# AXIAL ASYMMETRY IN HOLOGRAPHIC AND INCOHERENT CORRELATION IMAGING

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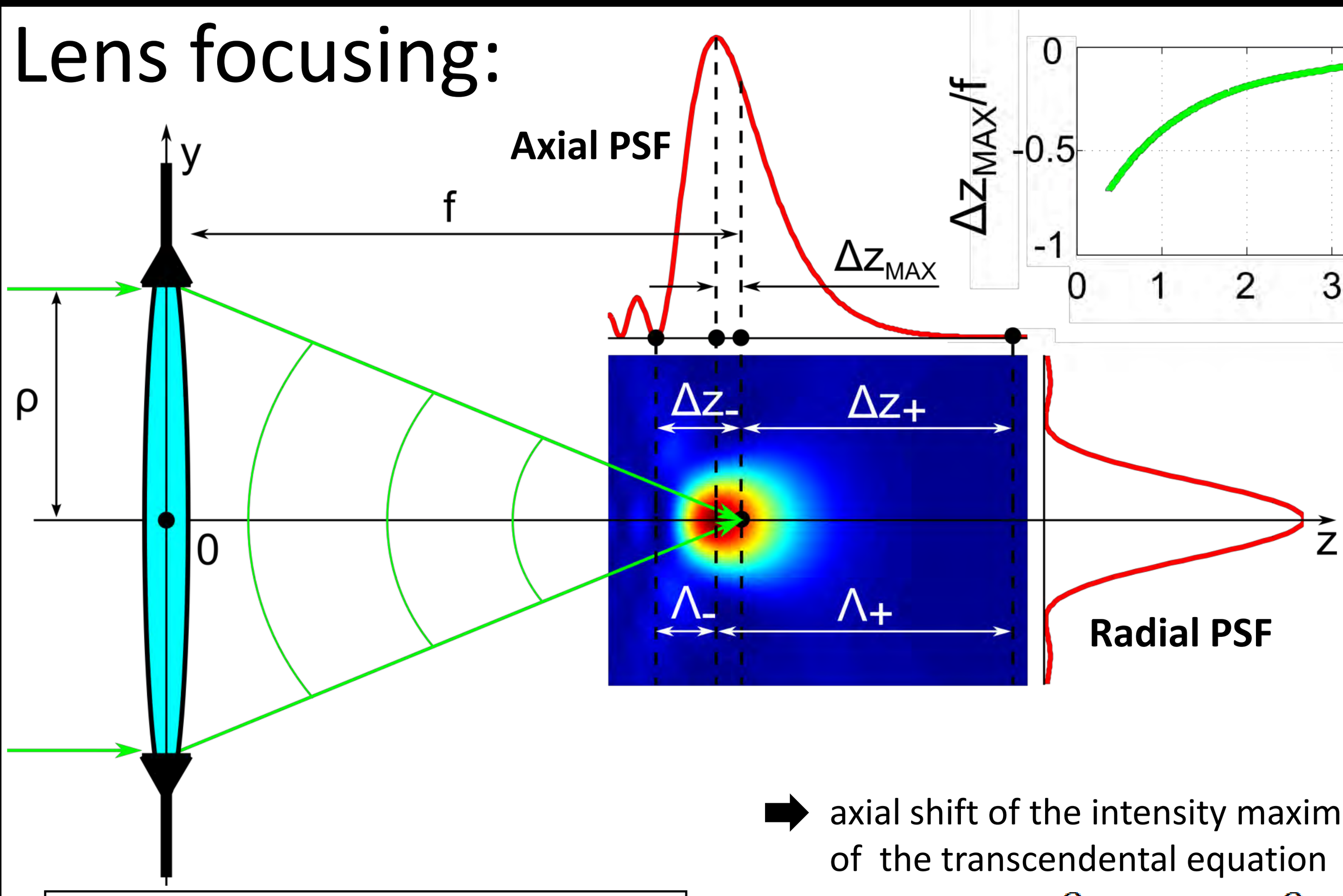
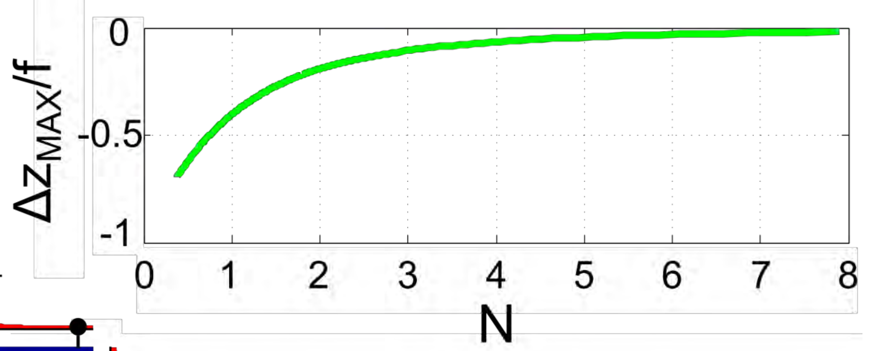
Axial asymmetry and focal shift occurring in lens focusing have been thoroughly investigated in many studies [1-3]. Here we present an extended analysis devoted to three-dimensional (3D) diffraction-limited Point Spread Function (PSF) in digital holography [4-6]. The axial profile and shift of the intensity maximum of the digitally reconstructed PSF are examined for geometries of recording waves commonly used in Digital In-line Holographic Microscopy (DIHM) and Fresnel Incoherent Correlation Holography (FINCH). Experimental configuration and critical parameters resulting in axial imaging asymmetry are assessed in both simulations and experiments.

## Lens focusing:




**Focusing with a low Fresnel number N:**
- nonsymmetry of the axial PSF
- shift of intensity maximum of the axial PSF

**Axial intensity:**

$$I_{0N} = \left(1 - \frac{\Omega}{\pi N}\right)^2 sinc^2\left(\frac{\Omega}{2}\right)$$

$$\Omega = \frac{\pi N \Delta z}{f + \Delta z}, N = \frac{\rho^2}{\lambda f}, \Delta z = z - f$$

➡ axial shift of the intensity maximum: $\Delta z_{MAX}$ – the root of the transcendental equation

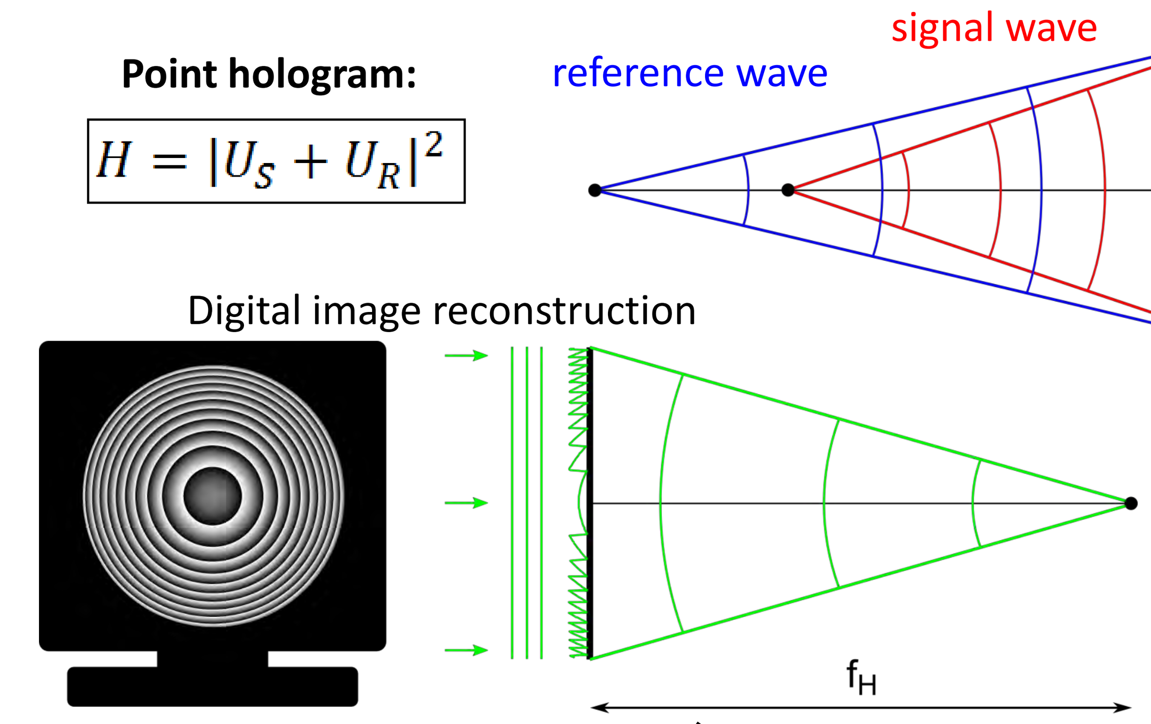
$$tan\left(\frac{\Omega}{2}\right) = \left(1 - \frac{\Omega}{\pi N}\right)^2 \frac{\Omega}{2}, \quad \Omega \epsilon(-2\pi, 0)$$

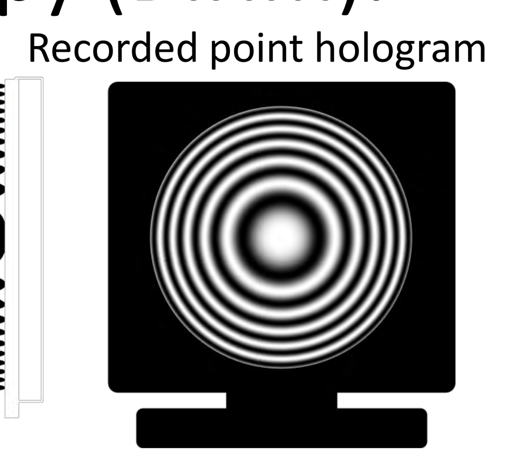
➡ axial PSF nonsymmetry – position of the first zero points

$$\Omega = \pm 2\pi \rightarrow \Delta z_+ = \frac{2f}{N-2}, \quad \Lambda_+ = \Delta z_+ - \Delta z_{MAX}$$

$$\rightarrow \Delta z_- = -\frac{2f}{N+2}, \quad \Lambda_- = \Delta z_- - \Delta z_{MAX}$$

## Digital in-line holographic microscopy (DIHM):




**Recording geometry:**
- lensless DIHM (coherent light)
- DIHM with lens (coherent light)
- FINCH - Fresnel Incoherent Correlation Holography (spatially incoherent light)

**Digital hologram processing:** (phase-shifting)

$$H = |U_S|^2 + |U_R|^2 + U_S U_R^* + U_S^* U_R$$

dc term, real image, virtual image

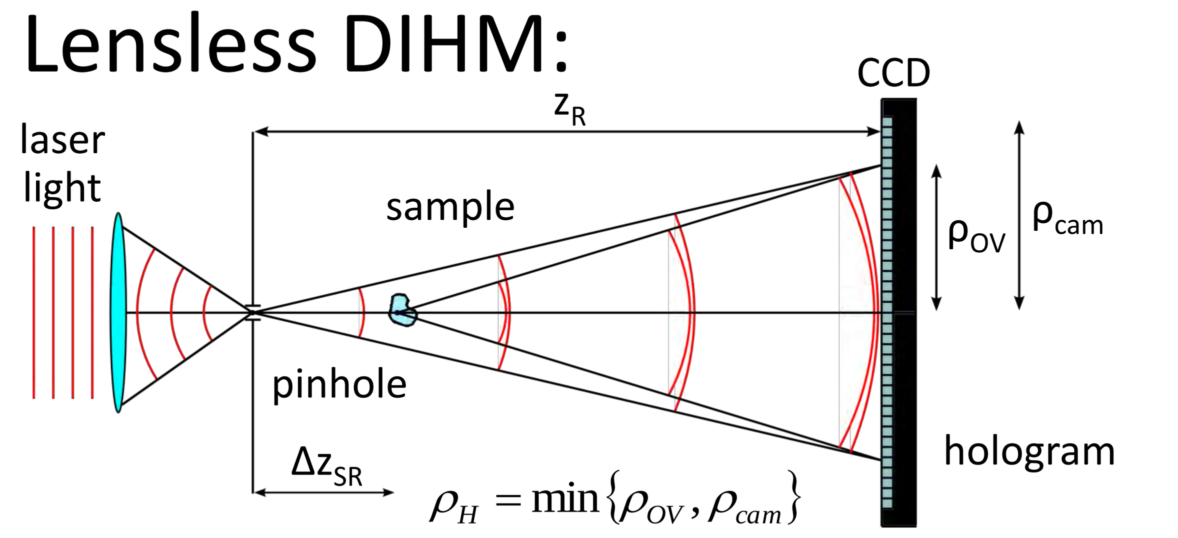
**Image reconstruction = focusing by a diffractive lens** (focal length $f_H$, numerical aperture $NA_H \sim \rho_H / f_H$)

$$U = FrT\{U_S U_R^*\} =$$

$$= FrT\left\{circ\left(\frac{r}{\rho_H}\right) exp\left(-ik\frac{r^2}{2f_H}\right)\right\}$$

*FrT* – Fresnel transform

## Lensless DIHM:




**Hologram** = intensity pattern created by interference of divergent reference and signal waves originating from the pinhole and the individual point scatterers of the sample.

Hologram aperture limited by pinhole diffraction → $\rho_H = 0.61\frac{\lambda|z_R|}{R_P}$

**Resolution** (in object space)

$$\Delta r = R_P\left(1 - \frac{\Delta z_{SR}}{|z_R|}\right)$$

increasing $\Delta z_{SR}$ ⇩ improvement in resolution & PSF symmetry

**Fresnel number**

$$N = 0.61^2 \frac{\lambda \Delta z_{SR}}{R_P \Delta r}$$

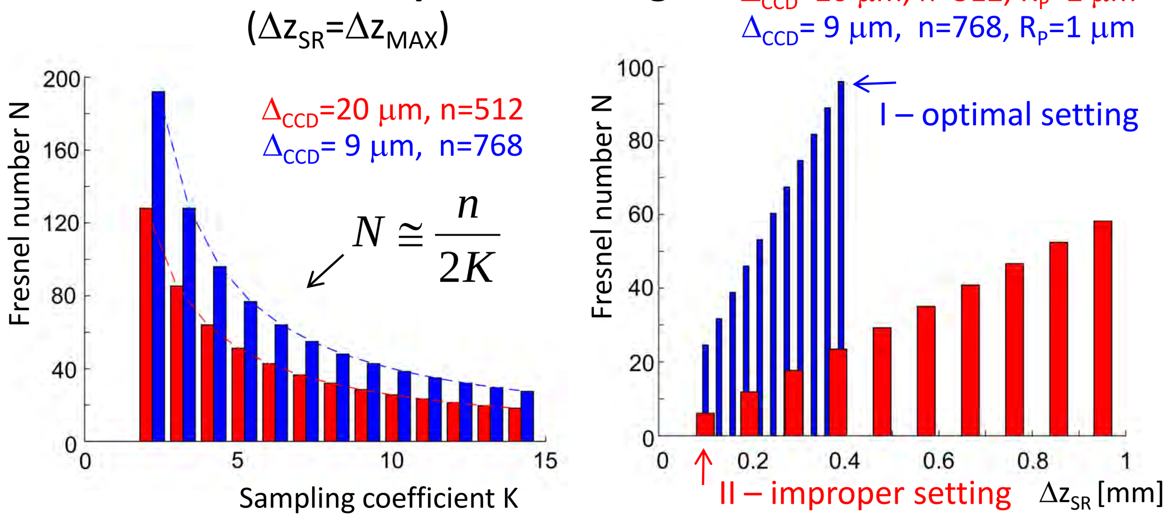
**Limits on sample-pinhole distance $\Delta z_{SR}$** (hologram sampling by CCD pixels)

$$\Delta z_{SR} < \frac{nR_P^2}{K\lambda} = \Delta z_{MAX} \qquad K = \frac{\nu_S}{\nu_H}$$

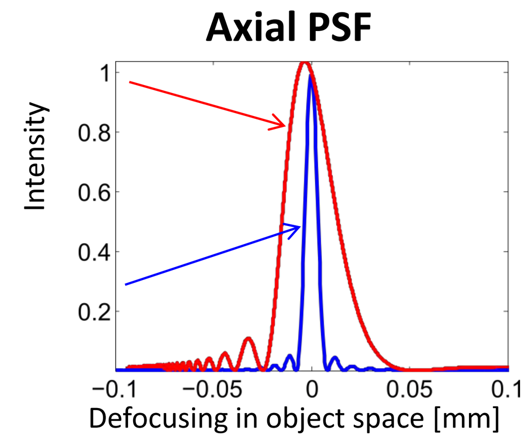
- $n$ ... number of CCD pixels
- $\nu_S$ ... sampling frequency
- $\nu_H$ ... highest hologram frequency

**Fresnel number for optimal setting** ($\Delta z_{SR} = \Delta z_{MAX}$)

$\Delta_{CCD}$=20 μm, n=512, $R_P$=2 μm
$\Delta_{CCD}$= 9 μm, n=768, $R_P$=1 μm

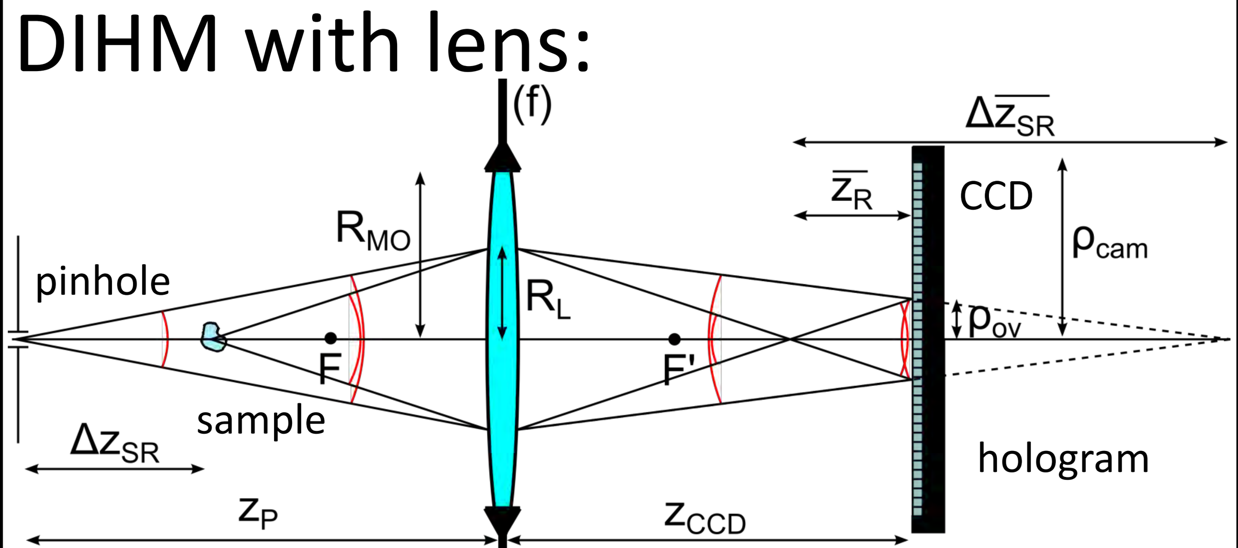



Radial resolution
$\Delta r \approx 2$ μm
$\Delta r \approx 1$ μm




**Design of lensless DIHM**
- when using the largest sample-pinhole distance $\Delta z_{SR} = \Delta z_{MAX}$, a perfect PSF symmetry is achieved with commonly available CCDs
- with improper sample-pinhole setting $\Delta z_{SR} < \Delta z_{MAX}$, the axial PSF is apparently asymmetrical and its maximum is shifted out of the paraxial image plane

## DIHM with lens:




**Hologram** created by interference of divergent reference wave (wavefront radius $\bar{z}_R$) and convergent signal wave (wavefront radius $\bar{z}_S$), originating from the pinhole and the point scatterer of the sample and transformed by a lens with the focal length *f*.

**Optimal CCD position** (full overlapping of interfering waves)

$$z_{CCD} = \frac{2|\bar{z}_R|\bar{z}_S}{\bar{z}_R + \bar{z}_S}$$

**Resolution** (in object space)

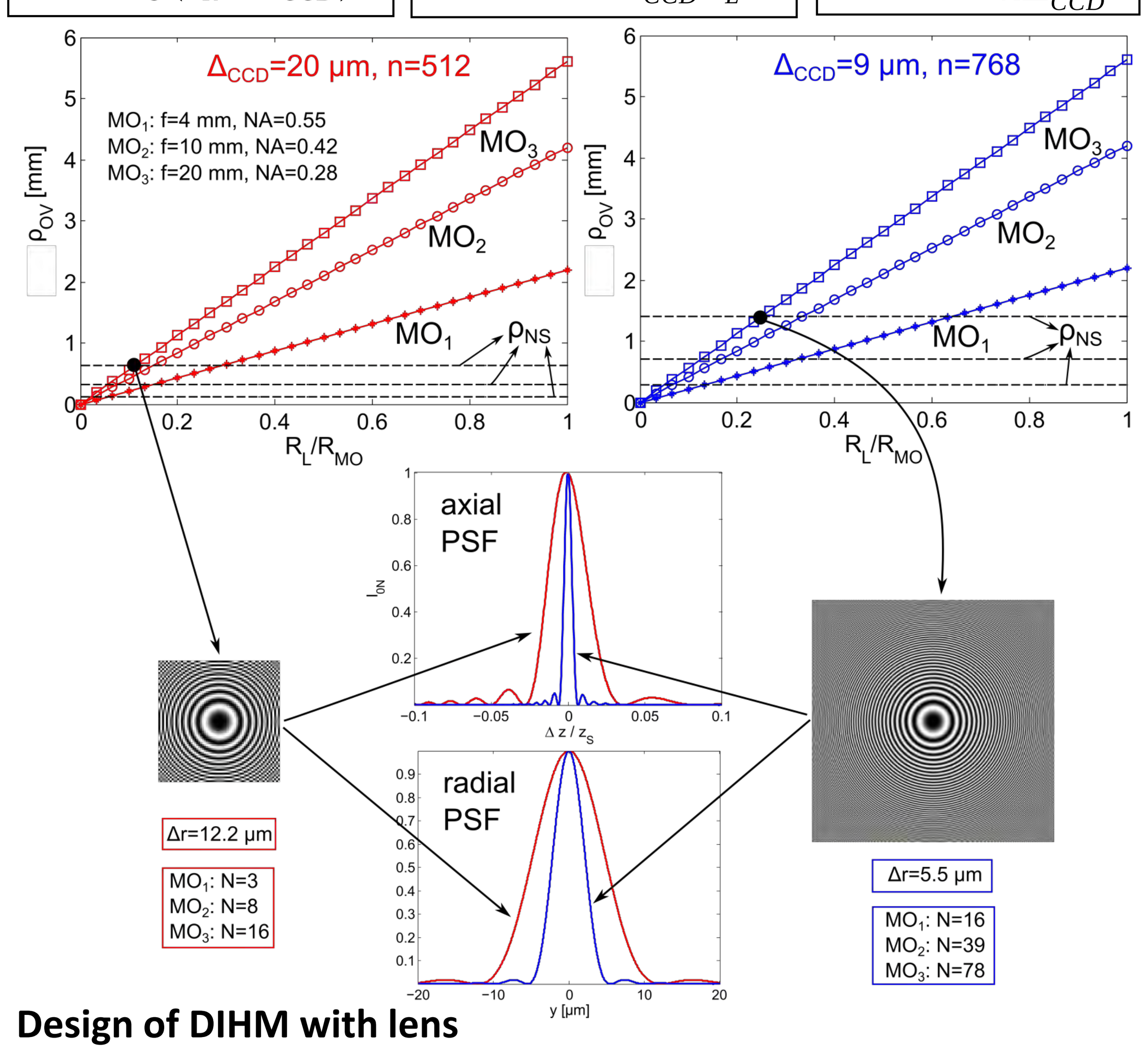
$$\Delta r = 0.61\frac{\lambda \bar{z}_S}{R_L}\left(1 + \frac{z_{CCD}}{|\bar{z}_R|}\right)$$

**Fresnel number**

$$N = \frac{R_L^2 |\bar{z}_R| \Delta \bar{z}_{SR}}{\lambda \bar{z}_S(\bar{z}_R + z_{CCD})^2}$$

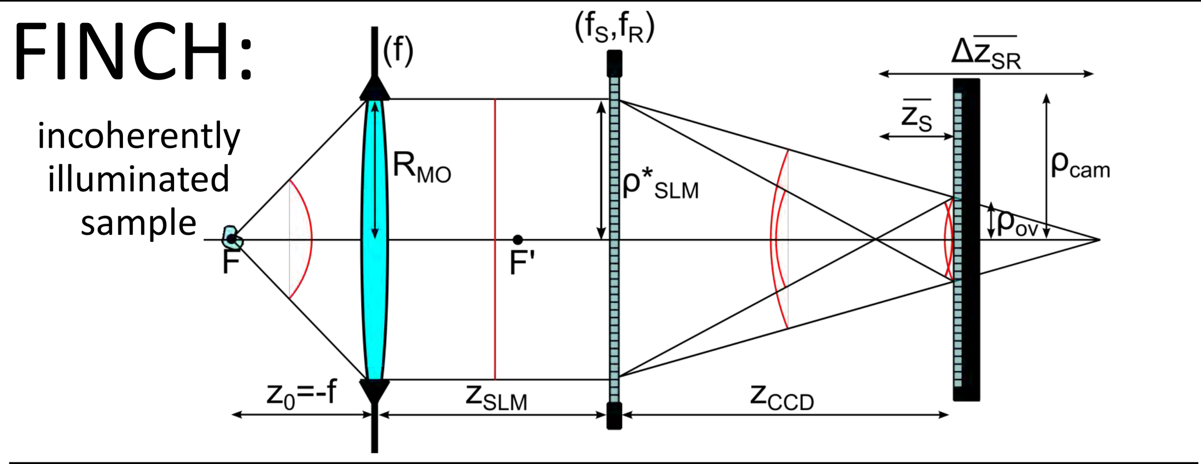
**Sampling conditions**

$$\Delta \bar{z}_{SR} < \frac{\lambda \bar{z}_S(\bar{z}_R + z_{CCD})}{K \Delta_{CCD} R_L} \qquad |\bar{z}_R| < \frac{z_{CCD}}{1 + \frac{2R_L}{n\Delta_{CCD}}}$$




**Design of DIHM with lens**
- due to hologram sampling, the lens aperture with radius $R_{MO}$ must be reduced to radius $R_L$ decreasing both resolution and Fresnel number (PSF asymmetry)
- sampling conditions do not allow the use of microscope objective with high magnification and numerical aperture
- hologram recording requires a high resolution CCD

## FINCH:




**Point hologram** is created by interference of reference and signal waves that originate from the same point of incoherently illuminated sample and are obtained by splitting the single wave by a Spatial Light Modulator (SLM).

**Particular FINCH properties**
- hologram recording in spatially incoherent quasi-monochromatic light [4] (fluorescence, LED illumination)
- violation of Lagrange invariant [5] for overcoming diffraction resolution limit [6]
- dependence of resolution and shape of PSF on temporal coherence of light [7]

**Optimal CCD position** (diffraction resolution limit exceeded twice)

$$z_{CCD} = \frac{2 f_S f_R}{f_S + f_R} \qquad \text{special case: } f_R \rightarrow \infty, \quad z_{CCD} = 2 f_S$$

**Reconstructed diffractive lens** (point hologram)

$$f_H = \frac{(z_{CCD} - f_S)(f_R - z_{CCD})}{f_R - f_S} \quad \text{⇦ focal length} \qquad \text{Fresnel number ⇨} \quad N = \frac{\rho_H^2}{\lambda f_H}$$

geometrical aperture (GA) $\quad \rho_H = \min\{\rho_{OV}, \rho_C\} \quad$ coherence aperture (CA)

$$\rho_{OV} \equiv \rho_{OV}(R_{MO}, \Delta \bar{z}_{SR}, z_{CCD}) \qquad \rho_C \equiv \rho_C(OPD, L_C)$$

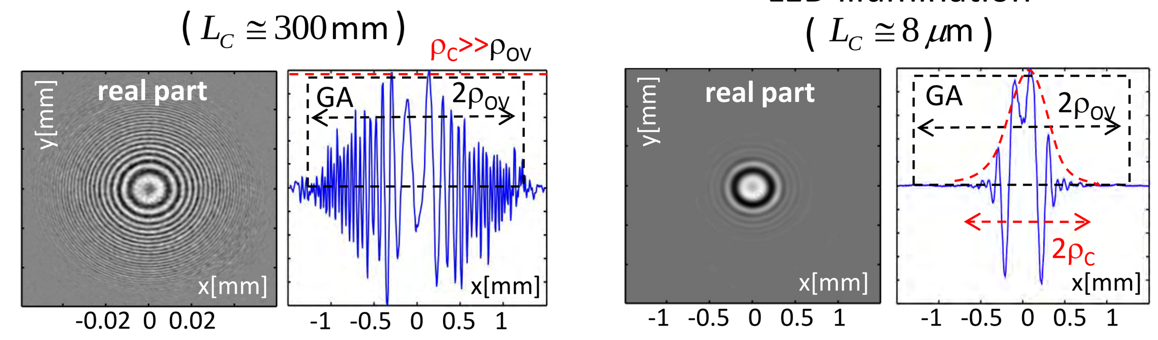
Optical Path Difference, coherence length

**Experimental results**

Point holograms

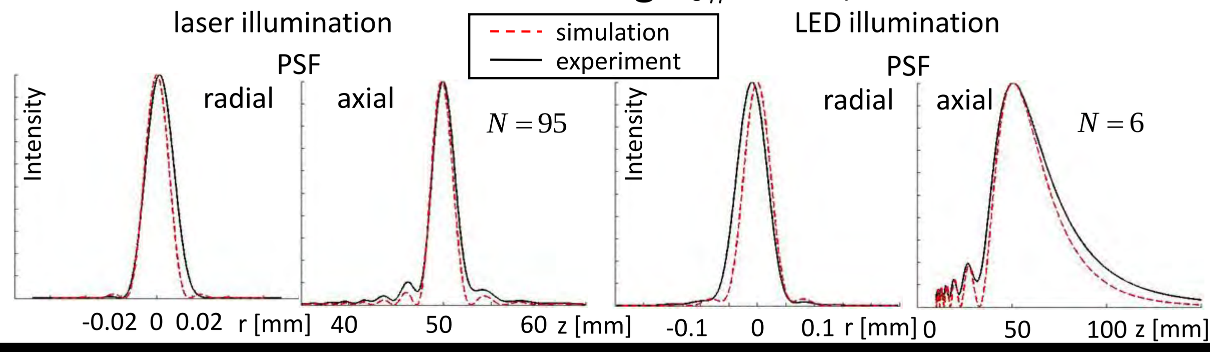
the same geometrical aperture x different temporal coherence ($f_S = 400$ mm, $f_R = 649$ mm, $z_{CCD} = 495$ mm, $R_{MO} = 5$ mm)

laser illumination ($L_C \cong 300$ mm) | LED illumination ($L_C \cong 8\,\mu m$)




Geometrical aperture given by overlapping of interfering waves is reduced by **coherence aperture**, whose width $\rho_C$ decreases with shortening of the coherence length of light $L_C$.

**Reconstructed image** ($f_H$=58 mm)




**Conclusions:**
- Based on previous studies on focusing systems with low Fresnel number, the axial asymmetry of the PSF was examined in experiments of digital holography.
- An optimal design of experiments on lensless in-line holography was made ensuring a nearly perfect axial symmetry of the PSF.
- Degradation of resolution caused by hologram sampling was revealed in systems of in-line holography using a high magnification microscope objective for transformation of interfering waves.
- Connection of the axial PSF asymmetry with the temporal coherence of light was demonstrated experimentally using a common-path interferometer enabling FINCH imaging.

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