Narrowing the temporal wavepacket of SPDC photons

Mikołaj Lasota, Karolina Sędziak,

Piotr Kolenderski

Faculty of Physics, Astronomy and Informatics, Nicolaus Copernicus University, Toruń, Poland

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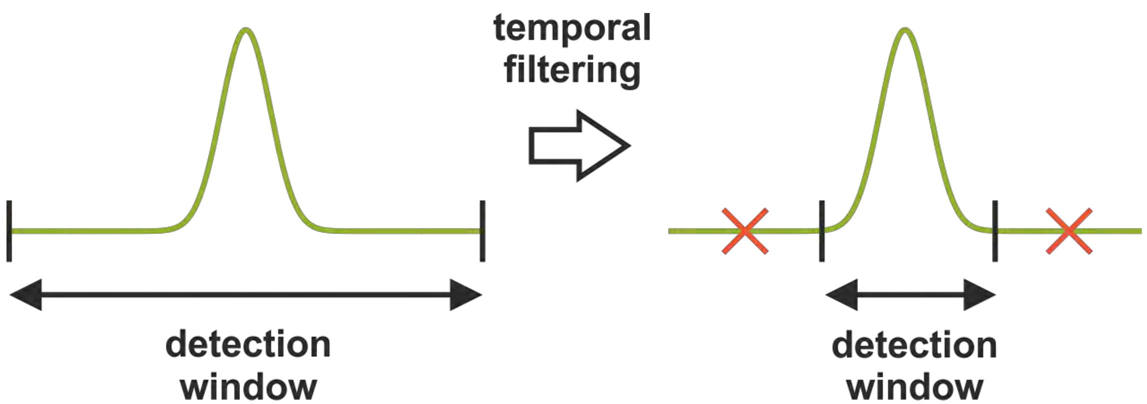
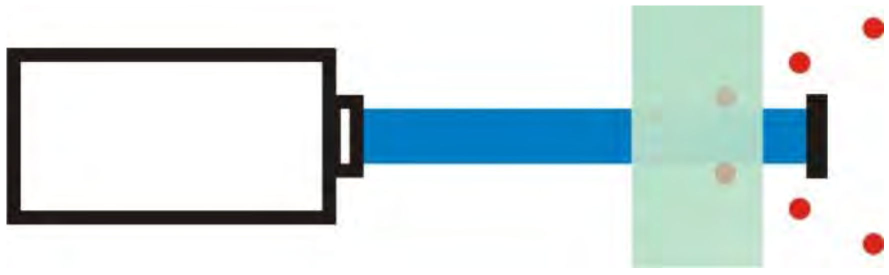
NATIONAL LABORATORY OF ATOMIC, MOLECULAR AND OPTICAL PHYSICS

Background

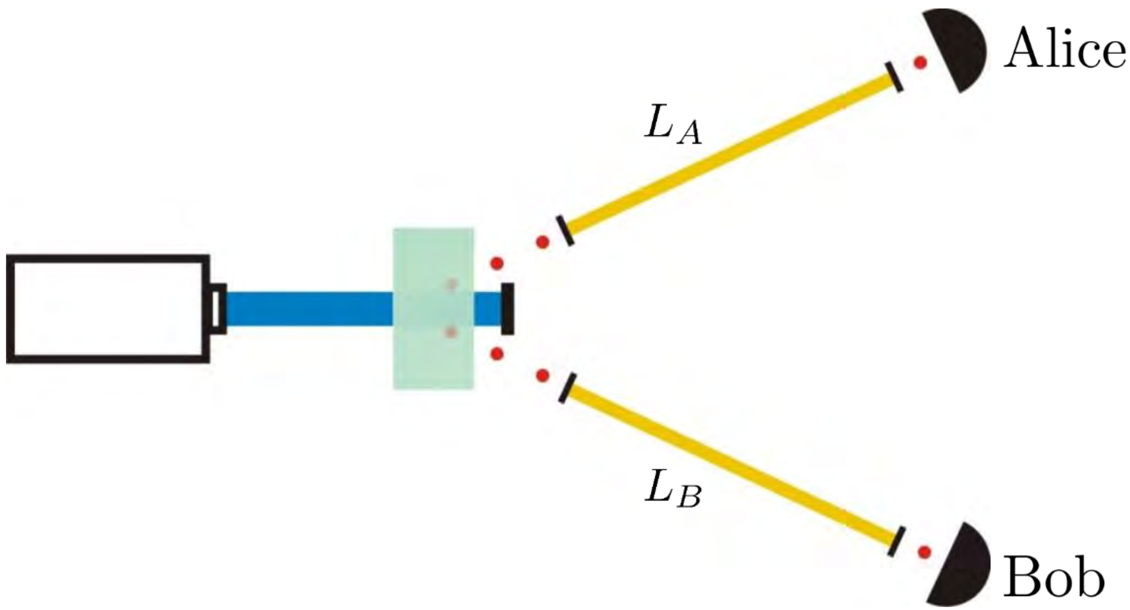
- Major problem of realistic quantum communication: errors
- An efficient way to reduce errors independent from the real signals: temporal filtering



- Main limitation on temporal filtering in long-distance communication: temporal broadening of signal



Detection scheme for SPDC photons

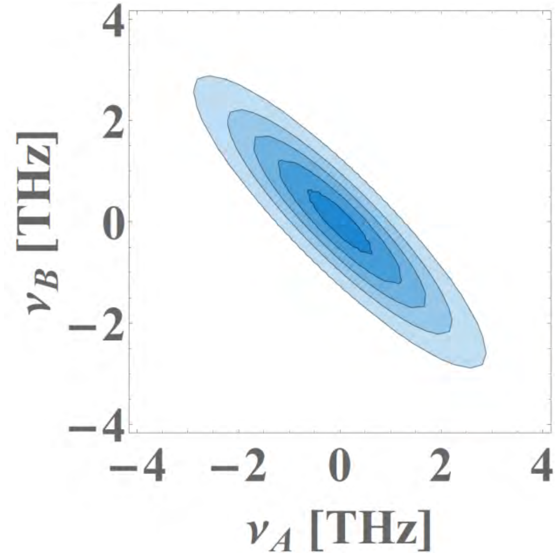


- Our goal: investigating the connection between the temporal width of a photon arriving at Alice’s detector and the type of spectral correlation between this photon and the corresponding photon travelling to Bob
- Temporal width of a given photon = standard deviation of the probability distribution function for the arrival time of this photon to the detector

Spectral correlation in SPDC process

- Types of spectral correlation:

  - **negative (typical)**



Spectral correlation in SPDC process

- Types of spectral correlation:
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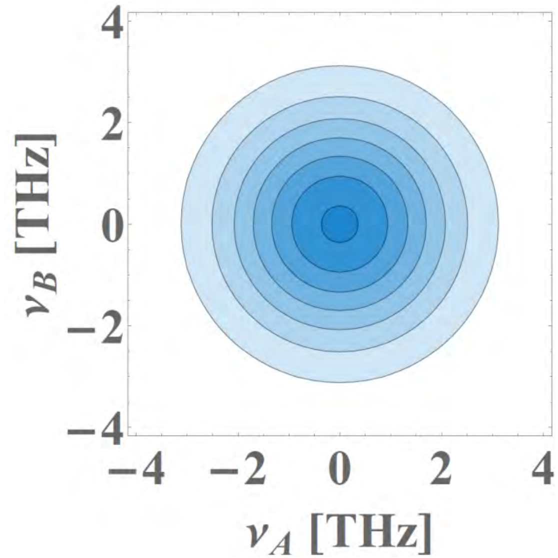
  - **no correlation**



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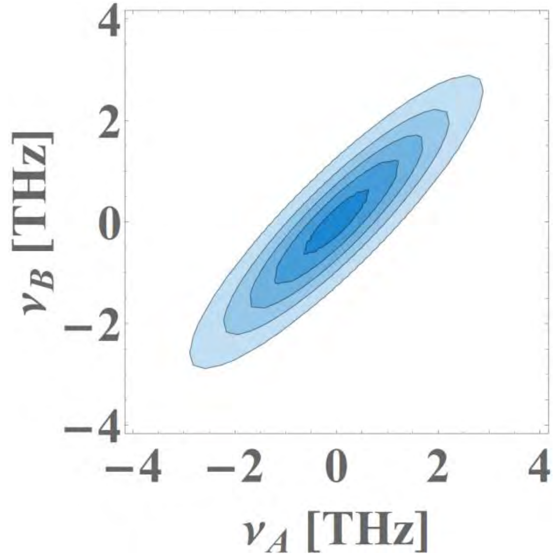

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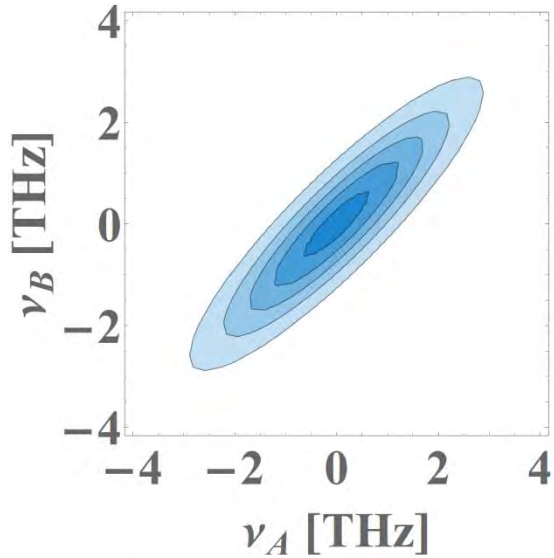

- The analytical approximation of the spectral wave function (at the input of the fibers):

$$\phi(\nu_A, \nu_B) = \frac{1}{\sqrt{\pi}\sqrt{\sigma_A \sigma_B \sqrt{1-\rho^2}}} \exp\left(-\frac{1}{2\left(1-\rho^2\right)}\left(\frac{\nu_A^2}{\sigma_A^2} + \frac{\nu_B^2}{\sigma_B^2} - \frac{2\nu_A \nu_B \rho}{\sigma_A \sigma_B}\right)\right)$$

where

σ_A, σ_B - spectral widths of the emitted photons

ρ – spectral correlation coefficient (negative: $-1 < \rho < 0$,

no correlation: $\rho = 0$, positive: $0 < \rho < 1$)

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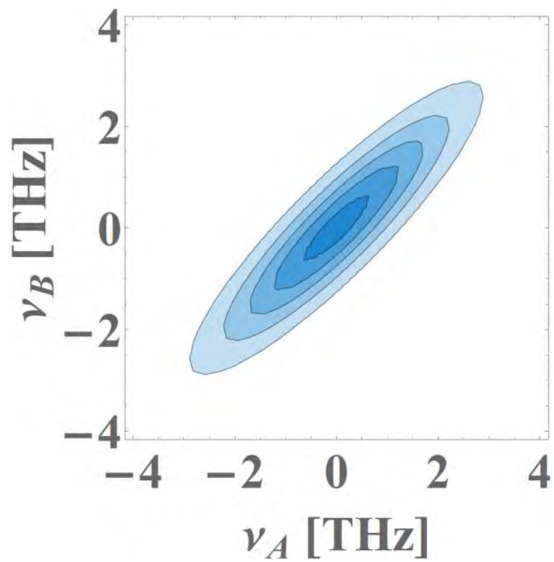

- The analytical approximation of the spectral wave function (at the input of the fibers) – for symmetric source:

$$\phi(\nu_A,\nu_B) = \frac{1}{\sigma_0\sqrt{\pi}\sqrt[4]{1-\rho^2}} \exp\left(-\frac{\nu_A^2+\nu_B^2-2\nu_A\nu_B\rho}{2\sigma_0^2\left(1-\rho^2\right)}\right)$$

where

σ_0 – spectral width of the emitted photons

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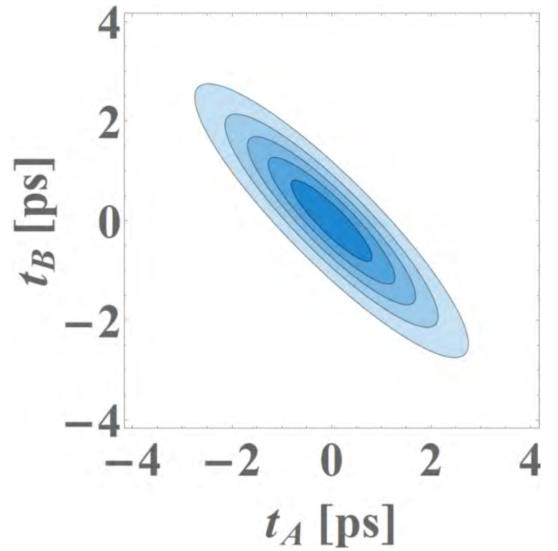
Temporal correlation in SPDC process

- Initial temporal correlation:
  opposite to the spectral correlation



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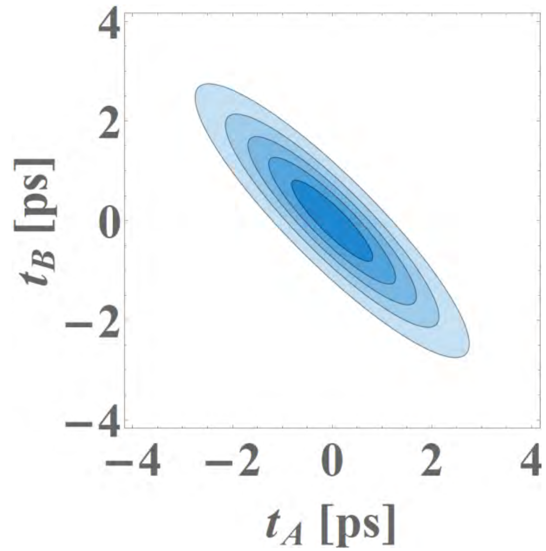

- Temporal wave function at the output of the fibers of length *L*:

$$\psi_{L_A L_B}(t_A, t_B) = \int \mathrm{d}t'_A\, \mathrm{d}t'_B\, \mathcal{S}_A(t_A, t'_A, L_A)\mathcal{S}_B(t_B, t'_B, L_B)\psi(t'_A, t'_B)$$

where

$\psi(t_A, t_B)$ - initial temporal wave function,

$$\mathcal{S}_k(t_k, t'_k, L_k) = \frac{1}{\sqrt{4\pi i \beta_k L_k}} \exp\left(\frac{i(t_k - t'_k)^2}{4\beta_k L_k}\right)$$

β_k – group velocity dispersion

Temporal correlation in SPDC process

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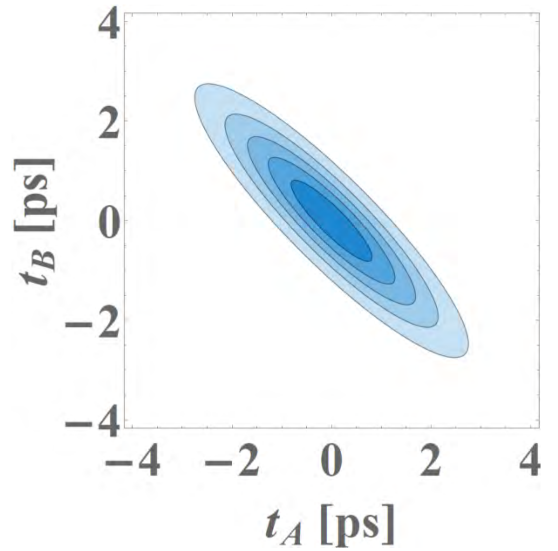
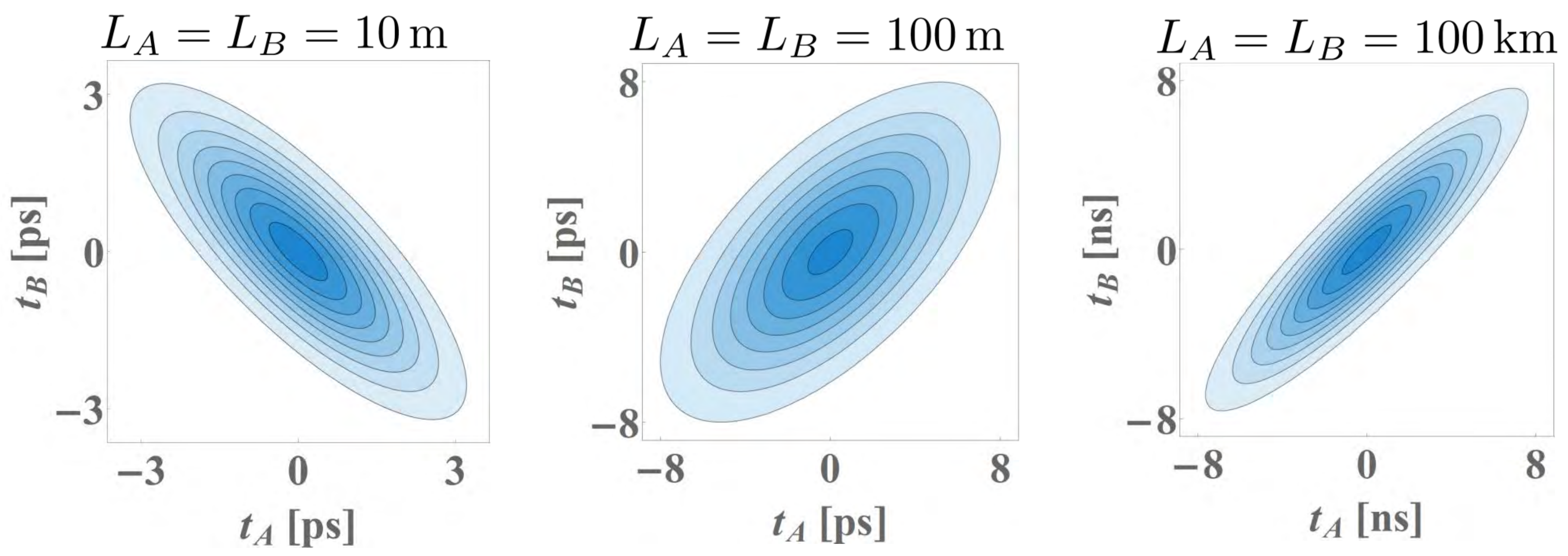


- Effect of the propagation:



Temporal correlation in SPDC process

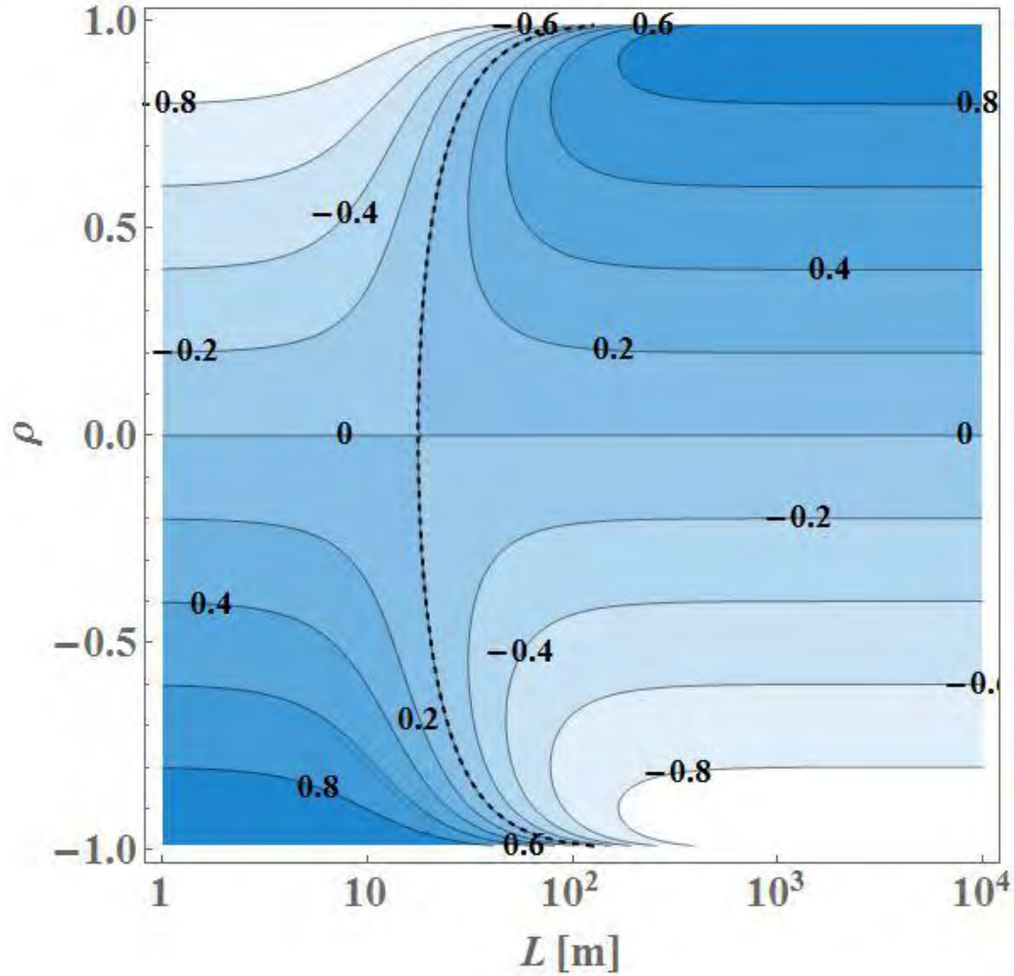
- Paerson correlation coefficient:

$$r_{t_A t_B} = \frac{E\left[(t_A - E[t_A])(t_B - E[t_B])\right]}{\sqrt{\left[E\left[(t_A - E[t_A])^2\right] E\left[(t_B - E[t_B])^2\right]\right]}}$$

- For the wavefunction $\psi_L(t_A, t_B)$:

$$r_{t_A t_B}(L, \rho) = -\rho \frac{1 - 4\sigma_0^4 \beta^2 L^2 (1 - \rho^2)}{1 + 4\sigma_0^4 \beta^2 L^2 (1 - \rho^2)}$$

Temporal correlation in SPDC process



- The Pearson coefficient as a function of the propagation distance and the spectral correlation coefficient. The values $\beta = -1.15 \times 10^{-26} \frac{\mathrm{s}^2}{\mathrm{m}}$ and $\sigma_0 = 1.57\ \mathrm{THz}$ are assumed.

Temporal correlation in SPDC process

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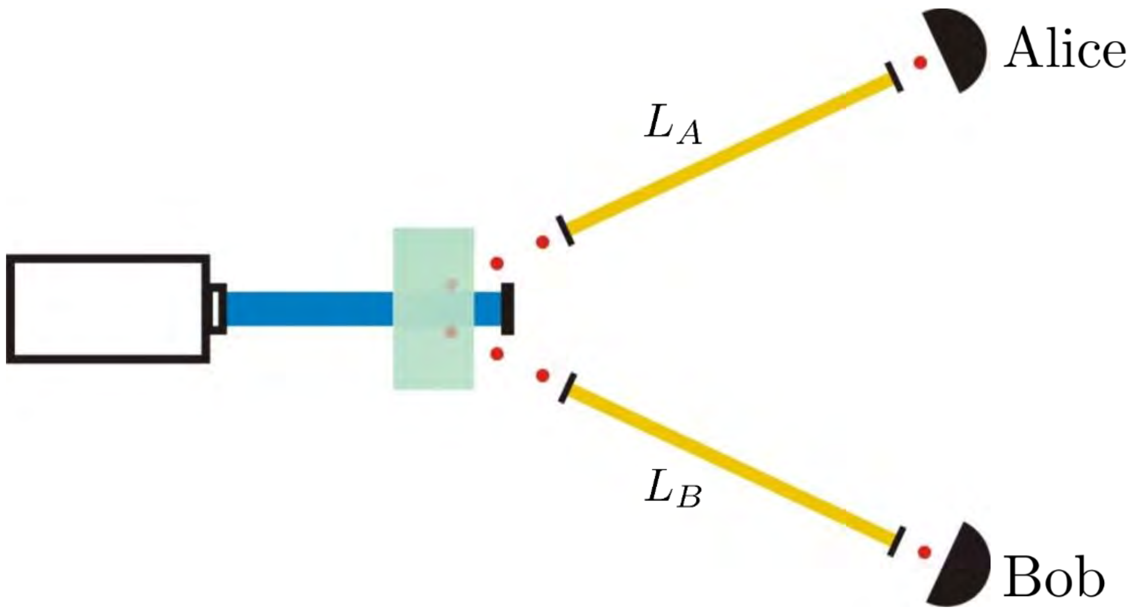
- The distance at which $r_{t_A t_B} = 0$:

$$L_0 = \frac{1}{2\sigma_0^2 \beta \sqrt{1 - \rho^2}}$$

Detection scheme for SPDC photons



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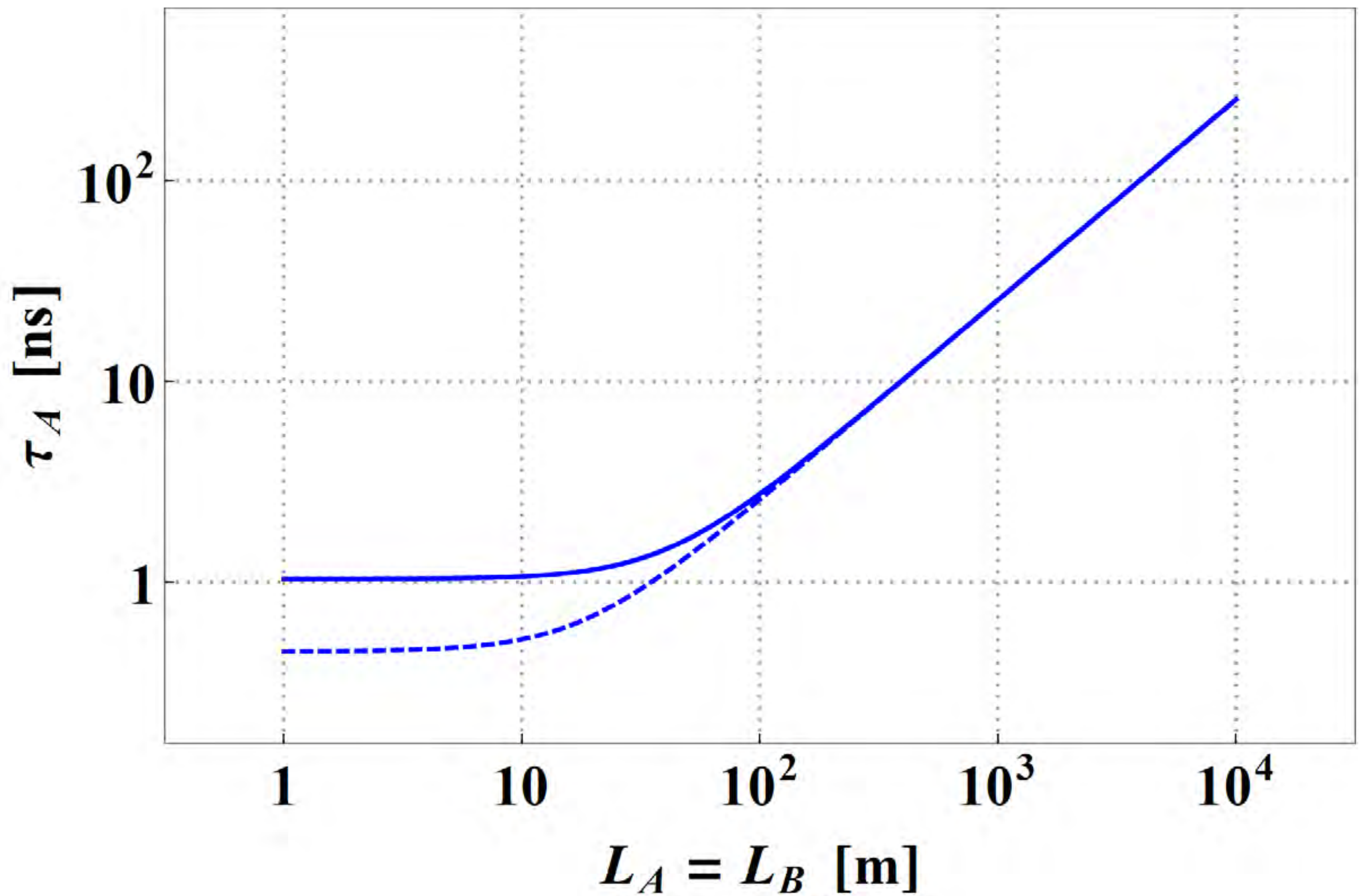
Detection time characteristics

- Temporal width of Alice's photon if the **global time reference** is **available** but the **detection time of Bob's photon** is **not known**:

$$\tau_A(\sigma_A) = \sqrt{\frac{g(x_A^2)}{2\sigma_A^2(1-\rho^2)}}$$ where $g(x) = 1 + x(1-\rho^2)$ and $x_Y = 2\sigma_Y^2 \beta_Y L_Y$

Detection time characteristics



Results for $\rho = 0.9$ and $\rho = -0.9$ (solid) and for $\rho = 0$ (dotted).

Assumptions: $\beta_A = \beta_B = -1.15\ \times 10^{-26}\,\frac{\mathrm{s}^2}{\mathrm{m}}$ and $\sigma_A = \sigma_B = 1.57\ \mathrm{THz}$

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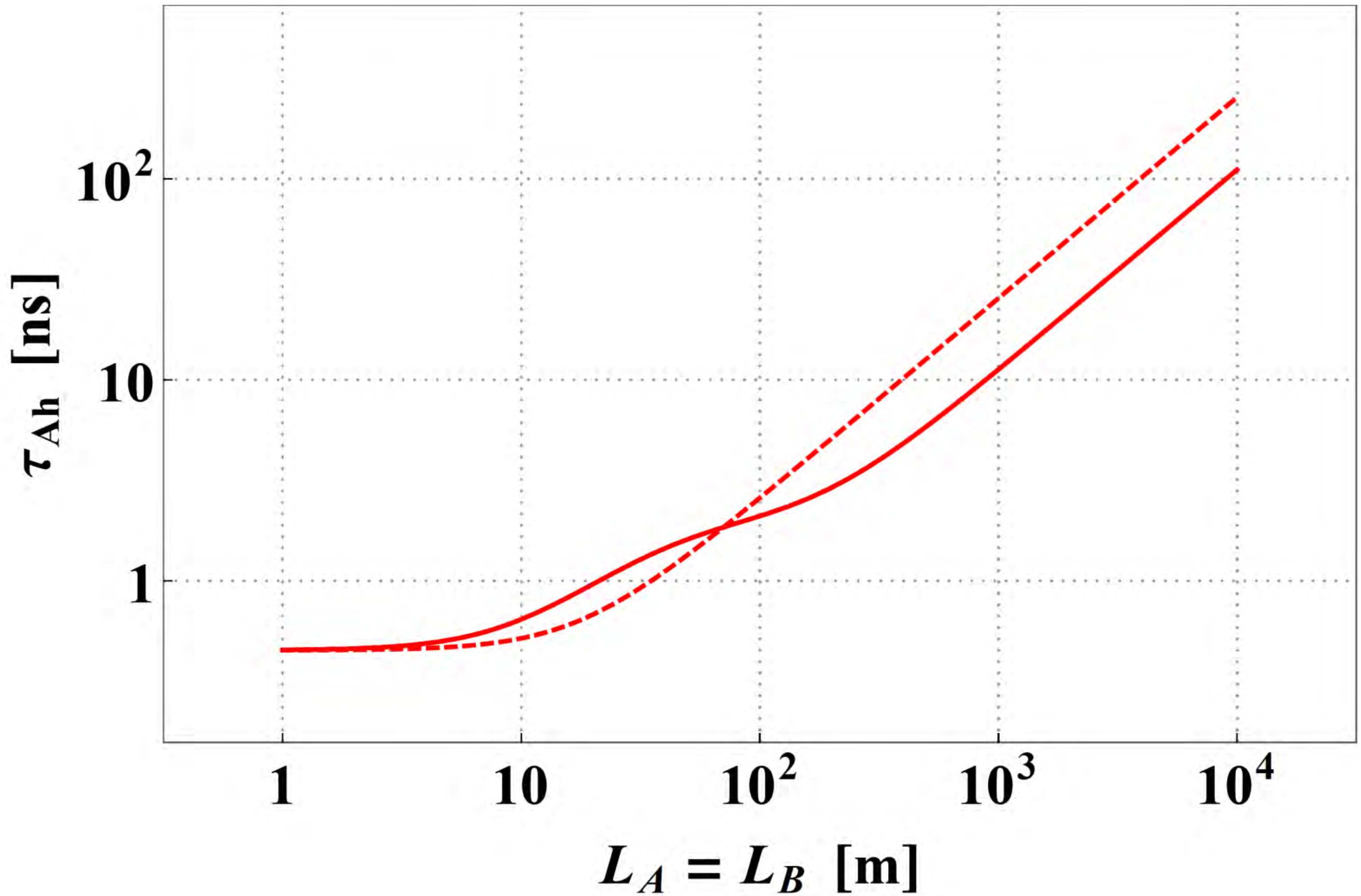
- Temporal width of Alice's photon if the **global time reference** is **available** and the **detection time of Bob's photon** is **known**:

$$\tau_{Ah}(\sigma_A, \sigma_B) = \sqrt{\frac{[g(-x_A x_B)]^2 + (x_A + x_B)^2}{2\sigma_A^2 g(x_B^2)}}$$

Detection time characteristics



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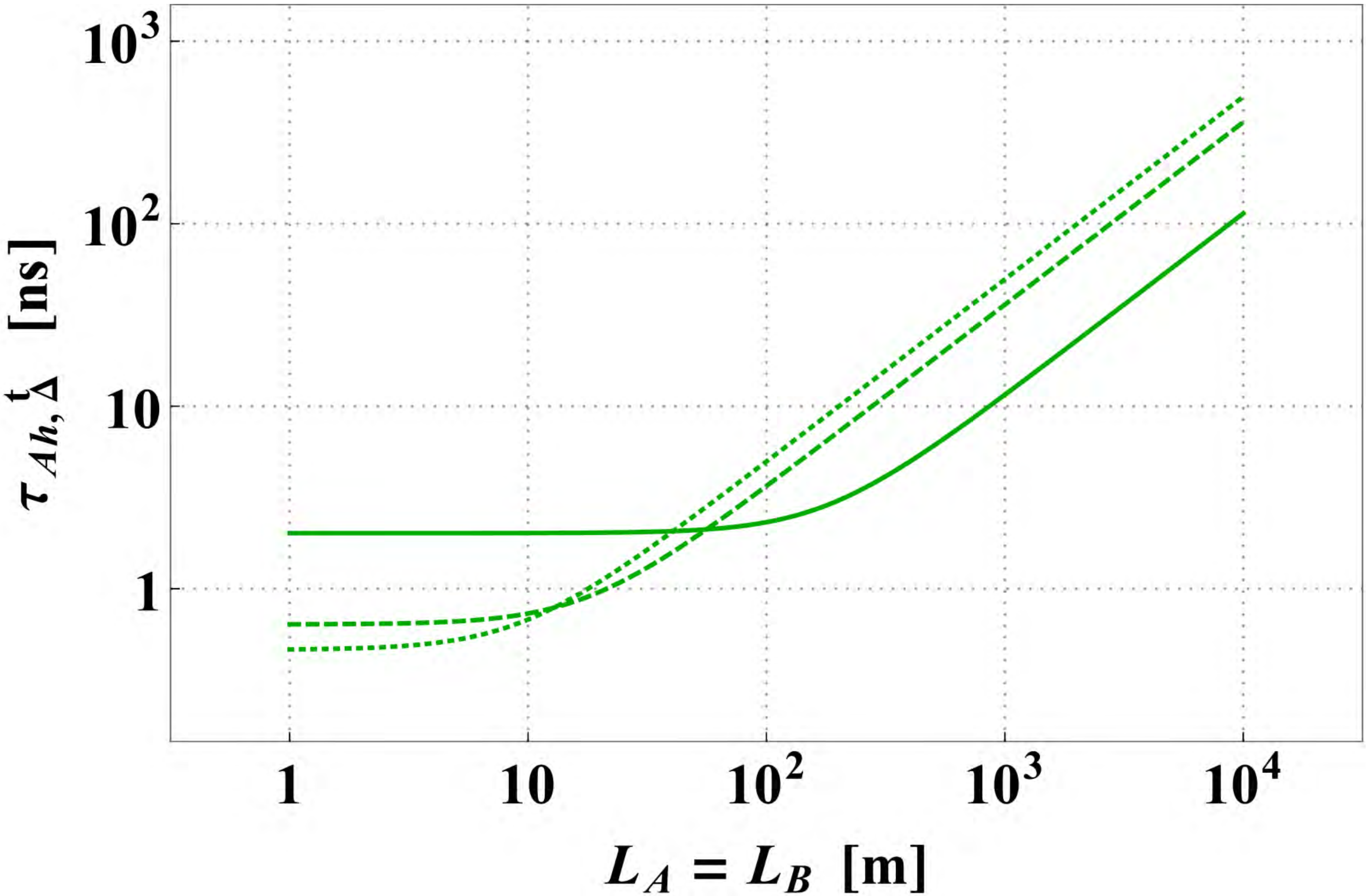
- Temporal width of Alice's photon if the **global time reference** is **not available**, but **she can measure** $\Delta t = t_A - t_B$:

$$\tau_{Ah,\Delta t}(\sigma_A, \sigma_B) = \sqrt{\frac{\{g(x_A^2)\sigma_B^2 + g(x_B^2)\sigma_A^2 + 2g(-x_A x_B)\sigma_A\sigma_B\rho\}\left\{\left[g(-x_A x_B)\right]^2 + (x_A + x_B)^2\right\}}{2\sigma_A^2\sigma_B^2\left[g(x_A^2)g(x_B^2) - \left[g(-x_A x_B)\right]^2\rho^2\right]}}$$

Detection time characteristics



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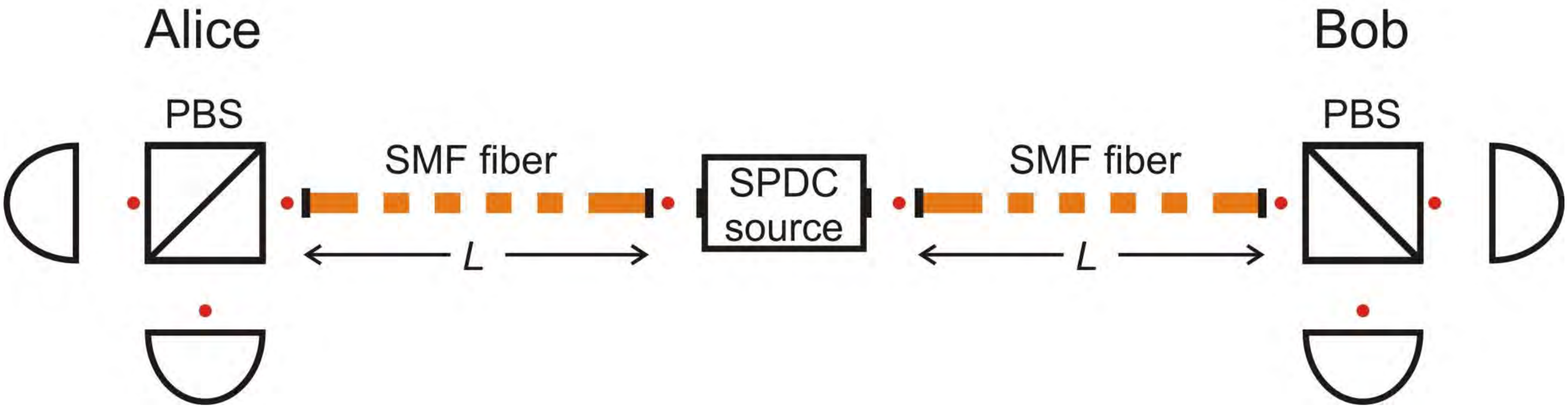
Symmetric quantum key distribution

- Discrete-variable QKD setup with source of photons located in the middle:



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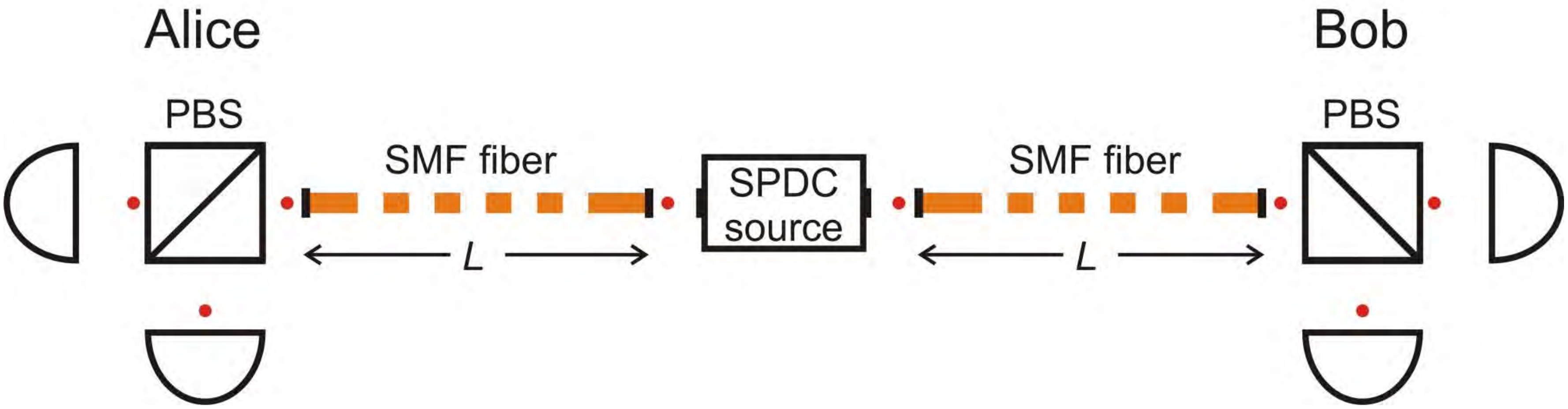

- Assumptions:

  - BB84 protocol

  - duration time of a single detection window = six temporal widths
    (99,73% probability for successful detection)

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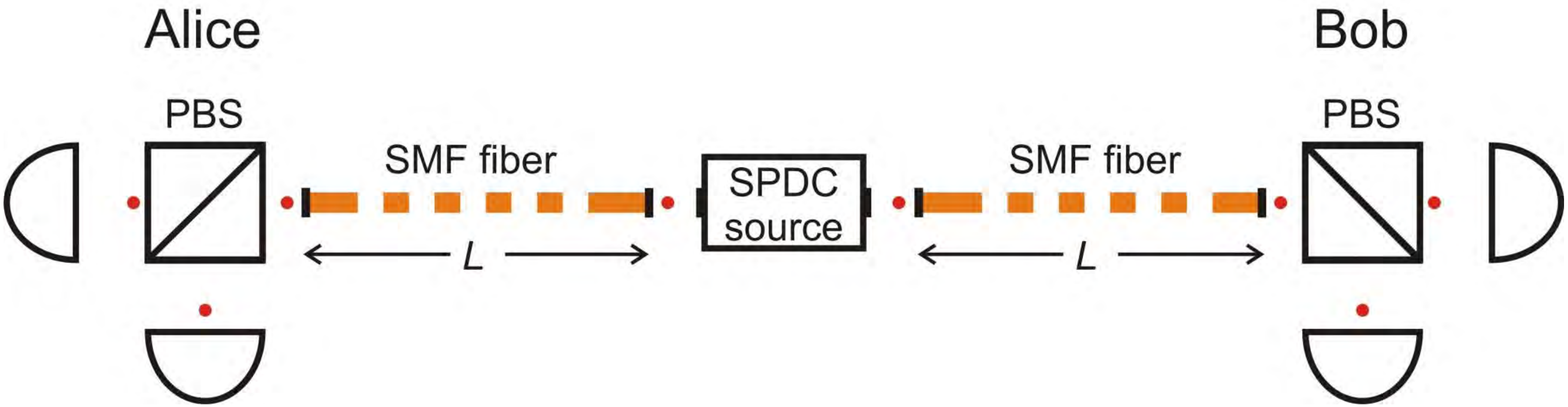
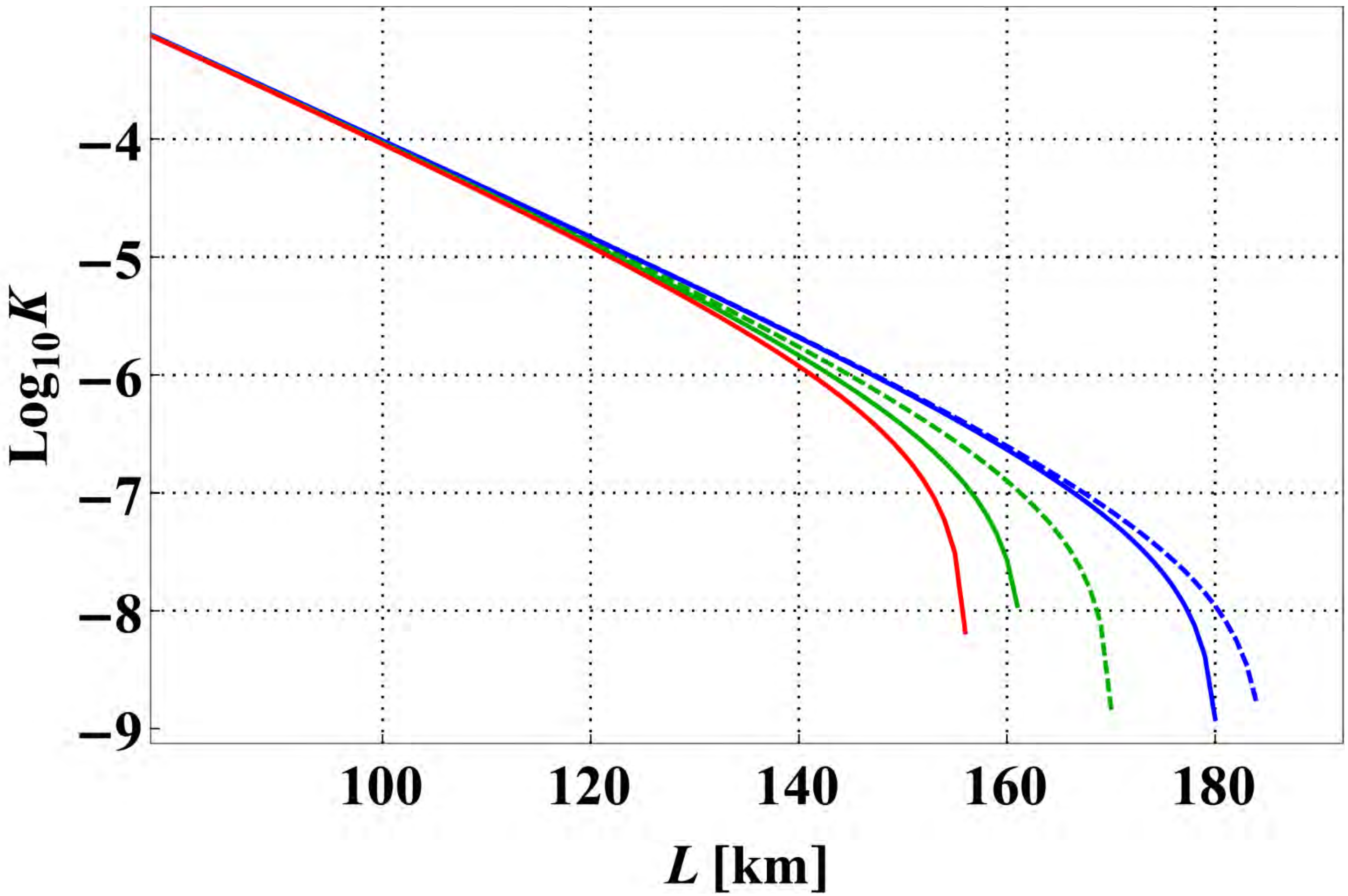


- Technical assumptions:

  - the only source of errors: dark counts (rate d = 1kHz)
  - transmittance of the fibers: $T = 10^{-\alpha L/10}$, where $\alpha = 0.2\,\mathrm{dB/km}$
  - group velocity dispersion (GVD) for the SMF fibers:
    $2\beta_A = 2\beta_B = -2.3 \times 10^{-26}\mathrm{s}^2/\mathrm{m}$
  - no other setup imperfections

Symmetric quantum key distribution



- Spectral widths: $\boldsymbol{\sigma_A = \sigma_B =}$ **1.57 THz**
- Spectral correlation: $\rho = -0.9$ (red), $\rho = 0$(green), $\rho = 0.9$ (blue)
- Availability of global time reference: yes (dashed), no (solid)

Symmetric quantum key distribution

- Conclusions:

  - strong spectral correlation (positive or negative) is better than no correlation at all for the case with global time reference

  - positive spectral correlation is the best and negative spectral correlation is the worst for the case without global time reference

Symmetric quantum key distribution

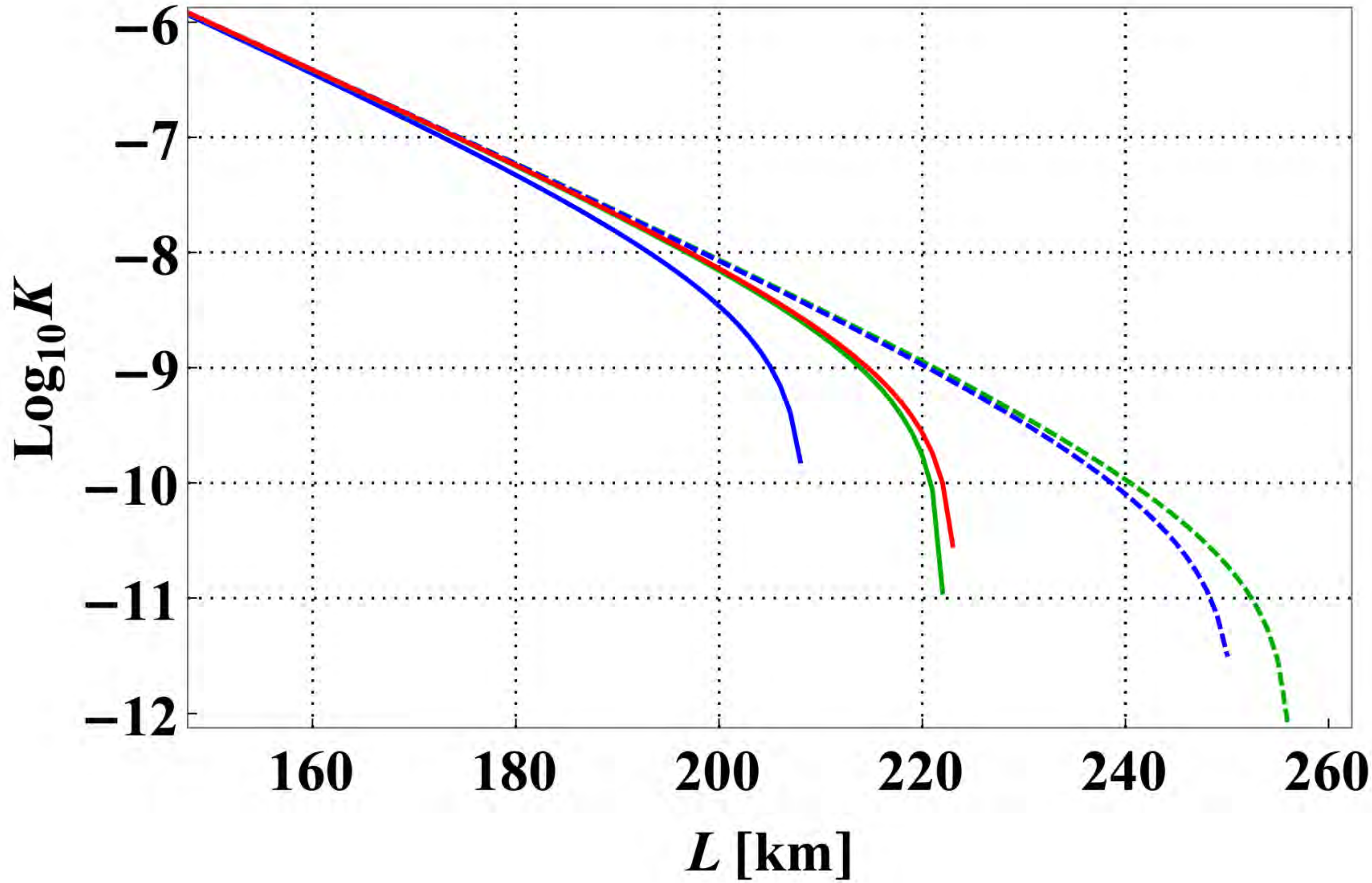
- Conclusions:

  - strong spectral correlation (positive or negative) is better than no correlation at all for the case with global time reference

  - positive spectral correlation is the best and negative spectral correlation is the worst for the case without global time reference

  - for spectrally narrow pairs the situation can be totally different!

Symmetric quantum key distribution



- Spectral widths: $\boldsymbol{\sigma_A = \sigma_B =}$ **10 GHz**
- Spectral correlation: $\rho = -0.9$ (red), $\rho = 0$(green), $\rho = 0.9$ (blue)
- Availability of global time reference: yes (dashed), no (solid)

SPDC spectral wavefunction in another parametrization

- Spectral wavefunction at the input of the fibers – „old parametrization”:

$$\phi(\nu_A, \nu_B) = \frac{1}{\sigma_0 \sqrt{\pi} \sqrt[4]{1-\rho^2}} \exp\left(-\frac{\nu_A^2 + \nu_B^2 - 2\nu_A \nu_B \rho}{2\sigma_0^2 \left(1-\rho^2\right)}\right)$$

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- Spectral wavefunction at the input of the fibers – „new parametrization” (collinear case):

$$\phi(\nu_A, \nu_B) = M \exp\left(-\frac{(\nu_A - \nu_B)^2}{\sigma^2} - \frac{(\nu_A + \nu_B)^2 \tau_p^2}{4}\right)$$

where
σ – characteristic width of the effective phase-matching function
τ_p – pump laser pulse duration

Optimizing a given SPDC source

- Temporal width of Alice's photon in the new parametrization:

  - when Alice knows only the emission time of a given pair of photons

$$\tau_A(\sigma,\tau_p)=\frac{\sqrt{\left(\tau_p^2+\beta_A^2L_A^2\sigma^2\right)\left(4+\sigma^2\tau_p^2\right)}}{2\sigma\tau_p}$$

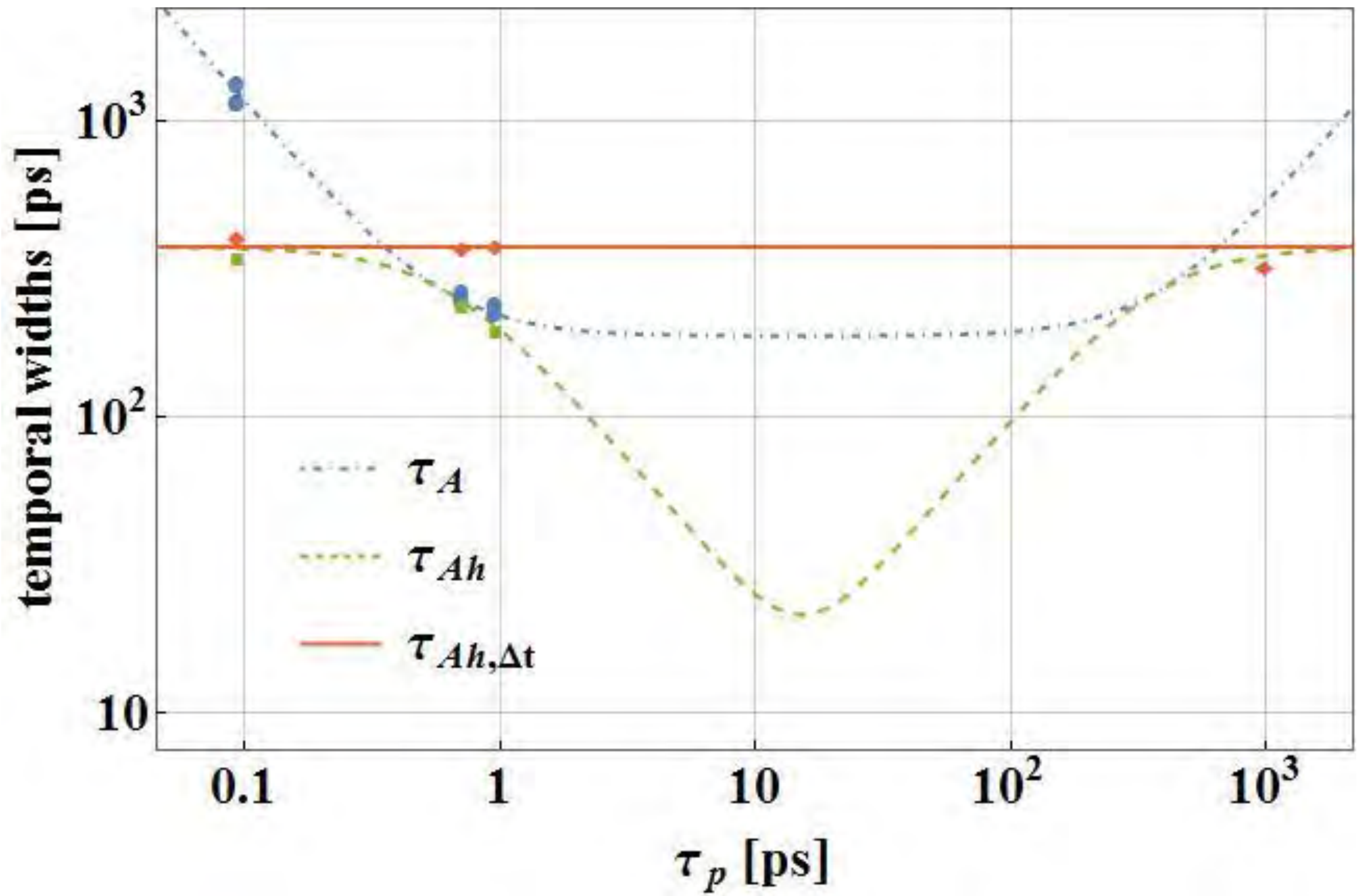
  - when Alice knows both the emission time of a given pair of photons and the detection time of Bob's photon

$$\tau_{Ah}(\sigma,\tau_p)=\sqrt{\frac{16\left(\tau_p^2-\beta_AL_A\beta_BL_B\sigma^2\right)^2+\left(\beta_AL_A+\beta_BL_B\right)^2\left(4+\sigma^2\tau_p^2\right)^2}{4\left(4+\sigma^2\tau_p^2\right)\left(\tau_p^2-\beta_AL_A\beta_BL_B\sigma^2\right)}}$$

  - when Alice knows only the detection time of Bob's photon

$$\tau_{Ah,\Delta t}(\sigma,\tau_p)=\frac{\sqrt{\left[16+\left(\beta_AL_A+\beta_BL_B\right)^2\sigma^4\right]\tau_p^2+4\sigma^2\left(\beta_AL_A-\beta_BL_B\right)^2}}{2\sigma\tau_p}$$

Optimizing a given SPDC source



- Minimization of Alice's photon temporal width in the case of symmetric setup (two SMFs of 10 km length, $\sigma = 3.25$ THz):

Optimizing a given SPDC source

- Minimization of Alice's photon temporal width in the case of symmetric setup ($\beta_A L_A = \beta_B L_B \equiv \beta L$):

- $\tau_A^{\min}(\sigma) = \dfrac{|\beta| L\sigma^2 + 2}{2\sigma}$ for $\tau_p^{\mathrm{opt}} = \sqrt{2|\beta| L}$

- $\tau_{Ah}^{\min}(\sigma) = \dfrac{2\sqrt{|\beta| L\left(\beta^2 L^2 \sigma^4 + 4\right)}}{|\beta| L\sigma^2 + 2}$ for $\tau_p^{\mathrm{opt}} = \sqrt{2|\beta| L}$

- $\tau_{Ah,\Delta t}(\sigma) = \dfrac{\sqrt{\beta^2 L^2 \sigma^4 + 4}}{\sigma}$ independent from τ_p

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- $\tau_{Ah,\Delta t}(\sigma) = \dfrac{\sqrt{\beta^2 L^2 \sigma^4 + 4}}{\sigma}$ independent from τ_p

- Optimal value of spectral correlation coefficient for the symmetric setup:

$$\rho^{opt} = \frac{2 - |\beta| L\sigma^2}{2 + |\beta| L\sigma^2}$$

Designing optimal SPDC source

- Absolute minimum of Alice's photon temporal width in the case of symmetric setup:

  - $\tau_A^{\mathrm{abs}} = \tau_{Ah}^{\mathrm{abs}} = \sqrt{2|\beta|L}$ for $\tau_p^{\mathrm{opt}} = \sqrt{2|\beta|L}$ and $\sigma^{\mathrm{opt}} = \sqrt{2/|\beta|L}$

  - $\tau_{Ah,\Delta t}^{\mathrm{abs}} = 2\sqrt{|\beta|L}$ for $\sigma^{\mathrm{opt}} = \sqrt{2/|\beta|L}$ and arbitrary τ_p

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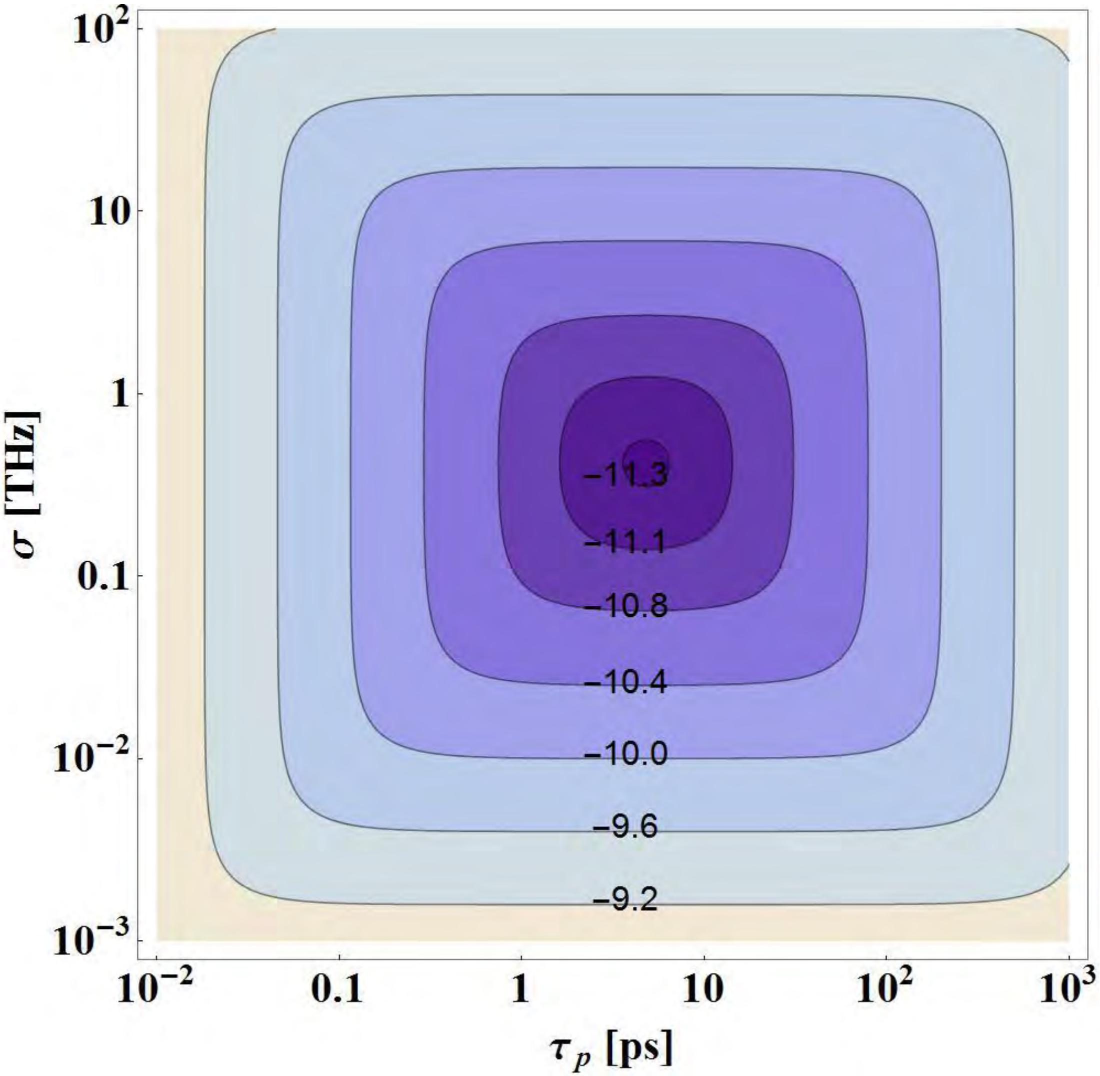
  - $\tau_{Ah,\Delta t}^{\text{abs}} = 2\sqrt{|\beta|L}$ for $\sigma^{\text{opt}} = \sqrt{2/|\beta|L}$ and arbitrary τ_p

- Optimal type of spectral correlation between SPDC photons:

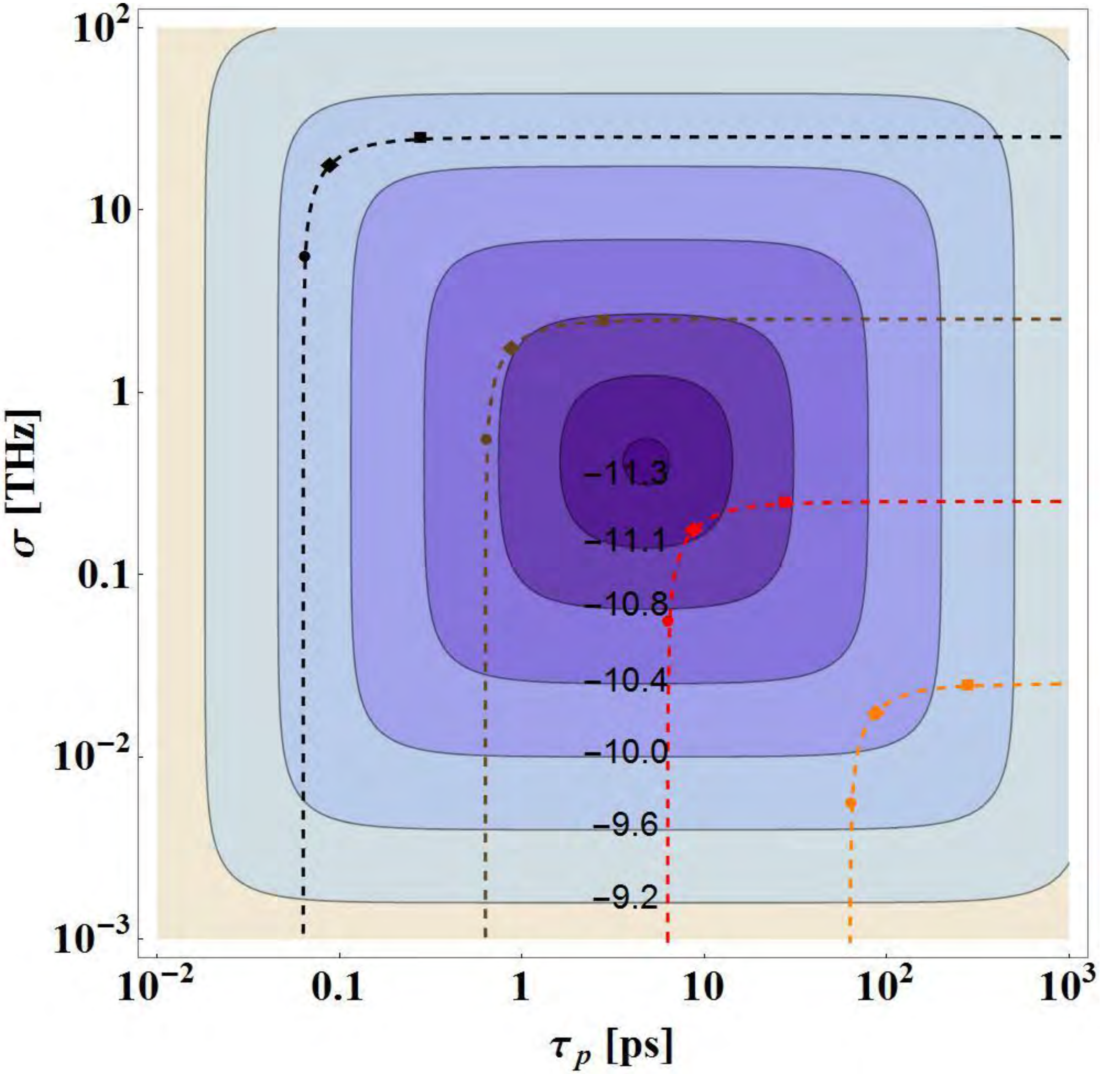
  no correlation

Designing optimal SPDC source



- $\log_{10} \tau_A$ plotted for symmetric setup with two 1km-long SMFs

Designing optimal SPDC source



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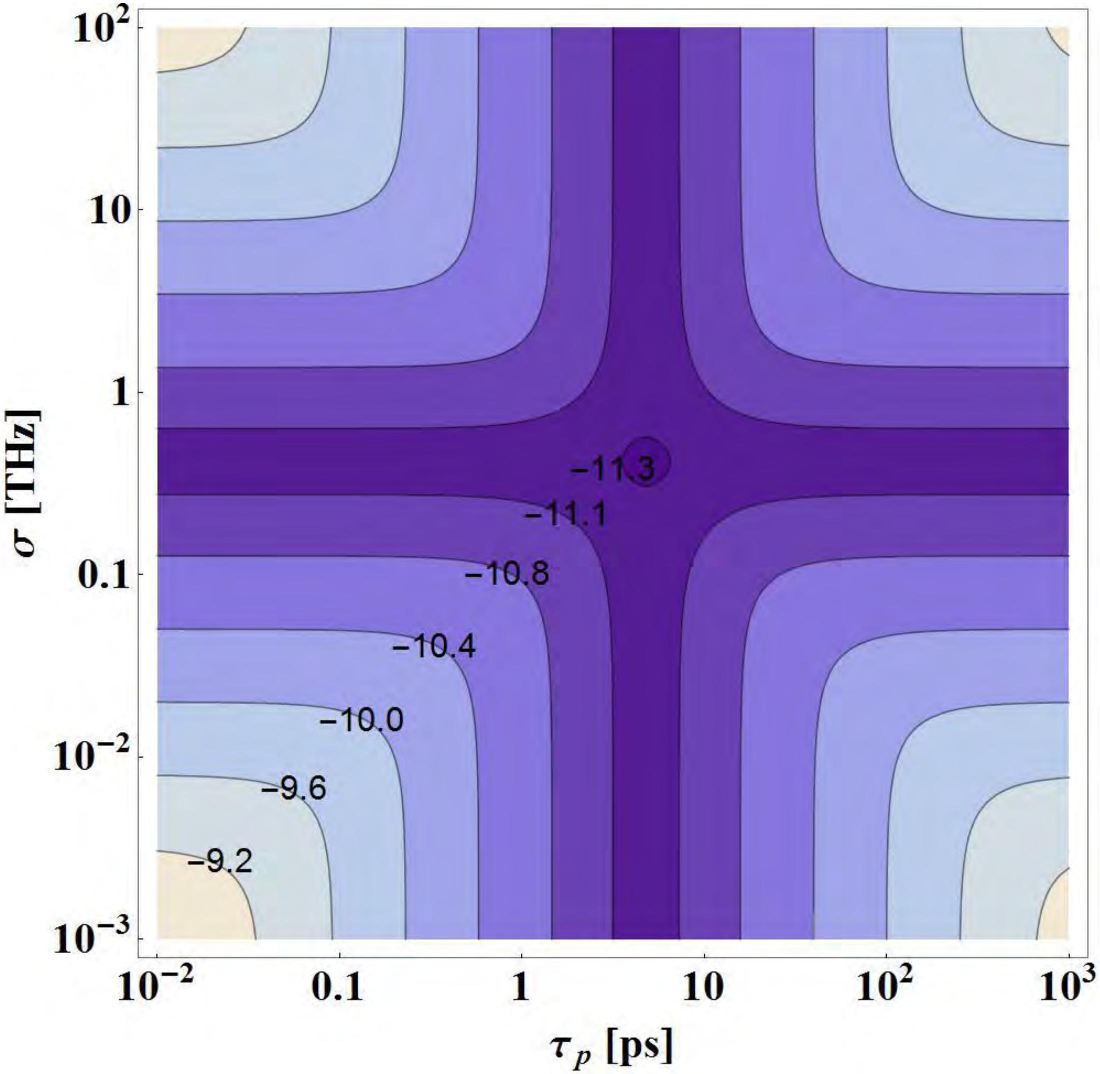

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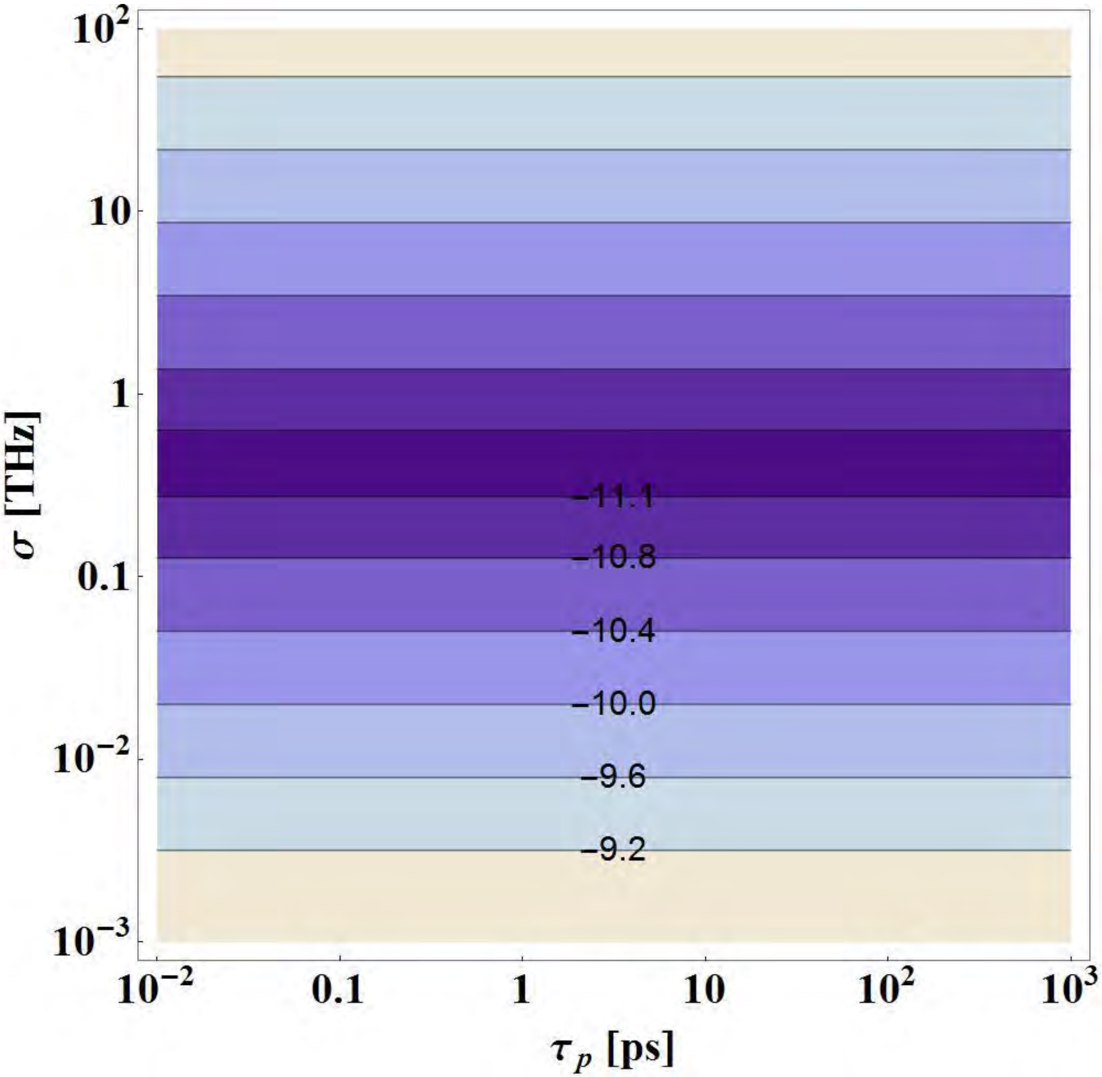
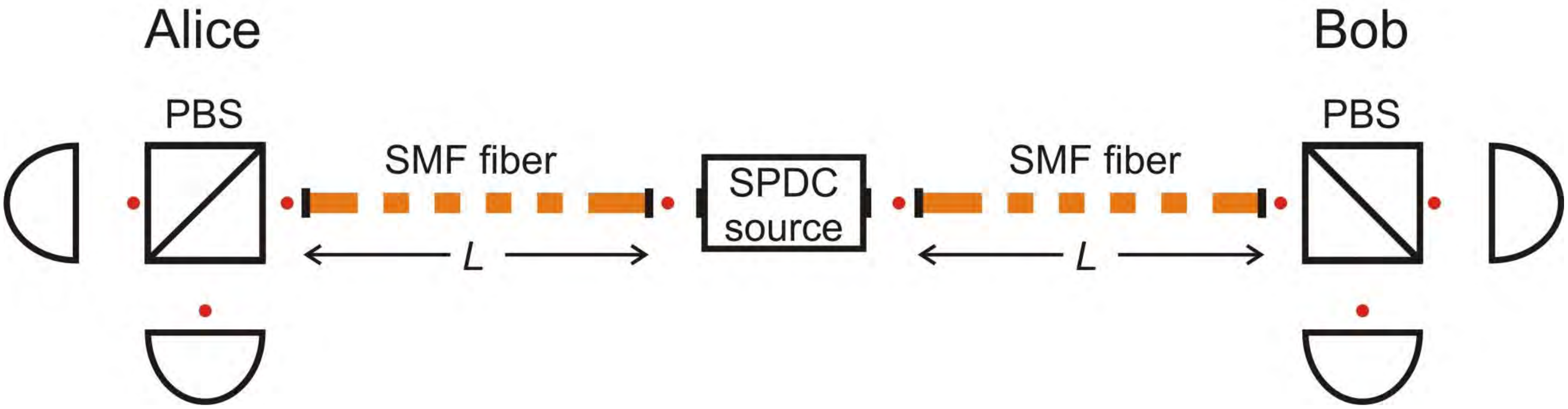


- $\log_{10} \tau_{Ah,\Delta t}$ plotted for symmetric setup with two 1km-long SMFs

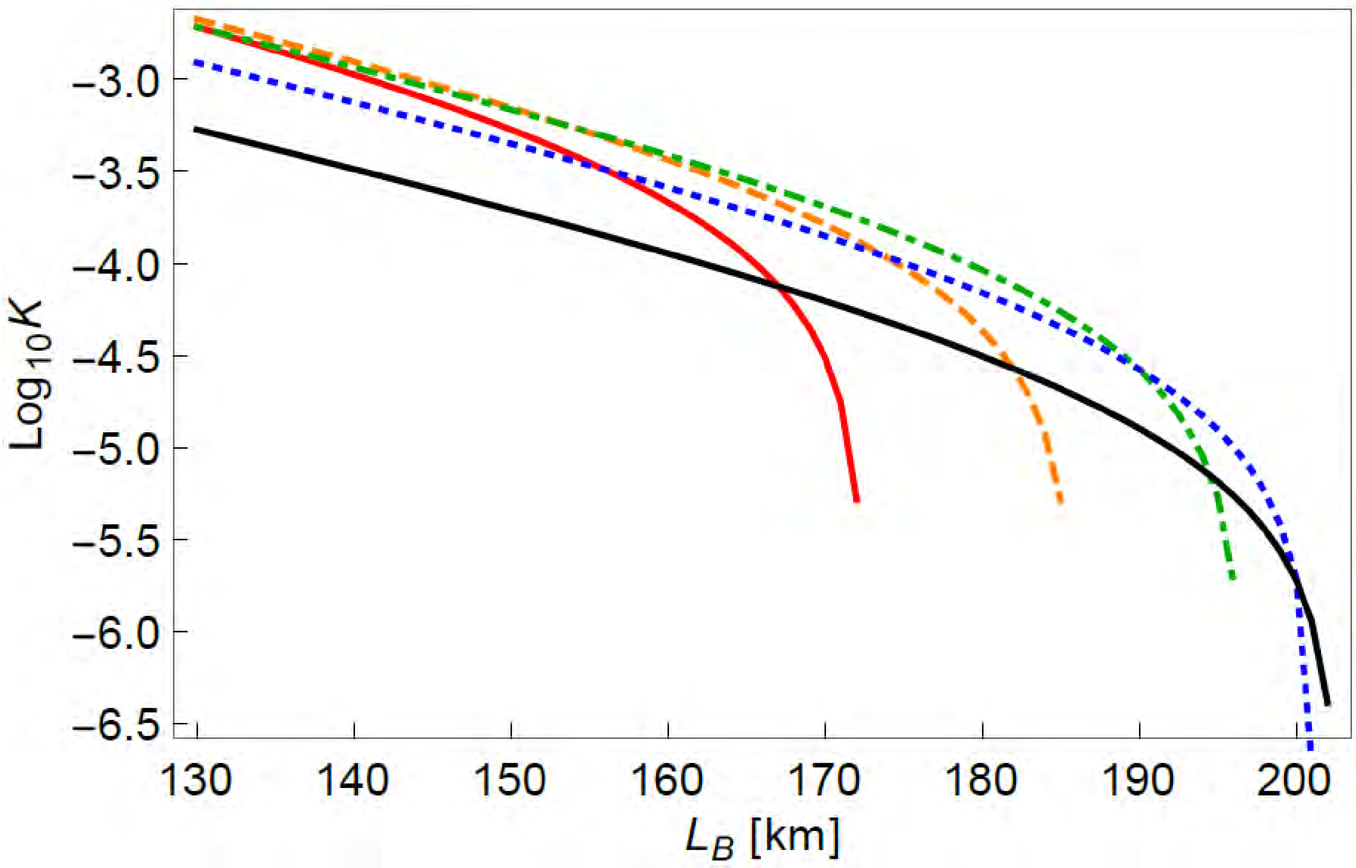
Symmetric quantum key distribution

- Discrete-variable QKD setup with source of photons located in the middle:



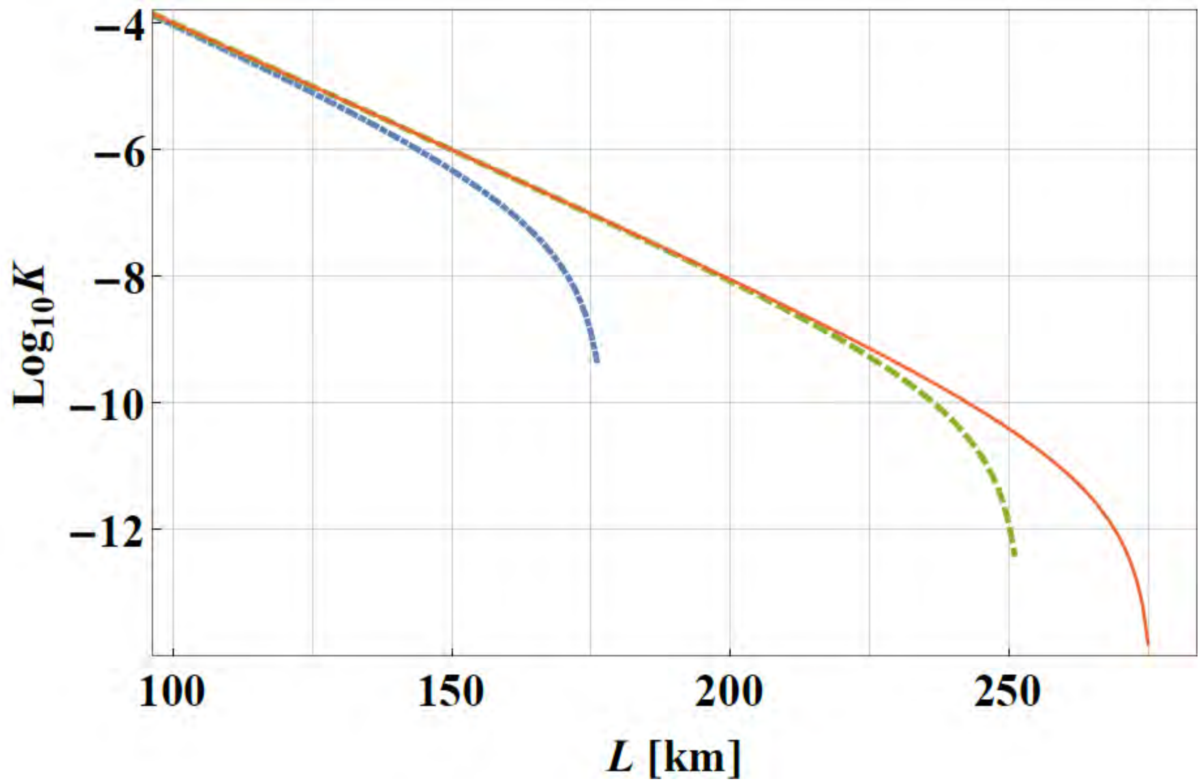
- Assumptions:
  - BB84 protocol
  - optimized duration time of a single detection window

Influence of the duration time of detection windows



- Key generation rate for the asymmetric QKD setup plotted for the following duration time of the detection window: $12\tau_{Bh}$ (red), $6\tau_{Bh}$ (orange), $3\tau_{Bh}$ (green), $1.5\tau_{Bh}$ (blue), $0.6\tau_{Bh}$ (black)

Potential to improve QKD security



- Logarithm of secure key rate calculated for symmetric QKD setup with two SMF fibers and for optimized duration time of a single detection window
  - $\sigma = 3.25$ THz, $\tau_p = 0.1$ ps
  - $\sigma = 3.25$ THz, $\tau_p = \sqrt{2|\beta|L}$
  - $\sigma = \sqrt{2/|\beta|L}$, $\tau_p = \sqrt{2|\beta|L}$

Our papers

1. K. Sedziak, M. Lasota, P. Kolenderski, Optica **4**, 84-89 (2017):
   - investigation of the dependence of temporal width of SPDC photons propagated in SMF fibers on the type of spectral correlation between them
   - example of application: symmetric QKD setup

2. M. Lasota, P. Kolenderski, arXiv:1702.05165 (2017):
   - consideration of asymmetric QKD setup
   - investigation of the possibility of extending QKD security by adding chromatic dispersion to the shorter fiber

3. K. Sedziak, M. Lasota, P. Kolenderski, arXiv:1711.06131 (2017):
   - optimization of the pump laser for a given SPDC source for QC applications
   - designing optimal SPDC source for QC applications
   - experimental results regarding wavepacket narrowing of SPDC photons propagated in SMF fibers

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