Optimal entanglement-assisted discrimination of quantum measurements: experimental demonstration

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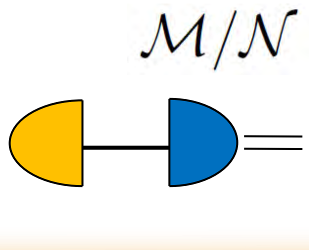
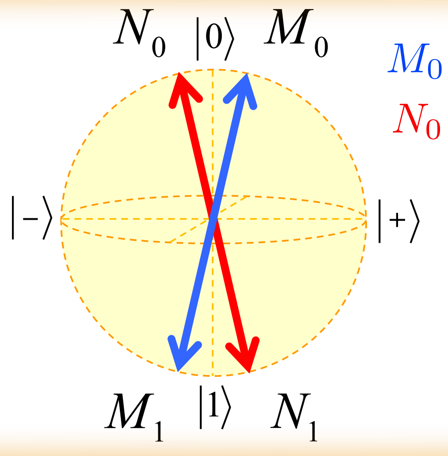
1) Introduction

We investigate optimal discrimination between two projective single-qubit measurements in a scenario where the measurement can be performed only once. We consider general setting involving a tunable fraction of inconclusive outcomes and we prove that the optimal discrimination strategy requires an entangled probe state for any nonzero rate of inconclusive outcomes. We experimentally implement this optimal discrimination strategy for projective measurements on polarization states of single photons. Our setup involves a real-time electro-optical feed-forward loop which allows us to fully harness the benefits of entanglement in discrimination of quantum measurements. The experimental data clearly demonstrate the advantage of entanglement-based discrimination strategy as compared to unentangled single-qubit probes.

2) Discrimination of quantum measurements

The impossibility to perfectly discriminate two non-orthogonal quantum states triggers the question what is the optimal approximate or probabilistic discrimination strategy. Here, we investigate the utility of entanglement for the canonical task of optimal discrimination between two projective measurements $\mathcal{M}$ and $\mathcal{N}$ on a single qubit provided that the measurement can be performed only once.





$$M_0 = |\phi\rangle\langle\phi|, \quad M_1 = |\phi^\perp\rangle\langle\phi^\perp|$$
$$N_0 = |\psi\rangle\langle\psi|, \quad N_1 = |\psi^\perp\rangle\langle\psi^\perp|$$

$$|\phi\rangle = \cos\theta|0\rangle + \sin\theta|1\rangle$$
$$|\phi^\perp\rangle = \sin\theta|0\rangle - \cos\theta|1\rangle$$
$$|\psi\rangle = \cos\theta|0\rangle - \sin\theta|1\rangle$$
$$|\psi^\perp\rangle = \sin\theta|0\rangle + \cos\theta|1\rangle$$

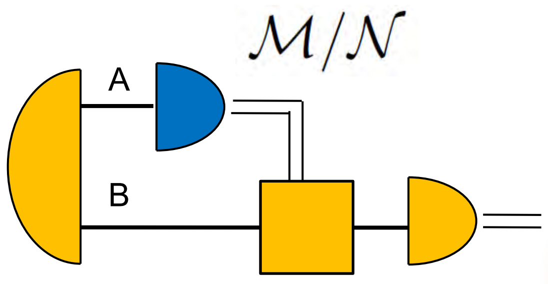
The projectors of the measurement bases $\mathcal{M}$ and $\mathcal{N}$ can be parametrized by single angle θ. Where θ denotes half of the angle between the states $|\psi\rangle$ and $|\phi\rangle$, $0 \leq \theta \leq \frac{\pi}{4}$.

3) The most general discrimination strategy, entanglement-assisted discrimination

A two-qubit entangled state $|\Psi_{AB}\rangle = |\Psi^-\rangle = (|01\rangle - |10\rangle)/\sqrt{2}$ is employed, the measurement that should be discriminated is performed on qubit **A**, and the measurement outcome (0 or 1) specifies which measurement is then performed on the other qubit **B**.

Outcome **0** at **A** heralds **B** in state $|\psi^\perp\rangle$ or $|\phi^\perp\rangle$
Outcome **1** at **A** heralds **B** in state $|\psi\rangle$ or $|\phi\rangle$

When the measurement outcome on **A** reads **0** we apply a suitable unitary operation to qubit **B**, which rotates states $|\psi^\perp\rangle$, $|\phi^\perp\rangle$ such that we end up with the task to discriminate between two fixed non-orthogonal states $|\phi\rangle$ and $|\psi\rangle$.



Perfect error-free discrimination between $|\psi\rangle$ and $|\phi\rangle$ is possible if we allow for a certain probability of inconclusive outcomes. It was shown by Ivanovic, Dieks, and Peres (IDP, 3-component POVM),

$$P_S + P_I = 1, \quad P_I = |\langle\psi|\phi\rangle| = cos(2\theta), \quad P_S = 2\sin^2\theta.$$

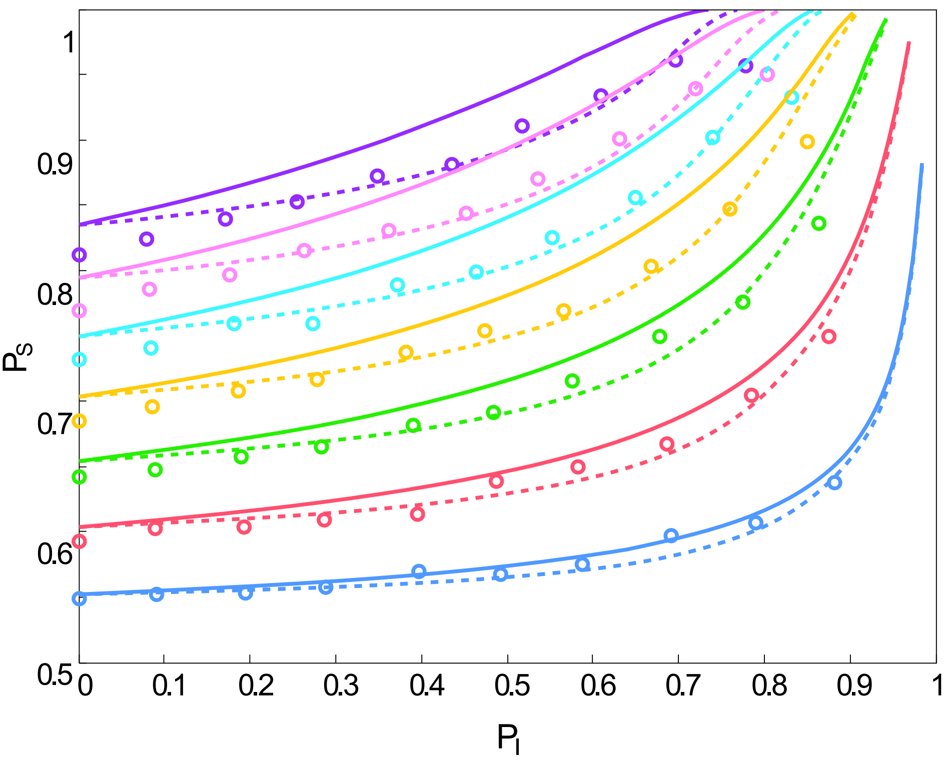
Due to various experimental imperfections, we will in practice encounter also erroneous conclusive results P_E , $P_S + P_E + P_I = 1$. Thus we consider a general discrimination scheme where **we maximize** P_S **(hence minimize** P_E**) for a fixed fraction of** P_I. This intermediate strategy optimally interpolates between IDP and Helstrom approaches.

Relative success probability: $\tilde{P}_S = \frac{P_S}{1-P_I}$, $P_S = \frac{1}{2}\left(1 - P_I + \sin(2\theta)\sqrt{1 - P_I/\cos^2\theta}\right)$

4) Results

Dependence of relative success probability $\tilde{P}_S$ on probability of inconclusive results P_I.

It is plotted for 7 values of $\theta_j = j\pi/30,\ j = 1,2,3,4,5,6,7$, where θ is half on the angle between states $|\psi\rangle$ and $|\phi\rangle$. The value of j increases from bottom to top.



○ experimental data (circles), errorbars are smaller than the symbols
— maximum $\tilde{P}_S$ achievable by the optimal scheme using entangled state (solid lines)
---- single-qubit probes only (dashed lines).

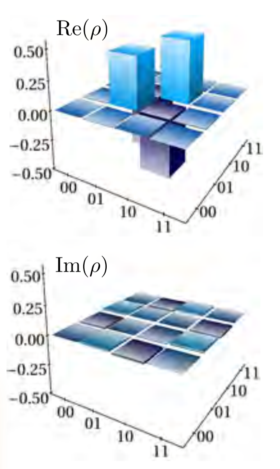
5) Experimental setup, Scheme and Photo

Photon source - our linear optical protocol requires photon pairs entangled in polarization. Photon pairs are generated by SPDC in BBO crystal, type II., degen. 810nm. Photons are entangeled at non-polarizing beam splitter 50:50 (BS).

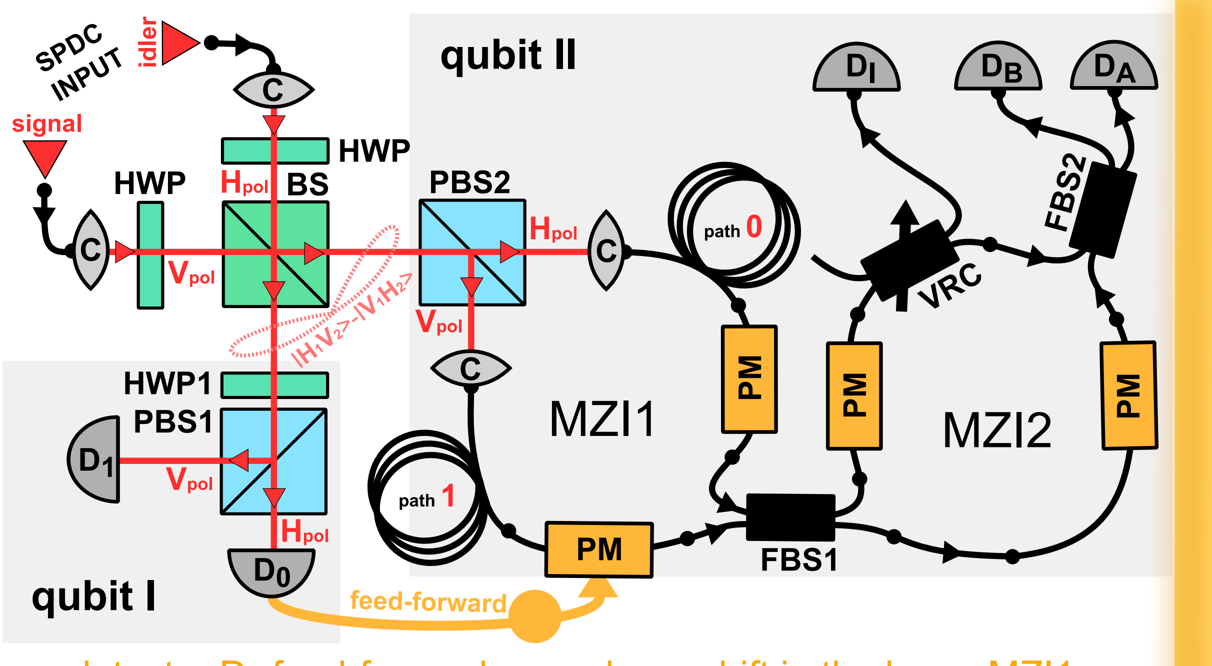
qubit I - One of the two known Von Neumann measurements $\mathcal{M}, \mathcal{N}$ is randomly chosen and applied on one of the photons of the entangled photon pair - qubit **I**. The measurement basis $\mathcal{M}, \mathcal{N}$ was set by rotating a half-wave plate (HWP1) in front of polarizing beamsplitter (PBS1).

qubit II - The state of the qubit **II**, is analyzed. Its polarization state was conditionally transformed in polarization maintaining fibre Mach-Zehneder intereferometr (MZI) by the real-time electro-optical feed-forward loop triggered by outcome **0** of the applied unknown measurement on the qubit **I**. The discrimination problem was transformed to the discrimination of two non-orthogonal states $|\psi\rangle, |\phi\rangle$. It is performed by a MZI with a variable ratio coupler (VRC). A detection of a photon leaving the MZI by the VRC corresponds to inconclusive result. The detection of a photon in any of MZI2 outputs is directly linked to the unknown measurement $\mathcal{M}$ or $\mathcal{N}$.

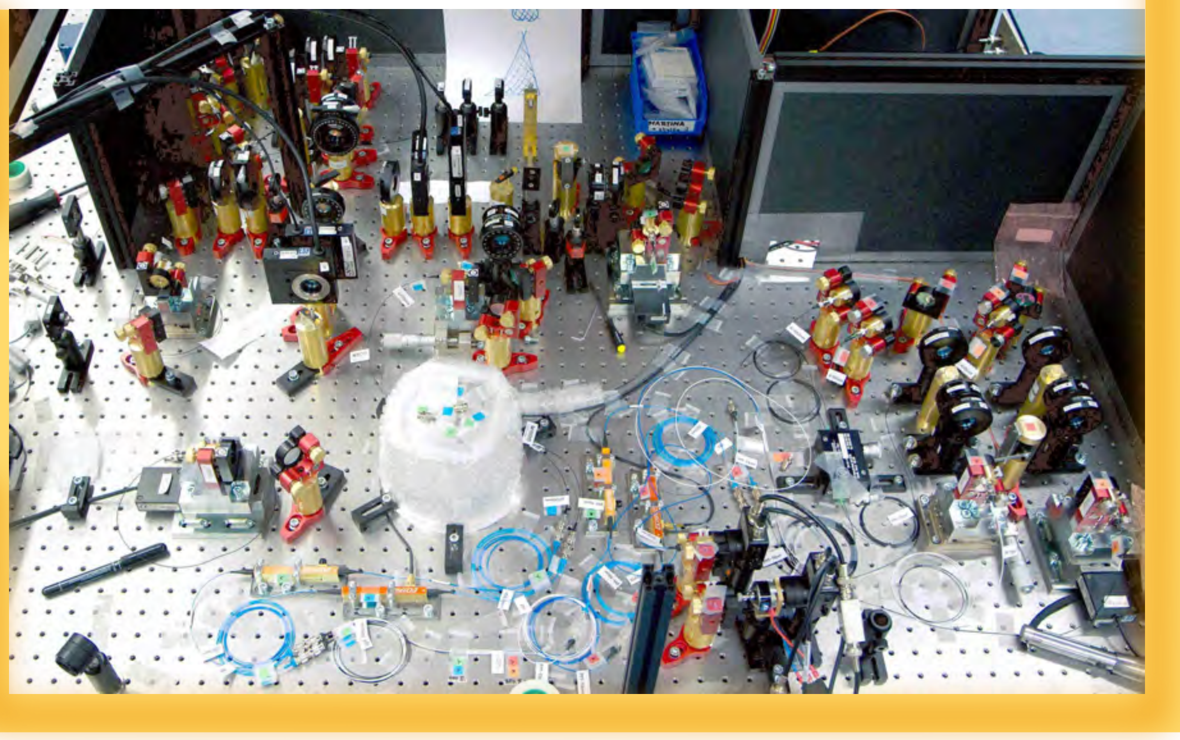
reconstructed density matrix



prepared state:
$|\mathbf{H_1 V_2}\rangle - |\mathbf{V_1 H_2}\rangle$
purity 98%, fidelity 99%,
HOM dip visibility 94%



detector D0 feed-forwards a π-phase shift in the lower MZI1 arm



6) Conclusion

In summary, we have determined theoretically and implemented experimentally optimal strategies for discrimination between two projective single-qubit quantum measurements. The experiment demonstrates that the quantum optical technology is mature enough to harness the benefits of entanglement in quantum device discrimination, although the entanglement-based scheme is much more demanding than the single-qubit probe scheme, as the former requires a real-time feedforward to fully explore the potential of entangled probes.

7) Acknowledgement

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8) References

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