Continuous- and Discrete-Variable Quantum Key Distribution with Nonclassical Light Over Noisy Channels

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Outline

- Motivation: why QKD?
- Discrete vs Continuous variables of light
- Model of channel noise
- Comparison of Discrete & Continuous variables

Motivation

Alice and Bob would like to communicate securely

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Asymmetrical cryptosystems are potentially vulnerable

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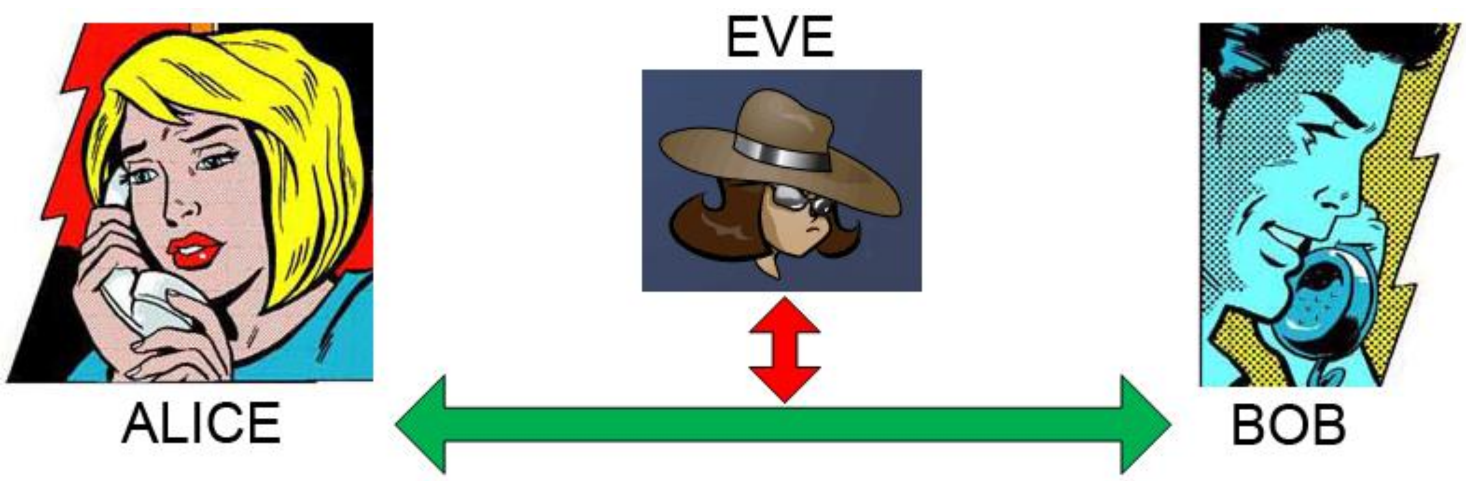
One-time pad (Vernam, 1919) is secure (Shannon, 1949), but needs secret keys

Motivation

Alice and Bob would like to communicate securely

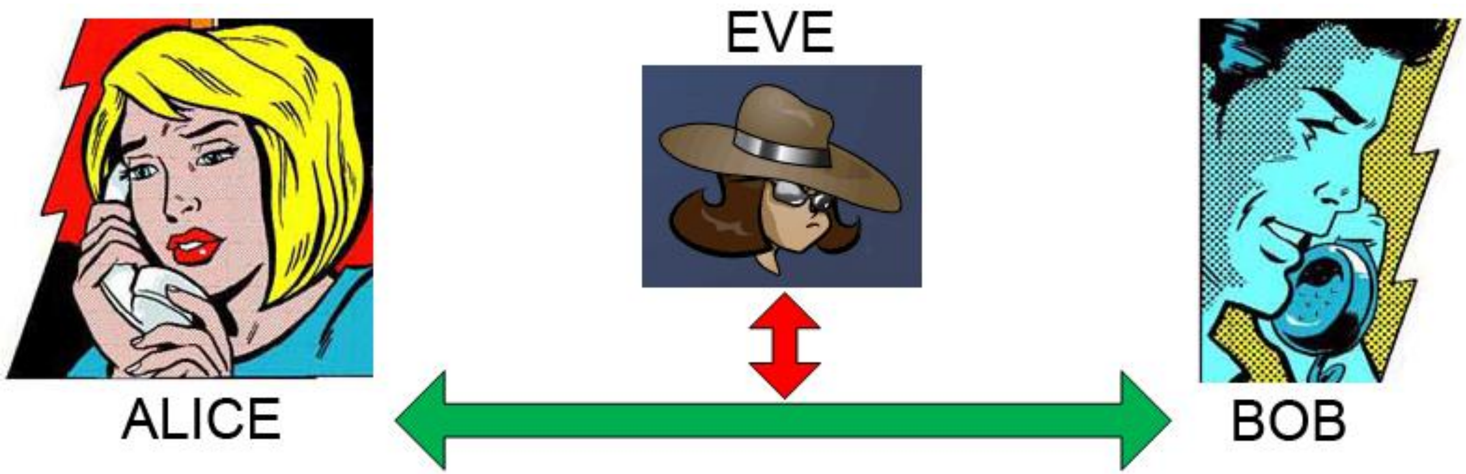
Key distribution: can be solved by mathematical methods
or by involving laws of physics -> **quantum key distribution**

Quantum key distribution



The idea of QKD: detect eavesdropping attempts and estimate security of the key.

Quantum key distribution



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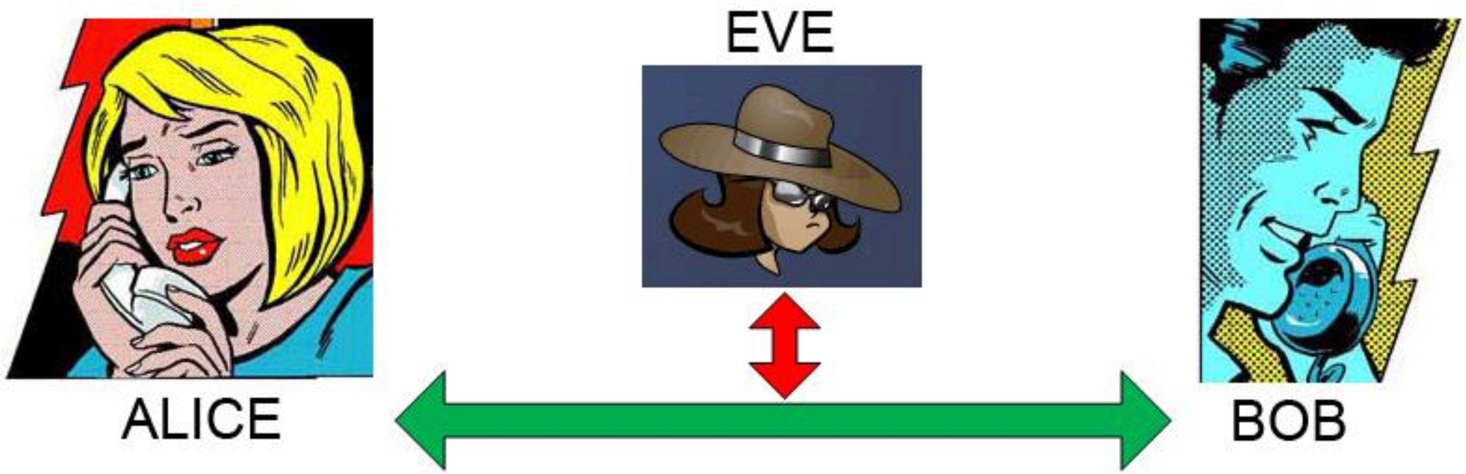
Two main families of QKD protocols:

- **Discrete-variable, DV**
- **Continuous-variable, CV**

Quantum key distribution

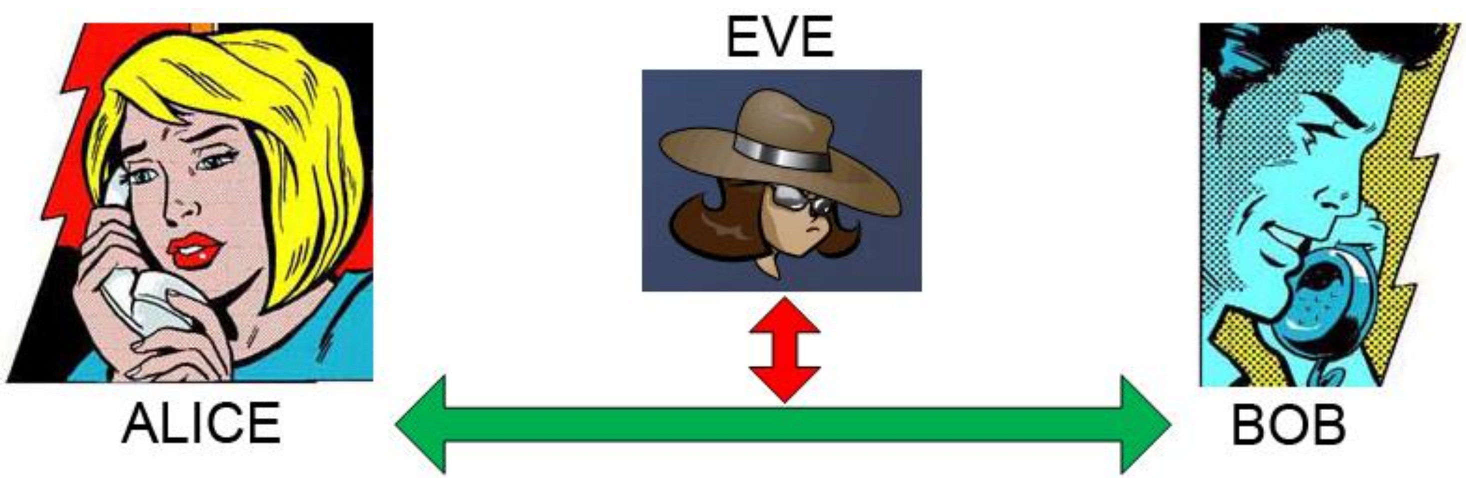


The idea of QKD: detect eavesdropping attempts and estimate security of the key.

Two main families of QKD protocols:

- **Discrete-variable, DV** (“particle-like” properties of light)
- **Continuous-variable, CV** (“wave-like” properties of light)

Quantum key distribution



Security analysis in QKD:

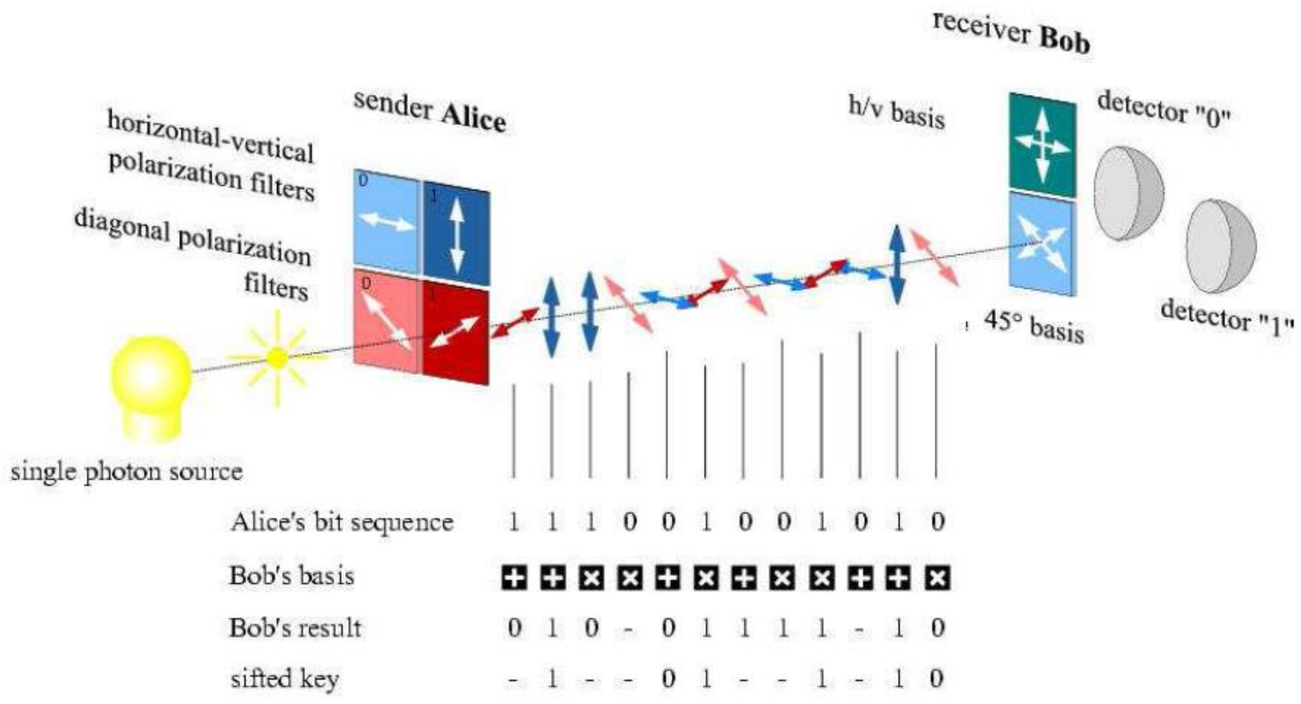
$$I_{AB} = H(A) + H(B) - H(A,B) = H(A) - H(A|B) = H(B) - H(B|A)$$

The secure key can be distilled if $I_{AB} > I_{BE}$ or $I_{AB} > I_{AE}$.

Lower bound on secure key: $K \geq \max(I_{AB} - I_{BE}, I_{AB} - I_{AE})$

[*Csiszár, Körner, IEEE Trans. Inf. Theor., IT-24, 339-348 (1978)*]

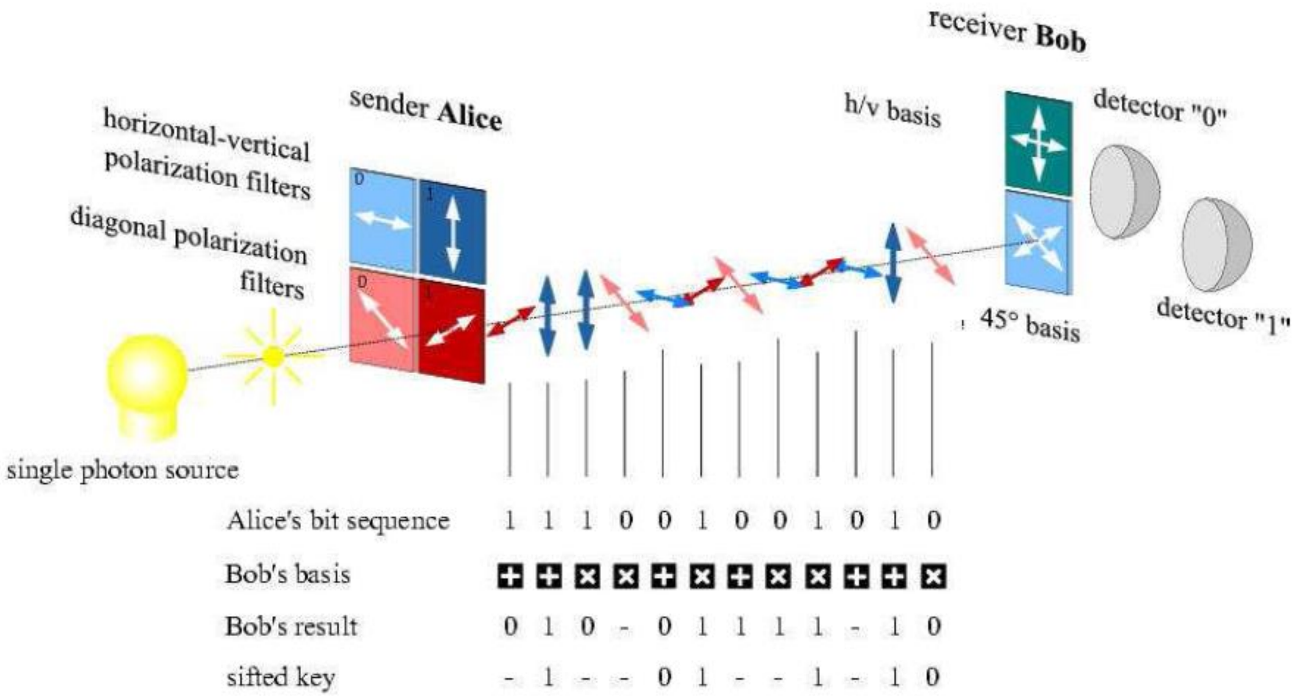
Discrete variables



Scheme of the BB84 protocol:

- Alice chooses a polarization basis
- Alice prepares a single photon in a given polarization state
- Bob chooses the detection basis
- Bob measures the state of the photon in a given basis
- Alice and Bob perform key sifting, error correction and privacy amplification

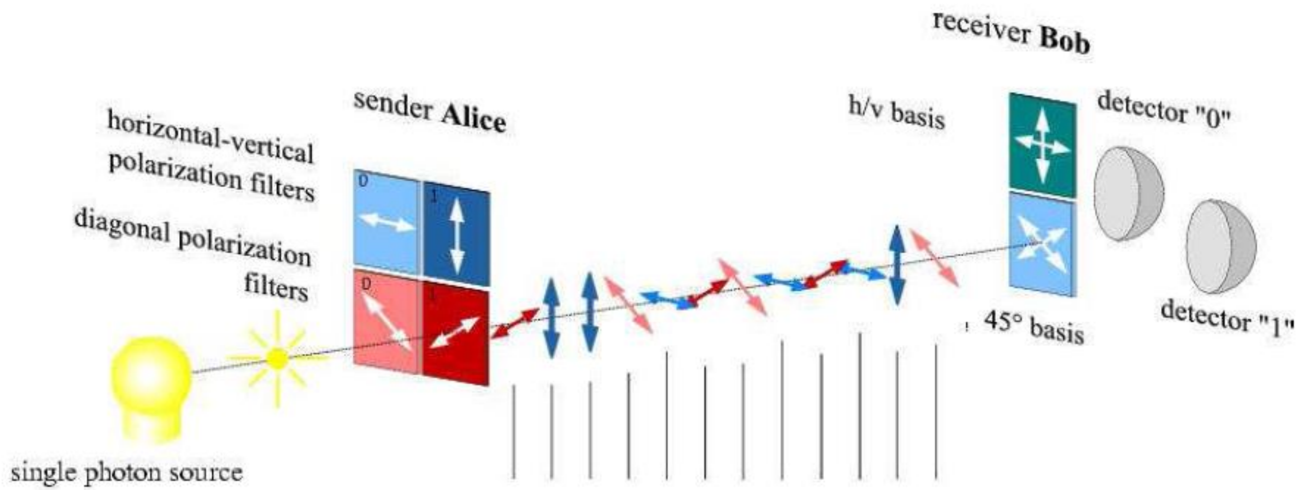
Discrete variables



Security analysis:

Estimate upper bound on Eve's information from the amount of errors (QBER).
For collective attacks bounds on QBER were derived (~12.6% for BB84)
[B. Kraus, N. Gisin, and R. Renner, Phys. Rev. Lett. 95, 080501 (2005)]

Discrete variables



Alice's bit sequence	1	1	1	0	0	1	0	0	1	0	1	0
Bob's basis	+	+	×	×	+	×	+	×	×	+	+	×
Bob's result	0	1	0	-	0	1	1	1	1	-	1	0
sifted key	-	1	-	-	0	1	-	-	1	-	1	0

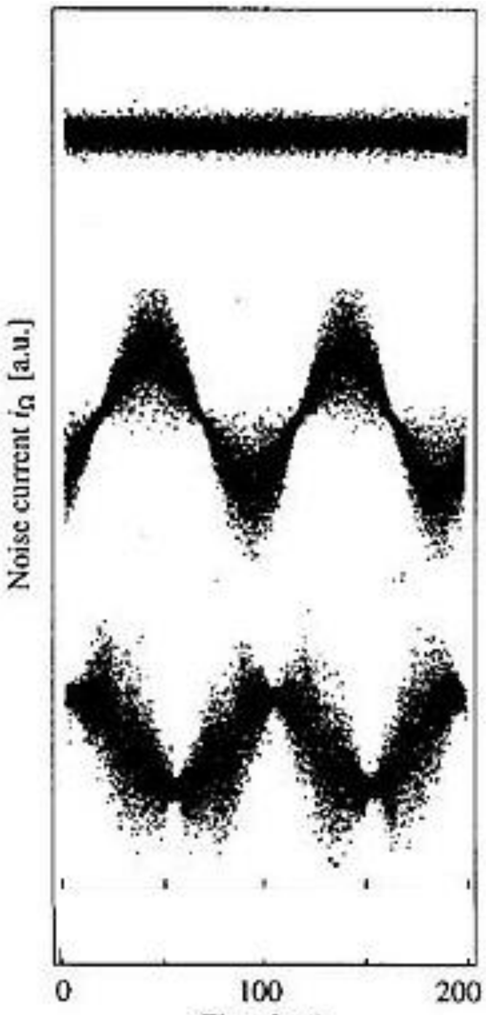
Physical systems: single photons (strongly nonclassical)
Detection method: photon counting
Issues:

- demanding and imperfect generation (in practice – weak laser pulses)
- imperfect detection (dark counts)
- lossy channels, stray light, implementation loopholes

Current achievements: tested in long-distance fiber and free-space channels (>100 km), devices are being sold and further developed

Continuous variables

Quadrature observables: in-phase and out-of-phase components of the electric field amplitude of a given mode (x- and p- quadratures).



Coherent/vacuum states: have the same noise (quantum fluctuations) in both the quadratures (called shot noise)

Squeezed states: have noise in one of the quadratures suppressed below shot noise

Continuous variables

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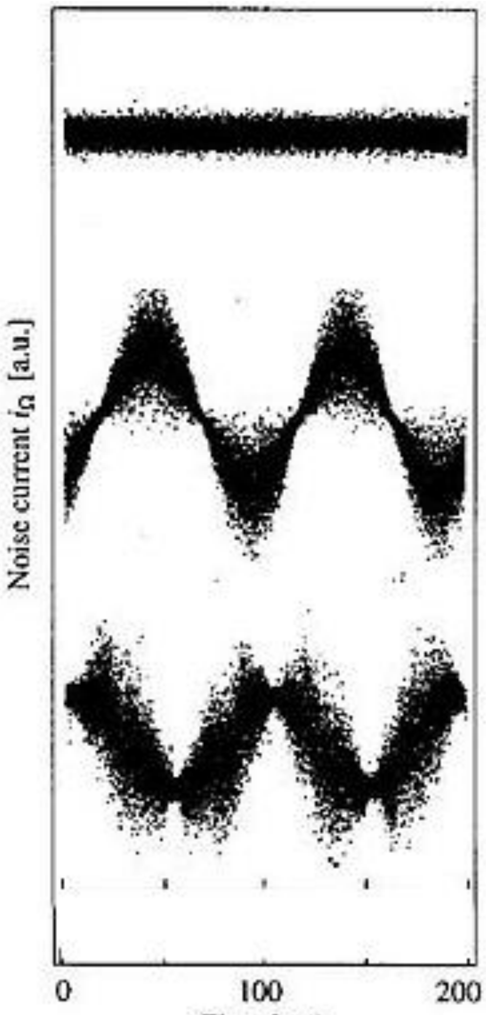
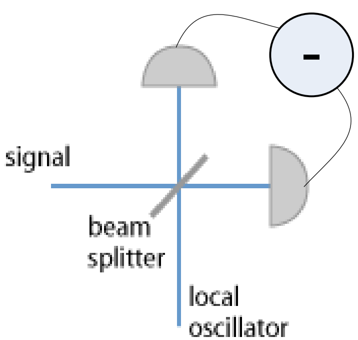


Coherent/vacuum states: have the same noise (quantum fluctuations) in both the quadratures (called shot noise)

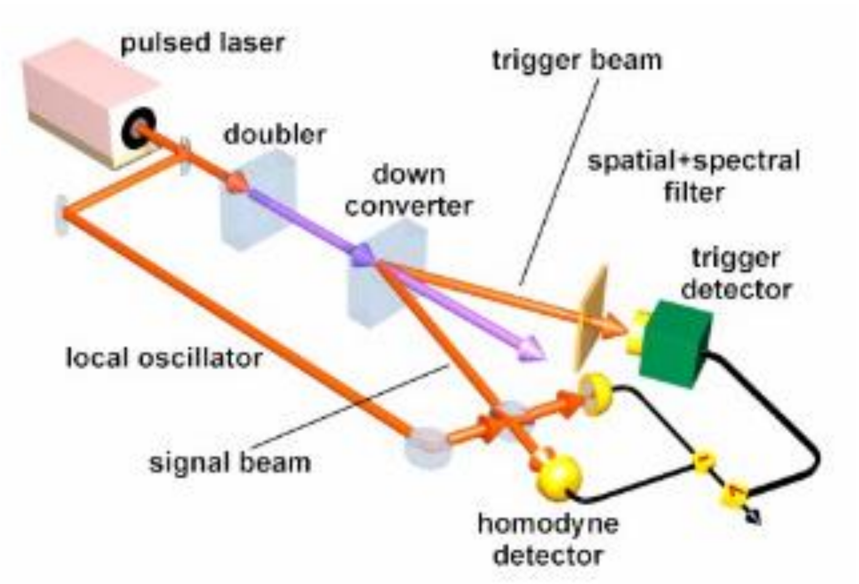
Squeezed states: have noise in one of the quadratures suppressed below shot noise

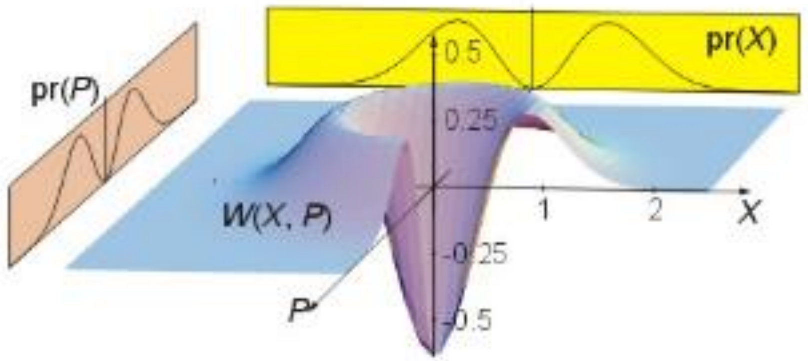
Quadratures can be measured using **homodyne detection**:



Continuous variables

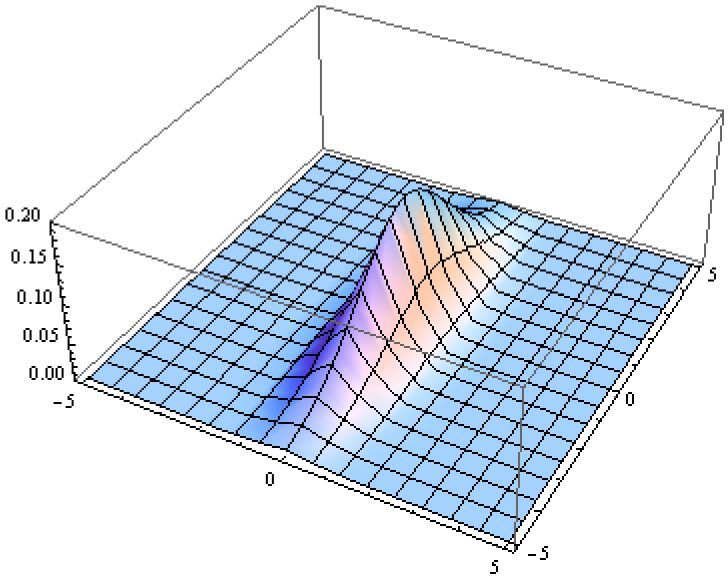
Quadrature distribution of a single-photon state:

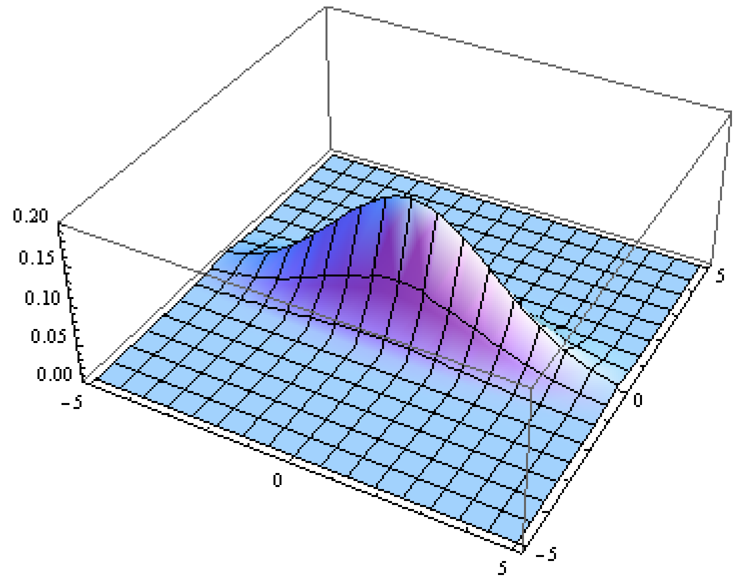




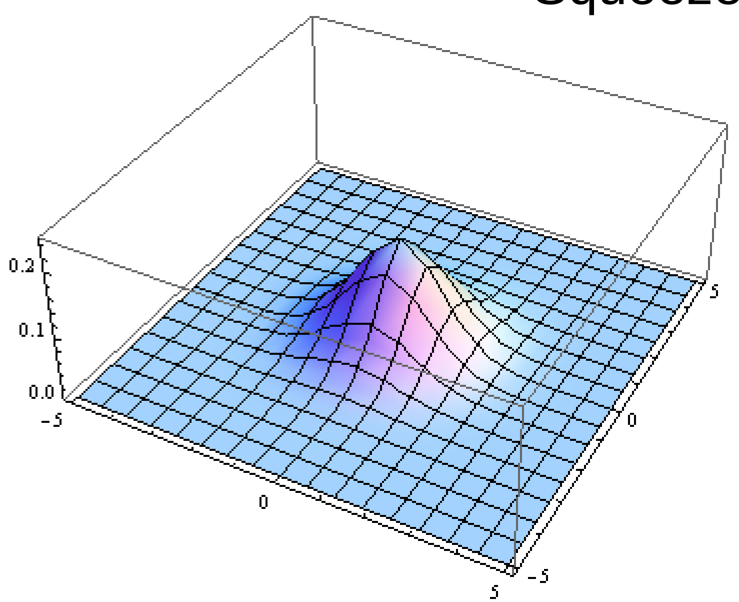
Negative quasiprobability distribution: clearly nonclassical feature

[Lvovsky et al., Phys. Rev. Lett. 87, 050402 (2001)]

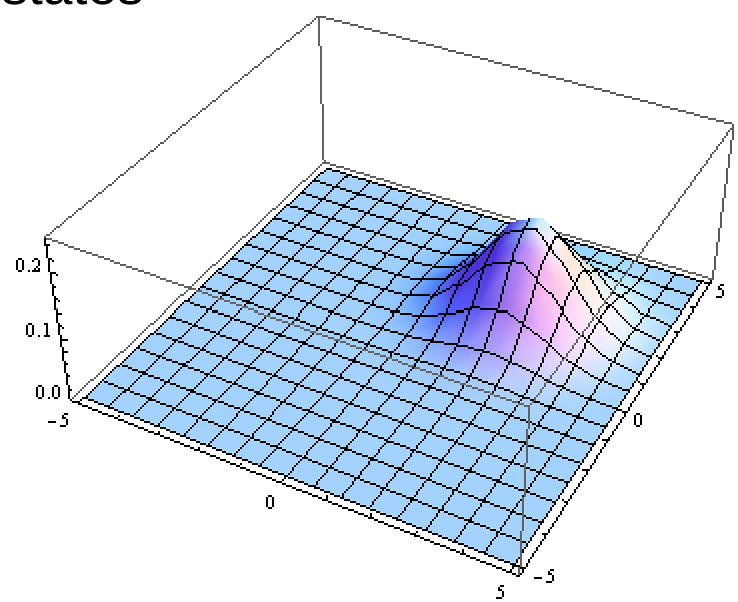




Squeezed states

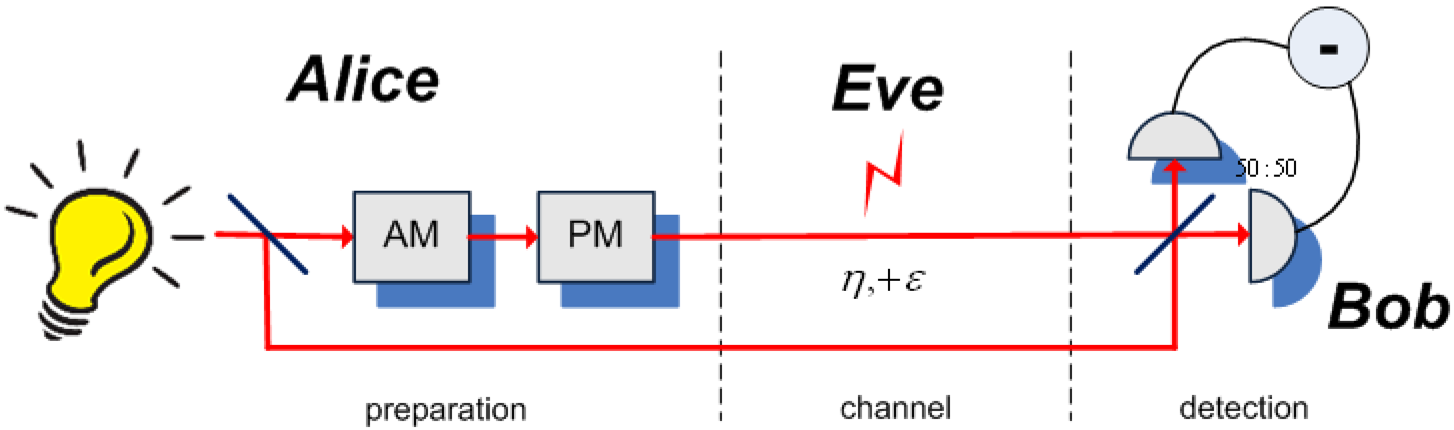


Vacuum state



Coherent state

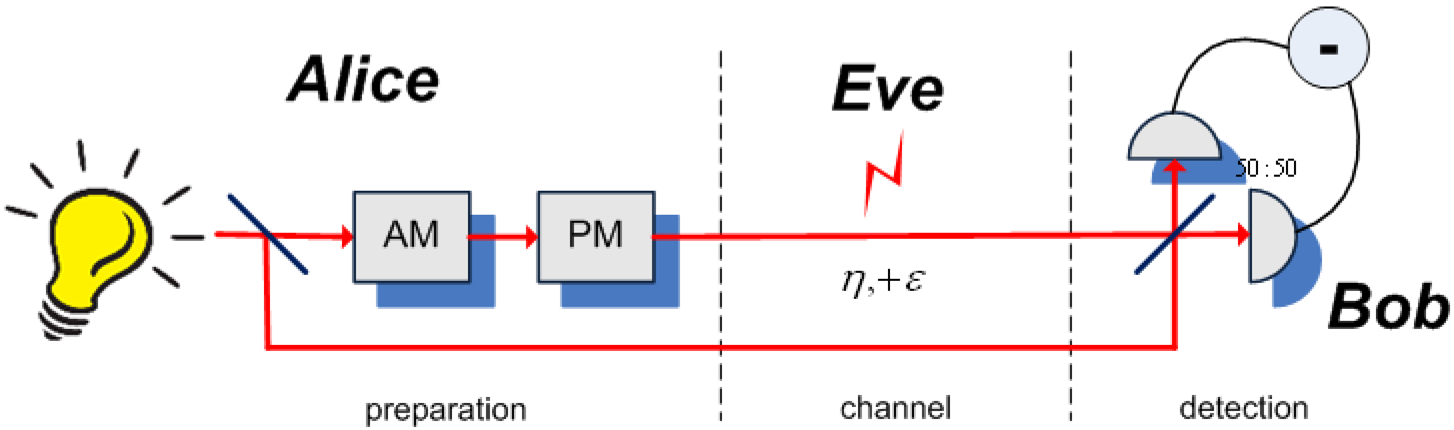
Continuous variables



Scheme of the squeezed-state protocol:

- Alice chooses a squeezing direction
- Alice prepares a respective squeezed state and displaces it randomly
- Bob chooses the detection basis
- Bob measures the state of the mode in a given basis
- Alice and Bob perform key sifting, error correction and privacy amplification

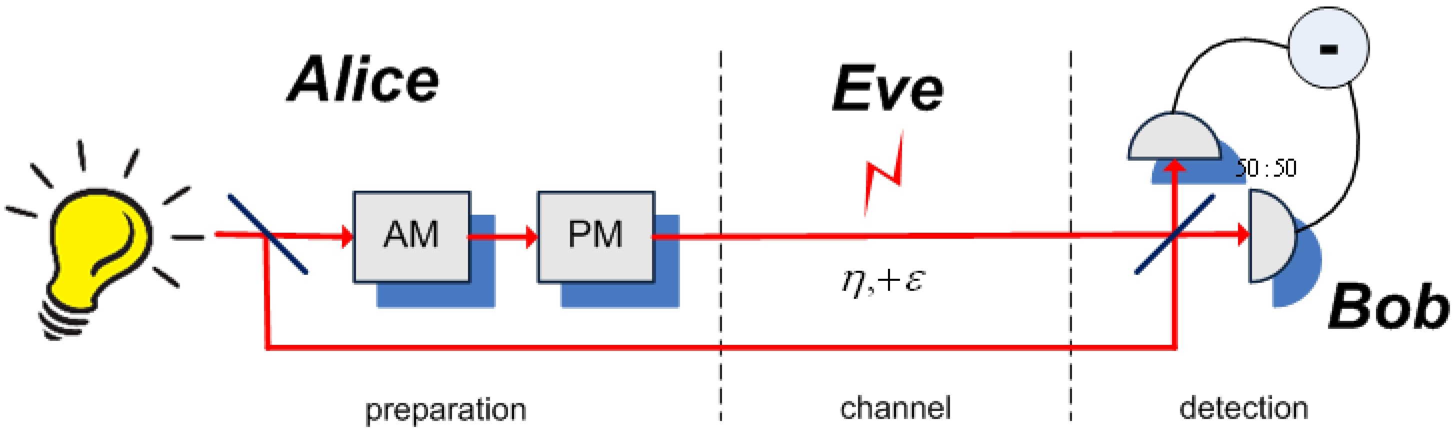
Continuous variables



Security analysis:

Estimate upper bound on Eve's information from the channel noise and loss.
Security against Gaussian collective attacks / general attacks was shown.
[M. Navascues, F. Grosshans, and A. Acin, Phys. Rev. Lett.
97, 190502 (2006); R. Garcia-Patron and N. J. Cerf, Phys. Rev. Lett. 97,
190503 (2006)]

Continuous variables

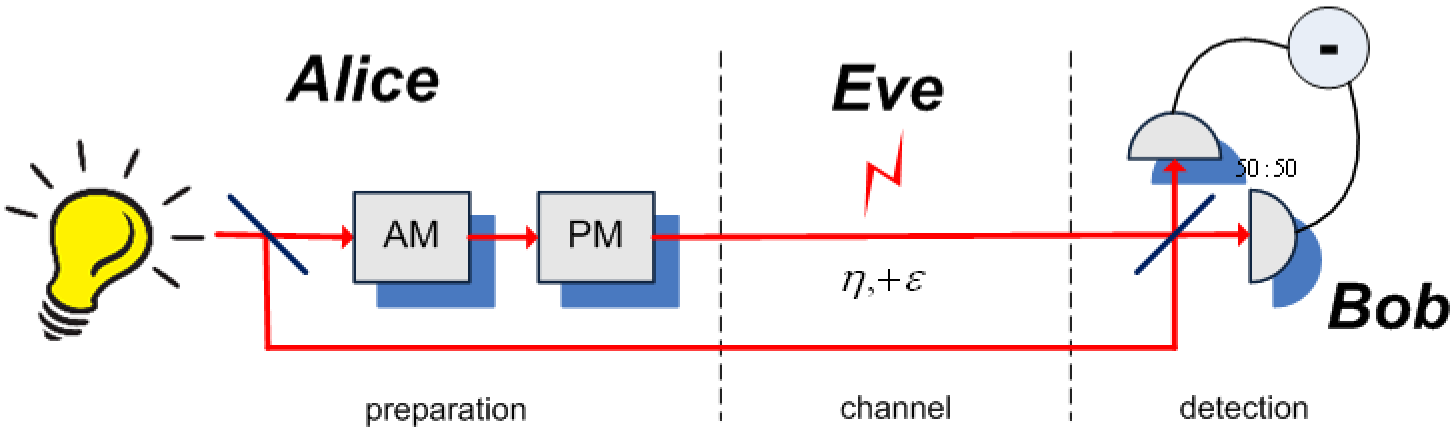


Physical systems: multiphoton states (weaker nonclassicality)
Detection method: homodyne detection
Issues:

- channel imperfections
- possible implementation loopholes

Current achievements: tested in fiber (up to 100 km) and free-space.
Prototypes in development.

Continuous variables



[Role of source noise: **Phys. Rev.** A 81, 022318 (2010),
Role of squeezing: **New J. Phys.** 13, 113007 (2011),
CV QKD over turbulent channels (exp.): **New J. Phys.** 14 (9), 093048 (2012),
Modulation-enhanced CV QKD (exp.): **Nature Communications** 3, 1083 (2012),
Optimization of channel estimation: **Phys. Rev. A** 90, 062310 (2014),
Multimode CV QKD: **Phys. Rev. A** 90, 062326 (2014),
Unidimensional protocol: **Phys. Rev. A** 92, 062337 (2015),
Role of “trusted noise” in CV QKD: **Entropy** 18, 20 (2016),
Effect of side-channels in CV QKD: **Phys. Rev. A** 93, 032309 (2016)]

Comparison between CV and DV?

For many years a comparison was either avoided or done in favor of any of the protocols.

We compare CV and DV in an perfect implementation and using the same channel parametrization.

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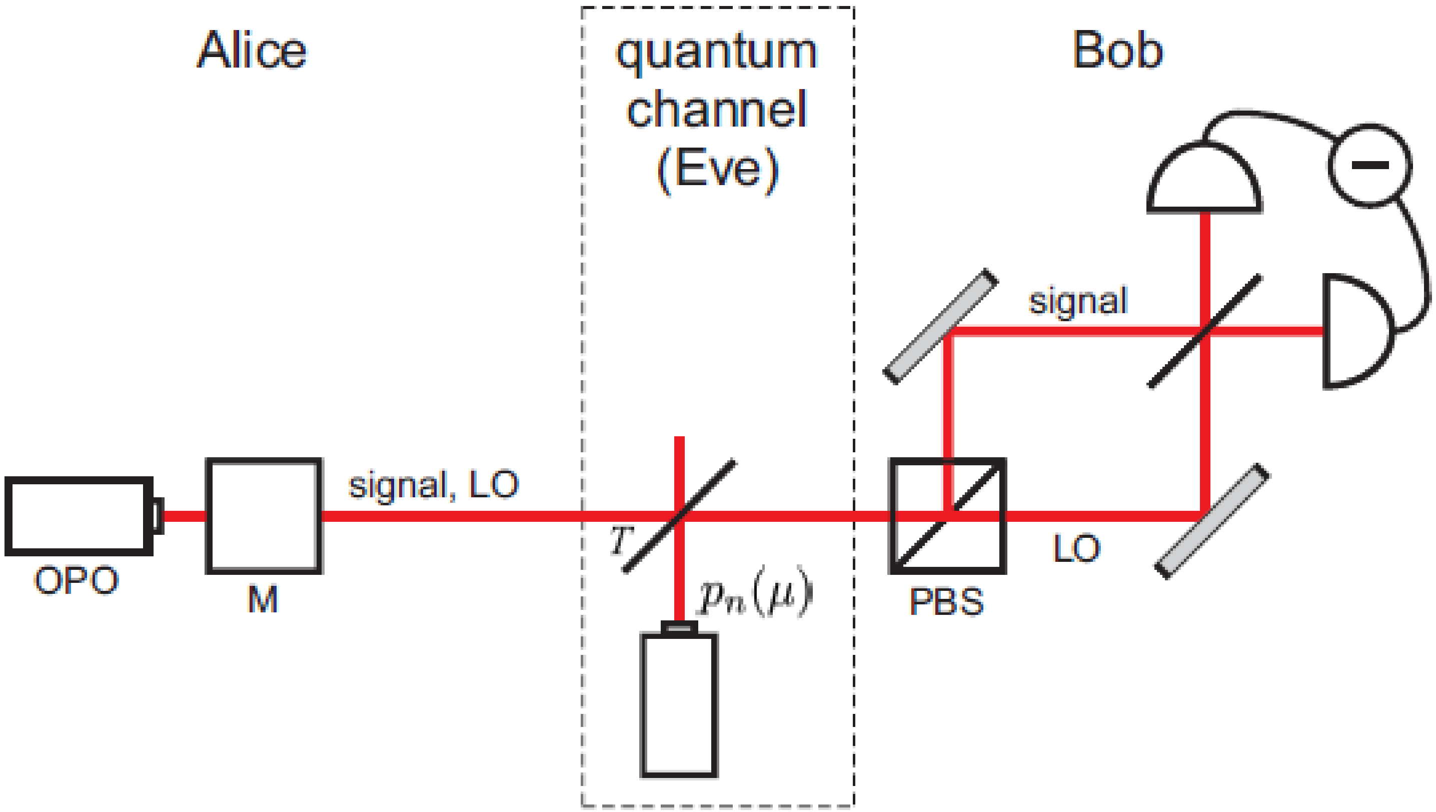
Perfect implementation:

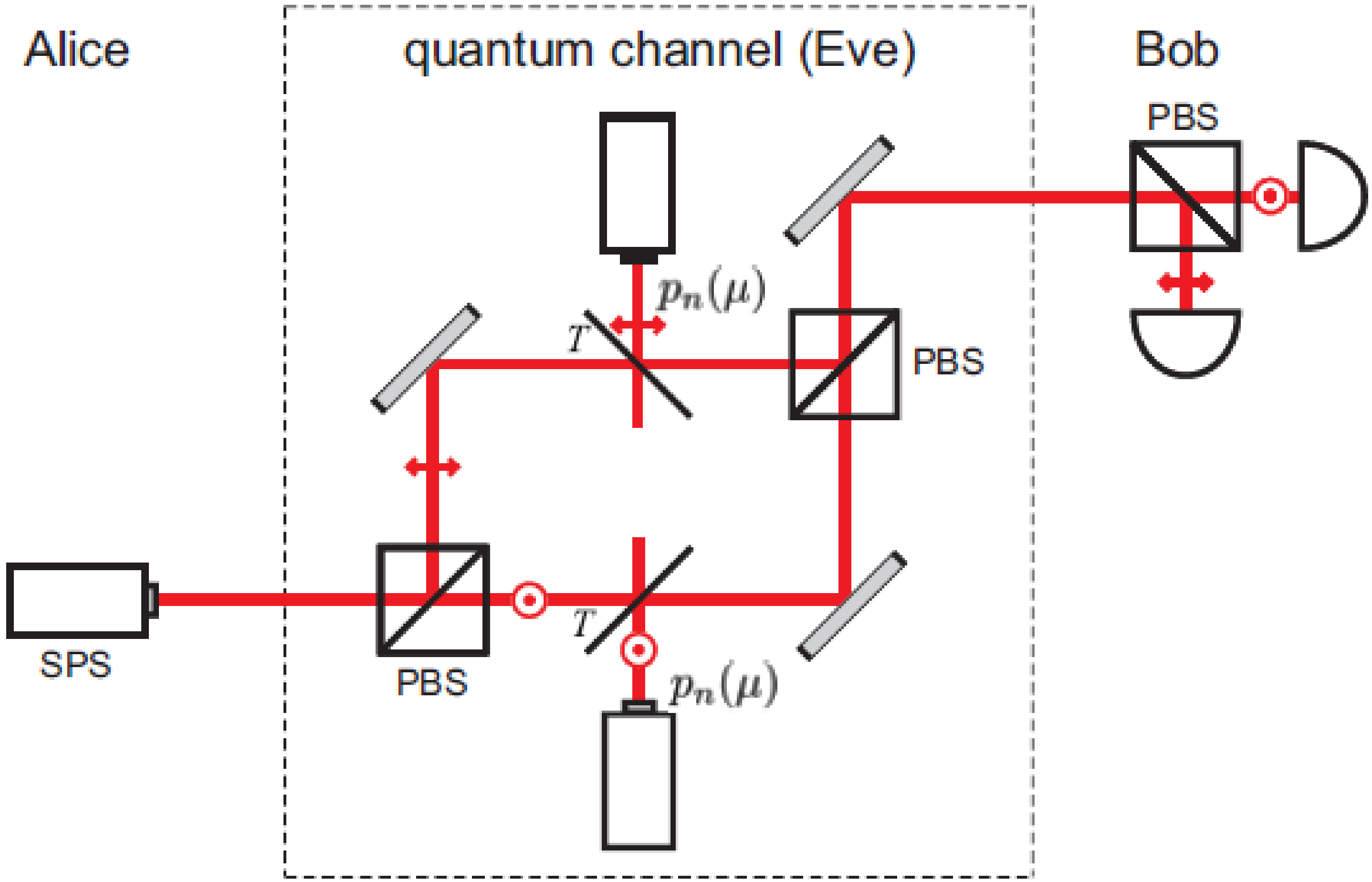
- Perfect single-photon source
- Arbitrary squeezed state generation
- Perfect detectors

CV vs DV



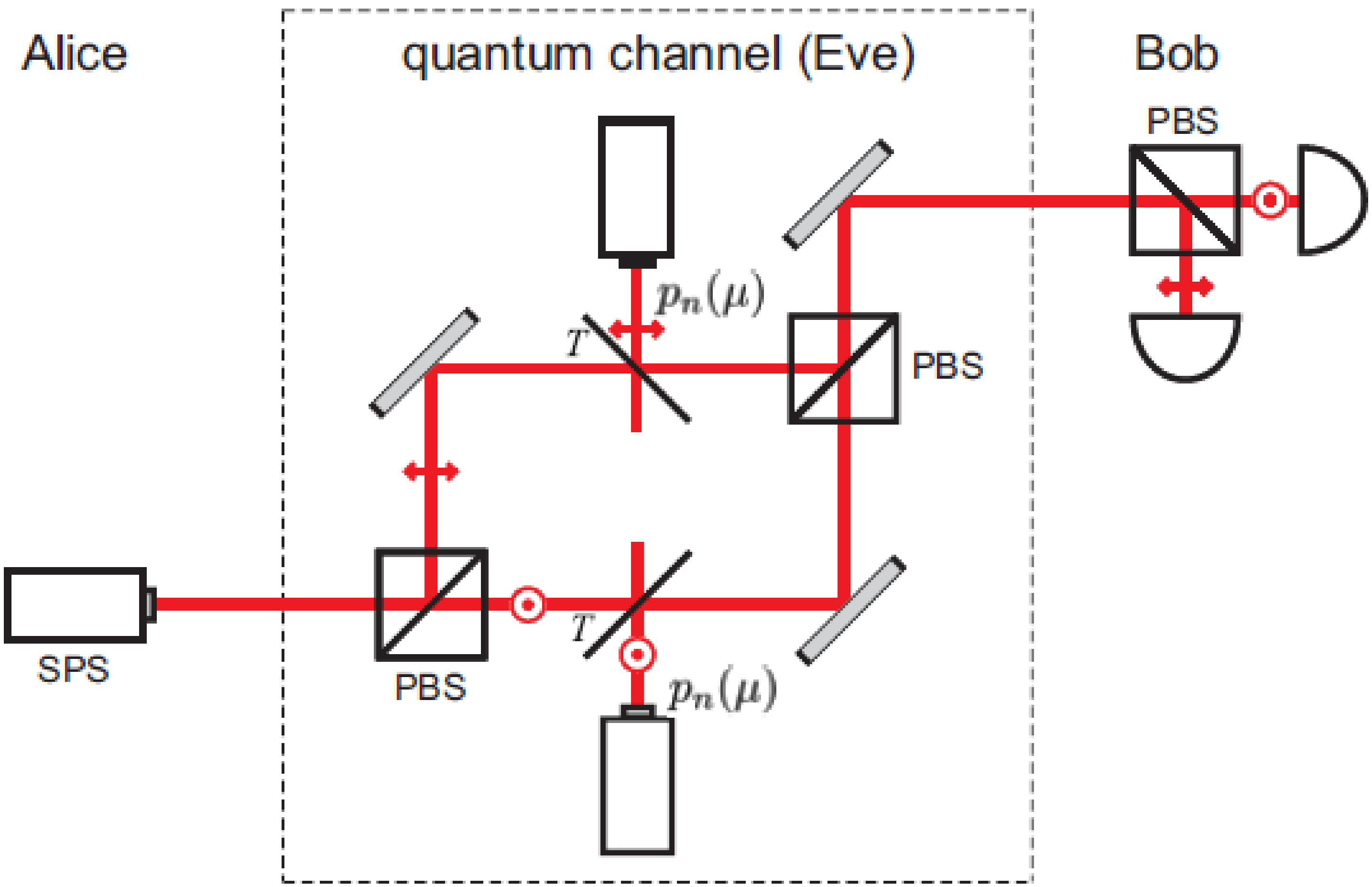
Typical noise model used in CV QKD and parametrized by a mean photon number

CV vs DV



The same noise model applied to DV QKD protocol

CV vs DV



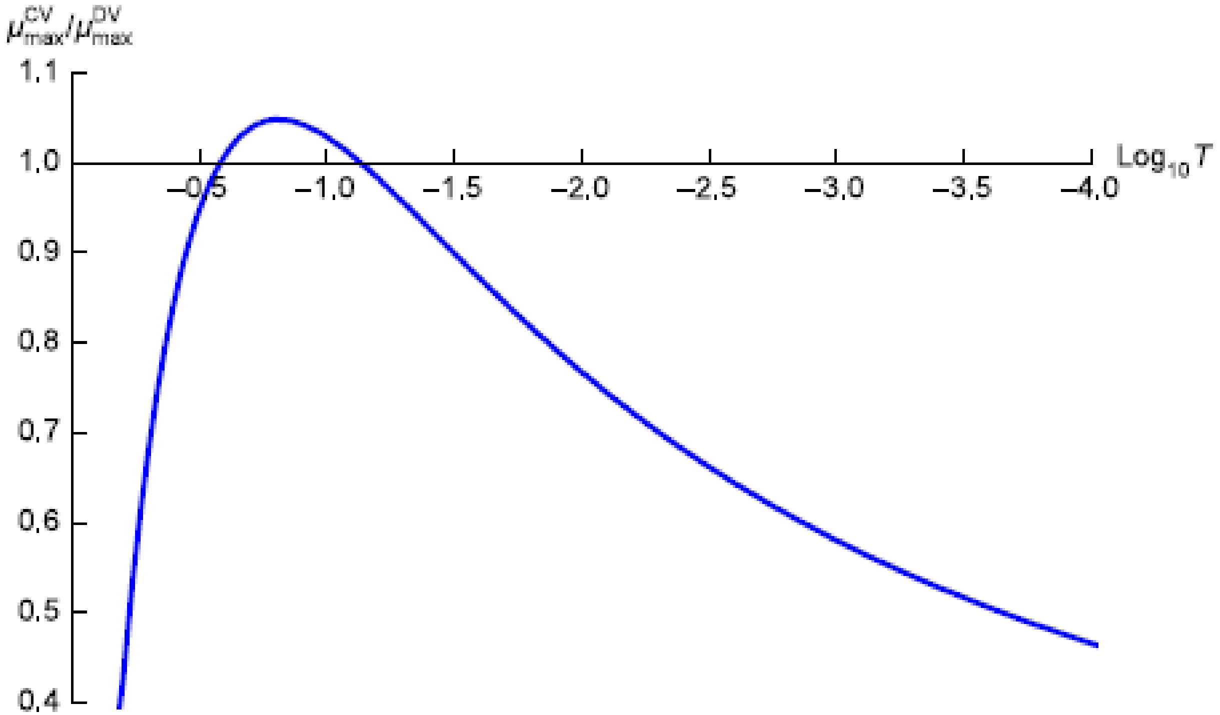
Photonic noise:

$$p_n(\mu) = \frac{\mu^n}{(\mu+1)^{n+1}}$$

Quadrature noise:

$$W = 2\mu + 1$$

CV vs DV



Comparison between robustness to noise in DV and CV

CV vs DV

Analytical result for CV:

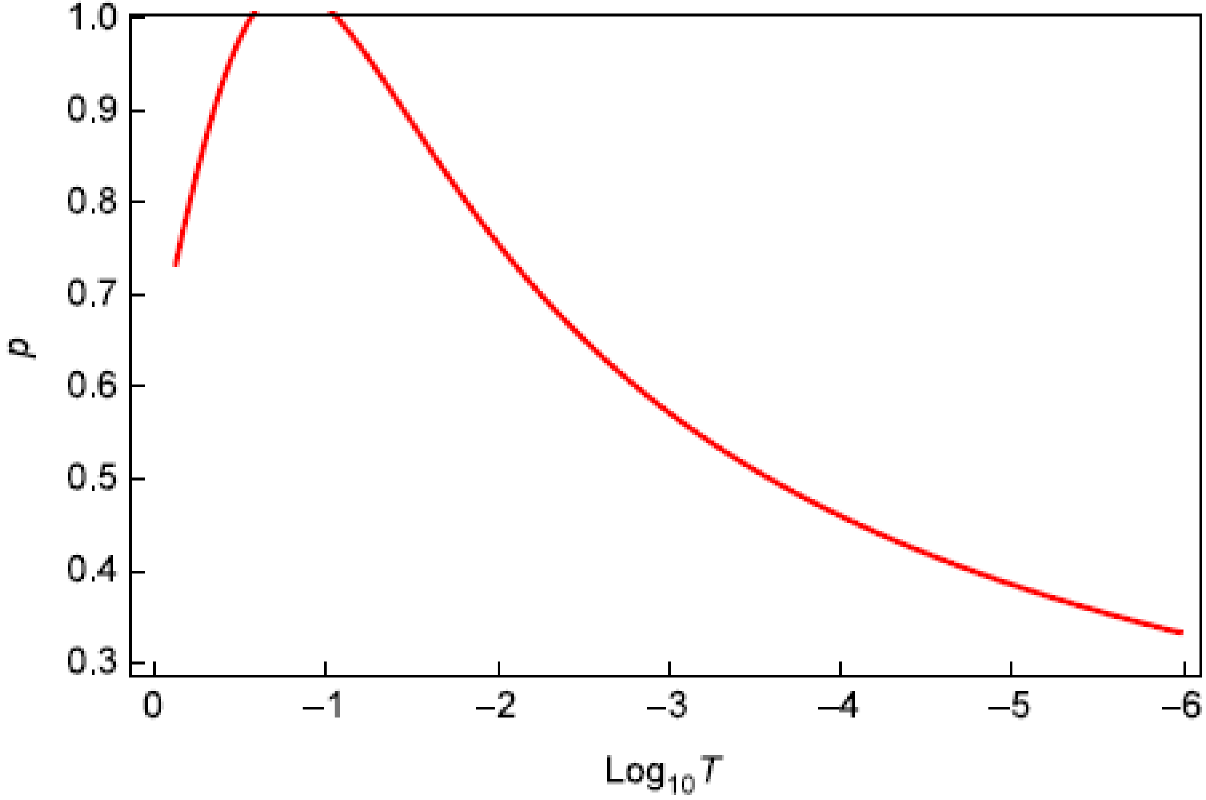
$$\mu_{max}(T) = \exp[1 + W_{-1}(-T/e)]$$

Analytical result for DV:

$$\mu_{\mathrm{max}}^{DV}(T) = \frac{TQ_{\mathrm{th}}}{1 - 2Q_{\mathrm{th}}}$$

($Q_{th} \approx 12.6\%$ for BB84)

CV vs DV



How good shall be the single-photon DV source to beat any CV protocol

Summary

• We developed the model of the channel noise allowing the same parametrization in CV and DV protocols

• Using the model we compared the robustness to channel noise of DV and CV protocols

• CV is more effective for mid-range channels, while DV is more effective for short-range or long-range channels with low or strong losses.

• The results are promising for planning QKD networks

See quant-ph **arXiv:1602.03122** for details.

Thank you for attention!

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