# Dissipative Generation of the Cubic Phase State in Optomechanics

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# Summary

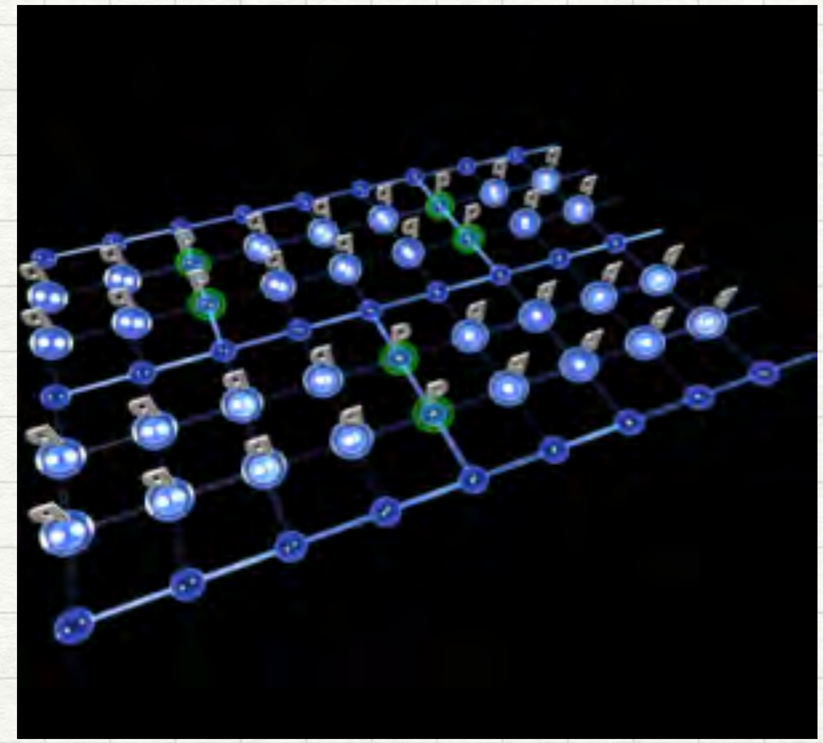
- Measurement Based Quantum Computation
- Cubic Phase State
- Optomechanics Setting
- Generation of the Nonlinear Resource
- Incorporation into Mechanical Cluster State
- Outlook & Conclusions

# Measurement Based Quantum Computation…

Cluster States are a class of highly entangled states

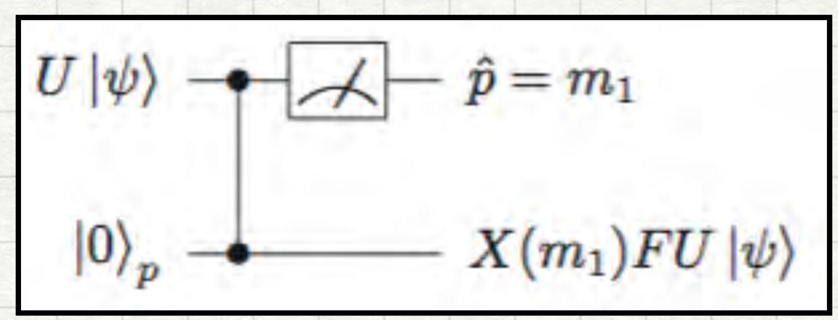
Form a resource for computing

Computation proceeds via local measurement on cluster nodes



Gu et al., PRA (2009)

Single mode unitary gates $U$ are implemented via measuring $U^{\dagger} p U$




Ref: Rev. Mod. Phys. 84 621 (2012)

# ...with Continuous Variables

Introduce CV operators

$$x = \frac{a + a^\dagger}{\sqrt{2}} \quad p = \frac{i(a + a^\dagger)}{\sqrt{2}}$$

$$[x, p] = i$$

$$[a, a^\dagger] = 1$$

Cluster State Vertices

$$|0\rangle_p = \int dx |x\rangle$$

Cluster State Edges

$$\mathrm{CZ}_{jk} = e^{i x_j x_k}$$

**Physical cluster states are Gaussian approximations to this ideal**

$$|0\rangle_p \simeq S(s)|0\rangle = e^{-\frac{i \ln s}{2}(xp + px)}|0\rangle$$

# Universal Quantum Computation

Lloyd-Braunstein criteria:

| | |
|---|---|
| Multimode Gaussian operations | ✓ |
| Single Non-Linear operation | |

**Cubic Phase Gate**

$$V = e^{i\gamma x^3} \longrightarrow$$

Requires a non-linear measurement i.e.

NOT homodyne detection (quadrature measurements)

$\Rightarrow$ Computation becomes adaptive

**Motivates the introduction of a non-Gaussian resource state**

✓ Ref: D. W. Moore, O. Houhou, A. Ferraro PRA (2017)

# Universal Quantum Computation

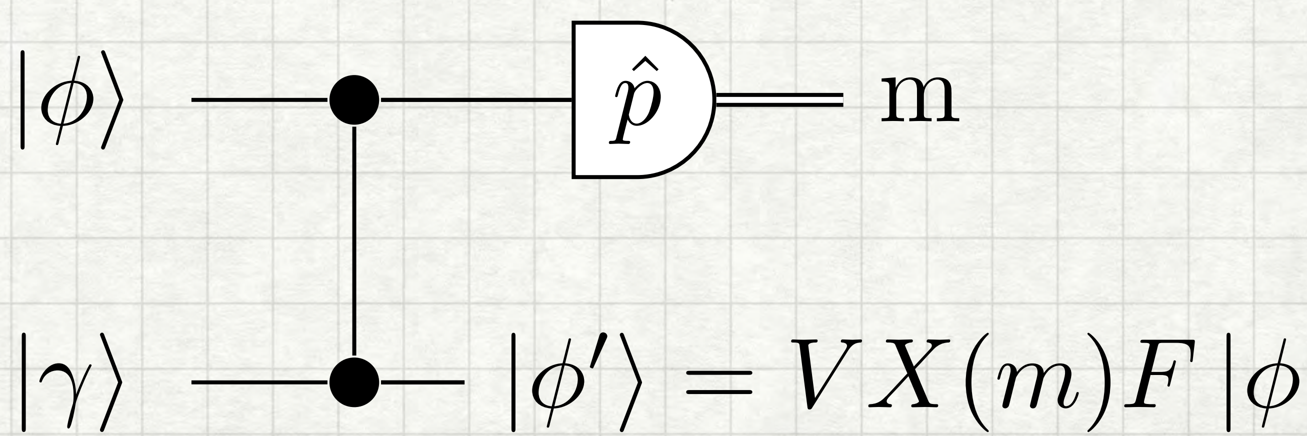
**Cubic Phase State**

$$|\gamma\rangle = V|0\rangle_p \quad \simeq \quad |\gamma, s\rangle = VS(s)|0\rangle$$

- Provides the non-linear operation
- Measurements are still Gaussian (quadrature measurements)
- Byproducts are always Gaussian operations

CPS is consumed by using it as

the ancilla in a teleportation scheme



# Optomechanics Setting

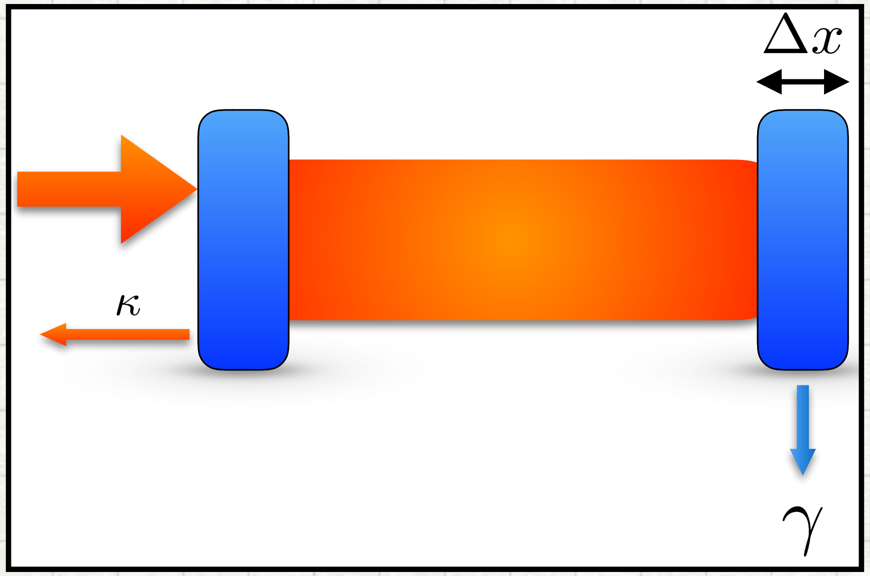
**Some standard assumptions:**

Cavity Decay rate $\kappa$

Mechanical Damping rate $\gamma_m$

Resolved Sideband regime $\kappa \ll \Omega$

Exploit intrinsic non-linearity in optomechanics




$$H = \omega(x) a^\dagger a + \Omega b^\dagger b$$

$$\omega(x) = \omega + \frac{\partial \omega}{\partial x} x + \frac{1}{2} \frac{\partial^2 \omega}{\partial x^2} x^2 + \ldots$$

# Optomechanics Setting

Set $G_L = \frac{1}{\sqrt{2}} \frac{\partial \omega}{\partial x}$

$$G_Q = \frac{1}{4} \frac{\partial^2 \omega}{\partial x^2}$$

as the linear and quadratic coupling coefficients

Then, in the rotating frame and

linearising with multiple tones $\longrightarrow$ $H_{\text{drive}} = \sum_k \epsilon_k e^{-i\omega_k t}$

the Hamiltonian is represented as

$$H = \sum_k \left(\alpha_k e^{-i\Delta_k t} a^\dagger + \alpha_k^* e^{i\Delta_k t} a\right) \left[G_L \left(e^{i\Omega t} b^\dagger + e^{-i\Omega t} b\right) + G_Q (e^{i\Omega t} b^\dagger + e^{-i\Omega t} b)^2\right]$$

# Optomechanics Setting

Taking the rotating wave approximation and choosing appropriate detunings we have

$$H = a^\dagger \left( g_1 b + g_2 b^\dagger + g_3 b^2 + g_4 {b^\dagger}^2 + g_5 \{b, b^\dagger\} \right) + \text{H.C.}$$

Detunings:

$$\Delta_1 = -\Omega \quad \Delta_2 = \Omega \quad \Delta_3 = -2\Omega \quad \Delta_4 = 2\Omega \quad \Delta_5 = 0$$

$$g_1 = \alpha_1 G_L \quad g_2 = \alpha_2 G_L \quad g_3 = \alpha_3 G_Q \quad g_4 = \alpha_4 G_Q \quad g_5 = \alpha_5 G_Q$$

Weak Coupling:

$$|\alpha_{1,2} G_L| \ll \Omega \qquad |\alpha_{3,4,5} G_Q| \ll \Omega$$

# Generating the Resource State

Define a new field operator $f$

$$H = a^\dagger f + f^\dagger a$$

Master Equation dynamics $(\gamma_m = 0)$

$$\dot{\rho}(t) = -i[H, \rho(t)] + \kappa D[a]\rho(t)$$

Steady State (pure product state) $|0\rangle \otimes |0\rangle_f$

# Generating the Resource State

Coefficients of $f$ chosen carefully

$$g_2 = -r g_1$$

$$s(r) = \sqrt{\frac{1+r}{1-r}}$$

$$g_3 = g_4 = g_5 = -\frac{3i}{2\sqrt{2}}\gamma(1+r)g_1$$

$$|0\rangle_f = VS(s)|0\rangle$$

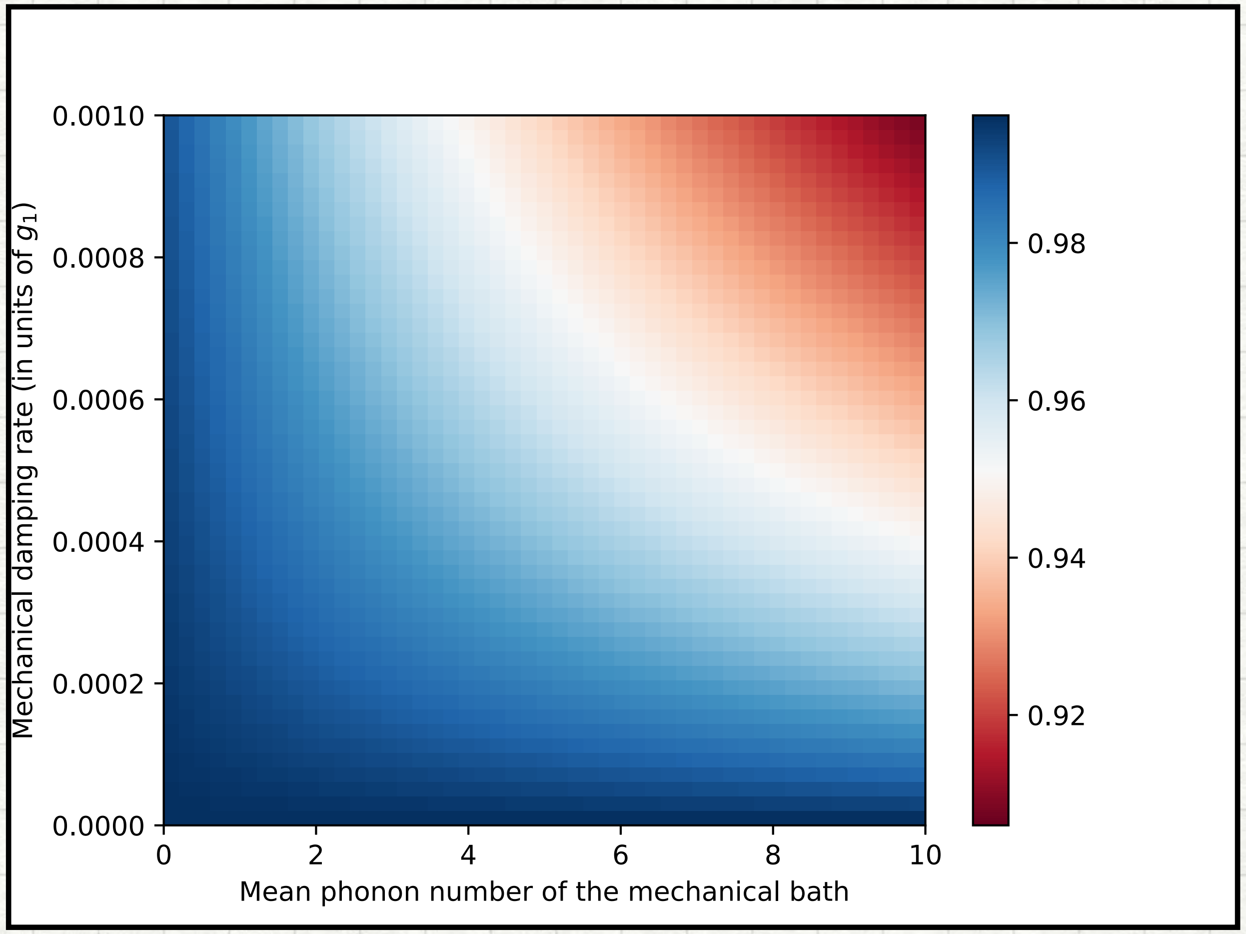
Restriction: $|g_1| \ll \Omega \quad |Rg_1| \ll \Omega \quad |R^{-1}g_1| \ll \Omega$

$$R = \frac{G_L}{G_Q}$$

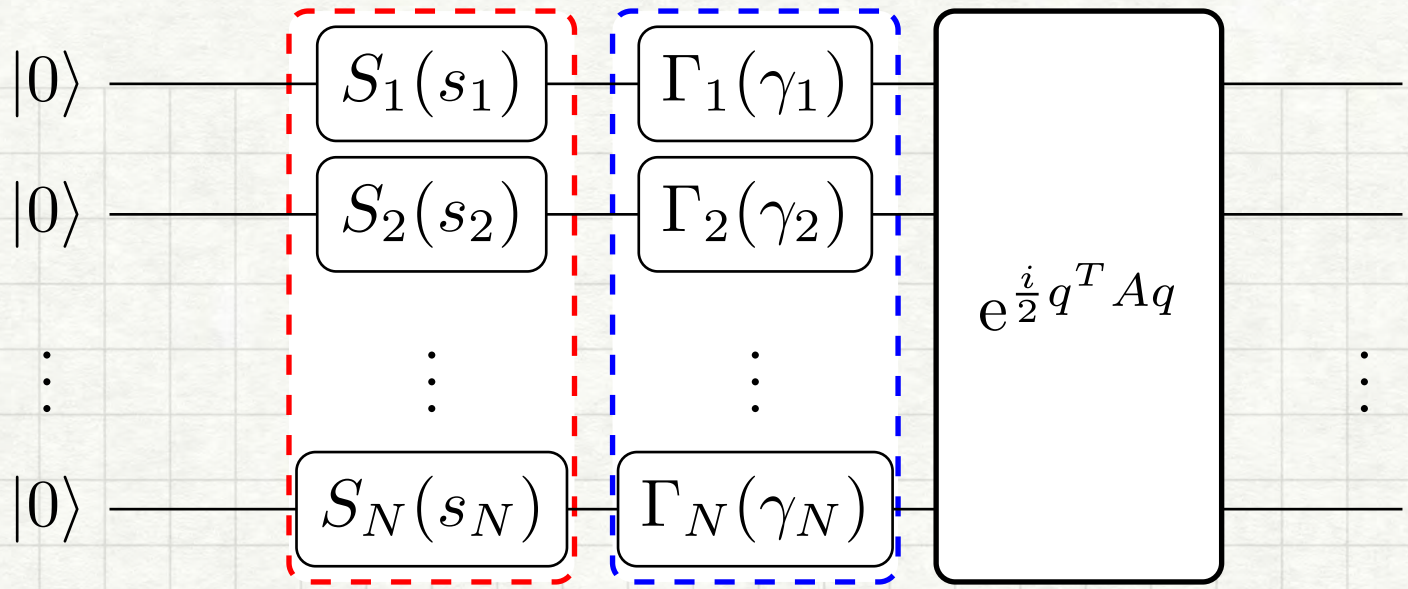
# Generating the Resource State (noisy)

Under mechanical dissipation, the fidelity of the output state is compromised

$$\dot{\rho}(t) = -i[H, \rho(t)] + \kappa D[a]\rho(t) + \gamma_m(\bar{n}+1)D[b]\rho(t) + \gamma_m \bar{n} D[b^\dagger]\rho(t)$$



# Incorporating the Resource in a Cluster State

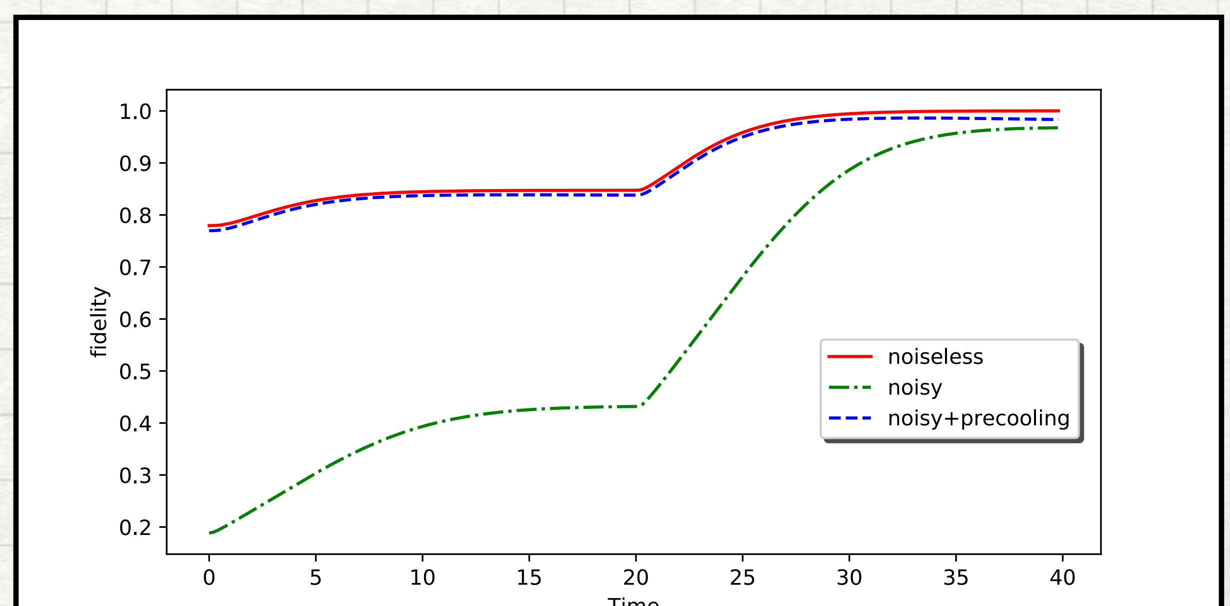



Each node is prepared in a squeezed state with a given non-linearity.

$A$ is the adjacency matrix for the cluster state

One can diminish the effect of noise

by pre cooling the oscillator




Ref: O. Houhou, A. Aissaoui, A Ferraro PRA (2015)

# Conclusions

- Prepare a non-linear resource for quantum computation
- Analysed the effect of noise
- Attached the resource to a cluster state

# Future

- More general dissipative generation of states

Freedom in $\{g_1, g_2, g_3, g_4, g_5\}$