



MEASUREMENT INDUCED QUANTUM NONLINEAR OPERATIONS FOR TRAVELING BEAMS OF LIGHT

Radim Filip, Petr Marek, Kimin Park
Department of Optics, Palacky University

University of Tokyo (Akira Furusawa Lab)

Laboratoire Kastler Brossel, Paris (Julien Laurat Lab)



QUANTUM OPTICS THEORY

Radim Filip

**Quantum Coherence
and Nonclassicality**

**Miroslav Gavenda
Petr Marek**

**Students:
Lukáš Lachman**

**Quantum Nonlinear
Operations**

**Petr Marek
Kimin Park**

**Students:
Petr Zapletal
Vojta Kupčík**

**Quantum
Communication**

**Vladyslav Usenko
Lazslo Ruppert
Mikolaj Lasota**

**Students:
Ivan Derkač**

**Quantum
Optomechanics**

Andrey Rakhubovsky

**Students:
Nikita Vostrosablin**

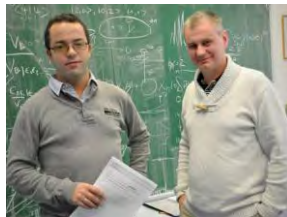
**Interaction of Light
with Atoms**

**Lukáš Slodička
Petr Marek**

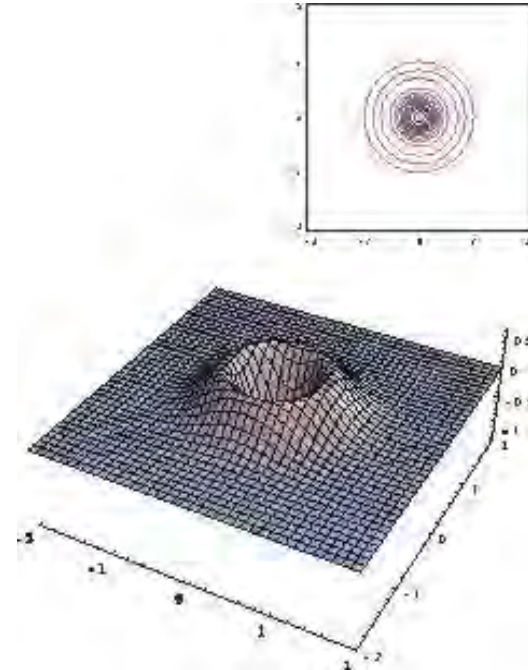
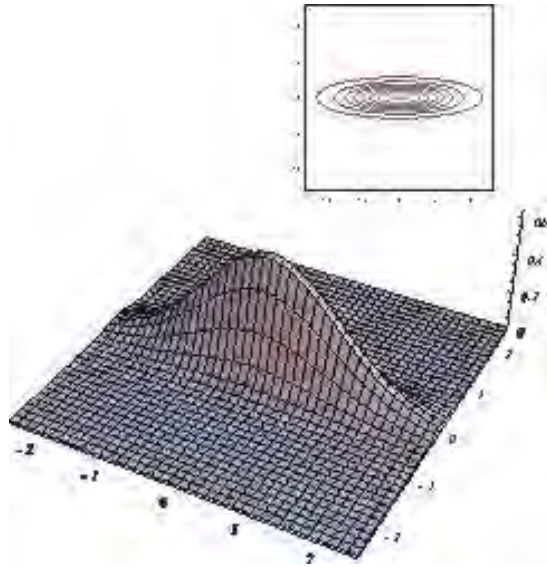
**Students:
Petr Obšil**

**Stochastic Dynamics
and Thermodynamics**

**Miroslav Gavenda
Michal Kolář**



NONCLASSICAL QUANTUM RESOURCES:



<http://qis.ucalgary.ca/quantech/wiggallery.php>

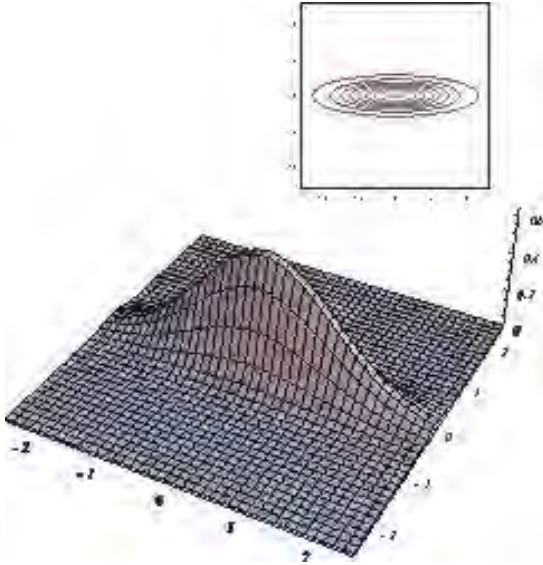
Gaussian squeezed state:

- positive Wigner function
- single quadrature variance below vacuum level

non-Gaussian Fock state:

- negative Wigner function
- all quadrature variances above vacuum level

NONCLASSICAL QUANTUM RESOURCES:

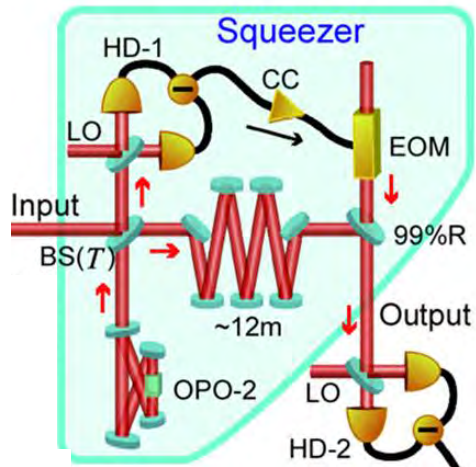


<http://qis.ucalgary.ca/quantech/wiggallery.php>

Gaussian squeezed state:

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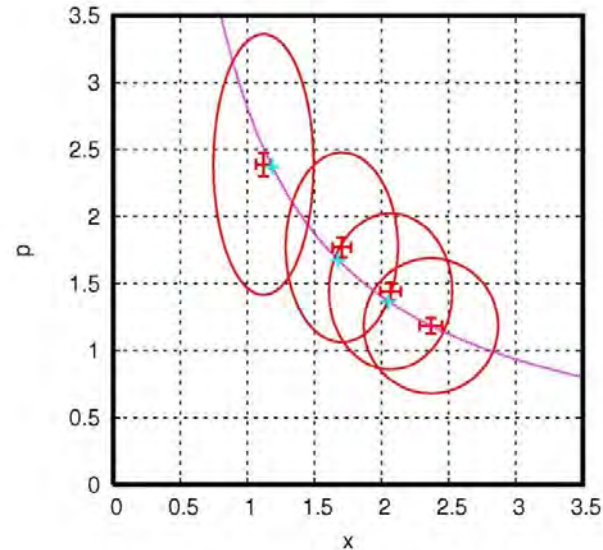
NOISE SQUEEZING = RESOURCE FOR SQUEEZER



$$\hat{x}_{\text{out}} = \frac{1}{\sqrt{T}} \hat{x}_{\text{in}},$$

$$\hat{p}_{\text{out}} = \sqrt{T} \hat{p}_{\text{in}} + \sqrt{1-T} \hat{p}_{\text{vac}} e^{-r}$$

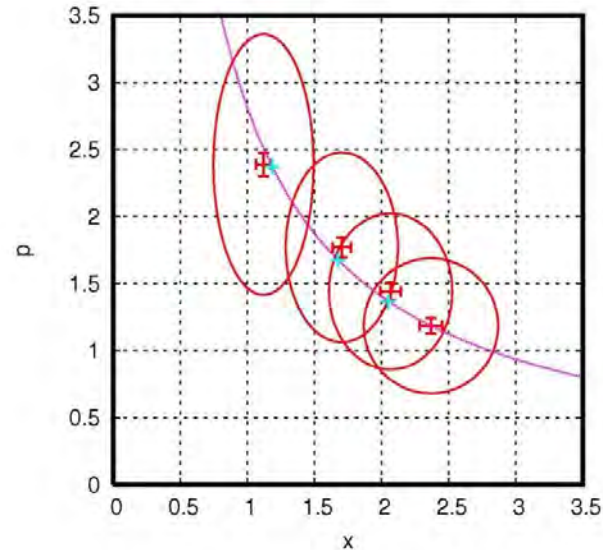
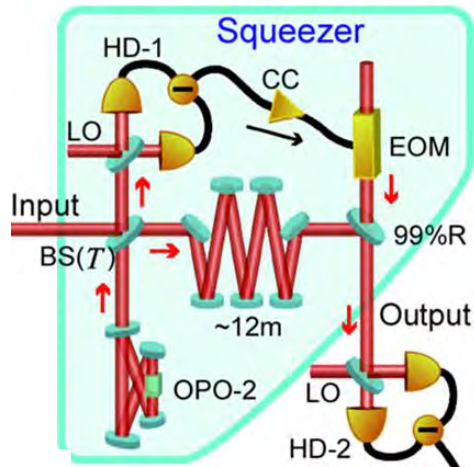
R. Filip, P. Marek and U.L. Andersen, Phys. Rev. A 71, 042308 (2005).



J. Yoshikawa et al., Phys. Rev. A 76, 060301(R) (2007)

- off-line squeezed state supplies on-line squeezer
- squeezer, amplifier, QND interaction

NOISE SQUEEZING = RESOURCE FOR SQUEEZER



J. Yoshikawa et al., Phys. Rev. A 76, 060301(R) (2007)

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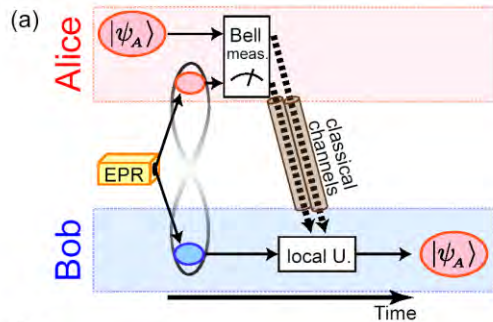
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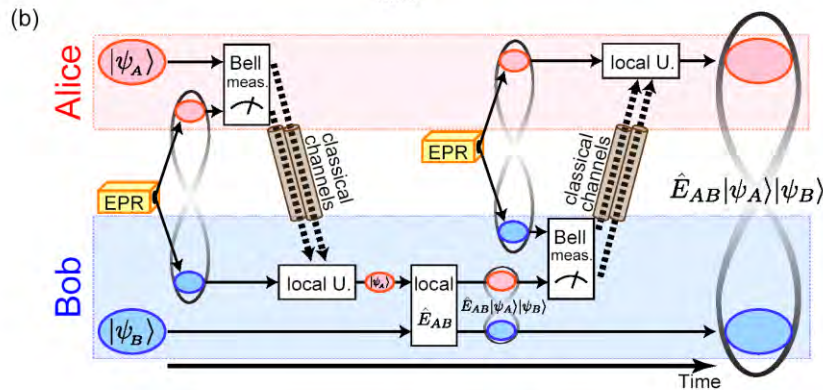


NONLOCAL QND OPERATION

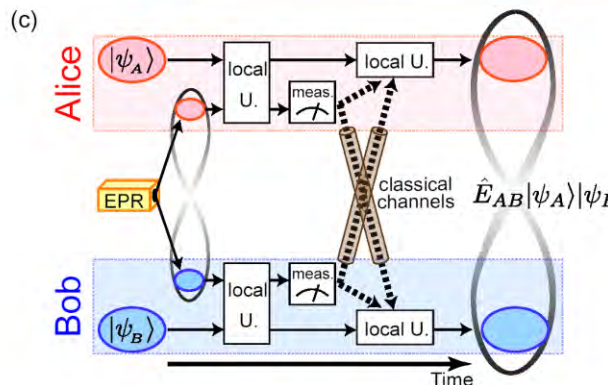
teleportation



double teleportation



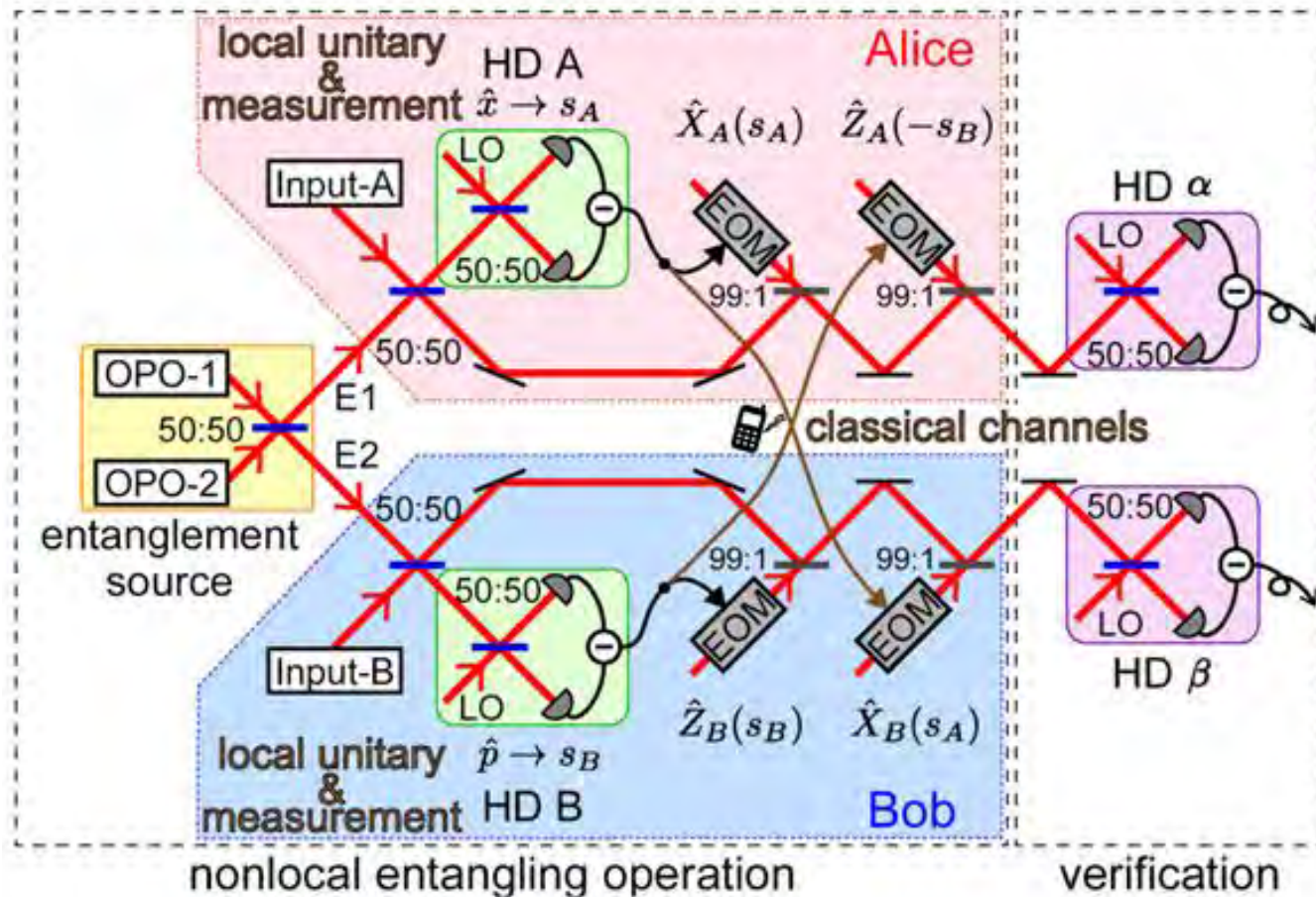
nonlocal parallel operation



$$\hat{\Sigma}_{AB} = e^{-2i\hat{x}_A\hat{p}_B}$$

$$\hat{\Sigma}_{AB}|x_A\rangle_A \otimes |x_B\rangle_B = |x_A\rangle_A \otimes |x_B + x_A\rangle_B$$

NONLOCAL QND OPERATION



Shota Yokoyama, Ryuji Ukai, Jun-ichi Yoshikawa, Petr Marek, Radim Filip, and Akira Furusawa, Phys. Rev. A 90, 012311 (2014).

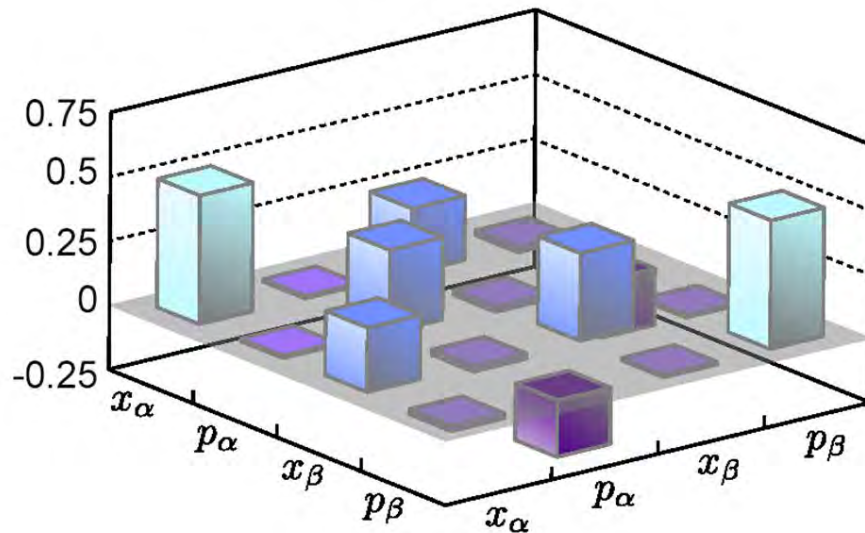
NONLOCAL QND OPERATION

$$\hat{\xi}_{\alpha\beta} = \begin{pmatrix} \sqrt{2} & 0 & 0 & 0 \\ 0 & \frac{1}{\sqrt{2}} & 0 & -\frac{1}{\sqrt{2}} \\ 1 & \sqrt{2} & 0 & 0 \\ \frac{1}{\sqrt{2}} & 0 & \frac{1}{\sqrt{2}} & 0 \\ 0 & 0 & 0 & \sqrt{2} \end{pmatrix} \hat{\xi}_{AB} + \hat{\delta}$$

$$\equiv \hat{E}_{AB}^\dagger \hat{\xi}_{AB} \hat{E}_{AB} + \hat{\delta},$$

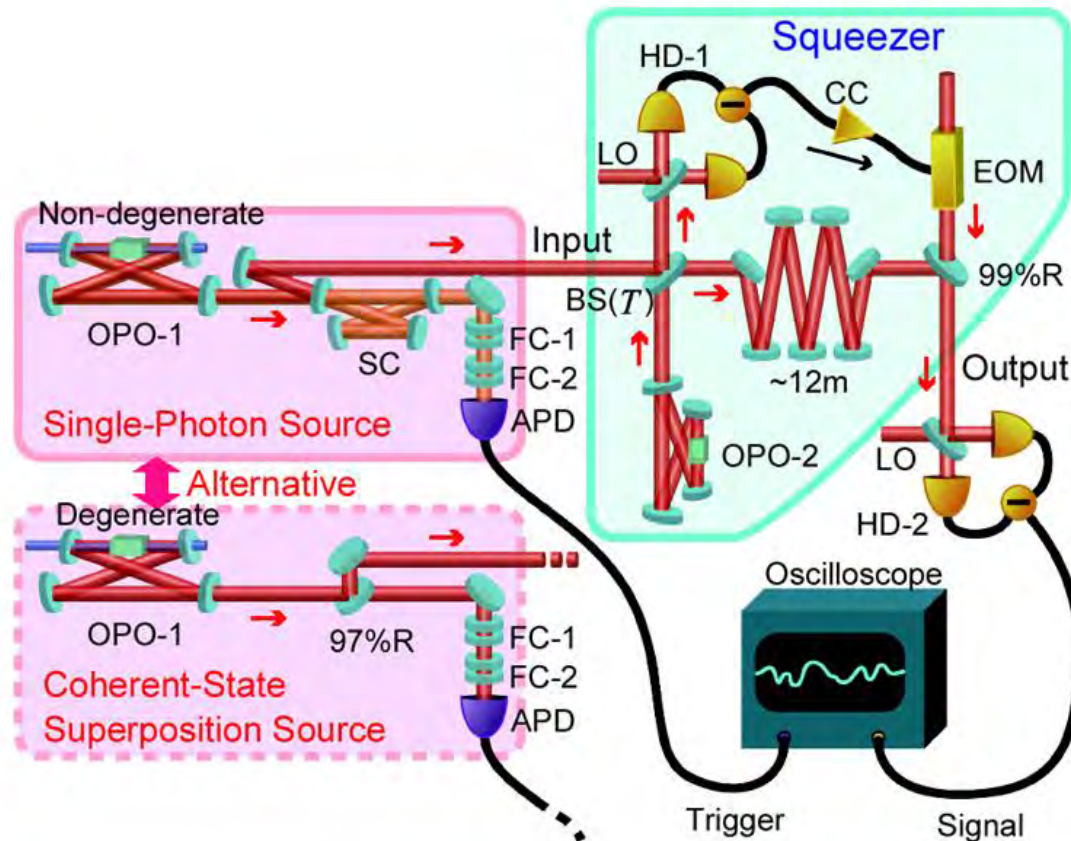
$$\hat{\xi}_{AB} = (\hat{x}_A, \hat{p}_A, \hat{x}_B, \hat{p}_B)^T, \quad \hat{\xi}_{\alpha\beta} = (\hat{x}_\alpha, \hat{p}_\alpha, \hat{x}_\beta, \hat{p}_\beta)^T$$

$$\hat{\delta} = (0, e^{-r} \hat{p}_2^{(0)}, e^{-r} \hat{x}_1^{(0)}, 0)^T$$



QND entanglement

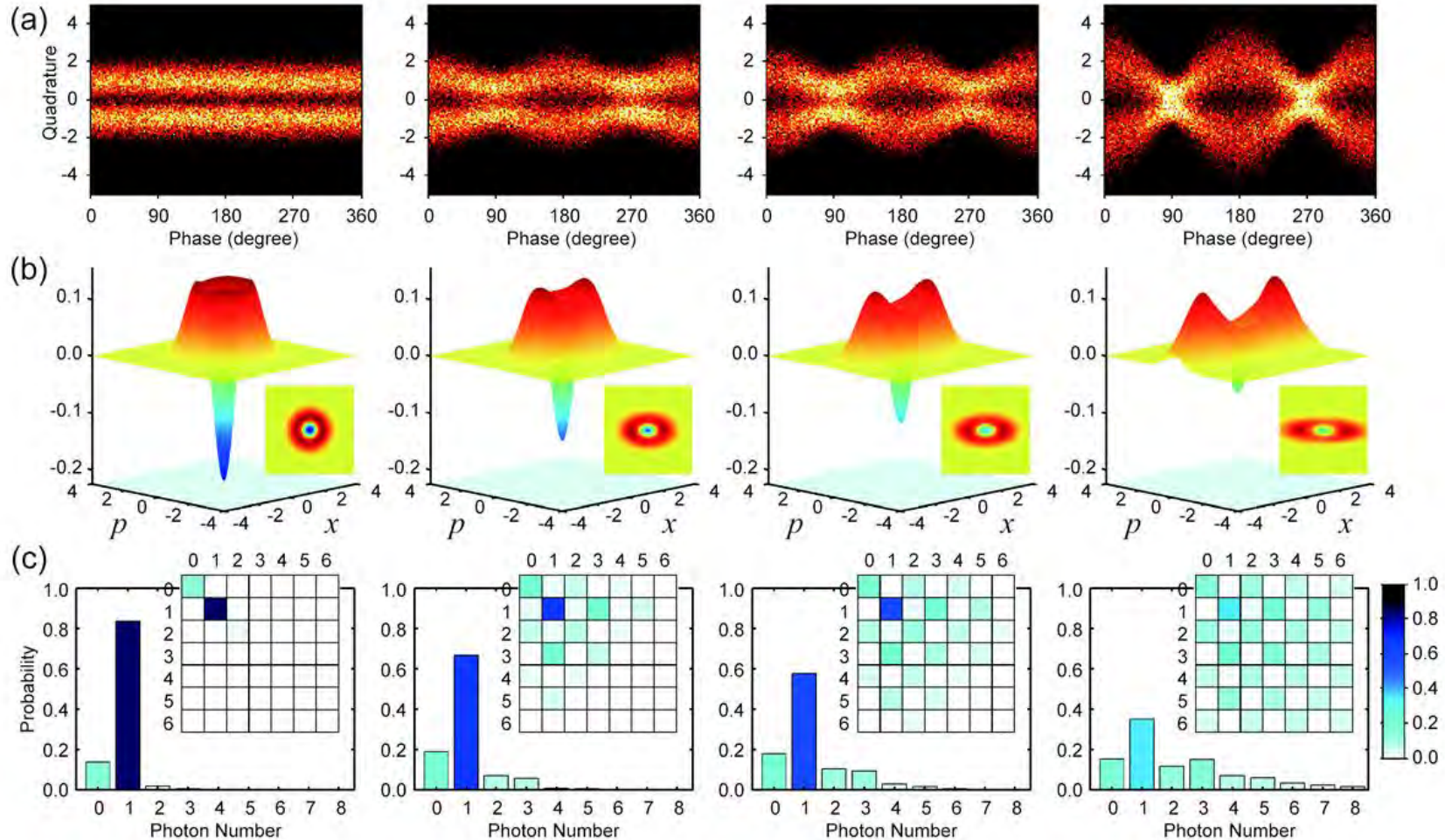
SQUEEZING OF SINGLE PHOTON



$$\hat{S}(\gamma) = e^{\gamma(\hat{a}^{\dagger 2} - \hat{a}^2)/2}$$

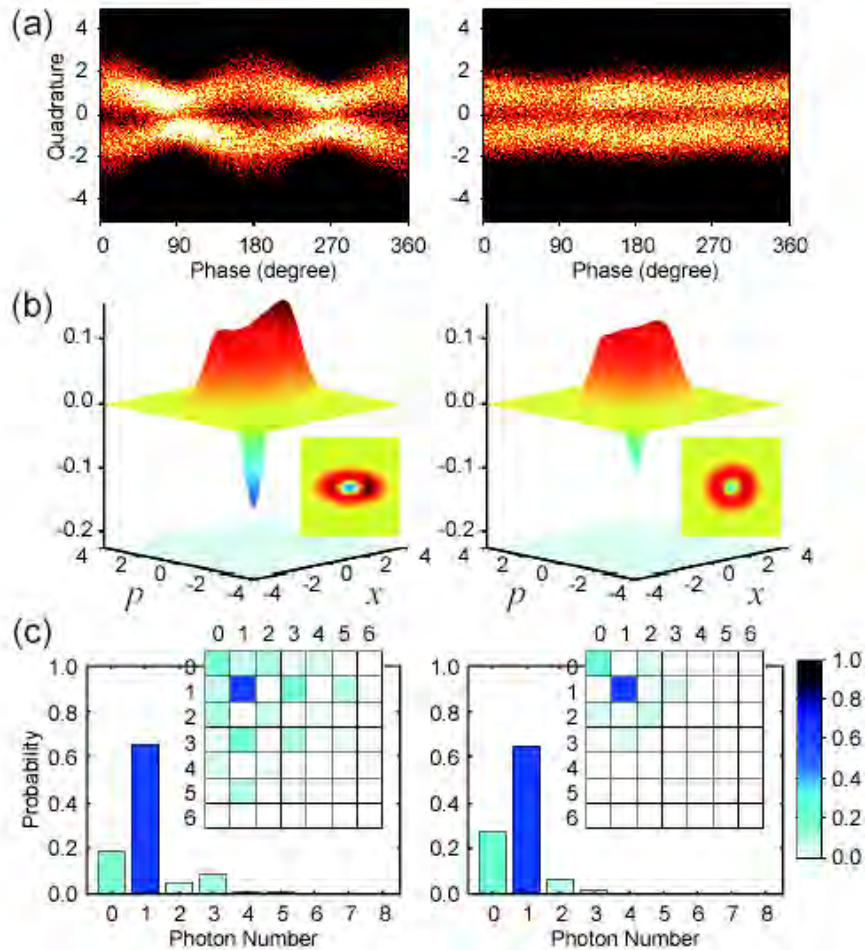
Yoshichika Miwa, Jun-ichi Yoshikawa, Noriaki Iwata, Mamoru Endo, Petr Marek, Radim Filip, Peter van Loock, and Akira Furusawa, Phys. Rev. Lett. 113, 013601 (2014).

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UNSCQUEEZING OF SQUEEZED PHOTON



Outcomes:

- reversible squeezer
- preserves negative Wigner function

Yoshichika Miwa, Jun-ichi Yoshikawa, Noriaki Iwata, Mamoru Endo, Petr Marek, Radim Filip, Peter van Loock, and Akira Furusawa, Phys. Rev. Lett. 113, 013601 (2014).

OPTIMAL STATE PREPARATION

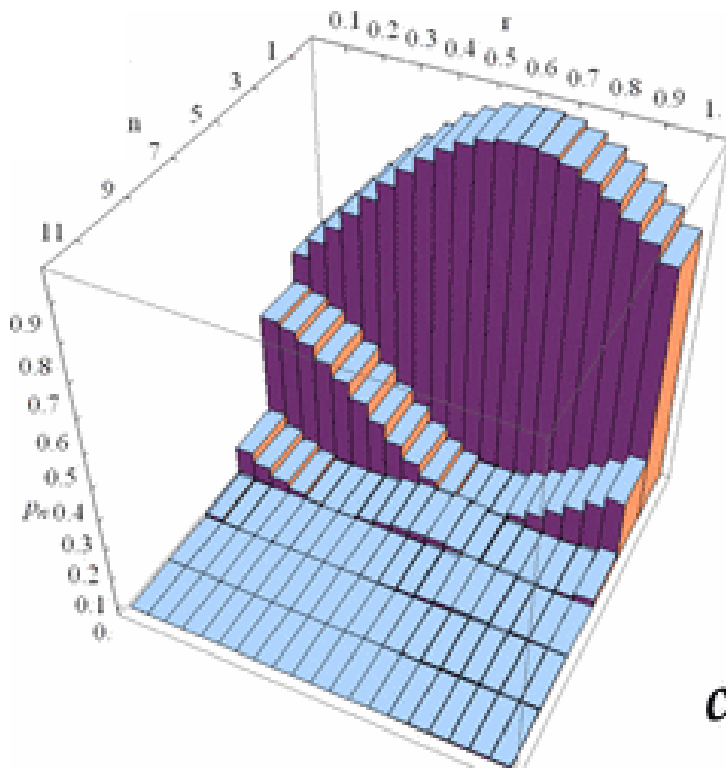
$$(|\alpha\rangle - |-\alpha\rangle) / \sqrt{2(1 - \exp(-2|\alpha|^2))}$$

OPTIMAL STATE PREPARATION

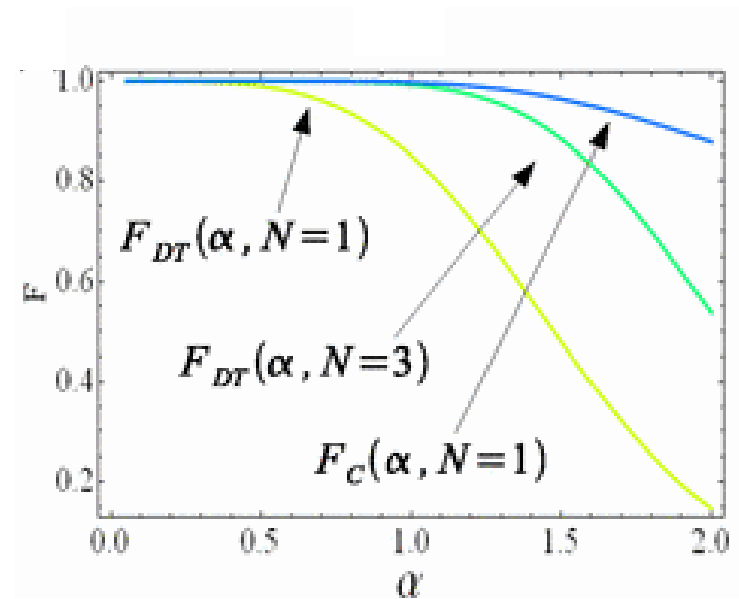
$$S(r) (|\alpha\rangle - |-\alpha\rangle) / \sqrt{2(1 - \exp(-2|\alpha|^2))}$$

OPTIMAL STATE PREPARATION

$$S(r) (|\alpha\rangle - |-\alpha\rangle) / \sqrt{2(1 - \exp(-2|\alpha|^2))}$$



$\alpha = 1.5$



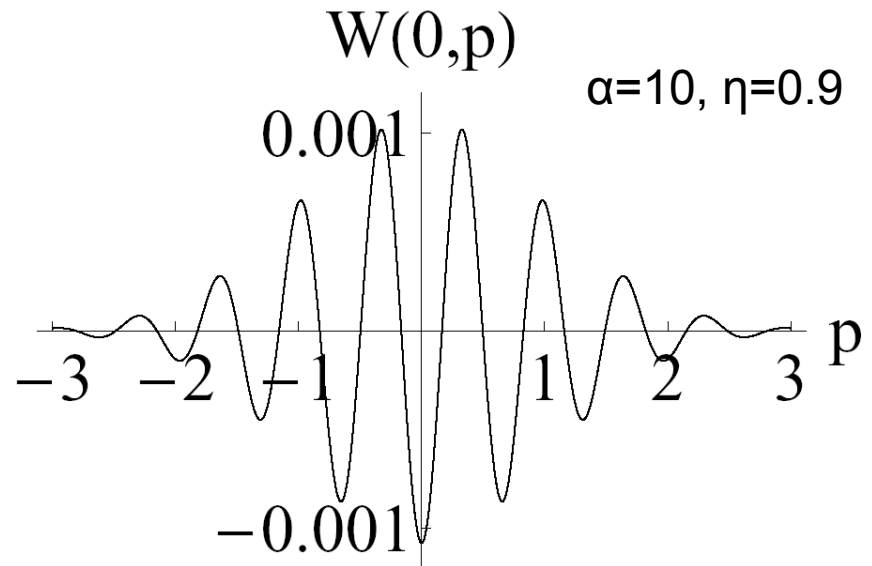
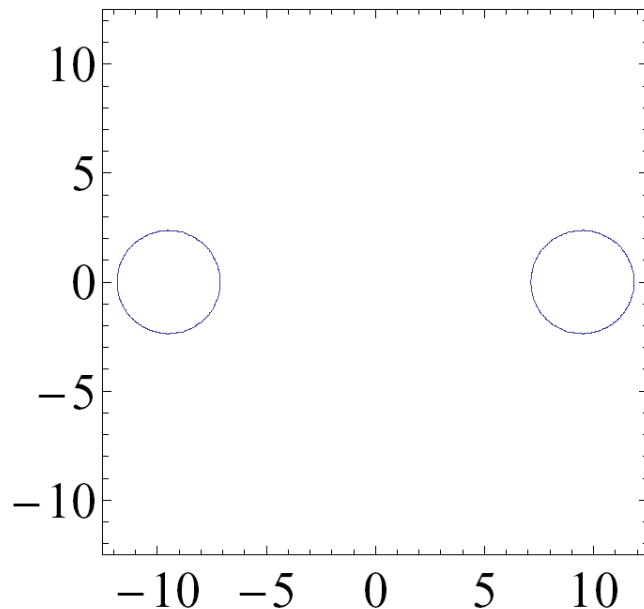
D. Menzies and R. Filip, Phys. Rev. A 79, 012313 (2009).

K. Huang, H. Le Jeannic, J. Ruaudel, V.B. Verma, M.D. Shaw, F. Marsili, S.W. Nam, E Wu, H. Zeng, Y.-C. Jeong, R. Filip, O. Morin, J. Laurat, arXiv:1503.08970, accepted in Phys. Rev. Lett.

QUANTUM DECOHERENCE

$$(|\alpha\rangle - |-\alpha\rangle) / \sqrt{2(1 - \exp(-2|\alpha|^2))}$$

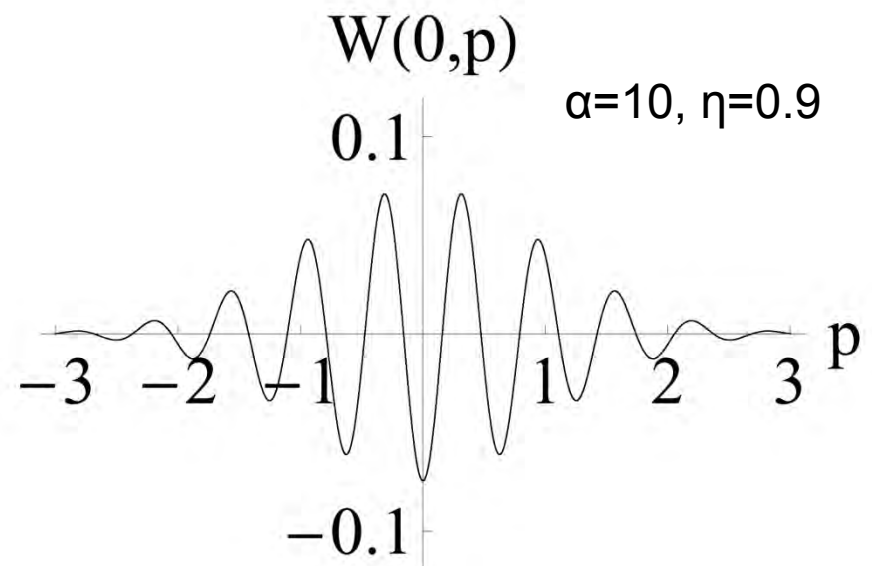
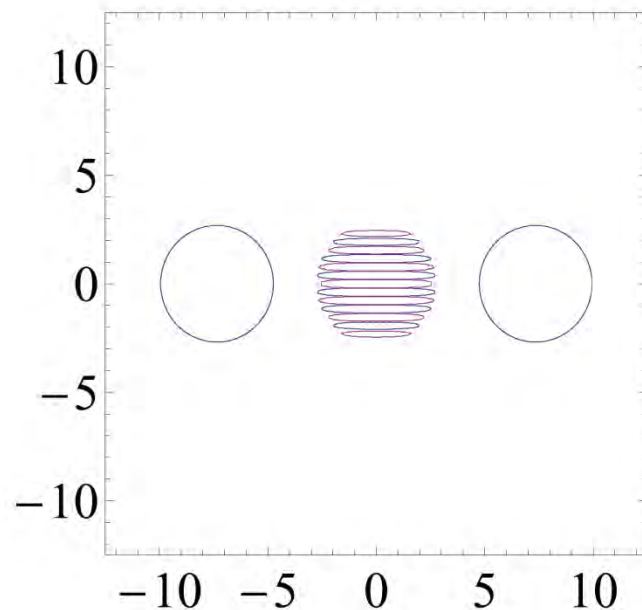
Losses with $\eta > 0.5$ does not vanish the oscillations, but they are hardly visible.



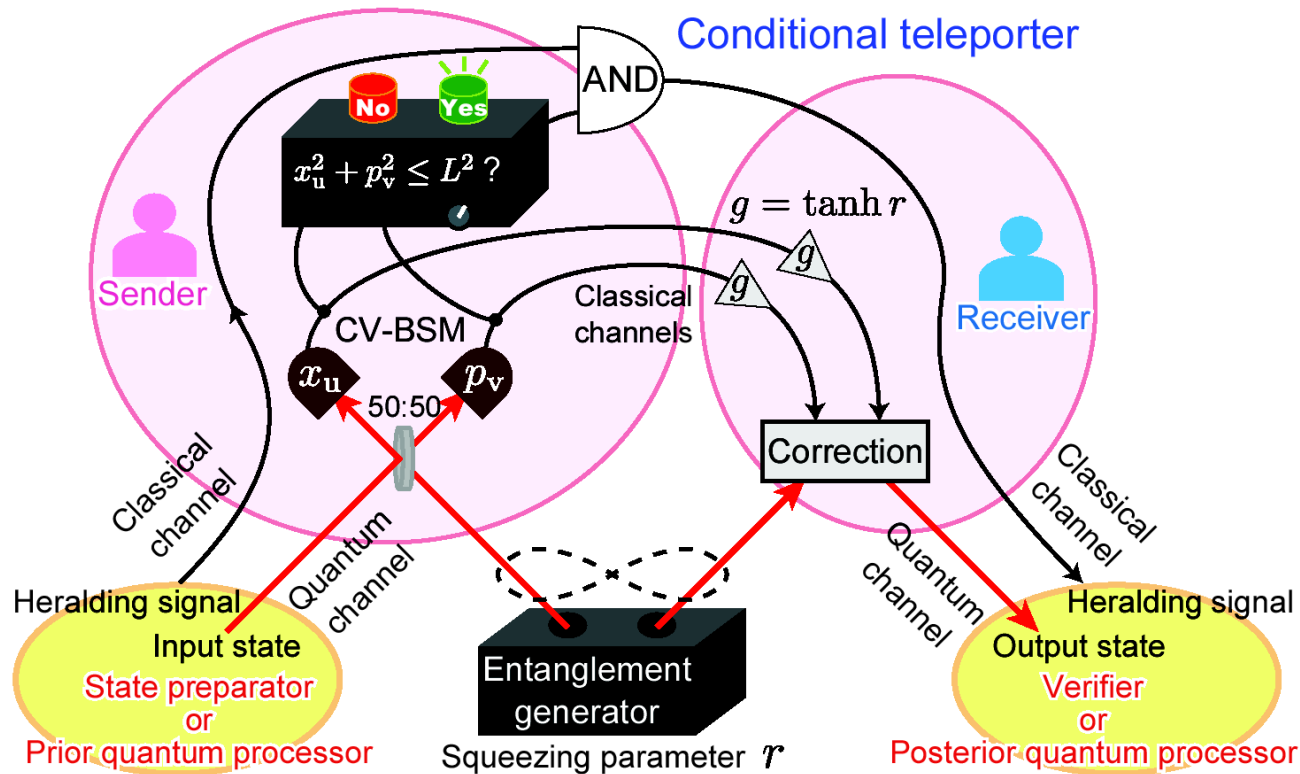
CONTROL OF DECOHERENCE

$$(|\alpha\rangle - |-\alpha\rangle) / \sqrt{2(1 - \exp(-2|\alpha|^2))}$$

For $\eta > 0.5$, visibility of the oscillations is significantly improved by **pre/post squeezing**.



NOISELESS TELEPORTATION OF SINGLE PHOTON



T. Ide, H. F. Hofmann, T. Kobayashi, and A. Furusawa, Phys. Rev. A 65, 012313 (2002).

Ladislav Mišta, Jr., Radim Filip, and Akira Furusawa, Phys. Rev. A 82, 012322 (2010)

LOSSY TELEPORTATION OF SINGLE PHOTON

$$|\psi\rangle \rightarrow \sum_{k=0}^{\infty} \frac{1}{k! \sinh^{2k} r} \hat{a}^k [(\tanh r)^{\hat{n}} |\psi\rangle \langle\psi| (\tanh r)^{\hat{n}}] \hat{a}^{\dagger k}$$

NOISELESS TELEPORTATION OF SINGLE PHOTON

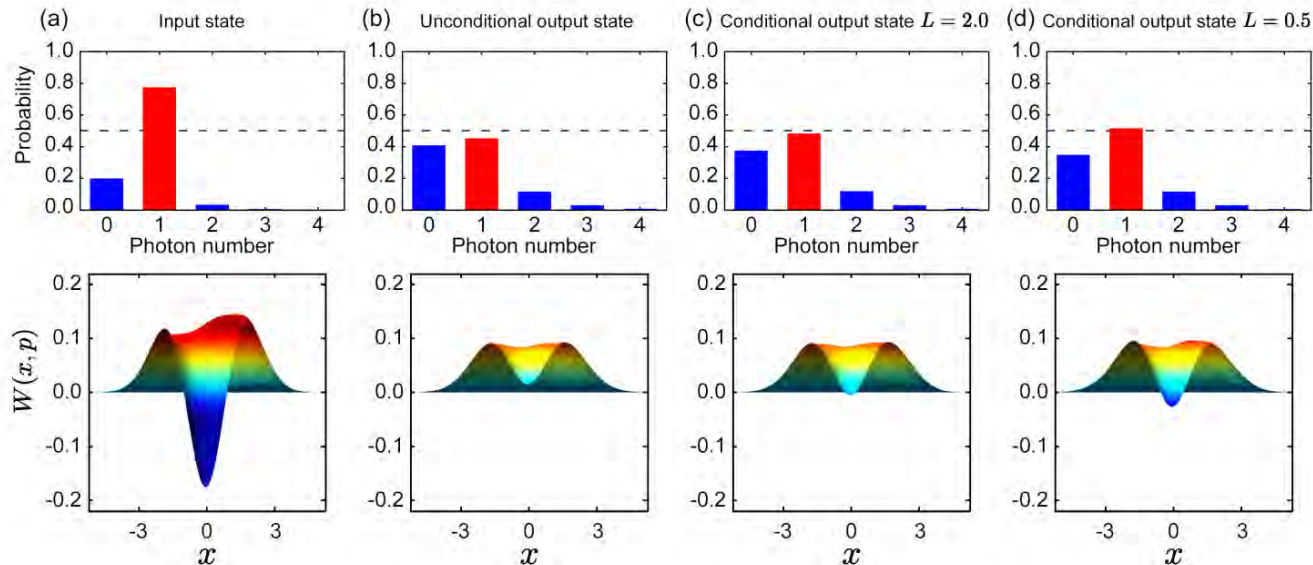
$$|\psi\rangle \rightarrow \sum_{k=0}^{\infty} \frac{1}{k! \cosh^k r} [(\tanh r)^{\hat{n}} |\psi\rangle \langle\psi| (\tanh r)^{\hat{n}}] c^k$$

$$|\psi\rangle \rightarrow (\tanh r)^{\hat{n}} |\psi\rangle$$

NOISELESS TELEPORTATION OF SINGLE PHOTON

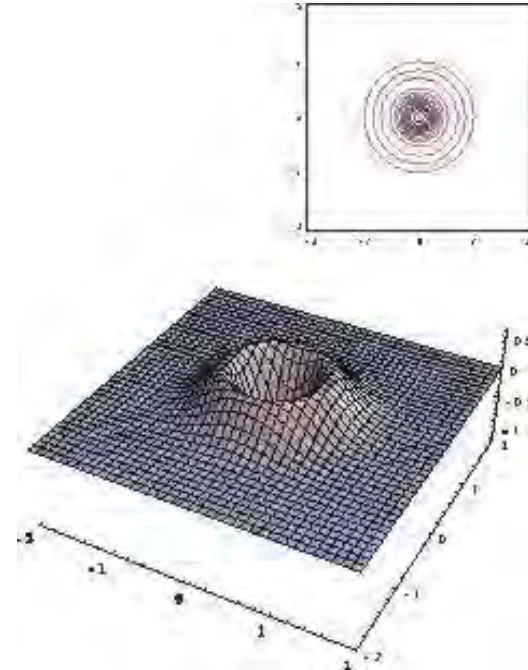
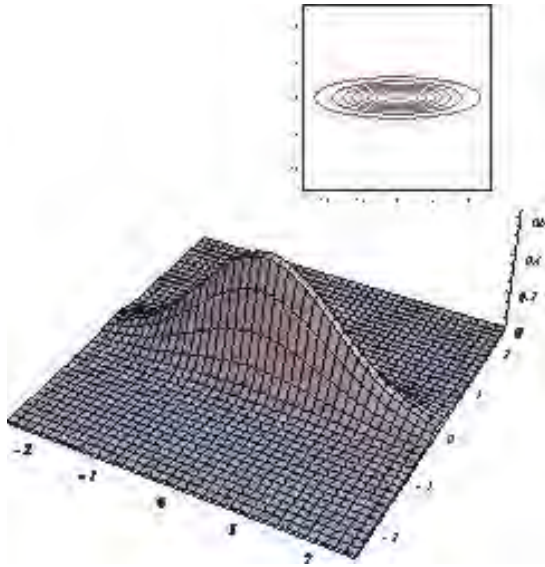
$$|\psi\rangle \rightarrow \sum_{k=0}^{\infty} \frac{1}{k! \sqrt{\cosh r}} \left[(\tanh r)^{\hat{n}} |\psi\rangle \langle\psi| (\tanh r)^{\hat{n}} \right]$$

$$|\psi\rangle \rightarrow (\tanh r)^{\hat{n}} |\psi\rangle$$



Maria Fuwa, Shunsuke Toba, Shuntaro Takeda, Petr Marek, Ladislav Mista Jr., Radim Filip, Peter van Loock, Jun-ichi Yoshikawa, Akira Furusawa, Phys. Rev. Lett. 113, 223602 (2014).

NONCLASSICAL QUANTUM RESOURCES:



<http://qis.ucalgary.ca/quantech/wiggallery.php>

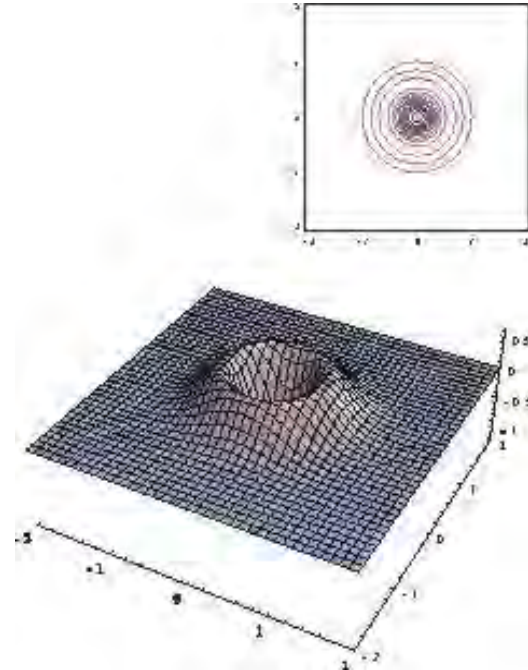
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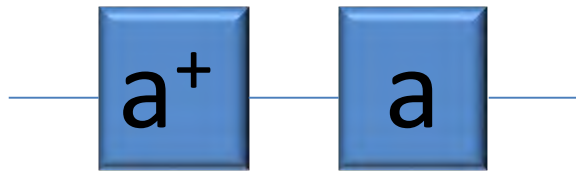
NONCLASSICAL QUANTUM RESOURCES:



non-Gaussian Fock state:

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- all quadrature variance above vacuum level

NOISELESS AMPLIFICATION



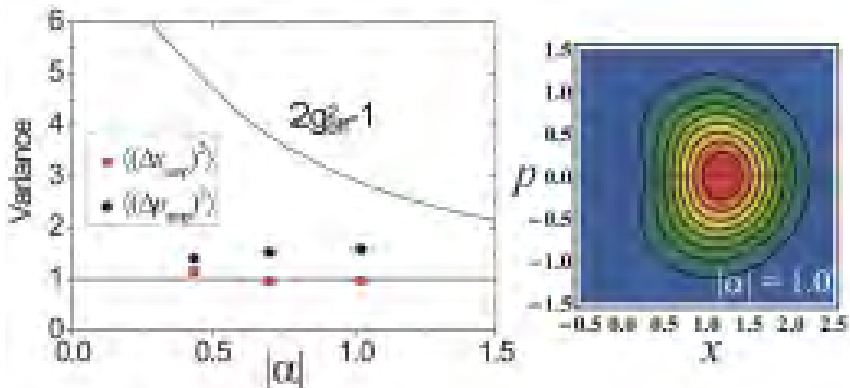
$$|\alpha\rangle = |0\rangle + \alpha |1\rangle + \dots$$

$$a^\dagger |\alpha\rangle = |1\rangle + 2^{1/2} \alpha |2\rangle + \dots$$

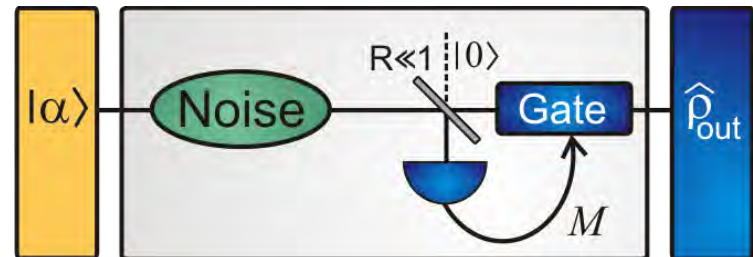
$$aa^\dagger |\alpha\rangle = |0\rangle + 2\alpha |1\rangle + \dots$$

P. Marek and R. Filip, Phys. Rev. A 81, 022302 (2010).

A. Zavatta, J. Fiurášek, M. Bellini, Nature Phot. 5, 52 (2011)



M.A. Usuga, Ch. R. Müller, Ch. Wittmann, P. Marek, R. Filip, Ch. Marquardt, G. Leuchs, U.L. Andersen, Nature Phys. 6, 767–771 (2010)



NONLINEAR POTENTIAL



$$\hat{H} = \hbar\omega \left(\hat{a}^\dagger \hat{a} + \frac{1}{2} \right) + V(\hat{X})$$

$$U(\hat{X}, \tau) = e^{-\frac{i}{\hbar} V(\hat{X}) \tau}$$

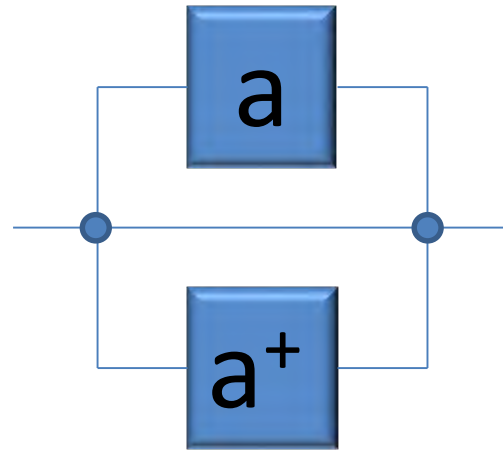
ANY NONLINEARITY BY X-GATES

$$U(\hat{X}, \tau) = \sum_{k=0}^{\infty} \frac{U^{(k)}(\bar{X})}{k!} (\hat{X} - \bar{X})^k$$

$$U(\hat{X}, \tau) = \prod_{k=0}^N \boxed{(1 + \lambda_k \hat{X})}$$

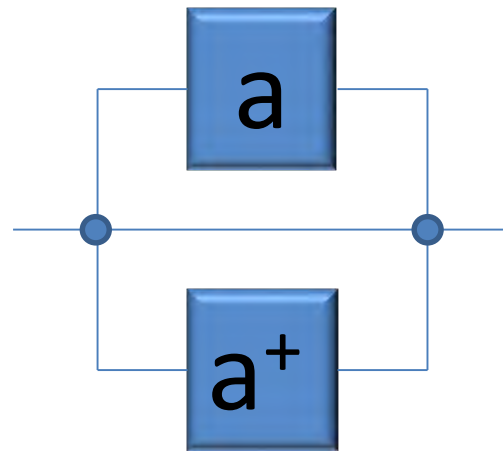
X-gate

ANY NONLINEARITY BY X-GATES



X-gate

ANY NONLINEARITY BY X-GATES

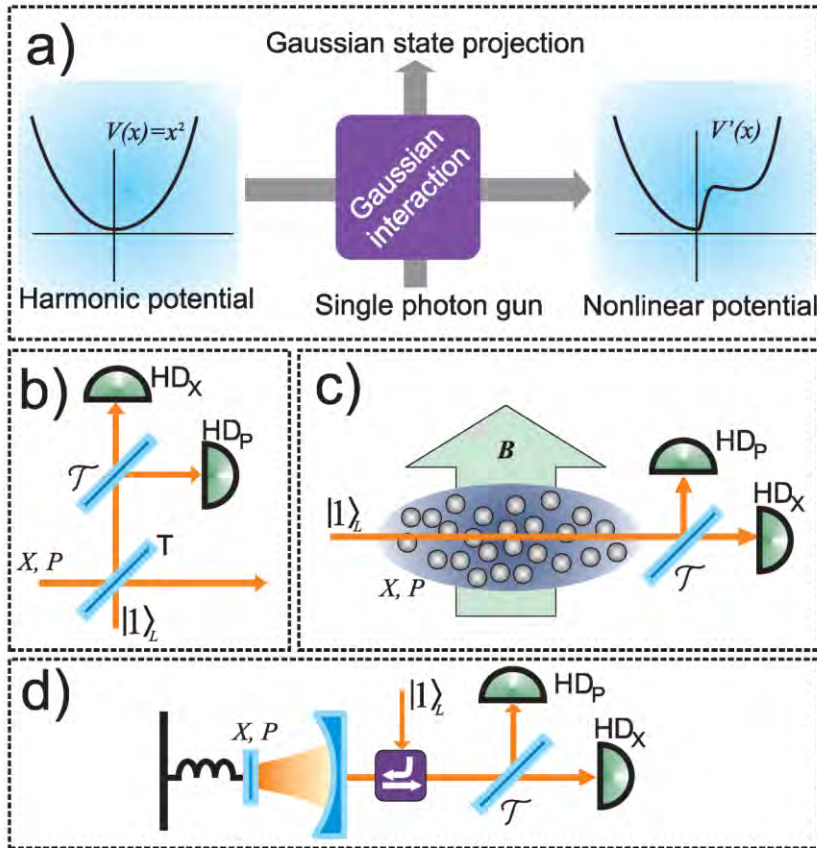


X-gate

Cubic nonlinearity as a sequence of X gates:

$$\begin{aligned} \exp[i\chi\hat{X}^3] &\approx 1 + i\chi\hat{X}^3 - \frac{\chi^2}{2}\hat{X}^6 \propto \\ &(1 - (\frac{\chi}{-1+i})^{1/3}\hat{X})(1 + (\frac{\chi}{1-i})^{1/3}\hat{X}) \\ &(1 - (-1)^{-2/3}(\frac{\chi}{-1+i})^{1/3}\hat{X})(1 - (\frac{\chi}{1+i})^{1/3}\hat{X}) \\ &(1 + (\frac{\chi}{-1-i})^{1/3}\hat{X})(1 - (-1)^{-2/3}(\frac{\chi}{1+i})^{1/3}\hat{X}) \end{aligned}$$

CONDITIONAL X-GATES



a) conditional emulation of quantum oscillator (light mode) in any nonlinear potential based on multiple sequential applications of elementary X gate. X gates exploit single photon gun to achieve highly non-classical affects from the nonlinear potential.

b) linear optical implementation

$$\hat{A}_{BS} = (T'T)^{\hat{n}} (A^* + 2B^* R^* \hat{a} + R \hat{a}^\dagger)$$

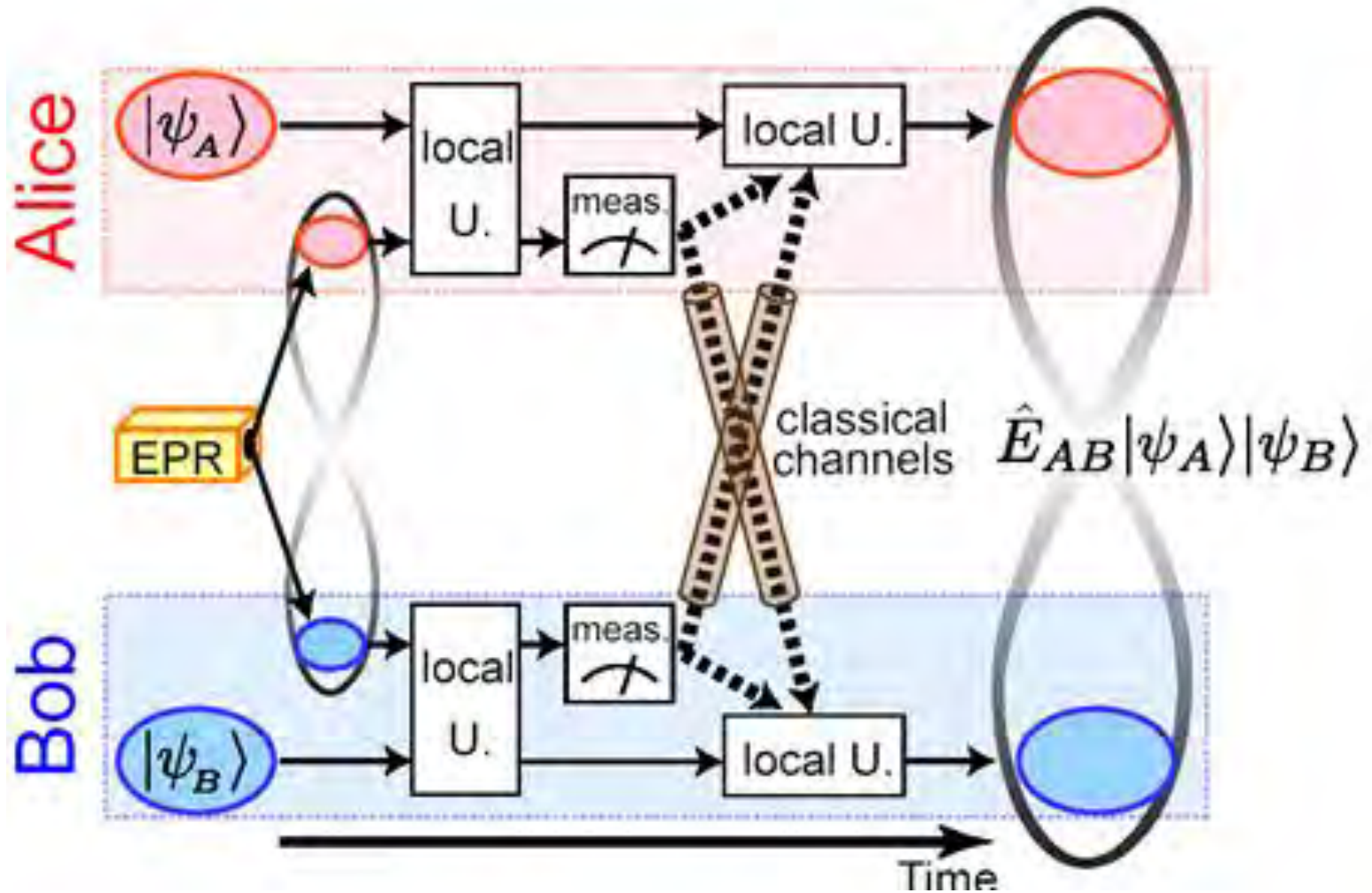
$$A = \sqrt{2}(x\mathcal{T} - ip\mathcal{R}) \quad B = 2^{-1}(\mathcal{R}^2 - \mathcal{T}^2)$$

c) implementation with atomic ensemble

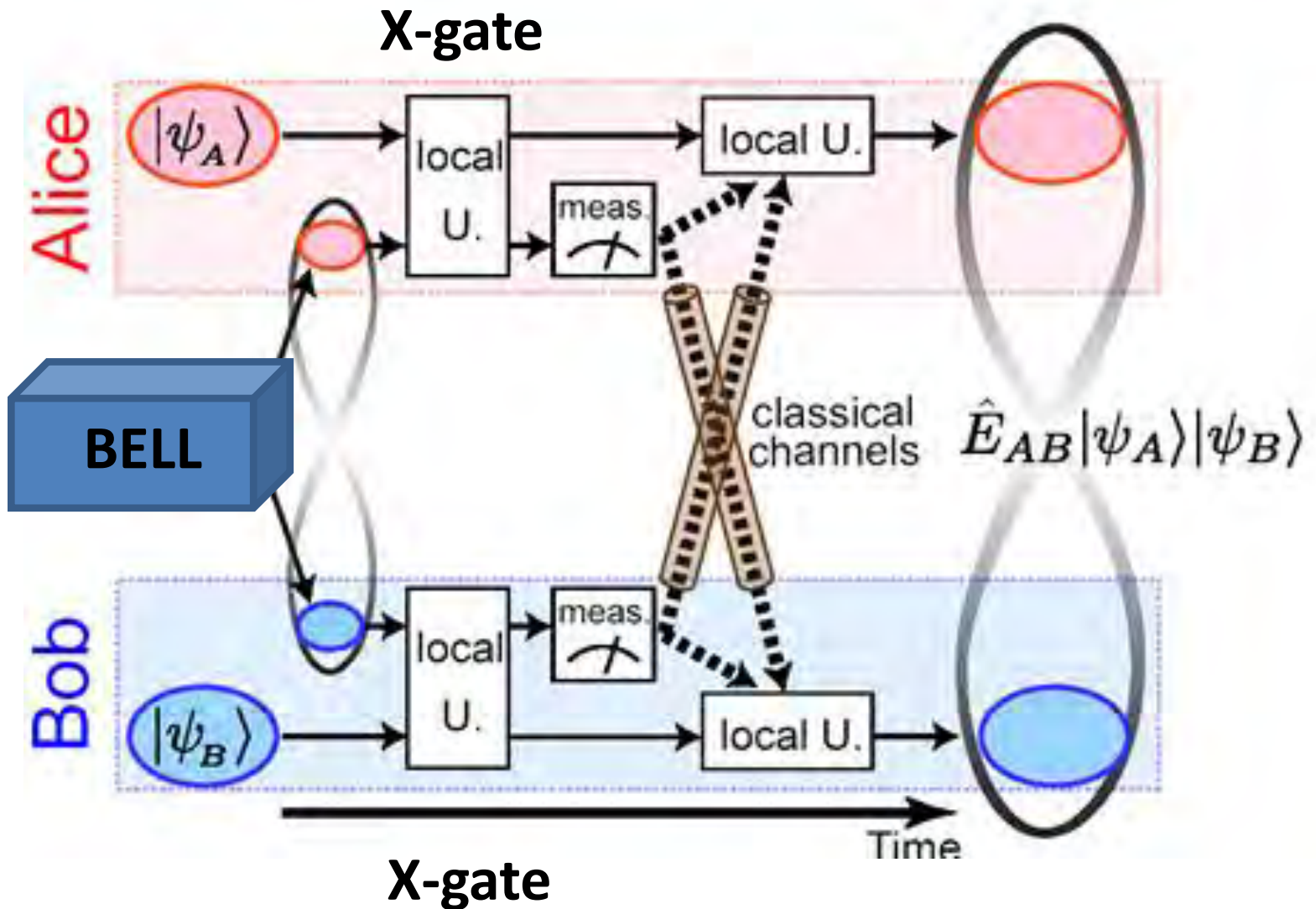
d) opto-mechanical implementation

$$\begin{aligned} {}_L \langle x_0 = 0 | U_{\text{QND}} f(\hat{X}_L) | 0 \rangle_L &= {}_L \langle x_0 = 0 | f(-\kappa \hat{X}) U_{\text{QND}} | 0 \rangle \\ &= f(-\kappa \hat{X})_L \langle x_0 = 0 | U_{\text{QND}} | 0 \rangle = F(\hat{X}) \exp[-\frac{1}{2} \kappa^2 \hat{X}^2]. \end{aligned}$$

NONLOCAL ENTANGLING X-GATES



NONLOCAL ENTANGLING X-GATES



NONLOCAL ENTANGLING X-GATES

$$\hat{O}_p[\alpha] = e^{i\sqrt{2}\alpha\hat{X}_1} - e^{i\sqrt{2}\alpha\hat{X}_2}$$

$$\hat{O}_p[\alpha] \approx e^{i\frac{\sqrt{2}\alpha}{2}(\hat{X}_1+\hat{X}_2)}(\hat{X}_1 - \hat{X}_2) \equiv \hat{O}_p^{(1)}[\alpha]$$

$$\hat{O}_p^{(1)}[\alpha]|\beta\rangle_1|\beta'\rangle_2 = e^{i\frac{\alpha}{\sqrt{2}}(\hat{X}_1+\hat{X}_2)}D_1[\beta]D_2[\beta']$$

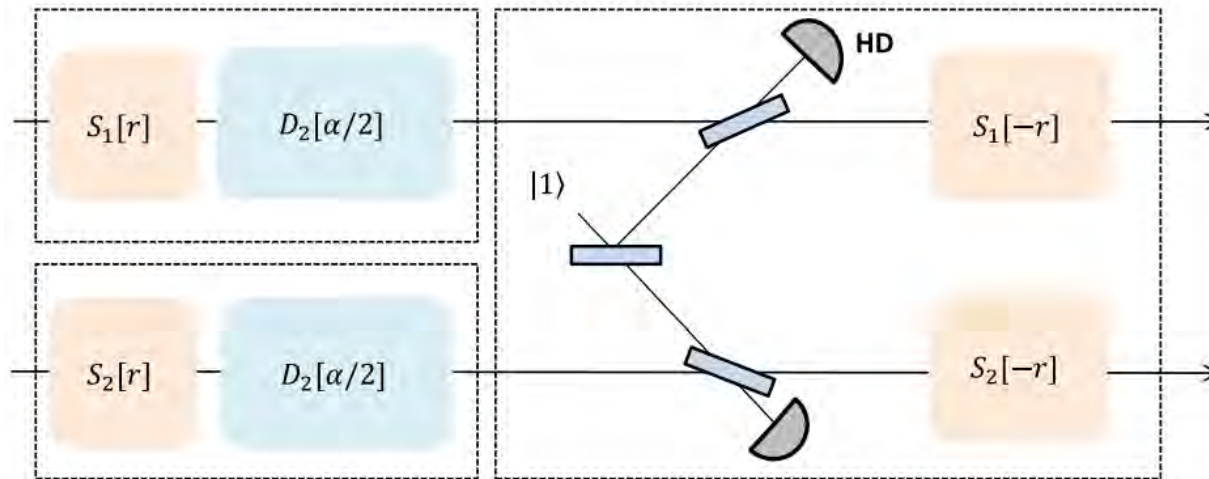
$$\times (\sqrt{2}(\beta - \beta') + \hat{X}_1 - \hat{X}_2)|00\rangle_{12}$$

$$\sqrt{2}(\beta - \beta')|00\rangle_{12} + \frac{1}{\sqrt{2}}(|10\rangle_{12} - |01\rangle_{12})$$

NONLOCAL ENTANGLING X-GATES

$$\hat{O}_p[\alpha] \approx e^{i\frac{\sqrt{2}\alpha}{2}(\hat{X}_1 + \hat{X}_2)}(\hat{X}_1 - \hat{X}_2) \equiv \hat{O}_p^{(1)}[\alpha]$$

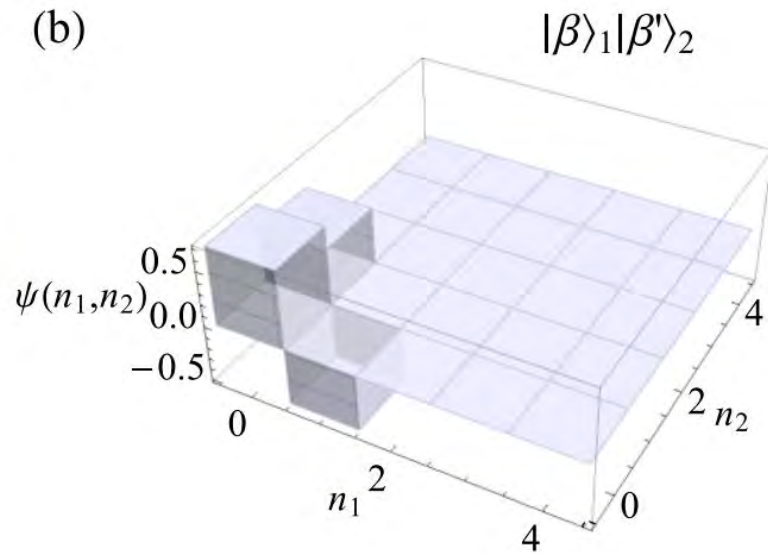
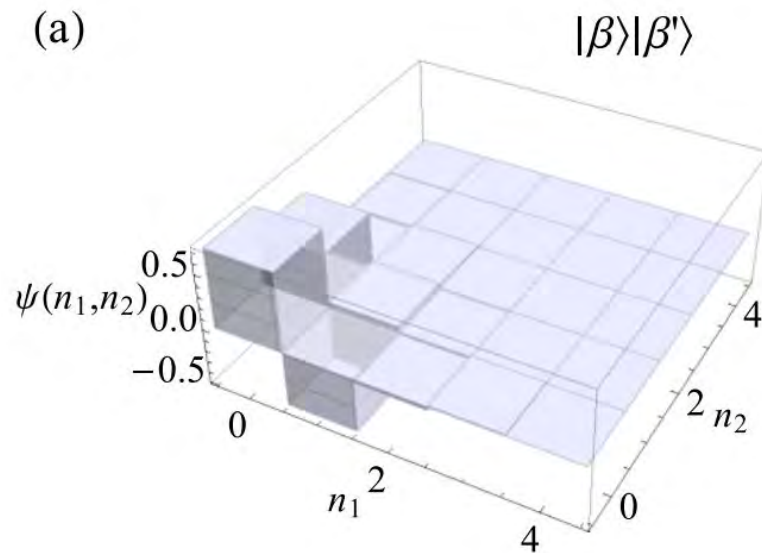
the simplest non-Gaussian quantum entangler by nonlocal X gates:



ENTANGLING COHERENT STATES

a) ideal operation:
$$N_- = \frac{1 - e^{-\alpha^2}}{1 - e^{-\alpha^2} \cos[2\alpha(\beta - \beta')]}$$

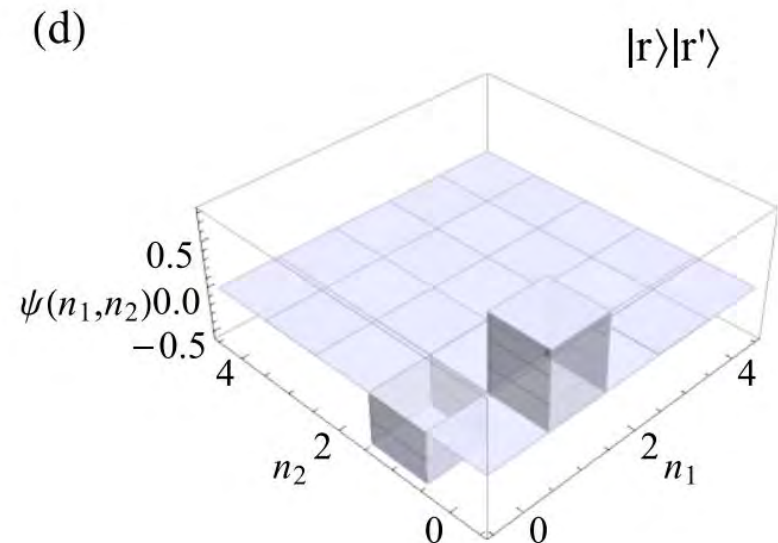
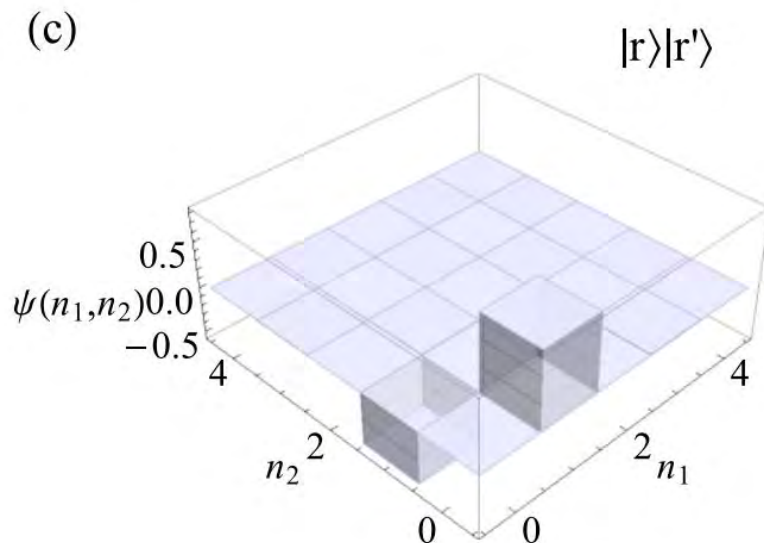
b) approximation:
$$N_- \approx \frac{1}{1 + 2(\beta - \beta')^2} + O[\alpha^2]$$



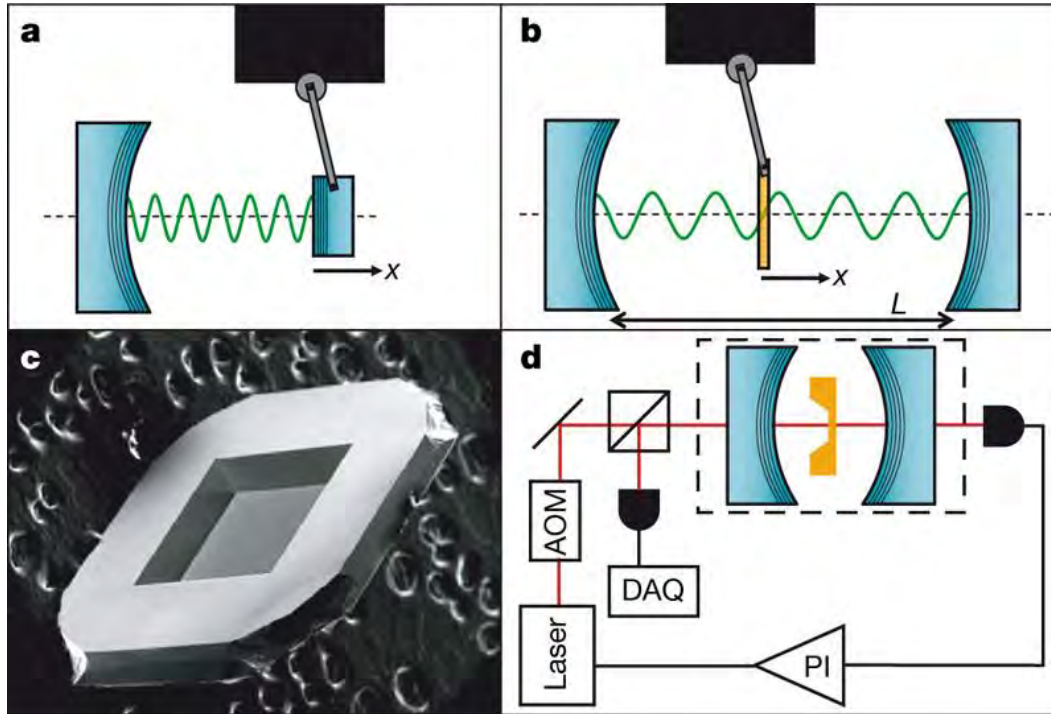
ENTANGLING SQUEEZED STATES

c) ideal operation:
$$N_- = \frac{\sqrt{(1 - e^{-\alpha^2 e^{2r}})(1 - e^{-\alpha^2 e^{2r'}})}}{1 - e^{-\alpha^2(e^{2r} + e^{2r'})}/2}}$$

d) approximation:
$$N_-^{(1)} = \frac{2}{e^{r-r'} + e^{r'-r}} = \text{sech}[r - r']$$



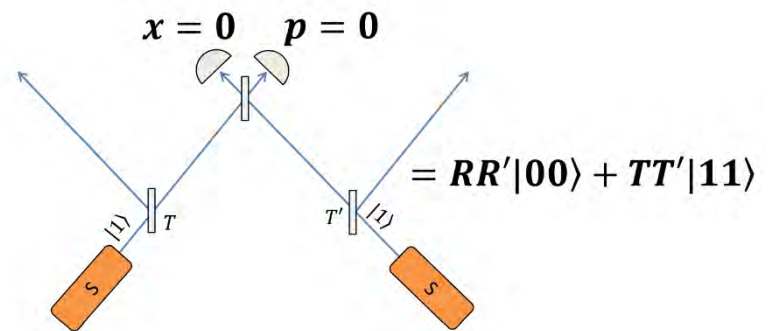
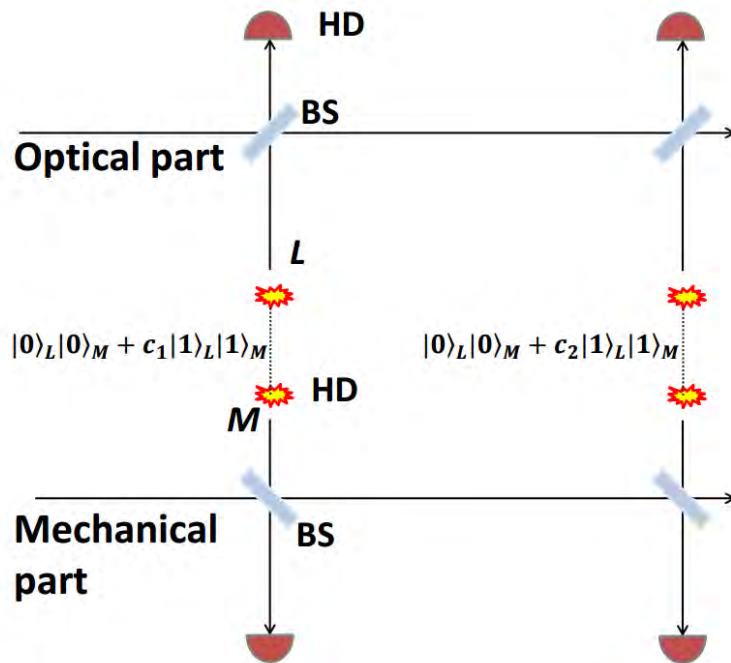
EMULATING THERMAL OPTOMECHANICS



membrane-
in-the-middle

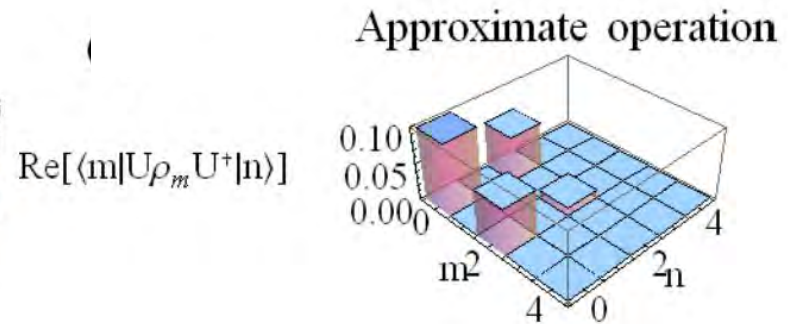
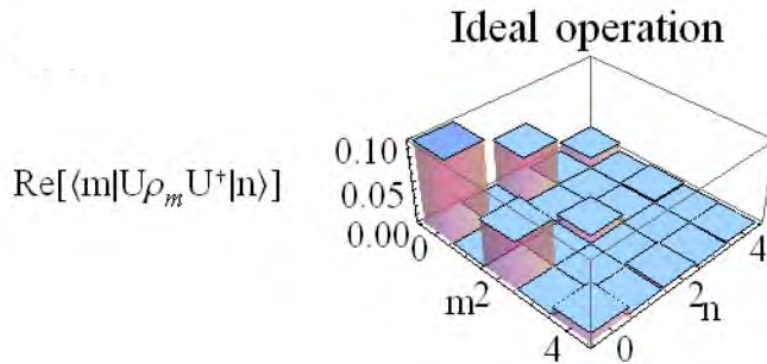
$$\hat{H} \propto \hat{n}_L \hat{X}_M^2$$

EMULATING THERMAL OPTOMECHANICS

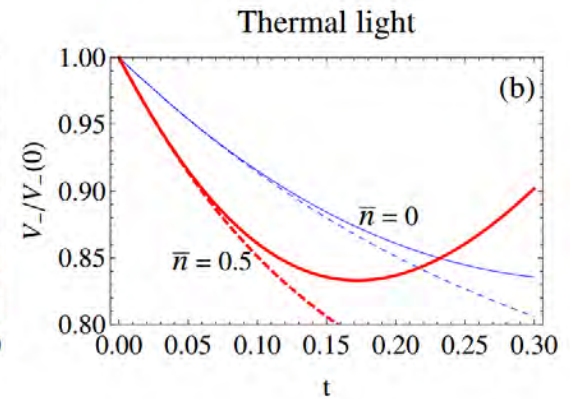
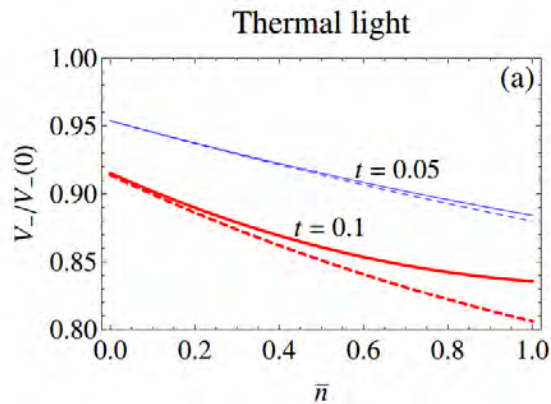


$$H = \chi \hat{X}_L^2 \hat{X}_M^2$$

EMULATING THERMAL OPTOMECHANICS



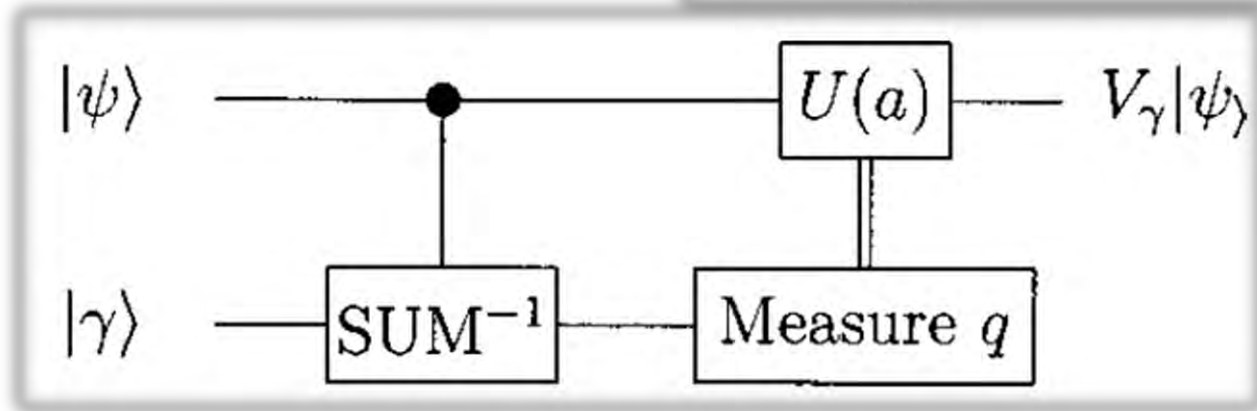
Thermal light generates nonclassical squeezing



DETERMINISTIC CUBIC NONLINEARITY

$$\hat{H}_3 = \omega_3 \hat{x}^3$$

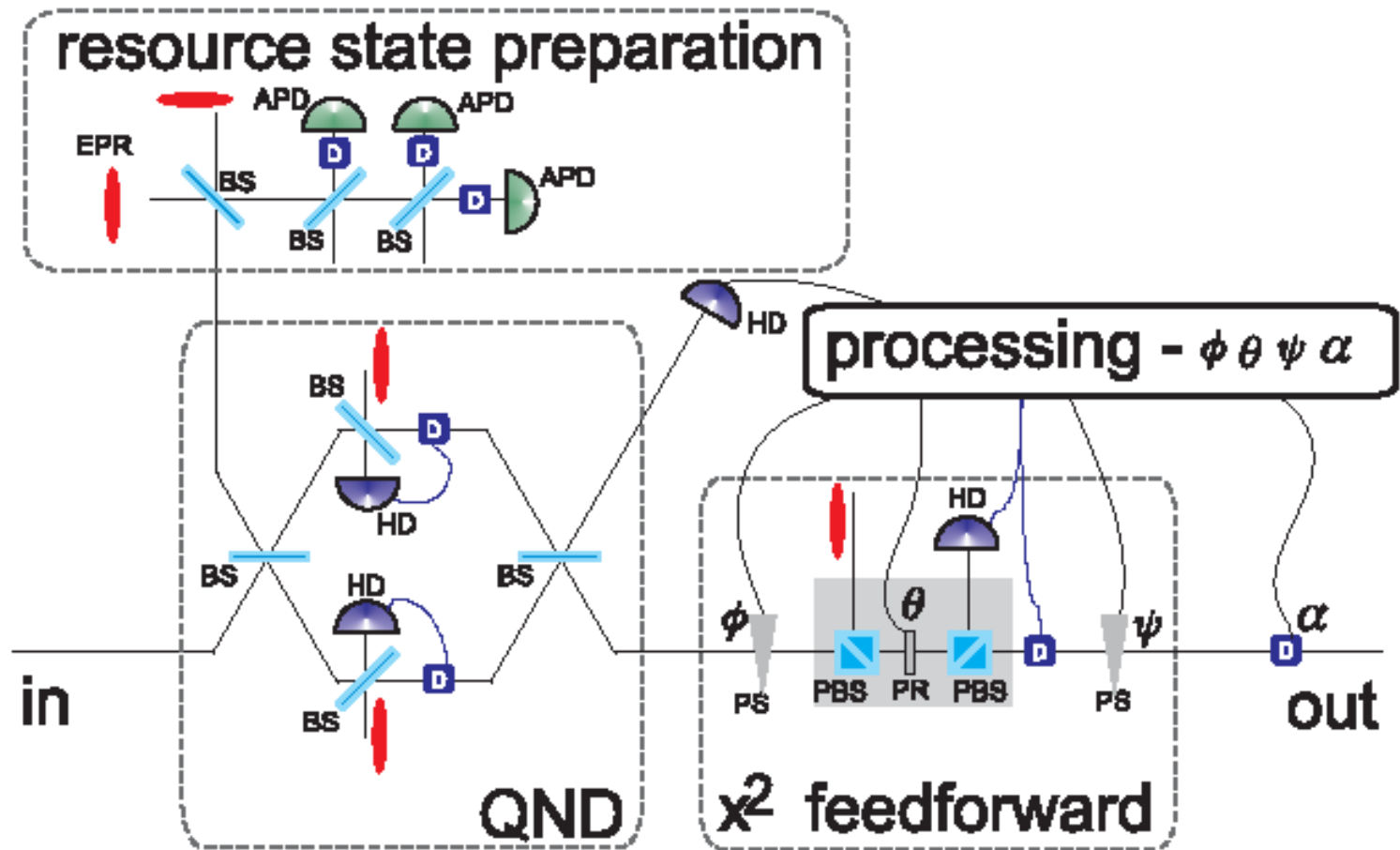
$$|\gamma\rangle = \int e^{i\chi x^3} |x\rangle dx$$



[Gottesman and Preskill, PRA 64
012310 (2001)]

We obtained QND gate and cubic state, we need X^2 feed-forward correction techniques and then do it.

FEASIBLE CUBIC INTERACTION

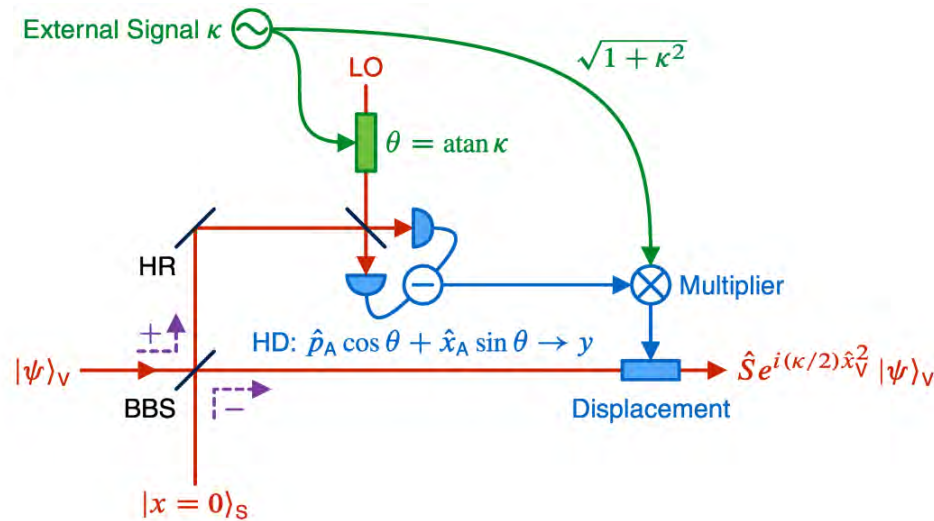
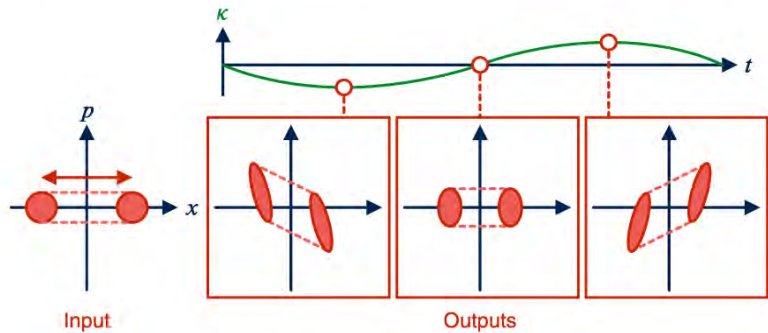


QUADRATIC X^2 FEEDFORWARD

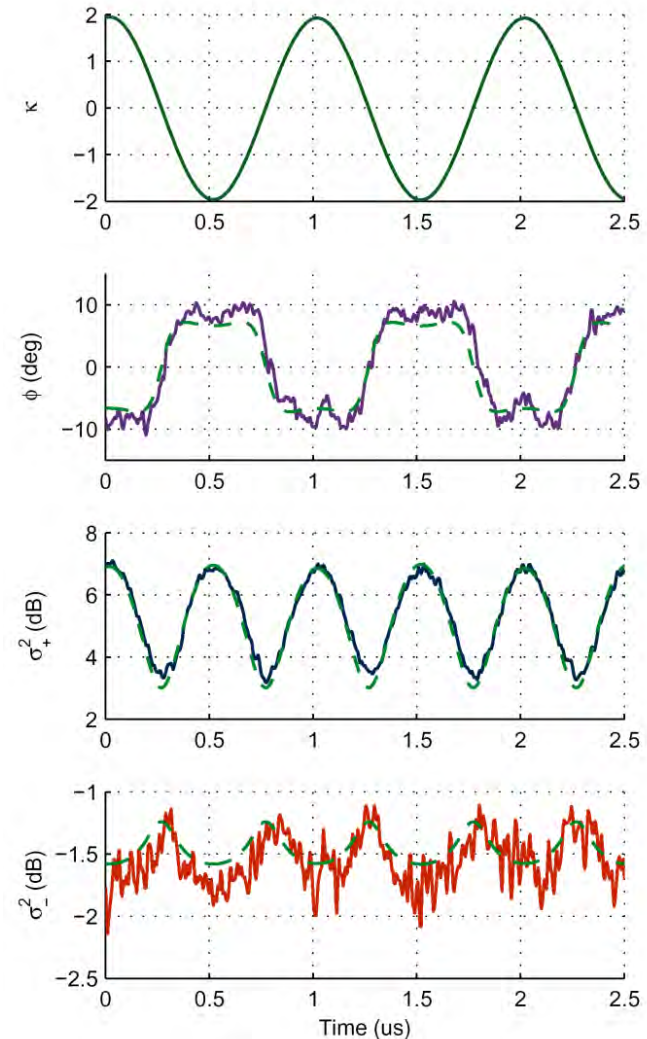
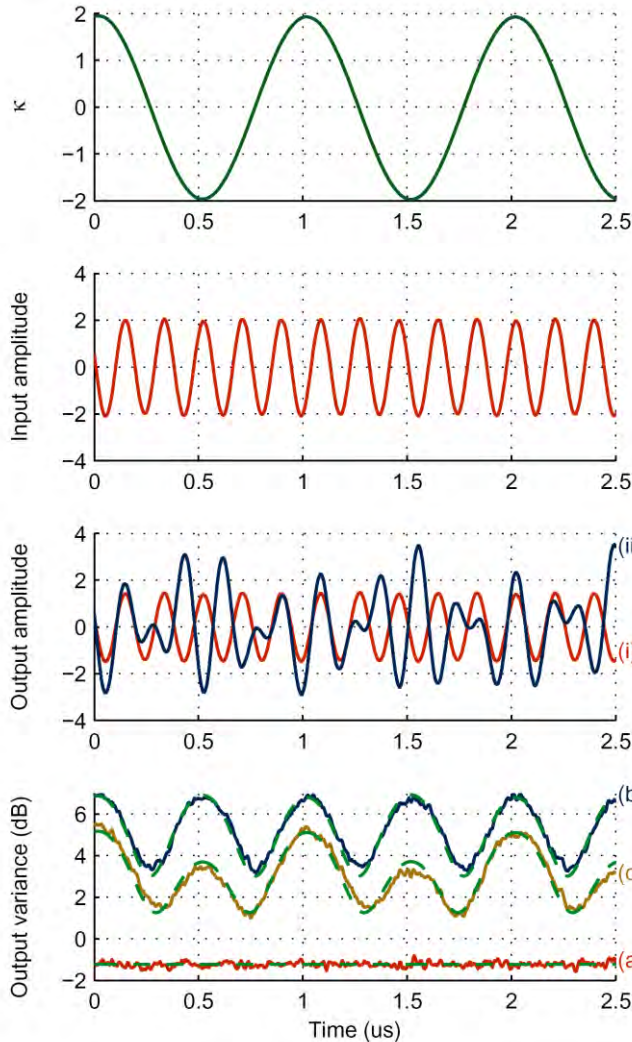
$$X \rightarrow X, P \rightarrow P + \kappa X$$

$$\hat{x} = \frac{1}{\sqrt{2}} \hat{x}_V - \frac{1}{\sqrt{2}} \hat{x}_S^{(0)},$$

$$\hat{p} = \sqrt{2} \left(\hat{p}_V + \frac{\kappa}{2} \hat{x}_V \right) + \frac{\kappa}{\sqrt{2}} \hat{x}_S^{(0)}$$

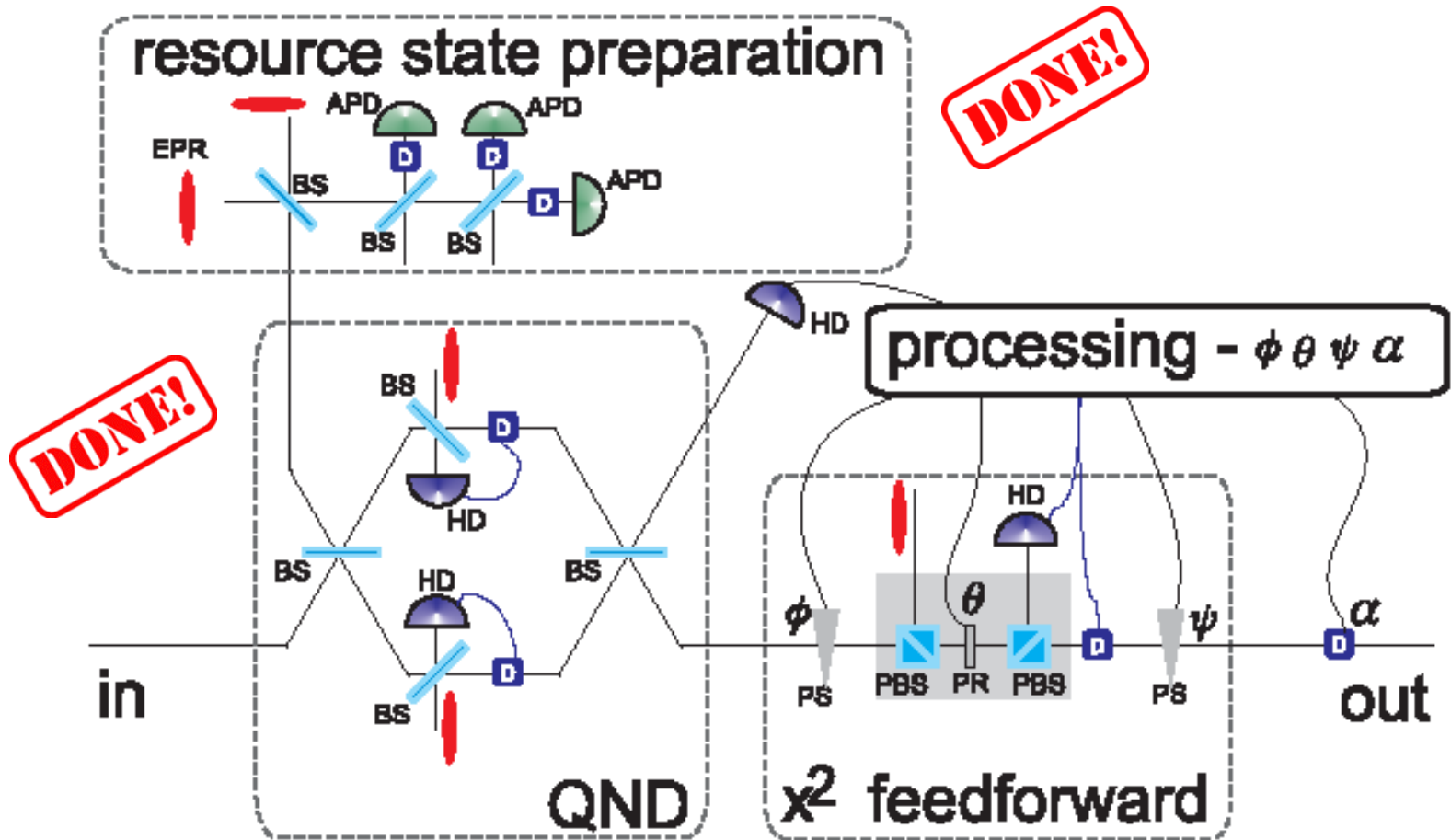


QUADRATIC X^2 FEEDFORWARD

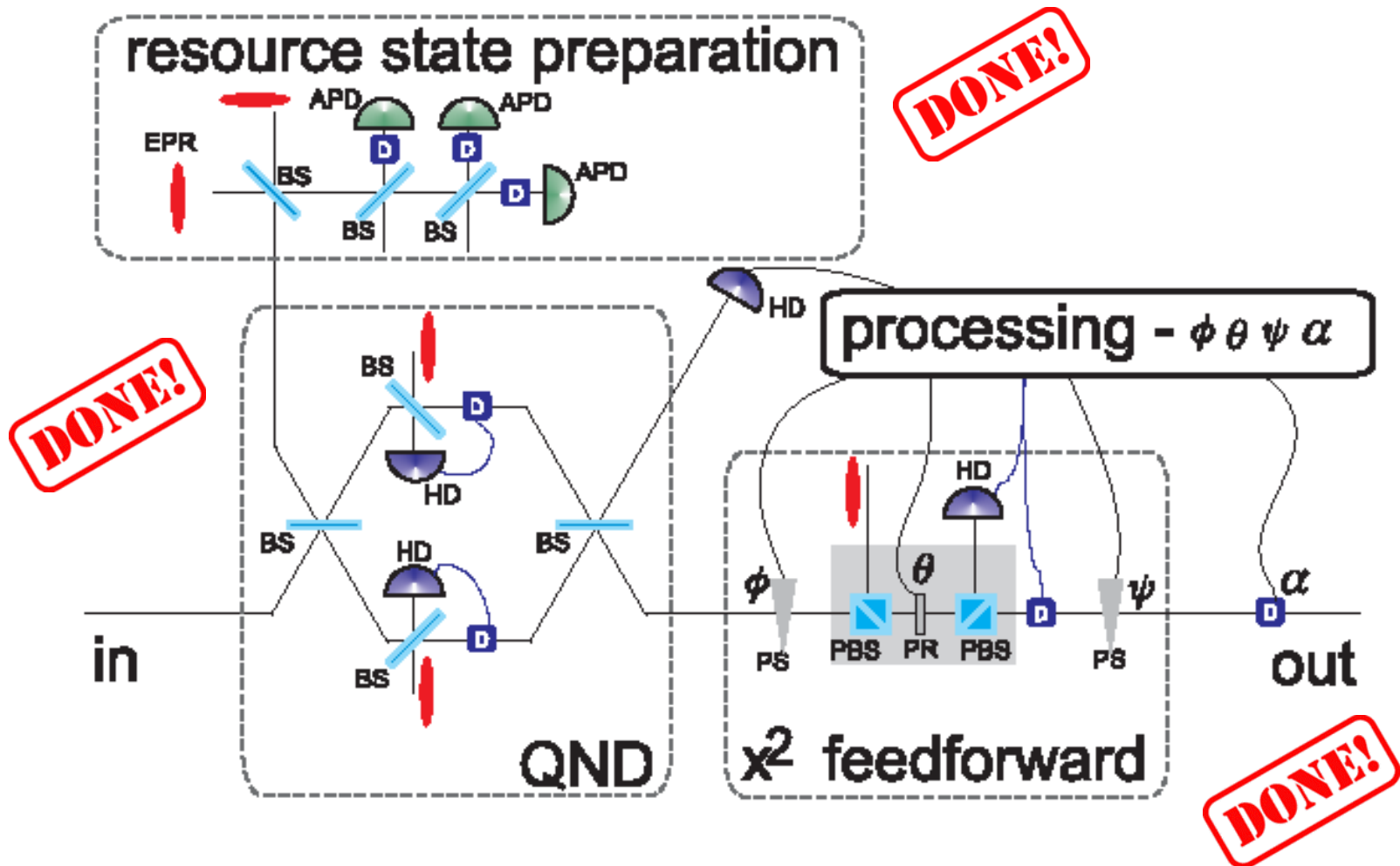


K. Miyata, H. Ogawa, P. Marek, R. Filip, H. Yonezawa, J. Yoshikawa, and A. Furusawa, Phys. Rev. A 90, 060302(R) (2014).

FEASIBLE CUBIC INTERACTION



FEASIBLE CUBIC INTERACTION



GOALS AND TARGETS

ICSSUR 2017

Exploiting the squeezed light and single photons:

- optimized entangled state preparation
- control of quantum decoherence
- conditional simulations of quantum nonlinear effects
- nonlocal quantum operations with different platforms
- dynamical quantum nonlinearities
- deterministic cubic nonlinearities

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