Optomechanical transfer of quantum light to classical thermodynamics

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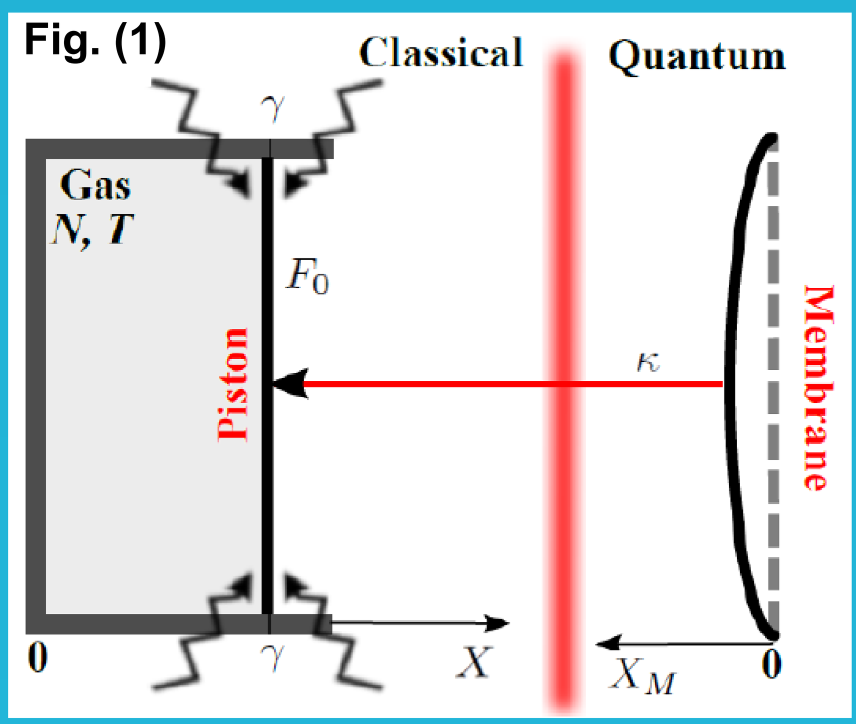
A one-way chain starting from quantum state of radiation swapped to sufficiently large mechanical oscillator pushing or pulling a classical almost mass-less piston, which is further pressing an ideal gas in small container is proposed and theoretically analyzed. We repeat such gedanken experiment on an ensemble of the same chains and observe strongly nonlinear and non-monotonous transfer of classical and quantum uncertainty of light to average mechanical work by isothermal process. Various classical and nonclassical quantum states of light are analyzed and compared for their ability to produce average classical equilibrium work.

The scheme:



Brownian (overdamped) dynamics:

$$\gamma\dot{X} = -\kappa X_M - F_0 + \frac{Nk_BT}{X} + \sqrt{2\gamma k_BT}\,\xi(t)$$

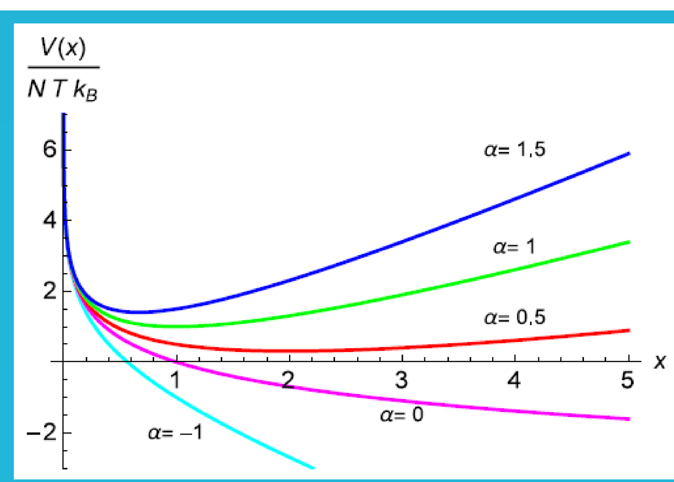
Equivalent Fokker-Planck equation:

$$\frac{\partial\rho}{\partial t} = -\frac{1}{\gamma}\frac{\partial}{\partial X}\left[\left(-\kappa X_M - F_0 + \frac{Nk_BT}{X}\right)\rho\right] + \frac{k_BT}{\gamma}\frac{\partial^2\rho}{\partial X^2}$$

$$V(x) = Nk_BT\,(\alpha x - \ln x)\,, \quad \alpha \equiv 1 + \frac{\kappa X_M}{F_0}$$

The notation:

- $\gamma\ldots$ effective damping constant of the piston
- $X\ldots$ piston position
- $X_M\ldots$ membrane position, possibly fluctuating with density $p(X_M)$
- $F_0\ldots$ background gas pressure-force
- $N\ldots$ compressed gas particle number
- $T\ldots$ background temperature
- $\xi(t)\ldots$ delta-correlated stationary Gaussian process with zero-mean



Stationary solutions of the Fokker-Planck equation

Membrane without position-uncertainty, large gas particle-number solution:

$$\rho(X_M) = \delta(X_M - X_0)$$

$$\alpha \equiv 1 + \frac{\kappa X_M}{F_0}$$

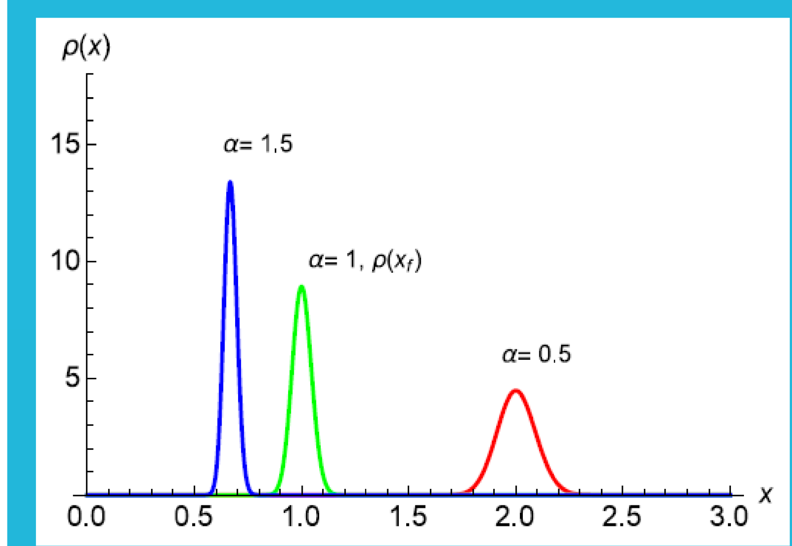
$$\rho(x) = \frac{1}{Z}x^N\exp\left[-\alpha N x\right], \quad x \geq 0$$

$$\lim_{N\to\infty}\rho(x_i) = \delta(x_i - 1/\alpha),$$

$$\lim_{N\to\infty}\rho(x_f) = \delta(x_f - 1),$$

$$W \equiv -\int_{x_i}^{x_f} PS\,\mathrm{d}x = -Nk_BT\ln\frac{x_f}{x_i},$$

$$w(\alpha) \equiv \frac{W}{Nk_BT} = -\ln\alpha, \quad \alpha = 1 + \frac{\kappa X_M}{F_0}.$$



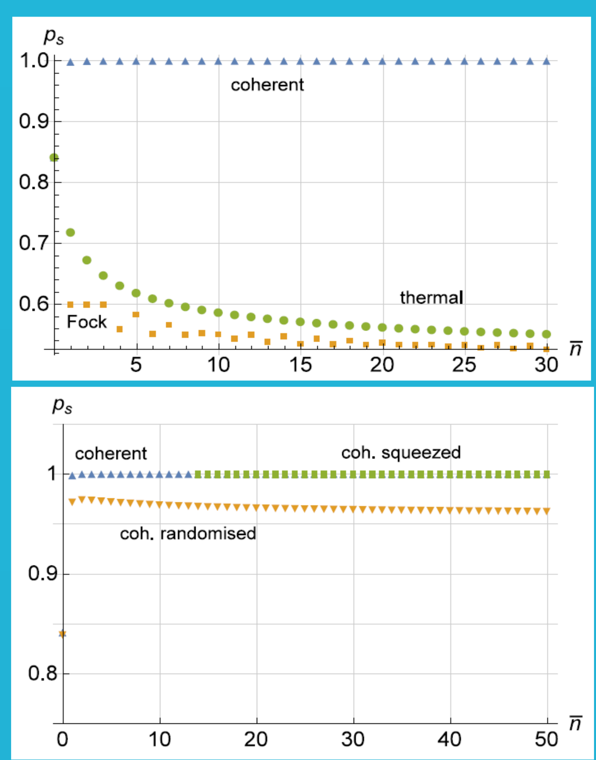
Membrane with position-uncertainty, Gaussian (thermal+coherent+nonclassical) states example:

$$\rho(X_M) = \frac{1}{\sqrt{2\pi\bar{\epsilon}}}\exp\left[-\frac{(X_M - X_0)^2}{2\bar{\epsilon}^2}\right] \Rightarrow$$

$$p(\alpha) = \frac{1}{\sqrt{2\pi}\epsilon}\exp\left[-\frac{(\alpha - \alpha_0)^2}{2\epsilon^2}\right],$$

$$\epsilon = \frac{\kappa\bar{\epsilon}}{F_0}, \quad \alpha_0 = \left(1 + \frac{\kappa X_0}{F_0}\right).$$

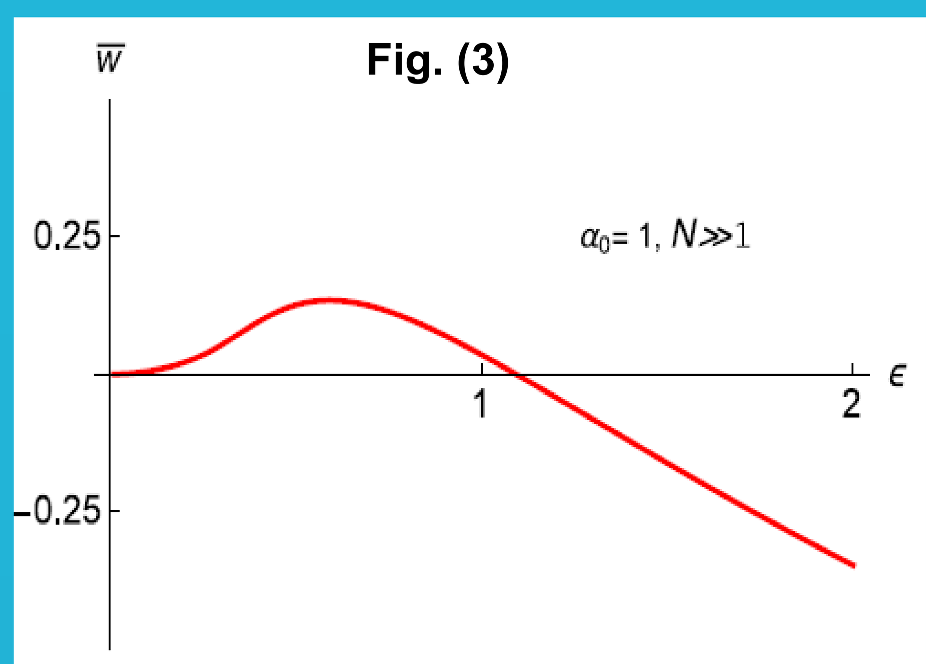
$$p_s \equiv \int_0^\infty p(\alpha)\mathrm{d}\alpha$$



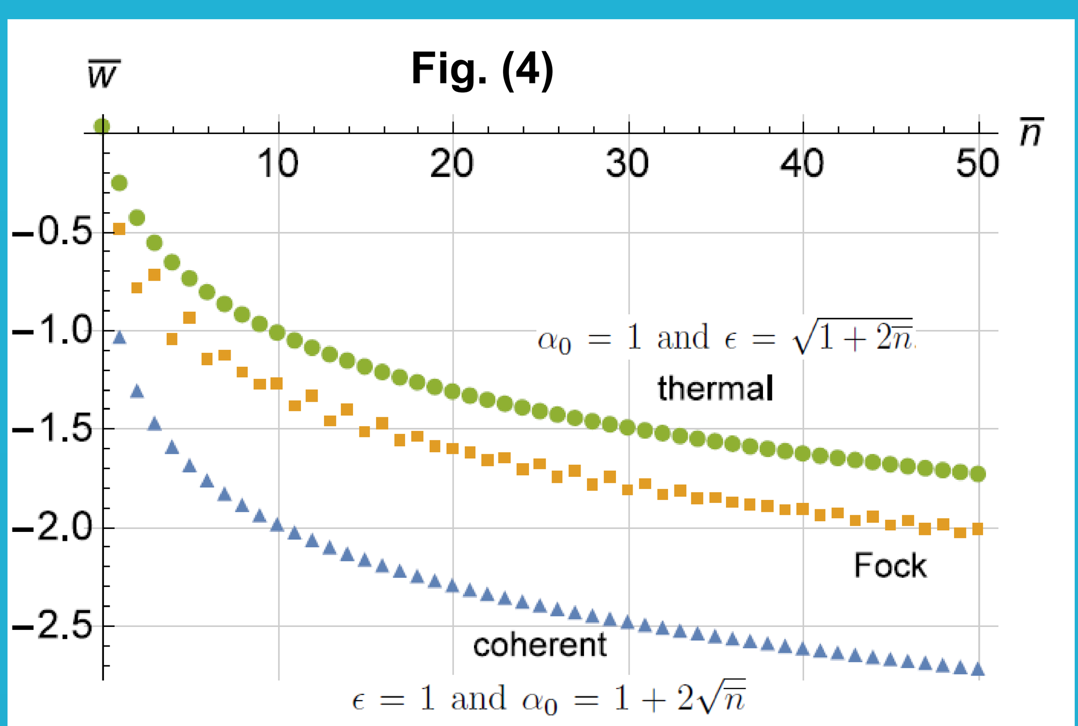
$$\rho(x_i) = \frac{\int_0^\infty p(\alpha)Z_i^{-1}x_i^N\exp\left[-\alpha N x_i\right]\mathrm{d}\alpha}{\int_0^\infty p(\alpha)\mathrm{d}\alpha}$$

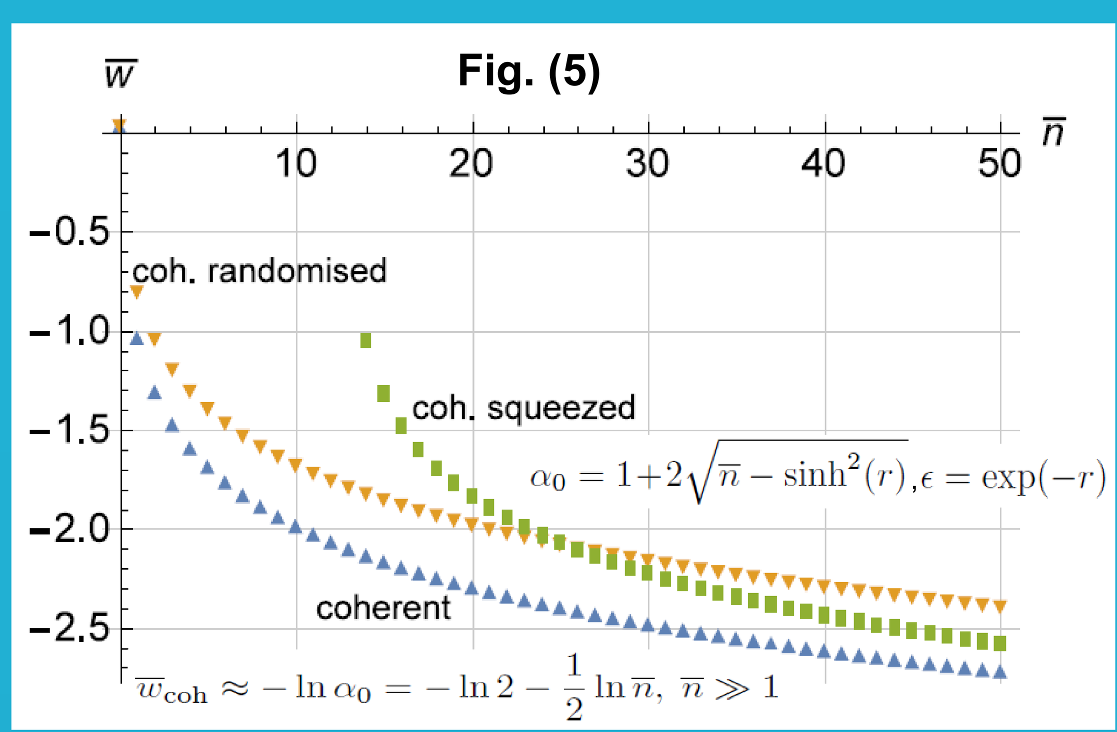
$$\overline{w} = -\overline{\ln\alpha} = -\int_{-\infty}^{\infty}\ln(\alpha)\overline{p}(\alpha)\mathrm{d}\alpha,$$

$$\overline{p}(\alpha) \equiv \frac{\theta(\alpha)p(\alpha)}{\int_{-\infty}^{\infty}\theta(\alpha)p(\alpha)\,\mathrm{d}\alpha},$$



Work on the „successful“ subensemble:





Conclusions:

- **We have described a toy model for conversion of mechanical membrane position-fluctuations into average work based on stochastic dynamics and equilibrium thermodynamics, Fig. (1).**
- **States of the membrane representing the same value of input energy stored in a different (coherent, thermal, etc.) form, yield different work outputs.**
- **Thermalized membrane with zero mean position yields nonzero work with nonmonotonic dependence on the external parameter F_0, Fig. (3).This is due to the logarithmic dependence of the work on the membrane position, Fig. (2).**
- **All assumed states of the membrane have the same (logarithmic) type of the work asymptotics for large input energies, Figs. (4-5).**
- **The coherent state position distribution seems to yield the most effective energy transfer through the assumed chain.**

Analysis of a simple example of mechanical position-fluctuations conversion into average work shows that states with the same energy, stored in a different form, can perform different average work. The average work-value may depend nontrivially on the external parameters as well as on the parameters characterizing the mechanical system.

References: * H.-P. Breuer, F. Petruccione: The Theory of Open Quantum Systems (Oxford, 2002).

* Feynman, Leighton, Sands: The Feynman Lectures on Physics (Addison-Wesley, 1977).

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