Complexity of multiphoton interferometry

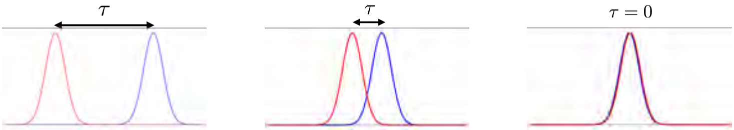
Hubert de Guise

Lakehead University

25 February 2016

What is the question?

- Distinguishability of photons results in different coincidence rates of the interfering photons.



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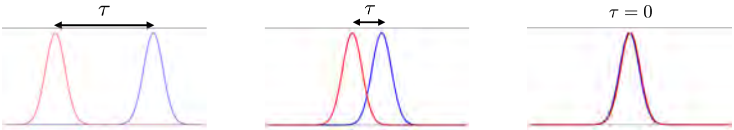

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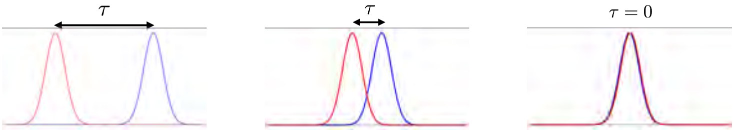
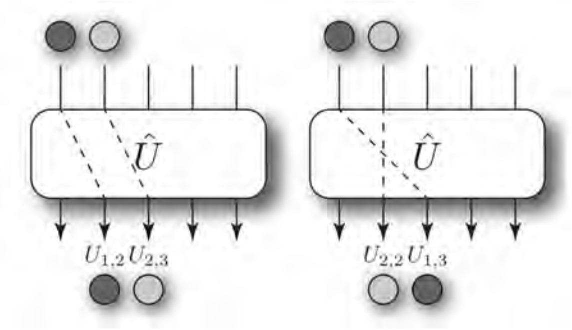


- Some coincidence rates in interferometry are related to matrix functions.
- Some of these matrix functions have known computational complexity.

distinguishability		computational complexity
$\downarrow$		$\uparrow$
rates	$\rightarrow$	matrix functions

Indistinguishability and permanents

- If two indistinguishable ($\tau = 0$) pulses enter input modes 1 and 2, and exit at output 2 and 3:



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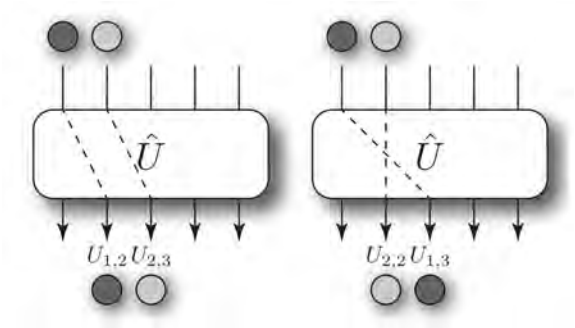

- The scattering amplitude is:

$$U_{12}U_{23} + U_{13}U_{22} = \text{Per}\begin{pmatrix} U_{12} & U_{13} \\ U_{22} & U_{23} \end{pmatrix}.$$

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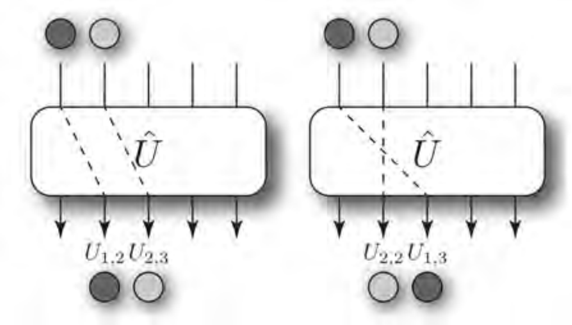

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- The probability $P((1,2) \to (2,3)) \sim |\mathrm{Per}|^2$.

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- If there exists a polynomial-time classical algorithm that samples from the same probability distribution as a linear-optical network (fed with indistinguishable photons), then lots of problems thought to be “hard” can be solved using this algorithm (unlikely!).
- Linear-optical networks function as “restricted” quantum computers to establish “quantum computational supremacy” over classical computers.

The Computational Complexity of Linear Optics

Citation	Scott Aaronson and Alex Arkhipov. 2011. The computational complexity of linear optics. In Proceedings of the 43rd annual ACM symposium on Theory of computing (STOC '11). ACM, New York, NY, USA, 333-342.
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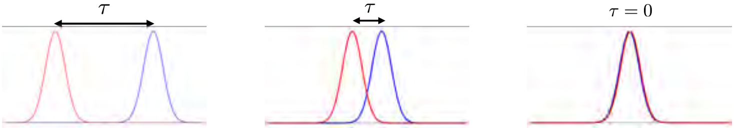

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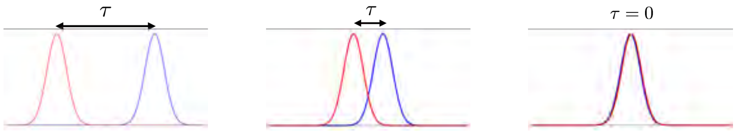

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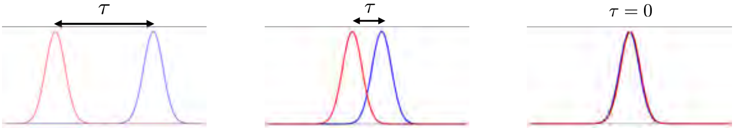

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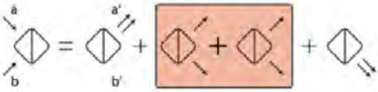
- The experiment counts the rate at which photons exit from two different output channels:



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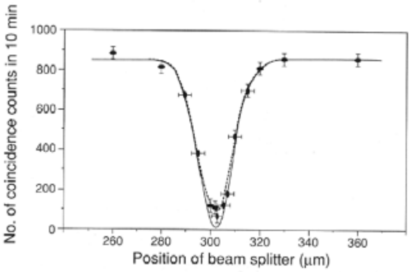
$$U = \frac{1}{\sqrt{2}} \begin{pmatrix} e^{-i\alpha} & -e^{-i\gamma} \\ e^{i\gamma} & e^{i\alpha} \end{pmatrix}$$ and the rate is Per(U)=0.



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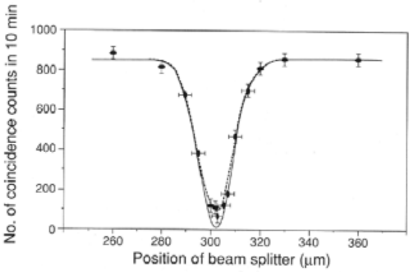

- This ties the indistinguishability features of the input pulses with the matrix function Per(U).

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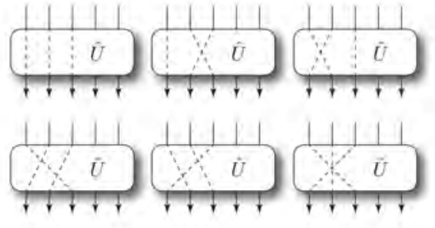

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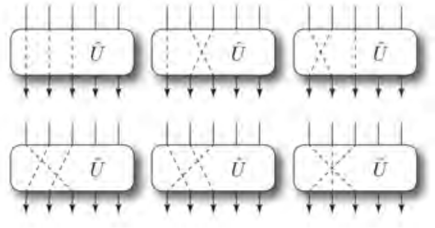

- It contains $6 = 3!$ terms and 18 multiplications.

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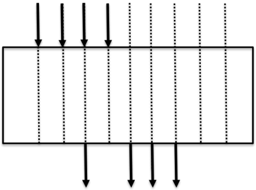
- Special cases exists if the matrix is not completely "arbitrary".
- Permanents of hermitian semi-definite matrices are well-studied (lots of connection with graph theory).

Interferometer: a (quantum) permanent calculator?

$$|k\rangle = |1111000\ldots\rangle$$



$$|s\rangle = |00101110\ldots\rangle$$

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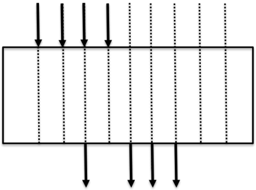

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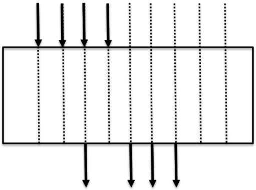

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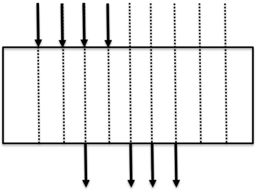

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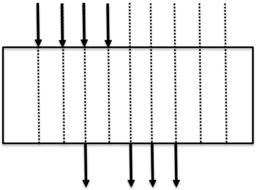

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  - For $m = 10$ and $n = 10$: $\sim 9 \times 10^4$ possible output modes,
  - For $m = 100$ and $n = 10$: $\sim 10^{23}$ possible output modes
- The probability of each output configuration is $|\mathrm{Per}(U_s)|^2$, the permanent of a specified $n \times n$ submatrix of the scattering matrix U.

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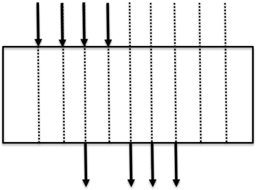

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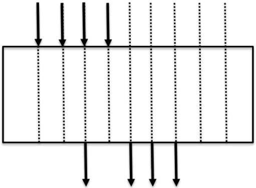

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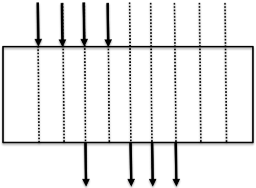

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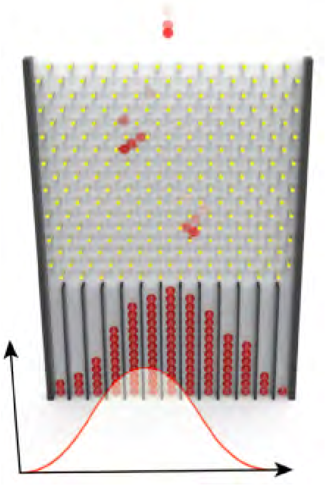
- Solution: transform the problem into a sampling problem where one looks at the distribution of photons in all output modes.

Sampling paradigm: Random walk



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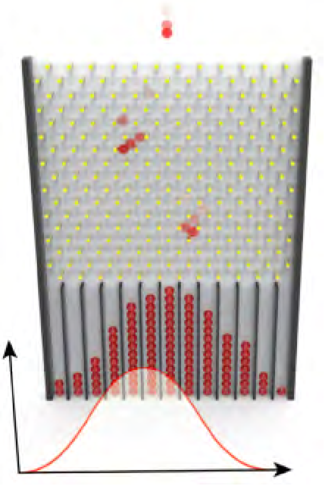

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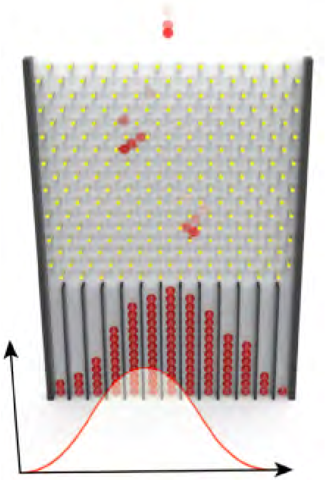

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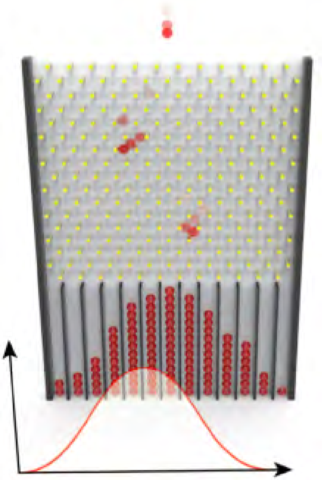

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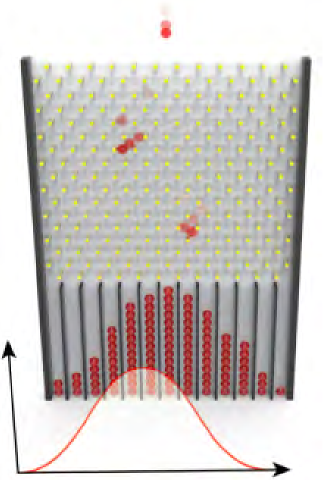
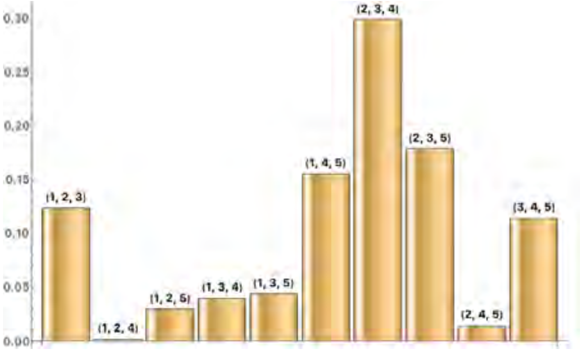
- The Galton board demonstrates classical random walk.



BosonSampling



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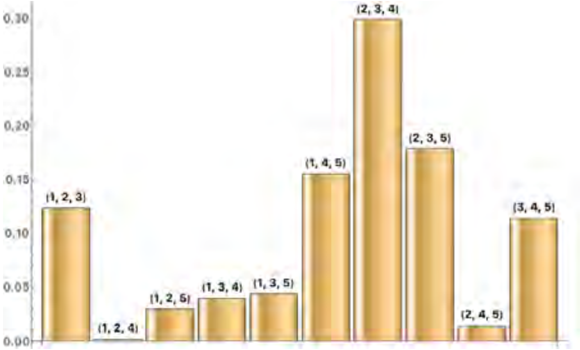

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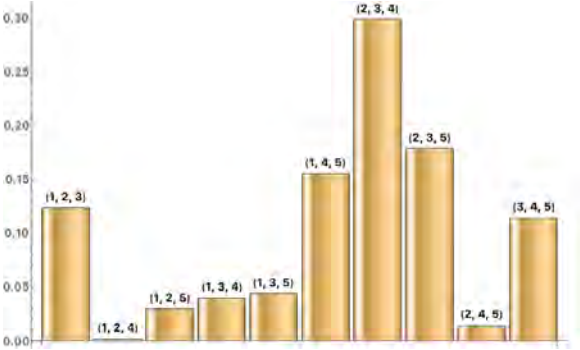

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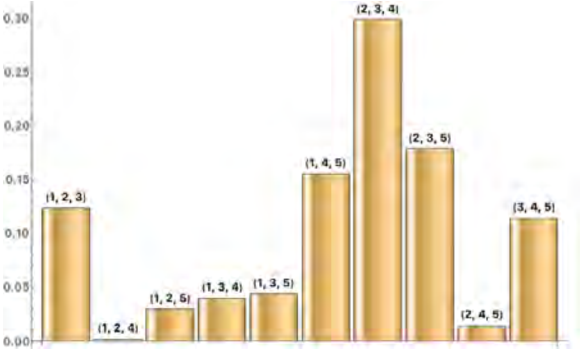
- This paradigm is BosonSampling.



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- Assume the unitary transformation is known.
  - Characterizing this unitary can be very time-consuming.

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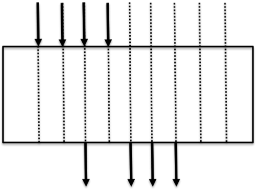

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  - Characterizing this unitary can be very time-consuming.

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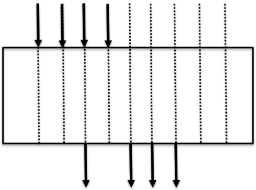

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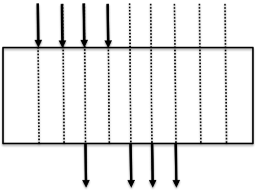

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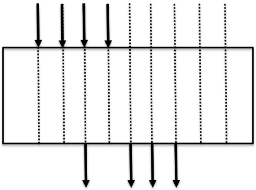

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  - if two (or more) photons come out the same channel reject: U_{ks} is not a submatrix of the original unitary.

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- This algorithm can be used to solve other "hard" problems.
- As of now, only brute force method known, i.e. actually calculate all the permanents.
  - given computer running at 100×10^{15} FLOPS, 15 photons in 275 channels implies $\sim 10^{29}$ multiplications and ~ 33000 years of runtime.

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- Requires only linear optics, photon sources and photon detectors.
- This is a rather "useless" practical problem but
- can still be used to demonstrate the superiority of the quantum vs classical computer.

Results

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LETTERS
PUBLISHED ONLINE: 12 MAY 2013 | DOI: [illegible]

nature photonics

Experimental boson sampling

Max Tillmann[1,2*], Borivoje Dakić[1], René Heilmann[3], Stefan Nolte[3], Alexander Szameit[3] and Philip Walther[1,2*]

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Photonic Boson Sampling in a Tunable Circuit

Matthew A. Broome,[1,2]* Alessandro Fedrizzi,[1,2] Saleh Rahimi-Keshari,[2] Justin Dove,[3] Scott Aaronson,[3] Timothy C. Ralph,[2] Andrew G. White[1,2]

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nature photonics
LETTERS
PUBLISHED ONLINE: 26 MAY 2013 | DOI: 10.1038/NPHOTON.2013.112

Integrated multimode interferometers with arbitrary designs for photonic boson sampling

Andrea Crespi[1,2], Roberto Osellame[1,2]*, Roberta Ramponi[1,2], Daniel J. Brod[3], Ernesto F. Galvão[3]*, Nicolò Spagnolo[4], Chiara Vitelli[4,5], Enrico Maiorino[4], Paolo Mataloni[4] and Fabio Sciarrino[4]*

Certification

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- How to verify that classically intractable quantum devices perform as expected?

Boson-Sampling in the light of sample complexity

C. Gogolin, M. Kliesch, L. Aolita, and J. Eisert

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nature COMMUNICATIONS

ARTICLE

Received 25 Dec 2014 | Accepted 27 Aug 2015 | Published 18 Nov 2015 DOI: [illegible] OPEN

Reliable quantum certification of photonic state preparations

Leandro Aolita[1,2], Christian Gogolin[1,3,4], Martin Kliesch[1] & Jens Eisert[1]

Certification

- There DOES exist an efficient test based on

$$P = \Pi_{i=1}^{n} \sum_{j=1}^{n} |U_{ij}|^2 > \left(\frac{n}{m}\right)^n .$$

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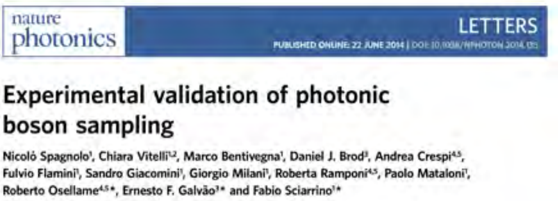
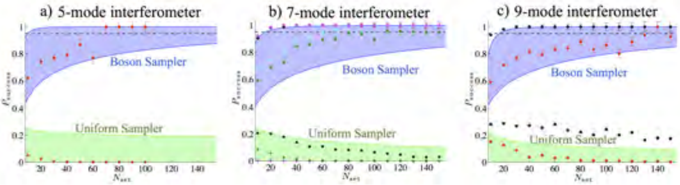

nature photonics

LETTERS

PUBLISHED ONLINE: 22 JUNE 2014 | DOI: 10.1038/NPHOTON.2014.[illegible]

Experimental validation of photonic boson sampling

Nicolò Spagnolo[1], Chiara Vitelli[1,2], Marco Bentivegna[1], Daniel J. Brod[3], Andrea Crespi[4,5], Fulvio Flamini[1], Sandro Giacomini[1], Giorgio Milani[1], Roberta Ramponi[4,5], Paolo Mataloni[1], Roberto Osellame[4,5]*, Ernesto F. Galvão[3]* and Fabio Sciarrino[1]*



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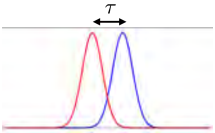

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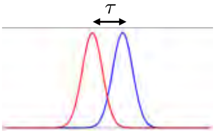

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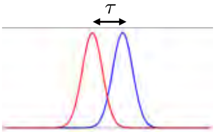

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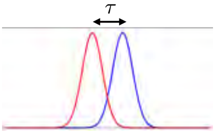

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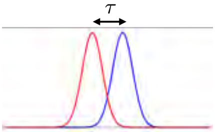

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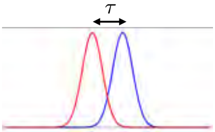

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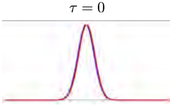

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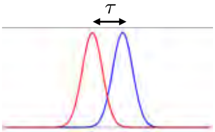

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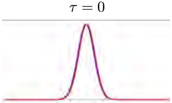
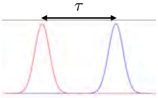
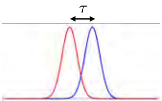


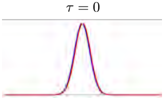
  - and an antisymmetric $\begin{smallmatrix}\square\\\square\end{smallmatrix}$ part
    $P_{12}\begin{smallmatrix}\square\\\square\end{smallmatrix} = -\begin{smallmatrix}\square\\\square\end{smallmatrix}$.

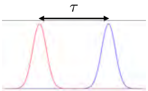


How does partial distinguishability work?

- Similar to writing 

as

  a τ-combination of 

and 

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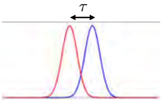
- Similar to writing 

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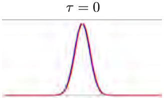
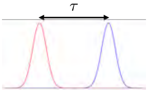
  a τ-combination of 

and 

- The coincidence rate R(τ) eventually yields

$$R(\tau) = \frac{1}{2}\left|\square\square\right|^2 \left(1 + \mathrm{e}^{-\sigma^2\tau^2}\right) + \frac{1}{2}\left|\begin{smallmatrix}\square\\\square\end{smallmatrix}\right|^2 \left(1 - \mathrm{e}^{-\sigma^2\tau^2}\right)$$

with $\square\square$ the permanent of the scattering matrix,
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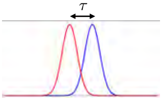
- Similar to writing 

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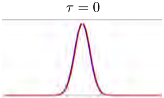
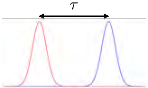
a τ-combination of 

and 

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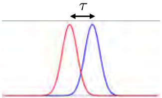
- Similar to writing 

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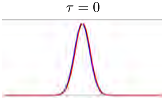
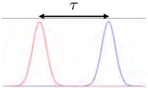
  a τ-combination of 

and 

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- Note that, for $\tau = 0$, only $|\Box\Box|^2$ survives.

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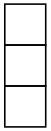
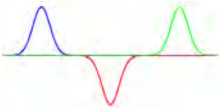
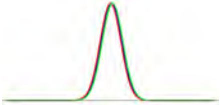
How does partial distinguishability work?

- for 3 photons there are 3! terms in the decomposition of $a^\dagger_\alpha(\omega_1)a^\dagger_\beta(\omega_2)a^\dagger_\gamma(\omega_3)$.
- 3! possible permutations: $\mathbb{1}, P_{12}, P_{23}, P_{13}, P_{123}, P_{132}$.
- 3! possible "basis configurations":

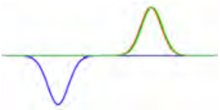
antisymmetric: 

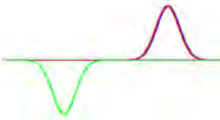


symmetric: 

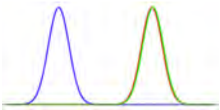


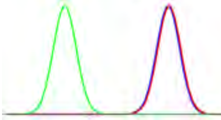
mixed symmetry:  1





mixed symmetry:  2





Relevance to BosonSampling

The proposal of A&A contains five sources of “errors” that will not make an experimental realization ideal.

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- Imperfect preparation of the input Fock state
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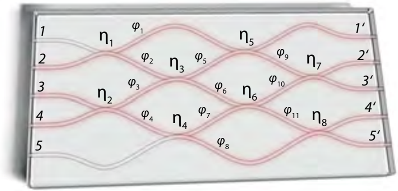
- imperfect detectors
- non-simultaneity of photon arrival times

3 photons in 5 channels (Vienna experiment)

- Inject 3 photons at the input of an interferometer.



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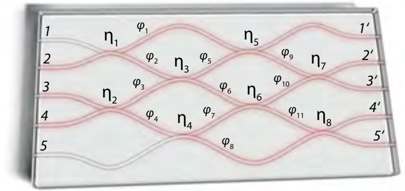
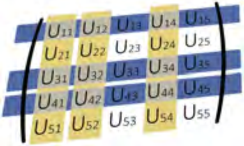


- Select 3 output channels



thereby selecting a 3×3 submatrix of the 5×5 scattering matrix.

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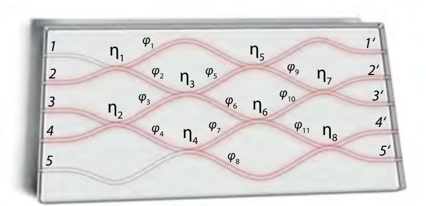
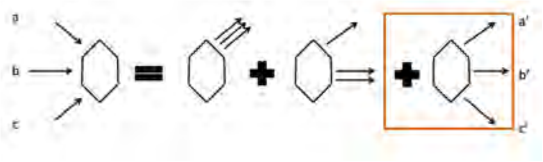


- Select 3 output channels

thereby selecting a 3×3 submatrix of the 5×5 scattering matrix.

- Record the coincidence counts as are function of the 2 relative delays w/r to a reference photon.



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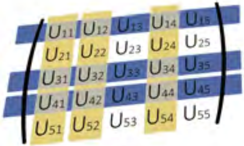
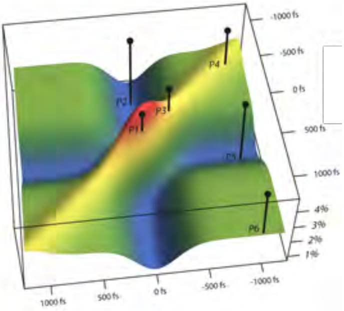
The result is a 2-D landscape in delay space





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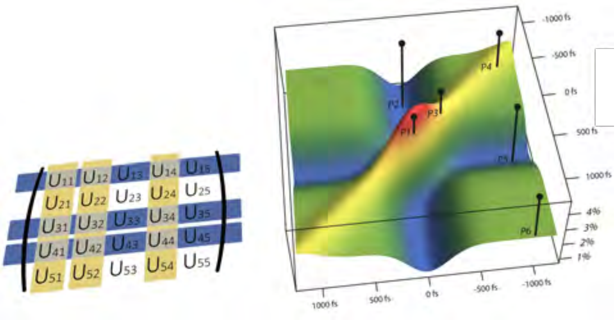

- There are plateaus in 2 corners:

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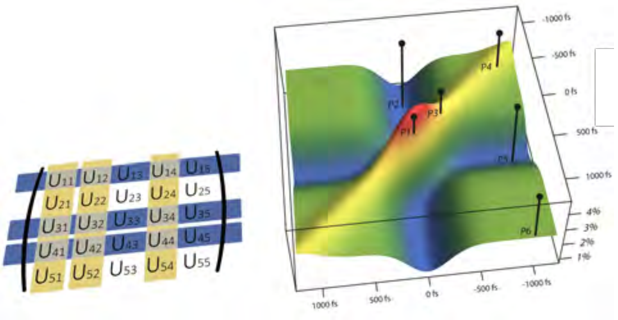

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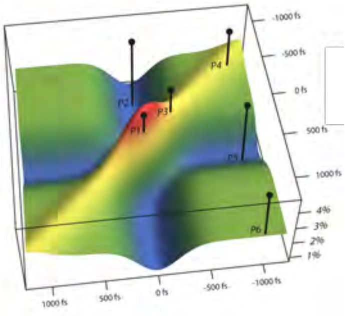
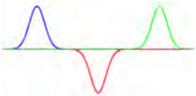


- There are plateaus in 2 corners:
  - These are areas where the two relative delays are large.
  - These correspond to 3 distinct pulses



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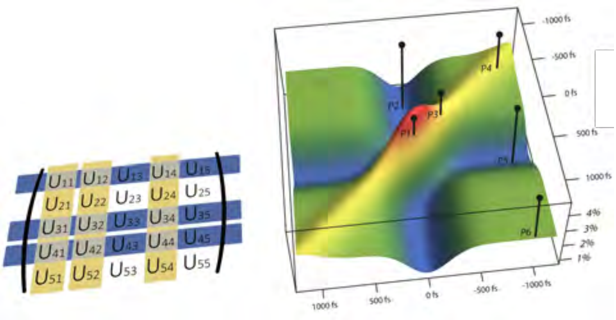

- There are two valleys and one ridge:

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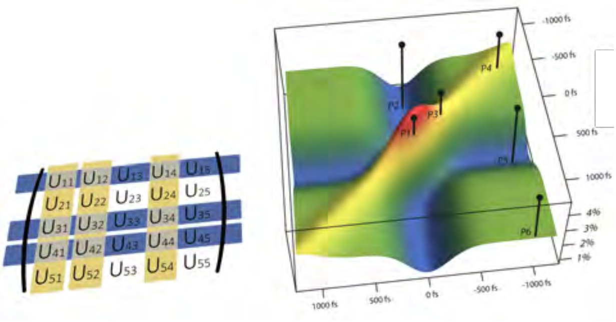

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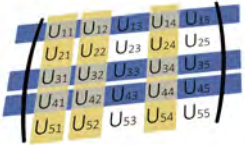
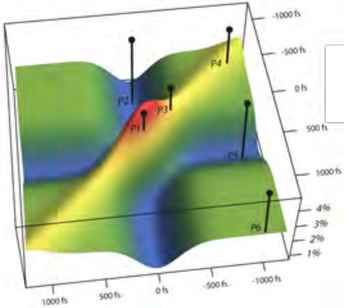
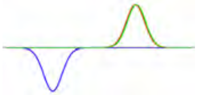




- There are two valleys and one ridge:
  - This occurs when one of the delay is 0.
  - This corresponds to 2 indistinguishable photons, with the third partially distinguishable



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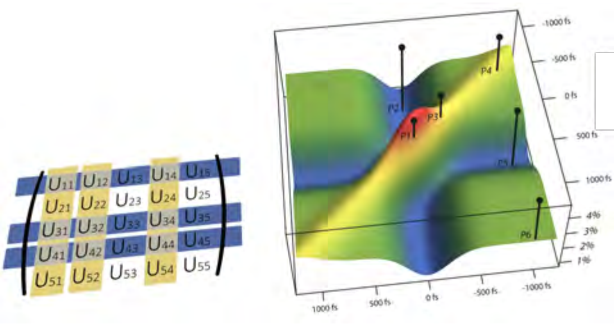
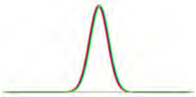
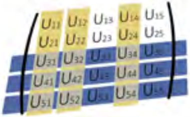


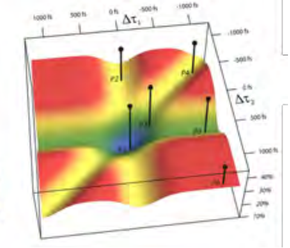
- There is a single point at the center:
  - This occurs when both delays are 0:.
  - This corresponds to 3 indistinguishable photons



Typical landscape features

All landscapes have the same typical features.





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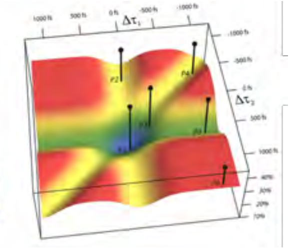

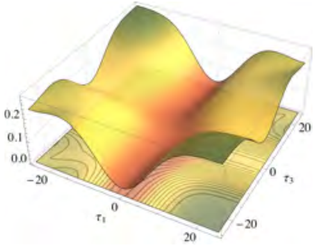

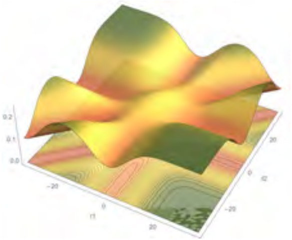

Mixed symmetry states

- First pair:

$$
\begin{aligned}
\left|\begin{array}{|c|c|}\hline 1 & 3 \\ \hline 2 \\ \cline{1-1}\end{array}\right\rangle_1 &= \left[a_1^\dagger(\omega_1)a_2^\dagger(\omega_2) - a_1^\dagger(\omega_2)a_2^\dagger(\omega_1)\right] a_3^\dagger(\omega_3)|0\rangle \\
&\quad + \left[a_1^\dagger(\omega_3)a_2^\dagger(\omega_2) - a_1^\dagger(\omega_2)a_2^\dagger(\omega_3)\right] a_3^\dagger(\omega_1)|0\rangle \\
\left|\begin{array}{|c|c|}\hline 1 & 2 \\ \hline 3 \\ \cline{1-1}\end{array}\right\rangle_1 &= P_{23}\left|\begin{array}{|c|c|}\hline 1 & 3 \\ \hline 2 \\ \cline{1-1}\end{array}\right\rangle_1 ,
\end{aligned}
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where P_{ij} interchanges $\omega_i \leftrightarrow \omega_j$

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- $P_{13}|\begin{array}{|c|c|}\hline 1 & 3 \\ \hline 2 \\ \cline{1-1} \end{array}\rangle_1 = +|\begin{array}{|c|c|}\hline 1 & 3 \\ \hline 2 \\ \cline{1-1} \end{array}\rangle_1$
  $P_{12}|\begin{array}{|c|c|}\hline 1 & 2 \\ \hline 3 \\ \cline{1-1} \end{array}\rangle_1 = +|\begin{array}{|c|c|}\hline 1 & 2 \\ \hline 3 \\ \cline{1-1} \end{array}\rangle_1$

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$$|\,\ytableaushort{13,2}\,\rangle_1 = \left[a_1^\dagger(\omega_1)a_2^\dagger(\omega_2) - a_1^\dagger(\omega_2)a_2^\dagger(\omega_1)\right] a_3^\dagger(\omega_3)|0\rangle$$
$$+ \left[a_1^\dagger(\omega_3)a_2^\dagger(\omega_2) - a_1^\dagger(\omega_2)a_2^\dagger(\omega_3)\right] a_3^\dagger(\omega_1)|0\rangle$$
$$|\,\ytableaushort{12,3}\,\rangle_1 = P_{23}|\,\ytableaushort{13,2}\,\rangle_1\,,$$

  where P_{ij} interchanges $\omega_i \leftrightarrow \omega_j$

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  $P_{12}|\,\ytableaushort{12,3}\,\rangle_1 = +|\,\ytableaushort{12,3}\,\rangle_1$
- but

$$P_{12}|\,\ytableaushort{13,2}\,\rangle_1 = -|\,\ytableaushort{13,2}\,\rangle_1 - |\,\ytableaushort{12,3}\,\rangle_1$$

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  $P_{12}\left|\begin{array}{|c|c|}\hline 1 & 2 \\ \hline 3 \\ \cline{1-1}\end{array}\right\rangle_1 = +\left|\begin{array}{|c|c|}\hline 1 & 2 \\ \hline 3 \\ \cline{1-1}\end{array}\right\rangle_1$
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- Similar properties holds for the second pair of states

Mixed symmetry matrix functions

To go along with mixed symmetry states are mixed symmetry matrix functions called *immanants*.

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- The are constructed in a group-theoretical way using characters.

	$\mathbb{1}$	P_{12}	P_{13}	P_{23}	P_{123}	P_{132}	
	1	1	1	1	1	1	Permanent
	1	-1	-1	-1	1	1	Determinant
	2	0	0	0	-1	-1	(2,1)-Immanant

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- To compute the permanent:

$$\begin{aligned}\mathrm{Per}(U) &= U_{11}U_{22}U_{33} + U_{12}U_{21}U_{33} + U_{13}U_{22}U_{31} \\ &\quad + U_{11}U_{23}U_{32} + U_{12}U_{23}U_{31} + U_{13}U_{21}U_{32}\,, \\ &= \sum \chi^{\square\square\square}(\sigma)\left[U_{1\sigma(1)}U_{2\sigma(2)}U_{3\sigma(3)}\right]\end{aligned}$$

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- Note that $P_{ij}\mathrm{Per}(U) = +\mathrm{Per}(U)$: permuting any two columns of U does not change the permanent.

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	1	-1	-1	-1	1	1	Determinant
	2	0	0	0	-1	-1	(2,1)-Immanant

- To compute the determinant:

$$\begin{aligned}\mathrm{Det}(U) &= U_{11}U_{22}U_{33} - U_{12}U_{21}U_{33} - U_{13}U_{22}U_{31} \\ &\quad - U_{11}U_{23}U_{32} + U_{12}U_{23}U_{31} + U_{13}U_{21}U_{32}\,, \\ &= \sum \chi^{\square}(\sigma)\left[U_{1\sigma(1)}U_{2\sigma(2)}U_{3\sigma(3)}\right]\end{aligned}$$

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	$\mathbb{1}$	P_{12}	P_{13}	P_{23}	P_{123}	P_{132}	
$\yng(3)$	1	1	1	1	1	1	permanent
$\yng(1,1,1)$	1	-1	-1	-1	1	1	Determinant
$\yng(2,1)$	2	0	0	0	-1	-1	(2,1)-Immanant

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$$\mathrm{Per}(U) = 2U_{11}U_{22}U_{33} - U_{12}U_{23}U_{31} - U_{13}U_{21}U_{32}\,,$$
$$= \sum \chi^{\yng(2,1)}(\sigma)\left[U_{1\sigma(1)}U_{2\sigma(2)}U_{3\sigma(3)}\right]$$

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- Permuting any two columns of U does not change the immanant back to itself.

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- Permuting any two columns of U does not change the immanant back to itself.
- In fact there are 4 linearly independent immanants:

 $_{123}$,  $_{213}$,  $_{132}$,  $_{312}$.

Rates for partially distinguishable pulses

Suppose

- photons 2 and 3 have no relative delays (are made indistinguishable)

Rates for partially distinguishable pulses

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- photons 2 and 3 have no relative delays (are made indistinguishable)
- photon 1 is delayed by τ w/r to photons 2 and 3.

Rates for partially distinguishable pulses

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- this must be so because the rate is a scalar (a number).

Experimental results beyond permanents

PHYSICAL REVIEW X **5**, 041015 (2015)

Generalized Multiphoton Quantum Interference

Max Tillmann,[1,*] Si-Hui Tan,[2] Sarah E. Stoeckl,[1] Barry C. Sanders,[3,4] Hubert de Guise,[5]
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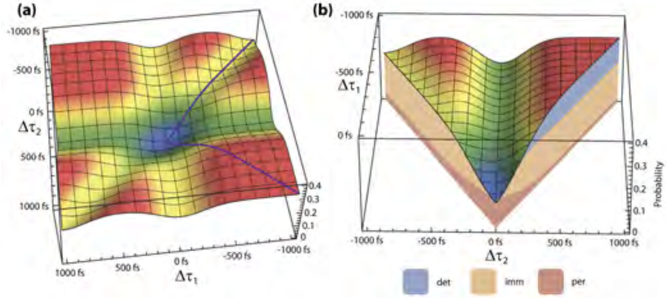
(Received 22 April 2015; revised manuscript received 21 July 2015; published 27 October 2015)



Computational complexity of immanants

THE COMPUTATIONAL COMPLEXITY OF IMMANANTS*

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Abstract. Permanents and determinants are special cases of immanants. The latter are polynomial matrix functions defined in terms of characters of symmetric groups and corresponding to Young diagrams. Valiant has proved that the evaluation of permanents is a complete problem in both the Turing machine model (#P-completeness) as well as in his algebraic model (VNP-completeness). We show that the evaluation of immanants corresponding to hook diagrams or rectangular diagrams of polynomially growing width is both #P-complete and VNP-complete.

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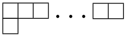
- Immanants of the type  are "just about equally hard" to compute as permanents.

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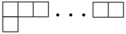
- Immanants of the type  are "just about equally hard" to compute as permanents.
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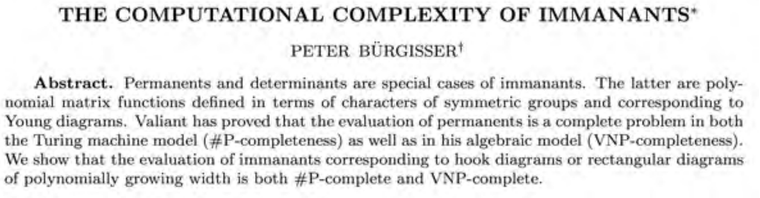

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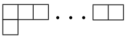
- Immanants of the type  are "just about equally hard" to compute as permanents.
- Immanants of the type  are "just about equally hard" to compute as permanents.
- There is a sliding scale of hardness but if one or more of the rows gets "very long" in comparison with the total number of rows then the associated immanant is "hard".

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PHYSICAL REVIEW A **89**, 063819 (2014)

Coincidence landscapes for three-channel linear optical networks

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- Hurdles to get clean “immanant signals”

Acknowledgments

- Collaborators:
  - Barry Sanders, Abdullah Khalid, Ish Dhand (Calgary)
  - Si-Hui Tan (Singapore)
  - Max Tillmann, Philip Walther, and the Vienna group.
- Special thanks to Max Tillmann and Philip Walther (Wien) for use of figures.