# Virtual implementation of noiseless amplification and entanglement distillation and their applications in continuous-variable quantum communication

J. Fiurášek[1], N.J. Cerf[2],

D. Abdelkhalek[3,4], M. Syllwasschy[4] and R. Schnabel[3,4]

*[1]Department of Optics, Palacky University, 17. listopadu 12, 77146 Olomouc, Czech Republic*

*[2]Quantum Information and Communication, Ecole Polytechnique de Bruxelles, CP 165, Universite libre de Bruxelles, 1050 Brussels, Belgium*

*[3]Institut fur Laserphysik, Universitat Hamburg, Luruper Chaussee 149, 22761 Hamburg, Germany*

*[4]Institut fur Gravitationsphysik, Leibniz Universitat Hannover and Max-Planck-Institut fur Gravitationsphysik (Albert-Einstein-Institut), Callinstr. 38, 30167 Hannover, Germany*

# Outline of the talk

**Part I:**

- Noiseless quantum amplification and attenuation
- CV QKD with eight port homodyne detection
- Virtual noiseless amplification in CV QKD

**Part II:**

- CV entanglement distillation by iterative Gaussification
- Single-photon detection vs. eight-port homodyne detection
- Emulation of iterative Gaussification by processing of experimental data
- Experimental setup and experimental results

# Part I: virtual noiseless amplification and attenuation

# Probabilistic noiseless quantum amplification

**Ideal unphysical noiseless quantum amplifier:**

$$|\alpha\rangle \rightarrow |g\alpha\rangle, \quad g \geq 1$$

**Ideal probabilistic noiseless quantum amplifier** – unbounded operator diagonal in Fock basis:

$$g^{\hat{n}}|\alpha\rangle = \exp\left[\frac{1}{2}\left(|g|^2 - 1\right)\right]|g\alpha\rangle \qquad \hat{I} \otimes g^{\hat{n}}\sqrt{1-\lambda^2}\sum_{n=0}^{\infty}\lambda^n|n,n\rangle = \sqrt{1-\lambda^2}\sum_{n=0}^{\infty}(g\lambda)^n|n,n\rangle$$

T. C. Ralph and A. P. Lund, in *Quantum Communication Measurement and Computing*, Proceedings of the 9th International Conference, edited by A. Lvovsky (AIP, New York 2009), pp. 155–160; arXiv:0809.0326.

# Probabilistic noiseless quantum amplification

**Ideal unphysical noiseless quantum amplifier:**

$$|\alpha\rangle \rightarrow |g\alpha\rangle, \quad g \geq 1$$

**Ideal probabilistic noiseless quantum amplifier** – unbounded operator diagonal in Fock basis:

$$g^{\hat{n}}|\alpha\rangle = \exp\left[\frac{1}{2}\left(|g|^2 - 1\right)\right]|g\alpha\rangle \qquad \hat{I} \otimes g^{\hat{n}}\sqrt{1-\lambda^2}\sum_{n=0}^{\infty}\lambda^n|n,n\rangle = \sqrt{1-\lambda^2}\sum_{n=0}^{\infty}(g\lambda)^n|n,n\rangle$$

**Possible applications:**

- Improved estimation and cloning of coherent states
- Entanglement distillation and concentration
- Breeding of Schrodinger cat-like states
- *Compensation of losses in quantum communication*

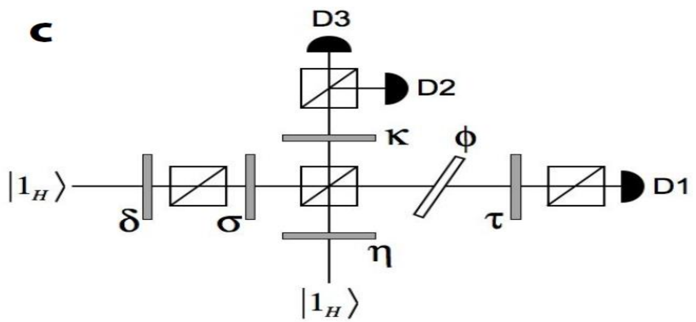
T. C. Ralph and A. P. Lund, in *Quantum Communication Measurement and Computing*, Proceedings of the 9th International Conference, edited by A. Lvovsky (AIP, New York 2009), pp. 155–160; arXiv:0809.0326.

# Experimental implementations of noiseless amplifiers

LETTERS
PUBLISHED ONLINE: 28 MARCH 2010 | DOI: 10.1038/NPHOTON.2010.35
nature photonics

## Heralded noiseless linear amplification and distillation of entanglement

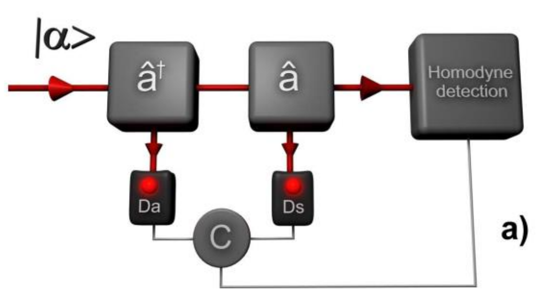
G. Y. Xiang[1], T. C. Ralph[2], A. P. Lund[1,2], N. Walk[2] and G. J. Pryde[1]*




nature photonics
ARTICLES
PUBLISHED ONLINE: XX XX 2010 | DOI: 10.1038/NPHOTON.2010.260

## A high-fidelity noiseless amplifier for quantum light states

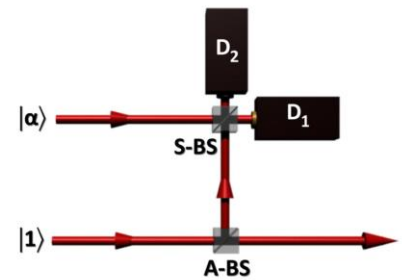
A. Zavatta[1,2], J. Fiurášek[3] and M. Bellini[1,2]*




PRL 104, 123603 (2010) PHYSICAL REVIEW LETTERS week ending 26 MARCH 2010

## Implementation of a Nondeterministic Optical Noiseless Amplifier

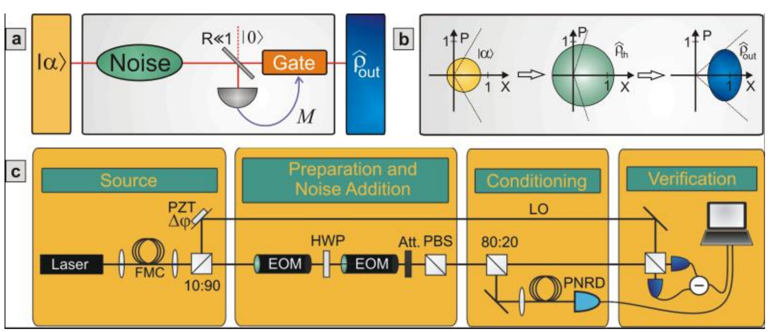
Franck Ferreyrol, Marco Barbieri, Rémi Blandino, Simon Fossier, Rosa Tualle-Brouri, and Philippe Grangier
*Groupe d'Optique Quantique, Laboratoire Charles Fabry, Institut d'Optique, CNRS, Université Paris-Sud, Campus Polytechnique, RD 128, 91127 Palaiseau cedex, France*
(Received 10 December 2009; published 24 March 2010)




nature physics
LETTERS
PUBLISHED ONLINE: XX MONTH XXXX | DOI: 10.1038/NPHYS1743

## Noise-powered probabilistic concentration of phase information

Mario A. Usuga[1,2†], Christian R. Müller[1,3†], Christoffer Wittmann[1,3], Petr Marek[4], Radim Filip[4], Christoph Marquardt[1,3], Gerd Leuchs[1,3] and Ulrik L. Andersen[2]*



# Probabilistic noiseless quantum attenuation

**Bounded operator diagonal in Fock basis:**

$$\nu^{\hat{n}}, \quad \nu \leq 1$$

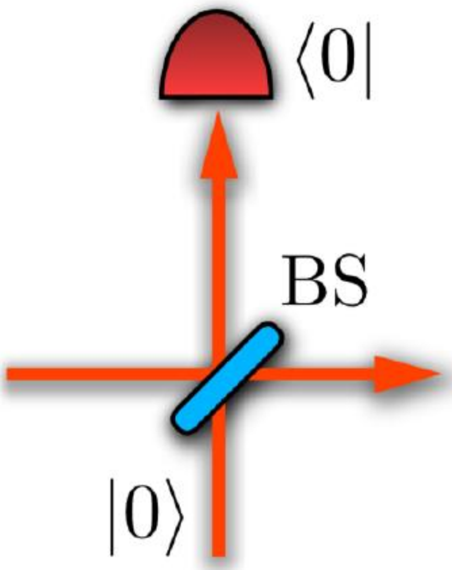
$$\nu^{\hat{n}}|\alpha\rangle = \exp\left[-\frac{1}{2}\left(1-|\nu|^2\right)\right]|\nu\alpha\rangle$$

# Probabilistic noiseless quantum attenuation

**Bounded operator diagonal in Fock basis:**

$$\nu^{\hat{n}}, \quad \nu \leq 1$$

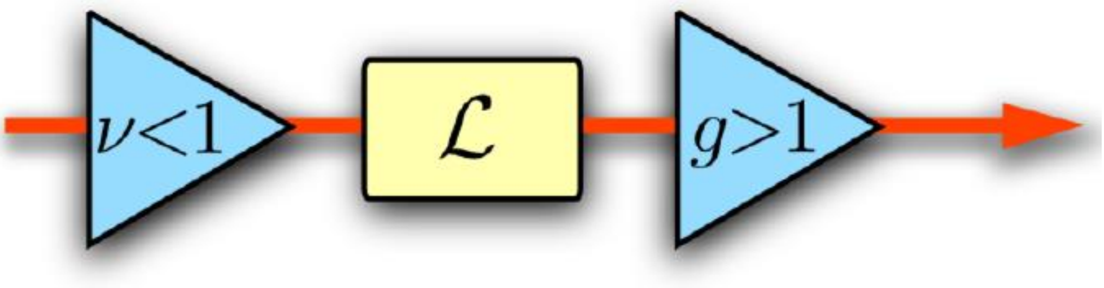
$$\nu^{\hat{n}}|\alpha\rangle = \exp\left[-\frac{1}{2}\left(1-|\nu|^2\right)\right]|\nu\alpha\rangle$$




**Possible implementation:**

- beam splitter with transmittance $\nu^2$
- projection of the auxiliary output mode onto vacuum

# Suppression of losses in quantum communication



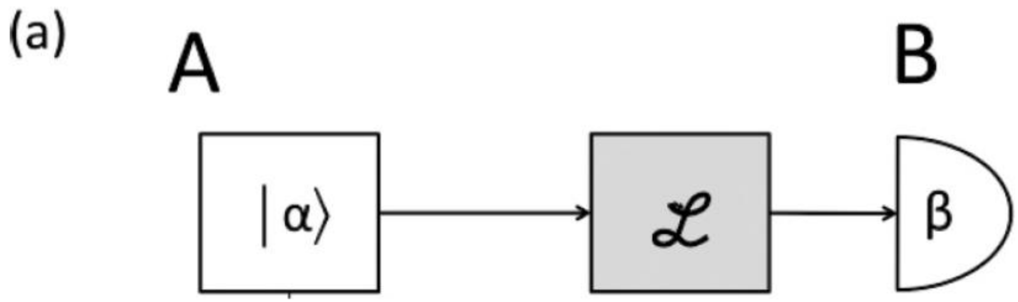

**Faithful conditional transfer of quantum states over lossy channels:**

- the input state is noiselessly attenuated
- the output state is noiselessly amplified


T. C. Ralph, Phys. Rev. A 84, 022339 (2011).

M. Mičuda, I. Straka, M. Miková, M. Dušek, N. J. Cerf, J. Fiurášek, and M. Ježek, Phys. Rev. Lett. 109, 180503 (2012).

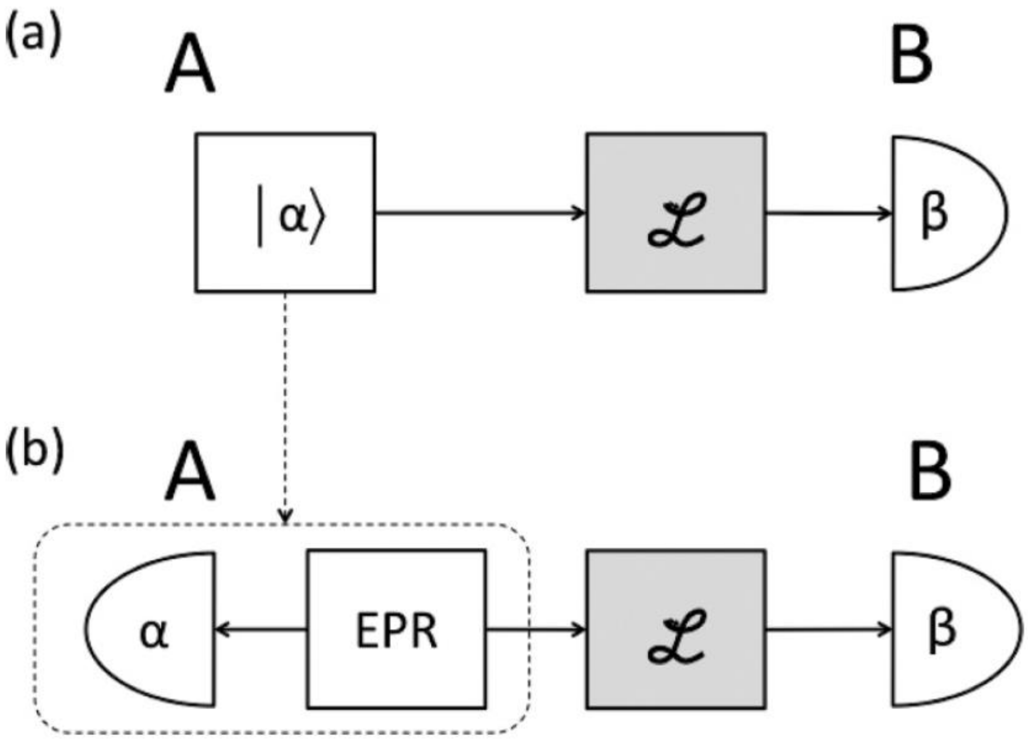
# CV QKD with heterodyne detection




Alice prepares coherent states with Gaussian modulated complex amplitude.

Bob measures the received states in the coherent-state basis.

# CV QKD with heterodyne detection




$$|\Psi_{EPR}\rangle = \sqrt{1-\lambda^2}\sum_{n=0}^{\infty}\lambda^n|n\rangle|n\rangle$$

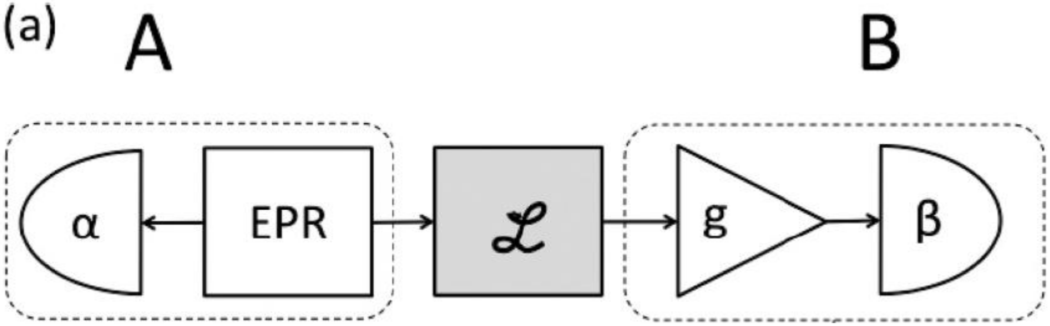
**Equivalent entanglement-based protocol**

Alice generates two-mode squeezed vacuum state and sends one mode to Bob.

Both Alice and Bob perform projections onto coherent states.

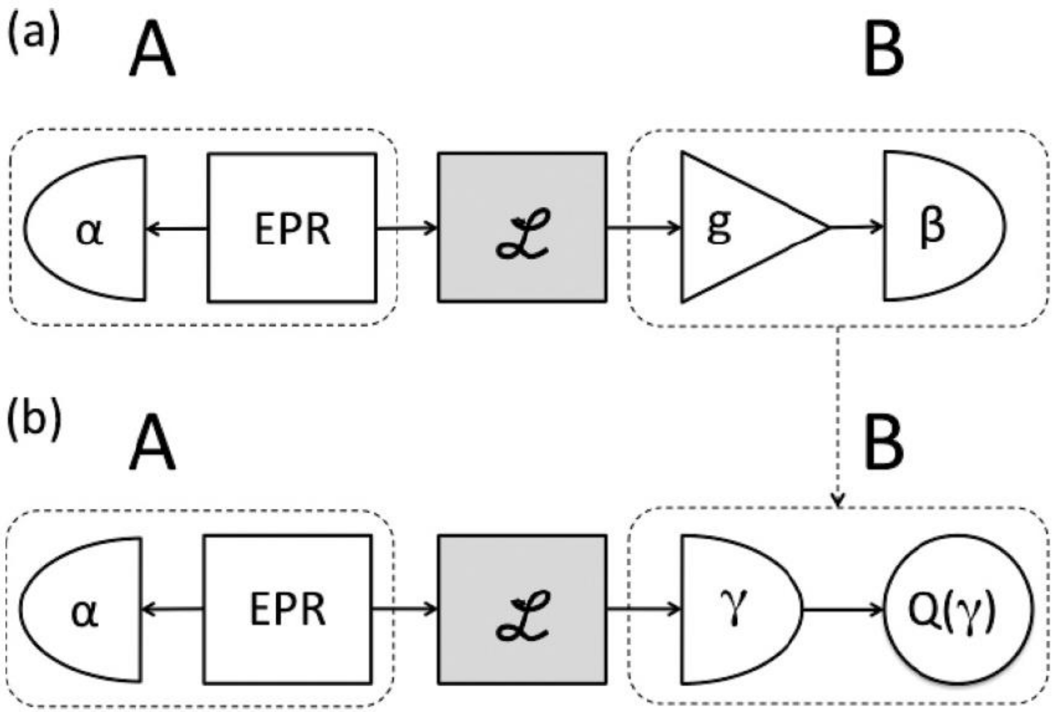
# CV QKD augmented with noiseless amplification

(a)




R. Blandino, A. Leverrier, M. Barbieri, J. Etesse, P. Grangier, and R. Tualle-Brouri, Phys. Rev. A **86**, 012327 (2012).

# Emulation of noiseless amplification in CV QKD




Noiseless amplification can be emulated by suitable processing of the experimental data.

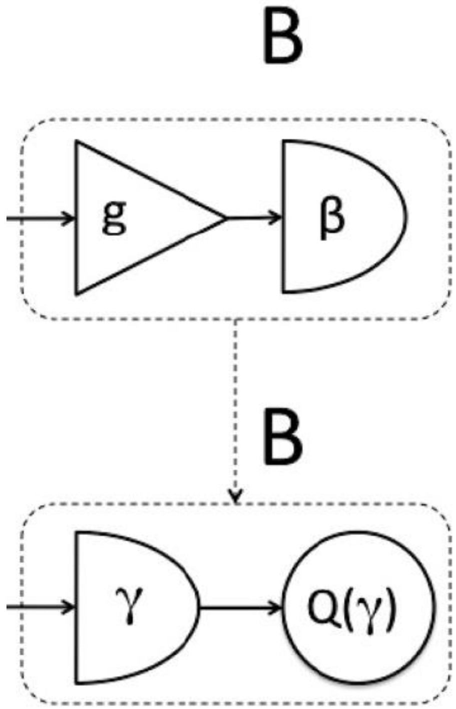

J. Fiurášek and N.J. Cerf, Phys. Rev. A 86, 060302(R) (2012).

N. Walk, T.C. Ralph, T. Symul, and P.K. Lam, Phys. Rev. A 87, 020303(R) (2013).

# Emulation of noiseless amplification by postselection

$$P(\beta) = \langle \beta | g^{n} \rho_B g^{n} | \beta \rangle$$

$$P(\beta) = e^{(g^2-1)|\beta|^2} \langle g\beta | \rho_B | g\beta \rangle$$



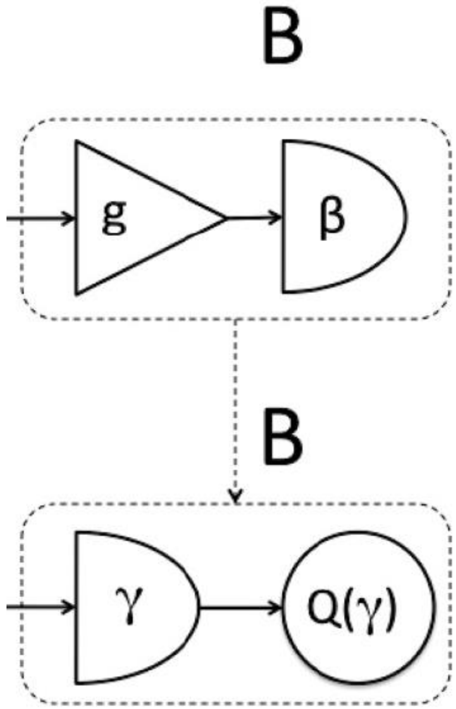
# Emulation of noiseless amplification by postselection

$$P(\beta) = \langle \beta | g^n \rho_B g^n | \beta \rangle$$

$$P(\beta) = e^{(g^2-1)|\beta|^2} \langle g\beta | \rho_B | g\beta \rangle$$

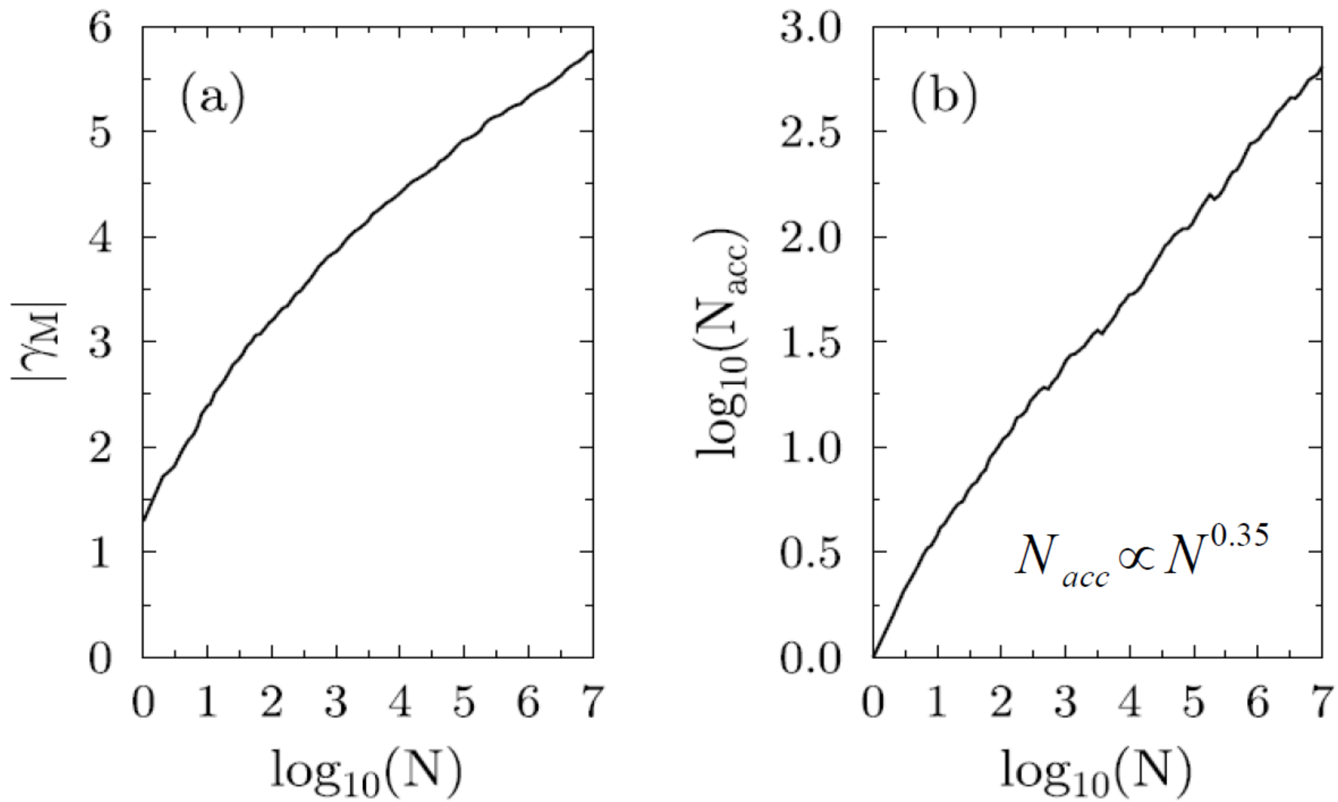
$$\beta = \frac{\gamma}{g}$$

$$Q(\gamma) = \exp\left[(1-g^{-2})|\gamma|^2\right]$$




**Postselection:** the data are accepted with probability proportional to Q($\gamma$)

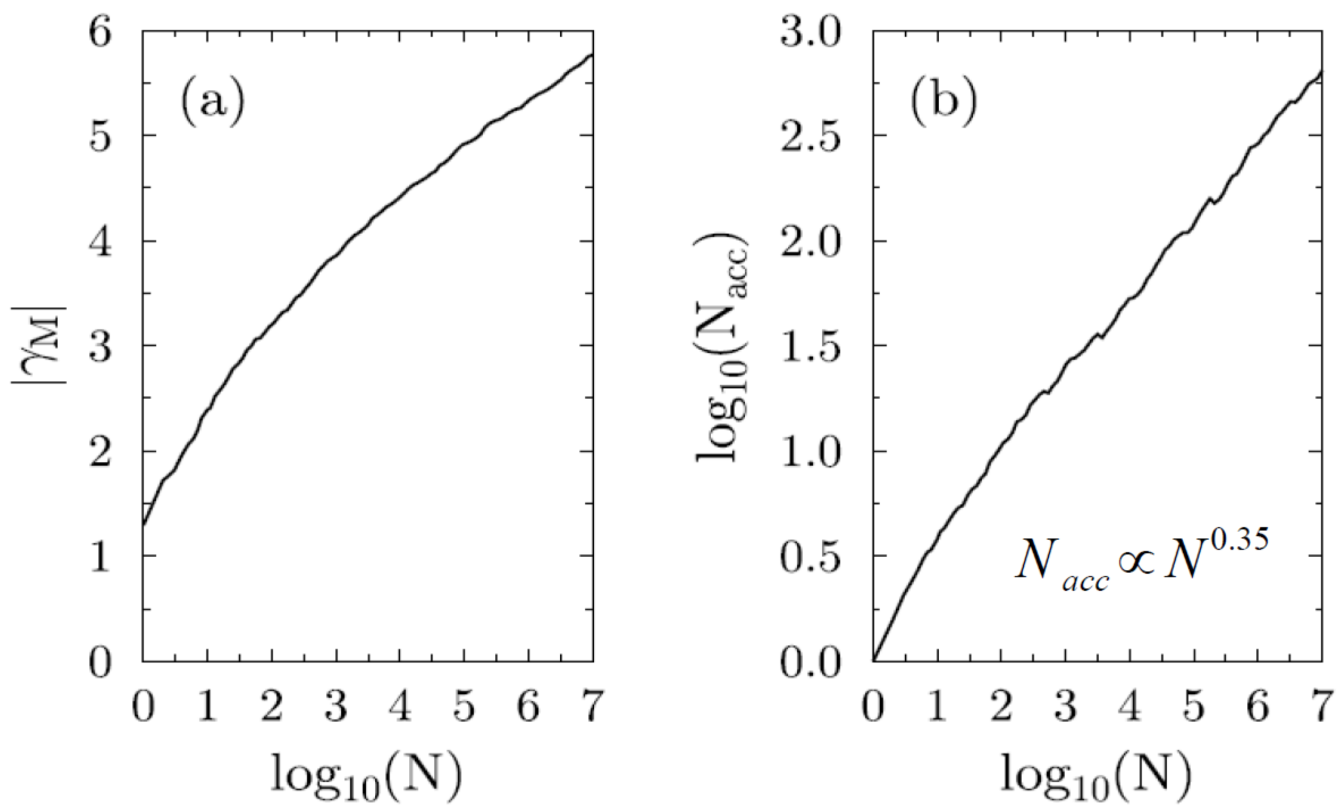
# Success rate of the postselection




$$Q(\gamma) = \exp\left[-(1-g^{-2})(|\gamma_M|^2 - |\gamma|^2)\right] \leqslant 1$$

$$|\gamma_M| = max_j |\gamma_j|$$

# Success rate of the postselection

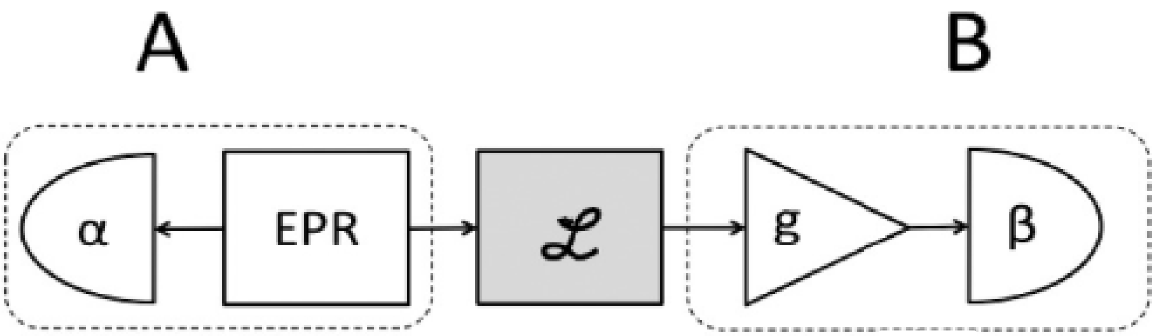



$$Q(\gamma) = \exp\left[-(1-g^{-2})(|\gamma_M|^2 - |\gamma|^2)\right] \leqslant 1 \qquad |\gamma_M| = max_j |\gamma_j|$$

Exact emulation of noiseless amplification

The number of accepted data scales sublinearly with N – inefficient procedure.

# Calculation of secret key rate




Virtual entanglement picture is employed.

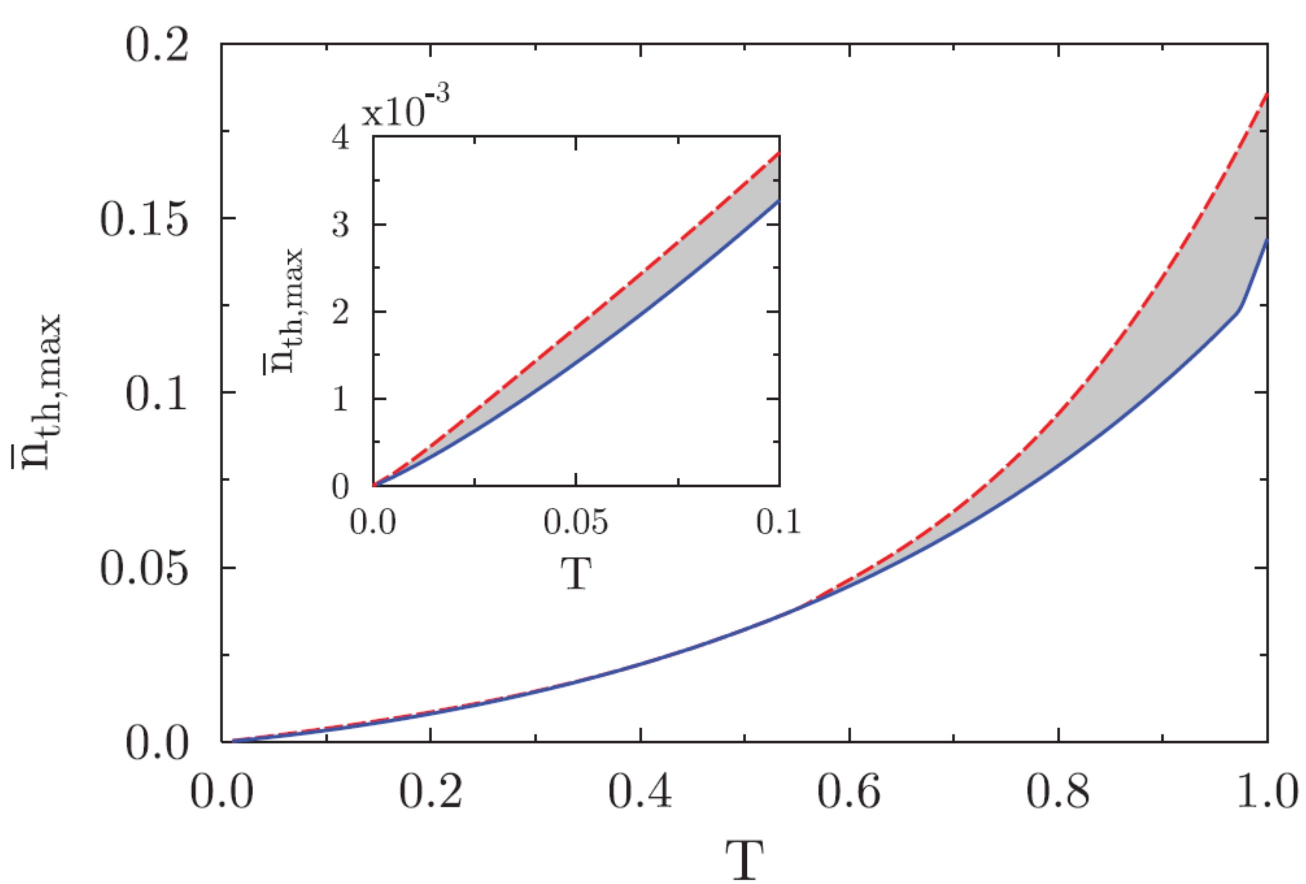
$$|\Psi_{EPR}\rangle = \sqrt{1-\lambda^2}\sum_{n=0}^{\infty} \lambda^n |n\rangle|n\rangle$$

Secret key is calculated as follows:

$$K = \max(\eta I_{AB} - \chi_{AE}, \eta I_{AB} - \chi_{BE})$$

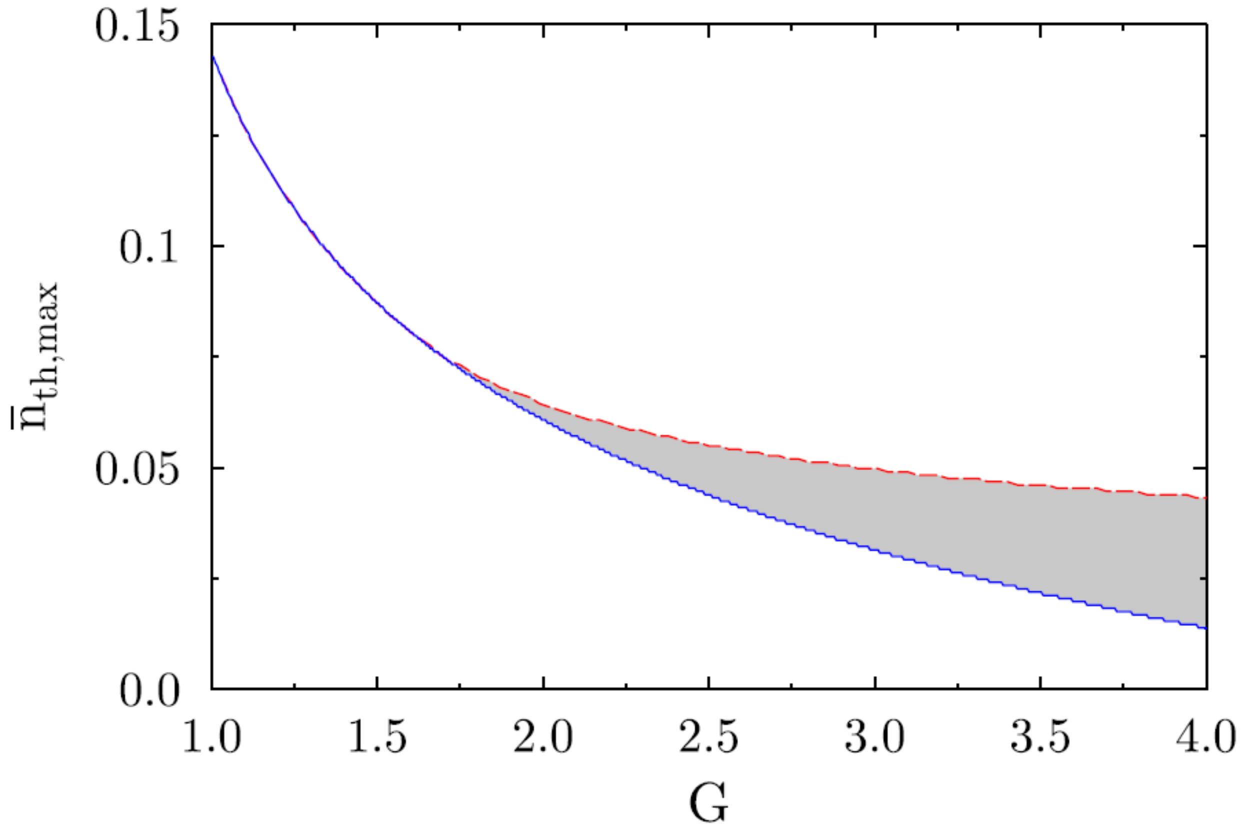
The key rate is maximized by optimizing Alice's modulation strength, i.e. the two-mode squeezing parameter in the entanglement picture.

# Lossy channel and noiseless amplification




Tolerable excess output thermal noise is plotted in dependence on channel transmittance

# Amplifying channel and noiseless attenuation




Tolerable excess output thermal noise is plotted in dependence on channel gain

# Achievable secret key rates

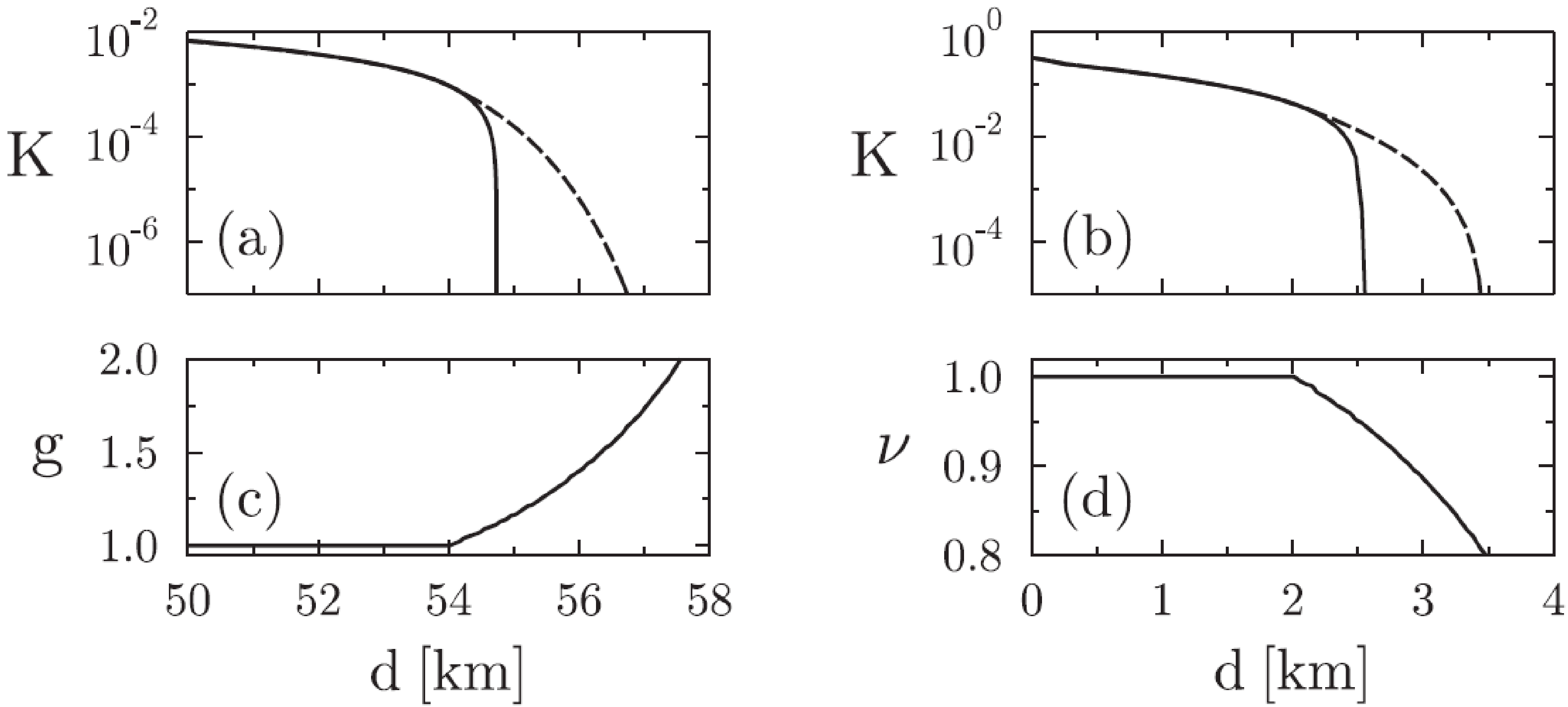



FIG. 4. Achievable secret key rate $K$ in CV QKD over a lossy channel with 0.2 dB losses per km. (a) Comparison of the protocol without Gaussian postselection (solid line) and with optimal noiseless amplification (dashed line), $n_{\mathrm{th}} = 2.5 \times 10^{-3}$, $\gamma_M = 3\sqrt{V_\gamma}$. (b) Comparison of the protocol without Gaussian postselection (solid line) and with optimal noiseless attenuation (dashed line), $n_{\mathrm{th}} = 0.1$. We assume $\eta = 0.9$, and the parameters $V$, $g$, and $\nu$ were optimized for each $d$ so as to maximize $K$. The resulting optimal $g$ and $\nu$ are plotted in panels (c) and (d), respectively.

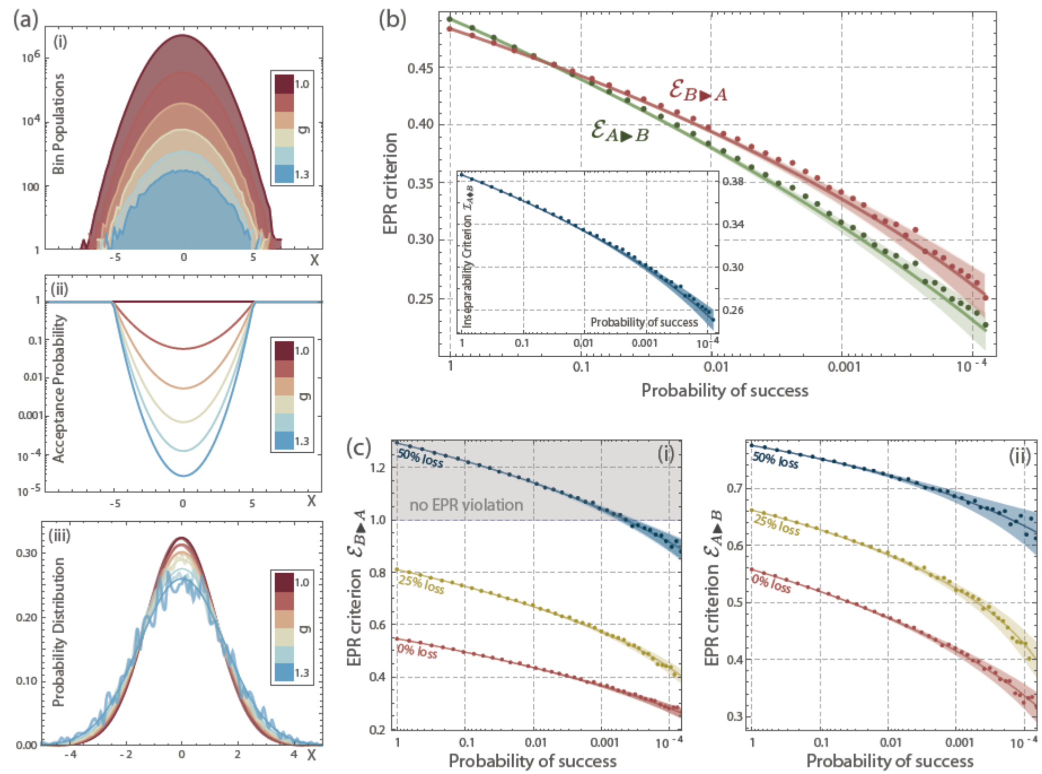
# Proof-of-principle experimental demonstration of virtual noiseless amplification

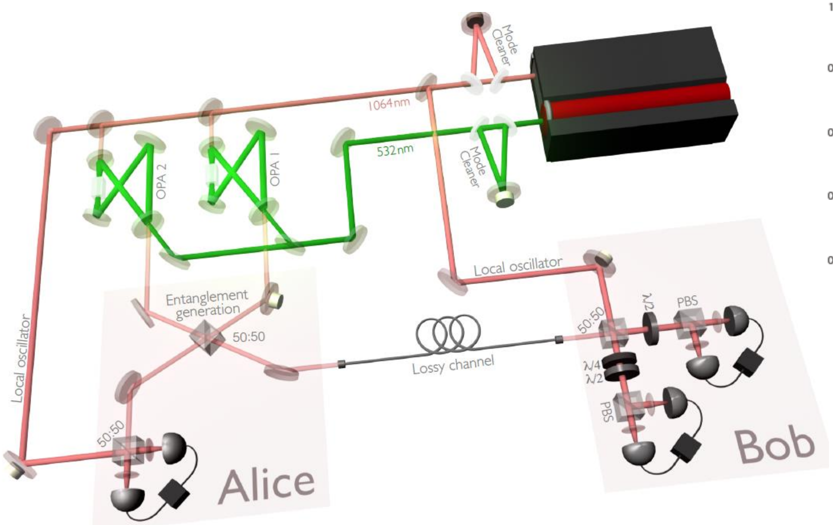
## Measurement-based noiseless linear amplification for quantum communication

Helen M. Chrzanowski, Nathan Walk, Syed M. Assad, Jiri Janousek, Sara Hosseini, Timothy C. Ralph, Thomas Symul & Ping Koy Lam

Affiliations | Contributions | Corresponding author

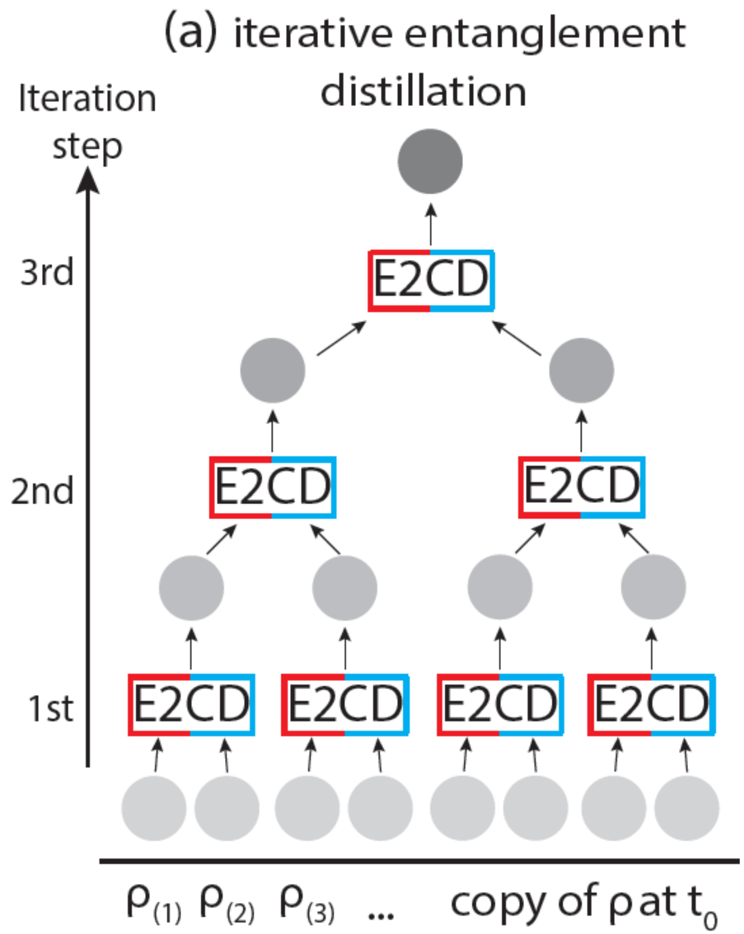
*Nature Photonics* **8**, 333–338 (2014)



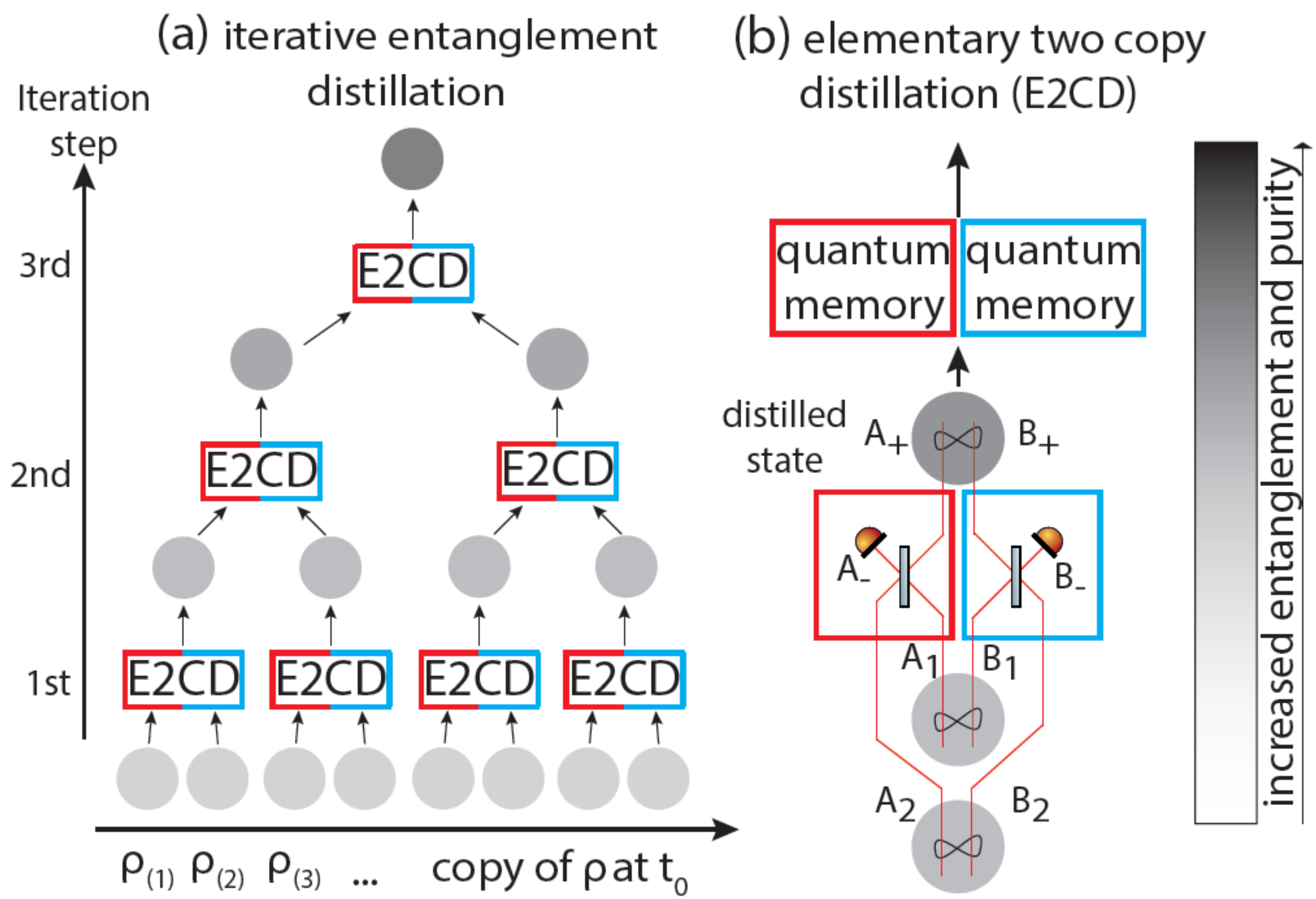
## Part II:
## emulation of iterative CV entanglement distillation

# CV entanglement distillation by Gaussification




C. H. Bennett, G. Brassard, S. Popescu, B. Schumacher, J. a. Smolin, and W. K.Wootters, Phys. Rev. Lett. 76, 722{725 (1996).
D. Deutsch, A. Ekert, R. Jozsa, C. Macchiavello, S. Popescu, and A. Sanpera, Phys. Rev. Lett. 77, 2818{2821 (1996).

# CV entanglement distillation by Gaussification

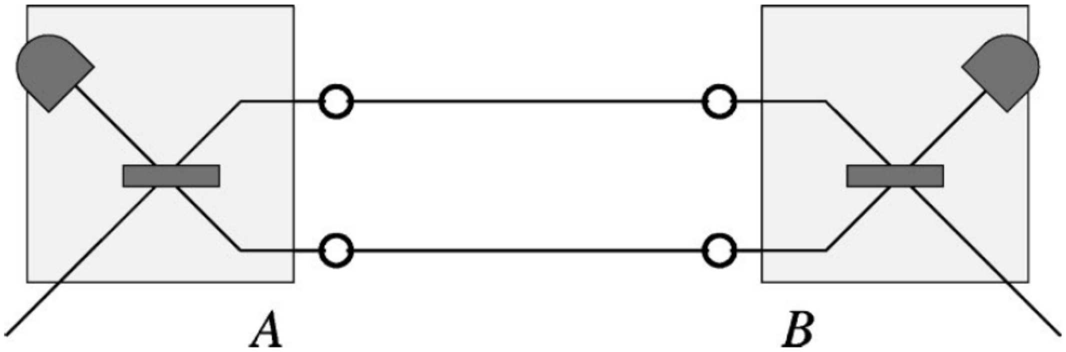




D. Browne, J. Eisert, S. Scheel, and M. Plenio, Phys. Rev. A 67, 062320 (2003).

J. Eisert, D. E. Browne, S. Scheel, and M. B. Plenio, Ann. Phys. 311, 431{458 (2004).

# Elementary Gaussification step



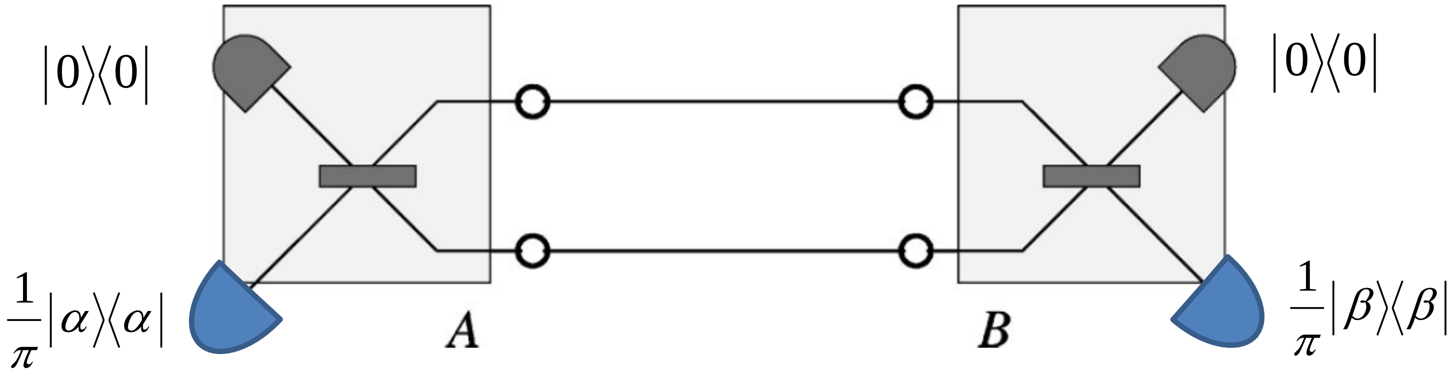

Modes on each side interfere on balanced beam splitters.

One output mode on each side is projected onto vacuum.


D. Browne, J. Eisert, S. Scheel, and M. Plenio, Phys. Rev. A 67, 062320 (2003).

J. Eisert, D. E. Browne, S. Scheel, and M. B. Plenio, Ann. Phys. 311, 431{458 (2004).

# Modified elementary Gaussification step

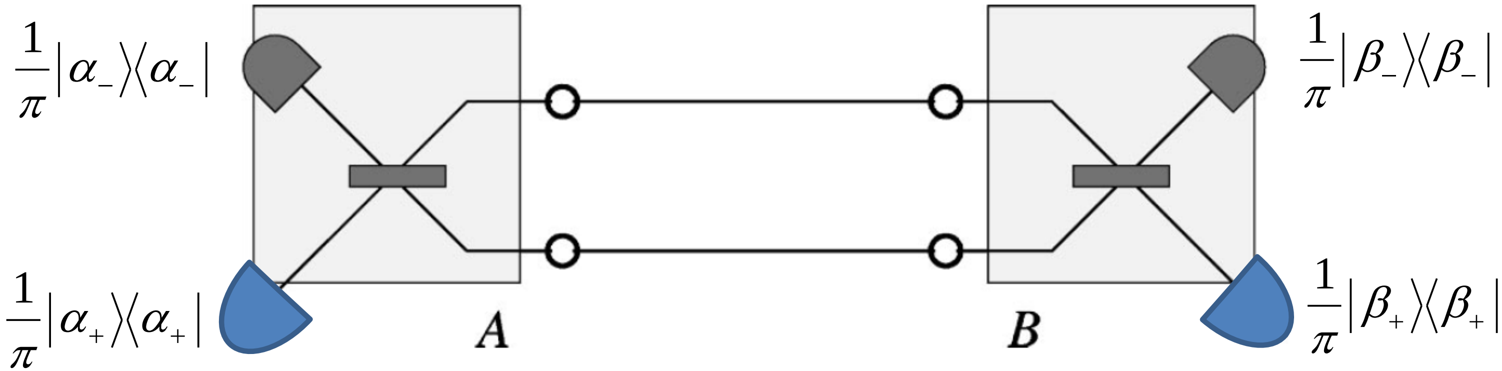



Assume that the Gaussified state is measured in the coherent state basis.


D. Browne, J. Eisert, S. Scheel, and M. Plenio, Phys. Rev. A 67, 062320 (2003).

J. Eisert, D. E. Browne, S. Scheel, and M. B. Plenio, Ann. Phys. 311, 431{458 (2004).

# Modified elementary Gaussification step



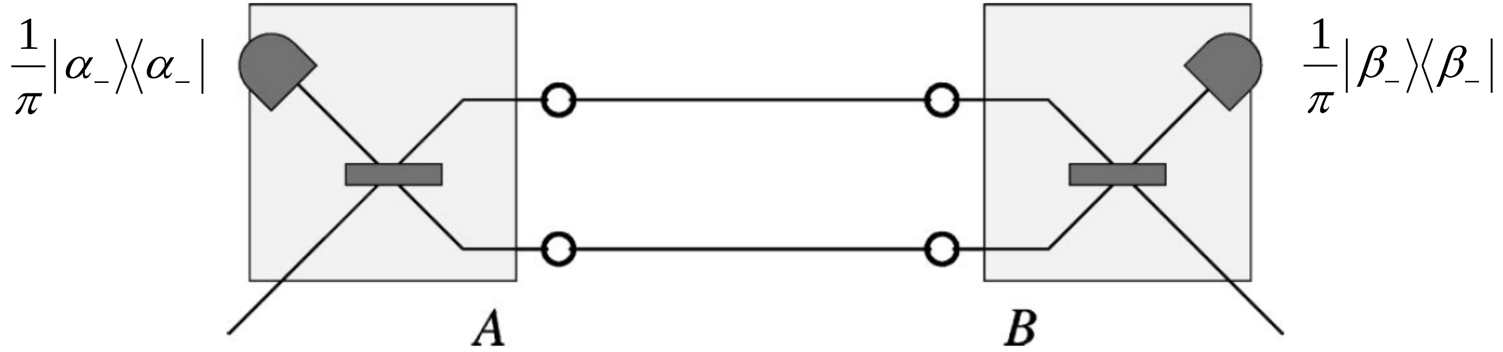

Assume that the Gaussified state is measured in coherent state basis.

Replace projection onto vacuum by conditioning on outcomes of eight-port homodyne detection.


D. Browne, J. Eisert, S. Scheel, and M. Plenio, Phys. Rev. A 67, 062320 (2003).

J. Eisert, D. E. Browne, S. Scheel, and M. B. Plenio, Ann. Phys. 311, 431{458 (2004).

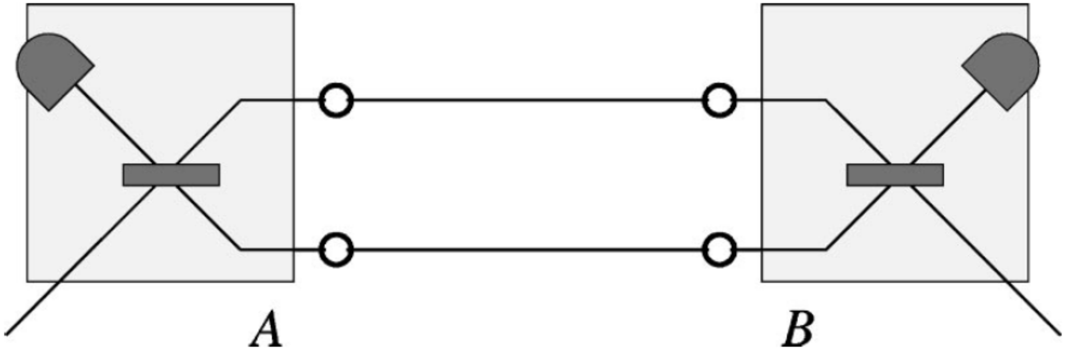
# Conditioning on outcomes of heterodyne detection




Projection of the "minus" modes onto vacuum states is equivalent to conditioning on

$$\alpha_- = 0, \quad \beta_- = 0$$

# Conditioning on outcomes of heterodyne detection




Projection of the "minus" modes onto vacuum states is equivalent to conditioning on

$$\alpha_- = 0, \quad \beta_- = 0$$

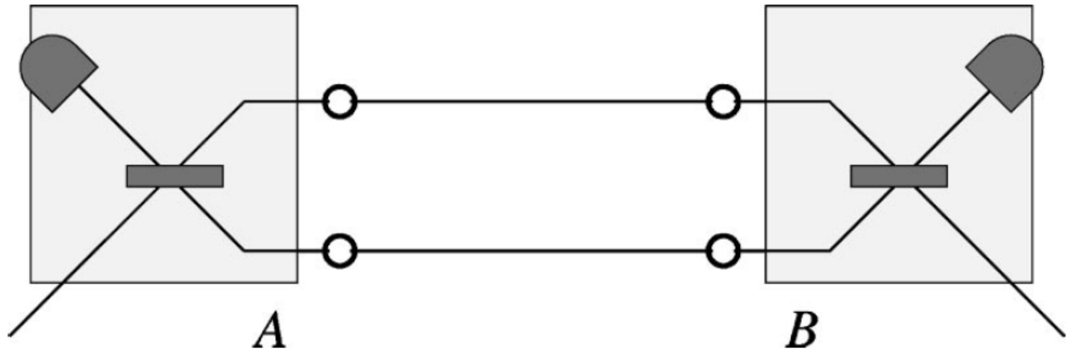
This has zero success probability in the proposed emulation protocol. Possible solutions:

(i) Conditioning on $\quad |\alpha_-| \le T_{acc}, \quad |\beta_-| \le T_{acc}$

(i) Accept the outcomes with a Gaussian probability

$$P_{acc} = \exp\left(-\frac{|\alpha_-|^2}{\bar{n}}\right)\exp\left(-\frac{|\beta_-|^2}{\bar{n}}\right)$$

# Emulation of a single step of the Gaussification protocol



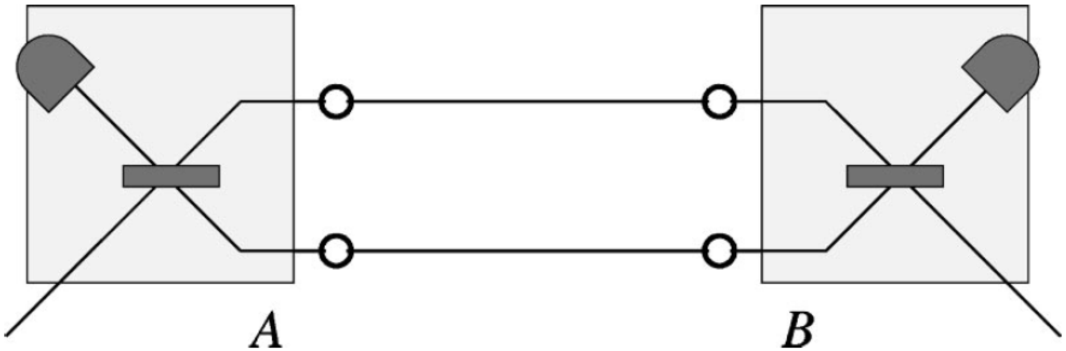

Interference of two copies of two-mode state on balanced beam splitters is emulated by linear transformations of the recorded complex amplitudes:

$$\alpha_{n,+} = \frac{1}{\sqrt{2}}(\alpha_{2n} + \alpha_{2n+1}), \qquad \alpha_{n,-} = \frac{1}{\sqrt{2}}(\alpha_{2n} - \alpha_{2n+1}),$$
$$\beta_{n,+} = \frac{1}{\sqrt{2}}(\beta_{2n} + \beta_{2n+1}), \qquad \beta_{n,-} = \frac{1}{\sqrt{2}}(\beta_{2n} - \beta_{2n+1}).$$


D. Browne, J. Eisert, S. Scheel, and M. Plenio, Phys. Rev. A 67, 062320 (2003).

J. Eisert, D. E. Browne, S. Scheel, and M. B. Plenio, Ann. Phys. 311, 431{458 (2004).

# Emulation of a single step of the Gaussification protocol




Interference of two copies of two-mode state on balanced beam splitters is emulated by linear transformations of the recorded complex amplitudes:

$$\alpha_{n,+} = \frac{1}{\sqrt{2}}(\alpha_{2n} + \alpha_{2n+1}), \qquad \alpha_{n,-} = \frac{1}{\sqrt{2}}(\alpha_{2n} - \alpha_{2n+1}),$$
$$\beta_{n,+} = \frac{1}{\sqrt{2}}(\beta_{2n} + \beta_{2n+1}), \qquad \beta_{n,-} = \frac{1}{\sqrt{2}}(\beta_{2n} - \beta_{2n+1}).$$

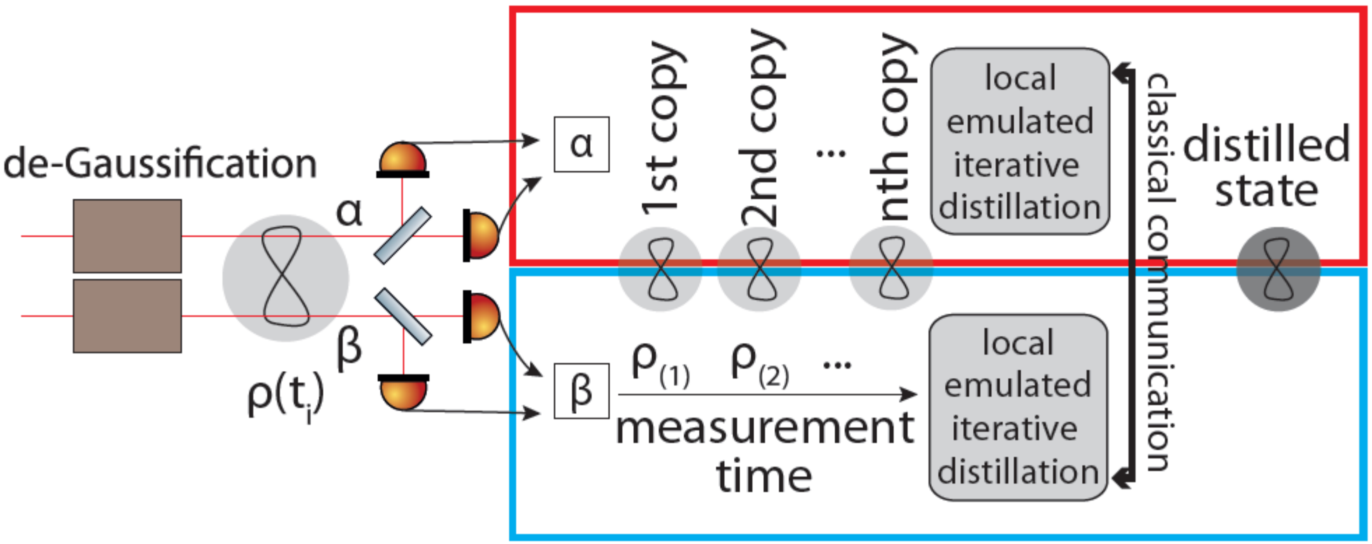
Conditioning on the values of $\alpha_{n,-}$ and $\beta_{n,-}$ is performed

The accepted values $\alpha_{n,+}$ and $\beta_{n,+}$ represent outcomes of eight port homodyne detection on the distilled state after one iteration of the Gaussification protocol.

D. Browne, J. Eisert, S. Scheel, and M. Plenio, Phys. Rev. A 67, 062320 (2003).

J. Eisert, D. E. Browne, S. Scheel, and M. B. Plenio, Ann. Phys. 311, 431{458 (2004).

# Emulation of CV entanglement distillation




Local measurements are performed on individual copies of the two-mode state

Each mode is projected onto coherent state by eight-port homodyne detection

Sequence of the measured complex amplitudes $\alpha_j$ and $\beta_j$ is recorded

# Properties of emulation of iterative Gaussification

**Advantages:**

The protocol does requires only single-copy measurements, yet it can emulate multi-copy iterative Gaussification.

Several iterations of the elementary Gaussification procedure are feasible, limited only by the amount of recorded data.

The scheme is inherently efficient, as if quantum memories were used.

# Properties of emulation of iterative Gaussification

**Advantages:**

The protocol does requires only single-copy measurements, yet it can emulate multi-copy iterative Gaussification.

Several iterations of the elementary Gaussification procedure are feasible, limited only by the amount of recorded data.

The scheme is inherently efficient, as if quantum memories were used.

**Disadvantages:**

We do not physically generate the distilled Gaussified state.

We only have access to outcomes of eight-port homodyne detection on the Gaussified state.

**The protocol is therefore particularly suitable for QKD and other similar applications, where measurements on the distilled state are performed.**

# Distillation of phase-diffused two-mode squeezed states

Initial two-mode Gaussian state with covariance matrix $\gamma_{AB}$

Random phase shifts $\phi_A$ and $\phi_B$ applied to modes A and B

Covariance matrix of non-Gaussian dephased state

$$\gamma'_{AB} = \left\langle R(\phi)\gamma R^T(\phi)\right\rangle_\phi$$

$$R(\phi) = \begin{pmatrix} \cos\phi_A & \sin\phi_A & 0 & 0 \\ -\sin\phi_A & \cos\phi_A & 0 & 0 \\ 0 & 0 & \cos\phi_B & \sin\phi_B \\ 0 & 0 & -\sin\phi_B & \cos\phi_B \end{pmatrix}$$

# Distillation of phase-diffused two-mode squeezed states

Initial two-mode Gaussian state with covariance matrix $\gamma_{AB}$

Random phase shifts $\phi_A$ and $\phi_B$ applied to modes A and B

Covariance matrix of non-Gaussian dephased state

$$\gamma'_{AB} = \left\langle R(\phi)\gamma R^T(\phi) \right\rangle_\phi$$

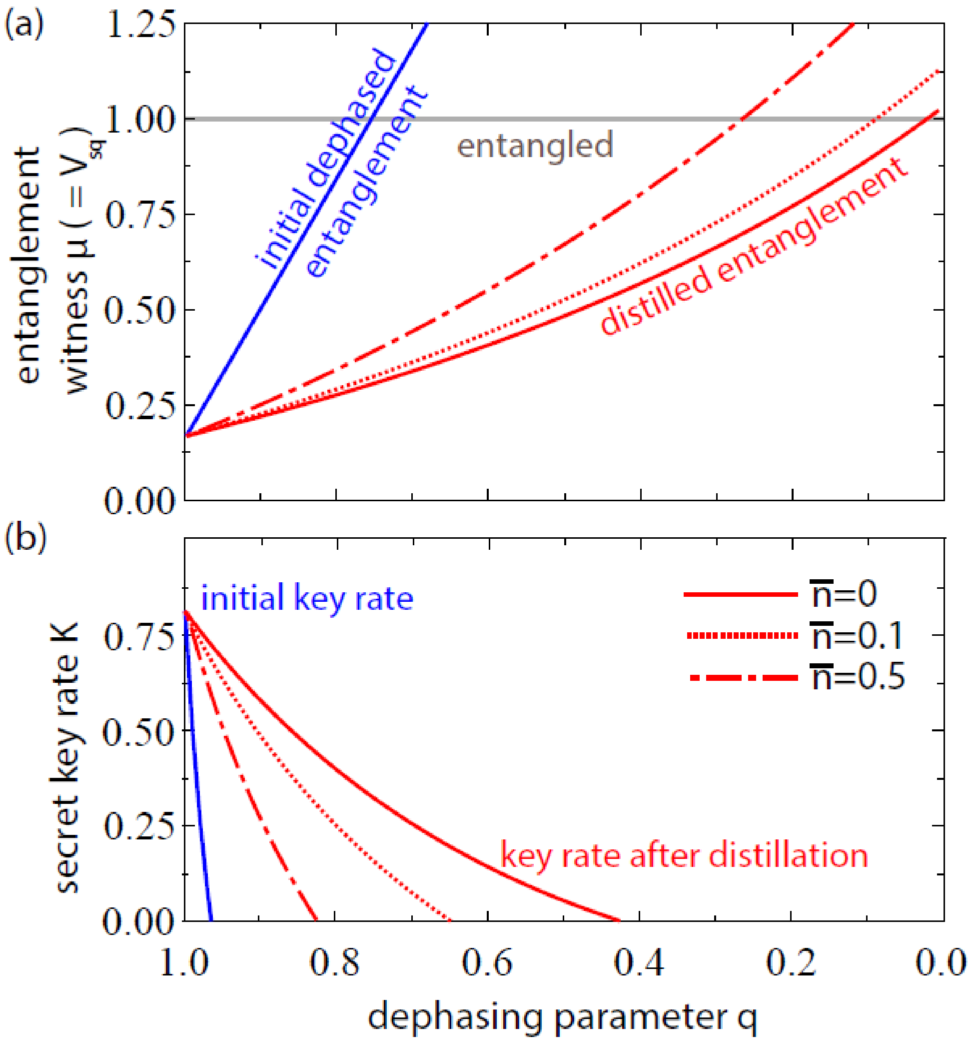
$$R(\phi) = \begin{pmatrix} \cos\phi_A & \sin\phi_A & 0 & 0 \\ -\sin\phi_A & \cos\phi_A & 0 & 0 \\ 0 & 0 & \cos\phi_B & \sin\phi_B \\ 0 & 0 & -\sin\phi_B & \cos\phi_B \end{pmatrix}$$

Covariance matrix of the asymptotic distilled Gaussian state

$$\gamma_{AB,\infty} = \langle R(\phi)[\gamma_{AB} + (2\bar{n}+1)I]^{-1} R^T(\phi) \rangle_\phi^{-1} - (2\bar{n}+1)I.$$

E. T. Campbell and J. Eisert, Phys. Rev. Lett. 108, 020501 (2012).

# Distillation of phase-diffused two-mode squeezed states




a=3.583 b=3.417

Initial covariance matrix:

$$\gamma_{AB} = \begin{pmatrix} a & 0 & b & 0 \\ 0 & a & 0 & -b \\ b & 0 & a & 0 \\ 0 & -b & 0 & a \end{pmatrix}$$

After dephasing:

$$a_{\mathrm{PD}} = a, \qquad b_{\mathrm{PD}} = qb,$$

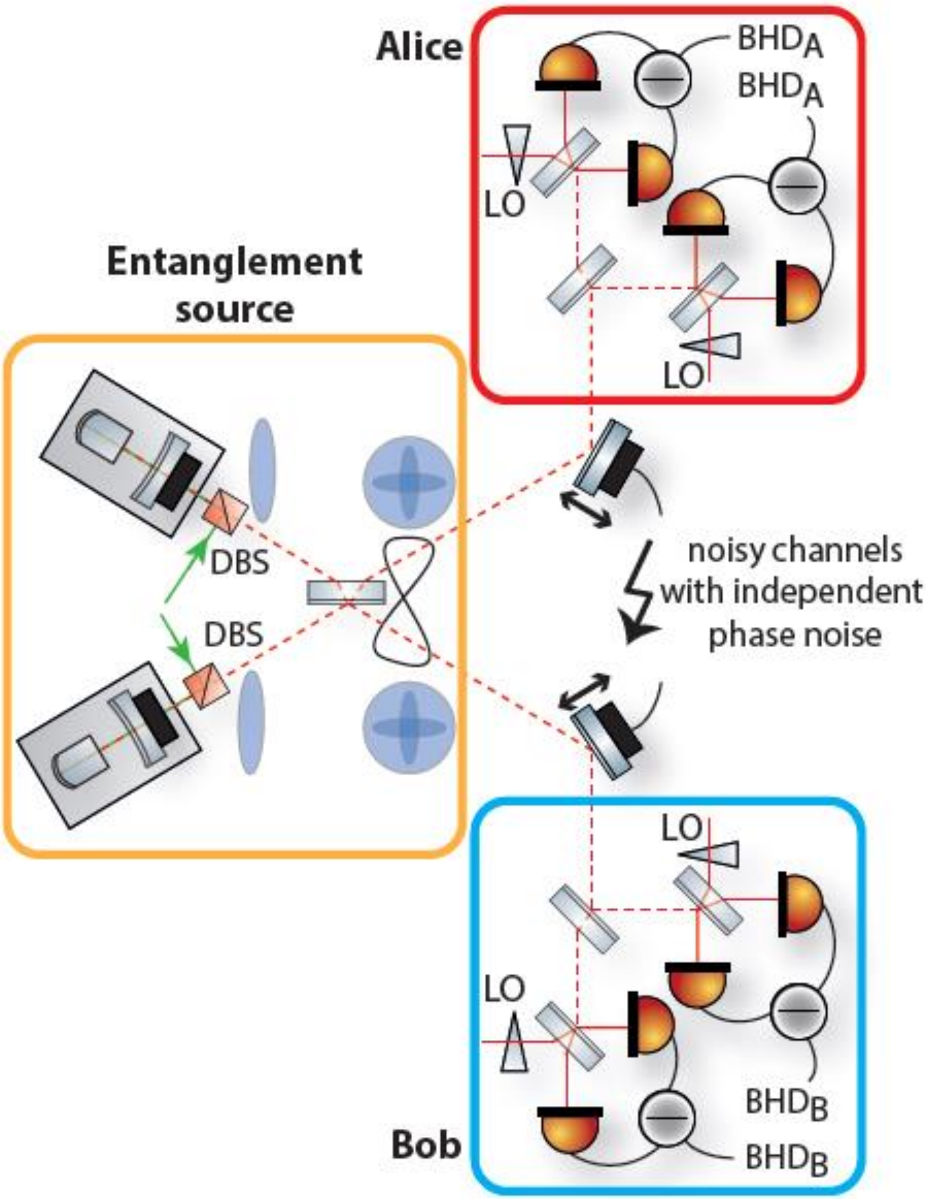
$$q = \langle \cos\phi_A \cos\phi_B \rangle$$

Asymptotic Gaussified state:

$$a_\infty = \frac{[(a+2\bar{n}+1)^2 - b^2]a - (2\bar{n}+1)b^2(1-q^2)}{(a+2\bar{n}+1)^2 - q^2b^2},$$

$$b_\infty = \frac{(a+2\bar{n}+1)^2 - b^2}{(a+2\bar{n}+1)^2 - q^2b^2}\, qb.$$

# Experimental setup



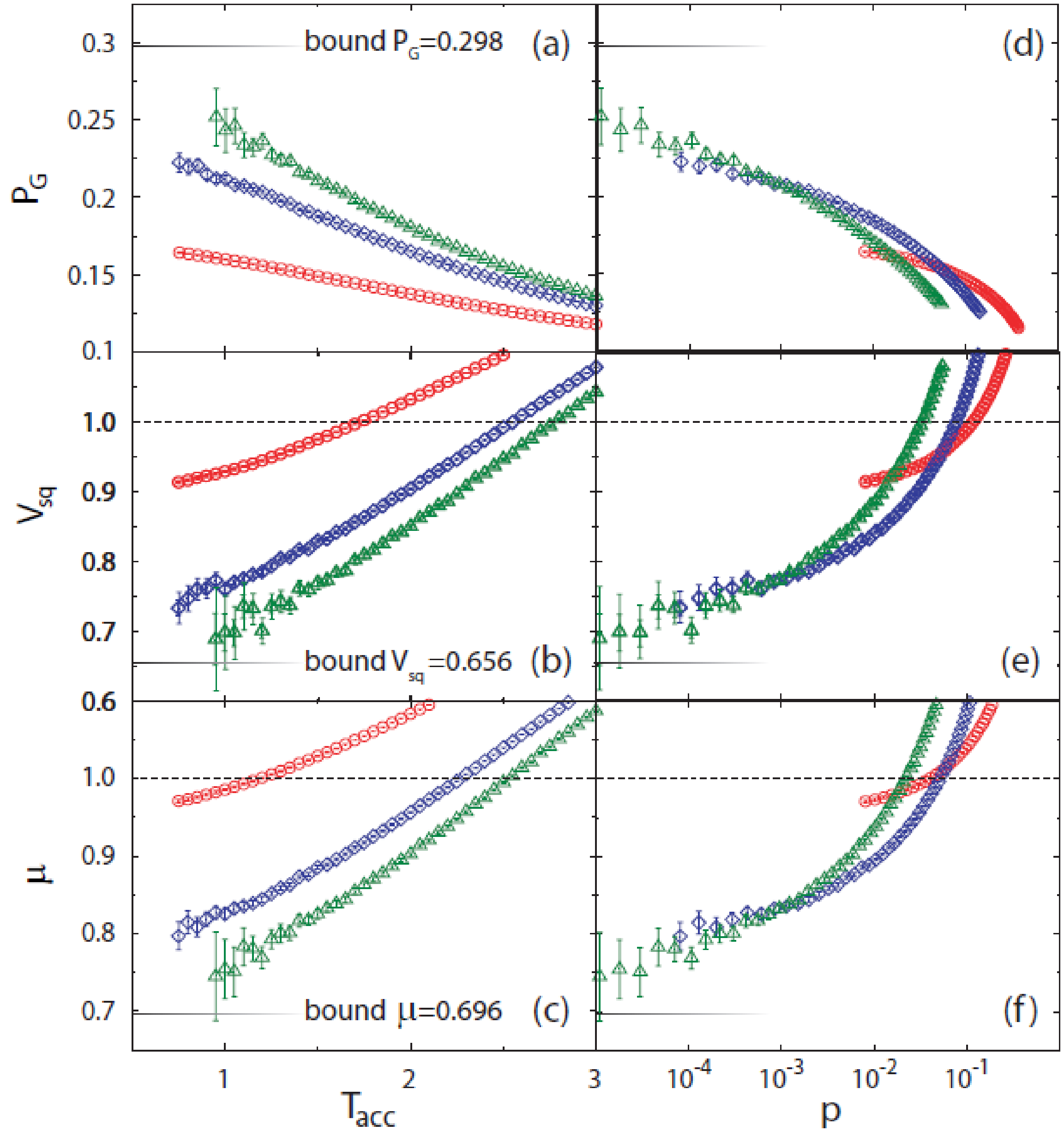

Covariance matrix of the input Gaussian state:

$$\gamma_{AB,exp} = \begin{pmatrix} 3.20 & -0.13 & -2.90 & -0.04 \\ -0.13 & 6.24 & -0.03 & 6.08 \\ -2.90 & -0.03 & 3.70 & -0.06 \\ -0.04 & 6.08 & -0.06 & 6.83 \end{pmatrix}$$

Estimated strength of phase noise:

$$q=0.78$$

# Experimental results




Quantification of state purity

$$P_G = 1/\sqrt{\det \gamma}$$

Quantification of squeezing

$$V_{sq} = \min(\mathrm{eig}(\gamma_{AB}))$$

Gaussian entanglement

# Conclusions

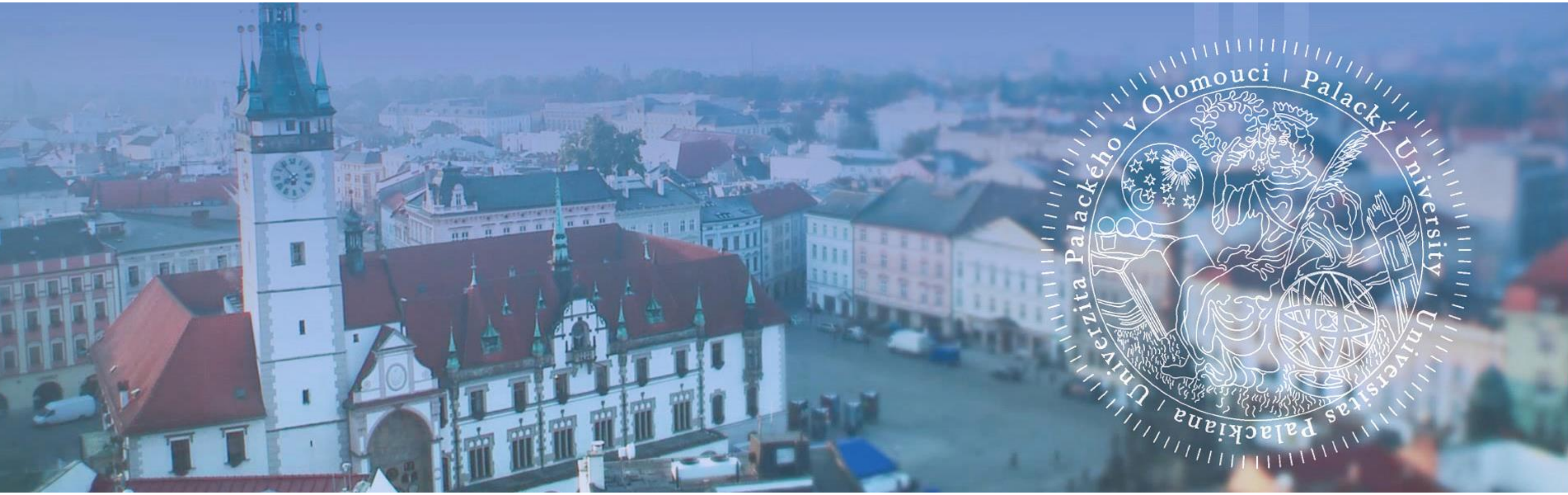
In certain protocols, physical implementation of complicated CV quantum operations can be replaced by processing of experimental data.

Significant simplification of the experiment implementation, makes high-quality noiseless amplification or iterative CV entanglement distillation experimentally feasible.

The approach is suitable for protocols, where Alice and Bob measure their states in the coherent state basis.

The technique is thus particularly suitable for CV QKD and related applications.

# Thank you for your attention!




optics.upol.cz

www.opticsolomouc.org