Twisting tensor and spin squeezing

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Motivation

- Spin squeezing: noise suppression in a sample of two-level systems (atomic internal states, atomic positions, photon polarizations, etc.).
- Application in precision metrology or in quantum information processing.
- Squeezing produced by particle interactions. How to optimize the process to achieve fastest and most efficient squeezing?
- Two basic processes by quadratic Hamiltonian interaction suggested by Kitagawa and Ueda [PRA 47, 5138 (1993)]:
  - one axis twisting (OAT): $H \propto J_z^2$,
  - two-axis counter-twisting (TACT): $H \propto J_x^2 - J_y^2$.

  Can there be a more general scheme? More axes to twist?
- Nonlinearity beyond quadratic?
- How to realize a general quadratic spin-squeezing scheme experimentally?

Quadratic interaction Hamiltonians

$$H = \omega_k J_k + \chi_{kl} J_k J_l + f(N),$$

where

$$\begin{aligned} J_x &= \frac{1}{2}(a^\dagger b + a b^\dagger), \\ J_y &= \frac{1}{2i}(a^\dagger b - a b^\dagger), \\ J_z &= \frac{1}{2}(a^\dagger a - b^\dagger b). \end{aligned}$$

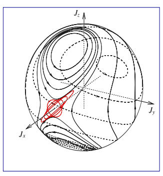
Parameters $\chi_{kl} = \chi_{lk}$: **Twisting Tensor**

$$\chi = \begin{pmatrix} \chi_{xx} & \chi_{xy} & \chi_{xz} \\ \chi_{yx} & \chi_{yy} & \chi_{yz} \\ \chi_{zx} & \chi_{zy} & \chi_{zz} \end{pmatrix}$$

Special case: One axis twisting (OAT)

$$H = \chi J_z^2 + \omega_x J_x$$

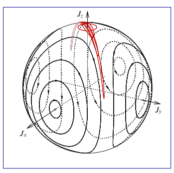
$$\chi^{(\mathrm{OAT})} = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & \chi \end{pmatrix}$$

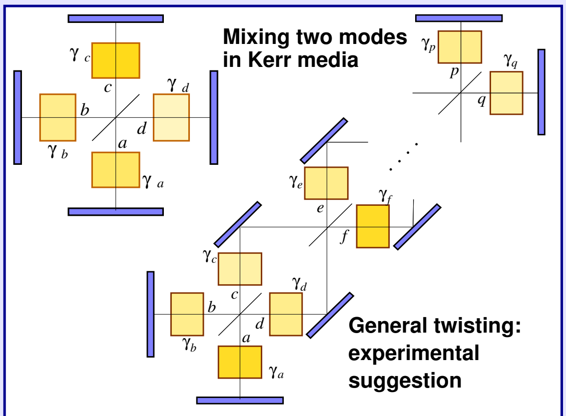


Special case: Two-axis counter-twisting (TACT)

$$H = \chi (J_x^2 - J_y^2)$$

$$\chi^{(\mathrm{TACT})} = \begin{pmatrix} \chi & 0 & 0 \\ 0 & -\chi & 0 \\ 0 & 0 & 0 \end{pmatrix}$$





Gaussian approximation, squeezing rate

For initial stages of squeezing (see [1,2]):

$$\frac{d\xi^2}{dt} = -Q\xi^2$$

where ξ^2 is squeezing parameter, Q squeezing rate:

$$\boxed{Q = \sqrt{\mathrm{Tr}\left(\mathcal{J} H'' \mathcal{J} H''\right) + \frac{\mathrm{Tr}\left(\mathcal{J}^2 H'' \mathcal{J}^2 H''\right)}{|J|^2}},}$$

where H expressed as *classical* function of $J_{x,y,z}$,

$$\begin{aligned} H'' &= \begin{pmatrix} H_{xx} & H_{xy} & H_{xz} \\ H_{xy} & H_{yy} & H_{yz} \\ H_{xz} & H_{yz} & H_{zz} \end{pmatrix}, \\ \mathcal{J} &= \begin{pmatrix} 0 & J_z & -J_y \\ -J_z & 0 & J_x \\ J_y & -J_x & 0 \end{pmatrix}, \end{aligned}$$

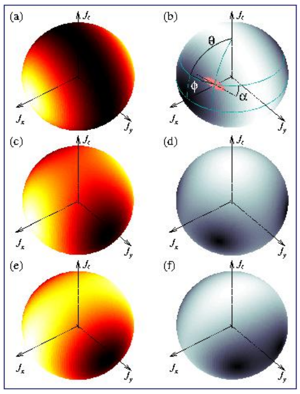
and $|J| = \sqrt{J_x^2 + J_y^2 + J_z^2}$.

Optimum rotation of the Bloch sphere:

$$H_{\mathrm{ad}} = \vec{\omega} \cdot \vec{J},$$

where

$$\vec{\omega} = -\mathrm{grad} H + \frac{1}{2}\left(\mathrm{Tr}H'' - \frac{\vec{J} H'' \vec{J}}{|\vec{J}|^2}\right)\vec{J}.$$



Hamiltonian H (a, c, e) and squeezing rate Q (b, d, f). The shades correspond to the mean value of the Hamiltonian and to the squeezing rate of a spin coherent state with the direction of $\vec{J}$, the lighter shade corresponds to higher values. Hamiltonian with $\vec{\omega} = 0$ and diagonal twisting tensor with $\{\chi_{xx}, \chi_{yy}, \chi_{zz}\} = \{1, 0, 0\}$ (OAT: a, b); $\{1, 0, 0.5\}$ (TACT: c, d); and $\{1, 0, 0.8\}$ (general twisting: e, f).

Example: optimum squeezing strategy for quadratic Hamiltonians

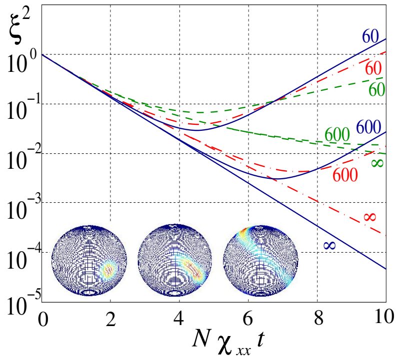
- With Hamiltonian $H = \chi_{kl} J_k J_l$, choose coordinate system to have diagonal χ with $\chi_{xx} \geq \chi_{zz} \geq \chi_{yy}$
- Place the initial state on the pole along J_z
- Rotate along J_z with frequency

  $$\omega = N\left(\frac{\chi_{xx} + \chi_{yy}}{2} - \chi_{zz}\right)$$

  (for $\chi_{zz} = (\chi_{xx} + \chi_{yy})/2$ no rotation, TACT)
- Achieved squeezing rate

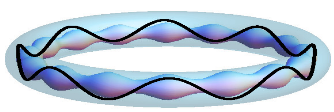
  $$Q = N\left(\chi_{xx} - \chi_{yy}\right)$$

Maximum achievable squeezing rate only depends on the difference between the largest and the smallest eigenvalues of χ.



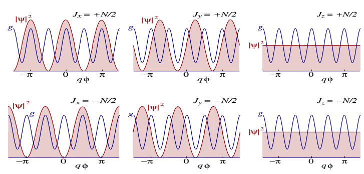
Time evolution of the squeezing parameter, $\chi_{xx} = 1$, $\chi_{yy} = 0$. OAT with $\chi_{zz} = 0$ (green broken line), TACT with $\chi_{zz} = 0.5$ (full blue line), and general twisting with $\chi_{zz} = 0.8$ (red dash-dotted line), the number at each line being the number of particles N.

Two mode squeezing and Lipkin-Meshkov-Glick model in a toroidal BEC with spatially modulated nonlinearity [3]



- Circular lattice couples two counter-rotating modes
- Squeezing generated by the nonlinear interaction spatially modulated at half the lattice period

$$\chi = \begin{pmatrix} \frac{\chi_1}{2}\cos\alpha & \frac{\chi_1}{2}\sin\alpha & 0 \\ \frac{\chi_1}{2}\sin\alpha & -\frac{\chi_1}{2}\cos\alpha & 0 \\ 0 & 0 & -\chi_0 \end{pmatrix}.$$



$$H = \hbar\left[u_x J_x + u_y J_y + 2q\omega J_z - \chi_0 J_z^2 + \frac{\chi_1 \cos\alpha}{2}\left(J_x^2 - J_y^2\right) + \frac{\chi_1 \sin\alpha}{2}\left(J_x J_y + J_y J_x\right)\right]$$

Conclusion

- General formula for squeezing rate with nonlinear Hamiltonians in J_k [1,2].
- All quadratic interactions in a two-level system can be described by matrix χ transforming as a tensor under $O(3)$ rotations ("twisting tensor").
- OAT corresponds to a single nonzero eigenvalue of χ, TACT to equidistant eigenvalues.
- Fastest possible squeezing determined by the difference between largest and smallest eigenvalues of χ.
- Possible physical realization: coupled optical modes in Kerr media [1], toroidal BEC with spatially modulated nonlinearity [3].
- Application in counterdiabatic driving and Dicke state preparation [4].

[1] T. Opatrný, *Twisting tensor and spin squeezing*, PRA **91**, 053826 (2015).

[2] T. Opatrný, *Squeezing with classical Hamiltonians*, PRA **92**, 033801 (2015).

[3] T. Opatrný, M. Kolář, and K. K. Das, *Spin squeezing by tensor twisting and Lipkin-Meshkov-Glick dynamics in a toroidal Bose-Einstein condensate with spatially modulated nonlinearity*, PRA **91**, 053612 (2015).

[4] T. Opatrný, H. Saberi, E. Brion, and K. Mølmer, *Counterdiabatic driving in spin squeezing and Dicke-state preparation*, PRA **93**, 023815 (2016).