



Palacký University
Olomouc

Non-Gaussian quantum optics

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MINISTERSTVO ŠKOLSTVÍ,
MLÁDEŽE A TĚLOVÝCHOVY



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Quantum Coherence and Nonclassicality

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Josef Hloušek

Quantum Nonlinear Operations

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Kimin Park**

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Petr Zapletal
Jan Provazník

Quantum Communication

**Vladyslav Usenko
Lazslo Ruppert**

Students:
Ivan Derkač
Olena Kovalenko

Quantum Optomechanics

Andrey Rakhubovsky

Students:
Nikita Vostrosablin

Interaction of Light with Atoms

**Lukáš Slodička
Petr Marek**

Students:
Petr Obšil
Lukáš Podhora

Stochastic Mechanics and Thermodynamics

**Michal Kolář
Giacomo Guarnieri**

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Luca Ornigotti
Maria Gumberidze





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Institute of Scientific Instruments of the CAS, Brno



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INTERNATIONAL COLLABORATION

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G. Leuchs, M. Chekhova, S. Götzinger



Danish Technical University, Lyngby
U.L. Andersen



University of Innsbruck
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J. Laurat, N. Treps



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NONCLASSICAL LIGHT

“non-classical state” = **not mixture of coherent states**

$$\rho \neq \int \mathcal{P}(\lambda) |\lambda\rangle \langle \lambda| d\lambda, \quad |\lambda\rangle = D(\beta) |0\rangle$$

R.J. Glauber, *Phys. Rev.* **131** 2766 (1963)



Nonclassical state cannot be prepared in **linear oscillator** driven by **classical external force** (with arbitrary fluctuations).

For low emission and collection efficiency, HBT measurement with two APDs can simply verify it.



QUANTUM NON-GAUSSIAN LIGHT

“quantum non-Gaussian state” = **not mixture of Gaussian states**

$$\rho \neq \int \mathcal{P}(\lambda) |\lambda\rangle \langle \lambda| d\lambda, \quad |\lambda\rangle = S(r, \psi) D(\beta) |0\rangle$$

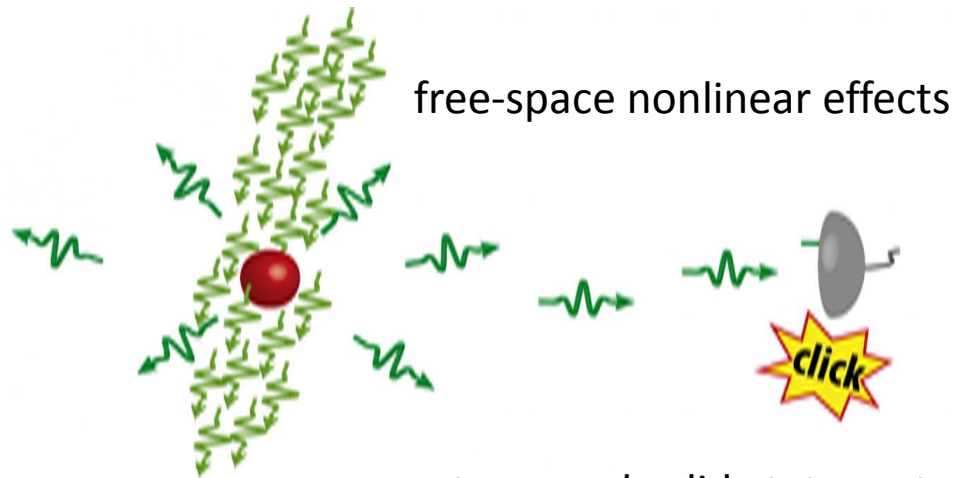
Quantum non-Gaussian state cannot be prepared by **linearized dynamics (maximally quadratic Hamiltonians)** with arbitrary fluctuating coupling coefficients (common in nonlinear optics, quantum optomechanics, etc.)

For low emission and collection efficiency, **direct verification** of quantum non-Gaussian light was always limited.

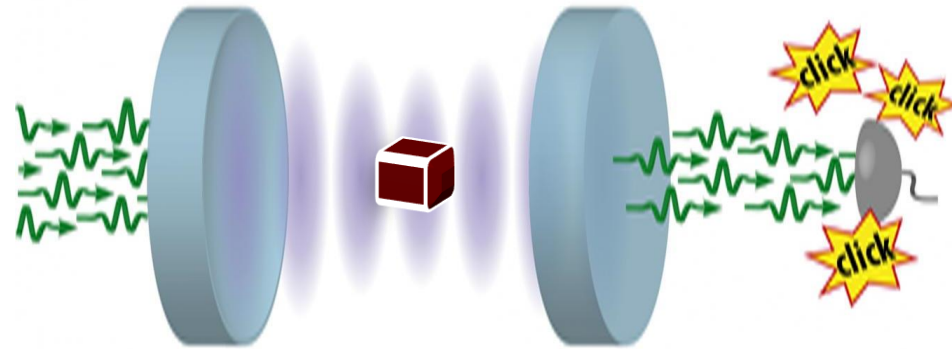


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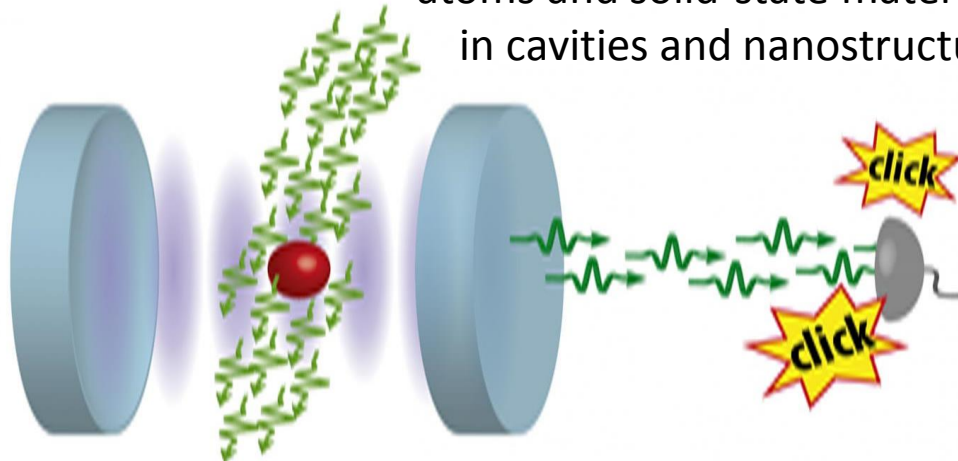
QUANTUM NON-GAUSSIAN LIGHT WITNESSES HIGHLY NONLINEAR QUANTUM EFFECTS



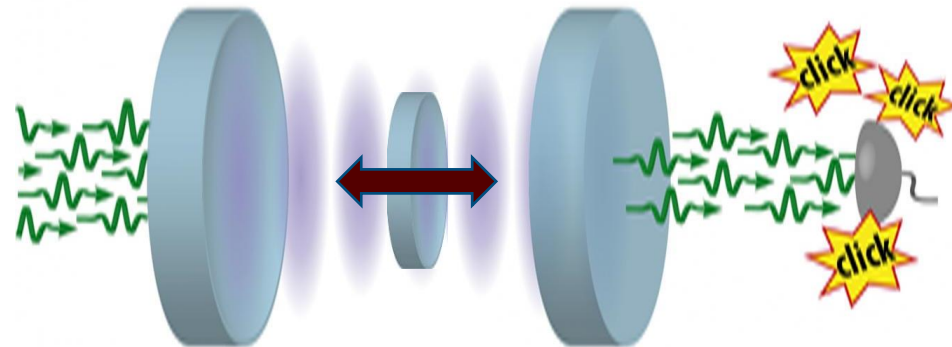
nonlinear optics in cavities and waveguides



atoms and solid-state materials
in cavities and nanostructures



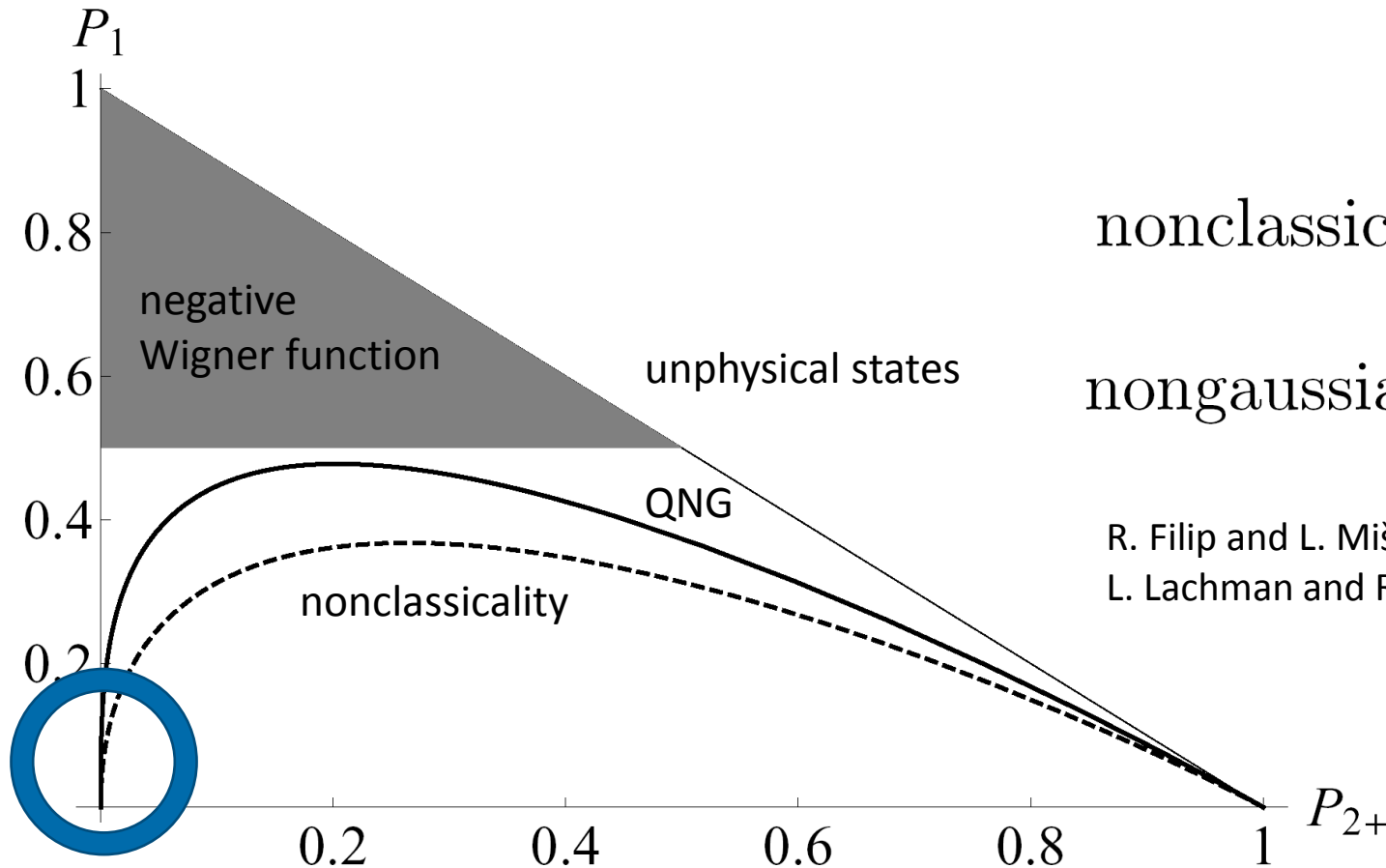
cavity quantum optomechanics





QUANTUM NON-GAUSSIAN LIGHT

$$\rho_c \neq \int \mathcal{P}(\lambda) |\lambda\rangle \langle \lambda| d\lambda, \quad |\lambda\rangle = S(r, \psi) D(\beta) |0\rangle$$



$$P_{2+} \ll P_1$$

nonclassicality: $P_{2+} < \frac{P_1^2}{2}$

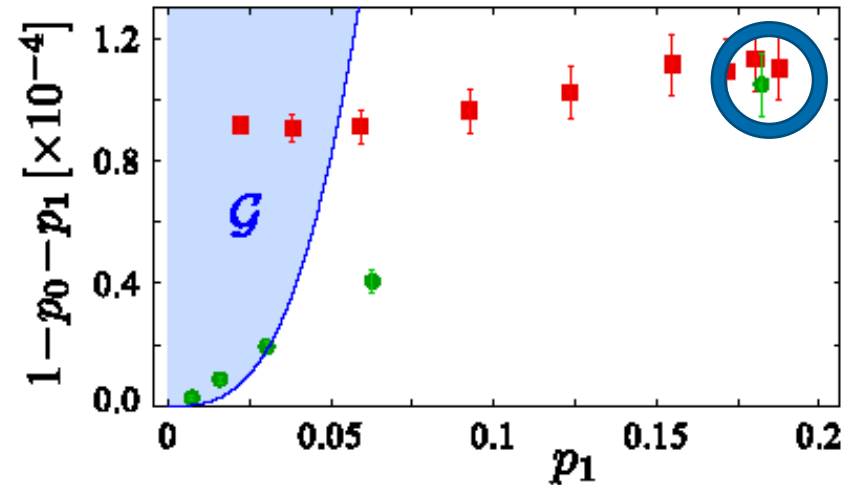
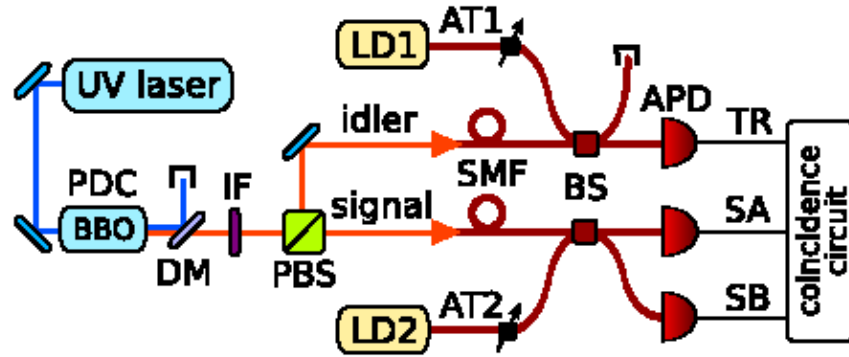
nongaussianity: $P_{2+} < \frac{2}{3} P_1^3$

R. Filip and L. Mišta, Phys. Rev. Lett. 106, 200401 (2011)

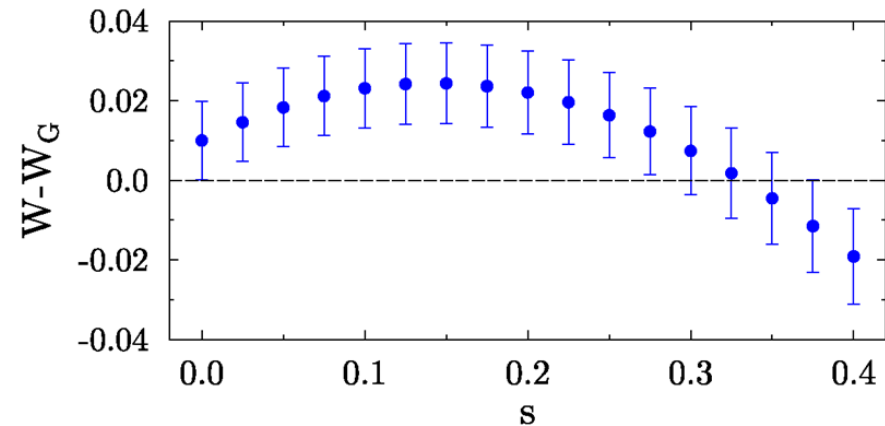
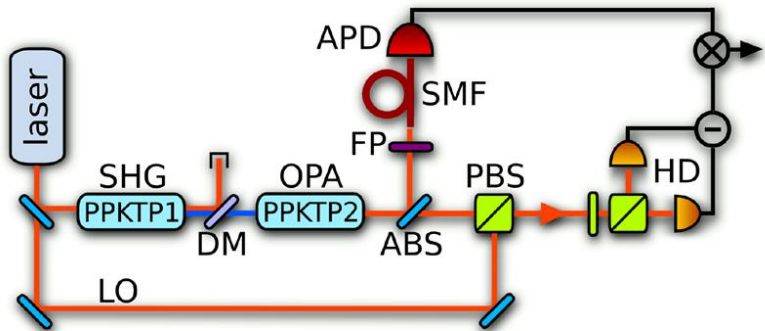
L. Lachman and R. Filip, Phys. Rev. A 88, 063841 (2013)



EXPERIMENT



M. Jezek, I. Straka, M. Micuda, M. Dusek, J. Fiurasek, R. Filip, Phys. Rev. Lett. 107, 213602 (2011).

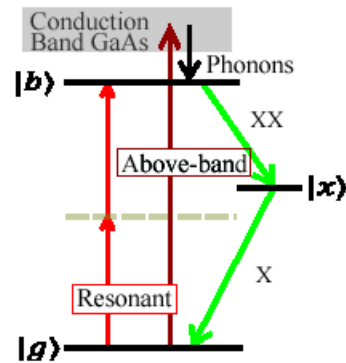


M. Ježek, A. Tipsmark, R. Dong, J. Fiurášek, L. Mišta Jr, R. Filip, and U.L. Andersen, Phys. Rev. A 86, 043813 (2012)

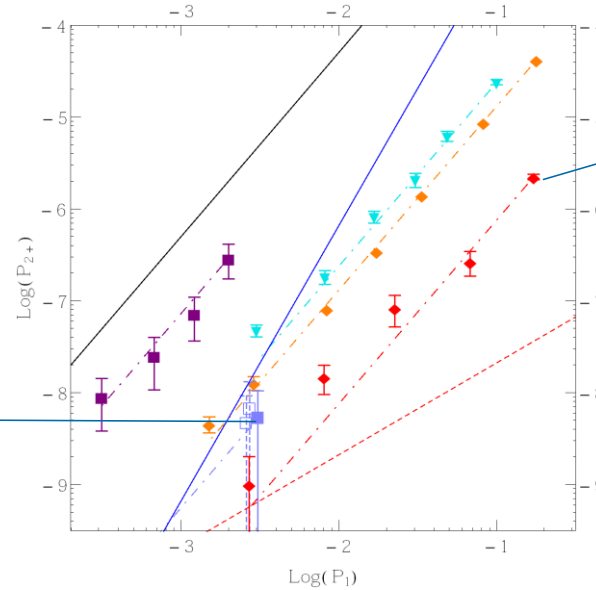
H. Song, K. Kuntz and E. Huntington, New J. Phys. 15, 1 (2013)



EXPERIMENT

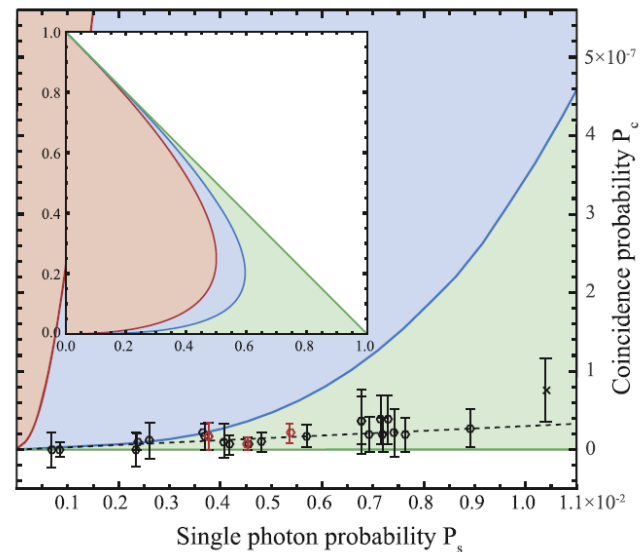
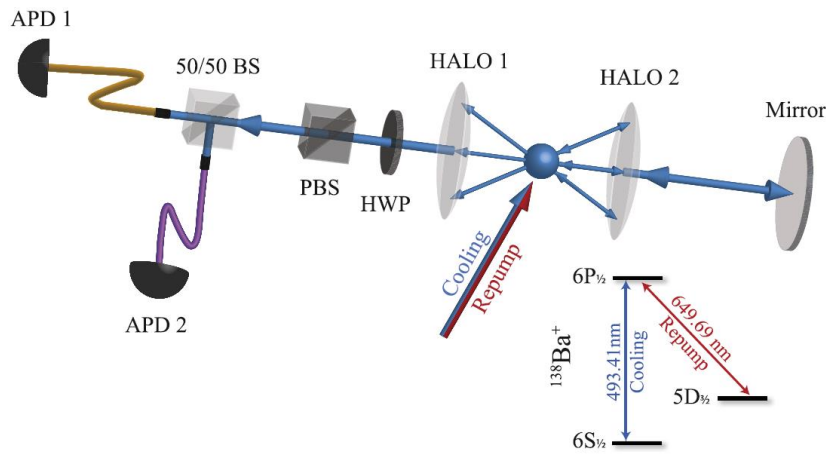


quantum dot



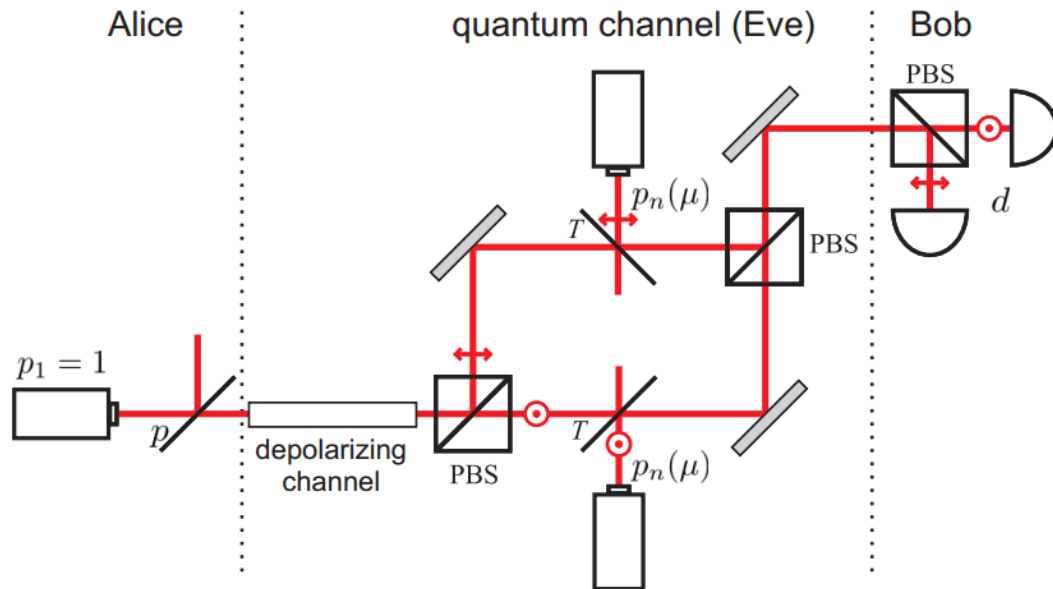
Depth:
18 dB

I. Straka, A. Predojević, T. Huber, L. Lachman, L. Butschek, M. Miková, M. Mičuda, G.S. Solomon, G. Weihs, M. Ježek, and R. Filip, Phys. Rev. Lett. 113, 223603 (2014).



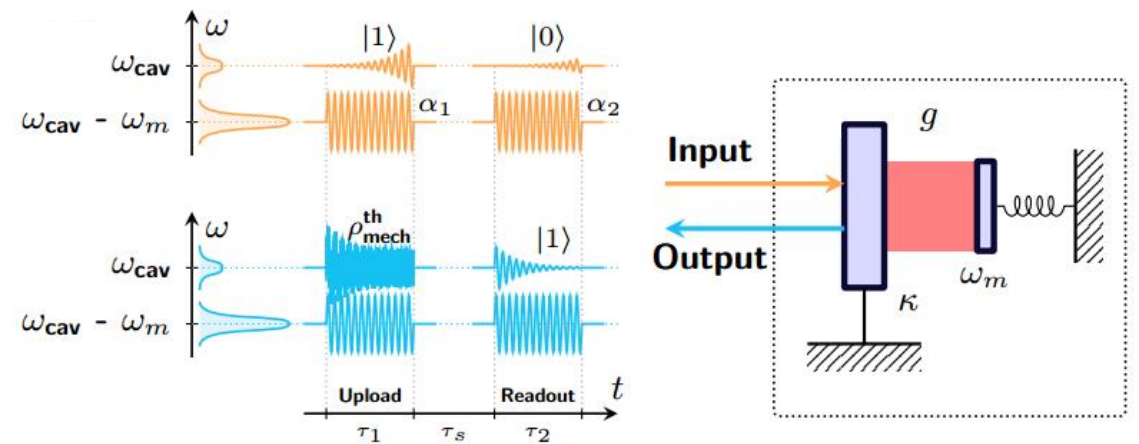
D.B. Higginbottom, L. Slodička, G. Araneda, L. Lachman, R. Filip, M. Hennrich and R. Blatt, New J. Phys. 18, 093038 (2016).

Security indicator for QKD BB84 single-photon protocol



M. Lasota, R. Filip, and V.C. Usenko, Phys. Rev. A 96, 012301 (2017)

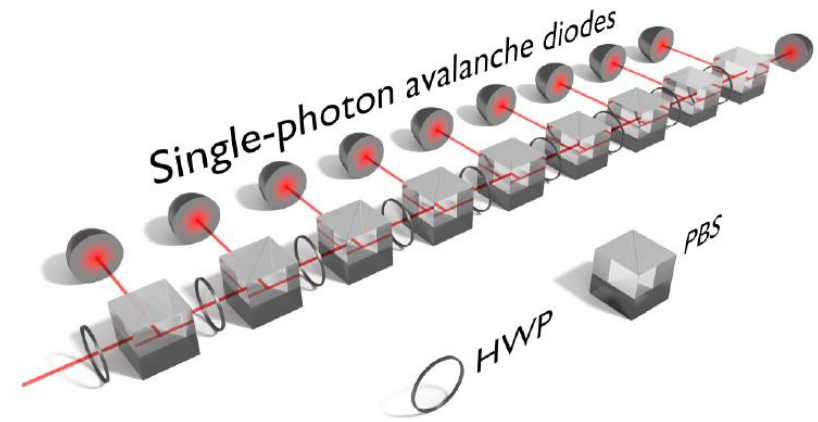
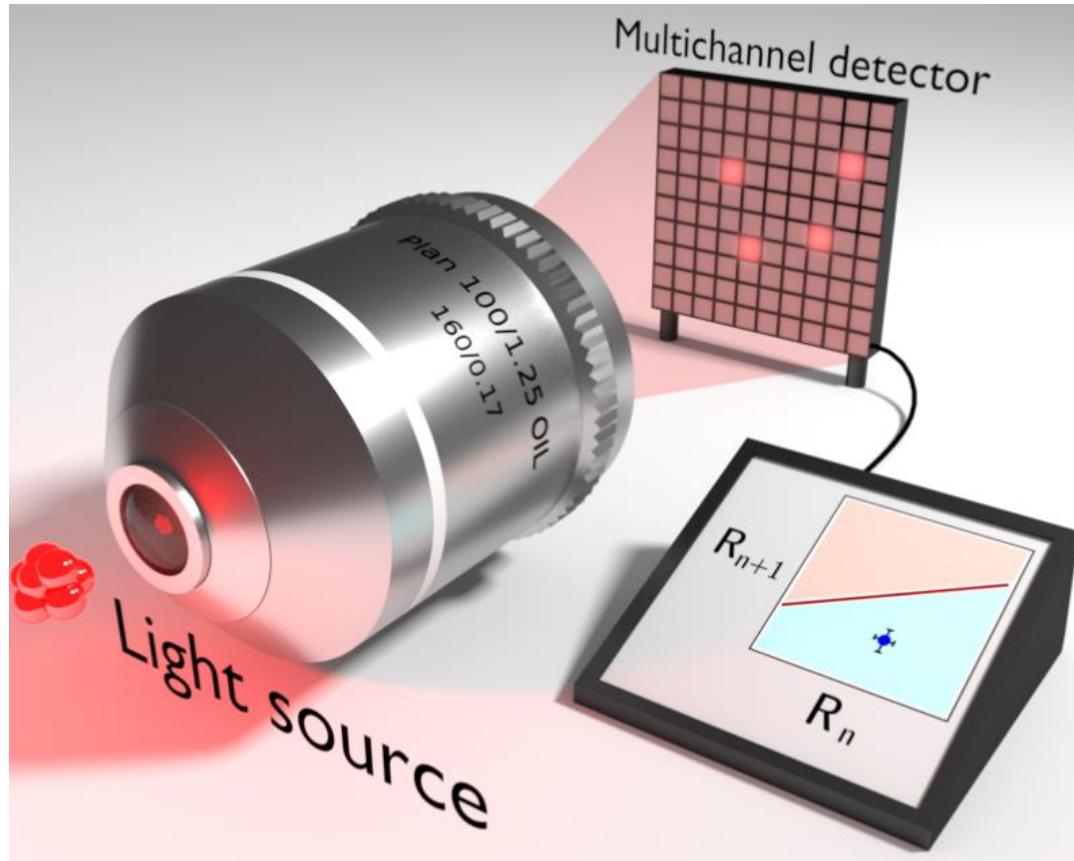
Verification of single photon-phonon-photon transfer



A.A. Rakhubovsky and R. Filip, Scientific Reports 7, 46764 (2017)



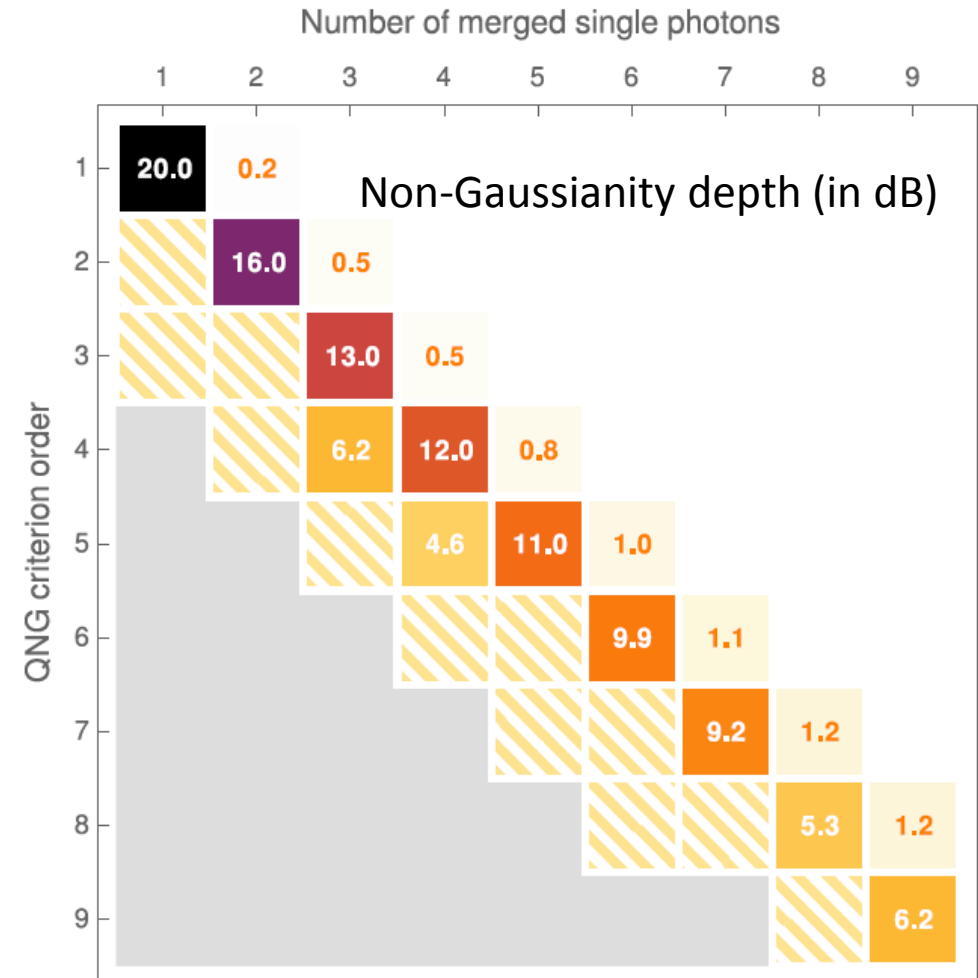
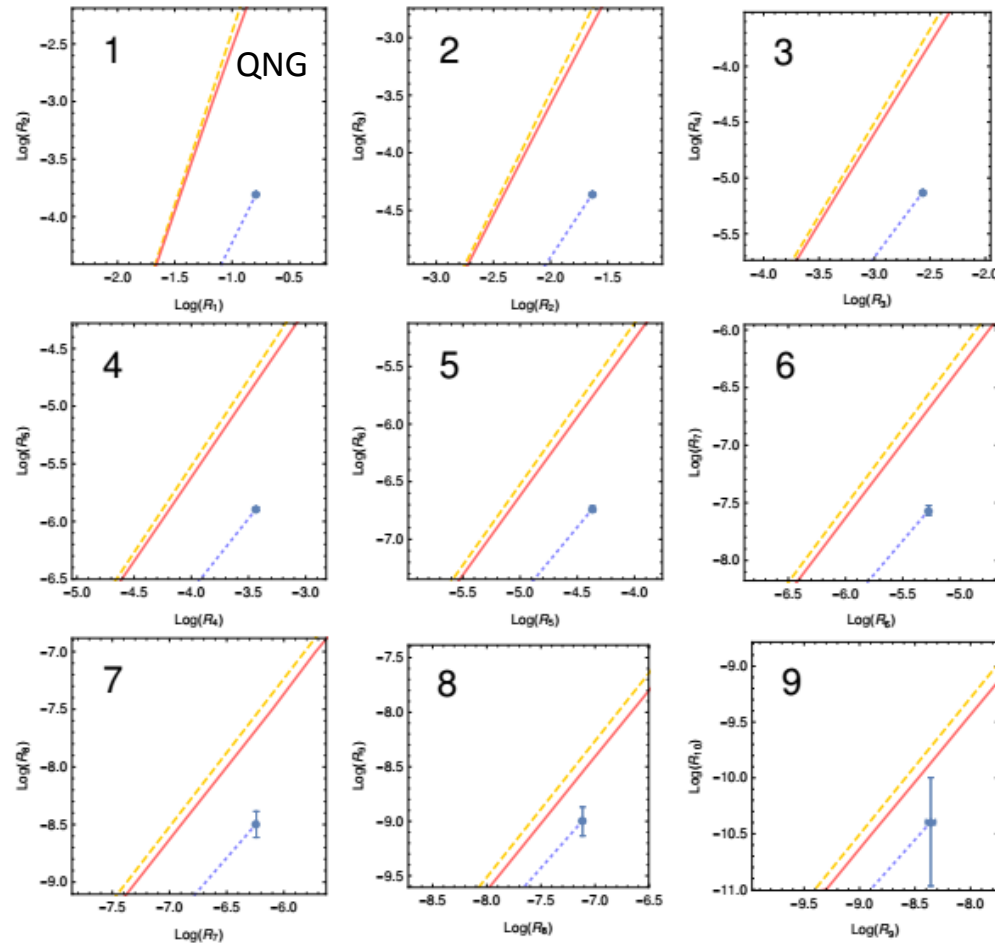
QUANTUM NON-GAUSSIANITY OF MANY PHOTONS



$$R_n^{n+2} > H_n^4(x) \left[\frac{R_{n+1}}{2(n+1)^3} \right]^n$$

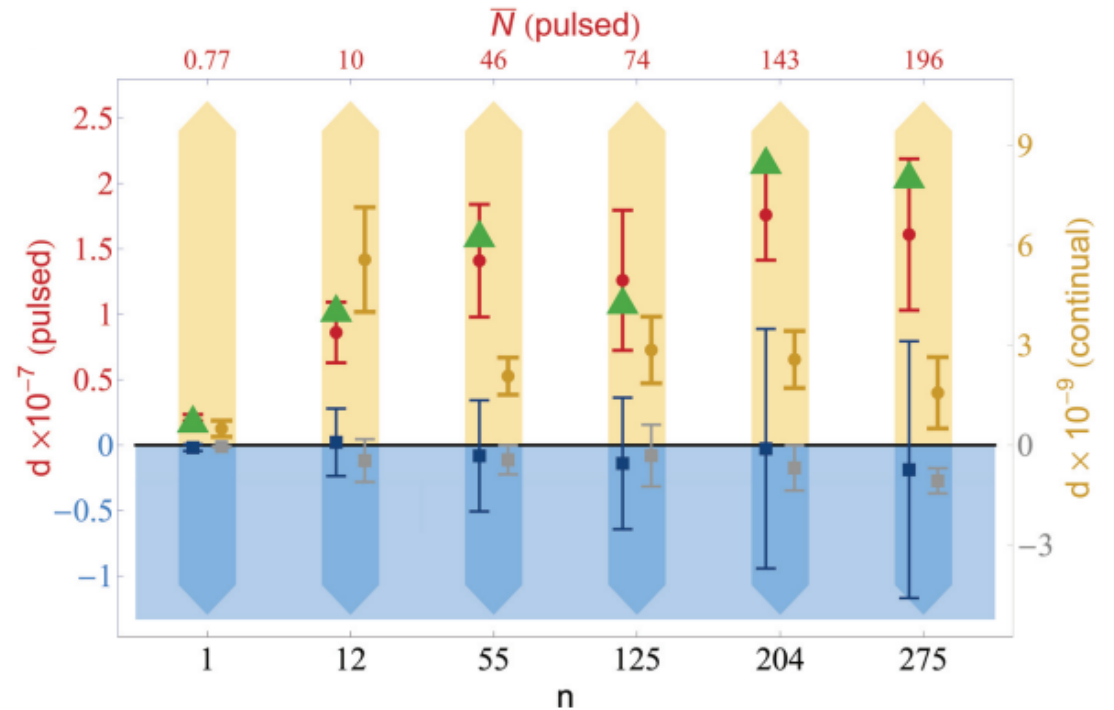
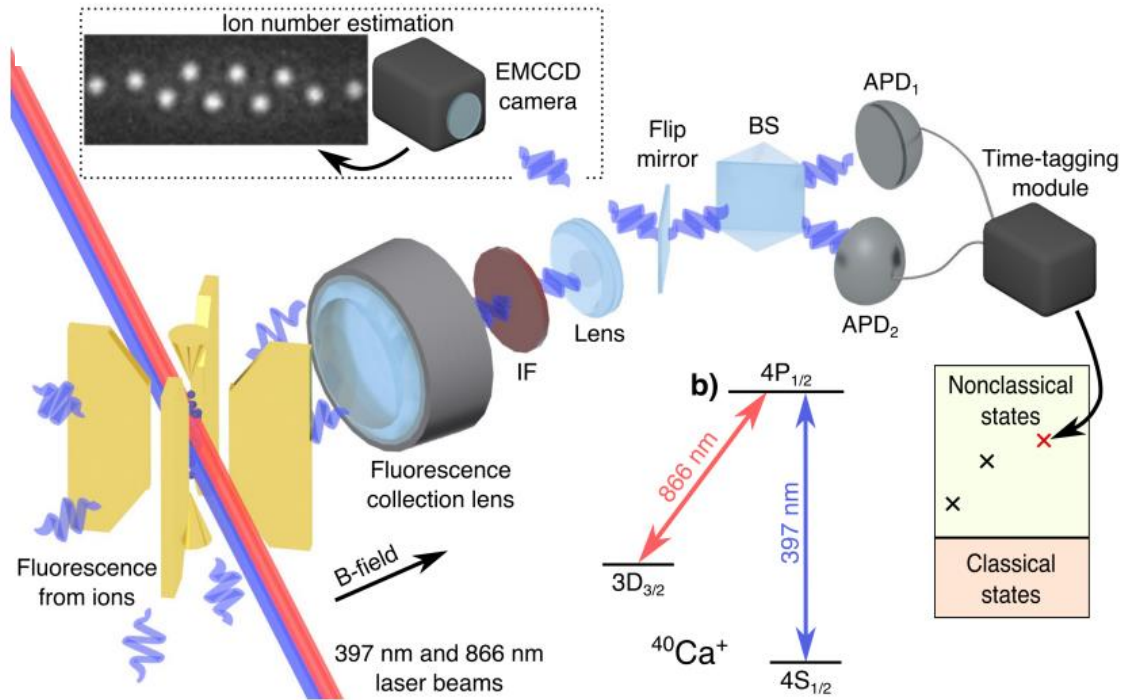


EXPERIMENT





NONCLASSICALITY OF MANY PHOTONS



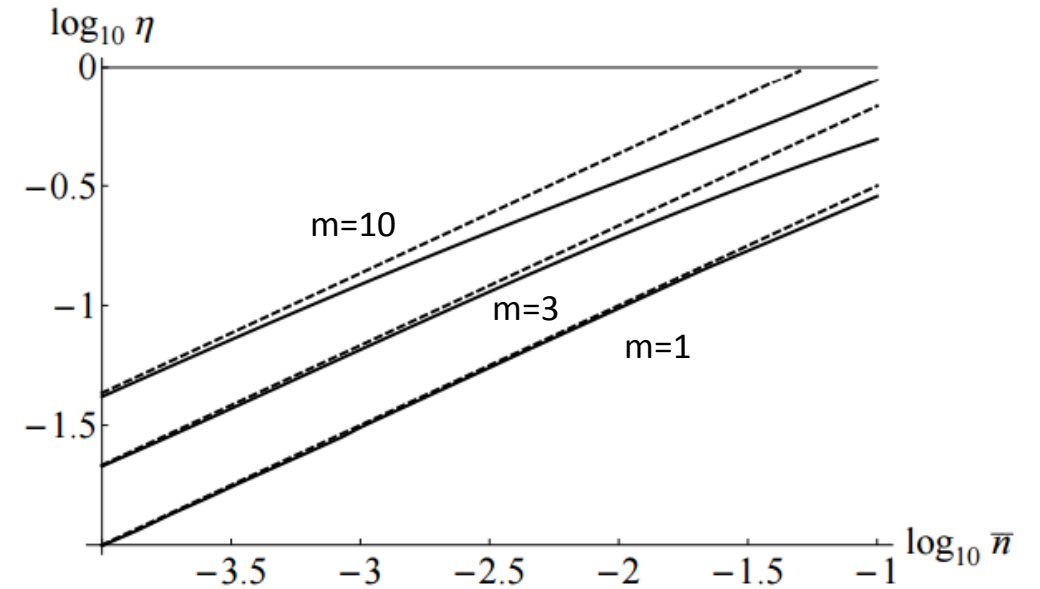
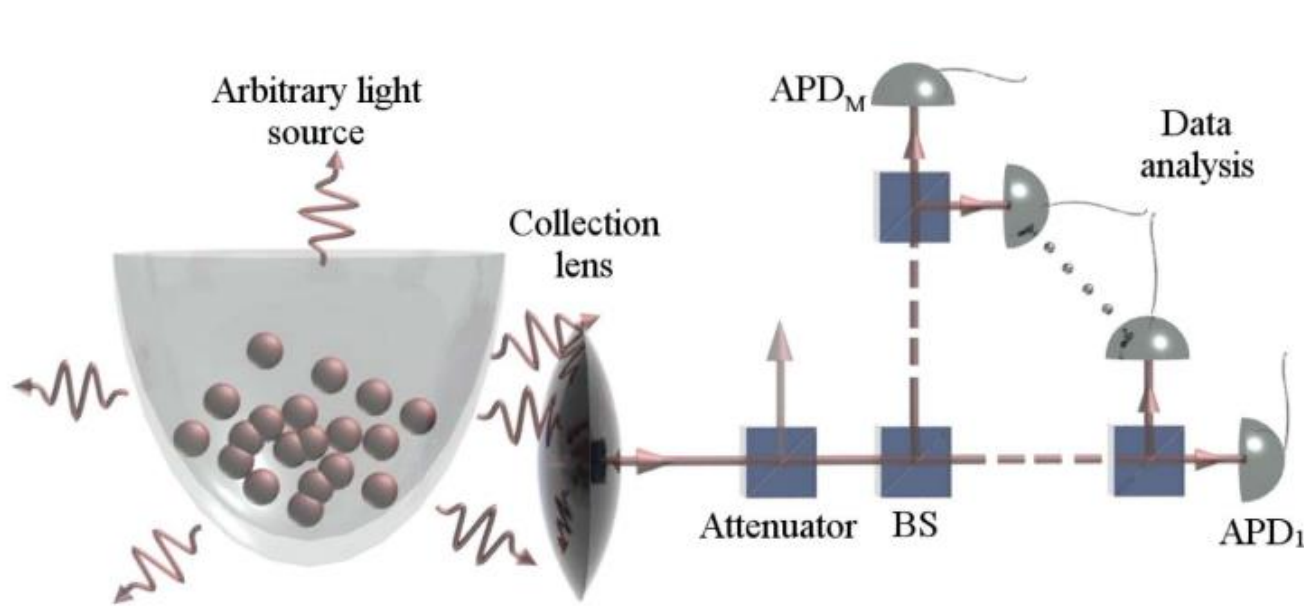
Nonclassicality of light can be detected for bright light from many emitters (>1000).

P. Obsil, L. Lachman, T. Pham, A. Lesundak, V. Hucl, M. Cizek, J. Hrabina, O. Cip, L. Slodicka and R. Filip, Nonclassical light from large ensemble of trapped ions, arXiv:1705.04472

E. Moreva, P. Traina, J. Forneris, I. P. Degiovanni, S. Ditalia Tchernij, F. Picollo, G. Brida, P. Olivero, M. Genovese, Direct experimental observation of nonclassicality in ensembles of single photon emitters, arXiv:1705.03079



QUANTUM NON-GAUSSIANITY OF MANY PHOTONS



Quantum non-Gaussianity from large ensemble is detectable!

$$\eta > \frac{H_m^{2/m}(x)}{\sqrt[m]{m!}} \sqrt{\frac{m\bar{n}}{2(m+1)}}$$

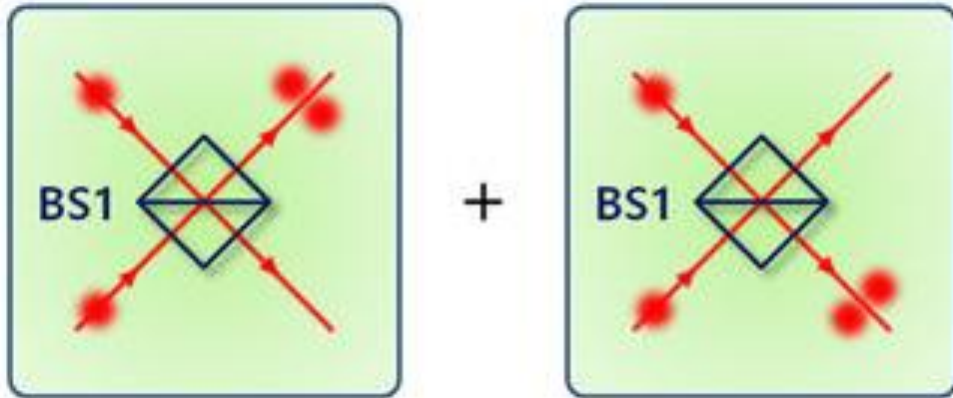
where x satisfies $H_{m+1}(x) = 0$ and $m\bar{n} \ll \eta$.

NEXT?

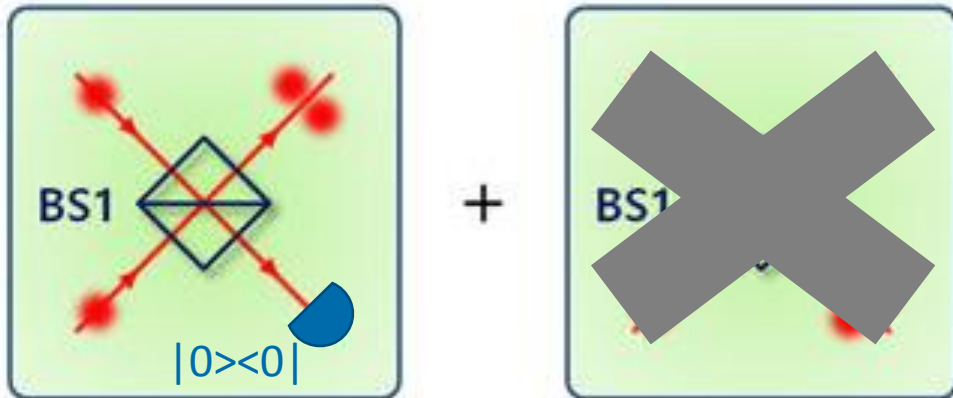


FOCK STATE CAPABILITY

Photon bunching

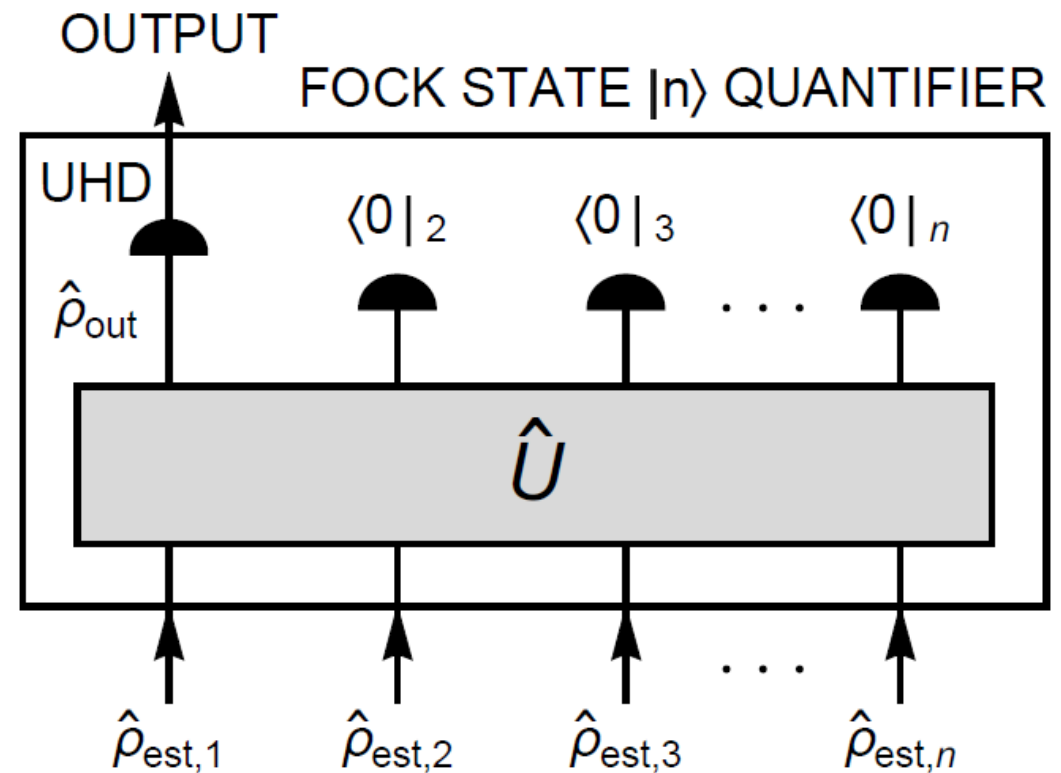


Fock state generation by photon bunching



C. K. Hong; Z. Y. Ou and L. Mandel, Phys. Rev. Lett. 59, 2044–2046 (1987)

Does output state exhibit oscillations of Wigner function corresponding to Fock state $|n\rangle$?

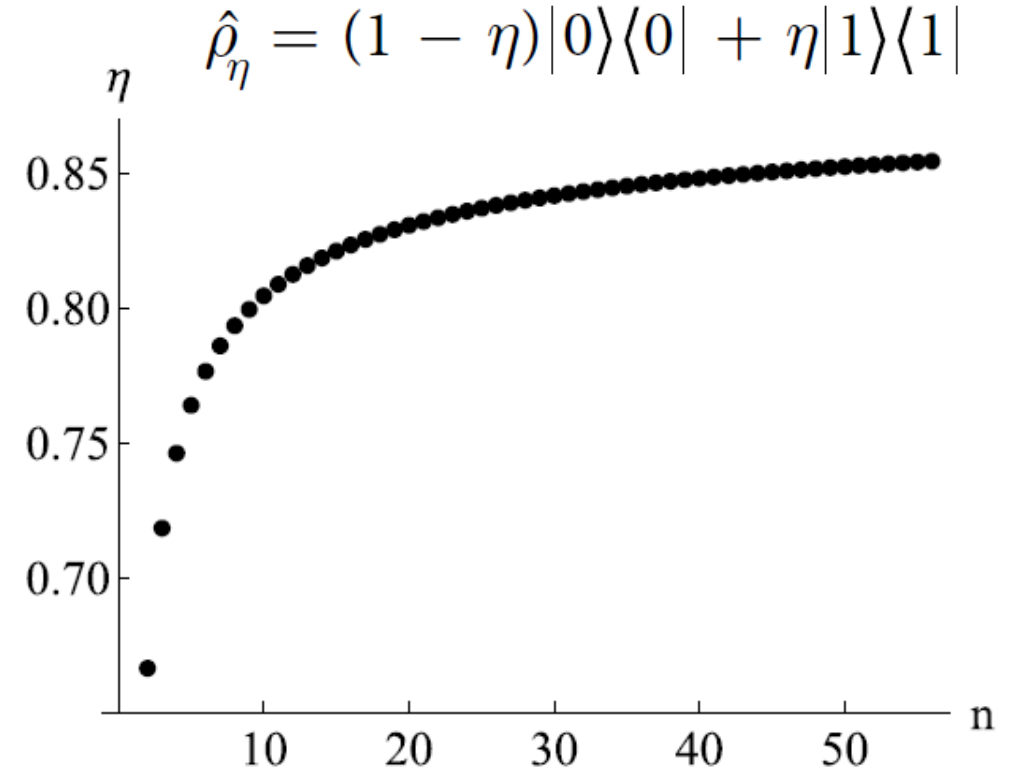
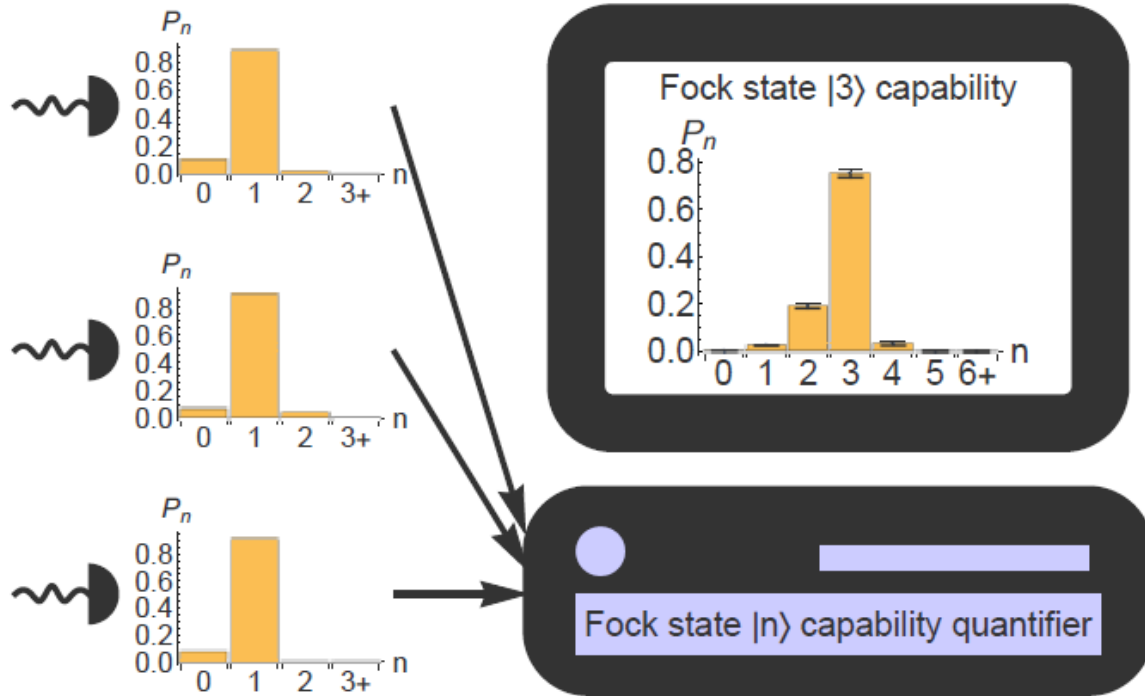


Independent estimated states from n single photon sources

P. Zapletal and R. Filip, Sci. Rep. 7, 1484 (2017)

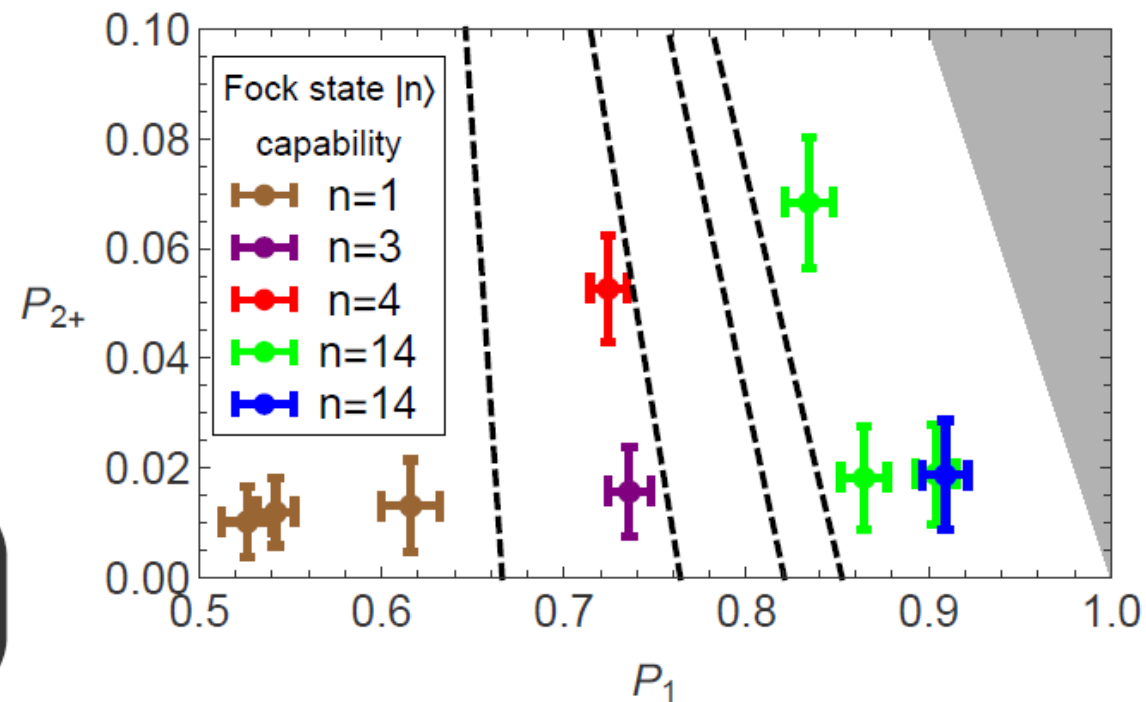
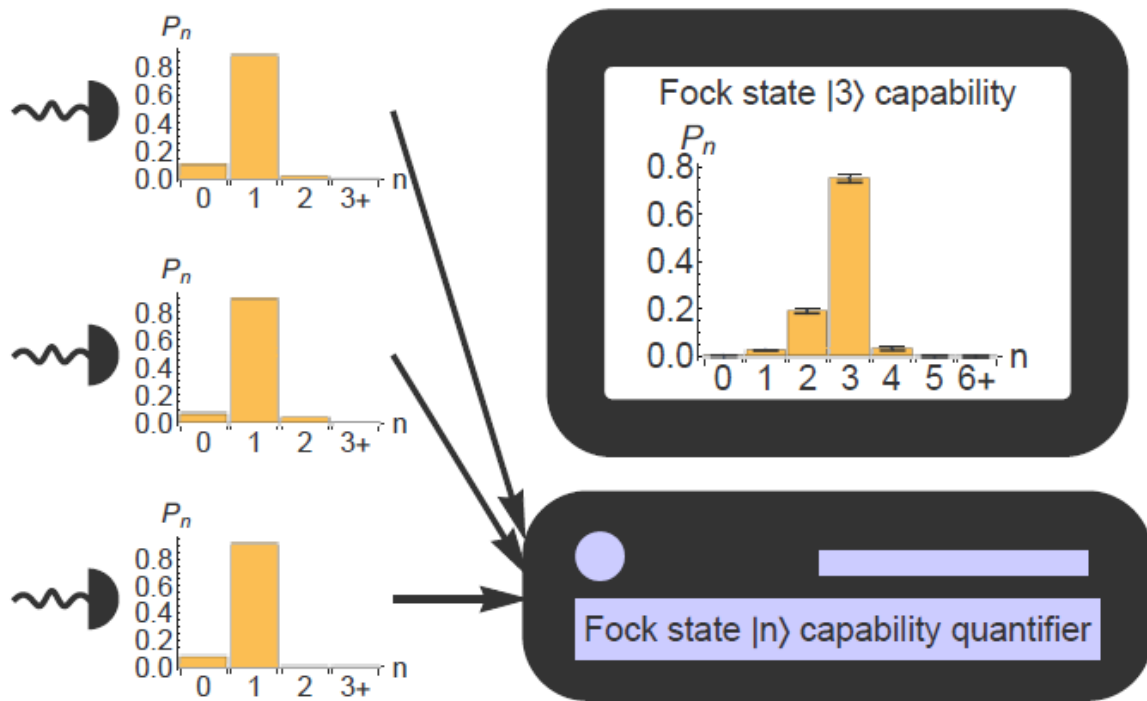


FOCK STATE CAPABILITY





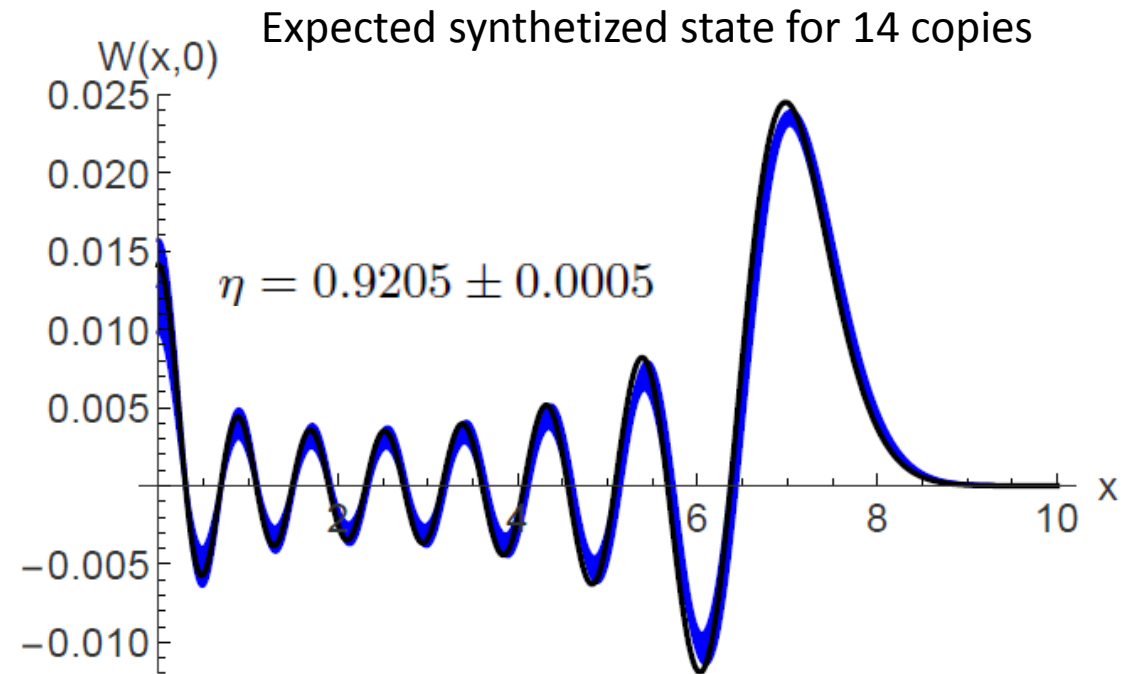
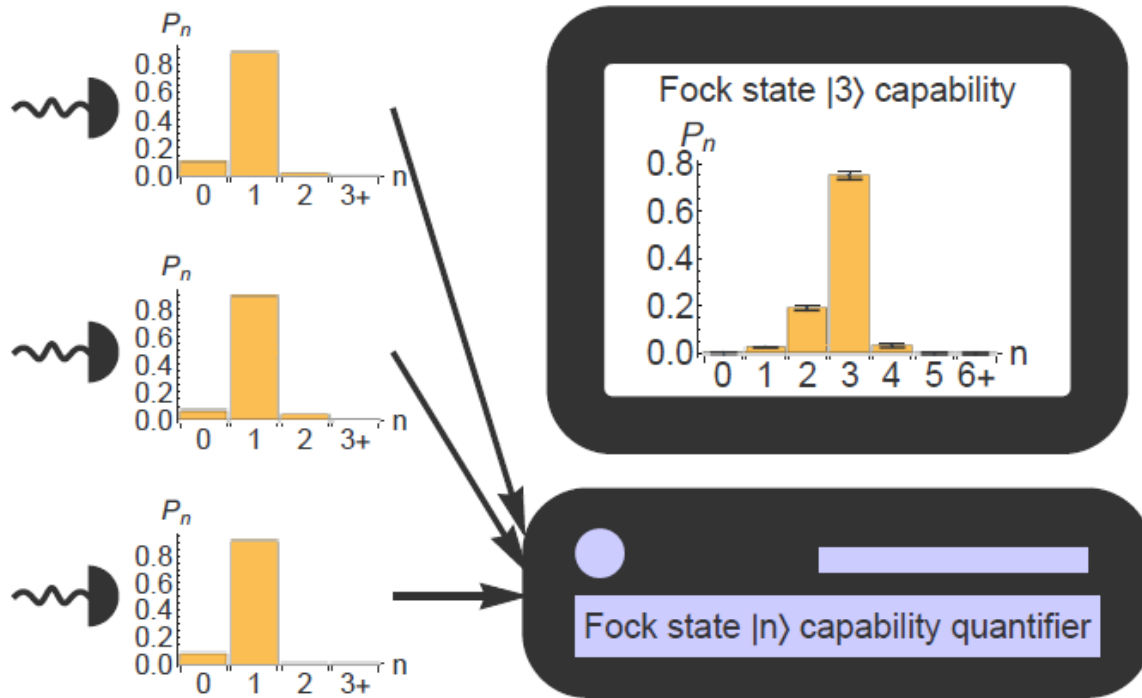
FOCK STATE CAPABILITY



NEXT?



FOCK STATE CAPABILITY



Almost perfectly attenuated Fock state $|14\rangle$.



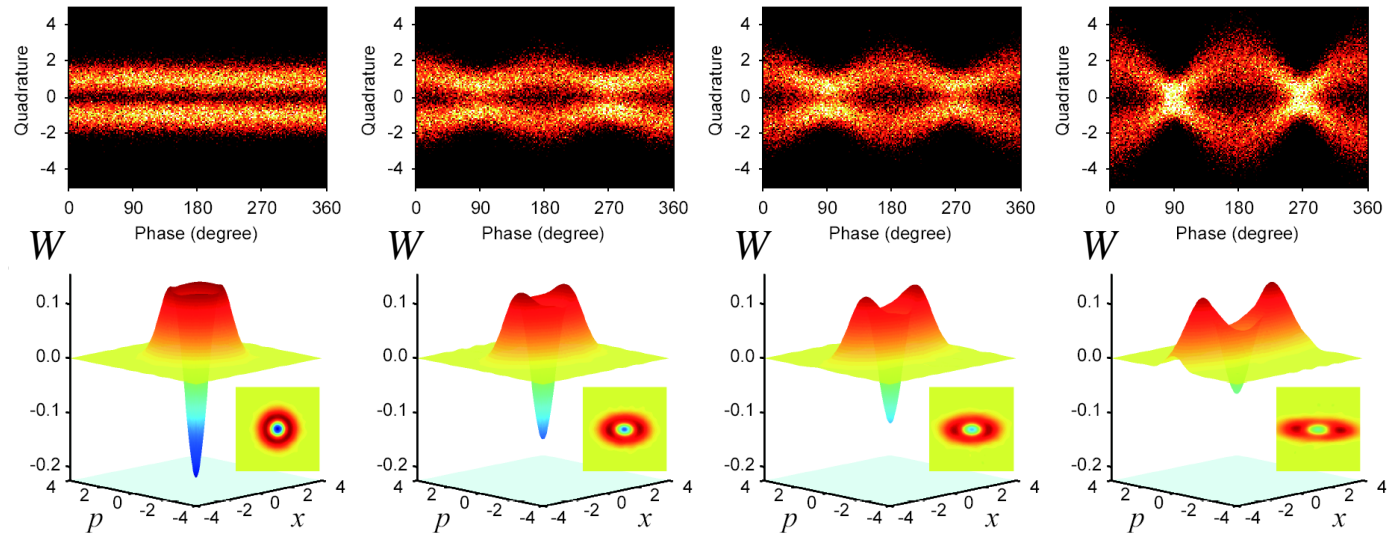
QUANTUM OPERATIONAL AMPLIFIER/SQUEEZER

Classical AMP



From off-line squeezed state to on-line linear amplifier/ squeezer for any travelling light.

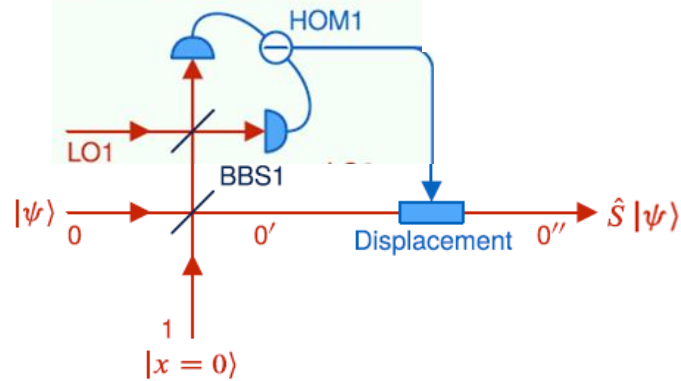
Single-photon amplification/squeezing



Y. Miwa, J. Yoshikawa, N. Iwata, M. Endo, P. Marek, R. Filip, P. van Loock, and A. Furusawa, Phys. Rev. Lett. 113, 013601 (2014)

Quantum AMP

Gaussian measurement





QUANTUM OPERATIONAL AMPLIFIER/SQUEEZER

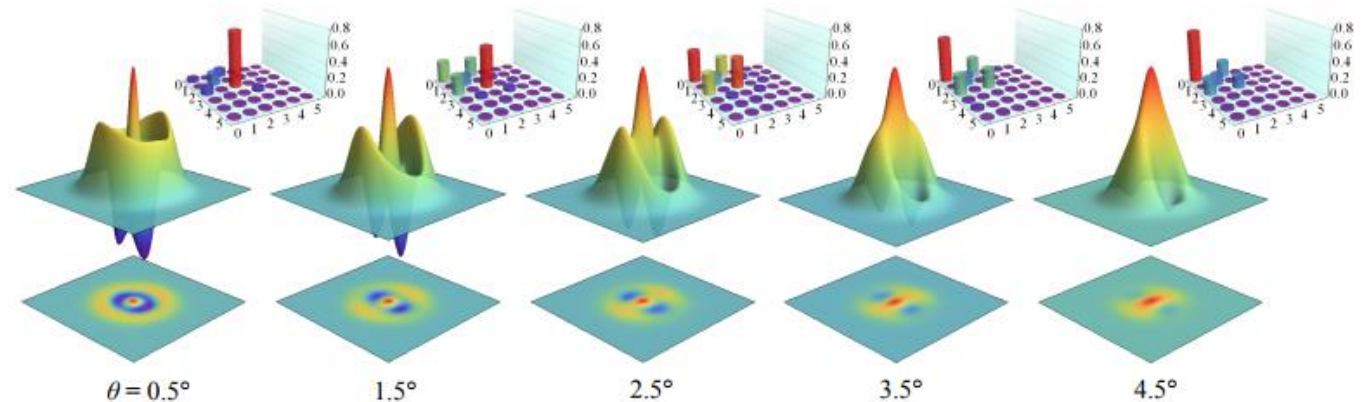
Classical AMP



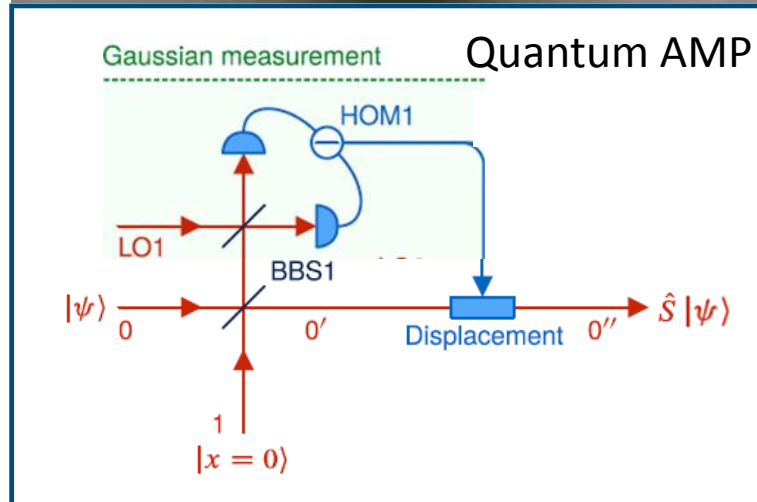
From off-line squeezed state to on-line linear amplifier/ squeezer for any travelling light.

Amplification reduces a demand on state preparation

D. Menzies and R. Filip, Phys. Rev. A 79, 012313 (2009)



K. Huang, H. Le Jeannic, J. Ruaudel, V.B. Verma, M.D. Shaw, F. Marsili, S.W. Nam, E Wu, H. Zeng, Y.-C. Jeong, R. Filip, O. Morin, J. Laurat, Phys. Rev. Lett. 115, 023602 (2015)



R. Filip, P. Marek and U.L. Andersen, Phys. Rev. A 71, 042308 (2005).

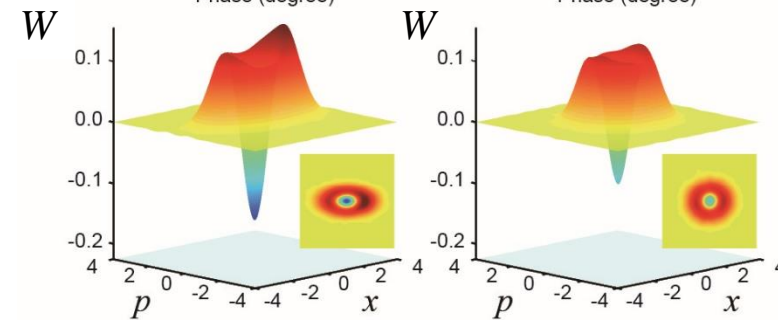
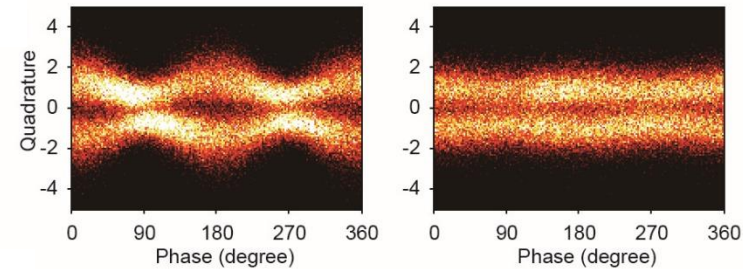
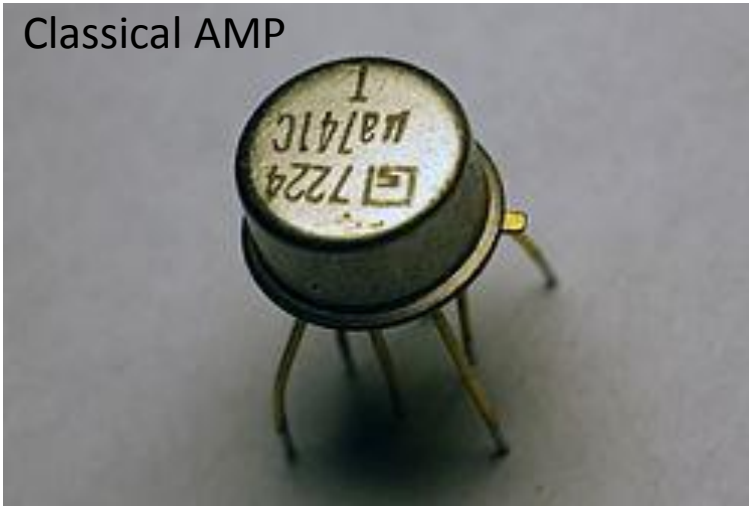


QUANTUM OPERATIONAL AMPLIFIER/SQUEEZER

From off-line squeezed state to on-line linear
amplifier/ squeezer for any travelling light.

Single-photon de-amplification/squeezing

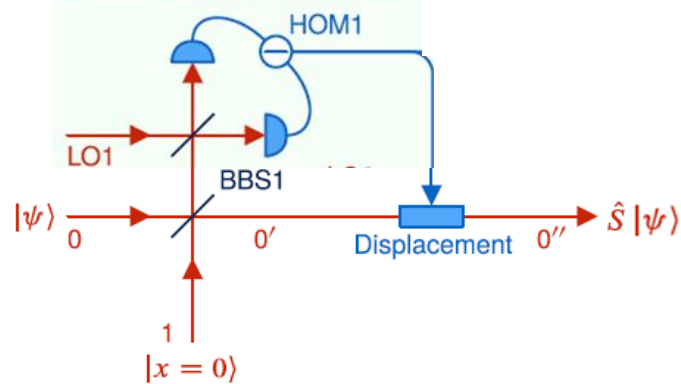
Classical AMP



Y. Miwa, J. Yoshikawa, N. Iwata, M. Endo, P. Marek, R. Filip, P. van Loock, and A. Furusawa, Phys. Rev. Lett. 113, 013601 (2014)

Quantum AMP

Gaussian measurement





QUANTUM OPERATIONAL AMPLIFIER/SQUEEZER

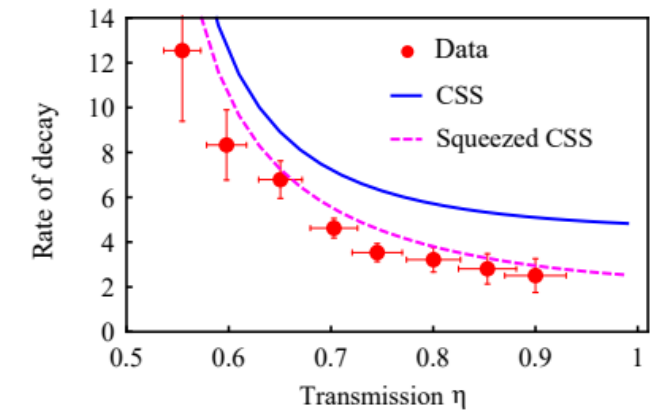
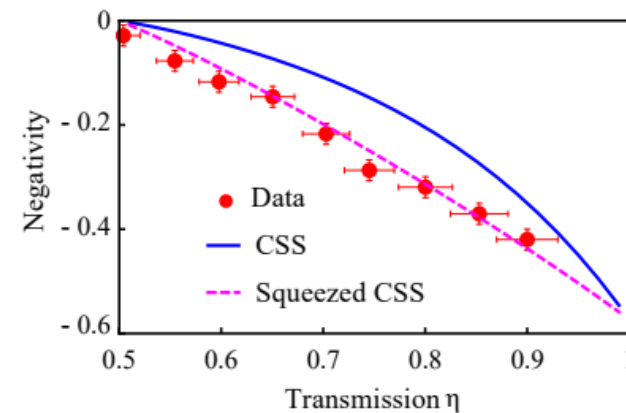
From off-line squeezed state to on-line linear
amplifier/ squeezer for any travelling light.

Classical AMP



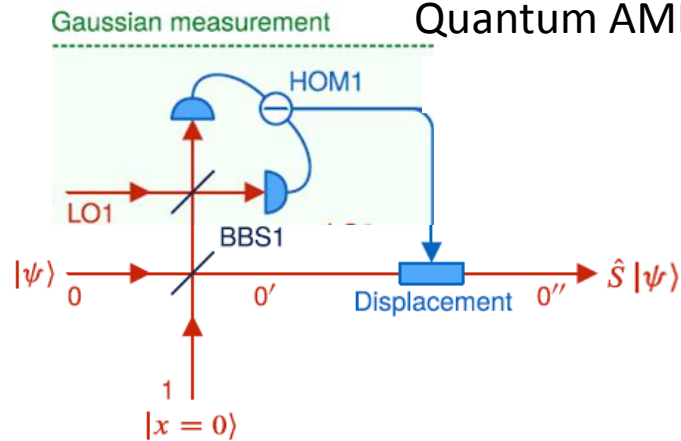
Amplification slows down quantum interference decay

R. Filip, Phys. Rev. A 87, 042308 (2013)



H. Le Jeannic, A. Cavallès, K. Huang, R. Filip, J. Laurat, arXiv:1707.06244

Quantum AMP



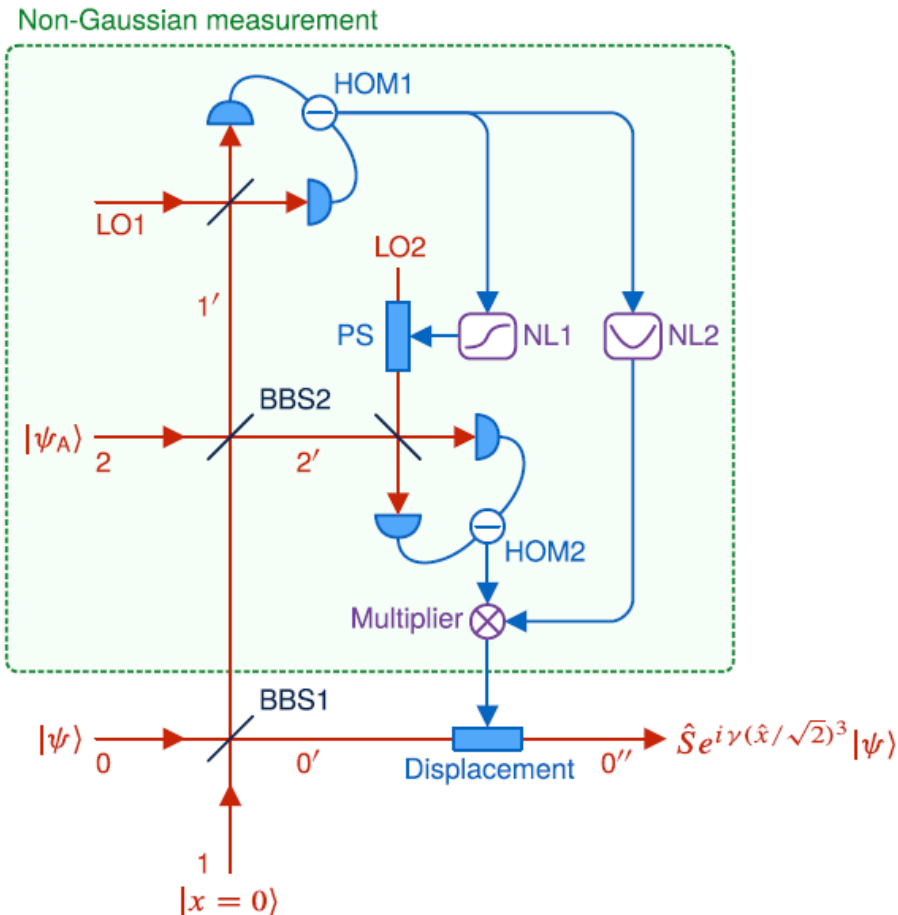
R. Filip, P. Marek and U.L. Andersen, Phys. Rev. A 71, 042308 (2005).

NEXT?



DETERMINISTIC CUBIC NONLINEARITY

$$\hat{U} = e^{i\gamma\hat{x}^3} \quad \hat{x}' = \hat{x}, \quad \hat{p}' = \hat{p} + 3\gamma\hat{x}^2$$



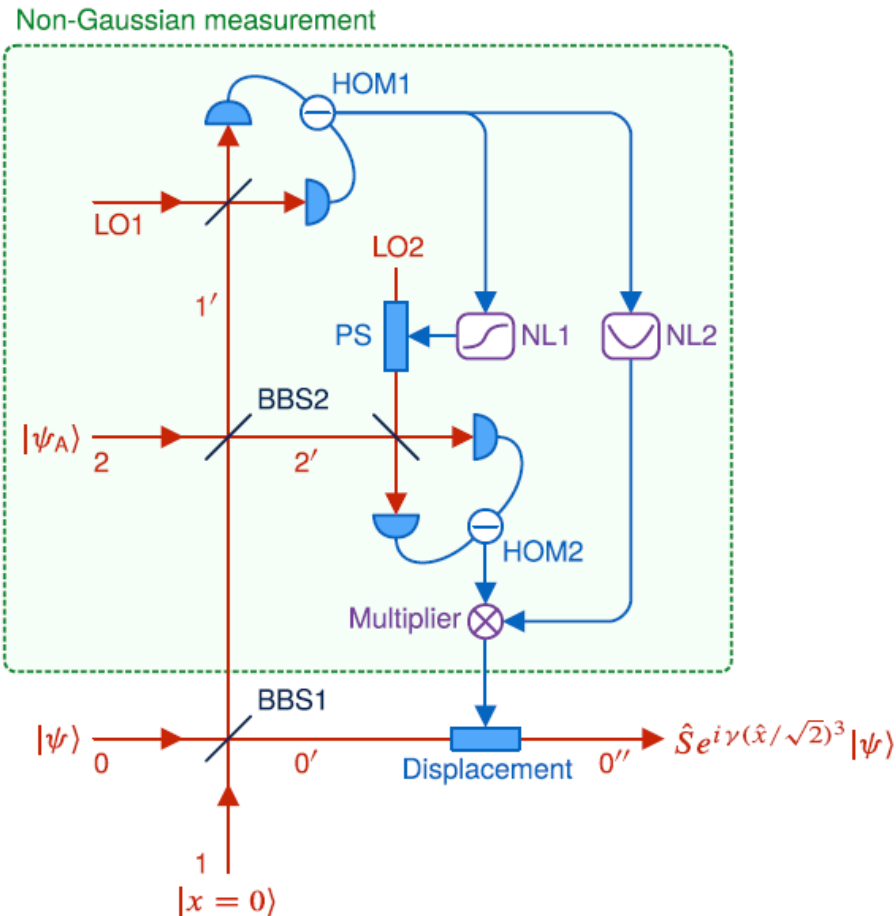
- adaptive measurement strategy
- nonlinear feedforward control
- fast time-resolved regime

$$\begin{aligned} \hat{x}_0'' &= \frac{1}{\sqrt{2}} \hat{x}_0 - \frac{1}{\sqrt{2}} \hat{x}_1, && \text{mode 1} \\ \hat{p}_0'' &= \sqrt{2} \left(\hat{p}_0 + \frac{3\gamma}{2\sqrt{2}} \hat{x}_0^2 \right) && \text{mode 0} \\ &+ (\hat{p}_2 - 3\gamma\hat{x}_2^2) + 3\gamma \left(\hat{x}_0\hat{x}_1 + \frac{1}{2}\hat{x}_1^2 \right) && \text{mode 2} \quad \text{mode 1} \end{aligned}$$



DETERMINISTIC CUBIC NONLINEARITY

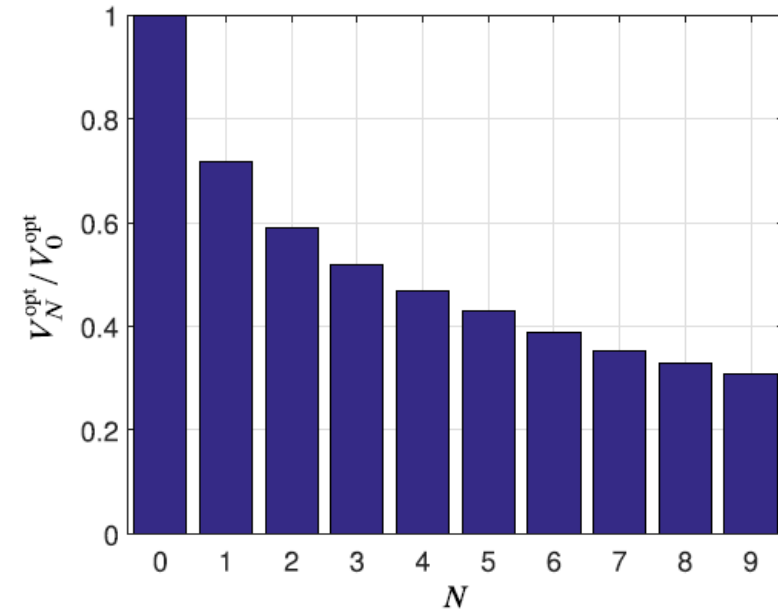
$$\hat{U} = e^{i\gamma\hat{x}^3} \quad \hat{x}' = \hat{x}, \quad \hat{p}' = \hat{p} + 3\gamma\hat{x}^2$$



- nonlinear squeezed states (cubic)

$$(\hat{p}_2 - 3\gamma\hat{x}_2^2)$$

variance of added noise

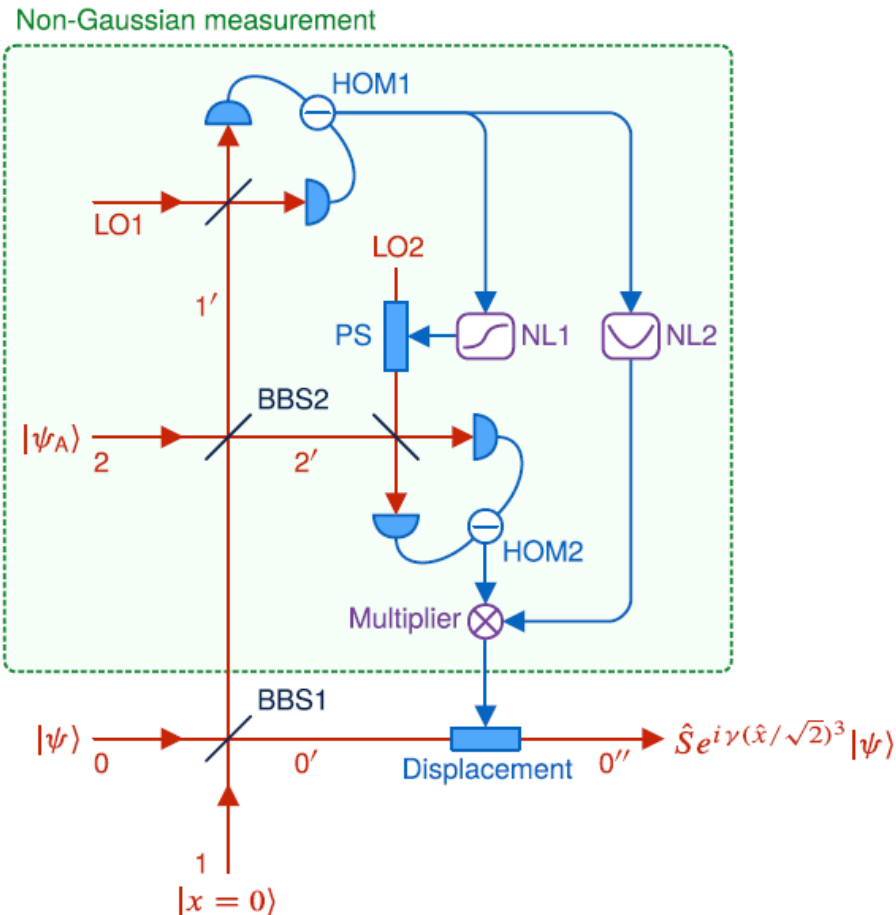


N-photon superposition for ancilla $|\psi_A\rangle$



DETERMINISTIC CUBIC NONLINEARITY

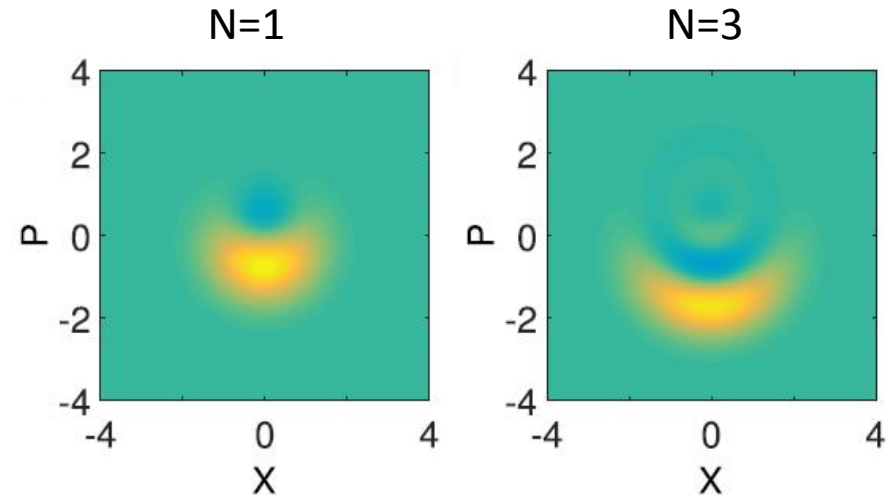
$$\hat{U} = e^{i\gamma\hat{x}^3} \quad \hat{x}' = \hat{x}, \quad \hat{p}' = \hat{p} + 3\gamma\hat{x}^2$$



- nonlinear squeezed states (cubic)

$$(\hat{p}_2 - 3\gamma\hat{x}_2^2)$$

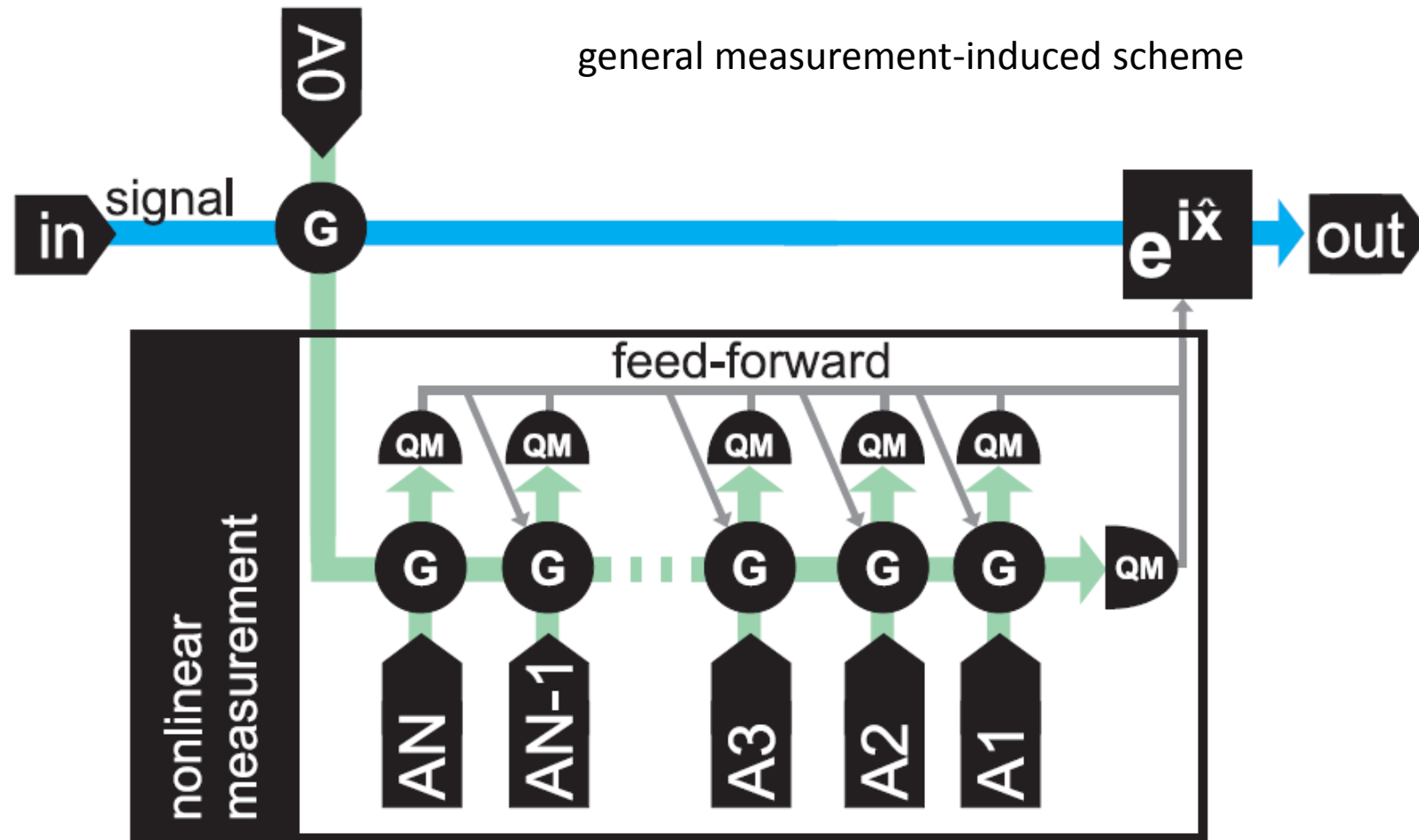
Wigner function



N-photon superposition for ancilla $|\psi_A\rangle$



DETERMINISTIC $V(x)$ NONLINEARITY



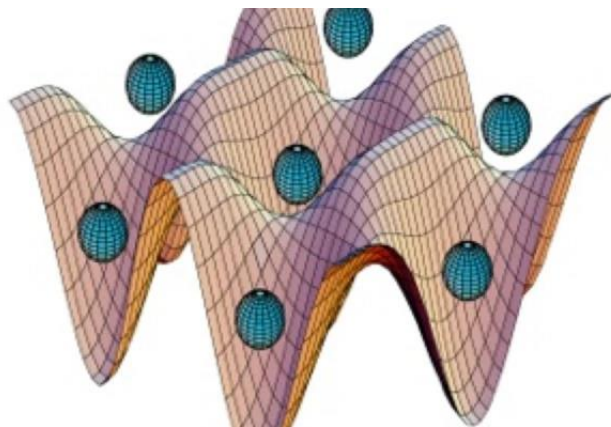


FUTURE APPLICATIONS



analog quantum simulators

I.H. Deutsch, Scientific American (2015)

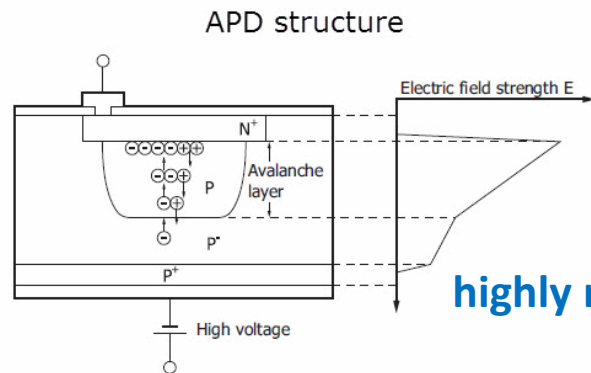


highly nonlinear



analog quantum sensing

L. Degen, F. Reinhard, P. Cappellaro
Rev. Mod. Phys. 89, 035002 (2017)

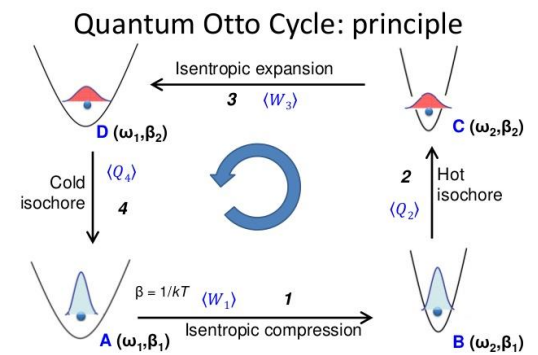


highly nonlinear



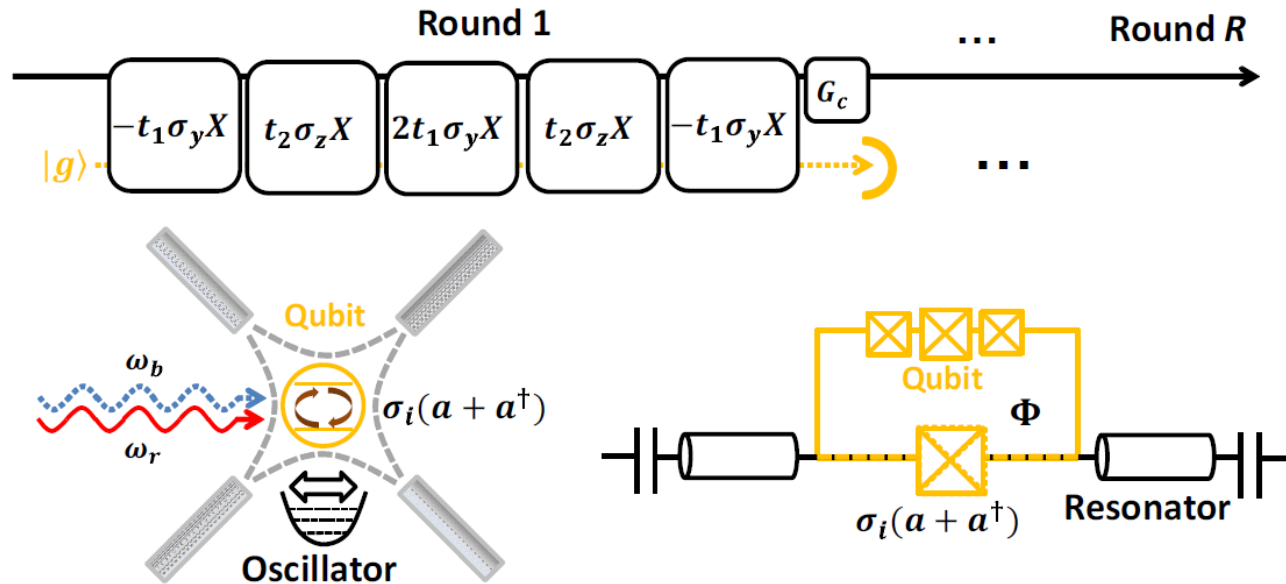
analog quantum engines

J. Millen, A. Xuereb, NJP (2016)



highly nonlinear

DETERMINISTIC CUBIC NONLINEARITY



$$\Gamma[\rho] = \hat{G}_c (\hat{O}_{3,s} \rho \hat{O}_{3,s}^\dagger + \hat{O}_f \rho \hat{O}_f^\dagger) \hat{G}_c^\dagger$$

$$\begin{aligned} \hat{O}_{3,s} &\approx \langle g | \exp[i2t_2 \hat{X} \cos[2t_1 \hat{X}] \sigma_z] | g \rangle \\ &= \exp[i2t_2 \hat{X} \cos[2t_1 \hat{X}]]. \end{aligned}$$

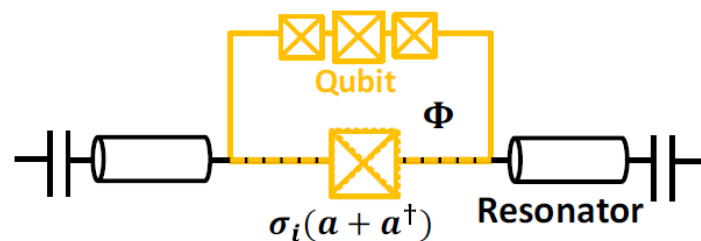
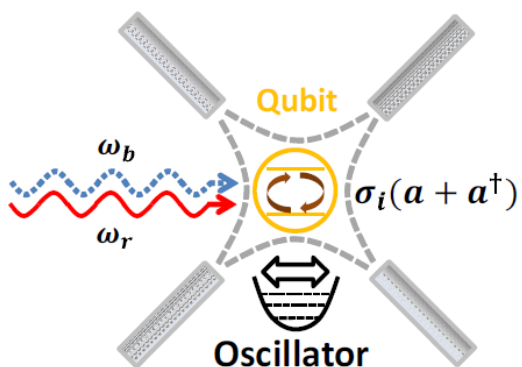
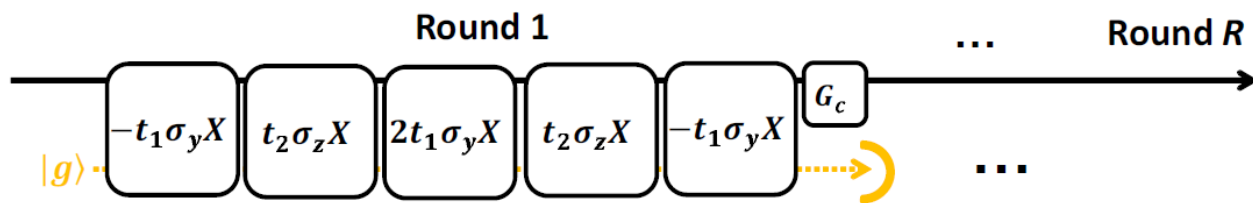
$$\exp[i2t_2 \hat{X} (1 - (2t_1 \hat{X})^2 / 2)]$$

$$\hat{O}_f = -\sin^2(t_2 \hat{X}) \sin(4t_1 \hat{X})$$

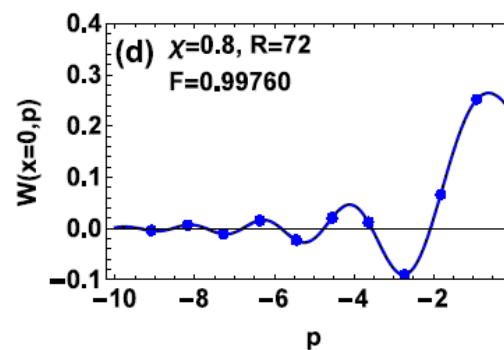
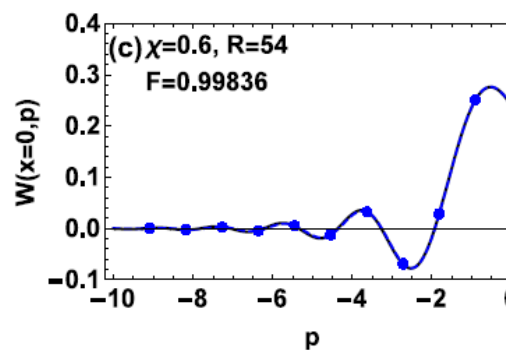
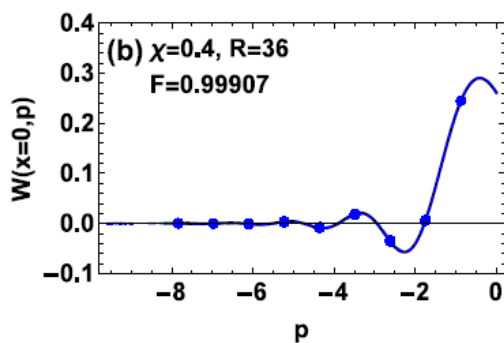
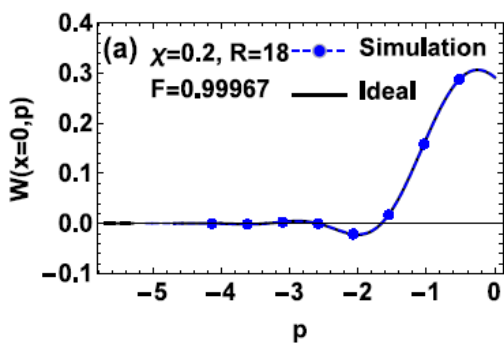
- deterministic without measurement
- many rounds with pulsed control (ions, cavity/circuit QED)
- feasible **Rabi coupling** (J-C interaction beyond RWA)
- correction by simple Gaussian transformation



DETERMINISTIC CUBIC NONLINEARITY

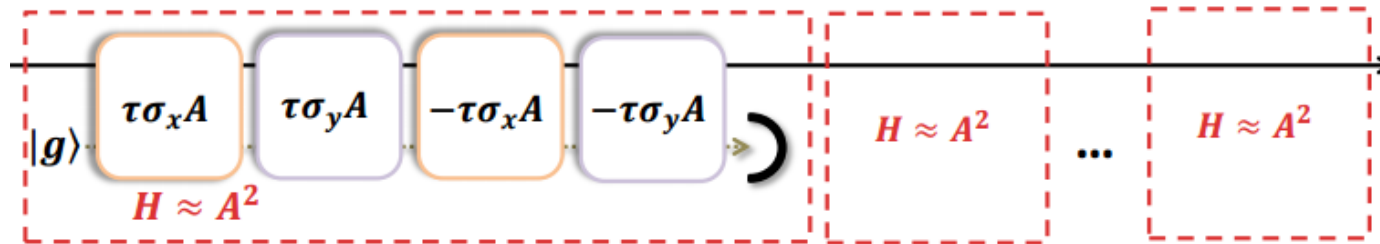


NEXT?



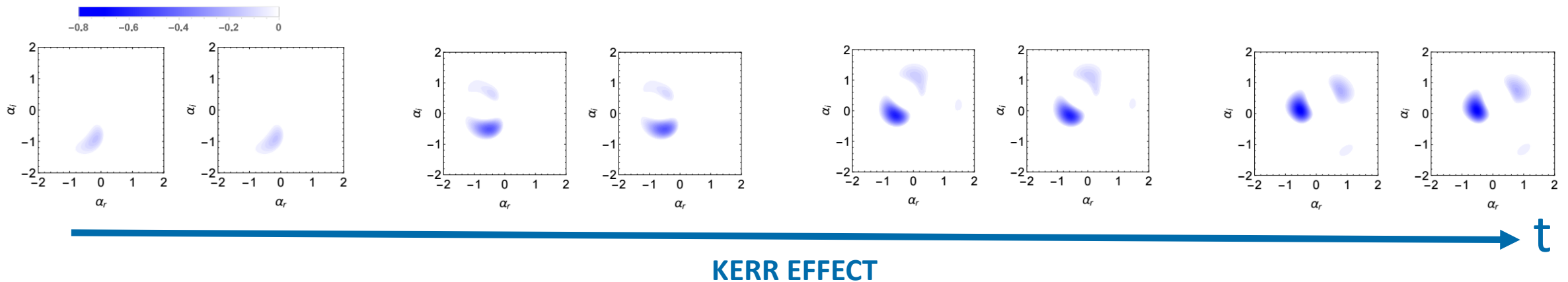


DETERMINISTIC KERR NONLINEARITY



- deterministic without measurement
- many rounds with pulse control
- we can already use dispersive and Rabi coupling

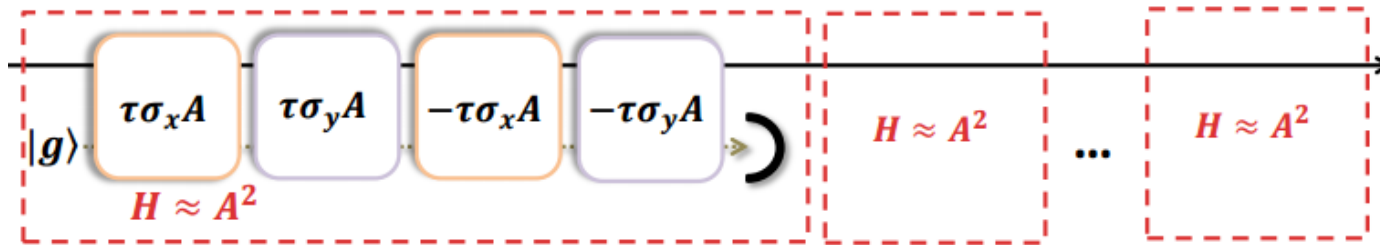
Dynamics of negative part of Wigner function: splitting of negativity



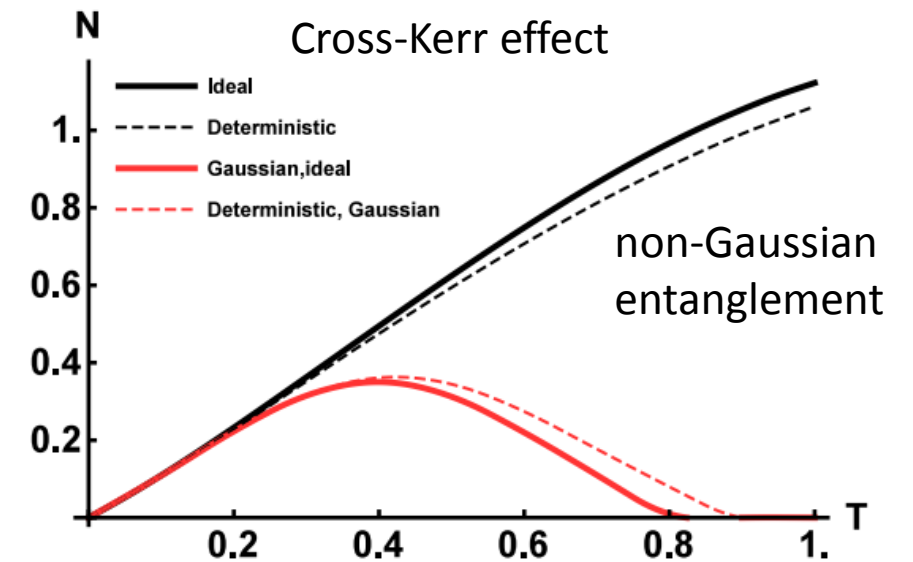
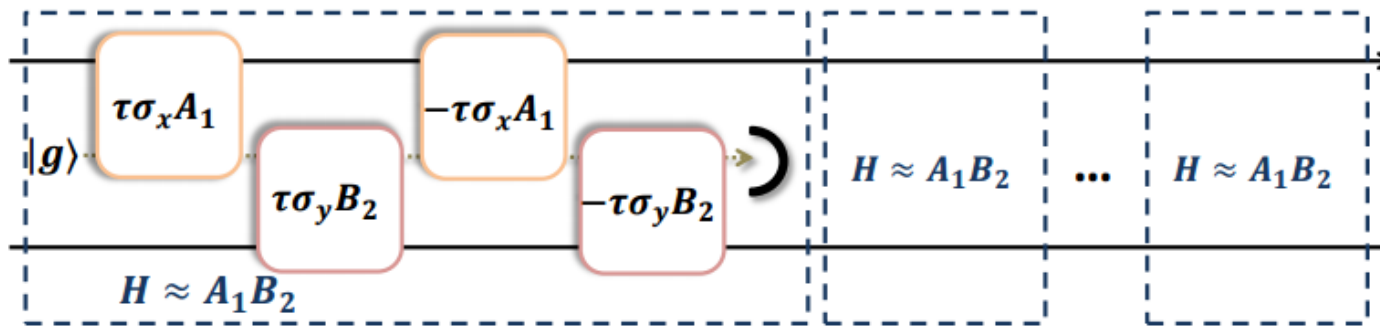
NEXT?



DETERMINISTIC CROSS KERR EFFECT



- deterministic without measurement
- many rounds with pulse control
- we can already use **dispersive** and Rabi coupling

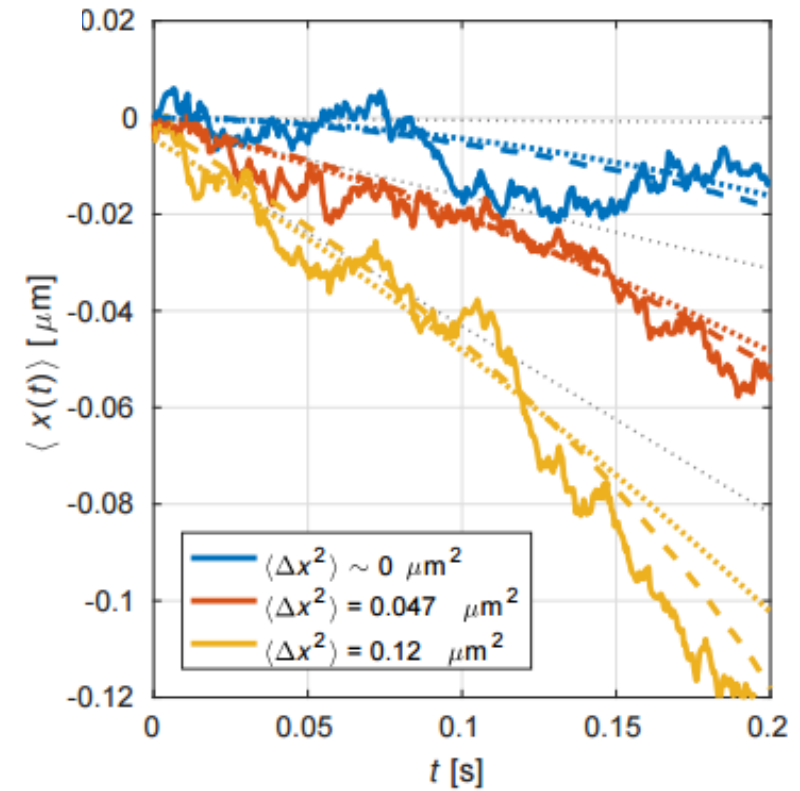
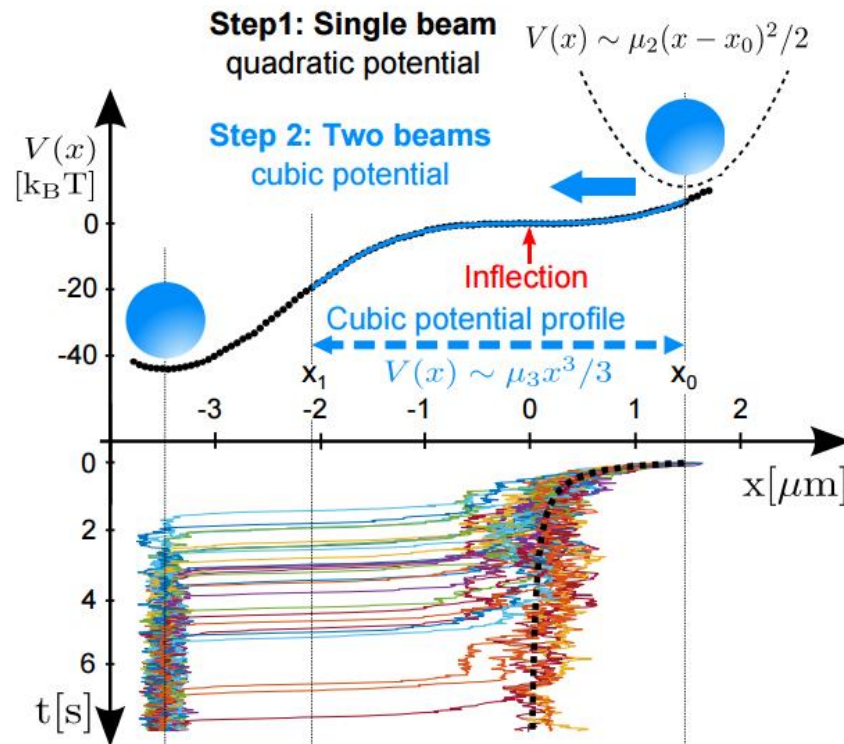


NEXT?



MECHANICAL CUBIC NONLINEARITY

$$\langle x(t) \rangle \simeq \langle x(0) \rangle - \kappa \langle x^2(0) \rangle t - \kappa \frac{k_B T}{\gamma} t^2 + \kappa^2 \langle x^3(0) \rangle t^2$$



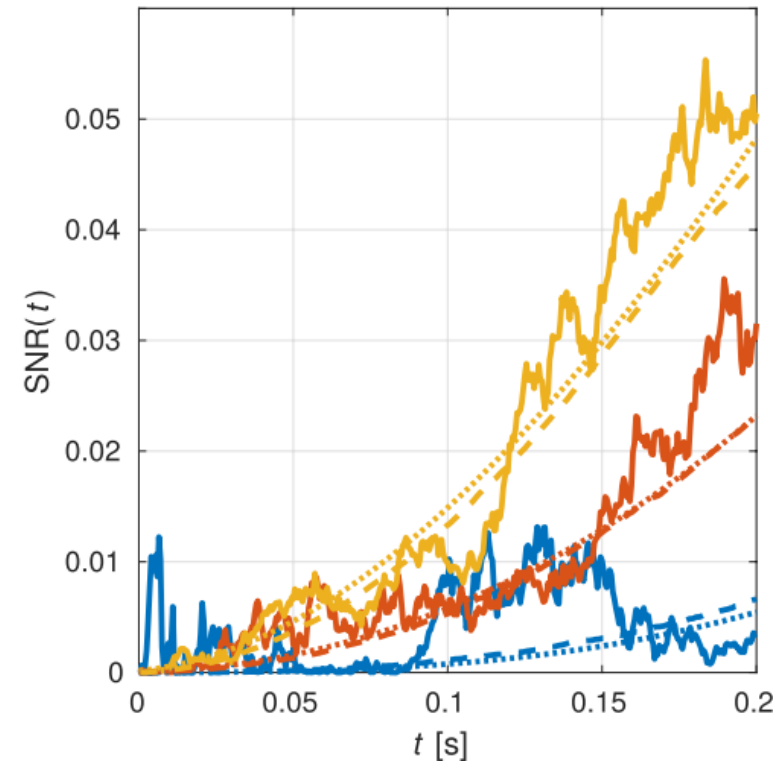
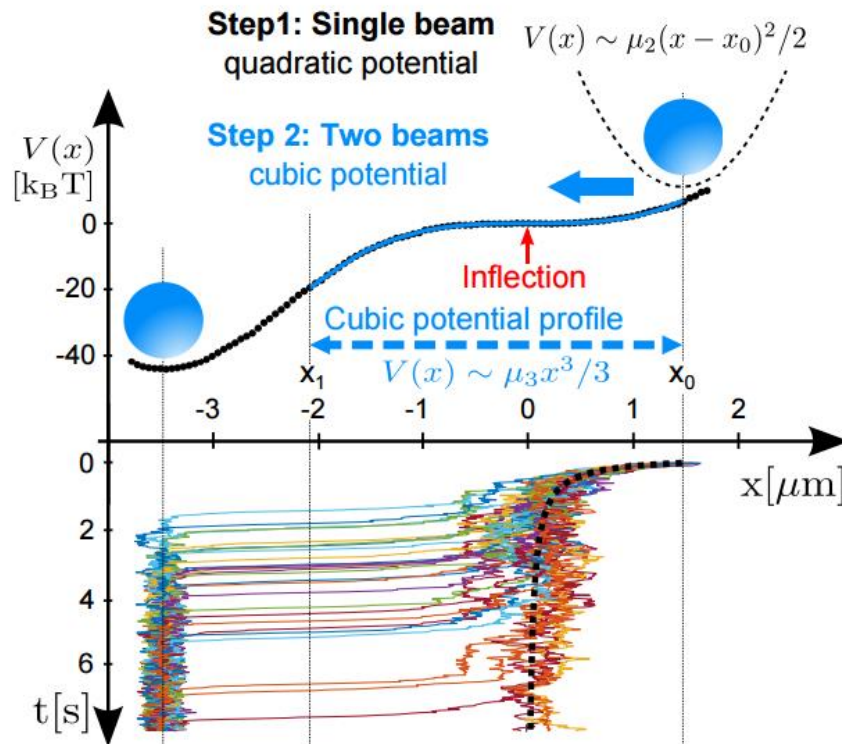
R. Filip and P. Zemánek, J. Opt. 18, 065401 (2016). A. Ryabov, P. Zemánek, and R. Filip, Phys. Rev. E 94, 042108 (2016)

M. Siler, P. Jakl, O. Brzobohaty, A. Ryabov, R. Filip and P. Zemanek, Sci. Rep. 7, 1697 (2017).



MECHANICAL CUBIC NONLINEARITY

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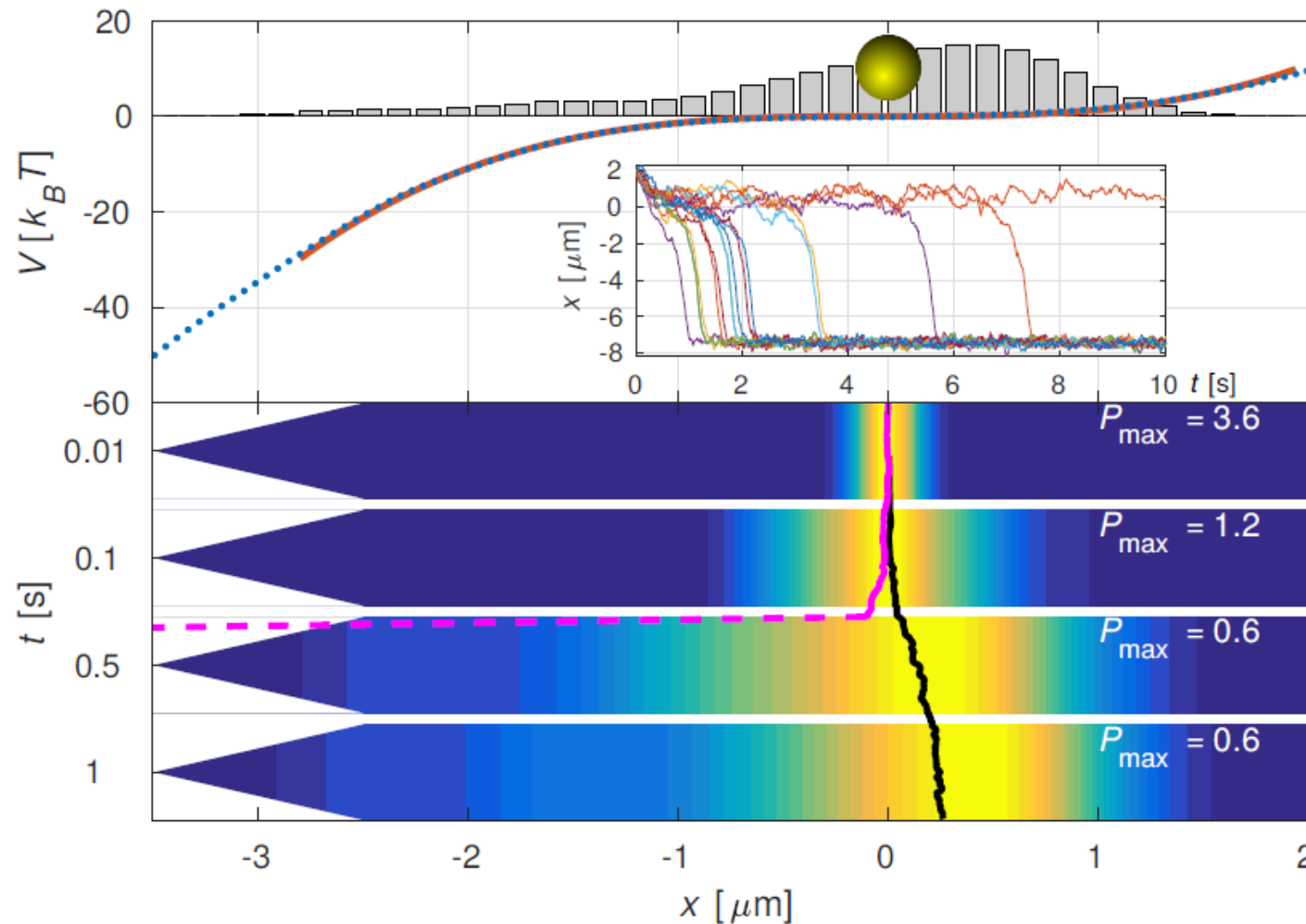


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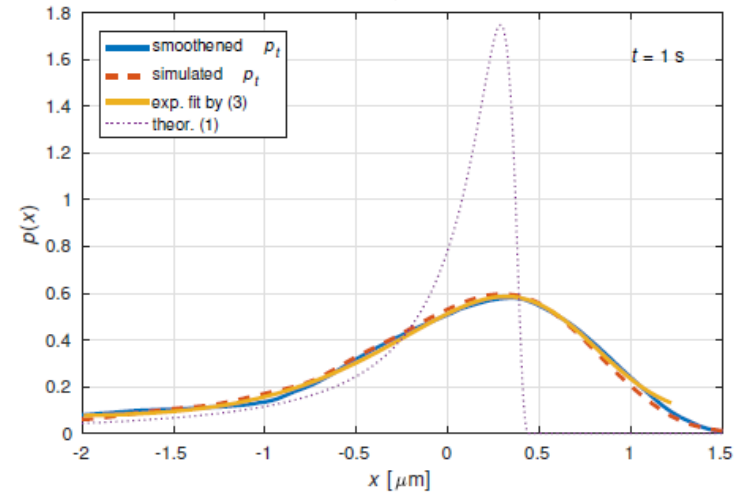
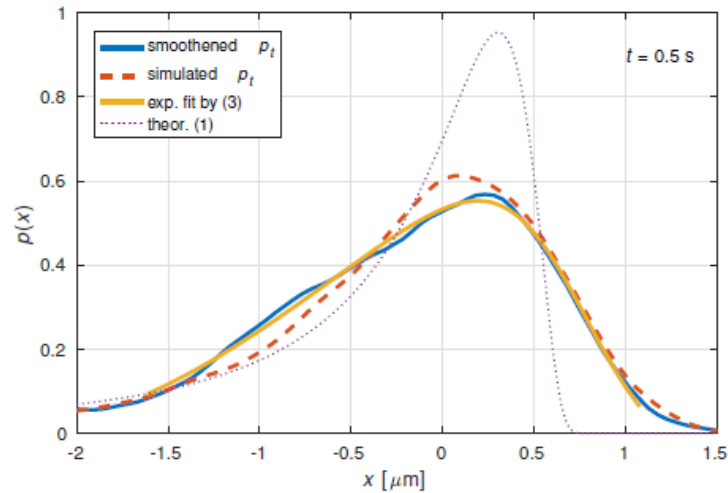
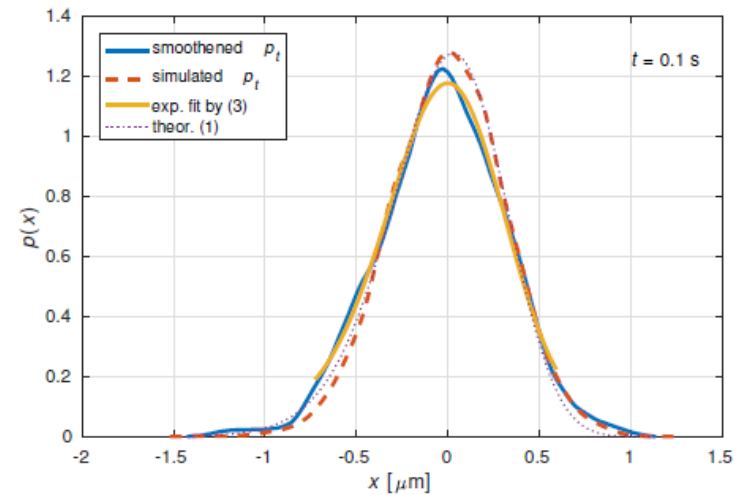
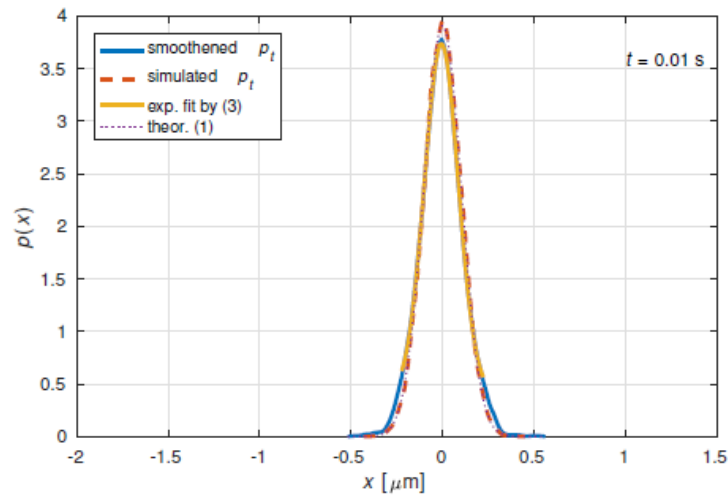
MECHANICAL CUBIC NONLINEARITY



M. Siler, L. Ornigotti, O. Brzobohaty, P. Jakl, A. Ryabov, P. Zemanek, and R. Filip, The-most-likely motion of Brownian particle in an instable cubic potential, submitted (2017).



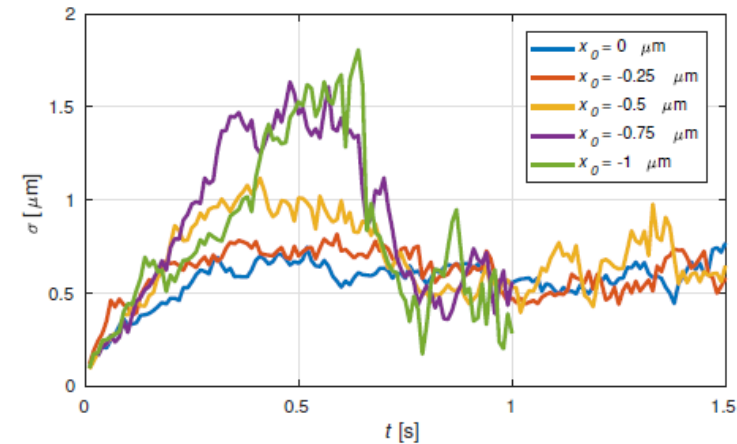
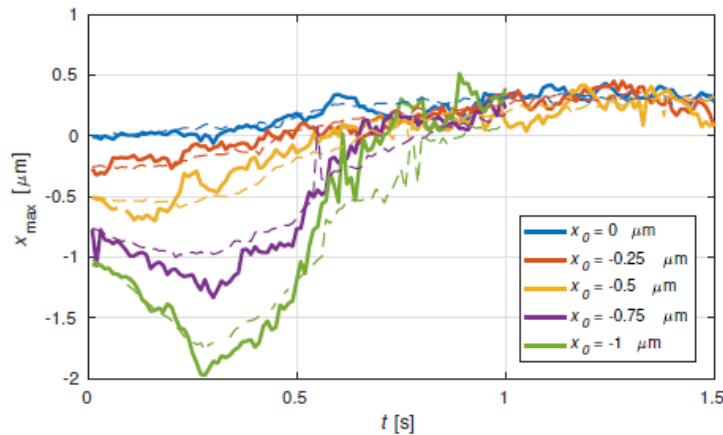
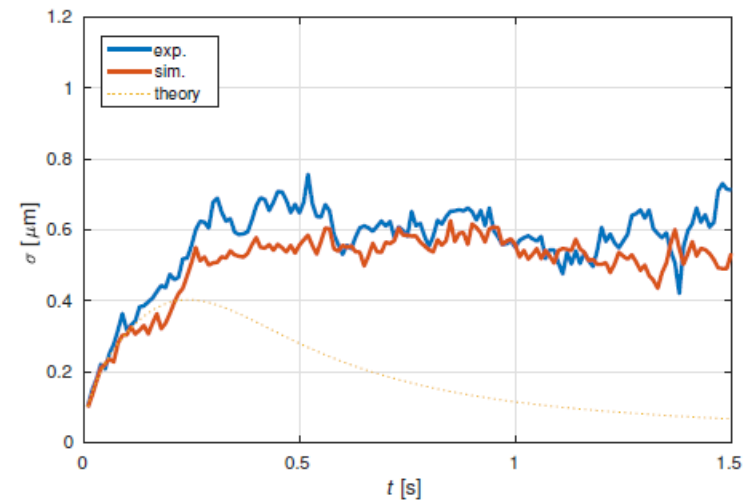
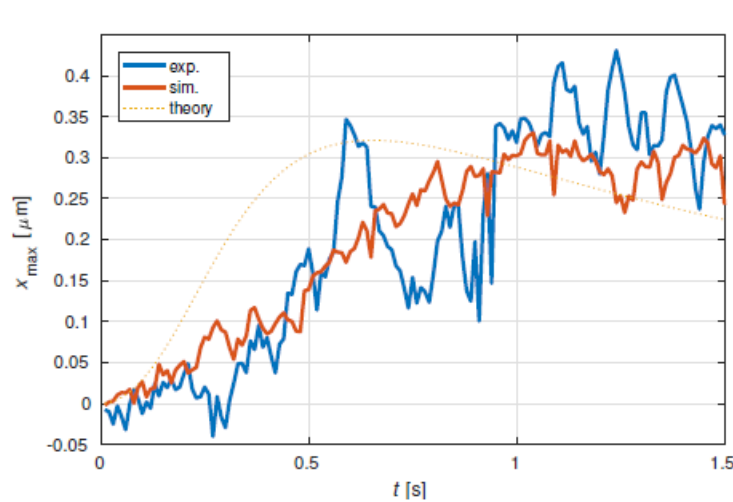
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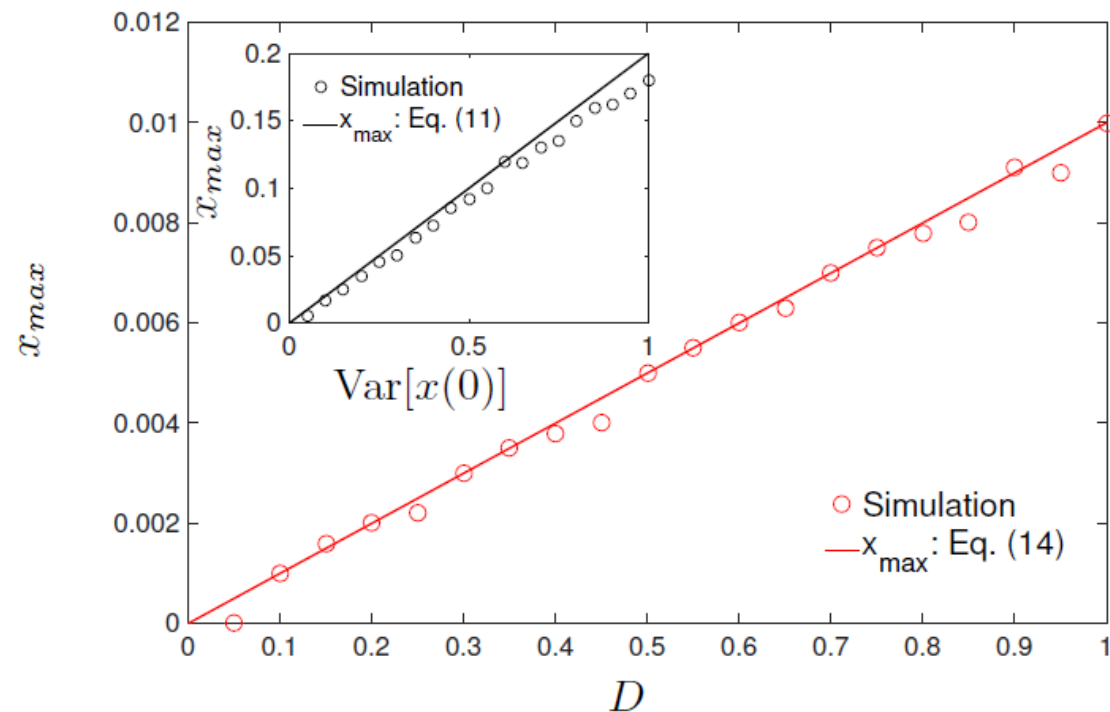
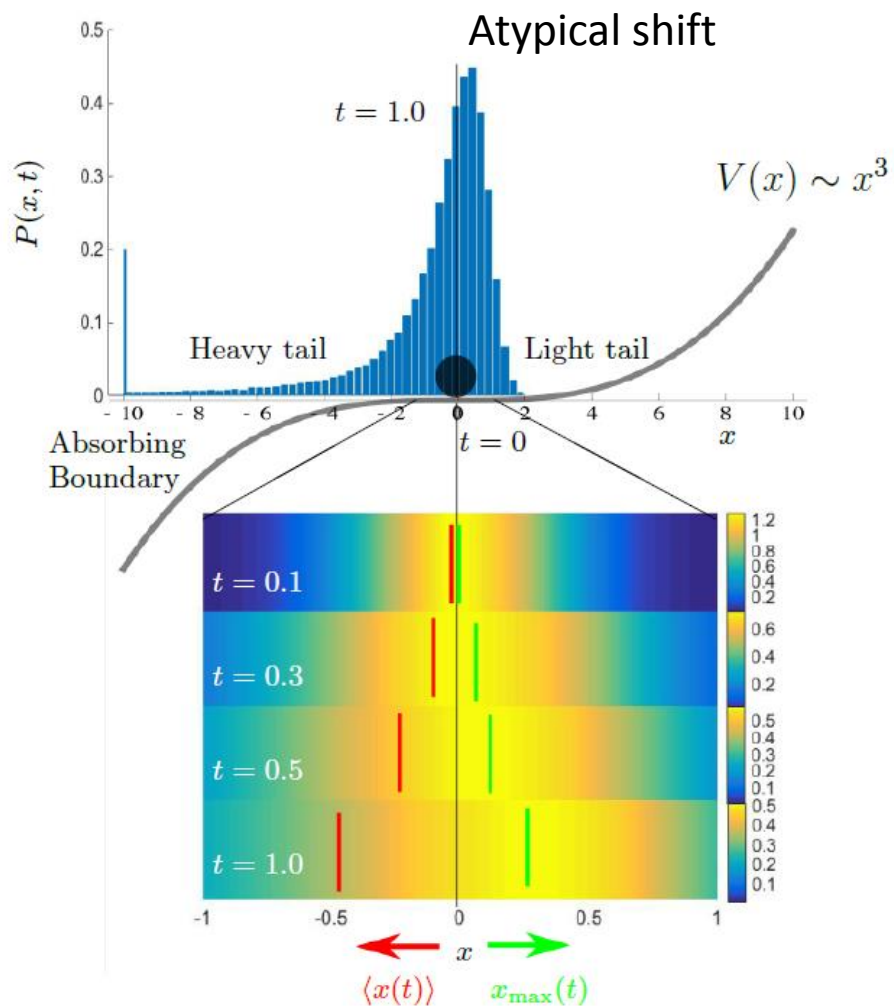
MECHANICAL CUBIC NONLINEARITY



NEXT?



MECHANICAL CUBIC NONLINEARITY



$$x_{\max}(t) \approx kDt^2 \quad \sigma^2(t) \approx 2Dt$$

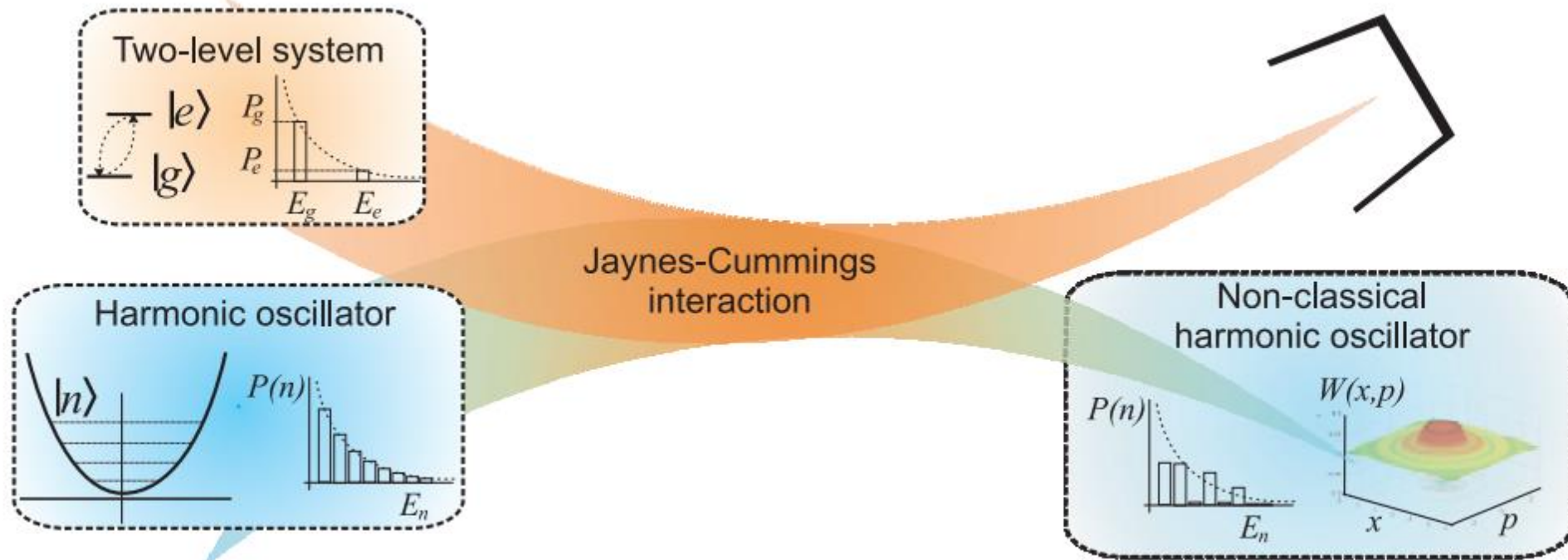
$$\langle x(t) \rangle \approx -kDt^2 \quad \text{Var}[x(t)] \approx 2Dt$$



NOISE-INDUCED NONCLASSICALITY

Energy-conserving resonant J-C interaction:

$$\hbar g(\sigma_+ a + a^\dagger \sigma_-)$$





NOISE-INDUCED NONCLASSICALITY

Energy-conserving resonant J-C interaction: $\hbar g(\sigma_+ a + a^\dagger \sigma_-)$

Nonclassicality test (Klyshko's criteria): $(n + 1)P_{n-1}P_{n+1} - nP_n^2 < 0$

Entanglement potential (BS coupling): $LN(\rho_{split}) = \log_2 \|\rho_{split}^{PT}\|$

$$\rho(t) = \sum_{n=0}^{\infty} \frac{\bar{n}^n}{(1 + \bar{n})^{1+n}} \cos^2(gt\sqrt{n}) |n\rangle\langle n| + \sum_{n=1}^{\infty} \frac{\bar{n}^n}{(1 + \bar{n})^{1+n}} \sin^2(gt\sqrt{n}) |n-1\rangle\langle n-1|$$

Simple analytical formula, but complex and not simply predictable behavior.

MIXTURE OF ANHARMONIC OSCILLATIONS

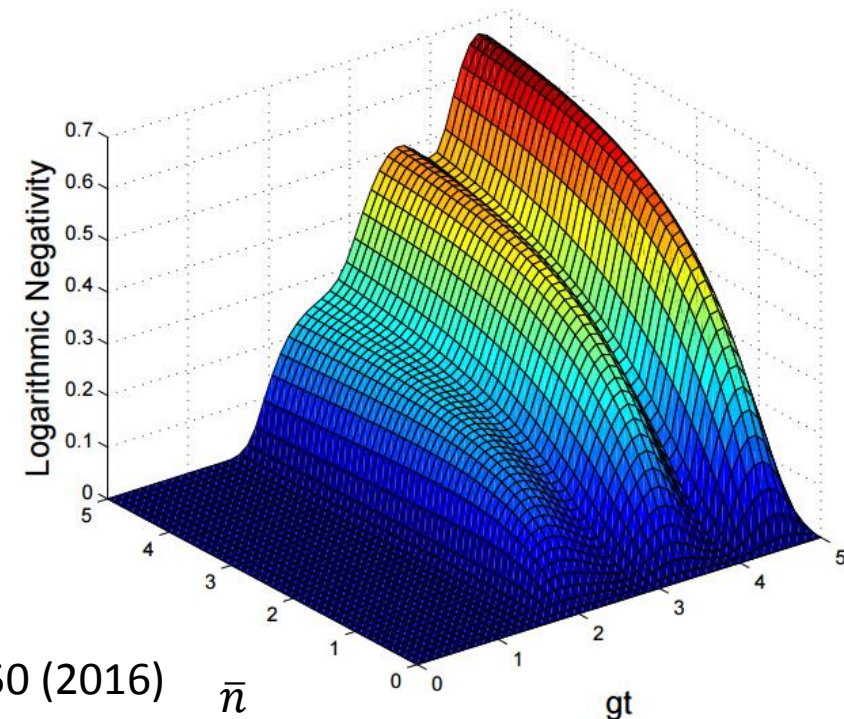
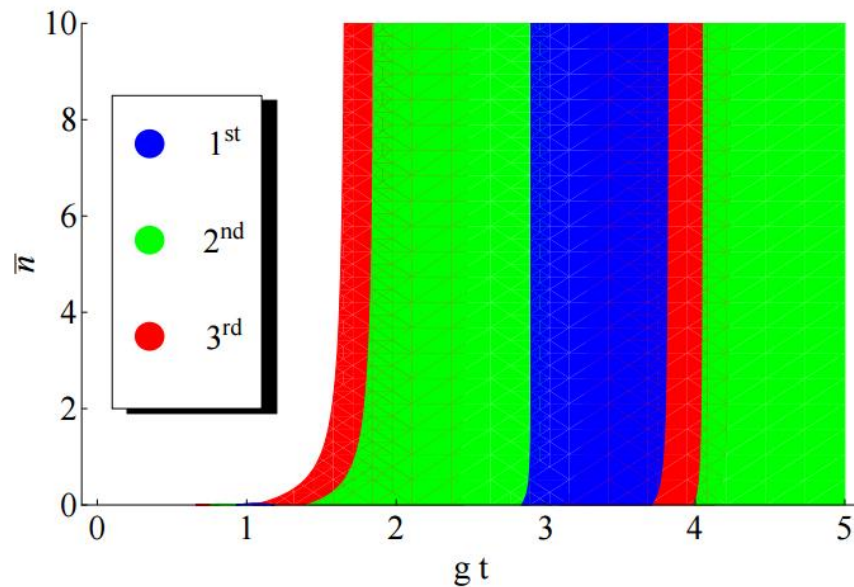


NOISE-INDUCED NONCLASSICALITY

Energy-conserving resonant J-C interaction: $\hbar g(\sigma_+ a + a^\dagger \sigma_-)$

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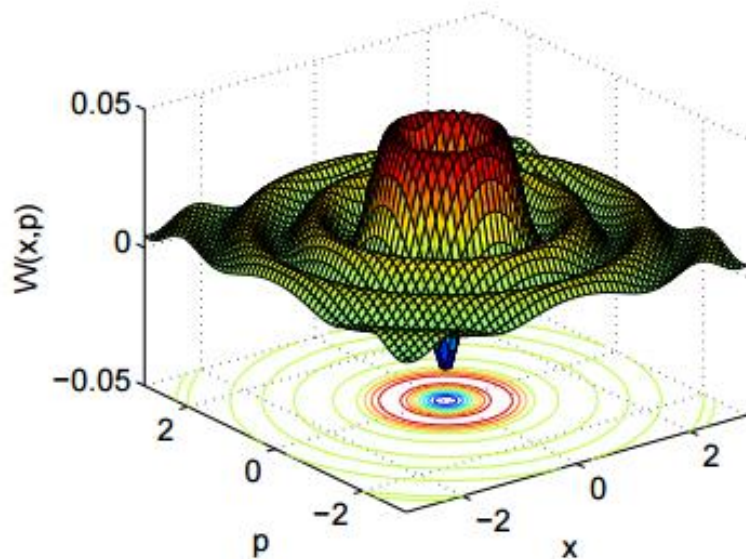
NOISE-ENHANCED NONCLASSICALITY

Energy-conserving resonant J-C interaction:

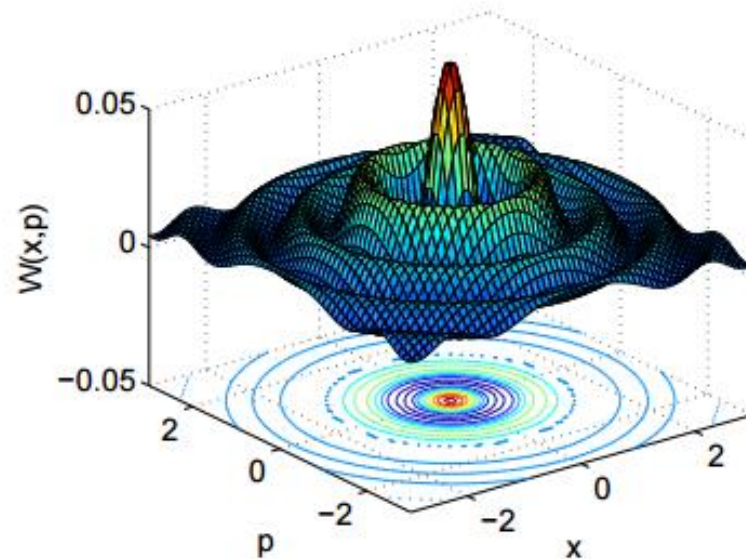
$$\hbar g(\sigma_+ a + a^\dagger \sigma_-)$$

Negative Wigner function:

$$\bar{n} = 10, gt = 5.5$$



$$\bar{n} = 10, gt = 2\pi$$



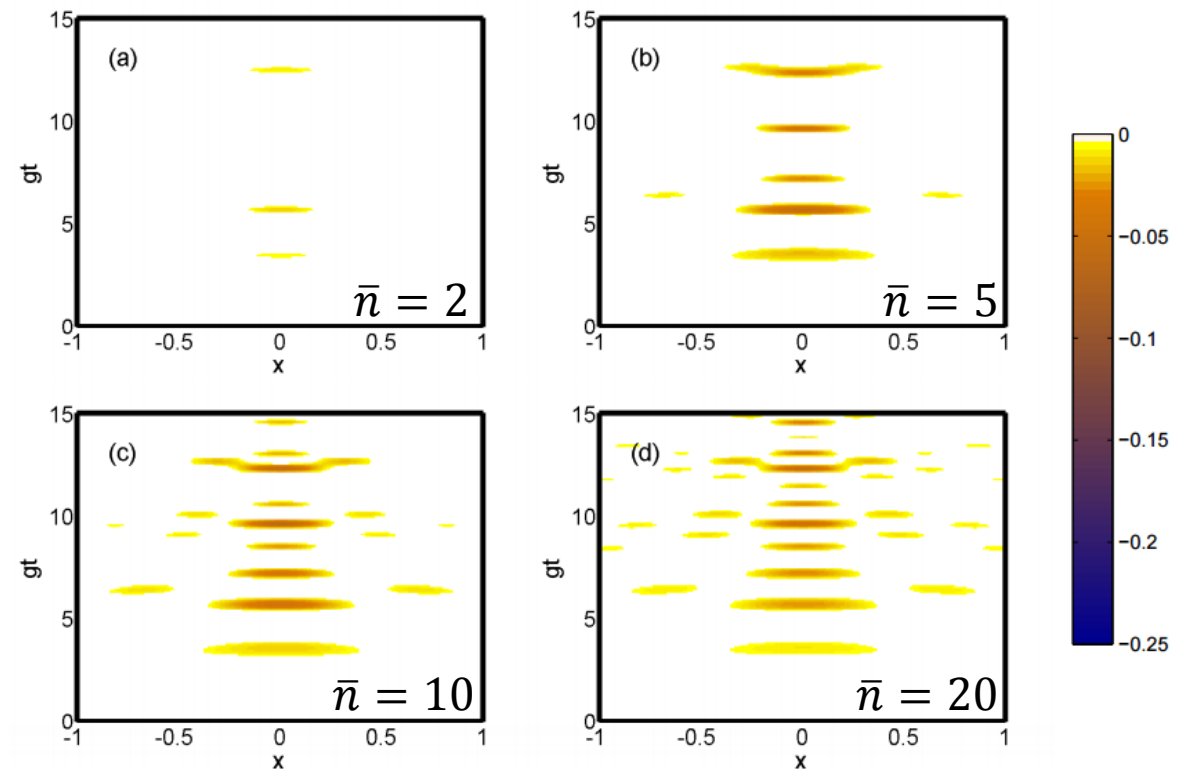


NOISE-ENHANCED NONCLASSICALITY

Energy-conserving resonant J-C interaction:

$$\hbar g(\sigma_+ a + a^\dagger \sigma_-)$$

Negative Wigner function:





FLIP - BASIC QUANTUM OPERATION

$$F(|\psi\rangle \otimes |\phi\rangle) = |\phi\rangle \otimes |\psi\rangle$$

Flip operator defines exchange of quantum states on two distinguishable particles, atoms, oscillators etc.

1. State transfer

$$F(|\psi\rangle \otimes |\phi\rangle) = |\phi\rangle \otimes |\psi\rangle$$

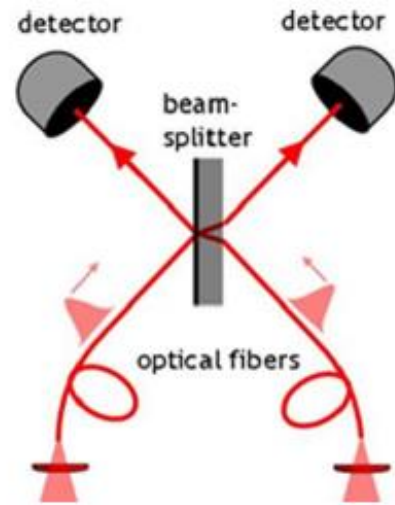
2. Overlap measurement

$$\text{Tr}F(\rho_A \otimes \rho_B) = \text{Tr}\rho_A\rho_B$$

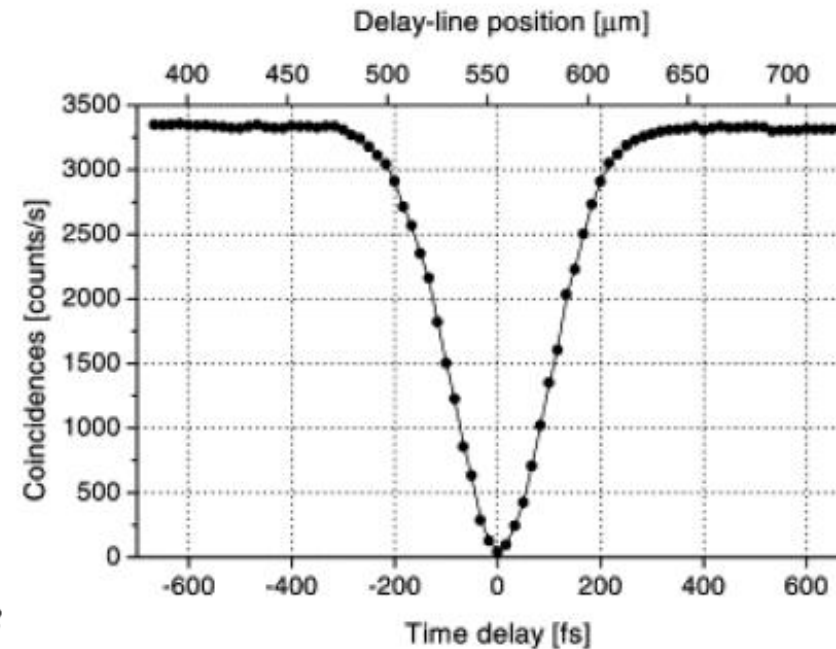
linear
nonlinear

OVERLAP MEASUREMENT

QUANTUM EXPERIMENT (HONG-OU-MANDEL 1987) :



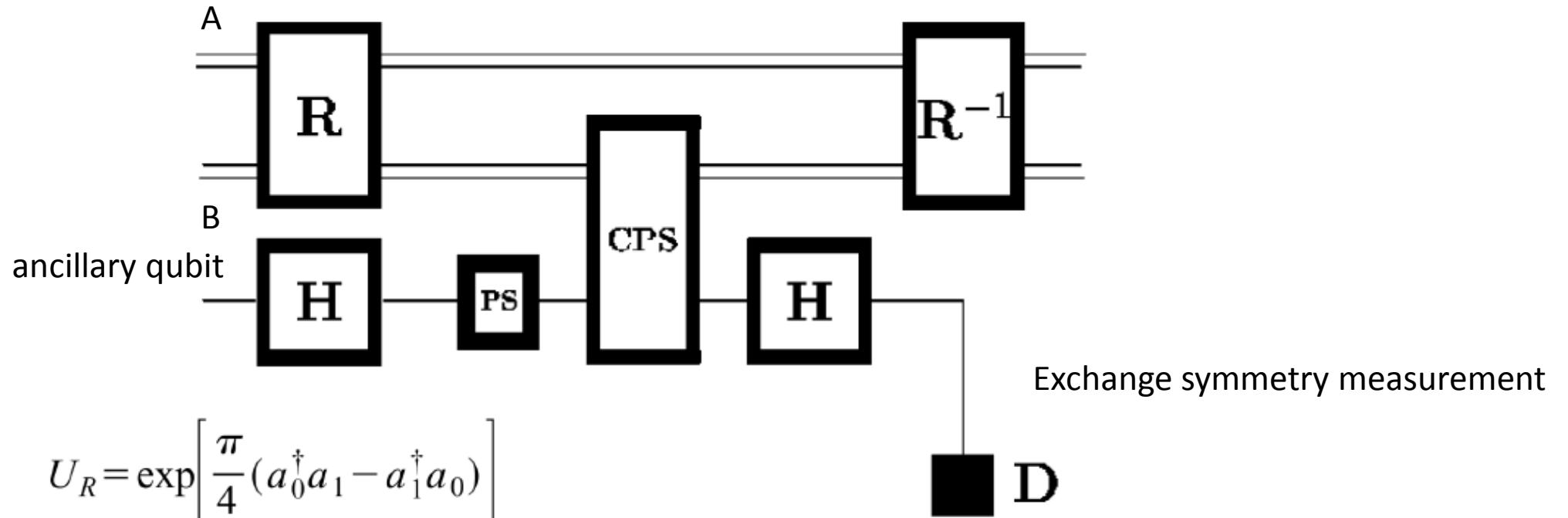
$$|1\rangle_A \langle 1| \otimes \rho_A \quad |1\rangle_B \langle 1| \otimes \rho_B$$



$$\text{Tr}(\rho_A \rho_B) = 1 - \frac{C_0}{C_{\text{off}}}$$



OVERLAP MEASUREMENT



modes k_0, k_1

$$U_R = \exp\left[\frac{\pi}{4}(a_0^\dagger a_1 - a_1^\dagger a_0)\right]$$

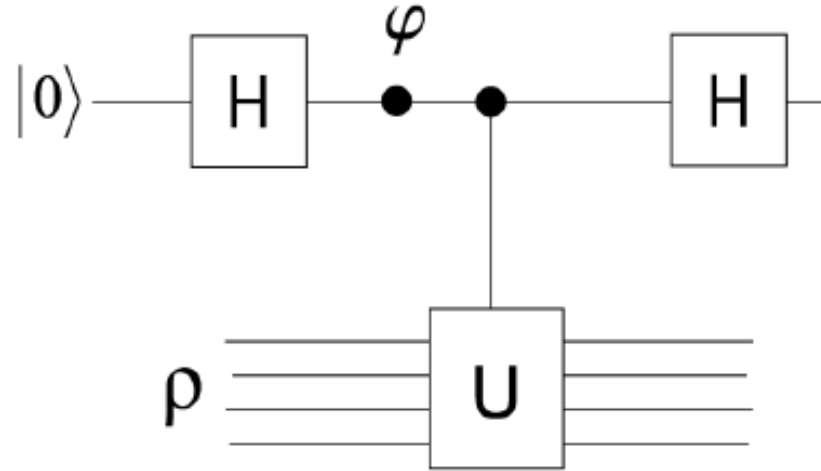
$$U_{CPS} = \exp(i\pi a_0^\dagger a_0 |\uparrow\rangle\langle\uparrow|)$$

Cavity QED, SQUIDS, trapped ions

$$V = \text{Tr}(\rho_A \rho_B) = \text{Tr}(F_{AB} \rho_A \otimes \rho_B)$$



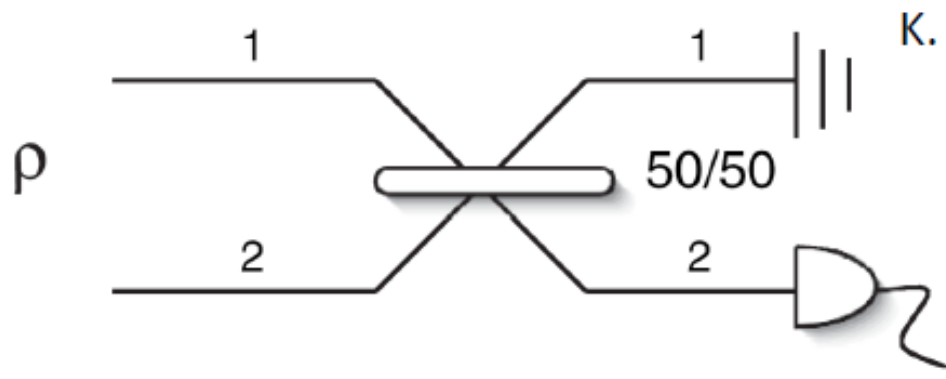
OVERLAP MEASUREMENT



Estimation of nonlinear functional of state

extendable to $\text{Tr } \rho^k = \sum_{i=1}^m \lambda_i^k$

A.K. Ekert et al., Phys. Rev. Lett. 88, 217901 (2002)
and many other publications.

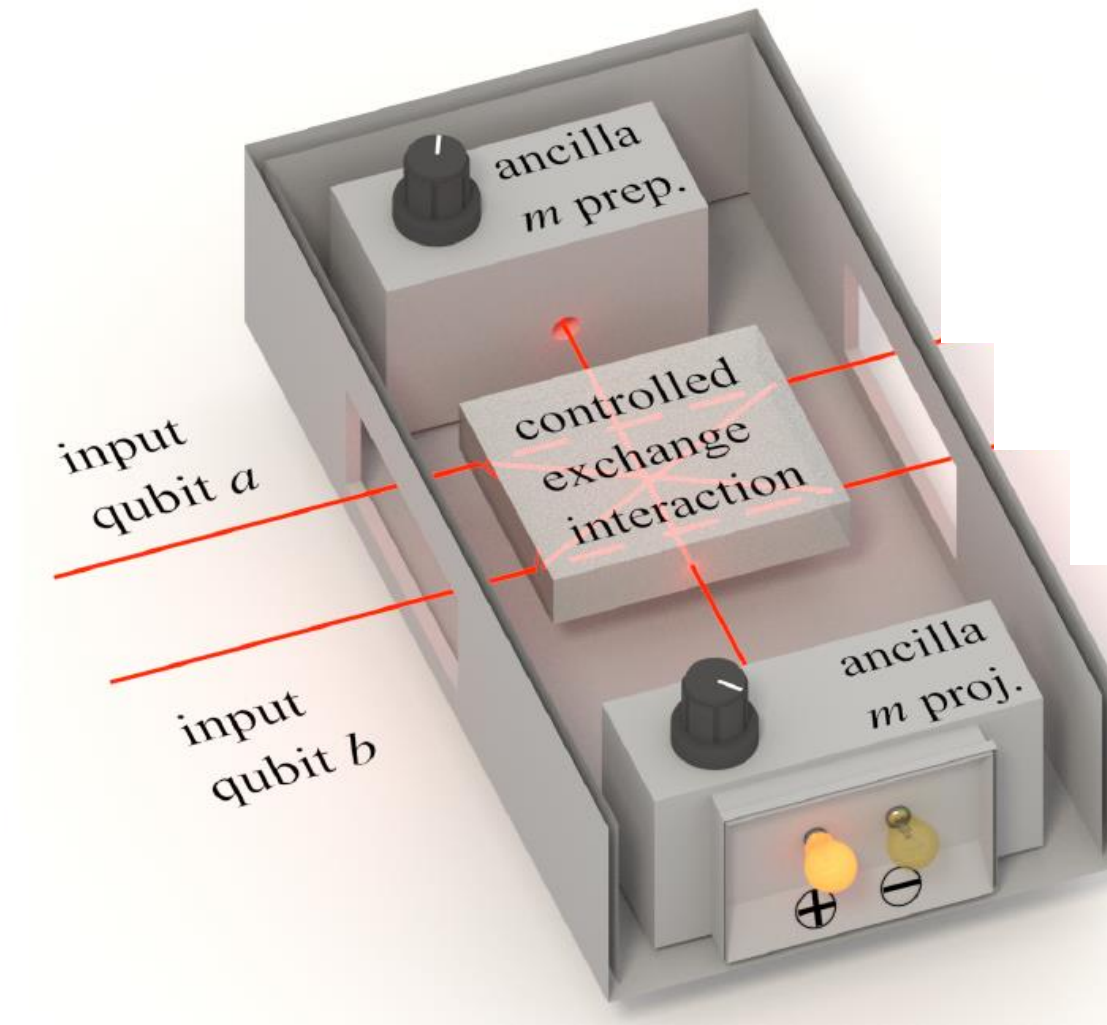


K. L. Pregnell, Phys. Rev. Lett. 96, 060501 (2006)

PARITY MEASUREMENT
(photon number measurement)

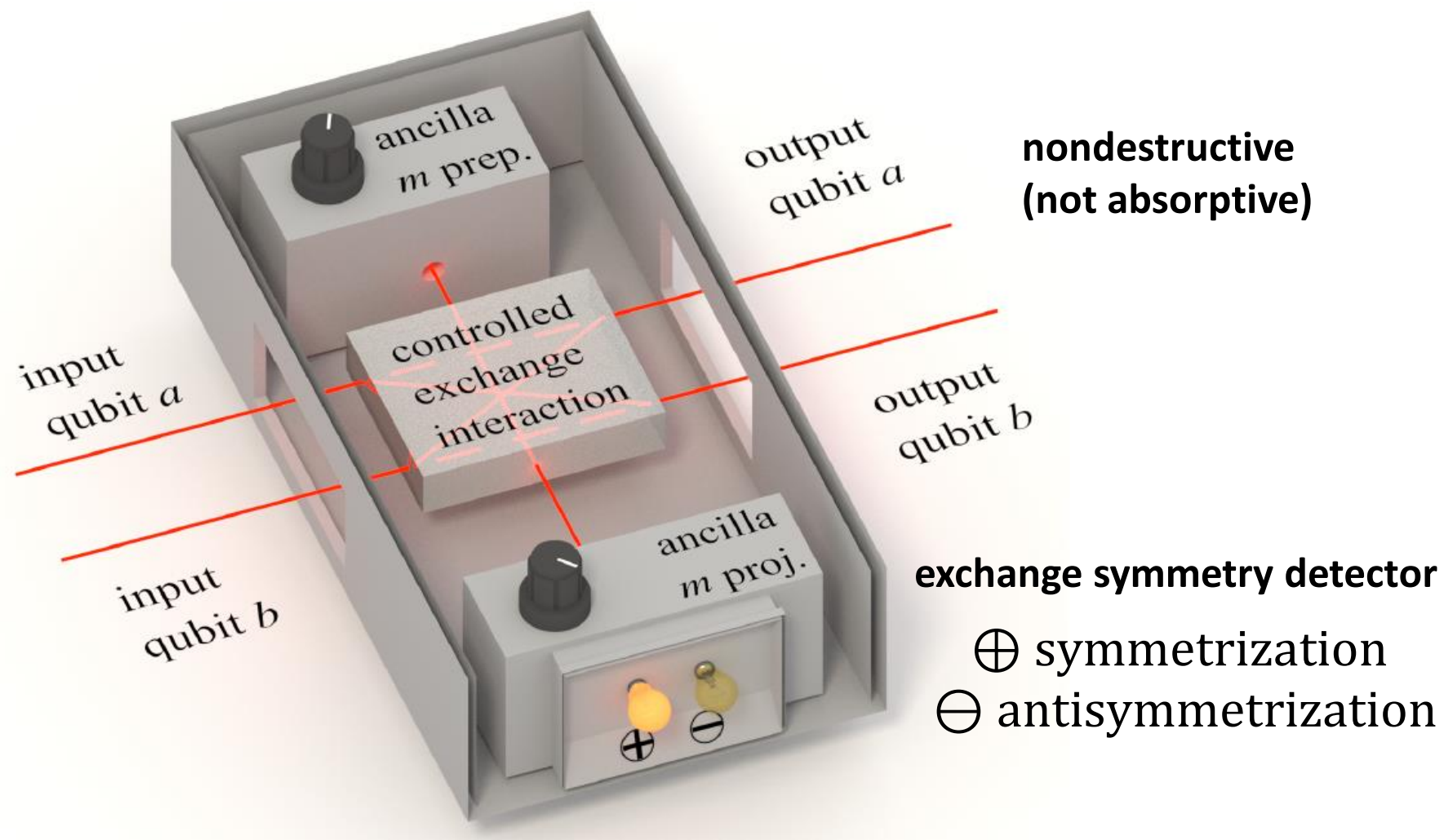


OVERLAP MEASUREMENT OF QUBITS





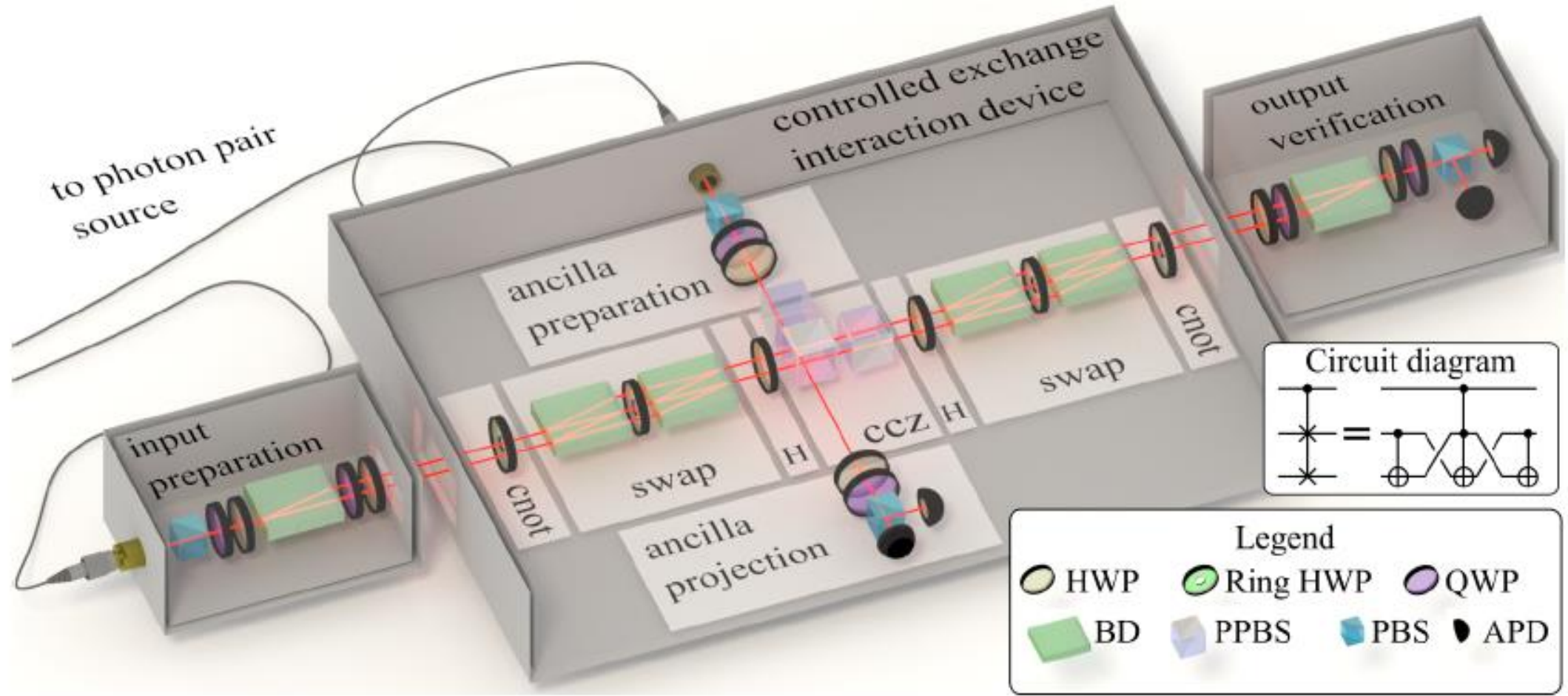
NONDESTRUCTIVE VERSION





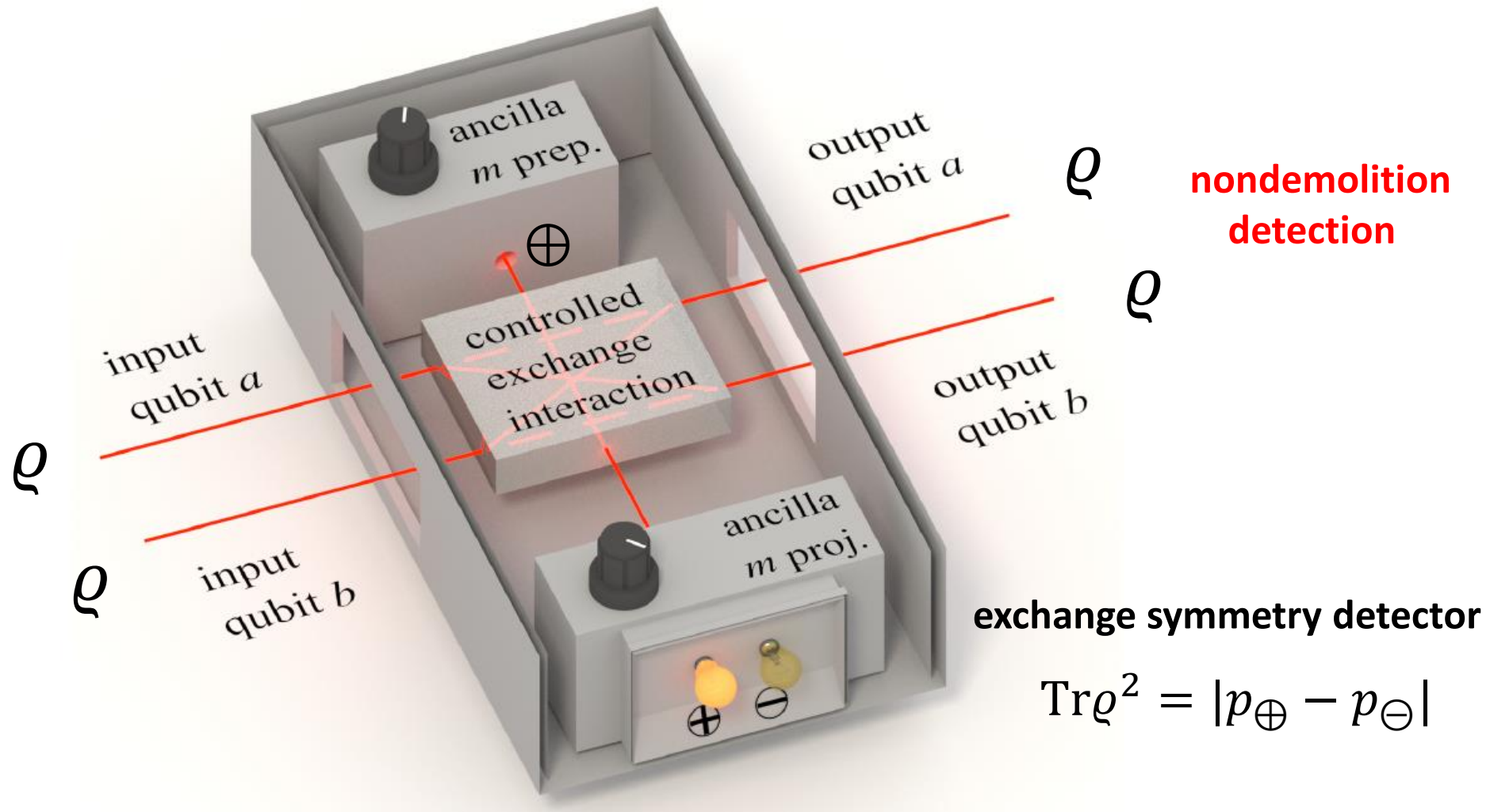
Palacký University
Olomouc

FOR TWO QUBITS



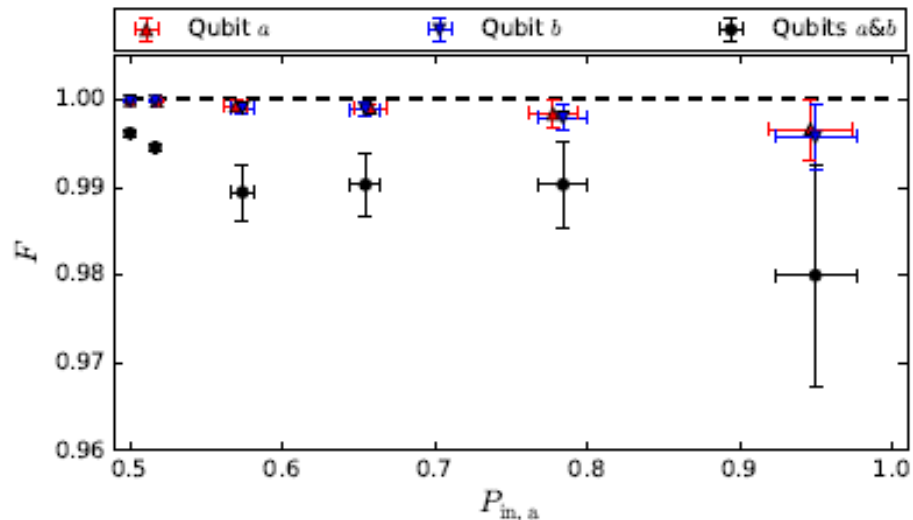
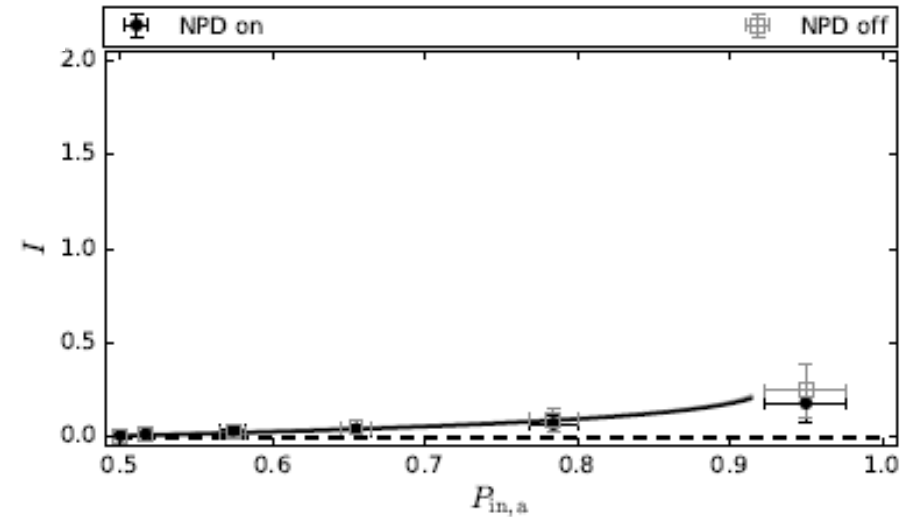
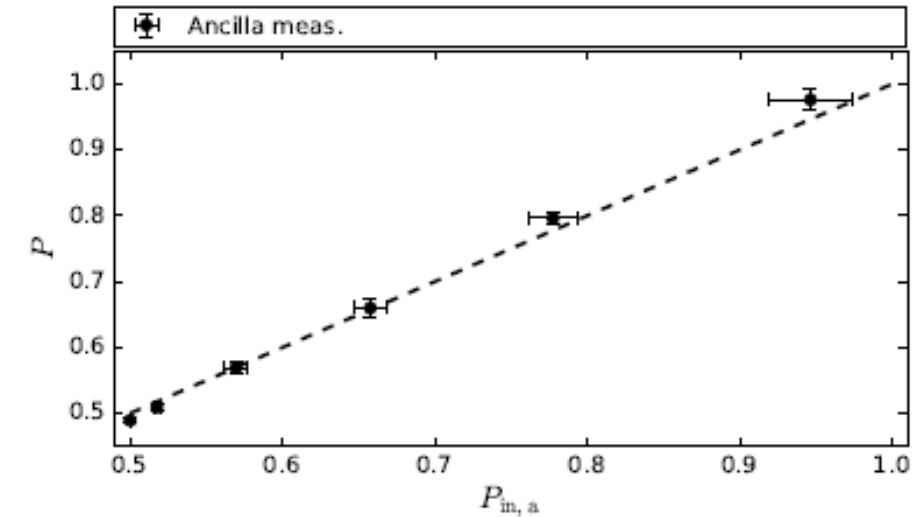


NONDEMOLITION PURITY DETECTION





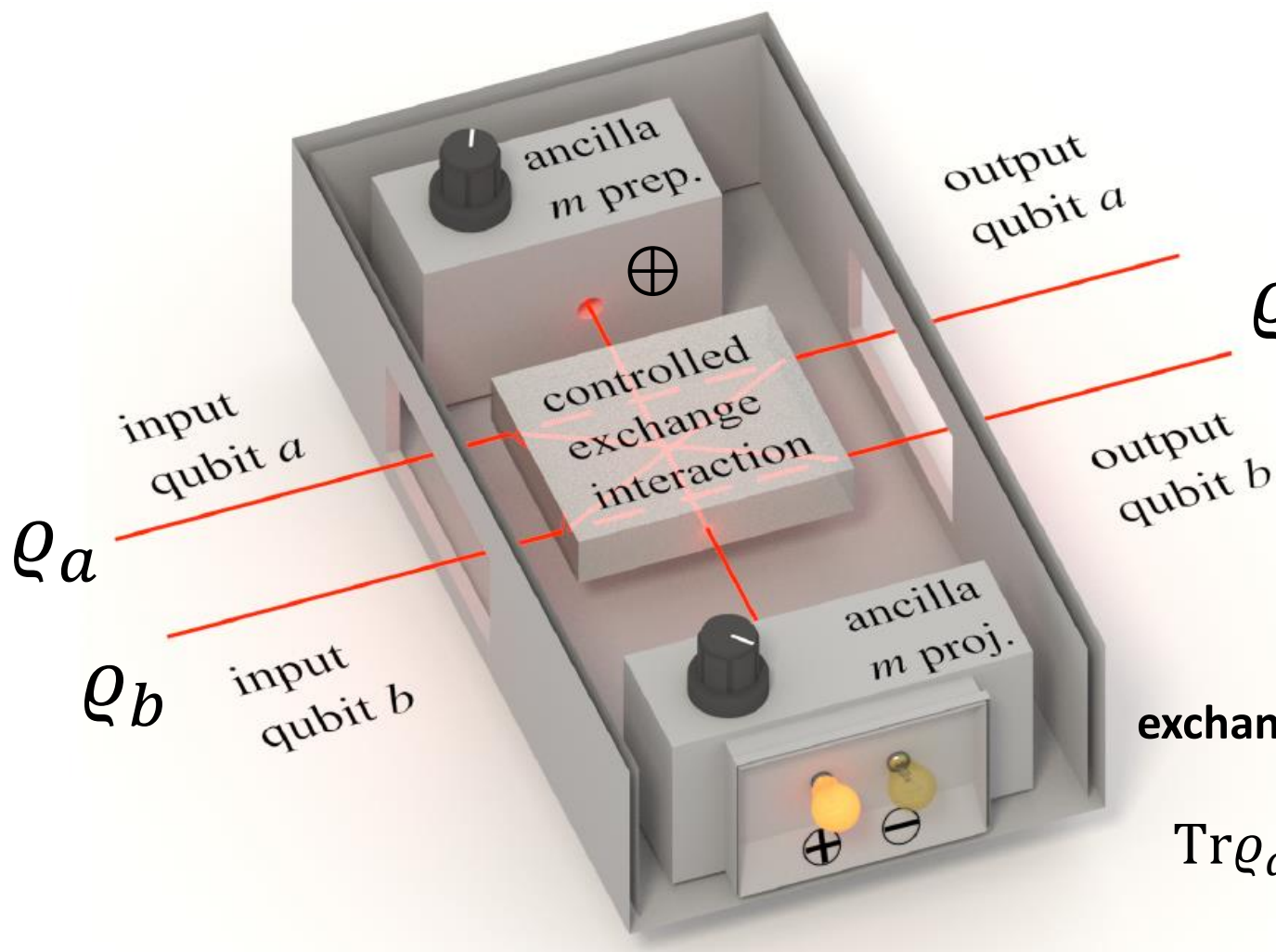
NONDEMOLITION PURITY DETECTION



- detector quality verified
- nondemolition feature verified
- no-correlation verified
- next step: sequential detection



OVERLAP DETECTOR



classically correlated states

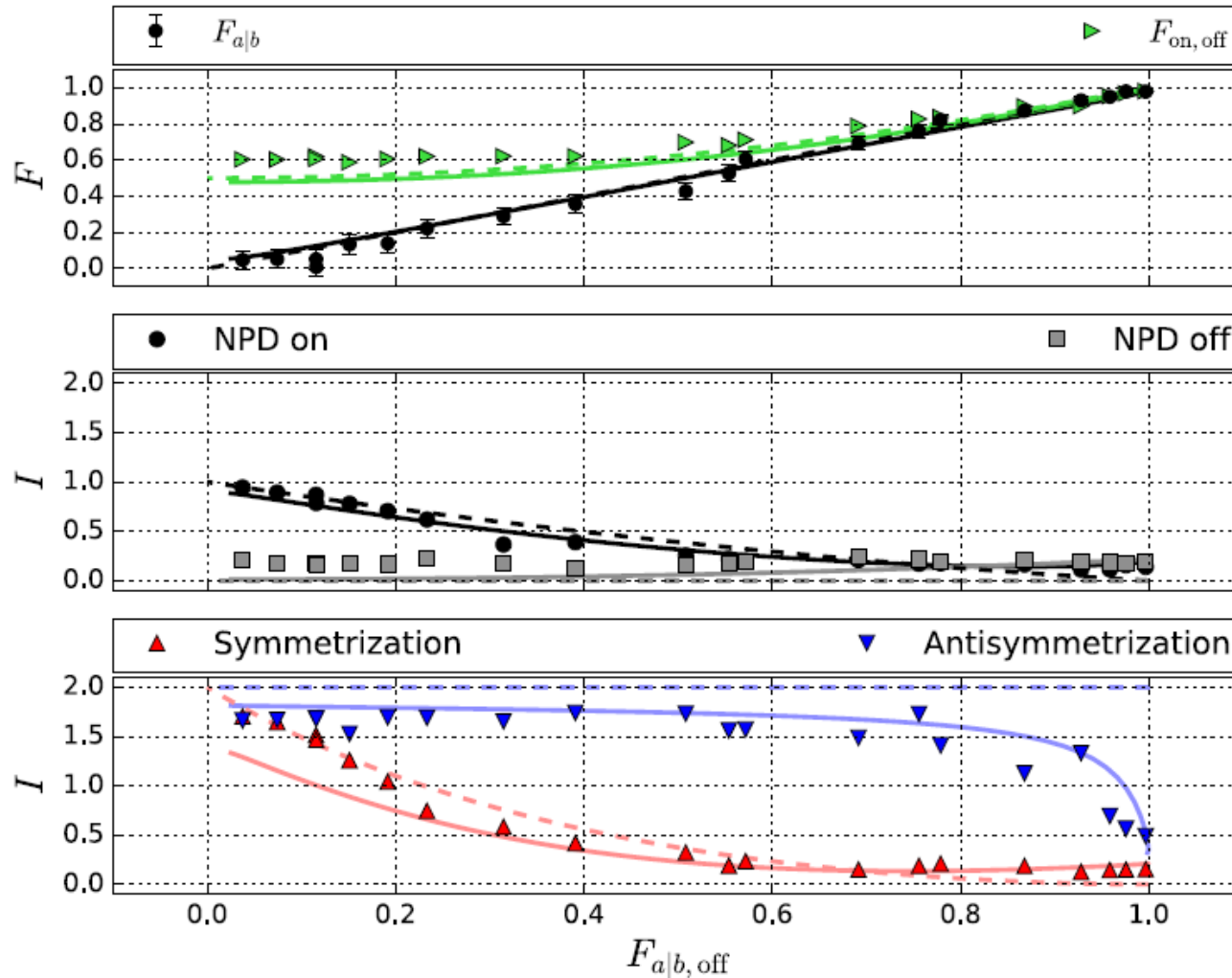
$$\rho'_{ab} = \frac{1}{2} (\rho_a \rho_b + \rho_b \rho_a)$$

exchange symmetry detector

$$\text{Tr} \rho_a \rho_b = |p^{\oplus} - p^{\ominus}|$$



NONDESTRUCTIVE OVERLAP DETECTION (PURE STATES)

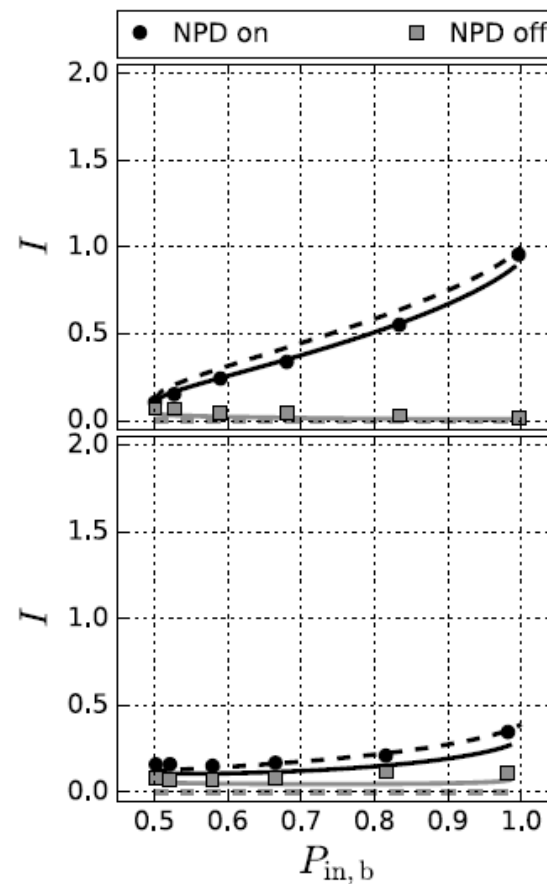
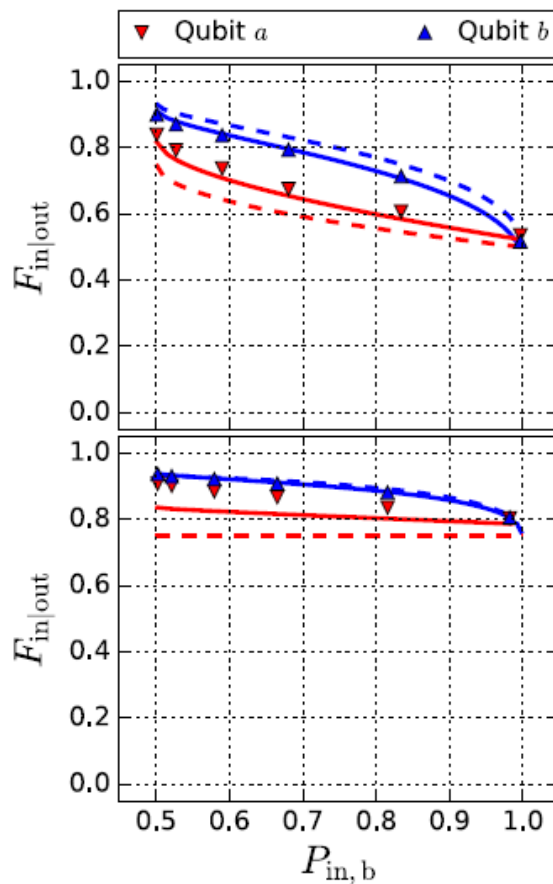
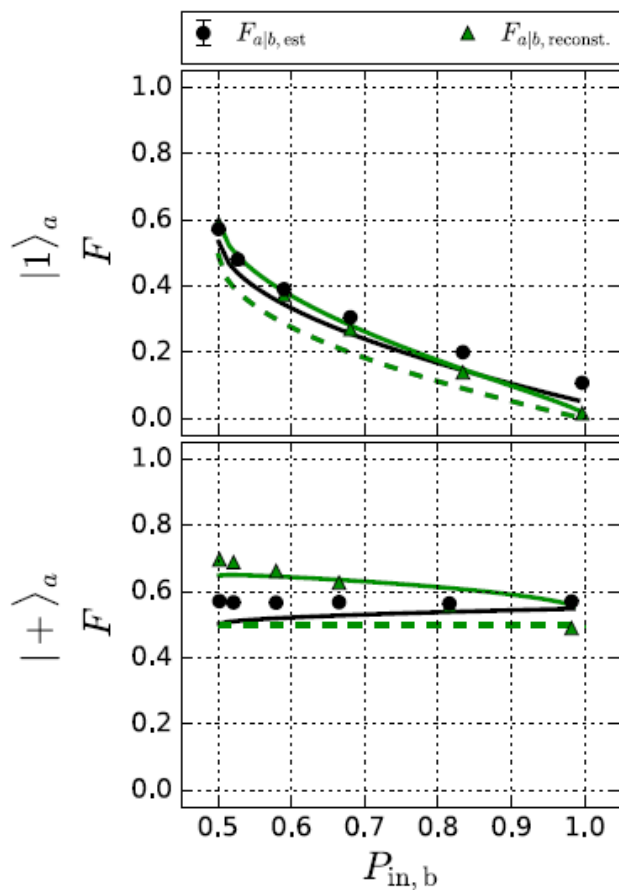


maximally entangled

no entanglement



NONDESTRUCTIVE OVERLAP DETECTION (MIXED STATES)



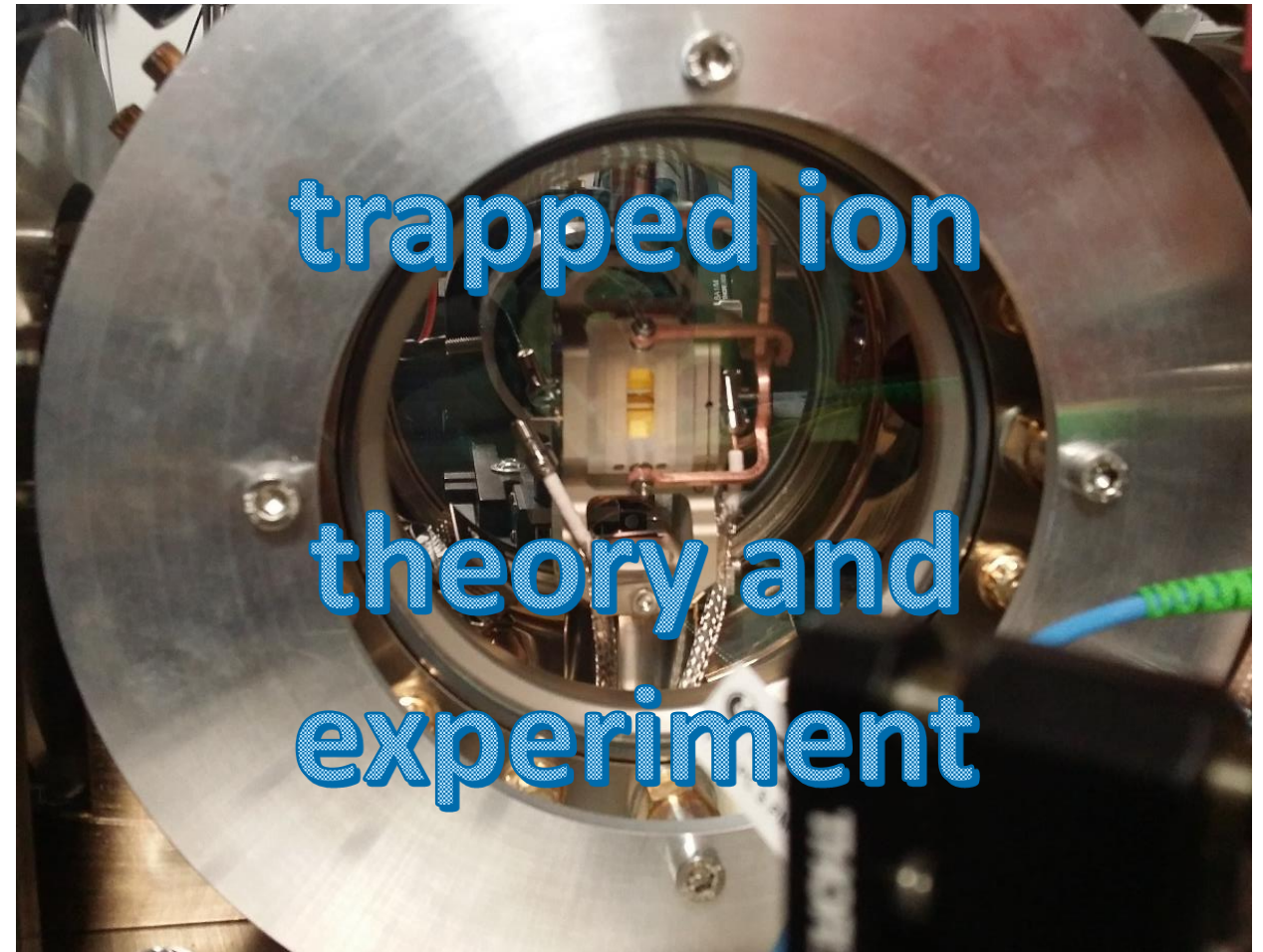
$$\rho_b = p|0\rangle\langle 0| + \hat{1}/2$$

NEXT?



NON-GAUSSIAN QUANTUM TASKS

- quantum non-Gaussianity for $n > 9$
- collective interference tests for complex quantum non-Gaussian states
- minimal decoherence for large squeezed cat states
- quantum cubic nonlinearity
- formation of interference fringes with negative Wigner function from Kerr effect
- non-Gaussian entanglement from cross-Kerr effect
- noise-induced nonclassical effects
- flip gate and overlap measurement for oscillators





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