

QUANTUM RESPONSE OF PLASMONIC SYSTEMS

MARK TAME

University of KwaZulu-Natal, South Africa





quantum.ukzn.ac.za



Durban, South Africa



INTRODUCTION

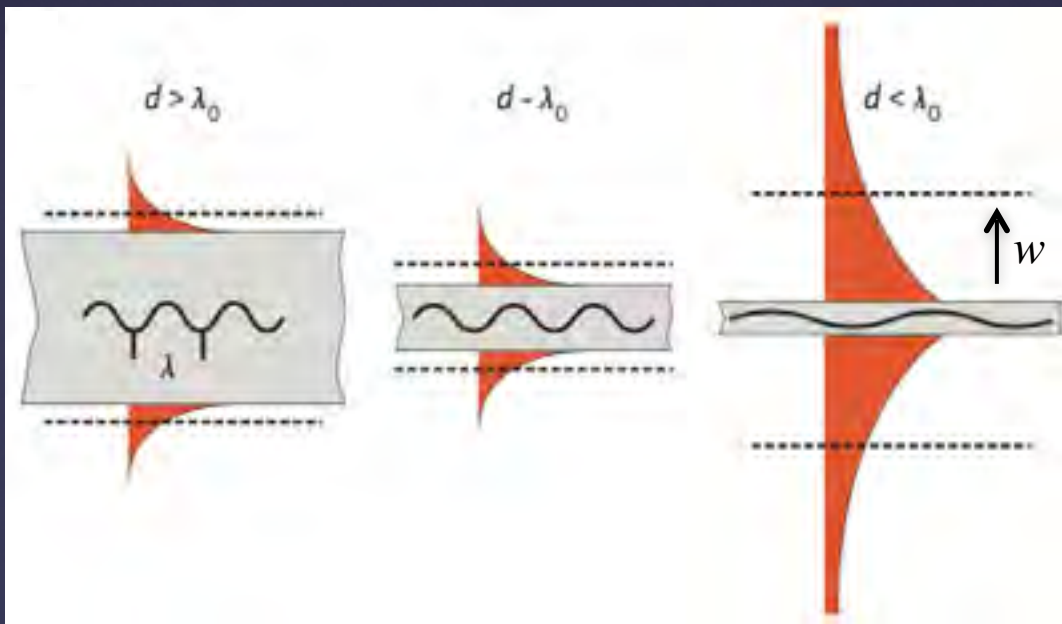
By how much can I squeeze light spatially?



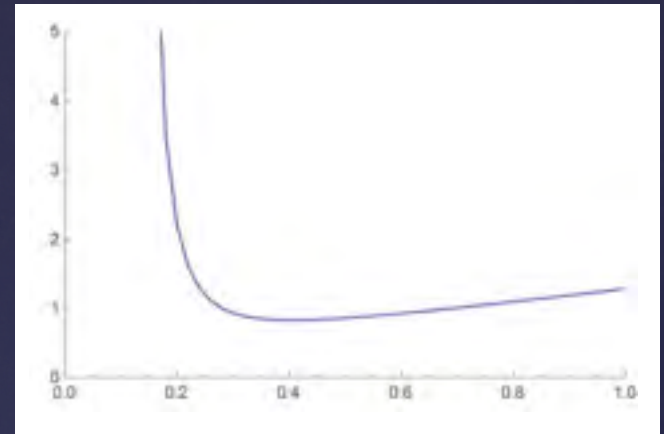


The 'Squishing of the Squash'

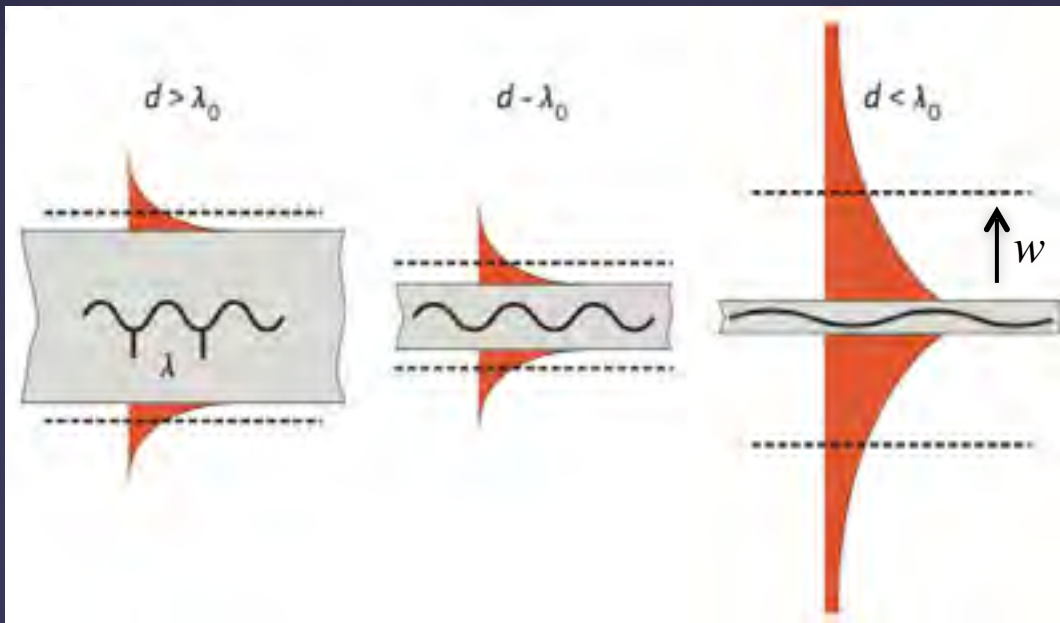




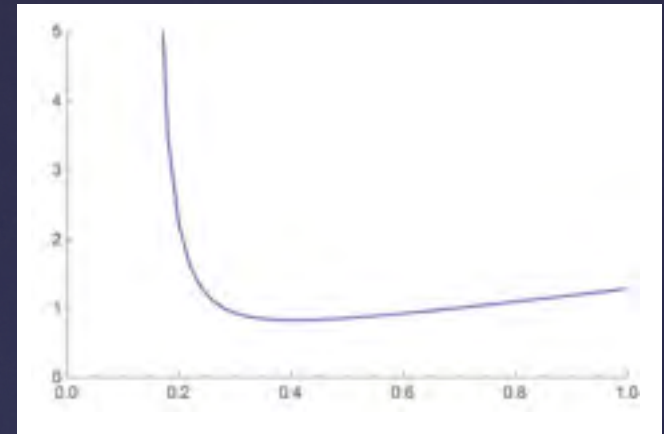
w/λ_0



d/λ_0

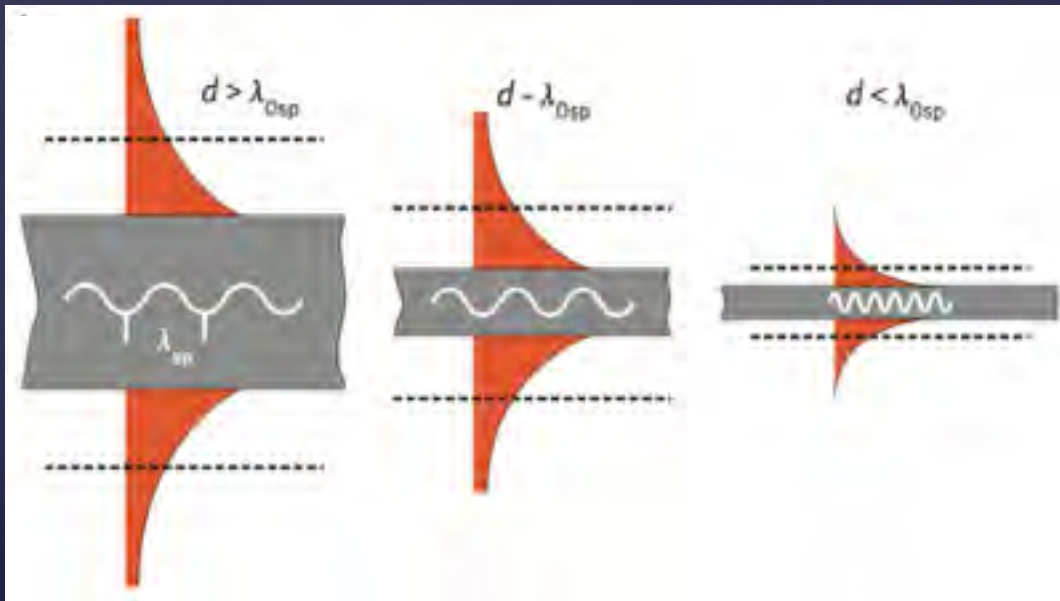


w/λ_0

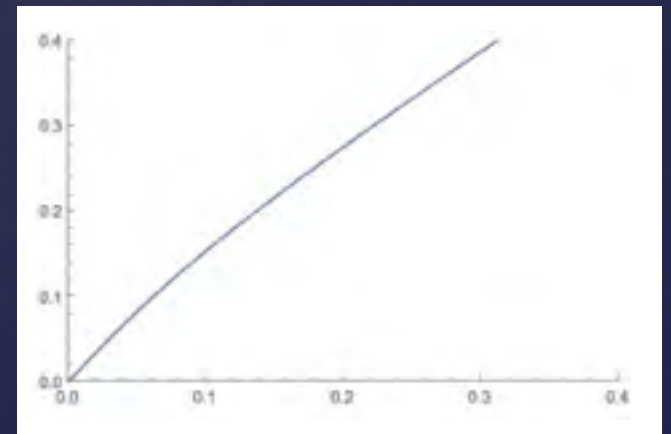


$\uparrow d$

d/λ_0



w/λ_0



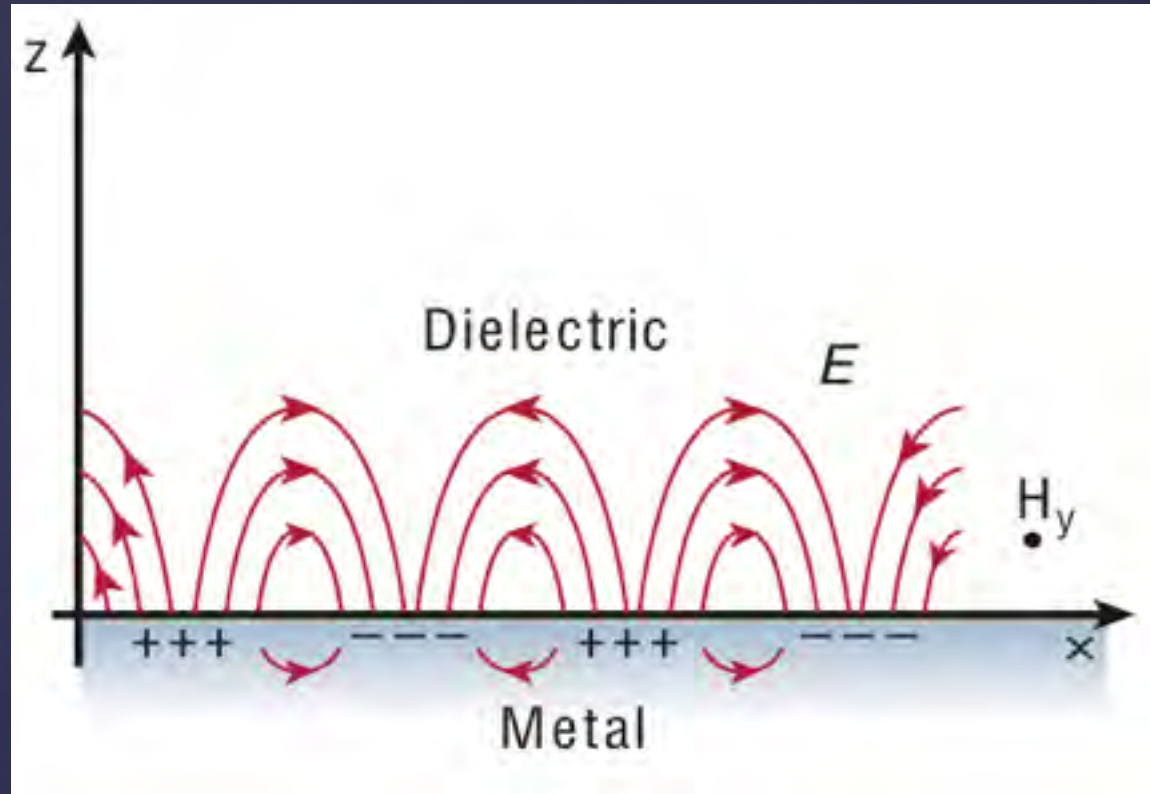
d/λ_0

Gramotnev and Bozhevolnyi, Nat. Phot. 4, 83 (2010)

Takahara et al., Opt. Lett. 22, 475 (1997)



Surface plasmon polariton

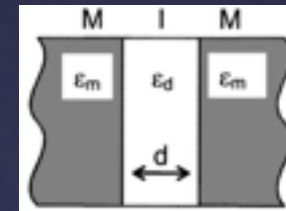


Barnes, Dereux and Ebbesen, Nature 424, 824 (2003)

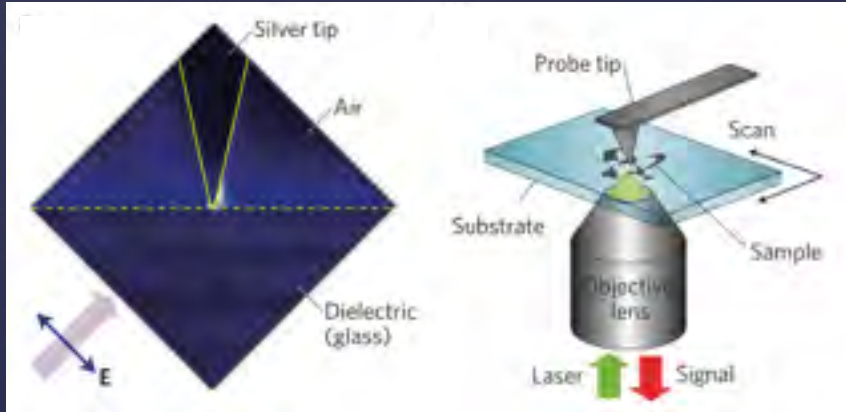
Applications



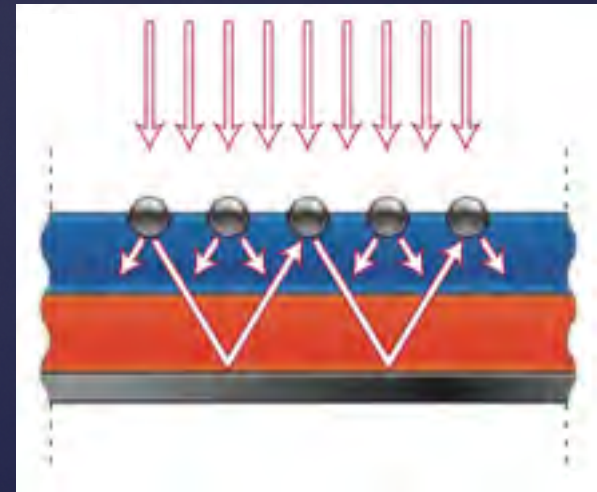
Takahara, Plasmonic Nanoguides and Circuits (2009)



Zia et al., J. Opt. Soc. Am. A 21, 2442 (2004)

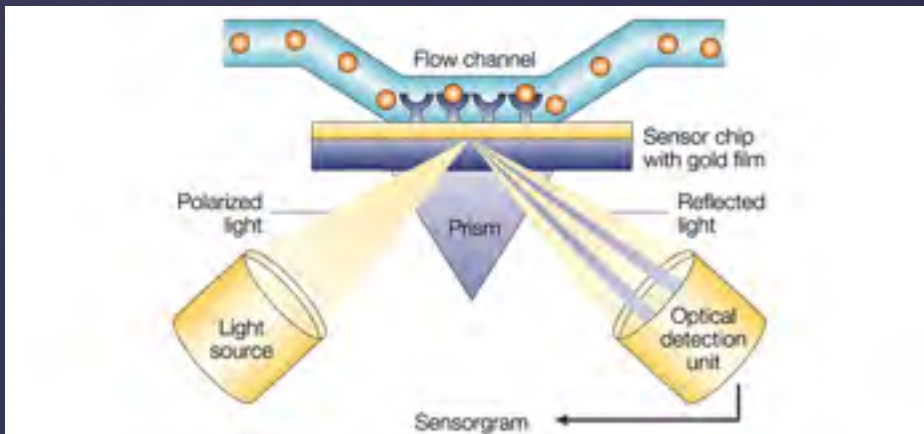


Kawata et al., Nature Phot. 3, 388 (2009)



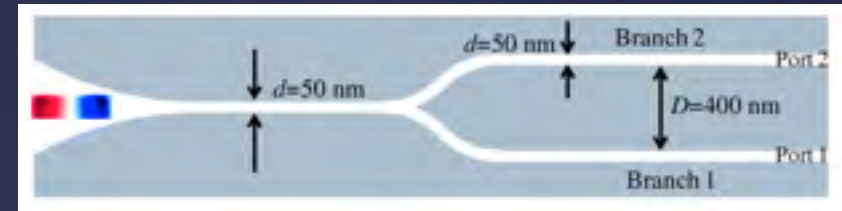
Atwater and Polman, Nature Mat. 9, 205 (2010)

More applications



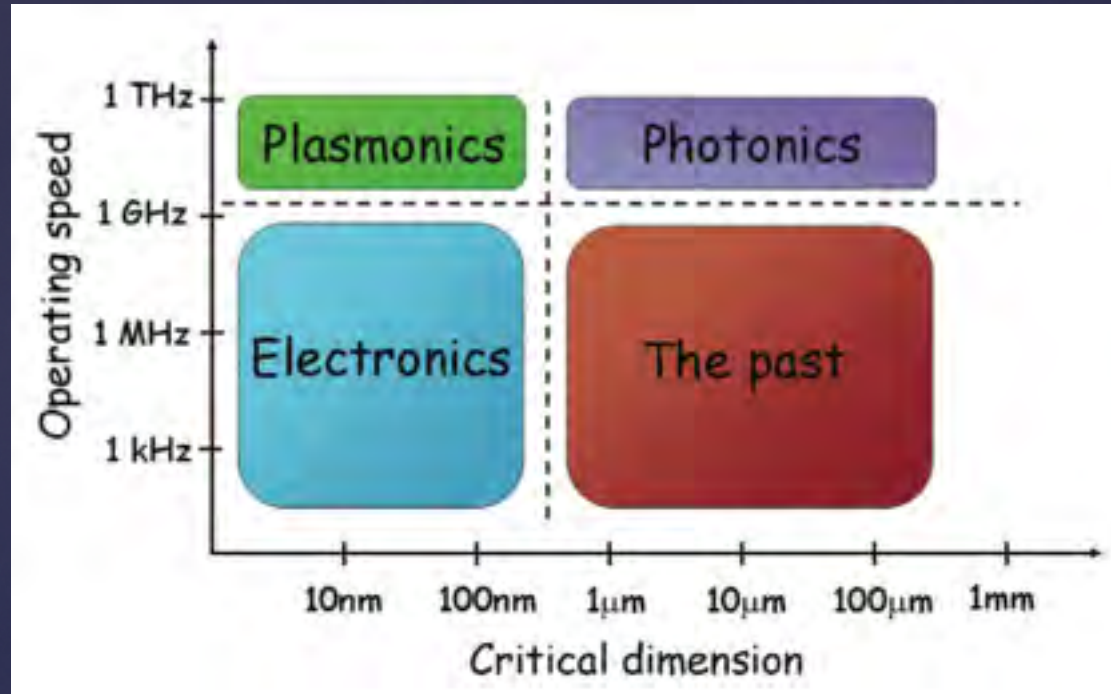
J. N. Anker et al., Nature Mat. 7, 442 (2008)

e.g. BIACORE, Dynamic Biosensors, Attana AB etc.



Gramotnev and Bozhevolnyi,
Nature Phot. 4, 83 (2010)

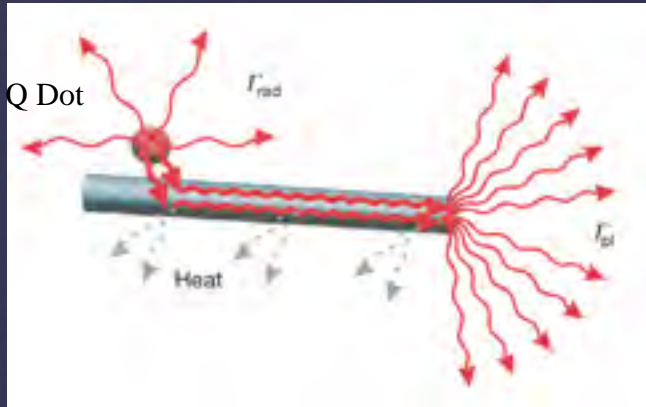
Technology perspective



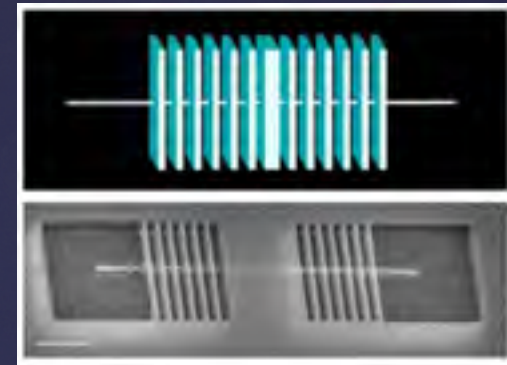
Zia et al., Materials Today 9, 20 (2006)

Single-photon sources

Enhancement of light-matter interaction



Akimov et al., Nature 450, 402 (2007)



de Leon et al., PRL 108, 226803 (2012)

waveguide

$$g \propto 1/\sqrt{A_{eff}}$$

cavity

$$g \propto 1/\sqrt{V_{eff}}$$

Purcell effect

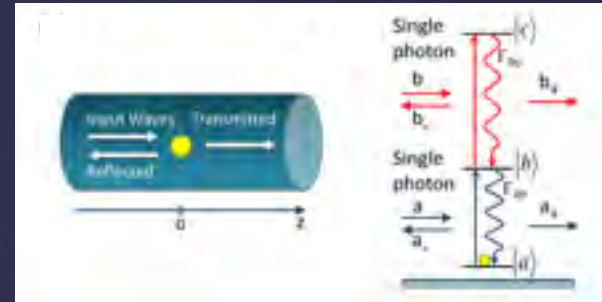
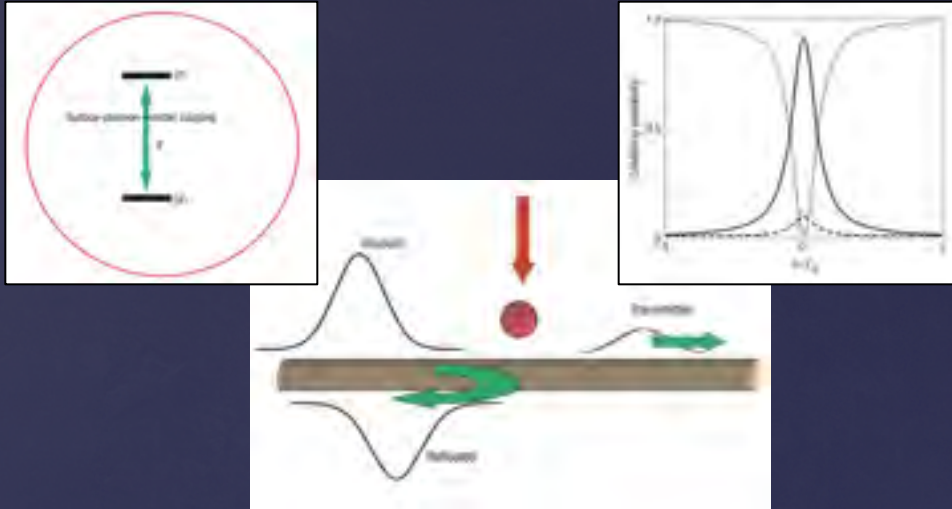
$$P \propto \frac{Q}{V}$$

dielectric: $P \lesssim 1$

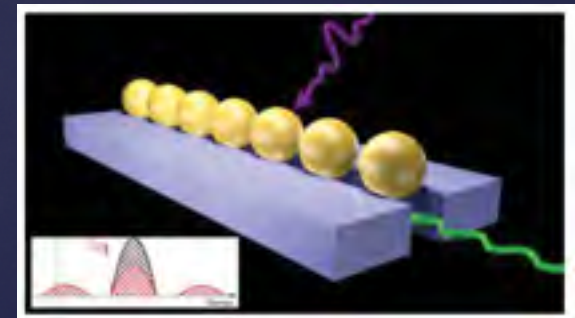
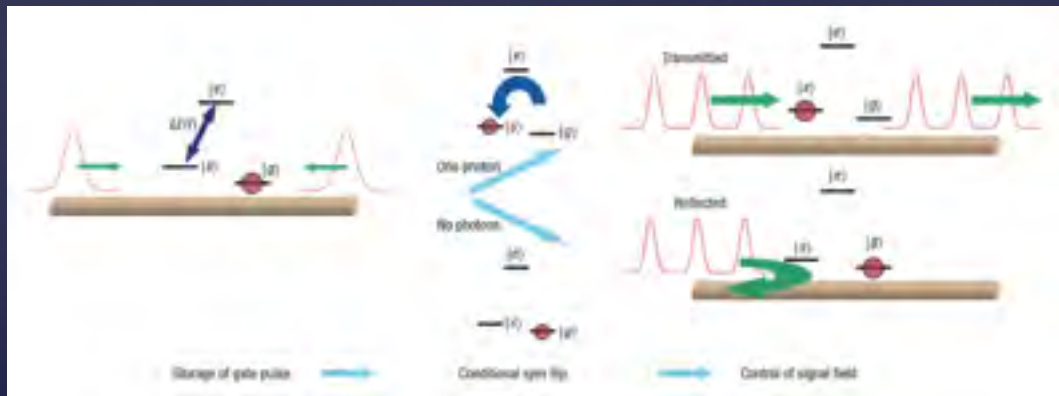
metal wire: $P \gtrsim 100$

metal cavity: $P \gtrsim 200$

Single-photon switches

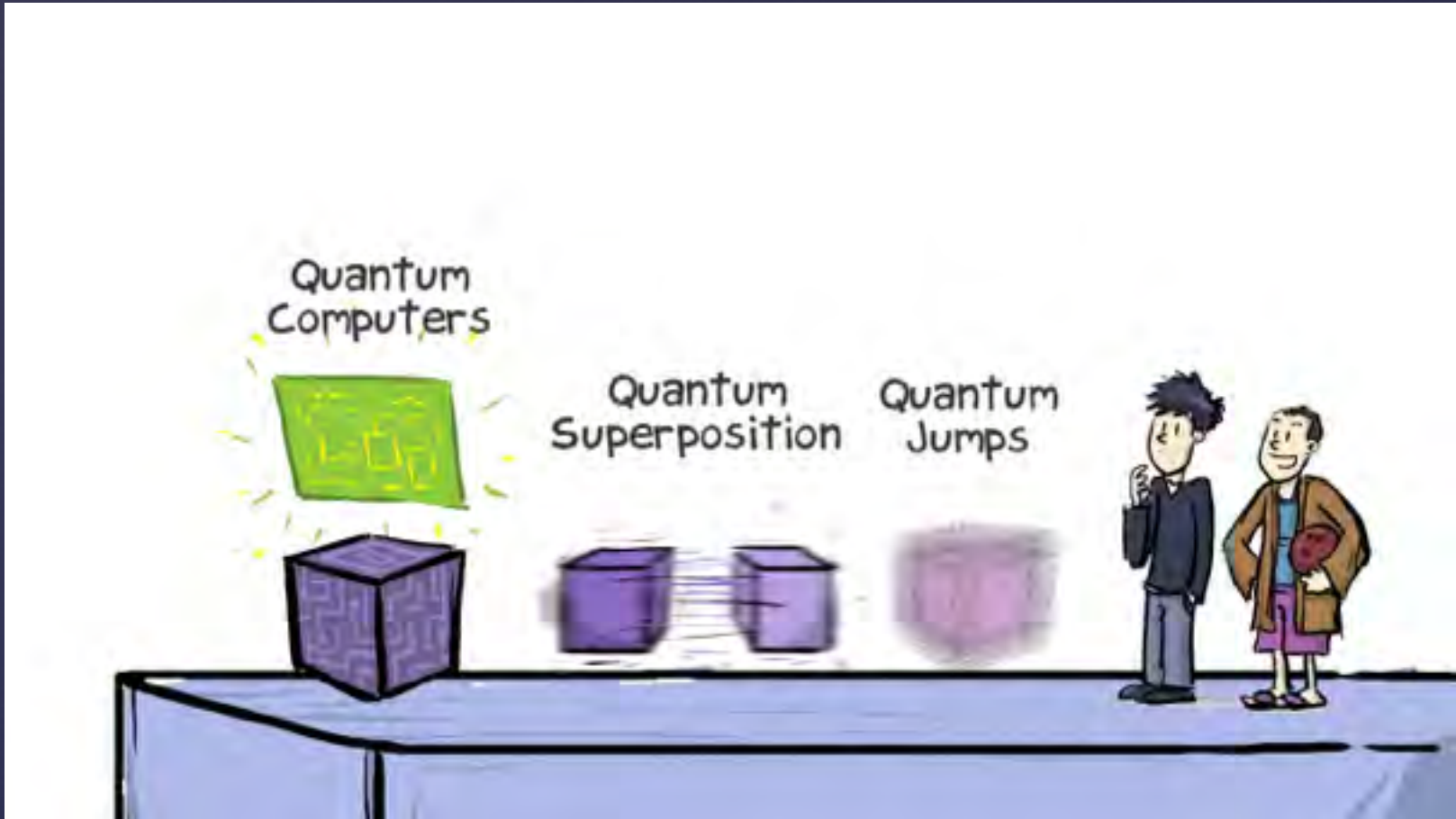


Kolchin et al., PRL 106, 113601 (2011)



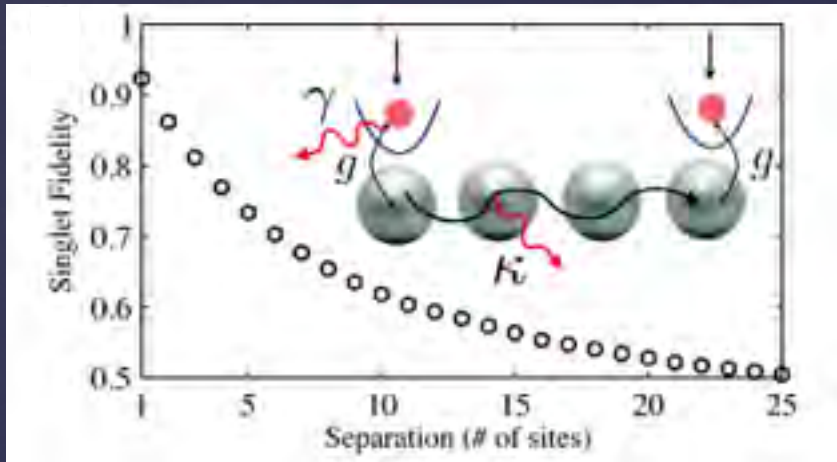
Frank, PRB 85, 195463 (2012)

Chang et al., Nature Phys. 3, 807 (2007)

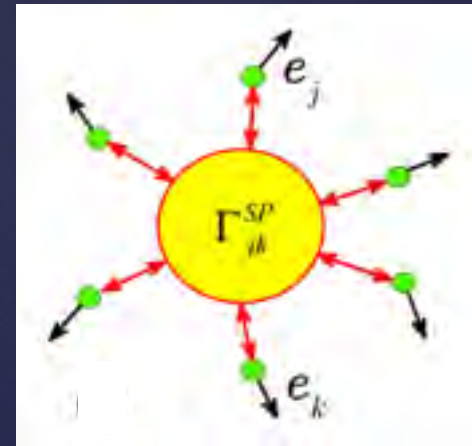


& Quantum Communication, Quantum Sensing...

Many-body quantum dynamics and simulation at nanoscale



Gullans et al., PRL 109, 235309 (2012)



Pustovit and Shahbazyan, PRB 82, 075429 (2010)

and more!

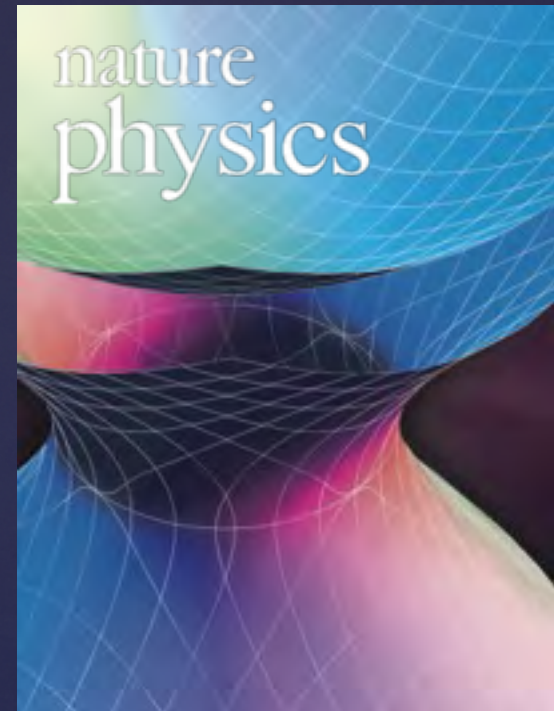
Quantum plasmonics

M. S. Tame^{1*}, K. R. McEnery^{1,2}, Ş. K. Özdemir³, J. Lee⁴, S. A. Maier^{1,5} and M. S. Kim²

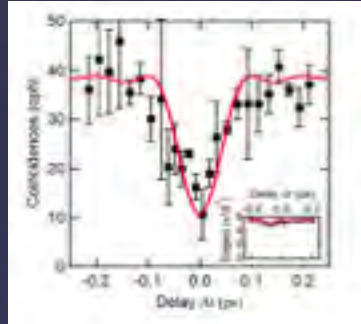
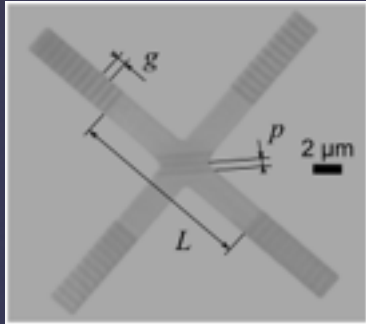
Quantum plasmonics is a rapidly growing field of research that involves the study of the quantum properties of light and its interaction with matter at the nanoscale. Here, surface plasmons—electromagnetic excitations coupled to electron charge density waves on metal-dielectric interfaces or localized on metallic nanostructures—enable the confinement of light to scales far below that of conventional optics. We review recent progress in the experimental and theoretical investigation of the quantum properties of surface plasmons, their role in controlling light-matter interactions at the quantum level and potential applications. Quantum plasmonics opens up a new frontier in the study of the fundamental physics of surface plasmons and the realization of quantum-controlled devices, including single-photon sources, transistors and ultra-compact circuitry at the nanoscale.

Plasmonics provides a unique setting for the manipulation of light via the confinement of the electromagnetic field to regions well below the diffraction limit^{1,2}. This has outlined one of the important challenges that remain to be addressed and new directions for the field.

Tame et al., Nature Phys. 9, 329 (2013)

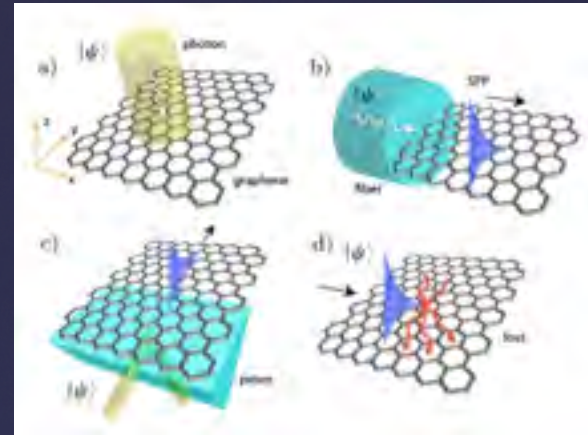


Quantum plasmonics



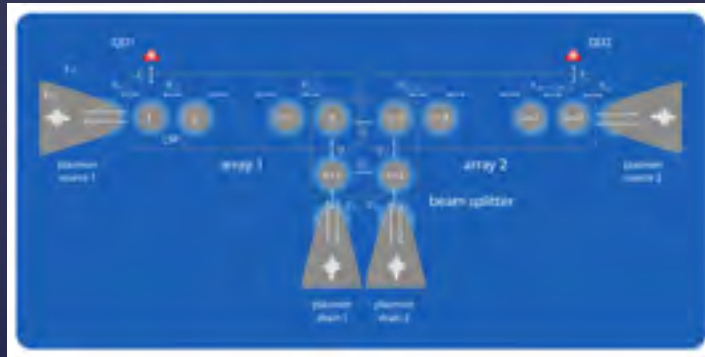
Plasmonic Hong-Ou-Mandel (e)

Di Martino et al., PR App. 1, 034004 (2014)



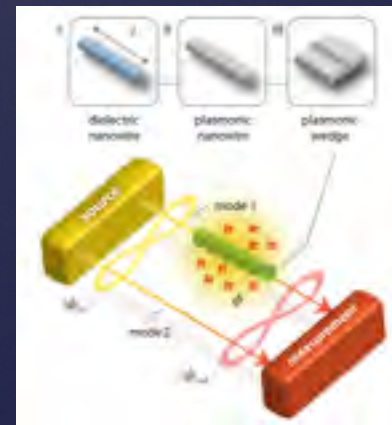
Loss-tolerant propagation in graphene (t)

Hanson et al., Phys. Rev. A 92, 013828 (2015)



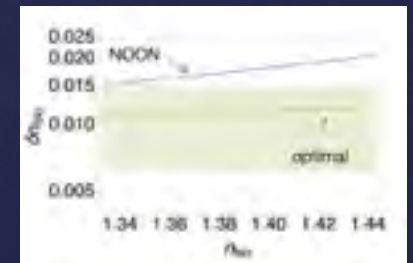
Robust-to-loss entanglement generation (t)

Lee et al., New J. Phys. 15, 083017 (2013)

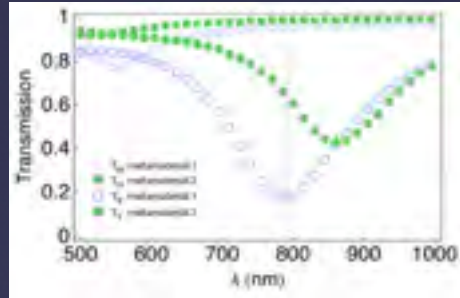
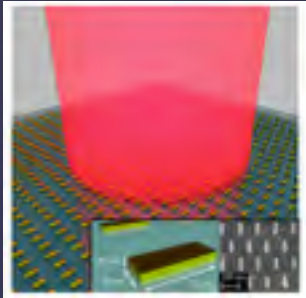


Quantum plasmonic sensing (t)

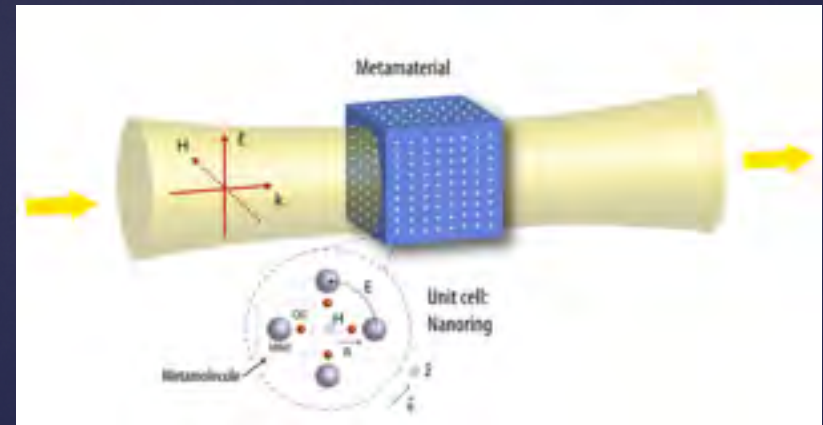
Lee et al. arXiv: 1601.00173 (2016)



Quantum plasmonics



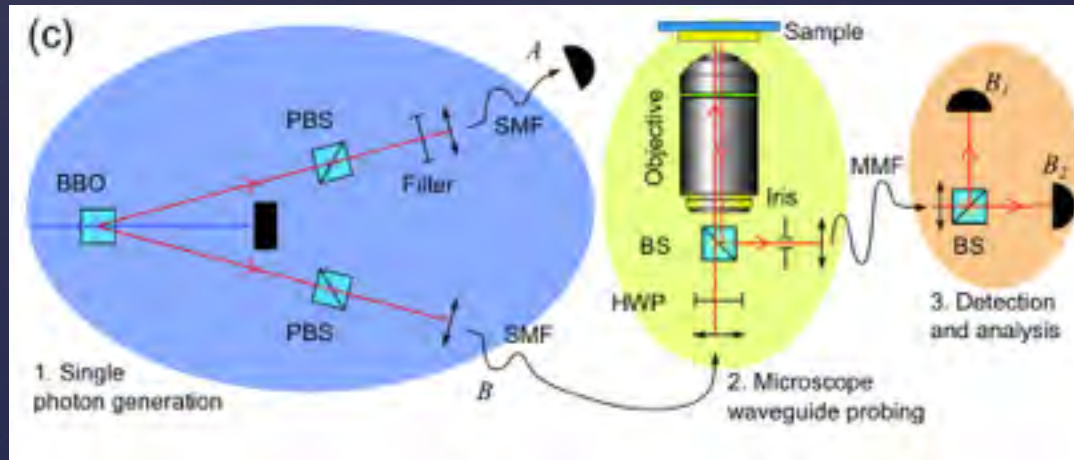
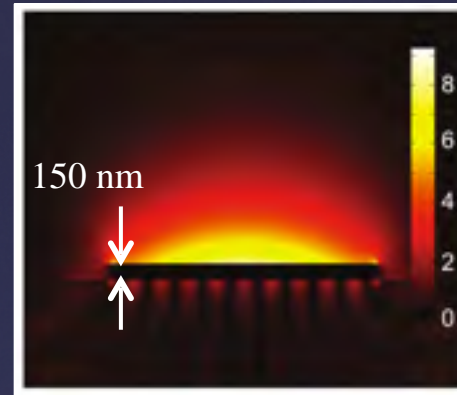
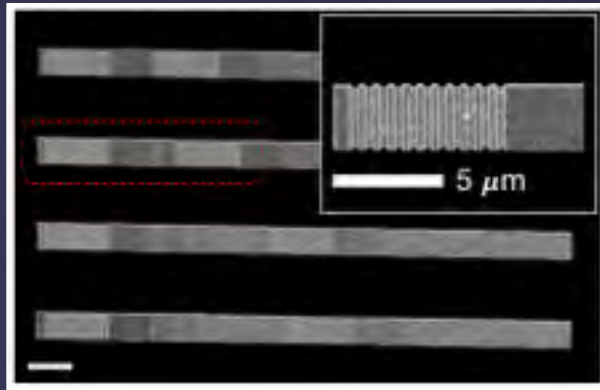
Entanglement distillation using a
plasmonic metamaterial (e)
Asano et al., Scientific Reports (2015)



Quantum plasmonic metamaterials (t)
McEneaney et al., PRA 89, 013822 (2014)

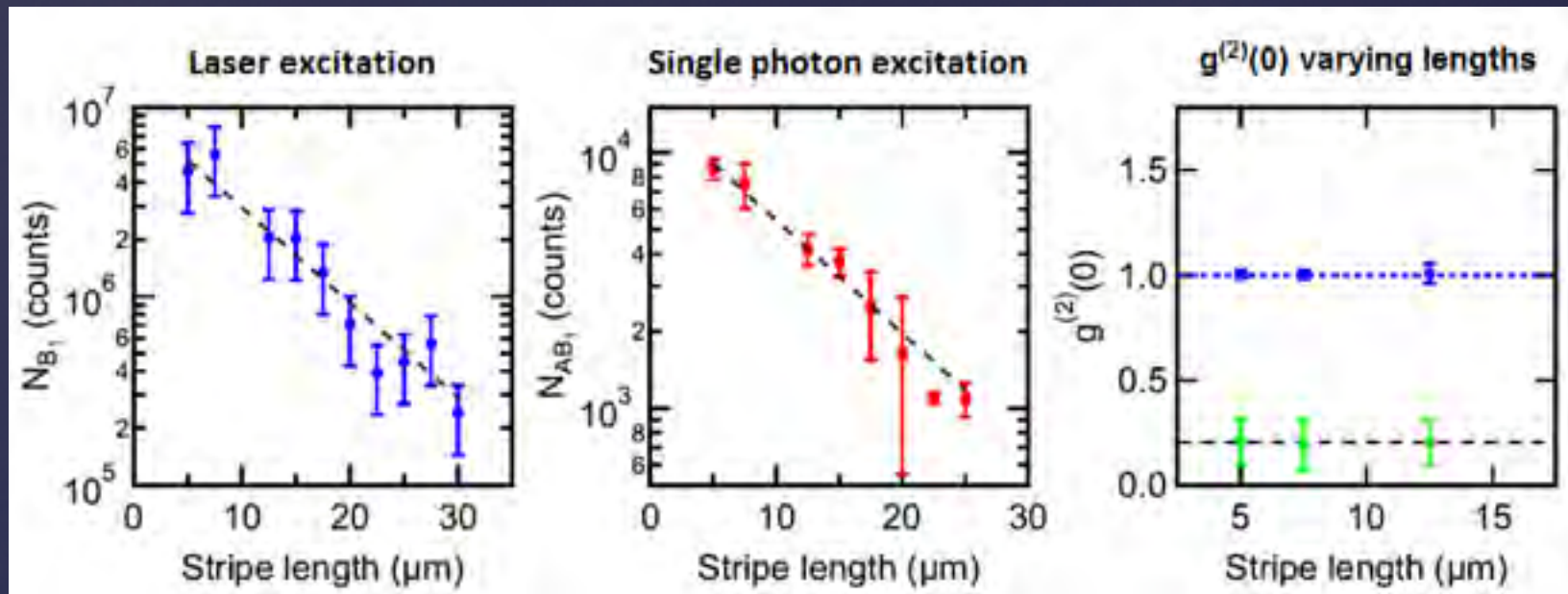
1. LOSS-TOLERANT PROPAGATION

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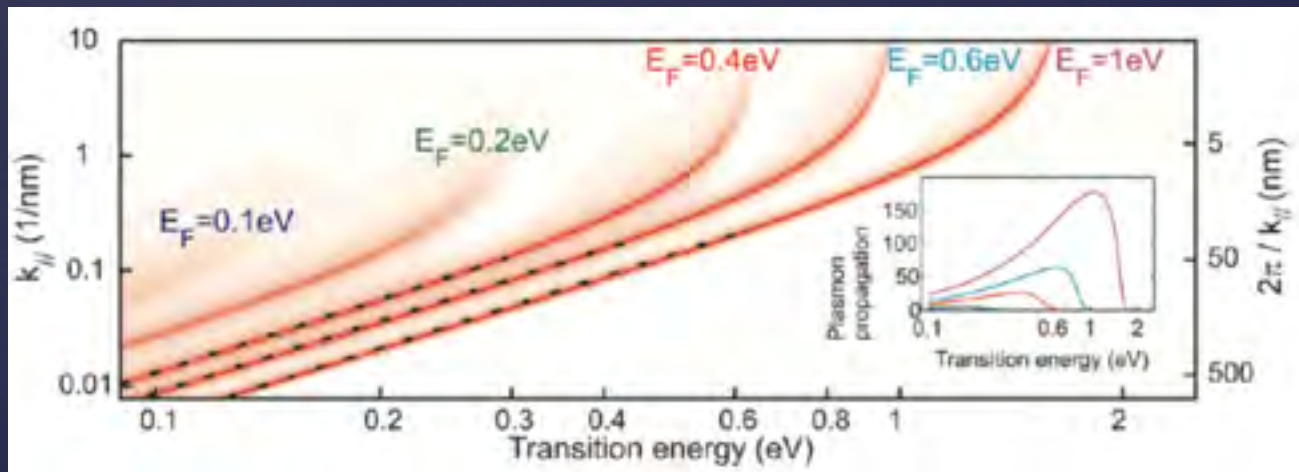
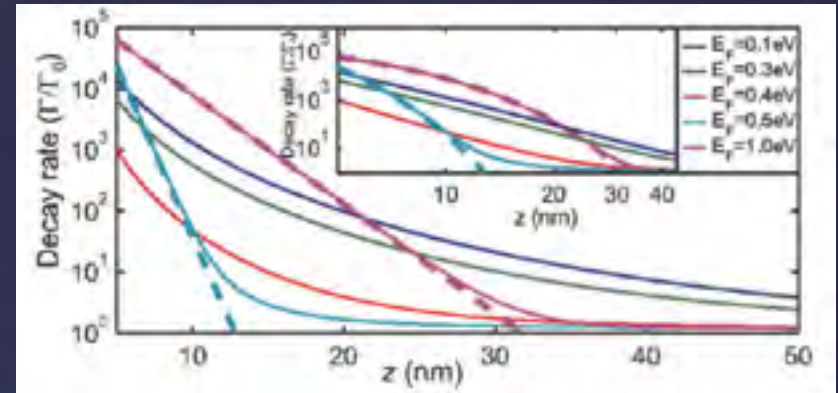
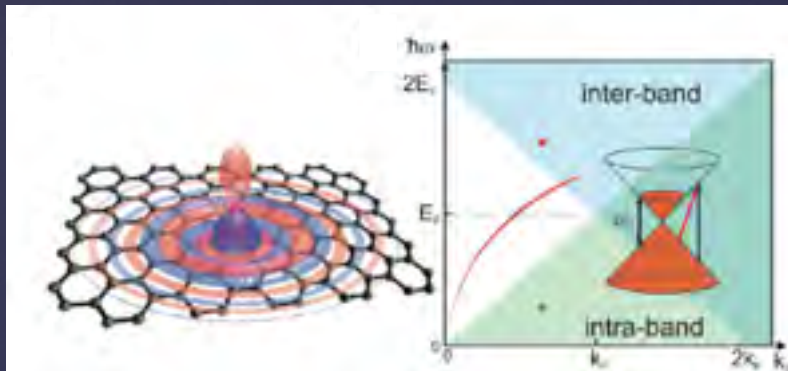
Di Martino et al., Nano Lett. 12, 2504 (2012)

1. LOSS-TOLERANT PROPAGATION



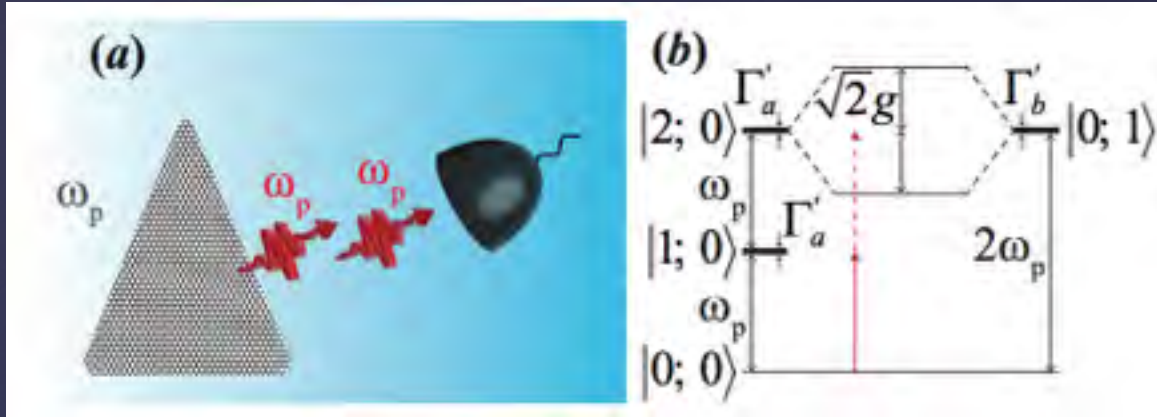
Di Martino et al., Nano Lett. 12, 2504 (2012)

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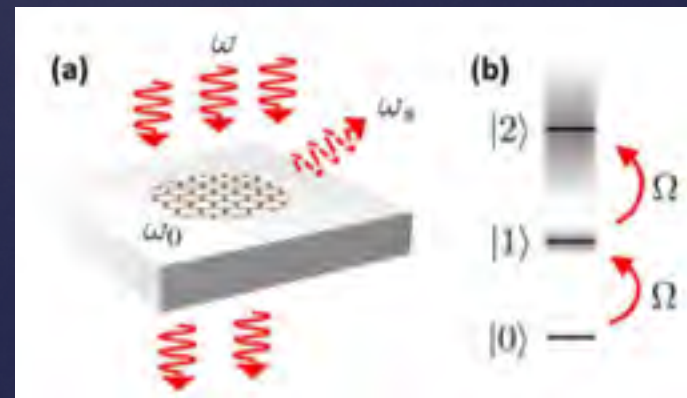


Koppens et al., NL 11, 3370 (2011)

1. LOSS-TOLERANT PROPAGATION

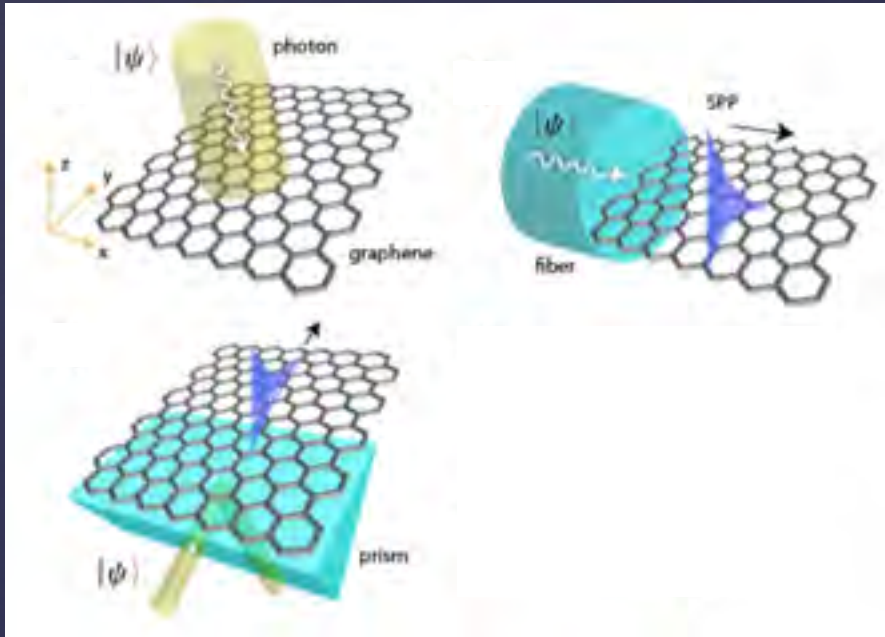


Manzoni et al., NJP 17, 083031 (2015)

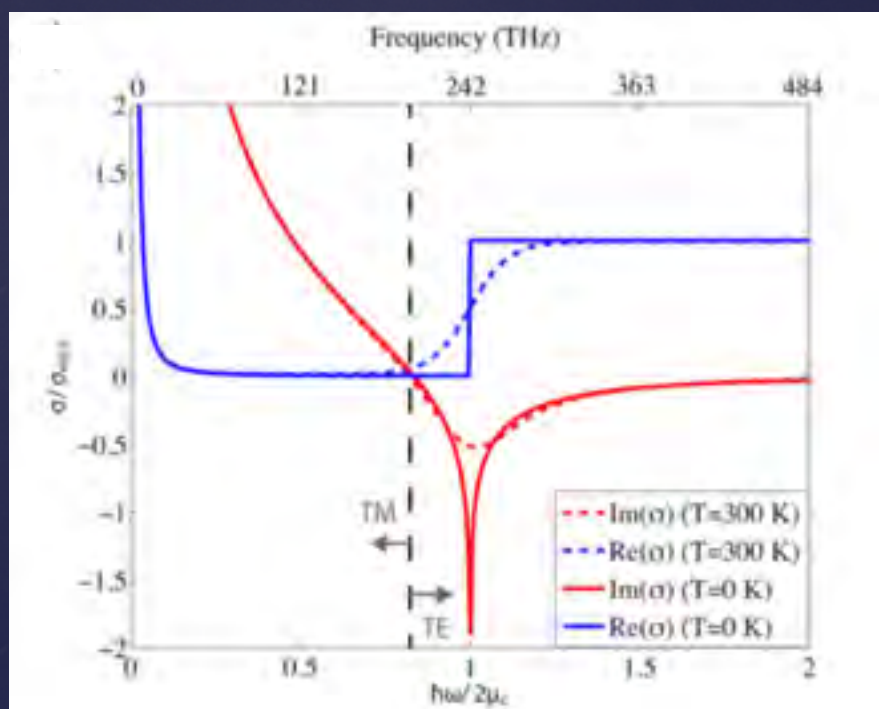


Jablan and Chang, PRL 114, 236801 (2015)

1. LOSS-TOLERANT PROPAGATION



$$\sigma(\omega) = \frac{ie^2 k_B T}{\pi \hbar^2 (\omega + i\Gamma)} \left(\frac{\mu_c}{k_B T} + 2 \ln(e^{-\frac{\mu_c}{k_B T}} + 1) \right) + \frac{ie^2 (\omega + i\gamma)}{\pi \hbar^2} \int_0^\infty \frac{f_d(-\epsilon) - f_d(\epsilon)}{(\omega + i\gamma)^2 - 4(\epsilon/\hbar)^2} d\epsilon$$

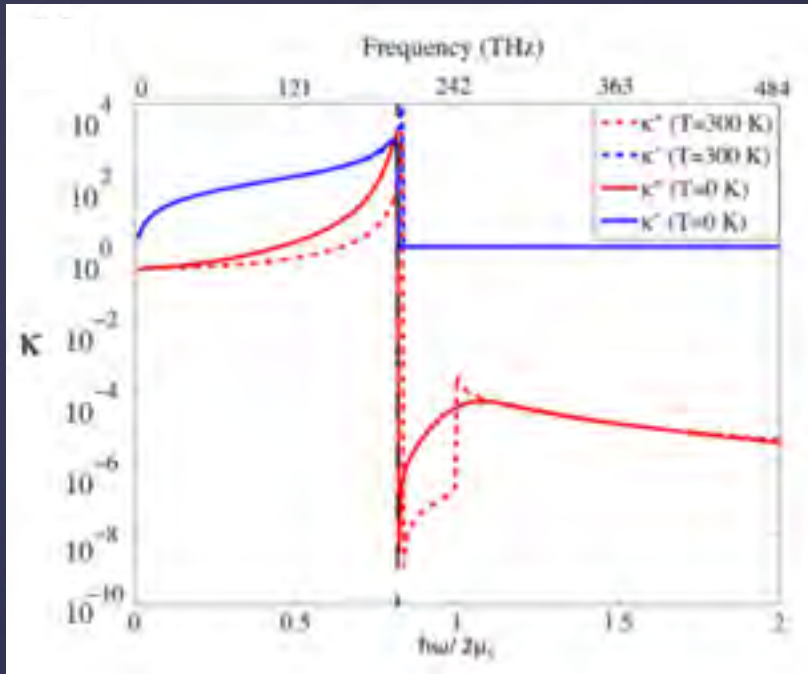


$$k^{\text{TM}} = k_x^{\text{TM}} = k_0 \sqrt{\epsilon_r \left(1 - \left(\frac{2}{\sigma \eta} \right)^2 \right)},$$

$$k^{\text{TE}} = k_x^{\text{TE}} = k_0 \sqrt{\epsilon_r \left(1 - \left(\frac{\sigma \eta}{2} \right)^2 \right)},$$

$$k_0 = \omega_k / c_0 \eta = \sqrt{\mu_0 \mu_r / \epsilon_0 \epsilon_r}$$

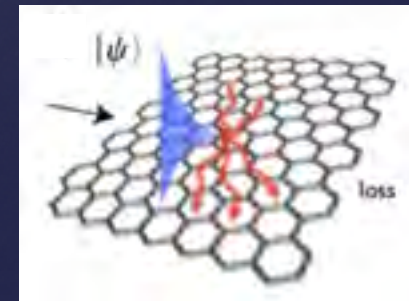
1. LOSS-TOLERANT PROPAGATION



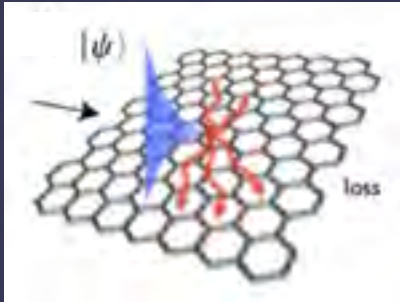
$$\hat{\mathbf{A}}_{\text{SPP}}^{\text{TE/TM}}(\mathbf{r}, t) = \frac{1}{2\pi} \int d\omega \sqrt{\frac{\hbar}{2\epsilon_0 W v_g \omega N^{\text{TE/TM}}}} \times \phi^{\text{TE/TM}}(z, \omega) e^{-i\omega(t-x/v_g)} \hat{b}(\omega) + \text{H.c.}$$

$$\hat{b}_{\text{out}}(\omega, x) = e^{ik_x x} \hat{b}_{\text{out}}(\omega) + i\sqrt{2k_0 \kappa''(\omega)} \int_0^x dx' e^{ik_x(x-x')} \hat{c}(\omega, x')$$

$$\kappa = k_x / k_0 = \kappa' + i\kappa''$$



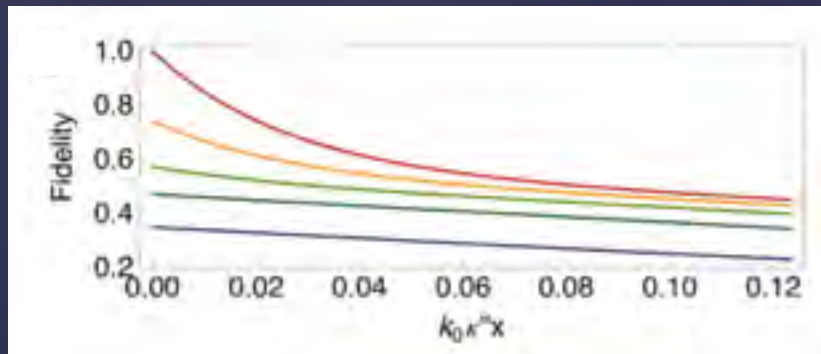
1. LOSS-TOLERANT PROPAGATION



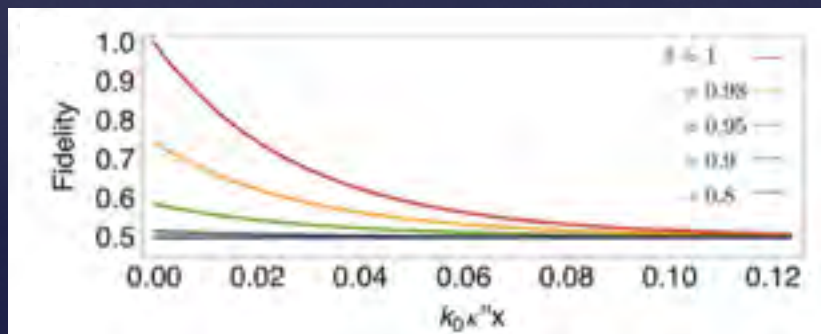
$$|\Psi\rangle_{\text{in}} = N(|\alpha\rangle + |-\alpha\rangle)$$

$$|\pm\alpha\rangle = \exp[-|\alpha|^2/2] \sum_{n=0}^{\infty} (\pm\alpha)^n / \sqrt{n!} |n\rangle$$

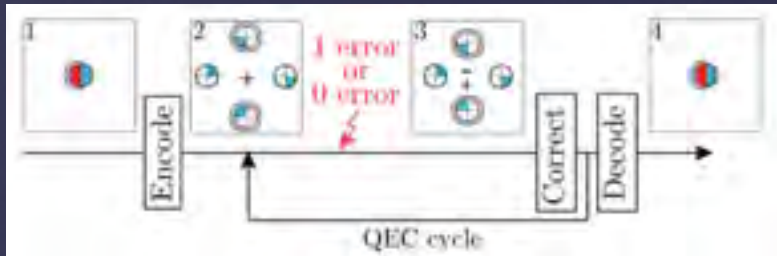
$$F = \langle\psi|\hat{\rho}|\psi\rangle$$



$\alpha = 3$



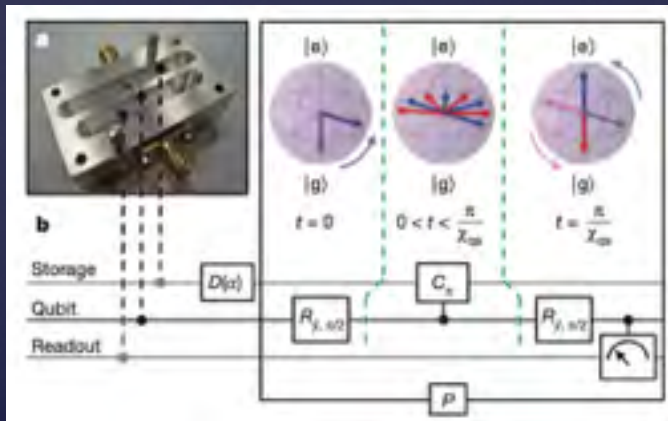
1. LOSS-TOLERANT PROPAGATION



Leghtas et al., PRL 111, 120501 (2013)

Mirrahimi et al., NJP 16, 045014 (2014)

Sun et al., Nature 511, 444 (2014)



$$|\bar{0}_{\pm}\rangle = \frac{1}{\sqrt{N_{\pm}}} (|\alpha\rangle \pm |-\alpha\rangle),$$

$$|\bar{1}_{\pm}\rangle = \frac{1}{\sqrt{N_{\pm}}} (|i\alpha\rangle \pm |-i\alpha\rangle),$$

$$N_{\pm} = 2(1 \pm e^{-2|\alpha|^2})$$

$$|\Psi\rangle = c_0|\bar{0}_+\rangle + c_1|\bar{1}_+\rangle$$

$$|c_0|^2 + |c_1|^2 = 1$$

1. LOSS-TOLERANT PROPAGATION

$$|\Psi\rangle = c_0|\bar{0}_+\rangle + c_1|\bar{1}_+\rangle$$

Encode

$$|c_0|^2 + |c_1|^2 = 1$$

$$\hat{\rho} \rightarrow \hat{\rho}' = \Delta P \hat{\rho}_{\text{jump}} + (1 - \Delta P) \hat{\rho}_{\text{no-jump}}$$

Loss

$$\begin{aligned} |\bar{0}_+\rangle\langle\bar{0}_+| &\rightarrow \Delta P |\bar{0}_-\rangle\langle\bar{0}_-| + (1 - \Delta P) |\bar{0}_{+,\Delta t}\rangle\langle\bar{0}_{+,\Delta t}| \\ |\bar{1}_+\rangle\langle\bar{1}_+| &\rightarrow \Delta P |\bar{1}_-\rangle\langle\bar{1}_-| + (1 - \Delta P) |\bar{1}_{+,\Delta t}\rangle\langle\bar{1}_{+,\Delta t}| \end{aligned}$$

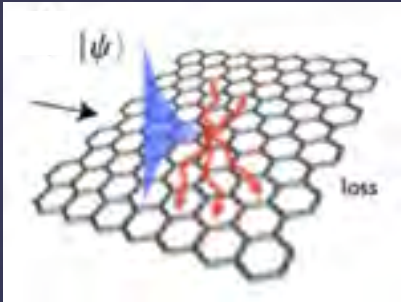
$$\begin{aligned} |\bar{0}_{\pm,\Delta t}\rangle &= \frac{1}{\sqrt{N_{\pm,\Delta t}}} (|\alpha e^{-\gamma\Delta t/2}\rangle \pm |-\alpha e^{-\gamma\Delta t/2}\rangle), \\ |\bar{1}_{\pm,\Delta t}\rangle &= \frac{1}{\sqrt{N_{\pm,\Delta t}}} (|i\alpha e^{-\gamma\Delta t/2}\rangle \pm |-i\alpha e^{-\gamma\Delta t/2}\rangle) \end{aligned}$$

$$\hat{P} = e^{i\pi} \hat{a}^\dagger \hat{a} = \sum_n e^{i\pi n} |n\rangle\langle n| \rightarrow$$

$$\begin{aligned} \langle\bar{0}_+|\hat{P}|\bar{0}_+\rangle &= \langle\bar{1}_+|\hat{P}|\bar{1}_+\rangle = +1 \\ \langle\bar{0}_-|\hat{P}|\bar{0}_-\rangle &= \langle\bar{1}_-|\hat{P}|\bar{1}_-\rangle = -1 \end{aligned}$$

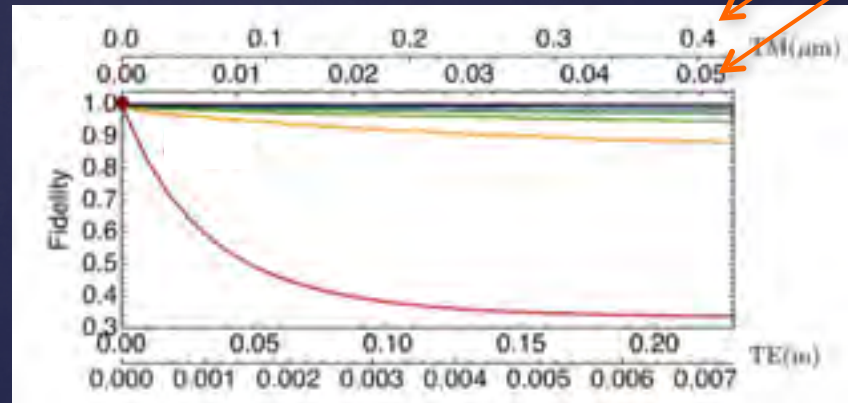
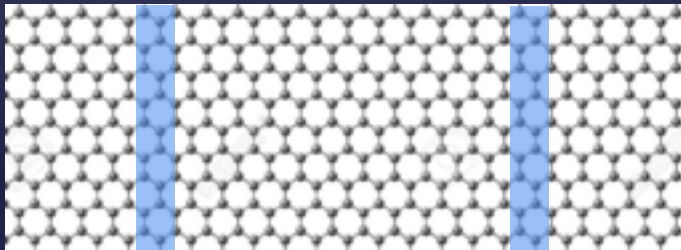
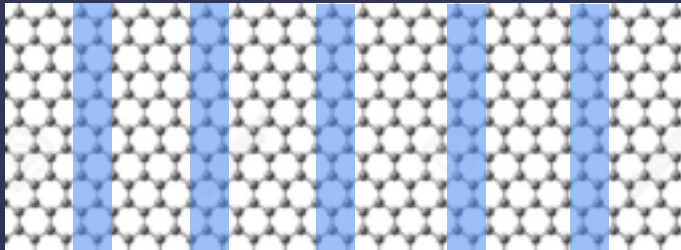
Parity
check

1. LOSS-TOLERANT PROPAGATION



Ideal case parity check is every

$$\Delta x \ll (2k_0\kappa''|\alpha|^2) \text{ m}$$



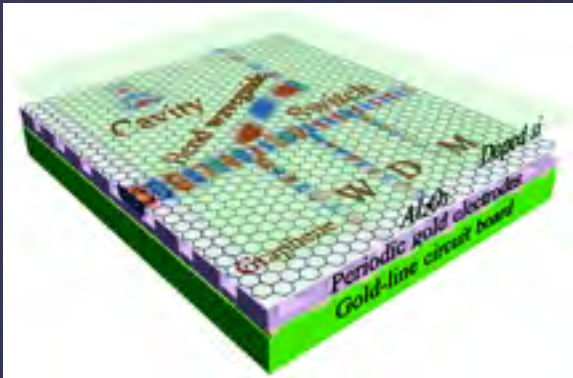
1500 nm
810 nm



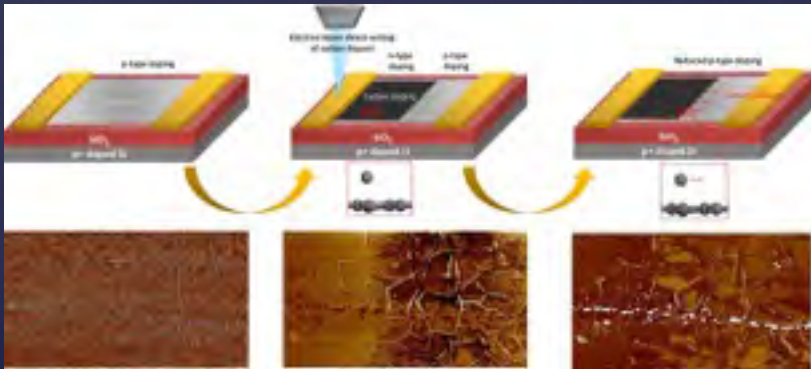
$\alpha = 3$

Fidelity averaged over Bloch sphere

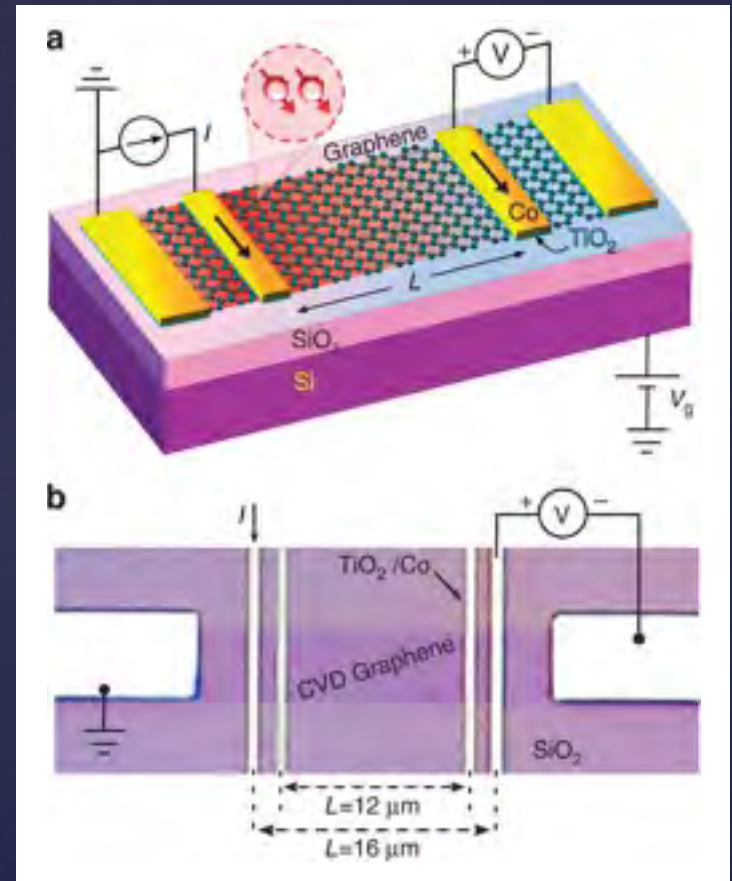
1. LOSS-TOLERANT PROPAGATION



Chen et al., *Nanoscale* 7, 10912 (2015)



Kim et al., *Nanoscale* 7, 14946 (2015)

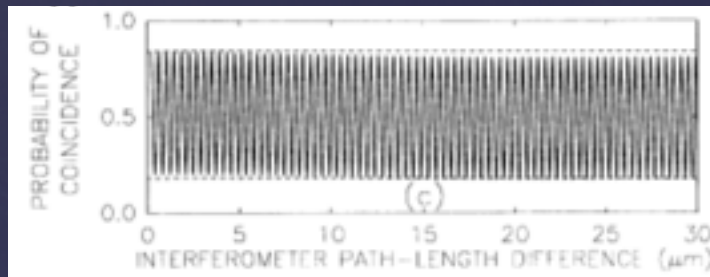


Kamalakar et al., *Nat. Comm.* 6, 6766 (2015)

2. *QUANTUM PLASMONIC SENSING*

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Caves, PRD 23, 1693 (1981)



N=2

$$|\psi_{\text{in}}\rangle = \frac{1}{\sqrt{2}} (|N0\rangle + |0N\rangle)_{12}$$

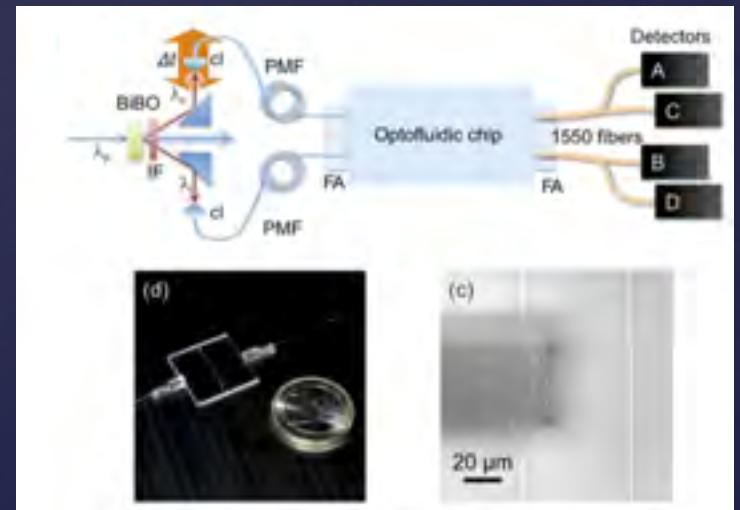
Rarity et al., PRL 65, 1348 (1990)

Lee et al., J. Mod. Opt. 49, 2325 (2002)

Giovannetti et al., Science 306, 1330 (2004)

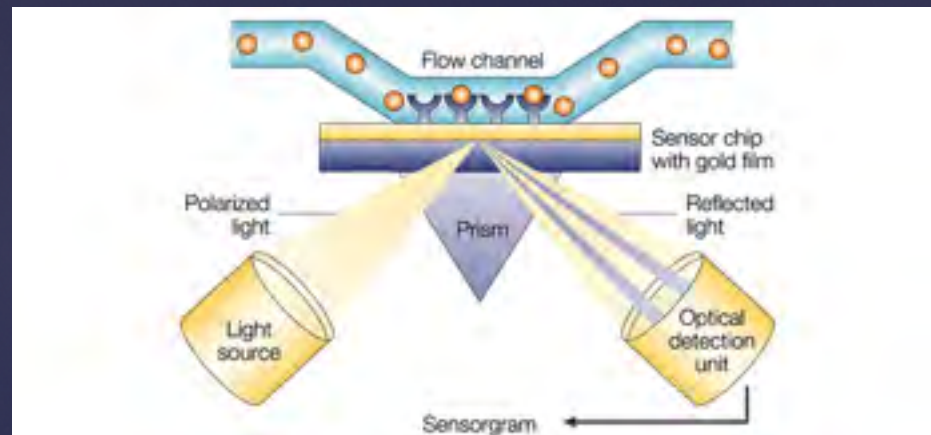
Nagata et al., Science 316, 5825 (2007) N=4

Demkowicz-Dobrzanski et al.,
Prog. Opt. 60, 345 (2015)



Crespi et al., APL 100, 233704 (2012)

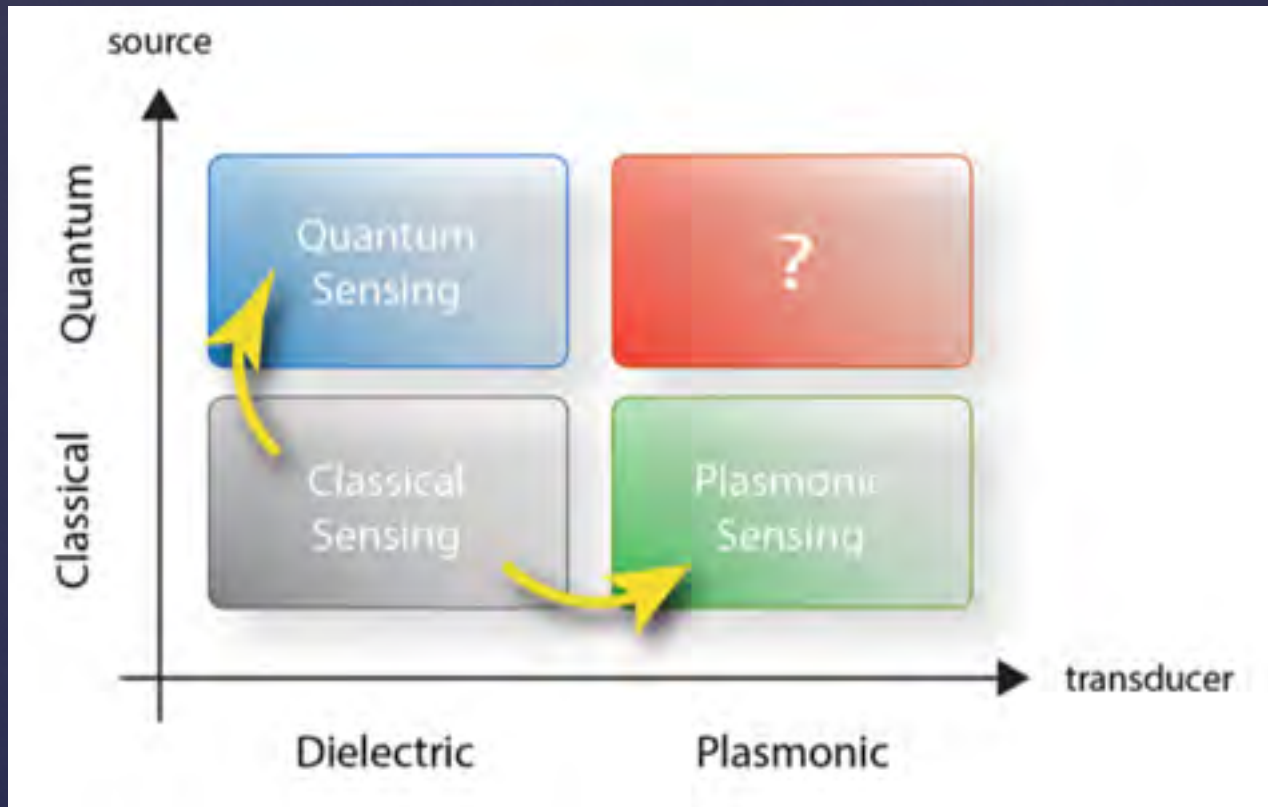
2. QUANTUM PLASMONIC SENSING



J. N. Anker et al., *Nature Mat.* 7, 442 (2008)

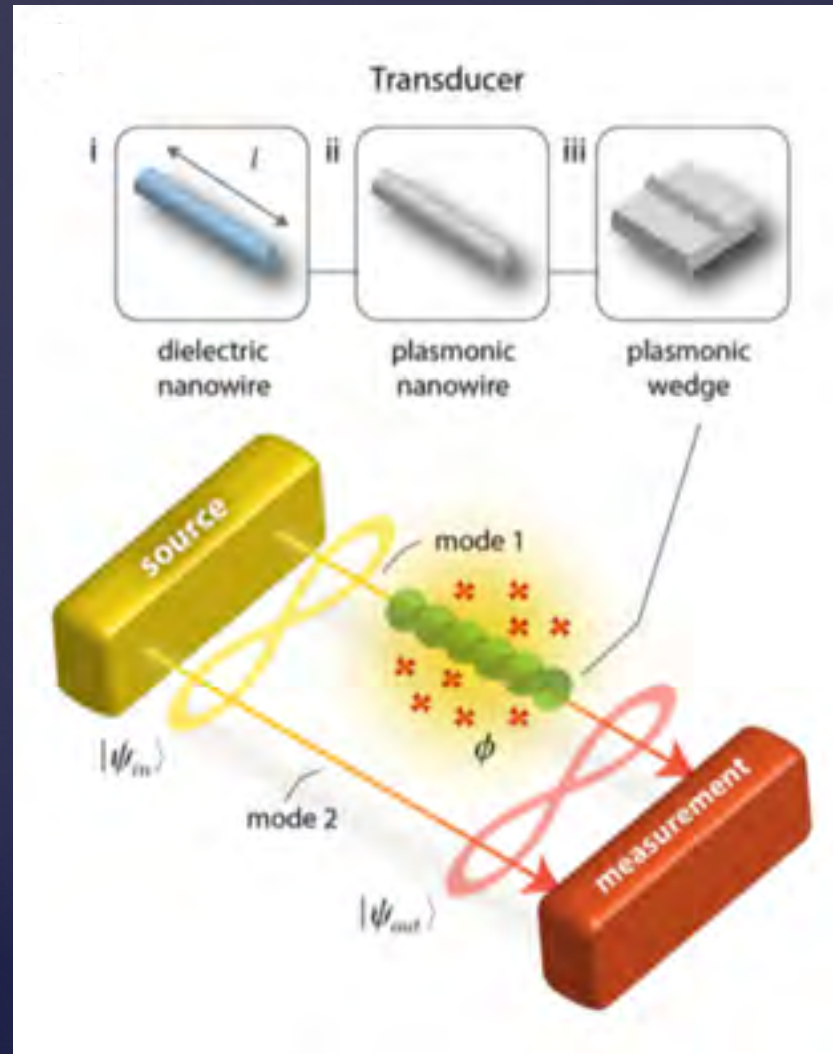
e.g. BIACORE, Dynamic Biosensors, Attana AB etc.

2. QUANTUM PLASMONIC SENSING

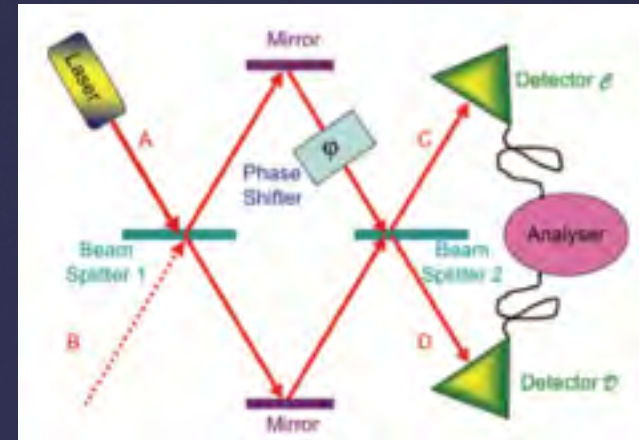


Lee et al. arXiv: 1601.00173 (2016)

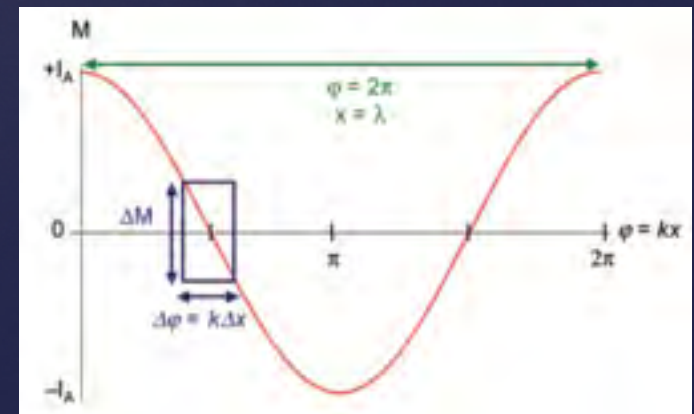
2. QUANTUM PLASMONIC SENSING



Dowling, Contem. Phys. 49, 125 (2008)



$$M(\varphi) \equiv I_D - I_C = I_A \cos(\varphi)$$



2. QUANTUM PLASMONIC SENSING

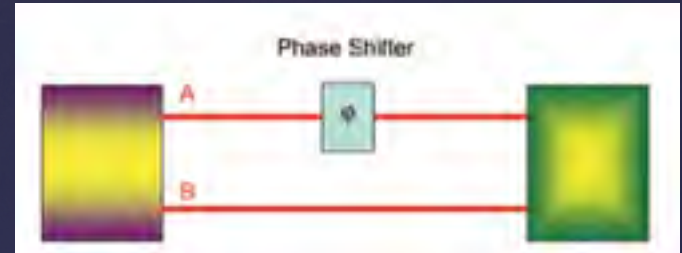
1. Source (fixed number of photons N)

(i) Classical:

$$|\alpha\rangle_A |0\rangle_B \xrightarrow{\text{BS}} \left| \frac{\alpha}{\sqrt{2}} \right\rangle_A \left| \frac{\alpha}{\sqrt{2}} \right\rangle_B \quad |\alpha|^2 = N$$

(ii) Quantum:

$$|N00N\rangle = \frac{1}{\sqrt{2}} (|N\rangle_A |0\rangle_B + |0\rangle_A |N\rangle_B)$$



source

measurement

2. Phase ϕ picked up in mode A

3. Measurement

(i) Classical: beamsplitter (BS) on modes, then measurement of intensity difference

$$M = I_B - I_A \rightarrow$$

\rightarrow minimum resolution

$$\delta\phi^{(\text{SNL})} = 1/\sqrt{N}$$

'shot noise' limit

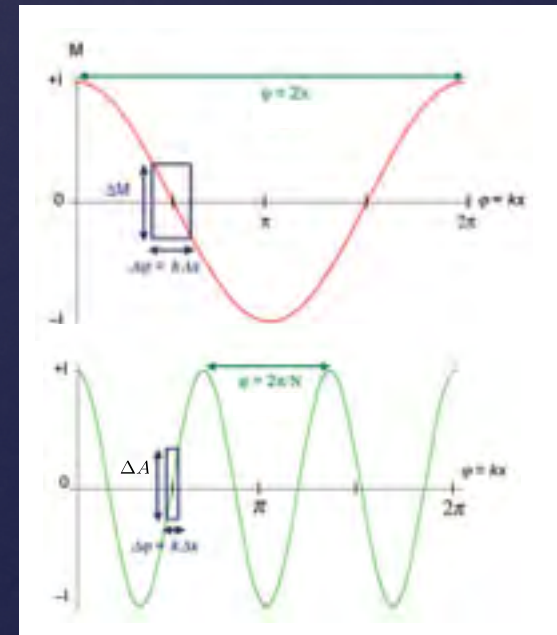
(ii) Quantum: measurement of operator

$$\hat{A} = |0, N\rangle \langle N, 0| + |N, 0\rangle \langle 0, N| \rightarrow$$

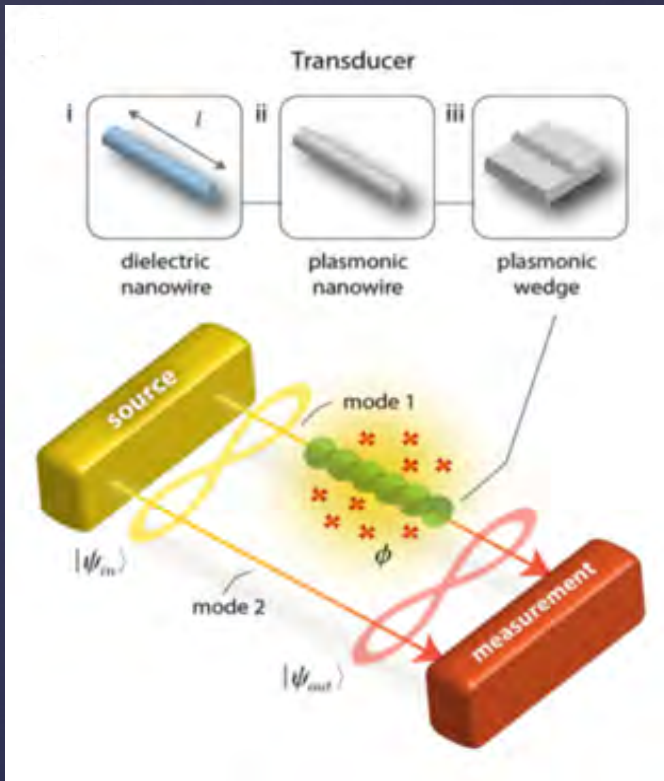
\rightarrow minimum resolution

$$\delta\phi^{(\text{HL})} = 1/N$$

'Heisenberg' limit



2. QUANTUM PLASMONIC SENSING

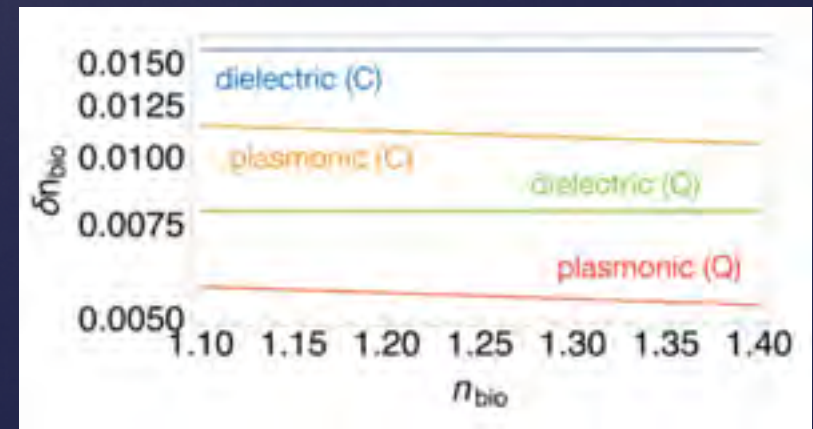
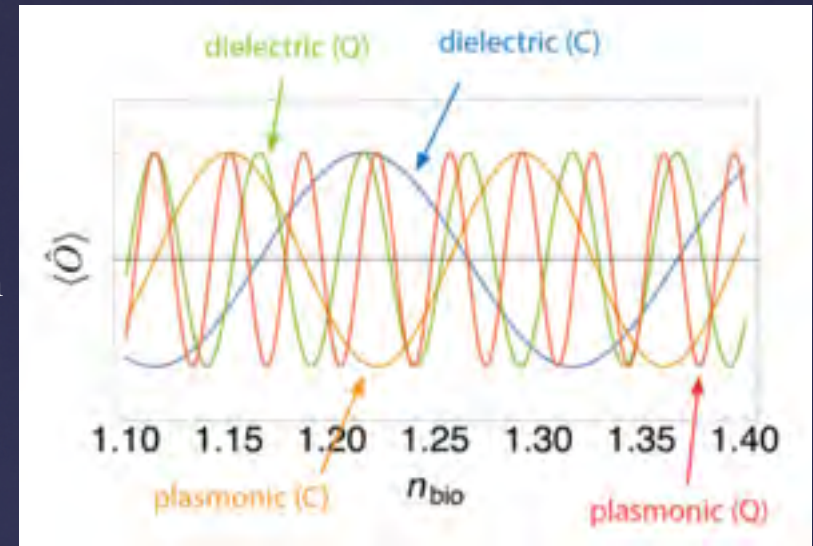


nanowire
 $l = 4\mu\text{m}$
 $\lambda_0 = 810\text{ nm}$
 $r = 50\text{ nm}$
 $N=4$

Minimum resolution

$$\delta n_{\text{bio}} = \frac{\Delta \hat{O}}{|\partial \langle \hat{O} \rangle / \partial n_{\text{bio}}|}$$

$$\phi(n_{\text{bio}}) = \beta(n_{\text{bio}}) \times l \quad \Delta \hat{O} = (\langle \hat{O}^2 \rangle - \langle \hat{O} \rangle^2)^{1/2}$$



2. QUANTUM PLASMONIC SENSING

- When loss is included the NOON state is no longer optimal
- Neither is the measurement operator A

Optimal states

$$|\psi_{\text{in}}\rangle = \sum_{n=0}^N c_n |n, N - n\rangle$$

for some set of coefficients c_n depending on loss

Dorner et al., PRL 102, 040403 (2009)

Optimal measurement

Hard to find but we can use the following relation:

$$\delta\phi = F_Q^{-1/2}$$

F_Q – Fisher information

(amount of info about ϕ that state contains)

and use

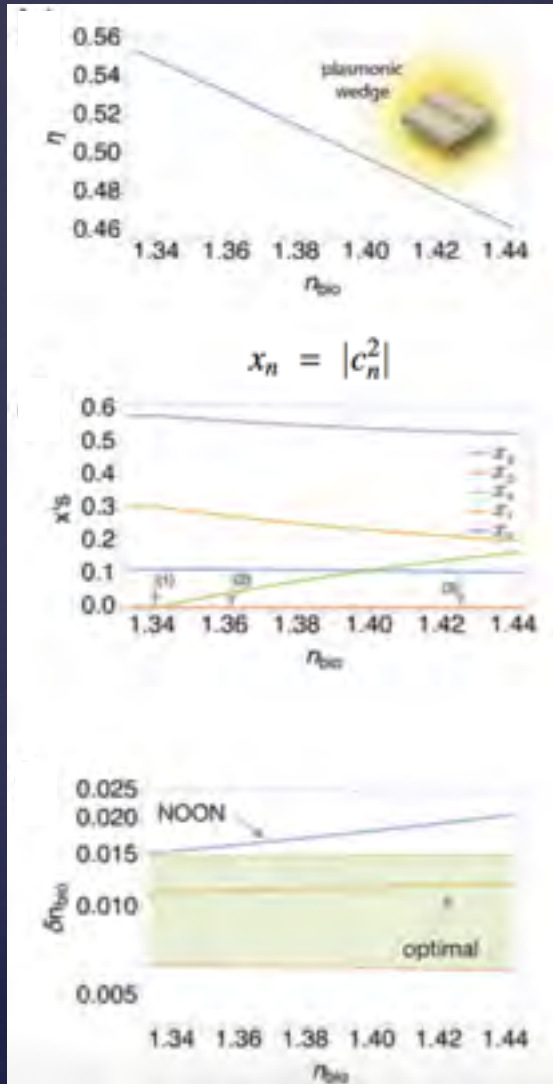
$$\delta n_{\text{bio}} = \delta\phi \left| \frac{\partial\phi}{\partial n_{\text{bio}}} \right|^{-1}$$

depends on medium (dielectric or plasmonic)

depends on state and measurement

$$\frac{\partial\phi}{\partial n_{\text{bio}}} (= l \times \frac{\partial\beta}{\partial n_{\text{bio}}})$$

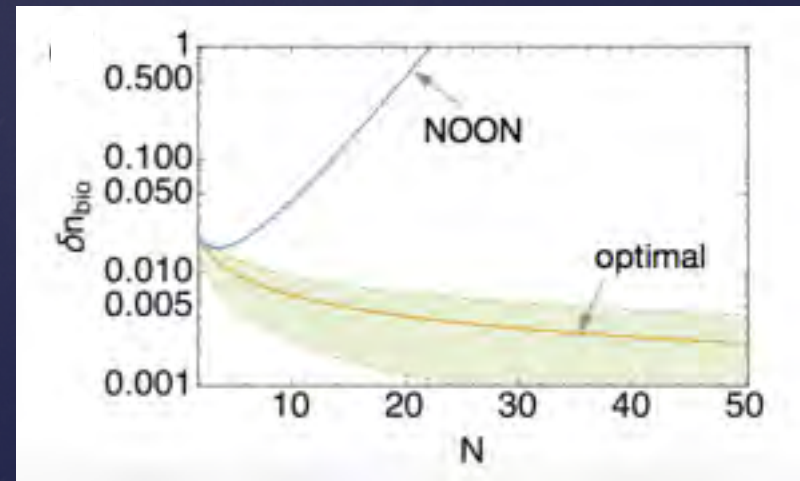
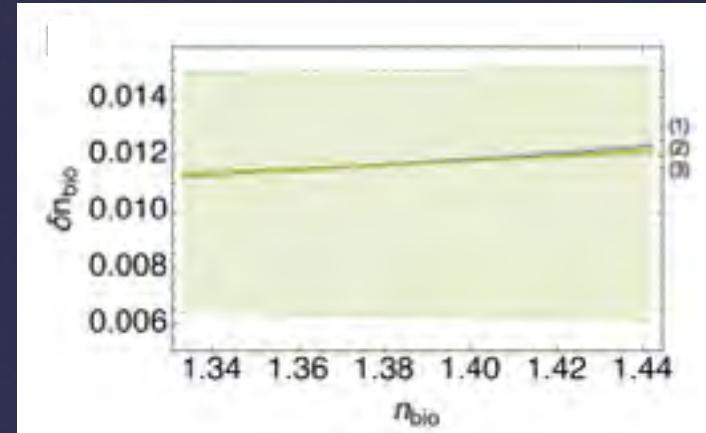
2. QUANTUM PLASMONIC SENSING



$l = 4\mu\text{m}$
 $\lambda_0 = 810\text{ nm}$
 $h = 50\text{ nm}$

SNL (SIL)

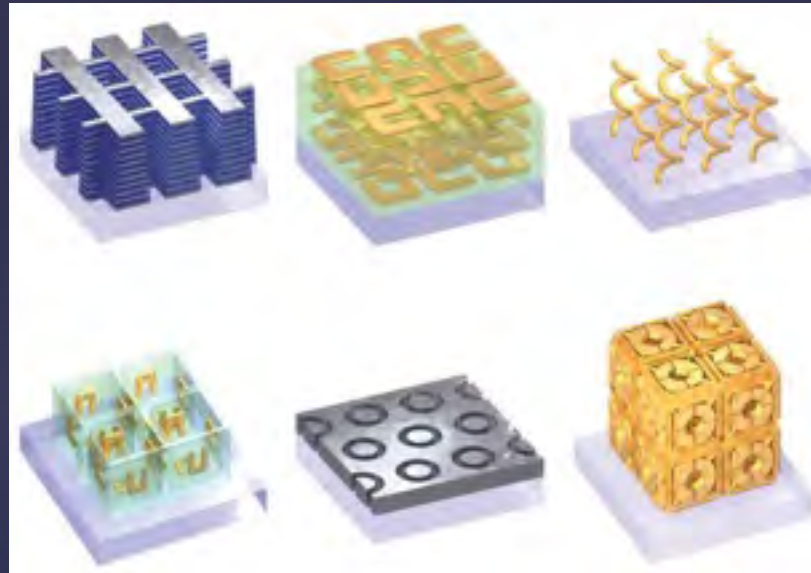
HL



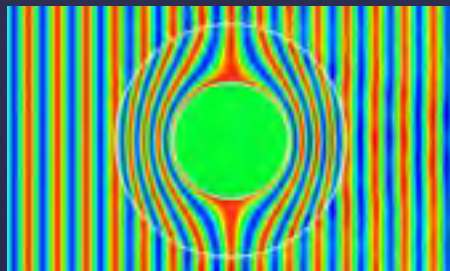
Quantum plasmonic sensing is useful for highly photosensitive material of which only a small quantity is available

3. ENGINEERED METAMATERIALS

3. ENGINEERED METAMATERIALS



Soukoulis and Wegener, Nat. Phot. 5, 523 (2011)

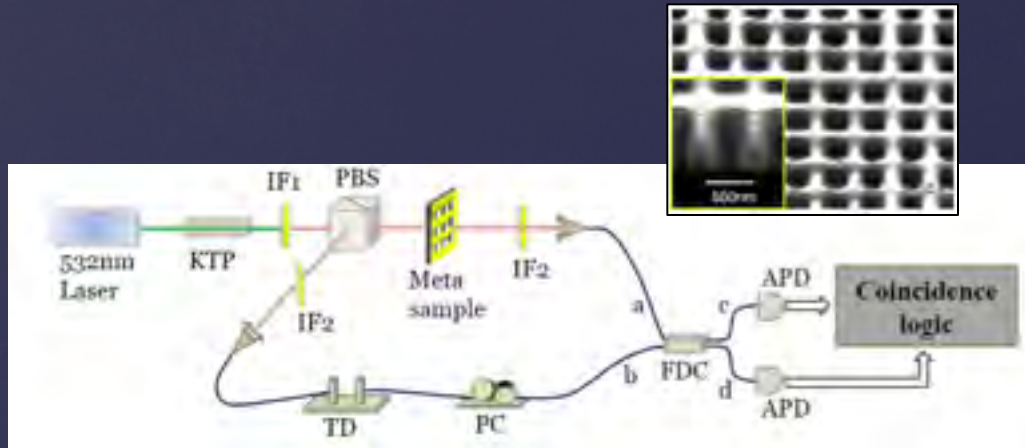


Cai et al., Nat. Phot. 1, 224 - 227 (2007)

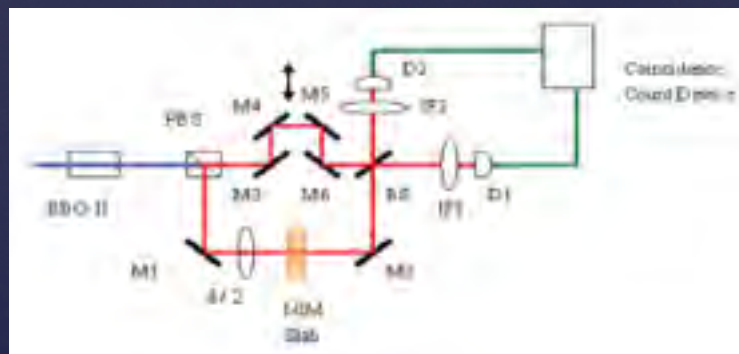


Casse et al., APL 96, 023114 (2010)

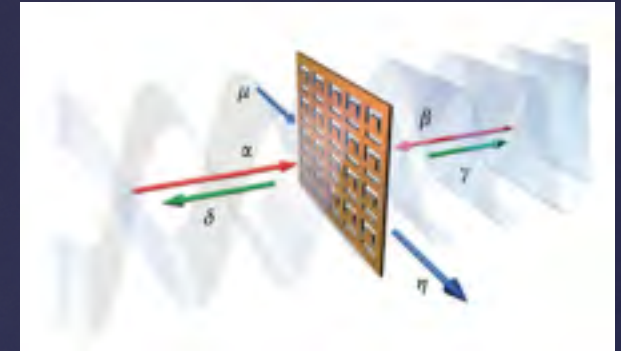
Quantum optical metamaterials



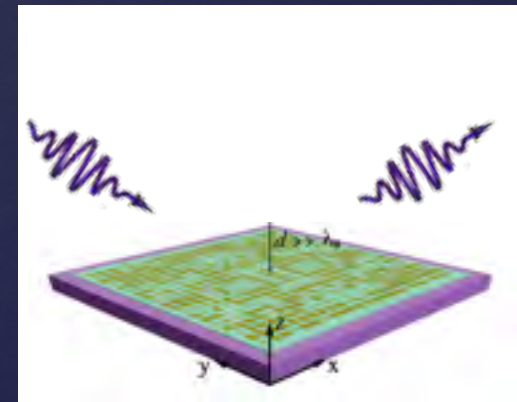
Wang et al., Opt. Exp. 20, 5213 (2012)



Zhou et al., PRA 85, 023841 (2012)

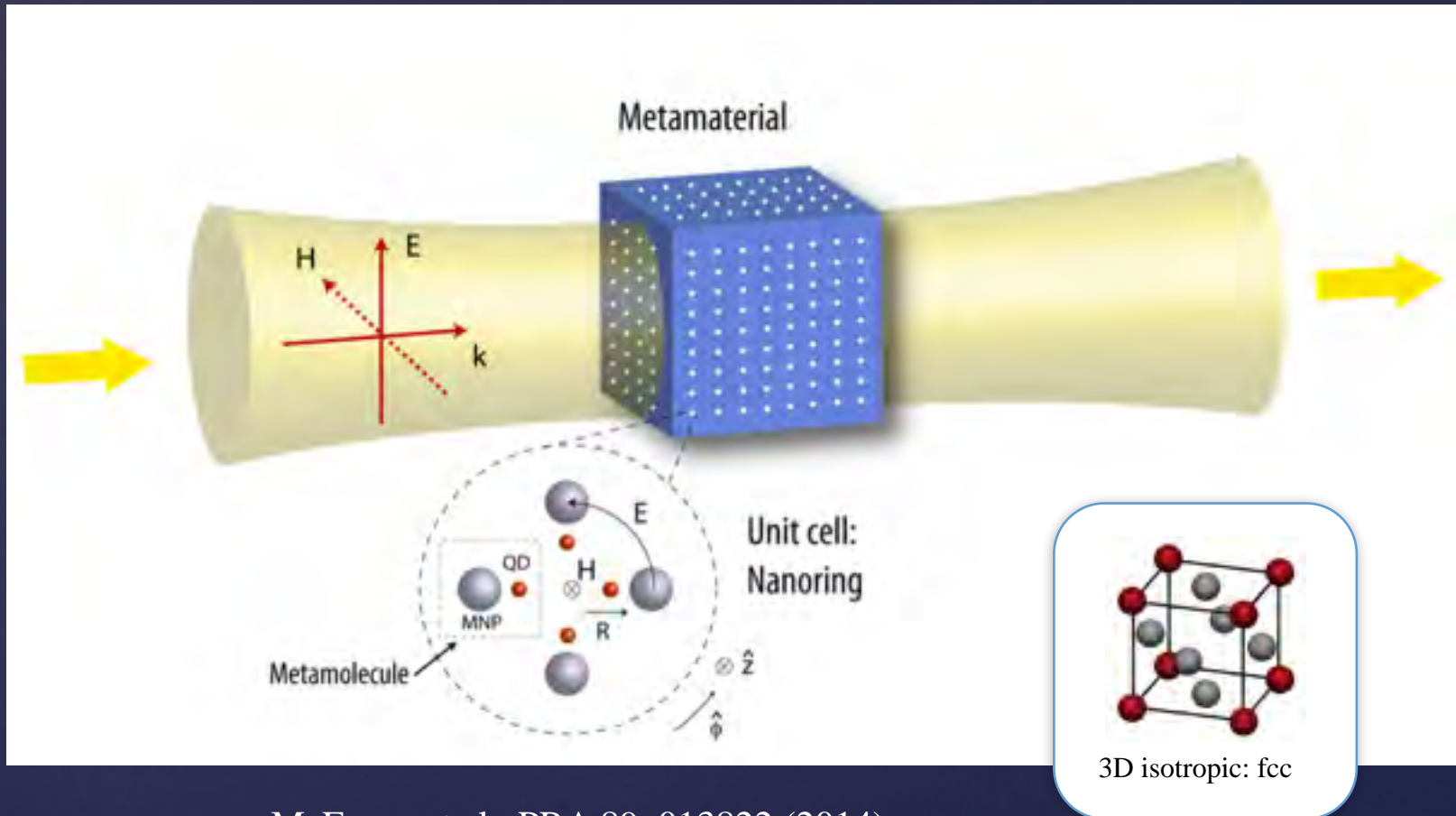


Roger et al., Nat. Comm. 6, 7031 (2015)



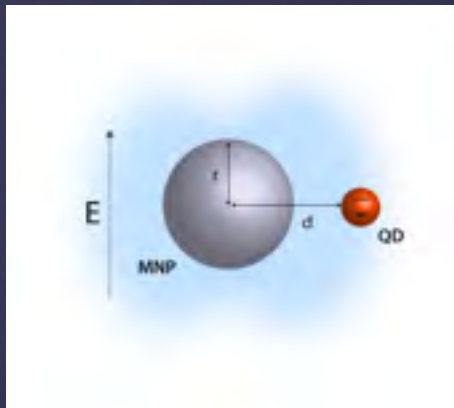
Jha et al., PRL 115, 025501 (2015)

Quantum optical metamaterials



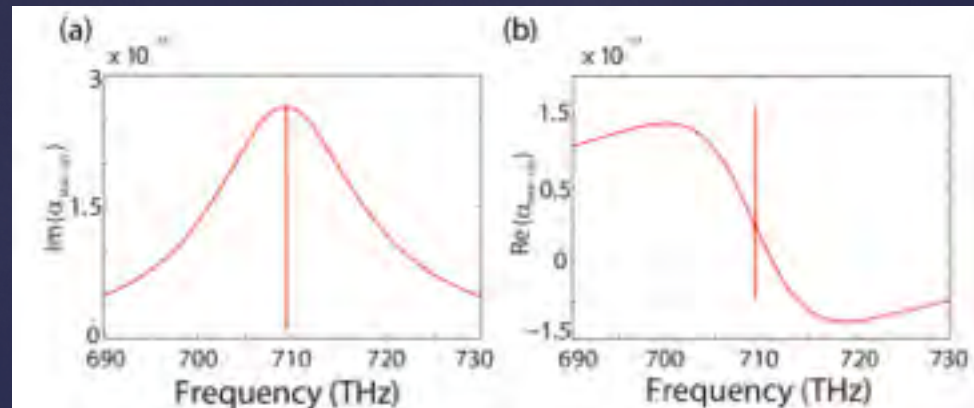
McEneaney et al., PRA 89, 013822 (2014)

Quantum optical metamaterials



Zhang, Govorov and Bryant, PRL 97, 146804 (2006)
 Ridolfo et al., PRL 105, 263601 (2010)
 Waks and Sridharan, PRA 82, 043845 (2010)

$$\hat{H} = \hat{H}_0 + \hat{H}_{int} + \hat{H}_{drive}$$

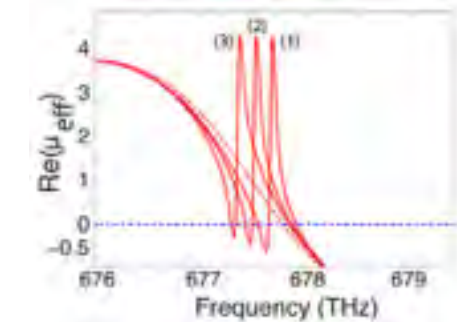
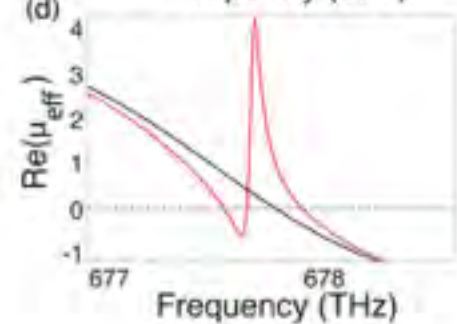
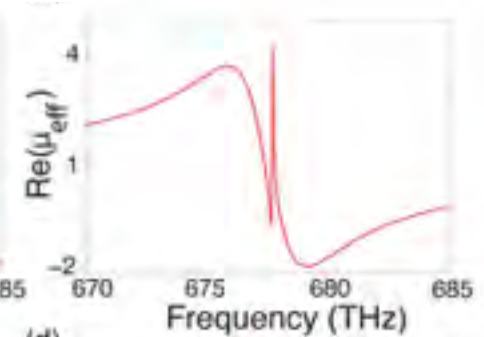
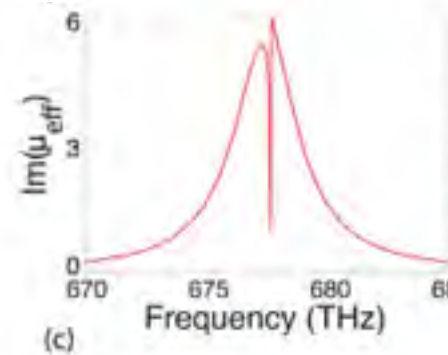
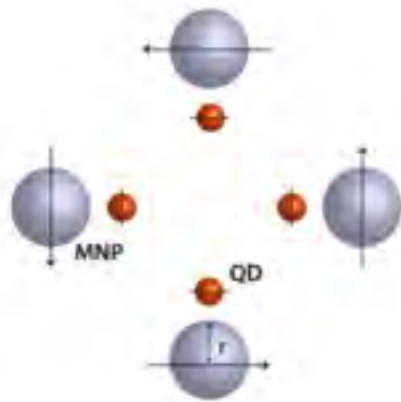


$$\begin{aligned} \hat{H}_0 &= \hbar\omega_0 \hat{a}^\dagger \hat{a} + \hbar\omega_x \hat{\sigma}^\dagger \hat{\sigma}, \\ \hat{H}_{int} &= i\hbar g (\hat{\sigma}^\dagger \hat{a} + \hat{\sigma} \hat{a}^\dagger), \\ \hat{H}_{drive} &= -E_0 \mu (\hat{\sigma} e^{-i\omega t} + \hat{\sigma}^\dagger e^{i\omega t}) \\ &\quad - E_0 (\chi^* \hat{a} e^{-i\omega t} + \chi \hat{a}^\dagger e^{i\omega t}) \end{aligned}$$

$$g = \frac{S\mu}{d^3} \sqrt{\frac{3\eta r^3}{4\pi\epsilon_0 \hbar}}$$

$$\chi = -i\epsilon_b \sqrt{12\eta\epsilon_0 \pi \hbar} r^3$$

Quantum optical metamaterials

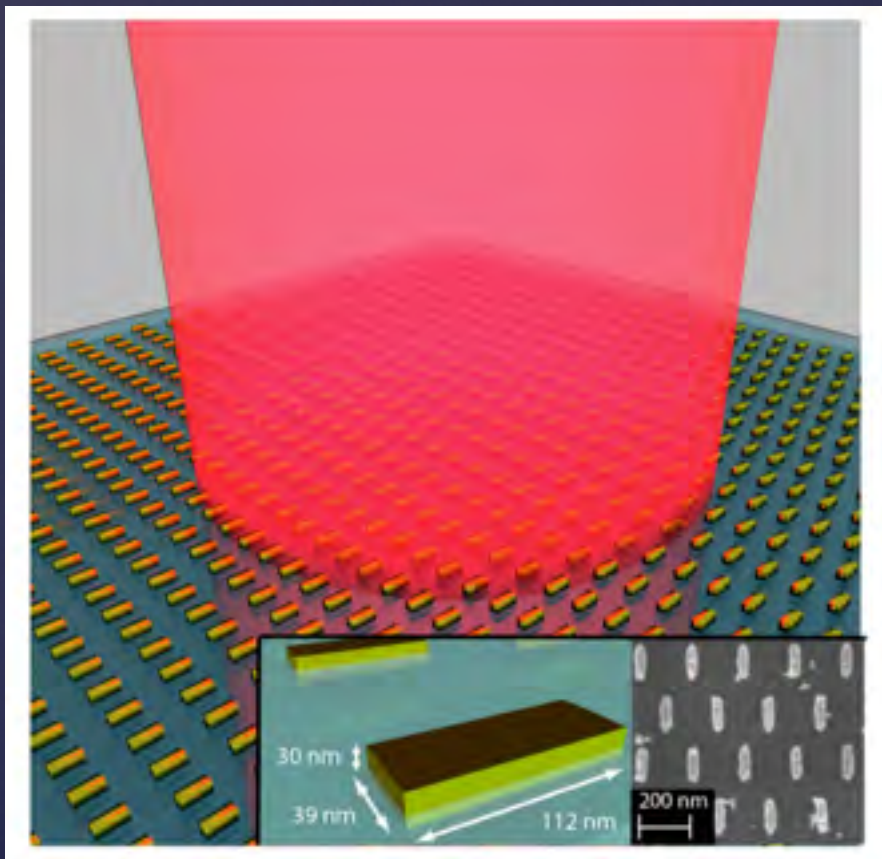


$$\hat{H}_0 = \sum_{n=0}^{N-1} \hbar\omega_0 \hat{a}_n^\dagger \hat{a}_n,$$

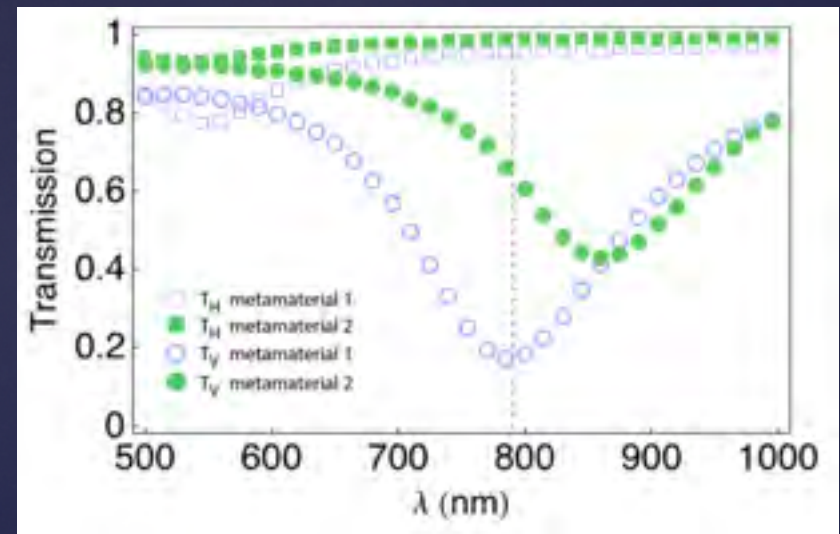
$$\hat{H}_{int} = \sum_{n,m=0}^{N-1} \hbar J_{nm} (\hat{a}_n^\dagger \hat{a}_m + \hat{a}_m^\dagger \hat{a}_n) \quad n \neq m$$

$$\hat{H}_{drive} = -E_0 \sum_{n=0}^{N-1} (\chi^* \hat{a}_n e^{-i\omega t} + \chi \hat{a}_n^\dagger e^{i\omega t}).$$

Quantum optical metamaterials



beam size $\sim 90 \mu\text{m}$



Asano et al., Sci. Rep. 5, 18313 (2015)

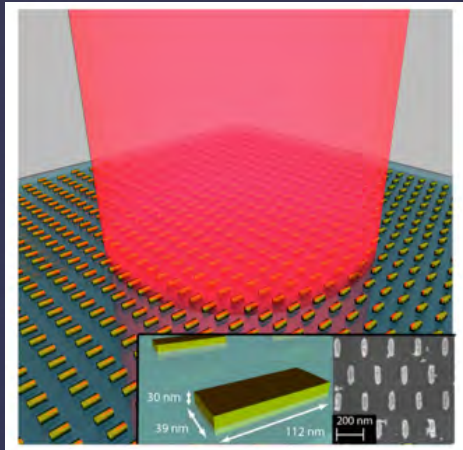
Quantum optical metamaterials

Bohren and Huffman,

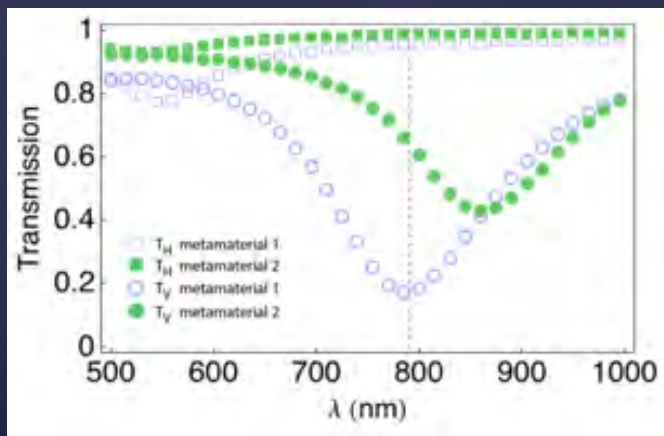
'Absorption and scattering of light by small particles' (1983)

Polarizability of individual nanorod:

$$\alpha_{ii} = \frac{\pi}{8} w z l \frac{\epsilon_m - \epsilon_d}{3\epsilon_d + 3L_{ii}(\epsilon_m - \epsilon_d)}$$

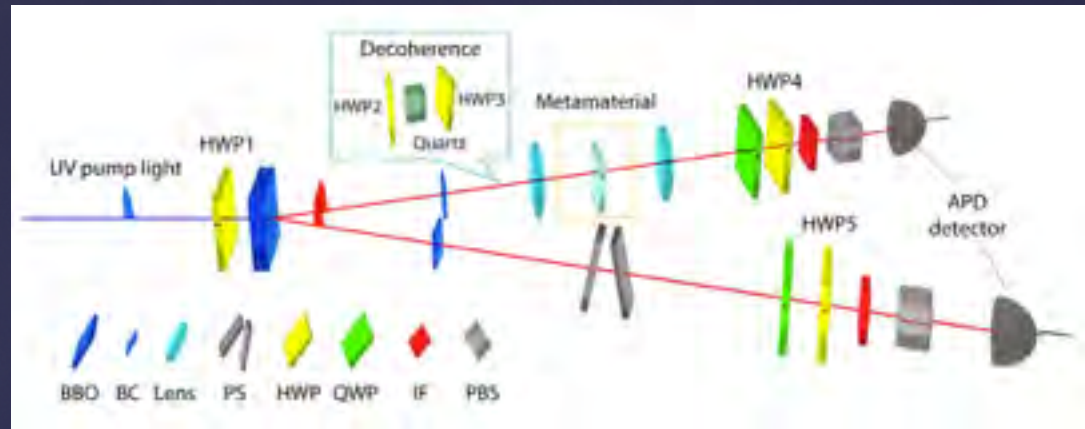
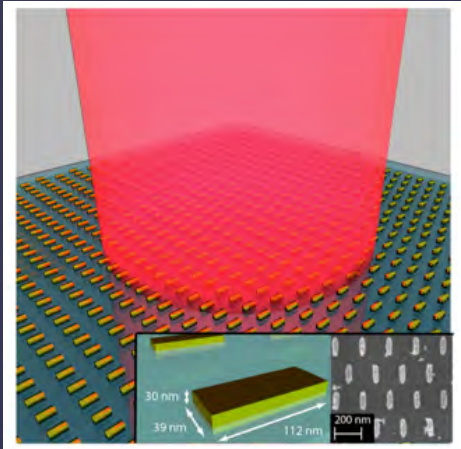


L_{ii} – shape factor



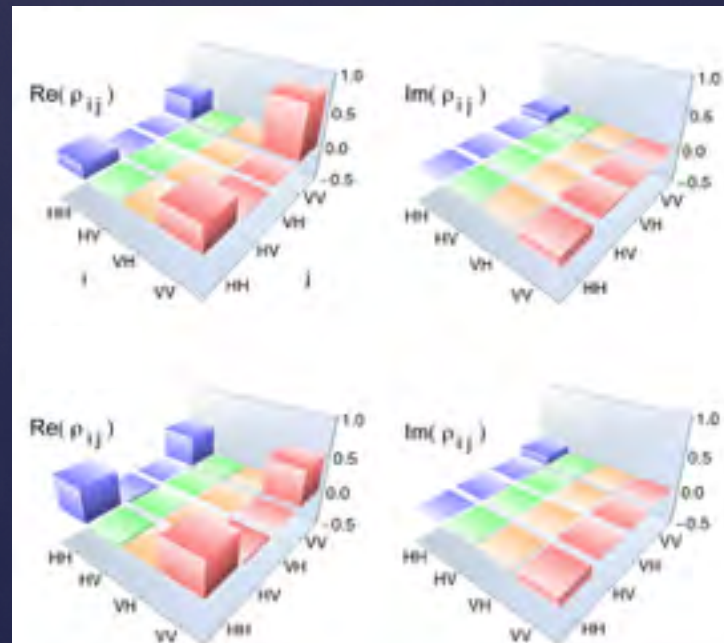
$$\mathbf{T} = \begin{pmatrix} T_{xx} & 0 \\ 0 & T_{yy} \end{pmatrix} = \begin{pmatrix} 1 - \frac{j\mu_0\pi f c}{d_x d_y} \frac{\alpha_{xx}}{1 - C_{xx}\alpha_{xx}} & 0 \\ 0 & 1 - \frac{j\mu_0\pi f c}{d_x d_y} \frac{\alpha_{yy}}{1 - C_{yy}\alpha_{yy}} \end{pmatrix}$$

Quantum optical metamaterials



Entanglement distillation of non-maximally entangled states:

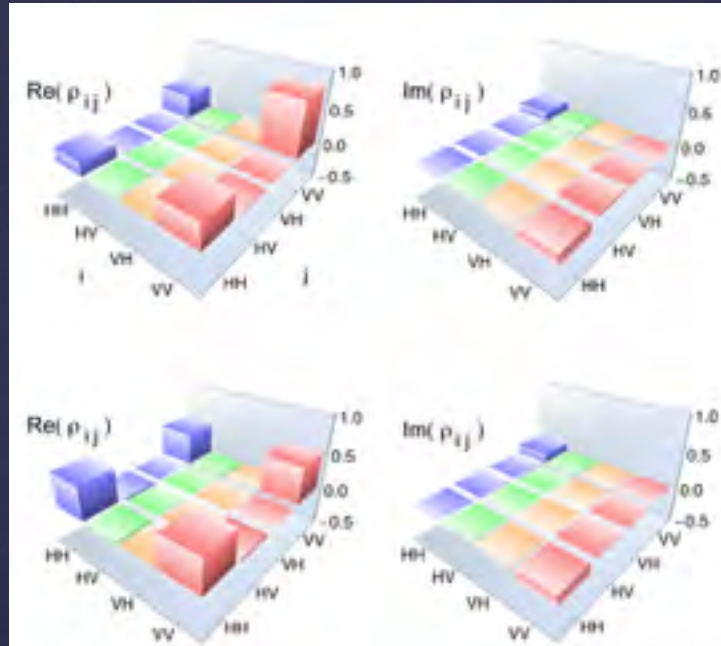
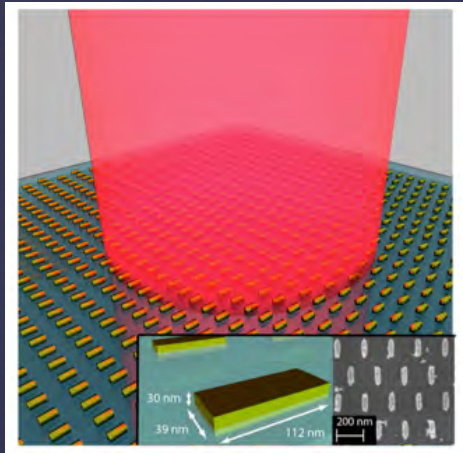
$$|\Phi_\epsilon\rangle = \frac{1}{\sqrt{1+\epsilon^2}} (\epsilon|H\rangle|H\rangle + |V\rangle|V\rangle)$$



before

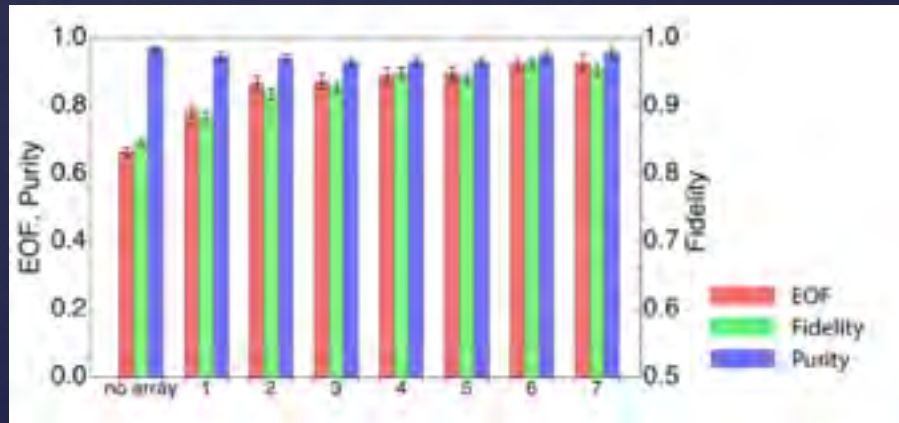
after

Quantum optical metamaterials



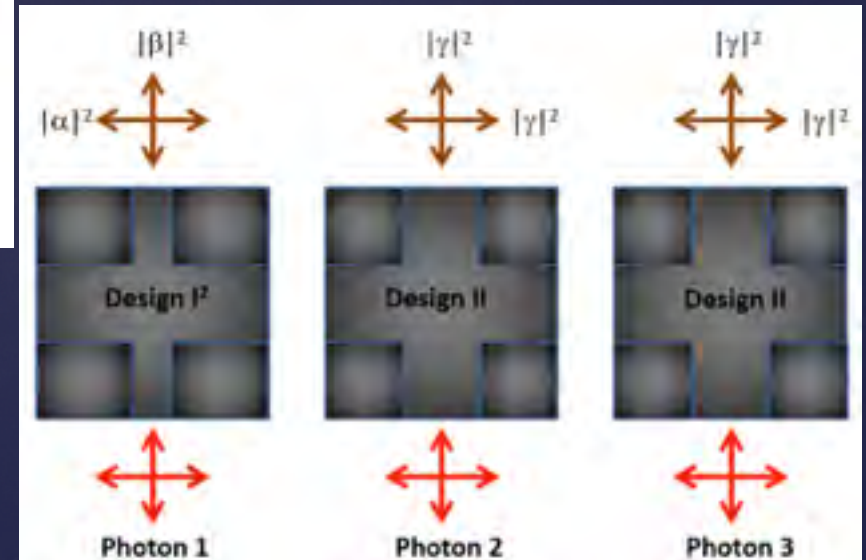
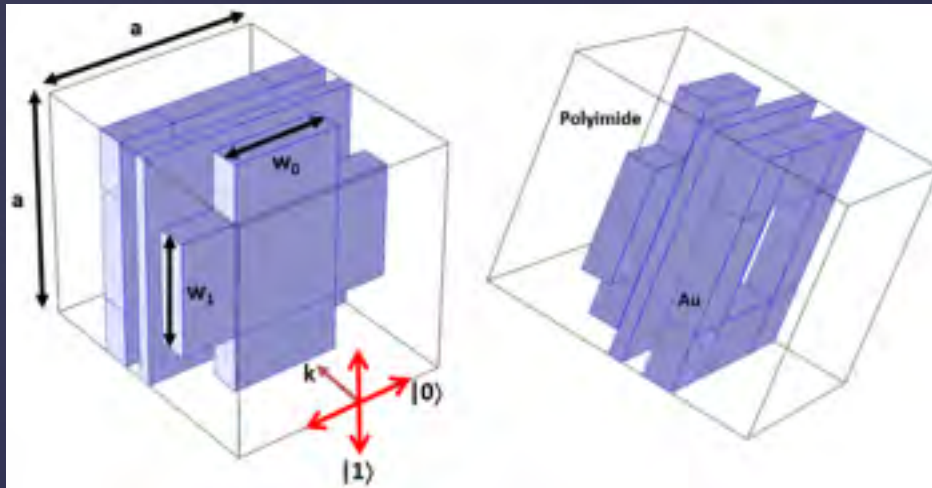
before

after



Quantum optical metamaterials

Abdullah al Farooqui et al., Opt. Exp. 23, 17941 (2015)



$$|\Phi_3\rangle = \alpha|1\rangle_1|0\rangle_2|0\rangle_3 + \beta(|0\rangle_1|1\rangle_2|0\rangle_3 + |0\rangle_1|0\rangle_2|1\rangle_3)$$

Collaborative network



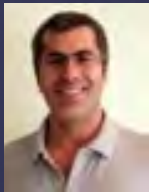
Changhyoup
Lee



Changsuk
Noh



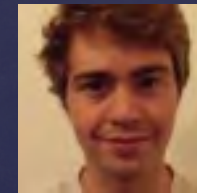
Dimitris
Angelakis



Sahin
Ozdemir



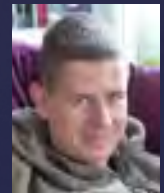
Jinhyoung
Lee



Frederik
Dieleman



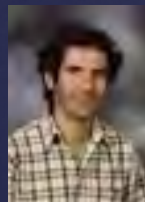
Myungshik
Kim



Stefan
Maier



Takashi
Yamamoto &
Motoki Asano



Durdu
Guney



Martin
Wegener &
Muriel Bechu



QUANTUM RESPONSE OF PLASMONIC SYSTEMS

MARK TAME

University of KwaZulu-Natal, South Africa

