

Experimental demonstration of perfect orthogonalization of partly unknown quantum states of light

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Outline of the talk

1. **Quantum universal NOT gate**
2. **Perfect orthogonalization of partly unknown quantum states**
3. **Experimental results for single-qubit states**
4. **Orthogonalization of partly unknown entangled states by local operations**

Quantum universal NOT gate

Perfect quantum U-NOT gate would map every input qubit state onto an orthogonal state:

$$\forall |\psi\rangle : |\psi\rangle \rightarrow |\psi_{\perp}\rangle, \quad \langle \psi | \psi_{\perp} \rangle = 0.$$

This transformation is forbidden by the laws of quantum physics.

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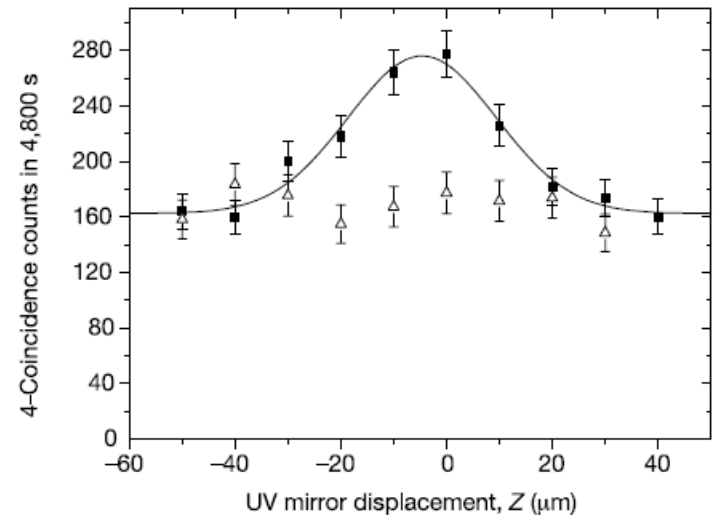
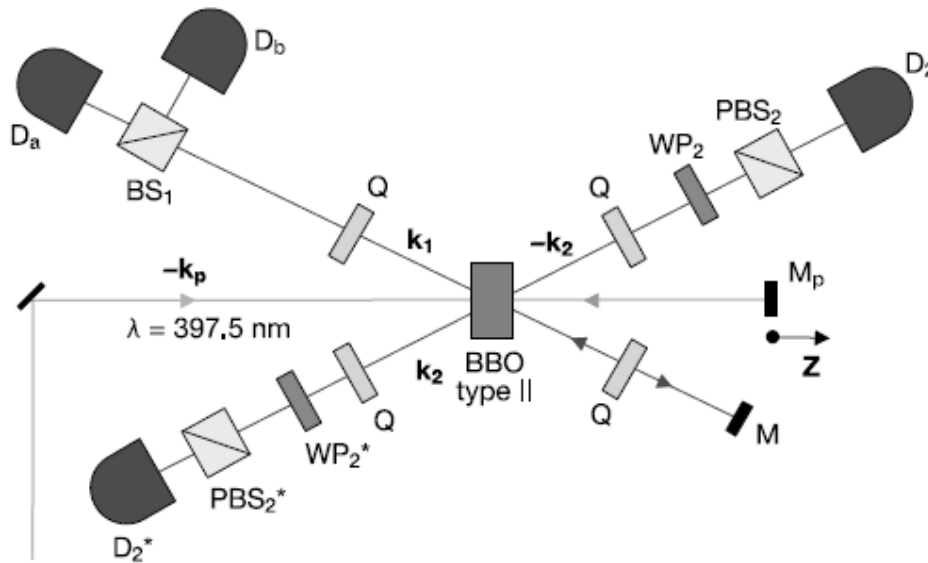
Optimal deterministic approximate U-NOT gate that minimizes the overlap between input and output states:

$$|\psi\rangle\langle\psi| \rightarrow \frac{2}{3}|\psi_{\perp}\rangle\langle\psi_{\perp}| + \frac{1}{3}|\psi\rangle\langle\psi|$$

Performance of the U-NOT gate cannot be improved even if we allow for probabilistic operations.

Experimental implementation of universal NOT gate

Stimulated parametric down-conversion, output obtained in the idler beam:



Measure and prepare scheme:

- Perform von Neumann measurement in a randomly chosen basis
- prepare output state orthogonal to the measurement outcome
- optimal U-NOT gate is an entanglement-breaking channel

Generalization of the quantum U-NOT gate for qudits

Defined as operation that minimizes the average overlap between input and output states. The averaging is performed over all pure input states with a measure induced by the Haar measure on $SU(d)$.

The optimal transformation:

$$\mathcal{G}_{\text{NOT}}(\rho) = (dI - \rho) / (d^2 - 1)$$

Minimum achievable average overlap between input and output states:

$$F_{\perp}(d) = \frac{1}{d+1}$$

P. Rungta, V. Buzek, C.M. Caves, M. Hillery, and G. J. Milburn, Phys. Rev. A **64**, 042315 (2001).

Perfect orthogonalization of partly unknown quantum states

Required prior information: a mean value a of some operator A :

$$a = \langle \psi | A | \psi \rangle$$

Conditional orthogonalization by quantum filtration:

$$|\psi_{\perp}\rangle \propto (A - aI) |\psi\rangle \quad \langle \psi_{\perp} | \psi \rangle = 0$$

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This orthogonalization procedure is perfect but probabilistic:

$$p_{\perp} \leq \lambda^{-2} \langle \Delta A^{\dagger} \Delta A \rangle, \quad \Delta A = A - aI$$

λ denotes the maximum singular value of ΔA .

M. R. Vanner, M. Aspelmeyer, and M. S. Kim, Phys. Rev. Lett. **110**, 010504 (2013).

M. Ježek, M. Mičuda, I. Straka, M. Miková, M. Dušek, and J. Fiurášek, Phys. Rev. A **89**, 042316 (2014).

Orthogonalization of single-qubit states

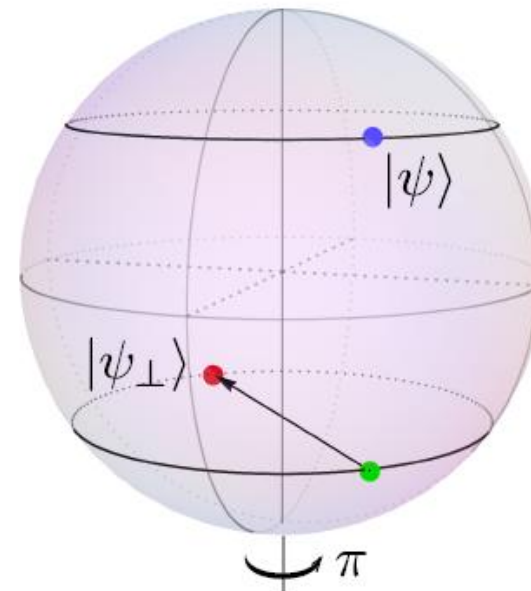
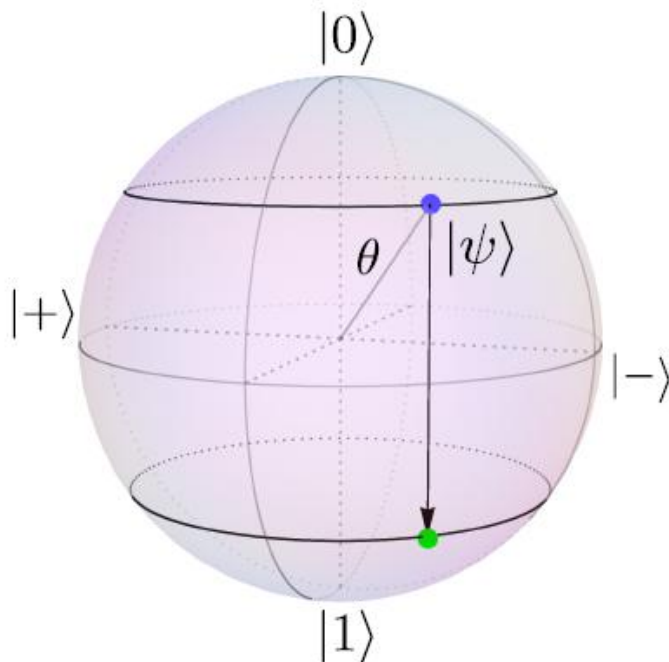
Bloch sphere parametrization: $|\psi\rangle = \cos \frac{\theta}{2} |0\rangle + e^{i\phi} \sin \frac{\theta}{2} |1\rangle$

Prior knowledge – mean value of σ_z :

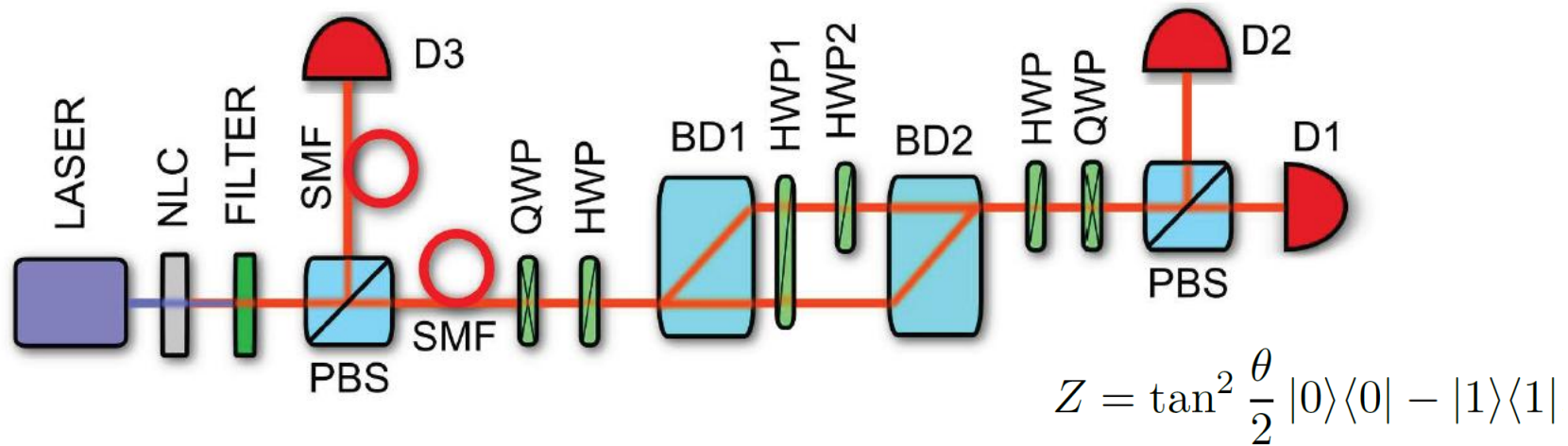
$$\sigma_Z = |0\rangle\langle 0| - |1\rangle\langle 1|.$$

Quantum filter:

$$Z = \tan^2 \frac{\theta}{2} |0\rangle\langle 0| - |1\rangle\langle 1|$$



Experimental setup

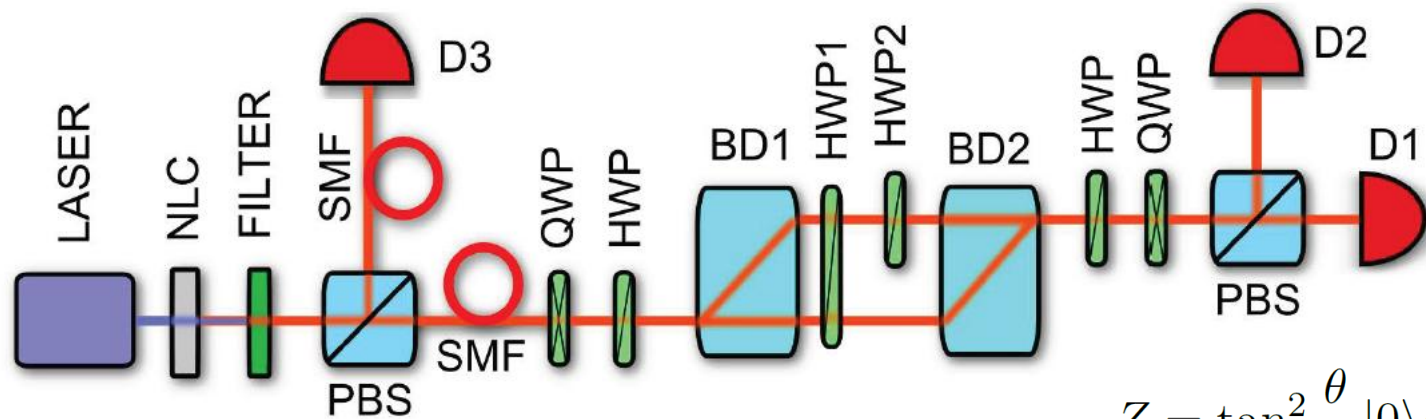


- Qubits are encoded into polarization states of single photons
- Correlated photon pairs are generated in the process of parametric downconversion
- Heralded single photons are generated by detection of idler photons
- Polarization dependent filtration is accomplished by combination of calcite beam-displacers and half-wave plates

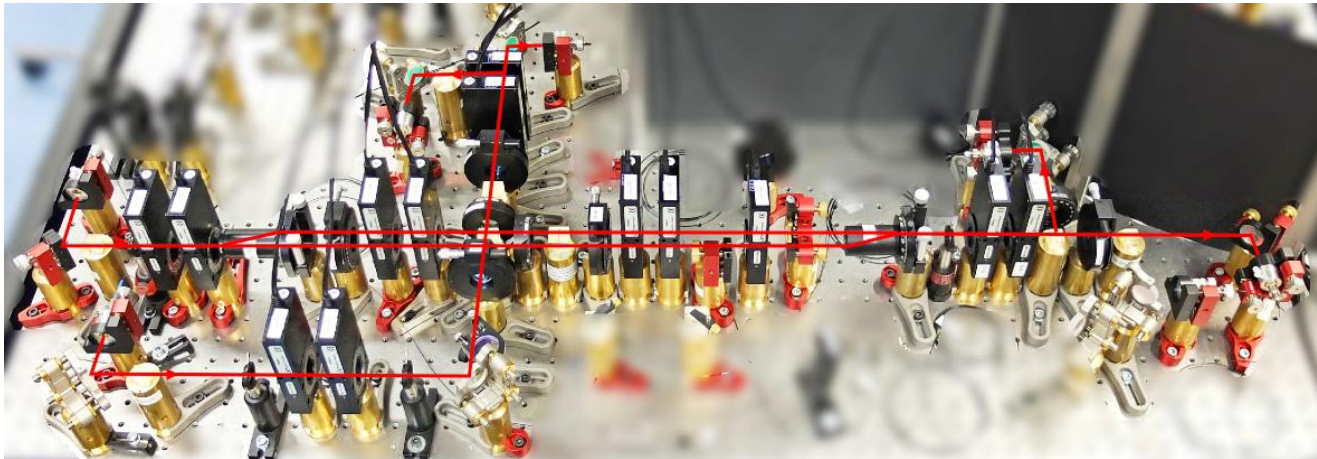
M. Ježek, M. Mičuda, I. Straka, M. Miková, M. Dušek, J. Fiurášek, Phys. Rev. A **89**, 042316 (2014).

M. Mičuda et al., Phys. Rev. Lett. **111**, 160407 (2013); Phys. Rev. A **89**, 042304 (2014).

Experimental setup

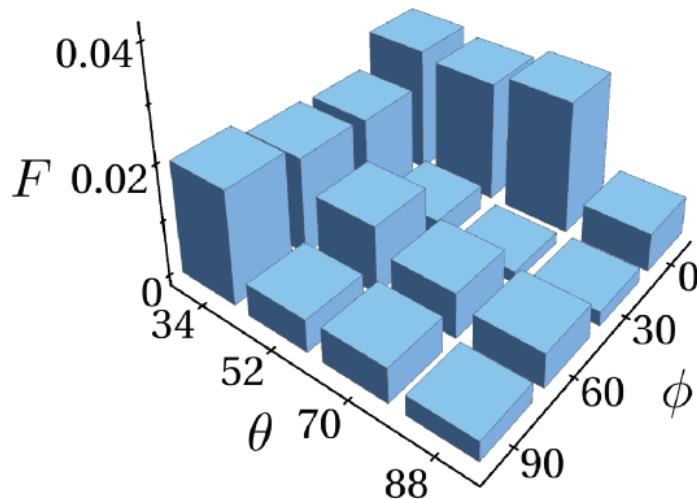


$$Z = \tan^2 \frac{\theta}{2} |0\rangle\langle 0| - |1\rangle\langle 1|$$



M. Ježek, M. Mičuda, I. Straka, M. Miková, M. Dušek, J. Fiurášek, Phys. Rev. A **89**, 042316 (2014).
M. Mičuda et al., Phys. Rev. Lett. **111**, 160407 (2013); Phys. Rev. A **89**, 042304 (2014).

Experimental results



Overlap between input and orthogonalized states

The overlap was determined from tomographically reconstructed density matrices of input and orthogonalized single-qubit states.

Single-qubit states:

$F=0$ only if both input and of input and orthogonalized states are pure and orthogonal.

$$F = \left[\text{Tr} \sqrt{\rho_1^{1/2} \rho_2 \rho_1^{1/2}} \right]^2$$

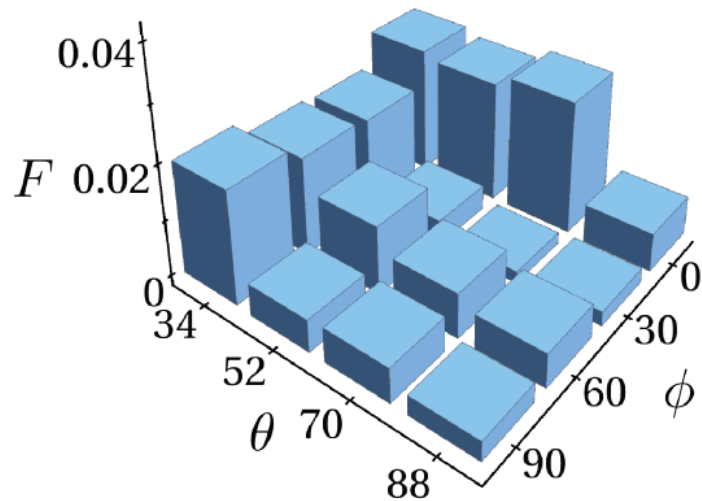
$$|\psi\rangle = \cos \frac{\theta}{2} |0\rangle + e^{i\phi} \sin \frac{\theta}{2} |1\rangle$$

Purity of input and output states:

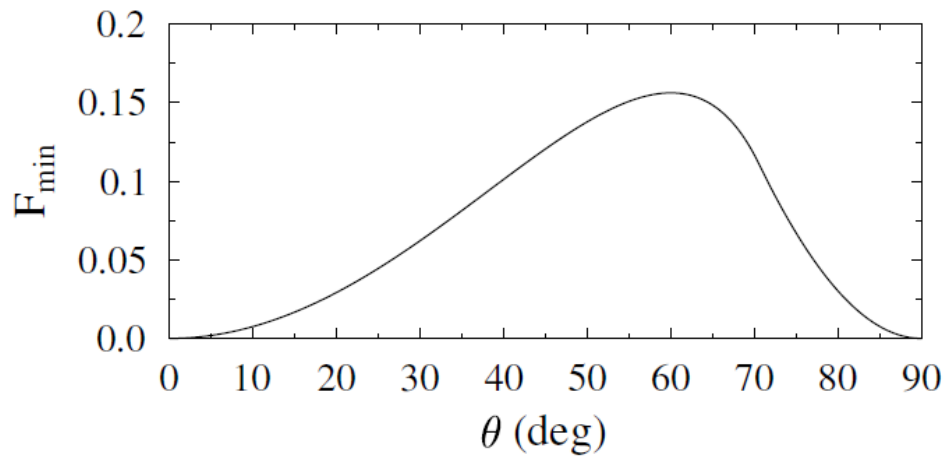
$$\mathcal{P}_I \geq 0.992$$

$$\mathcal{P}_O \geq 0.986$$

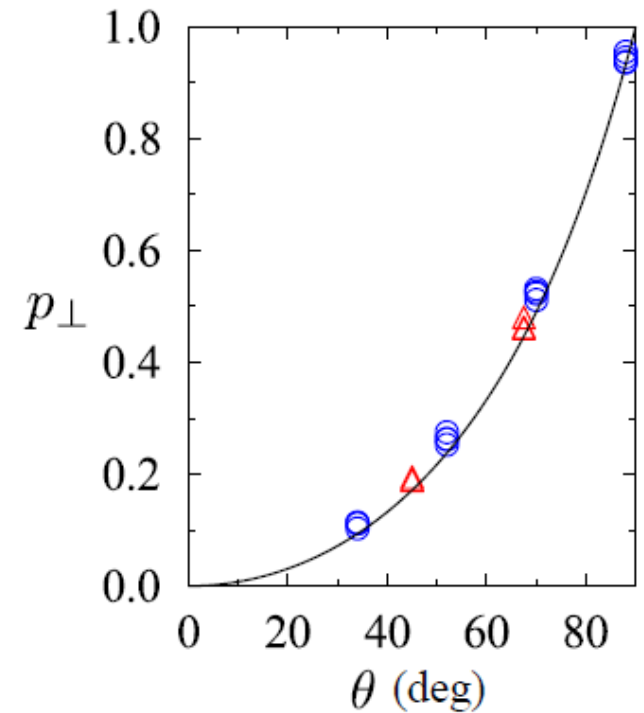
Experimental results



Overlap between input and orthogonalized states



Minimum overlap achievable by deterministic operations



Success probability

Orthogonalization of entangled two-qubit states

Consider pure bipartite state

$$|\Psi\rangle_{12}$$

Prior information – knowledge of a mean value of an operator A acting on subsystem 1:

$$a = \langle \Psi | A_1 \otimes I_2 | \Psi \rangle$$

Orthogonalization by local filtering on a single subsystem:

$$|\Psi_{\perp}\rangle_{12} \propto (A - aI)_1 \otimes I_2 |\Psi\rangle_{12}$$

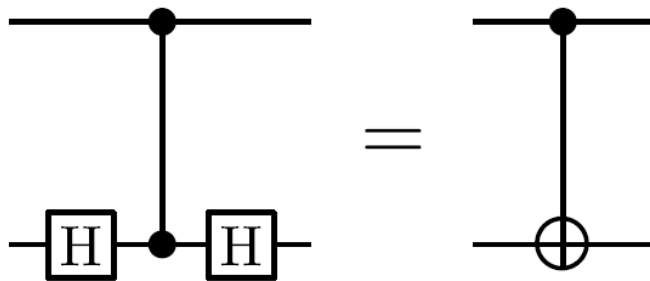
In our experiment, we prepare various entangled two-qubit two-photon states using a linear optical quantum CZ gate.

M. Ježek, M. Mičuda, I. Straka, M. Miková, M. Dušek, J. Fiurášek, Phys. Rev. A **89**, 042316 (2014).

Linear optical quantum CZ gate

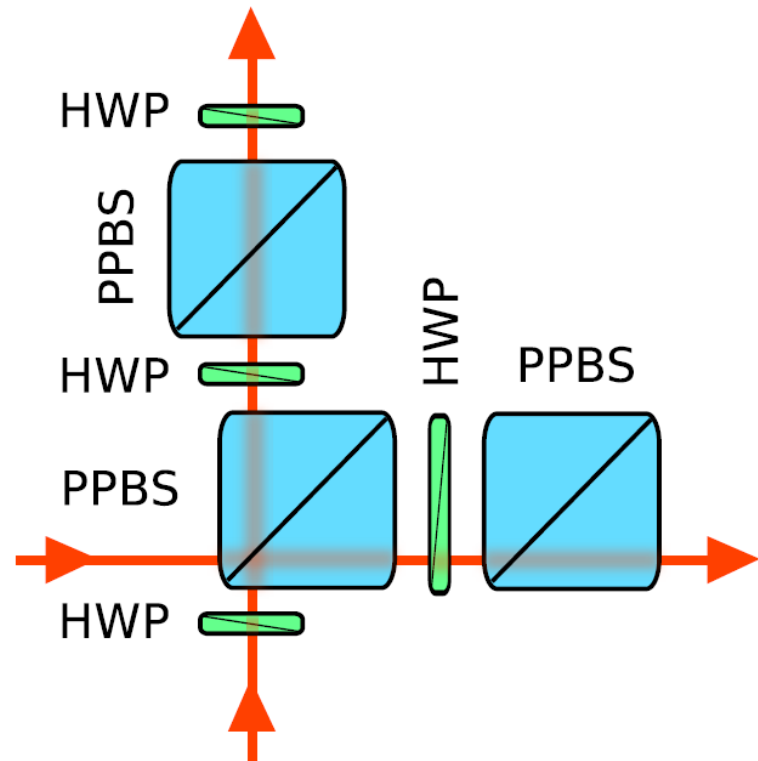
$$CZ = I - 2|11\rangle\langle 11|$$

$$CZ|jk\rangle = (-1)^{jk}|jk\rangle$$



$$|0\rangle = |H\rangle, |1\rangle = |V\rangle$$

$$PPBS: T_H = 1, T_V = 1/3$$

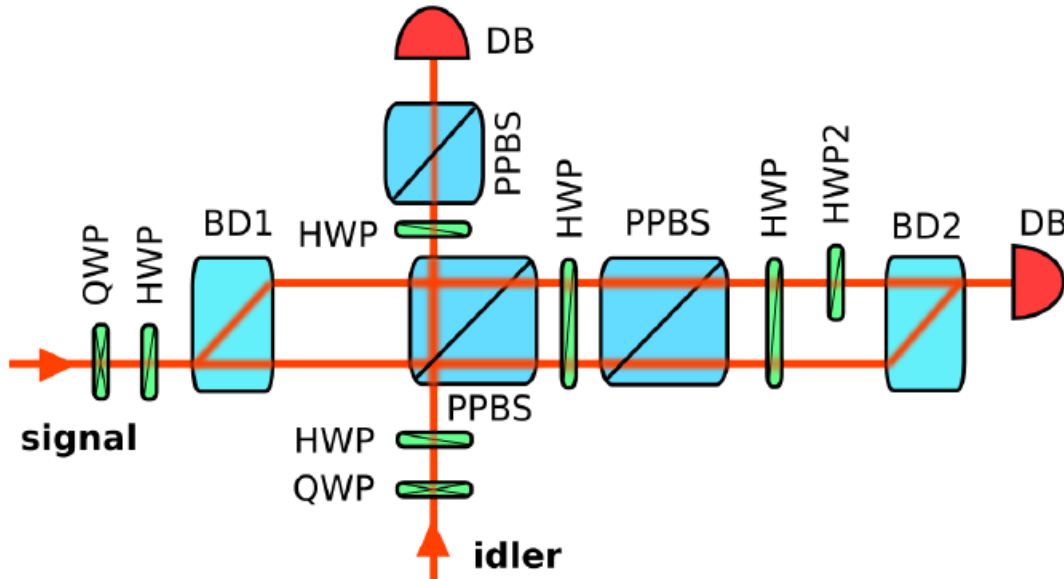


R. Okamoto, H.F. Hofmann, S. Takeuchi, and K. Sasaki, Phys. Rev. Lett. 95, 210506 (2005)

N. K. Langford, T.J. Weinhold, R. Prevedel, K. J. Resch, A. Gilchrist, J. L. O'Brien, G. J. Pryde, and A. G. White, Phys. Rev. Lett. 95, 210504 (2005)

N. Kiesel, C. Schmid, U. Weber, R. Ursin, and H. Weinfurter, Phys. Rev. Lett. 95, 210505 (2005)

Experimental setup and results



Signal:

$$|\psi_1\rangle = \cos \frac{\theta_1}{2} |0\rangle + e^{i\phi_1} \sin \frac{\theta_1}{2} |1\rangle$$

Idler:

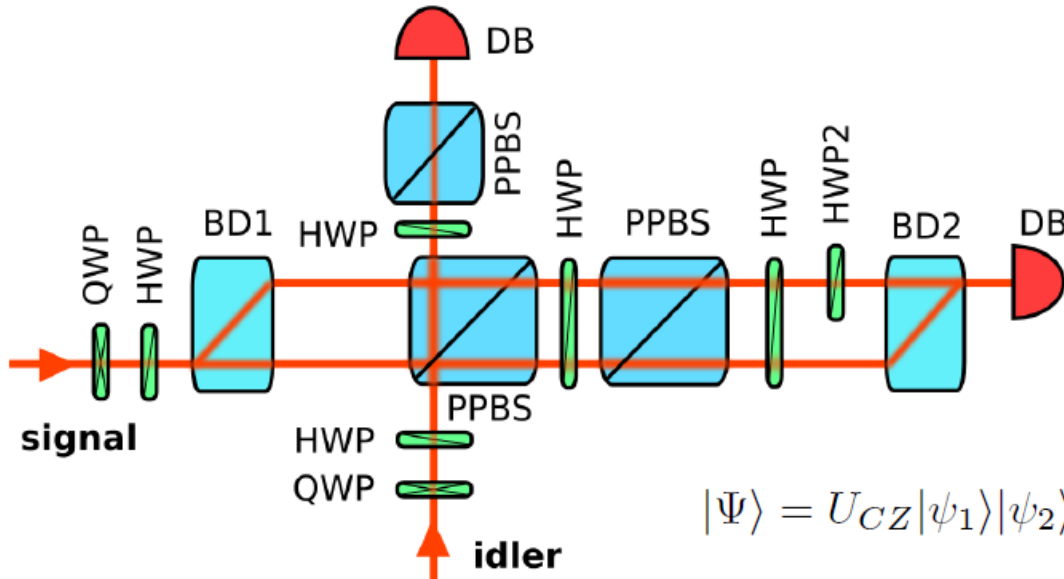
$$|\psi_2\rangle = \cos \frac{\theta_2}{2} |0\rangle + e^{i\phi_2} \sin \frac{\theta_2}{2} |1\rangle$$

Entangled two-photon states generated with the use of linear optical quantum CZ gate:

$$|\Psi\rangle = U_{CZ} |\psi_1\rangle |\psi_2\rangle = \cos \frac{\theta_1}{2} |0\rangle |\psi^+\rangle + e^{i\phi_1} \sin \frac{\theta_1}{2} |1\rangle |\psi^-\rangle$$

$$|\psi^\pm\rangle = \cos \frac{\theta_2}{2} |0\rangle \pm e^{i\phi_2} \sin \frac{\theta_2}{2} |1\rangle$$

Experimental setup and results



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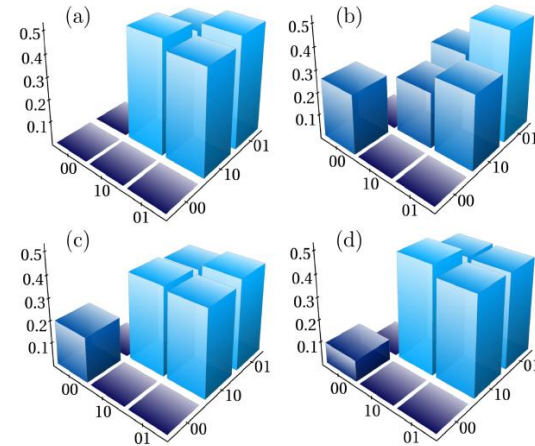
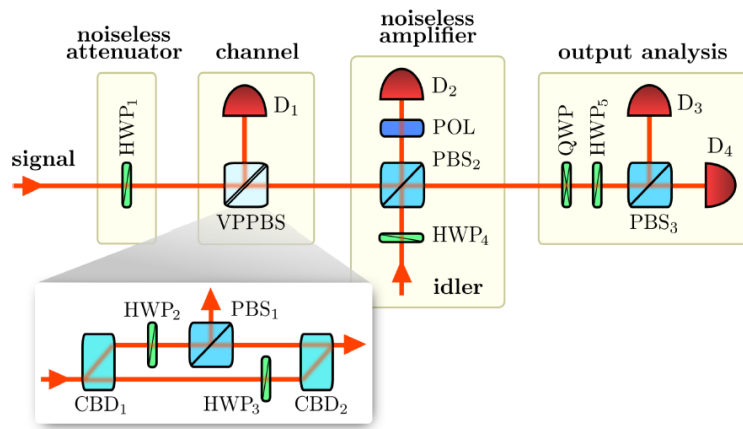
$$|\psi_2\rangle = \cos \frac{\theta_2}{2} |0\rangle + e^{i\phi_2} \sin \frac{\theta_2}{2} |1\rangle$$

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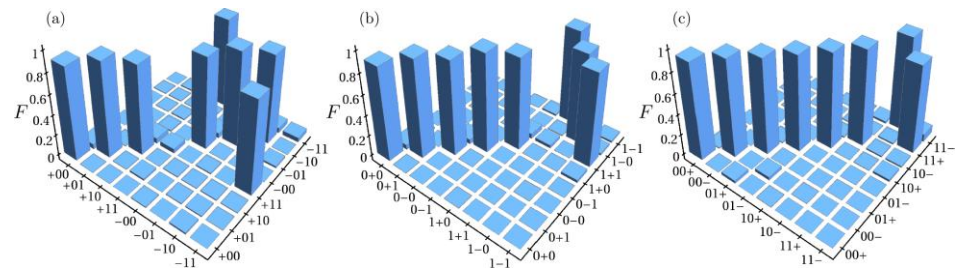
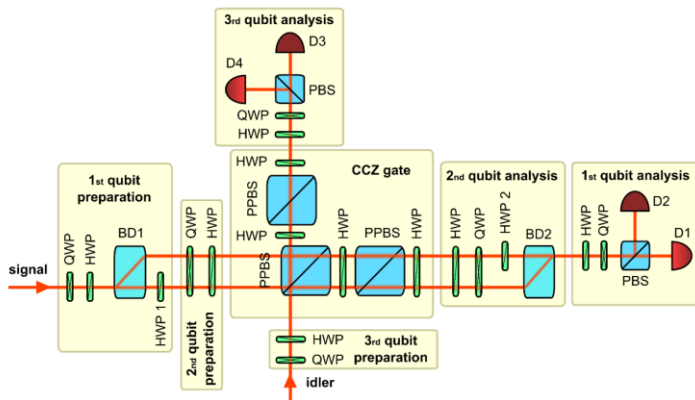
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θ_1	ϕ_1	θ_2	ϕ_2	F	\mathcal{P}_I	\mathcal{P}_O
45°	0°	90°	0°	0.040	0.964	0.890
67.5°	0°	90°	0°	0.031	0.961	0.891
45°	0°	45°	0°	0.021	0.936	0.944
67.5°	0°	45°	0°	0.008	0.975	0.952
67.5°	90°	45°	90°	0.041	0.971	0.946

Other experiments with calcite beam displacer-based interferometers



M. Mičuda, I. Straka, M. Miková, M. Dušek, N. J. Cerf, J. Fiurášek, and M. Ježek, *Noiseless Loss Suppression in Quantum Optical Communication*, Phys. Rev. Lett. **109**, 180503 (2012).



M. Mičuda, M. Sedlák, I. Straka, M. Miková, M. Dušek, M. Ježek, and J. Fiurášek, *Efficient Experimental Estimation of Fidelity of Linear Optical Quantum Toffoli Gate*, Phys. Rev. Lett. **111**, 160407 (2013).

Conclusions

We have experimentally demonstrated orthogonalization of partly unknown single-qubit and two-qubit states by quantum filtering.

The required prior information is a knowledge of a mean value of an arbitrary single operator A .

The probabilistic orthogonalization significantly outperforms the best deterministic orthogonalization procedure.

Entangled states can be orthogonalized by local filtering on one part of the state.

This procedure can be extended to preparation of superpositions of the input state and its orthogonal counterpart, see A.S. Coelho, L.S. Costanzo, A. Zavatta, C. Hughes, M. S. Kim, and M. Bellini, arXiv:1407.6644.

Thank you for your attention!



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