HIGH ORDER NONLINEARITY FROM SINGLE PHOTONS

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HIGH ORDER NONLINEAR OPERATIONS

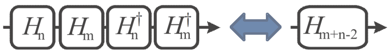
The ultimate goal of CV quantum information processing is the ability to perform an arbitary nonlinear quanutm operation. That is, an operation given by

$$\hat{H} = \sum \omega_{jk}\hat{x}^j\hat{p}^k$$

We could search for these operations in nature, or we could try to construct them from smaller elements using a simple trick [1]:

$$e^{iAt}e^{iBt}e^{-iAt}e^{-iBt} = e^{-[A,B]t^2} + O(t^3)$$

When A and B are Hamiltonians of orders m and n, the resulting operator is again a Hamiltonian:

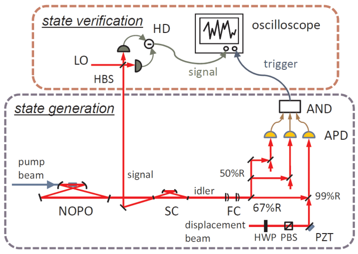


It is now apparent that the lowest order of operation allowing this procedure is the third order:

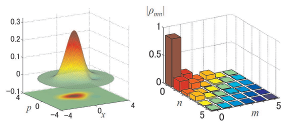
$$\hat{H}_3 = \omega_3\hat{x}^3$$

CREATING A CUBIC STATE [4]

$$|0\rangle + i\frac{\chi\sqrt{3}}{2\sqrt{2}}\left(\sqrt{3}|1\rangle + \sqrt{2}|3\rangle\right)$$



The experimentally produced state:



The fidelity with the ideal state is $F = 0.89$

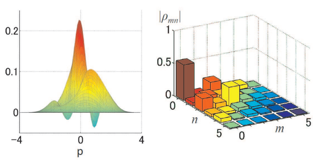
However, since the nonlinearity is fairly weak, the fidelity of the ideal state with vacuum is $F_{|0\rangle} = 0.98$

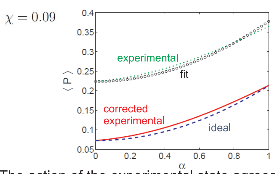
This suggests that fidelity is not the best figure of merit

We can look directly at the nonlinear part by analyzing the state after a virtual photon subtraction.

The state we expect is: $\sqrt{3}|0\rangle + \sqrt{6}|2\rangle$

The produced state has similar features, but it differs, slightly.





The action of the experimental state agrees with the ideal case up to constant displacement of $D(\Delta P) = 0.16$, which can be corrected.

To analyze whether the state possesses the cubic nonlinearity, we can take it and use it as the resource in a virtual cubic gate.

The cubic gate acts as

$$\hat{x} \to \hat{x} \quad \hat{p} \to \hat{p} + 3\chi\hat{x}^2$$

so when applied to a set of coherent states $|\alpha\rangle$ it should transform the p quadrature moment as

$$\langle p\rangle \to \langle p\rangle + 3\chi(2\alpha^2 + 1/2)$$

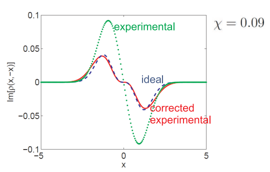
Alternatively, we can look directly to the density matrix in x representation.

$$\rho(x,x') = \langle x|\hat{\rho}|x\rangle$$

Its main anti-diagonal

$$\rho_{id}(x,-x) = \langle x|(1+i\chi\hat{x}^3)|0\rangle\langle 0|(1-i\chi\hat{x}^3)|-x\rangle = e^{-x^2}(1-\chi^2x^6+2i\chi x^3)$$

should reveal the cubic terms directly in its imaginary part.



The cubic nonlinearity is revealed after a correcting displacement of $D(\Delta P) = 0.17$

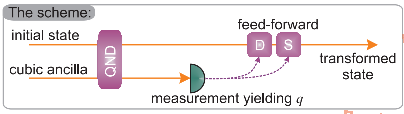
THE CUBIC NONLINEARITY [2]

How can we implement the cubic nonlinearity?
Not easily. In realistic conditions it is:

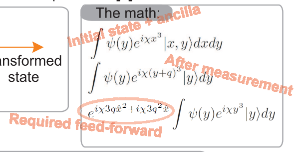
- too weak
- too noisy
- too weak and too noisy and hidden under effects coming from interactions of lower order

We need to create the operation artificialy.
One way is using the **ancilla and measurement** induced operations **[3]**.

The scheme:



The math:



$$\int \psi(y)e^{i\chi x^3}|x,y\rangle dxdy$$

$$\int \psi(y)e^{i\chi(y+q)^3}|y\rangle dy$$

$$e^{i\chi 3q\hat{x}^2 + i\chi 3q^2\hat{x}}\int \psi(y)e^{i\chi y^3}|y\rangle dy$$

The ancilla:

$\int e^{i\chi x^3}|x\rangle dx$ — Ideal but unphysical - infinite energy

$e^{i\chi\hat{x}^3}\hat{S}|0\rangle = \hat{S}e^{i\chi'\hat{x}^3}|0\rangle$ — Finite energy transformation. Squeezing can be disregarded

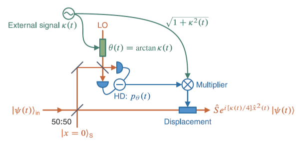
$(1+i\chi\hat{x}^3)|0\rangle$ — Weak nonlinearity approximation...

$|0\rangle + i\frac{\chi\sqrt{3}}{2\sqrt{2}}\left(\sqrt{3}|1\rangle + \sqrt{2}|3\rangle\right)$ — ... that can be sculpted from individual photons

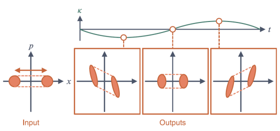
NONLINEAR FEED-FORWARD [5]

A necessary part of the complete cubic gate is the nonlinear feed-forward providing a variable squeezing operation driven by measurement results.

$$e^{i[\kappa(t)/4]\hat{x}^2(t)}$$

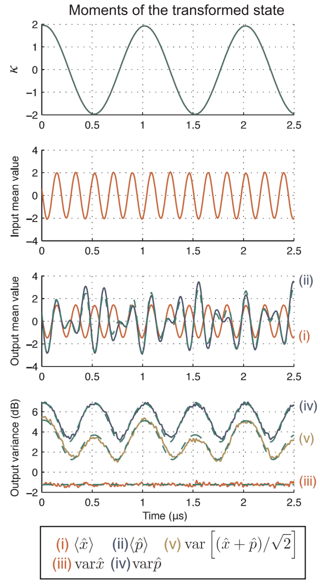


Experimental scheme implementing variable (as well as constant) squeezing

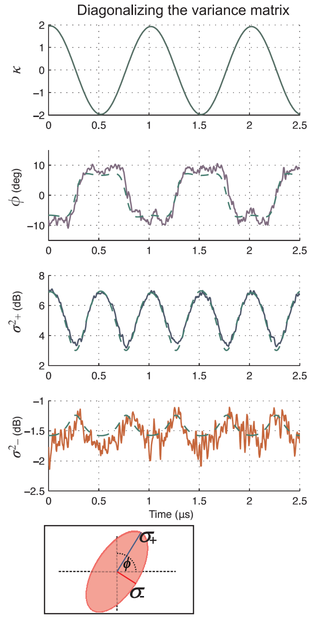


The feed-forward acting on a coherent state with fluctuating amplitude

Moments of the transformed state



Diagonalizing the variance matrix



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