev. Lett. **106**, 230501 (2011).

M. P. da Silva, O. Landon-Cardinal, and D. Poulin, Phys. Rev. Lett. **107**, 210404 (2011).

Generation of three-qubit GHZ state

$$\mathrm{CCZ} \cdot \mathrm{CNOT}_{12}\,|{+}0{+}\rangle = \mathrm{CCZ}\,|\Psi\rangle_{12}\,|{+}\rangle_3 = \frac{1}{\sqrt{2}}\left(|00{+}\rangle + |11{-}\rangle\right)$$



GHZ state purity 95%, fidelity 95%

M. Mičuda, M. Sedlák, I. Straka, M. Miková, M. Dušek, M. Ježek, J. Fiurášek, Phys. Rev. Lett. 111, 160407 (2013)

Outline of the talk

1. Linear optical quantum CCZ/Toffoli gate

2. **Perfect orthogonalization of partly unknown quantum states**

3. Optimal entanglement assisted discrimination of projective quantum measurements

Quantum universal NOT gate

Perfect quantum U-NOT gate is forbidden by the laws of quantum physics:

$$|\psi\rangle \not\rightarrow |\psi_\perp\rangle, \quad \langle\psi_\perp|\psi\rangle = 0$$

Optimal deterministic approximate U-NOT gate:

$$\mathcal{G}_{\mathrm{NOT}}(\rho) = (dI - \rho)\,/\,(d^2 - 1)$$

Minimum achievable average overlap between input and output states:

$$F_\perp(d) = \frac{1}{d+1}$$

V. Bužek, M. Hillery, and R.F. Werner, Phys. Rev. A **60**, 2626(R) (1999).

P. Rungta, V. Buzek, C.M. Caves, M. Hillery, and G. J. Milburn, Phys. Rev. A **64**, 042315 (2001).

F. De Martini, V. Bužek, F. Sciarrino, and C. Sias, Nature **419,** 815 (2002).

J. Fiurášek, Phys. Rev. A **70,** 032308 (2004).

Perfect orthogonalization of partly unknown quantum states

Required prior information: a mean value *a* of some operator *A*:

$$a = \langle\psi|A|\psi\rangle$$

Conditional orthogonalization by quantum filtration:

$$|\psi_\perp\rangle \propto (A - aI)\,|\psi\rangle \qquad \langle\psi_\perp|\psi\rangle = 0$$

M. R. Vanner, M. Aspelmeyer, and M. S. Kim, Phys. Rev. Lett. **110**, 010504 (2013).
M. Ježek, M. Mičuda, I. Straka, M. Miková, M. Dušek, J. Fiurášek, Phys. Rev. A **89**, 042316 (2014).

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This orthogonalization procedure is perfect but probabilistic:

$$p_\perp \leq \lambda^{-2}\langle\Delta A^\dagger \Delta A\rangle, \qquad \Delta A = A - aI$$

λ denotes the maximum singular value of ΔA.

M. R. Vanner, M. Aspelmeyer, and M. S. Kim, Phys. Rev. Lett. **110**, 010504 (2013).
M. Ježek, M. Mičuda, I. Straka, M. Miková, M. Dušek, J. Fiurášek, Phys. Rev. A **89**, 042316 (2014).

Orthogonalization of single-qubit states

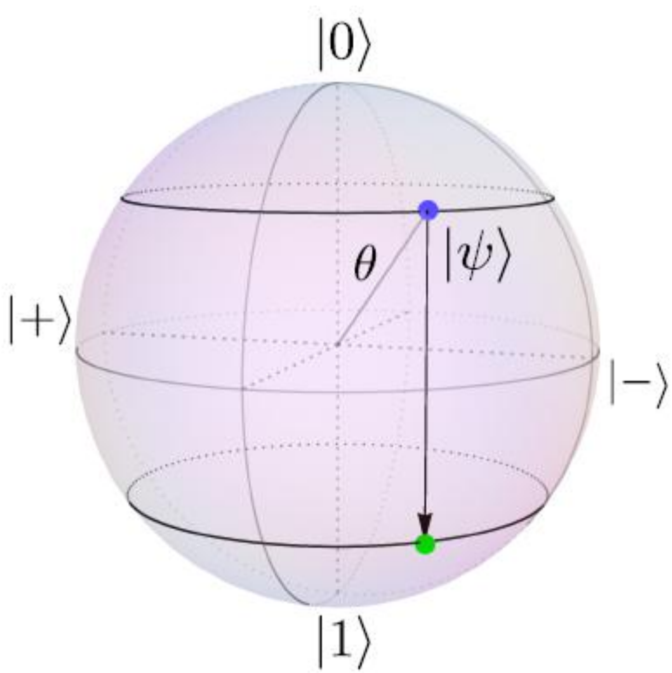
Bloch sphere parametrization: $$|\psi\rangle = \cos\frac{\theta}{2}|0\rangle + e^{i\phi}\sin\frac{\theta}{2}|1\rangle$$

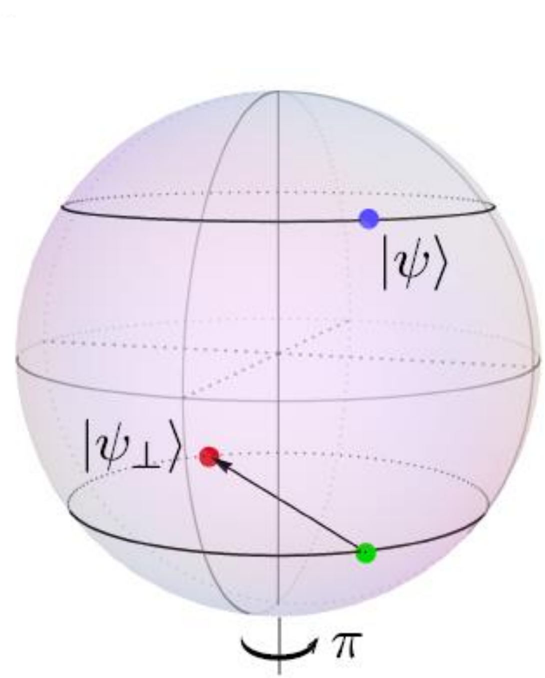
Prior knowledge – mean value of σ_Z:

$$\sigma_Z = |0\rangle\langle 0| - |1\rangle\langle 1|$$

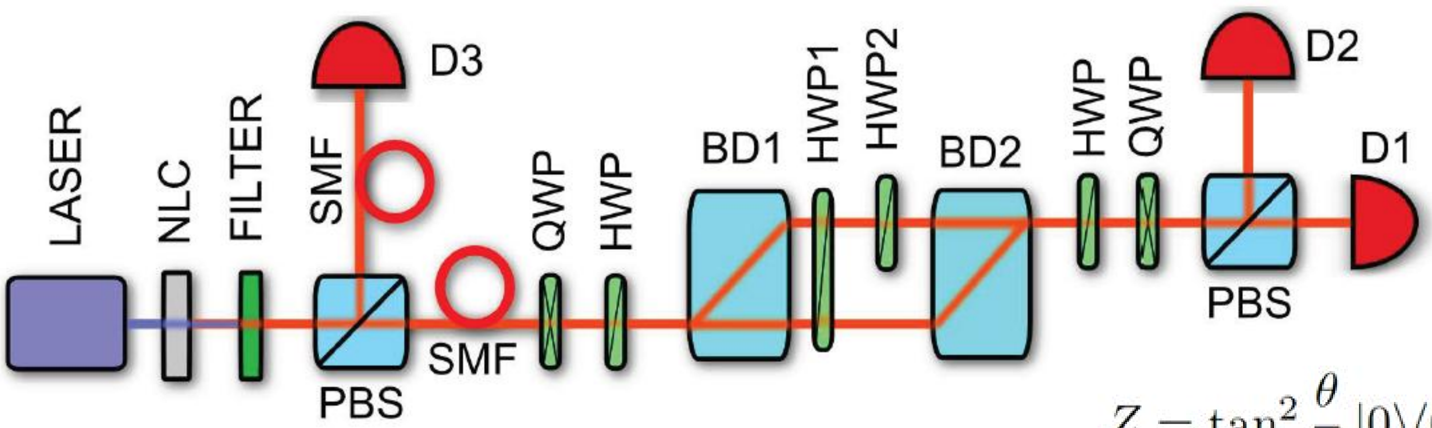
Quantum filter:

$$Z = \tan^2\frac{\theta}{2}\,|0\rangle\langle 0| - |1\rangle\langle 1|$$

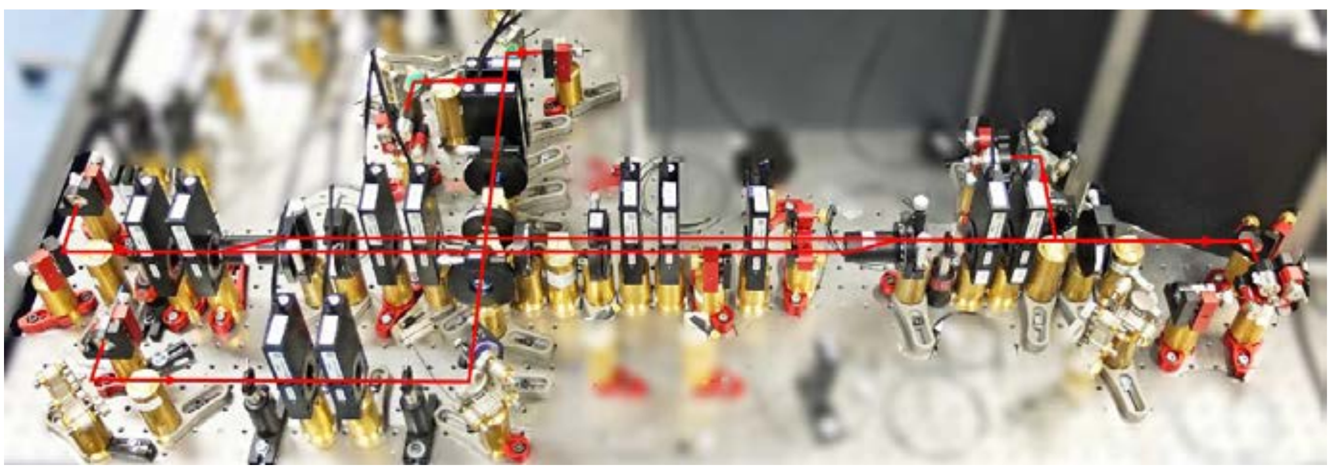




Experimental setup

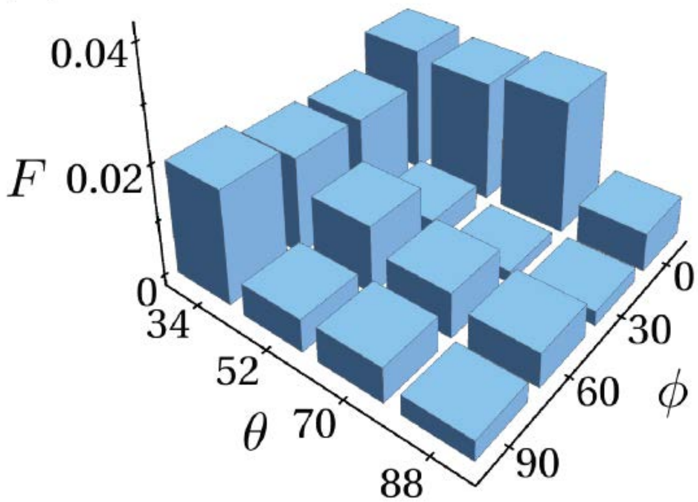


$$Z = \tan^2 \frac{\theta}{2} |0\rangle\langle 0| - |1\rangle\langle 1|$$



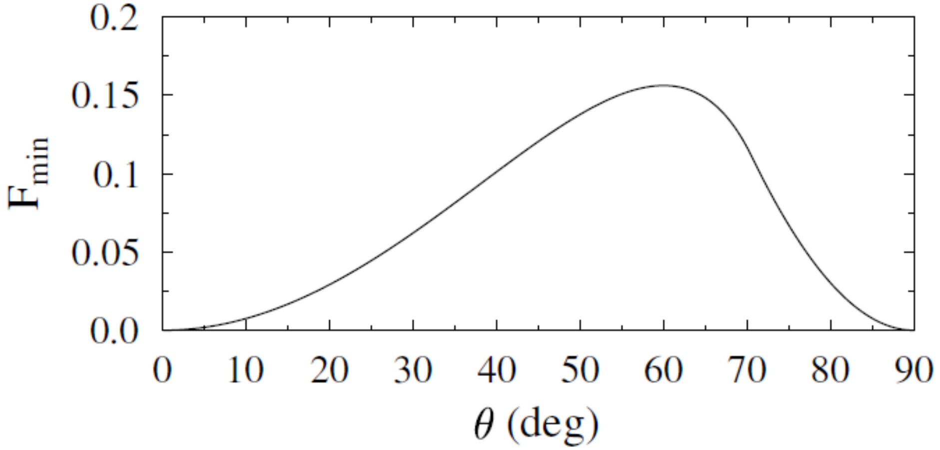
M. Ježek, M. Mičuda, I. Straka, M. Miková, M. Dušek, J. Fiurášek, Phys. Rev. A **89**, 042316 (2014).
M. Mičuda et al., Phys. Rev. Lett. **111**, 160407 (2013); Phys. Rev. A 89, 042304 (2014).

Experimental results



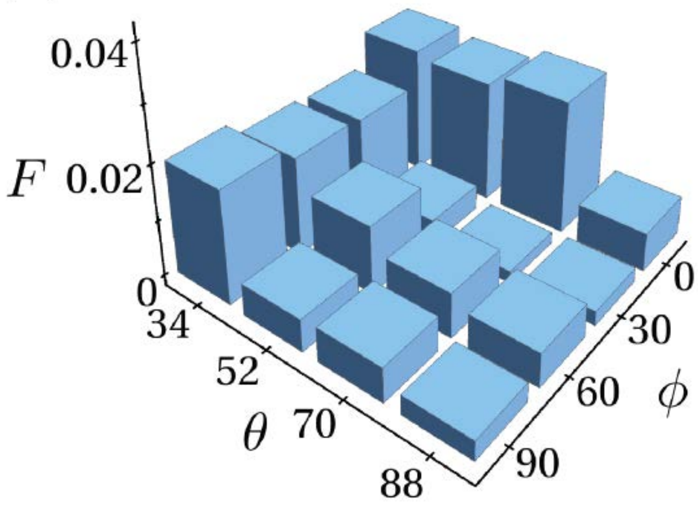
$$F = \left[\mathrm{Tr}\sqrt{\rho_1^{1/2}\rho_2\rho_1^{1/2}} \right]^2$$

Overlap between input and orthogonalized states



Minimum overlap achievable by deterministic operations

Experimental results



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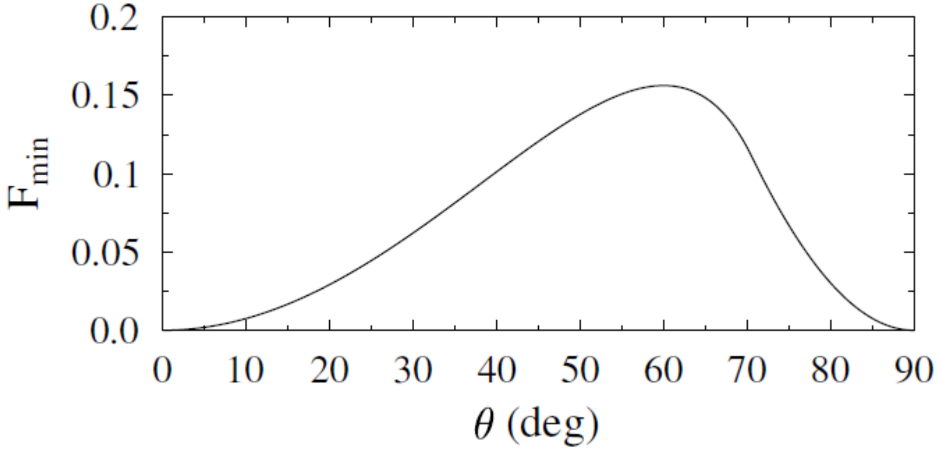

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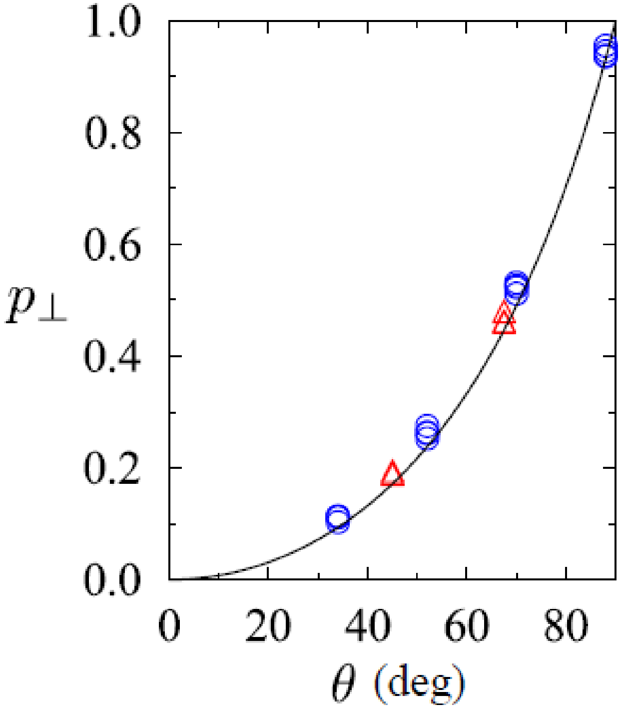


Success probability

Orthogonalization of entangled two-qubit states

Consider pure bipartite state

$$|\Psi\rangle_{12}$$

Prior information – knowledge of mean value of an operator A acting on subsystem 1:

$$a = \langle\Psi|A_1 \otimes I_2|\Psi\rangle$$

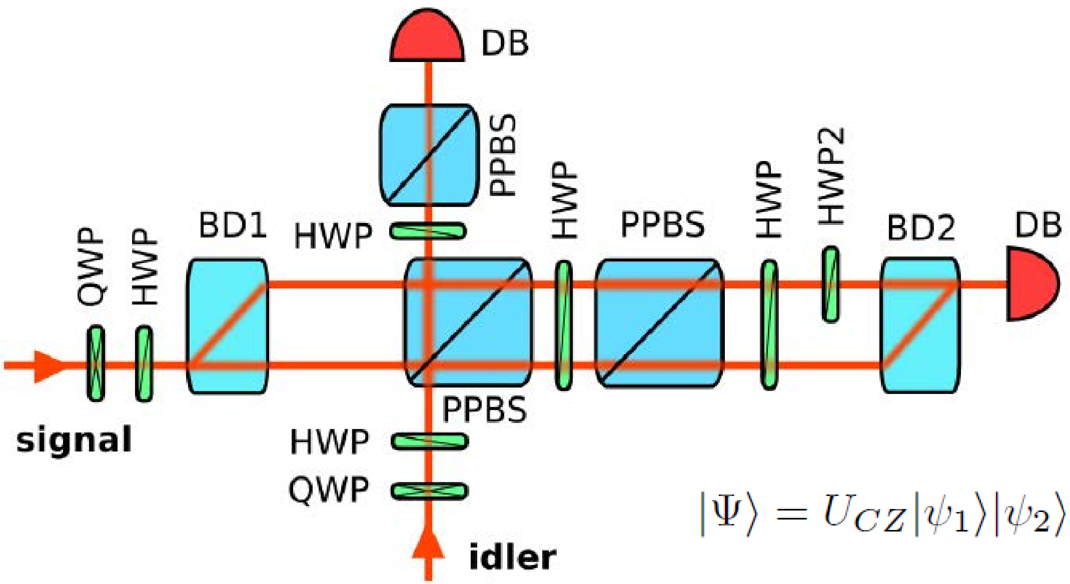
Orthogonalization by local filtering on a single subsystem:

$$|\Psi_\perp\rangle_{12} \propto (A - aI)_1 \otimes I_2|\Psi\rangle_{12}$$

In our experiment, we prepare various entangled two-qubit two-photon states using a linear optical quantum CZ gate.

M. Ježek, M. Mičuda, I. Straka, M. Miková, M. Dušek, J. Fiurášek, Phys. Rev. A **89**, 042316 (2014).

Experimental setup and results



Signal:

$$|\psi_1\rangle = \cos\frac{\theta_1}{2}|0\rangle + e^{i\phi_1}\sin\frac{\theta_1}{2}|1\rangle$$

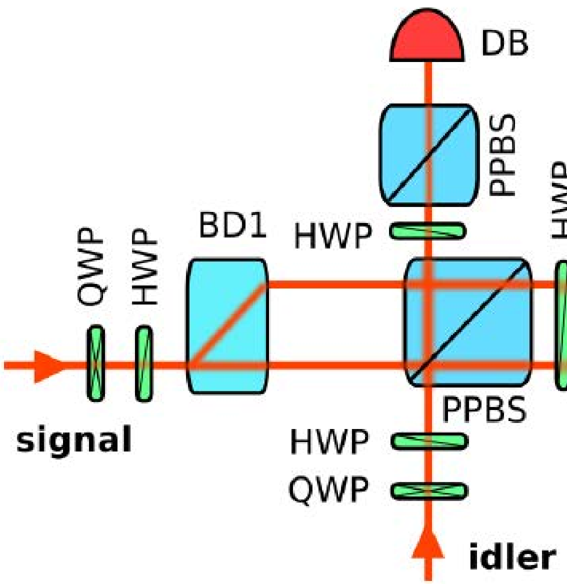
Idler:

$$|\psi_2\rangle = \cos\frac{\theta_2}{2}|0\rangle + e^{i\phi_2}\sin\frac{\theta_2}{2}|1\rangle$$

$$|\Psi\rangle = U_{CZ}|\psi_1\rangle|\psi_2\rangle = \cos\frac{\theta_1}{2}|0\rangle|\psi^+\rangle + e^{i\phi_1}\sin\frac{\theta_1}{2}|1\rangle|\psi^-\rangle$$

$$|\psi^\pm\rangle = \cos\tfrac{\theta_2}{2}|0\rangle \pm e^{i\phi_2}\sin\tfrac{\theta_2}{2}|1\rangle$$

Experimental setup and results



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$$|\Psi\rangle = U_{CZ}|\psi_1\rangle|\psi_2\rangle = \cos\frac{\theta_1}{2}|0\rangle|\psi^+\rangle + e^{i\phi_1}\sin\frac{\theta_1}{2}|1\rangle|\psi^-\rangle$$

$$|\psi^\pm\rangle = \cos\tfrac{\theta_2}{2}|0\rangle \pm e^{i\phi_2}\sin\tfrac{\theta_2}{2}|1\rangle$$

θ_1	ϕ_1	θ_2	ϕ_2	F	$\mathcal{P}_I$	$\mathcal{P}_O$
45°	0°	90°	0°	0.040	0.964	0.890
67.5°	0°	90°	0°	0.031	0.961	0.891
45°	0°	45°	0°	0.021	0.936	0.944
67.5°	0°	45°	0°	0.008	0.975	0.952
67.5°	90°	45°	90°	0.041	0.971	0.946

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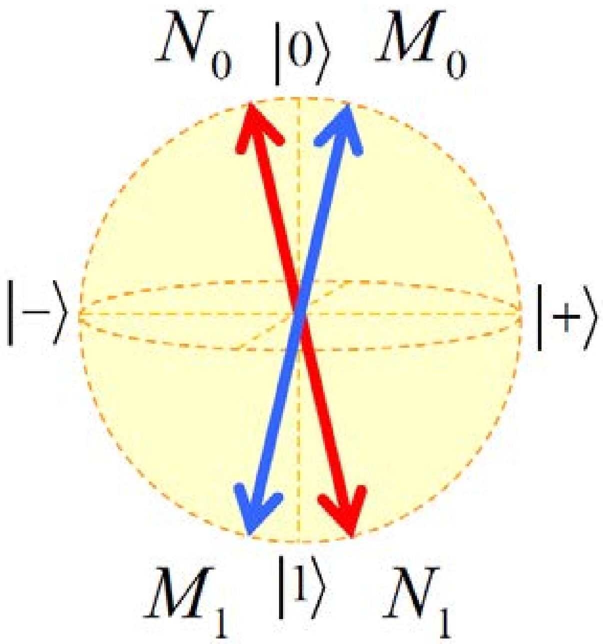
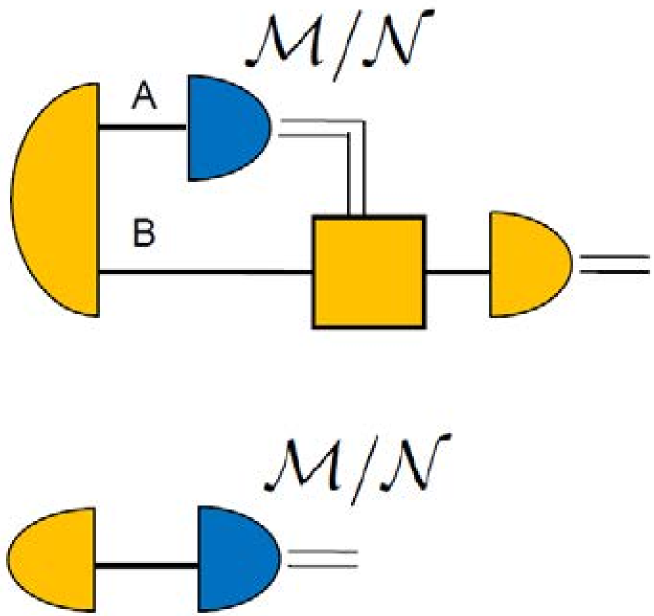
Discrimination of quantum measurements



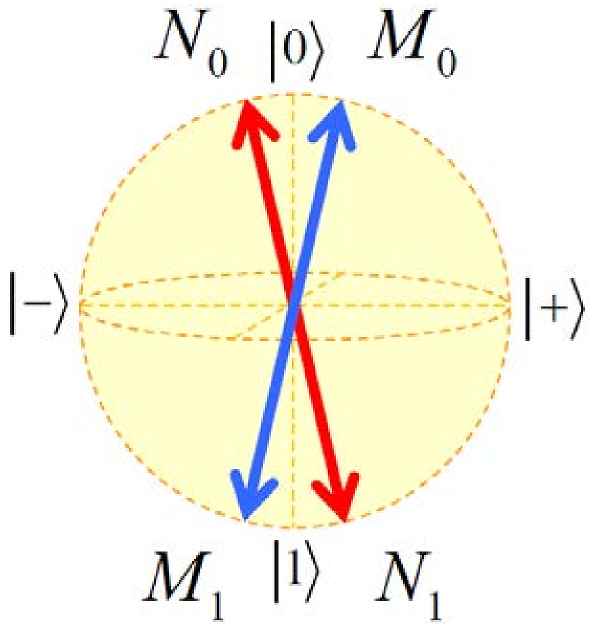


The task is to discriminate between two single-qubit projective measurements M and N when the measurement can be performed only once.

We consider a general discrimination strategy that can involve certain fraction of inconclusive outcomes.

M. Miková, M. Sedlák, I. Straka, M. Mičuda, M. Ziman, M. Ježek, M. Dušek, and J. Fiurášek (2014).

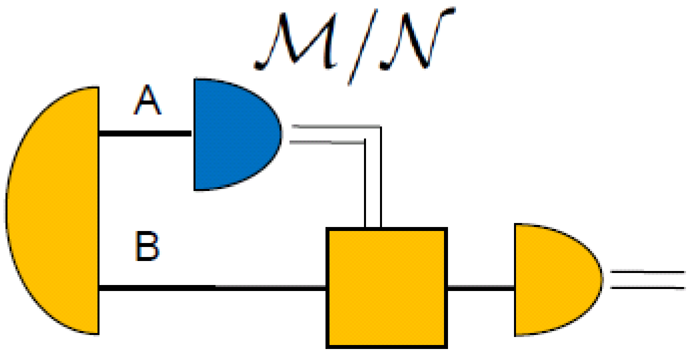
Measurement bases



$$M_0 = |\phi\rangle\langle\phi|, \qquad M_1 = |\phi^\perp\rangle\langle\phi^\perp|,$$
$$N_0 = |\psi\rangle\langle\psi|, \qquad N_1 = |\psi^\perp\rangle\langle\psi^\perp|,$$

$$|\phi\rangle = \cos\theta|0\rangle + \sin\theta|1\rangle, \quad |\phi^\perp\rangle = \sin\theta|0\rangle - \cos\theta|1\rangle,$$
$$|\psi\rangle = \cos\theta|0\rangle - \sin\theta|1\rangle, \quad |\psi^\perp\rangle = \sin\theta|0\rangle + \cos\theta|1\rangle,$$

Entanglement-assisted discrimination procedure



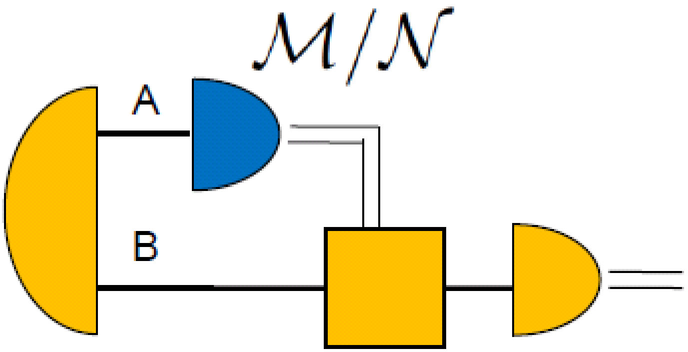
Prepare an entangled state of qubits A and B.

Perform the measurement M/N on qubit A.

Measure qubit B in a basis determined by the outcome of measurement on qubit A.

Guess M, N, or declare an inconclusive outcome depending on the measurement outcomes.

Entanglement-assisted discrimination procedure



Prepare an entangled state of qubits A and B.

Perform the measurement M/N on qubit A.

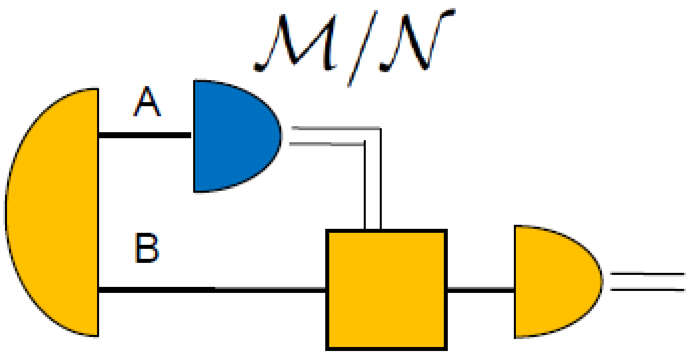
Measure qubit B in a basis determined by the outcome of measurement on qubit A.

Guess M, N, or declare an inconclusive outcome depending on the measurement outcomes.

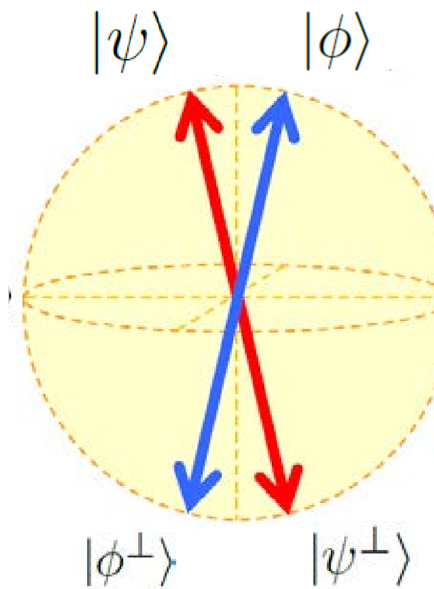
We assume equal a-priori probabilities of M and N. In this case it is optimal to employ a maximally entangled probe state:

$$|\Psi^-\rangle = \frac{1}{\sqrt{2}}(|01\rangle - |10\rangle)$$

Entanglement-assisted discrimination procedure



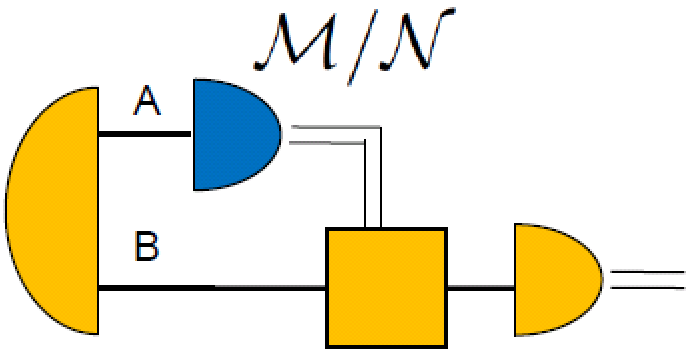
$$|\Psi^-\rangle = \frac{1}{\sqrt{2}}(|01\rangle - |10\rangle)$$



Outcome of measurement on A	State of qubit B if the measurement was M	State of qubit B if the measurement was N
0	$\|\phi^\perp\rangle$	$\|\psi^\perp\rangle$
1	$\|\phi\rangle$	$\|\psi\rangle$

We apply unitary σ_Y operation if the measurement outcome is 0. Discrimination of quantum measurements is thus reduced to discrimination of quantum states ϕ and ψ.

Entanglement-assisted discrimination procedure



$$|\phi\rangle = \cos\theta|0\rangle + \sin\theta|1\rangle,$$
$$|\psi\rangle = \cos\theta|0\rangle - \sin\theta|1\rangle,$$

General discrimination strategy with a three-component POVM – we allow for a tunable probability of inconclusive outcomes P_I.

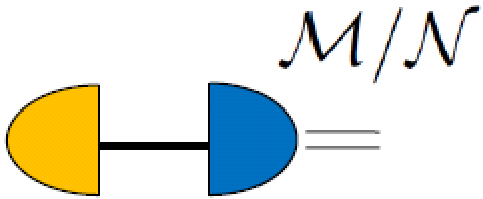
Maximum probability of a successful guess for a fixed P_I:

$$P_S = \frac{1}{2}\left(1 - P_I + \sin(2\theta)\sqrt{1 - \frac{P_I}{\cos^2\theta}}\right)$$

Optimality of this procedure can be proved using the formalism of process POVM.

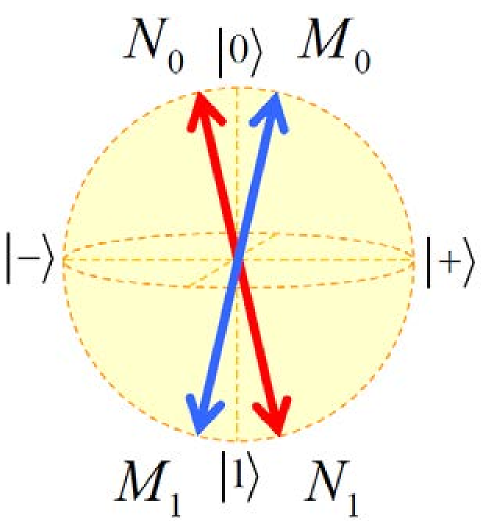
A. Chefles and S.M. Barnett, J. Mod. Opt. 45, 1295 (1998).
C.W. Zhang, C.F. Li, and G.C. Guo, Phys. Lett. A 261, 25 (1999).
M. Miková, M. Sedlák, I. Straka, M. Mičuda, M. Ziman, M. Ježek, M. Dušek, and J. Fiurášek (2014).

Single-qubit probe



Pure probe state

$$|\vartheta\rangle = \cos\vartheta|0\rangle + \sin\vartheta|1\rangle$$



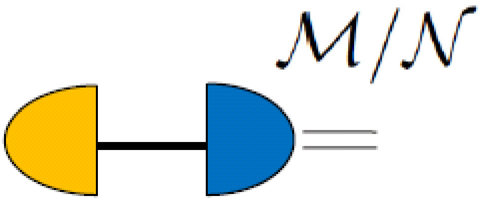
Minimum error discrimination:

$$|\vartheta\rangle = |+\rangle = \frac{1}{\sqrt{2}}(|0\rangle + |1\rangle)$$

$$P_S = \frac{1}{2}[1 + \sin(2\theta)] \qquad P_I = 0$$

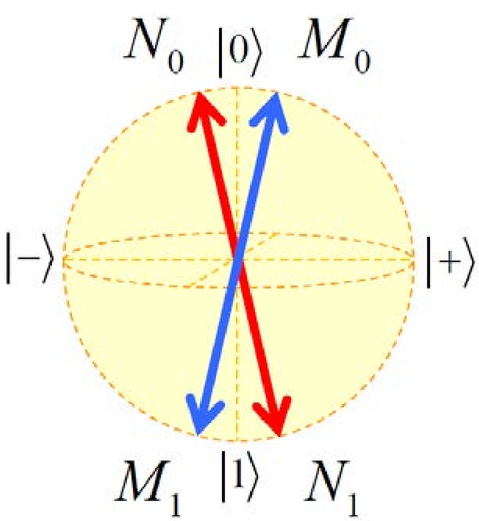
Globally optimal strategy. Entanglement is not needed for minimum error discrimination.

Single-qubit probe



Pure probe state

$$|\vartheta\rangle = \cos\vartheta|0\rangle + \sin\vartheta|1\rangle$$



Unambiguous discrimination:

$$|\vartheta\rangle = |\psi^{\perp}\rangle$$

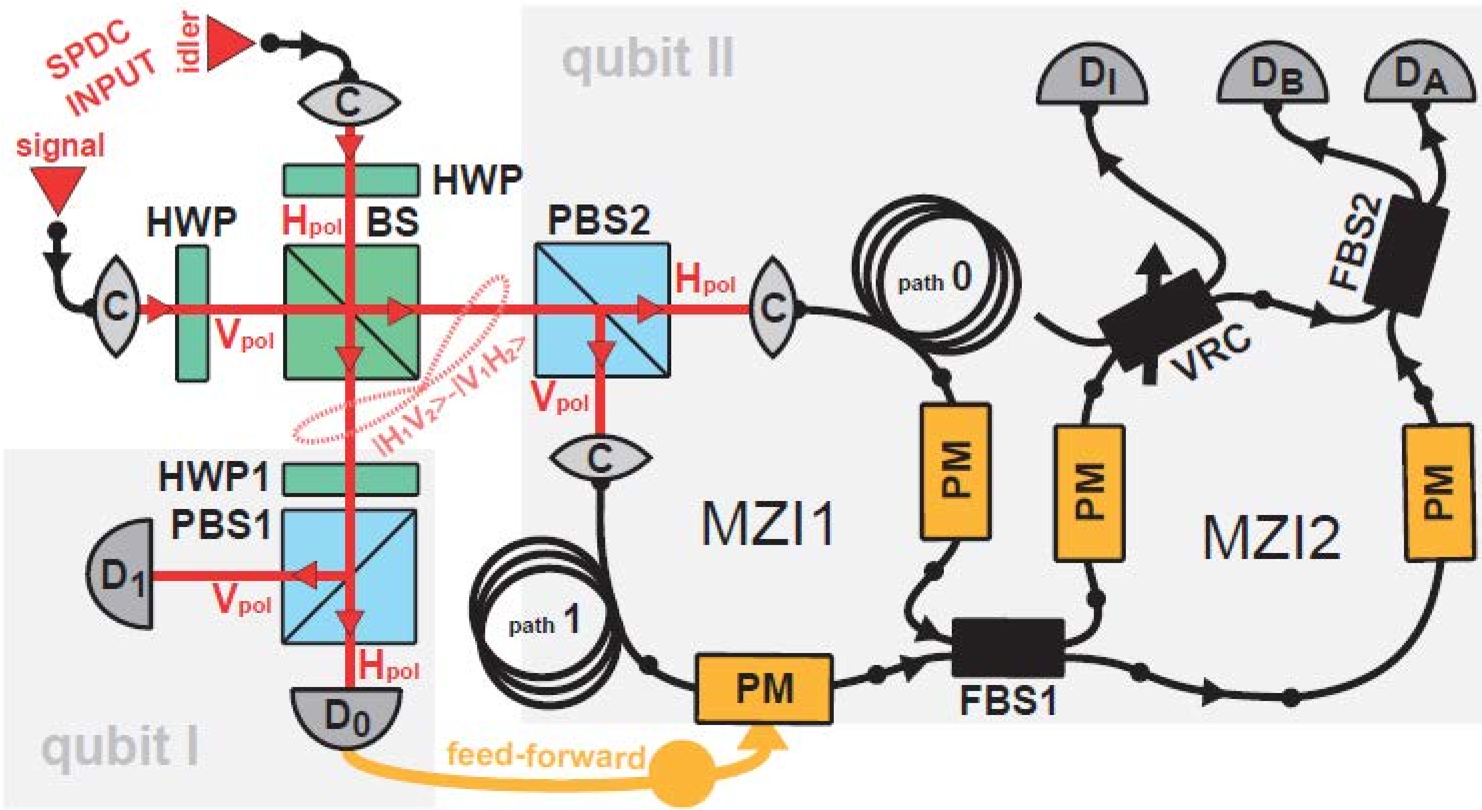
$$P_S = \frac{1}{2}\sin^2(2\theta) \qquad P_I = 1 - P_S$$

The optimal entanglement-assisted unambiguous discrimination achieves:

$$P_{S,\mathrm{ent}} = 1 - \cos(2\theta) > P_S$$

In fact, one can prove that entanglement helps for any $P_I>0$.

Experimental setup



Qubits encoded into polarization states of single photons.

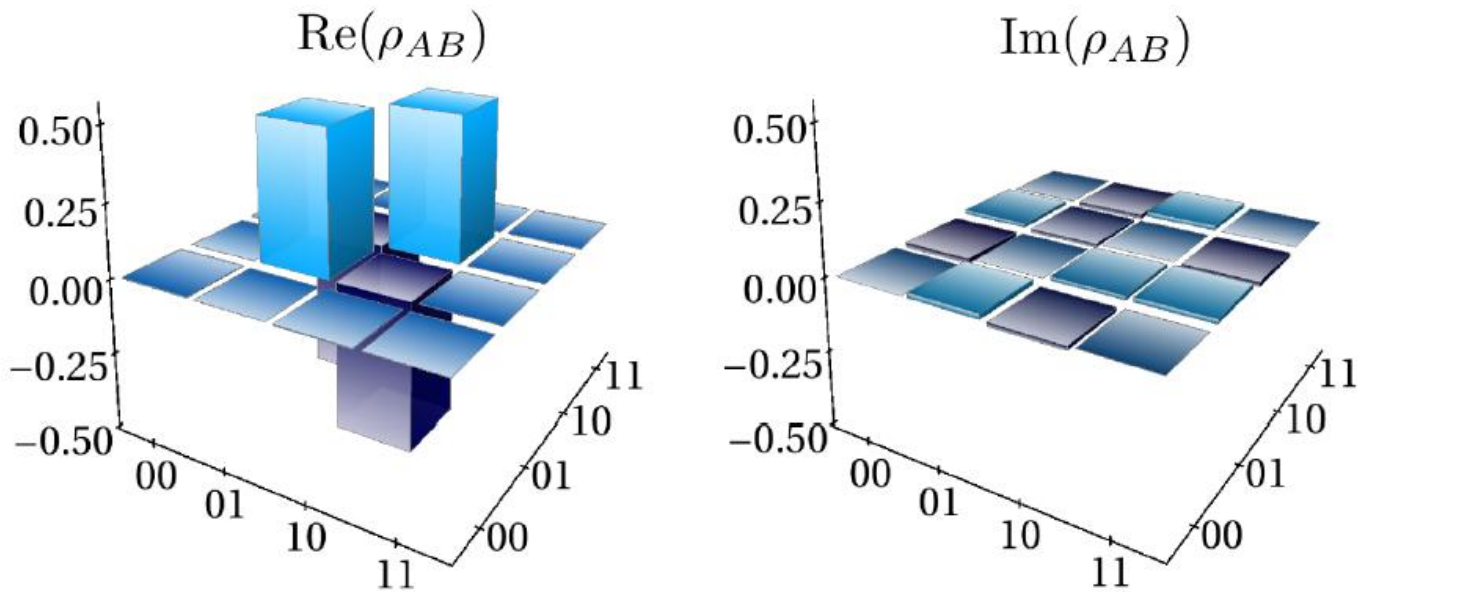
Two-qubit entangled state is conditionally generated by interference on a BS.

The conditional unitary on qubit B is applied using a real-time electronic feed-forward loop.

The POVM on qubit B is determined by the transmittance of VRC.

Characterization of entangled probe state

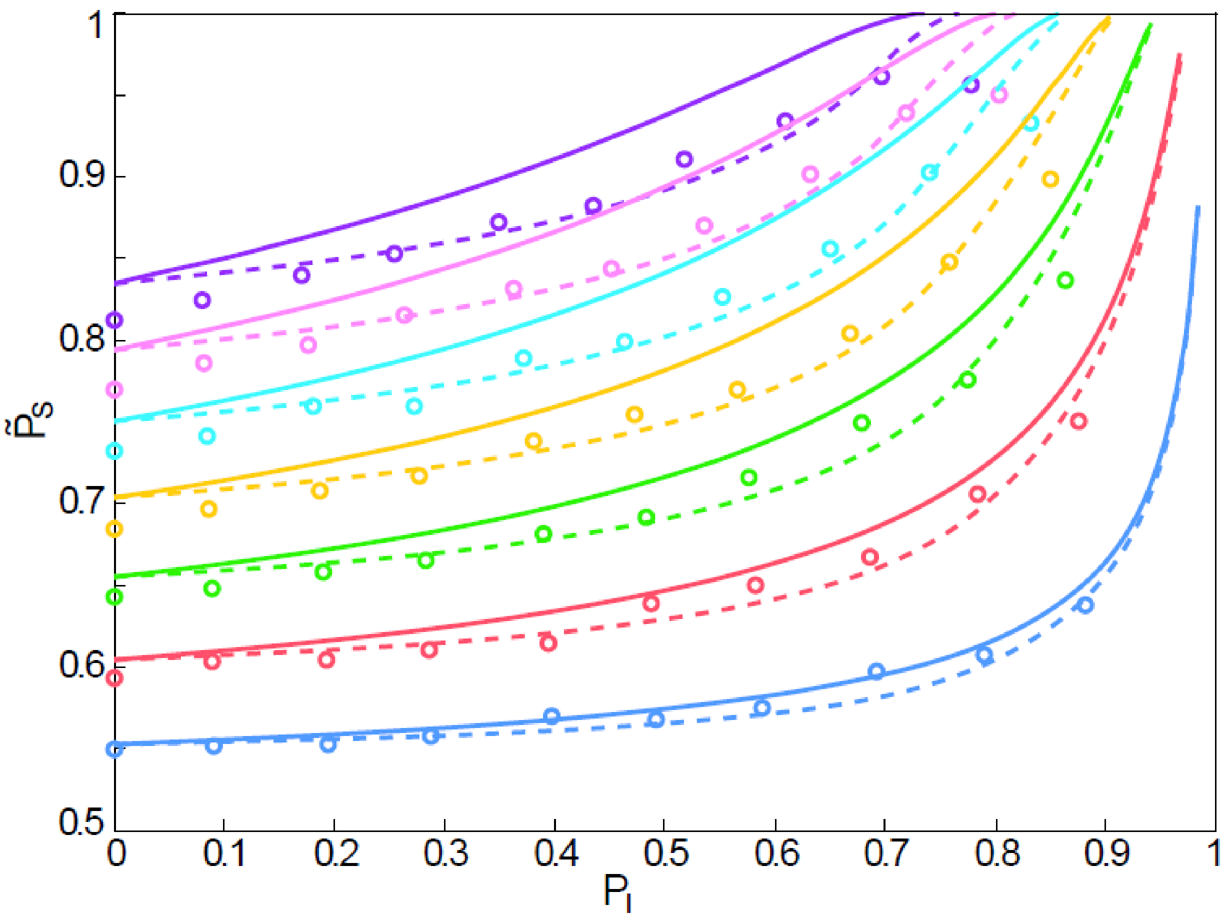
Target singlet state: $$|\Psi^-\rangle = \frac{1}{\sqrt{2}}(|01\rangle - |10\rangle)$$



purity 98 %, fidelity 99 %, concurrence 98%, ent. of formation 97%

Experimental results I

Dependence of relative success probability $\tilde{P}_S$ on probability of inconclusive results P_I for 7 values of $\theta_j = j\pi/30, \quad j = 1, \dots, 7$



$$\tilde{P}_S = \frac{P_S}{1 - P_I}$$

Circles – experiment | Solid lines – theory entangled probe | Dashed lines – theory single-qubit probe

Experimental results II

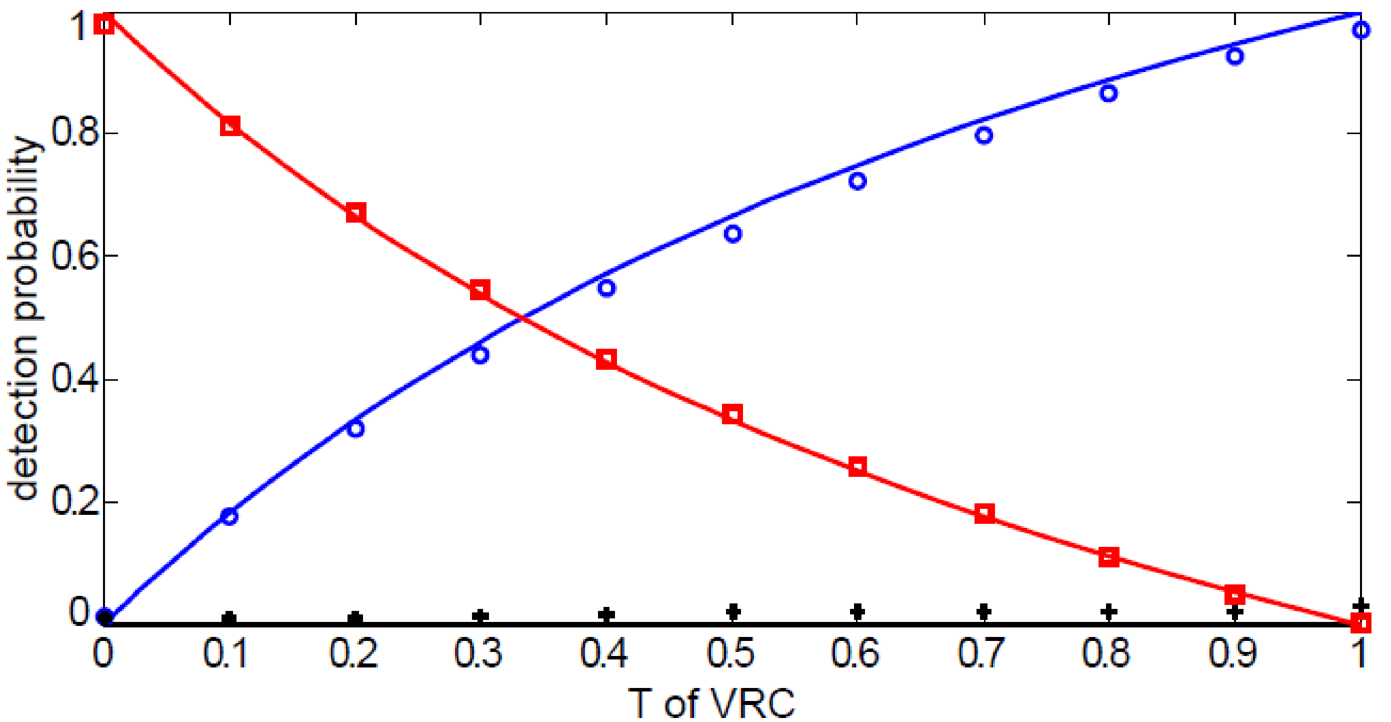


FIG. 4: Unambiguous discrimination of quantum measurements. The probabilities P_S (blue circles), P_I (red squares), and P_E (black crosses) are plotted as functions of the VRC splitting ratio T. The lines represent theoretical predictions.

Thank you for your attention!

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