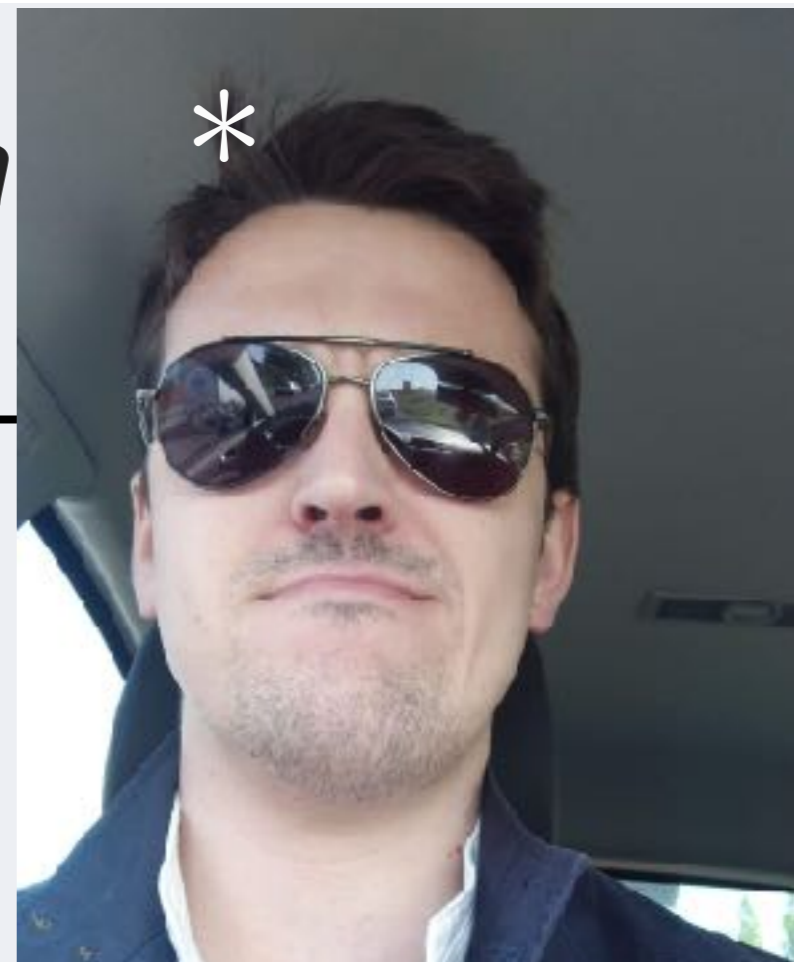




# *A self-contained quantum harmonic engine*

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Project in a nutshell:

**Using three quantum harmonic oscillators, we aim to realise an Otto cycle**

Project in a slightly bigger nutshell:

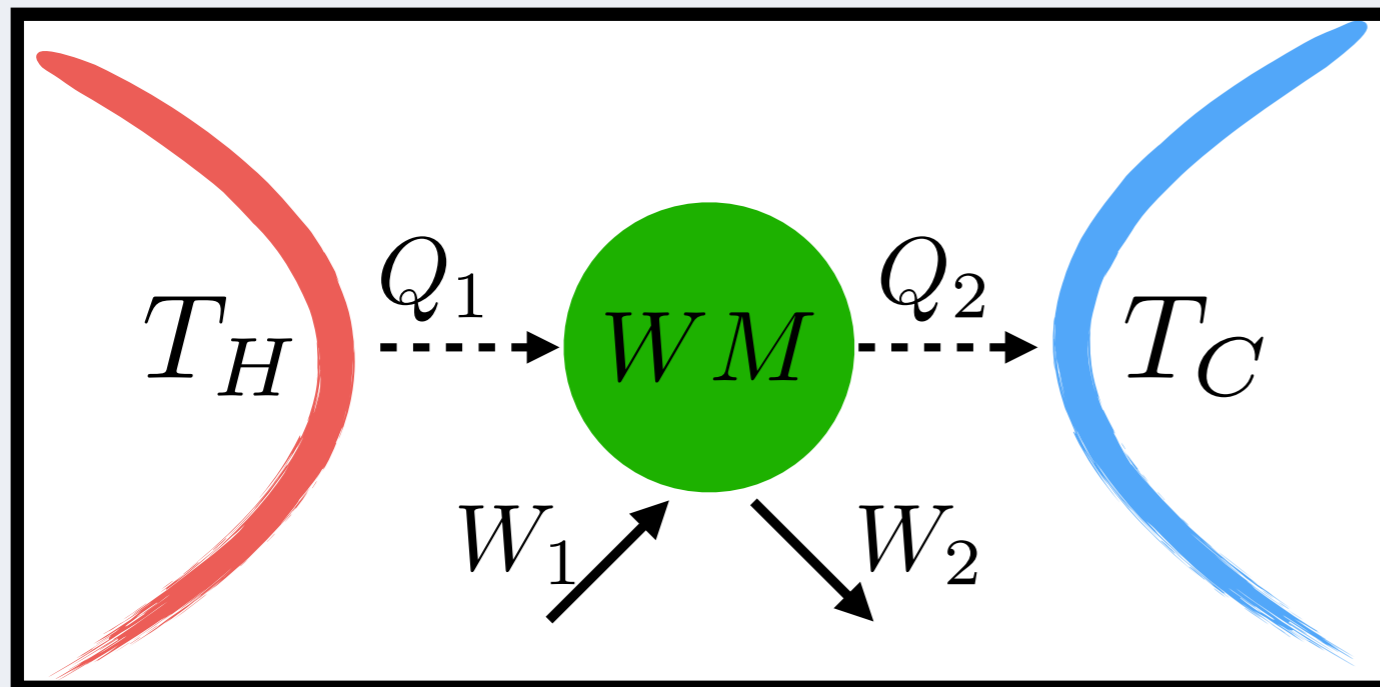
**Can we replicate a thermodynamic engine (i.e. extract work) from a quantum system consisting of only three QHO?**

*Okay, fine, but why?*

Quantum thermodynamics is a study of how the laws of thermodynamics apply at the microscopic quantum level

Up until recently quantum engines were studied **exclusively** with infinitely sized reservoirs

Dephasing and dissipation destroyed quantum coherences!



The state of the thermal reservoirs is assumed constant for all  $t$  — however the  $WM$  state necessarily changes.

*Okay, fine, but why? cont.*

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Technological implications—work extraction in a localised region of space.

Finitely sized system evolves unitarily—no Markovian approximation!

Built-in protection for non-classical features: any entanglement or discord will remain

**Can we find a connection between engine performance and non-classical features?**

# *Quantum heat engine literature (is extensive)*

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- Kosloff & Rezek, NJP **8**, 86 (2006) - *harmonic oscillators*.
- Friedenberger & Lutz, arXiv:1508.04128 (2015) - *trapped ions*
- Nori et al, PRE **76**, 031105 (2007) - *qubits*.
- Skrzypczyk et al, PRL **105**, 130401 (2010) - *qubits*.
- Singer et al, Science **352**, 6283 - *trapped ions*
- Matsukevich et al, arXiv:1702.08672 - *trapped ions*

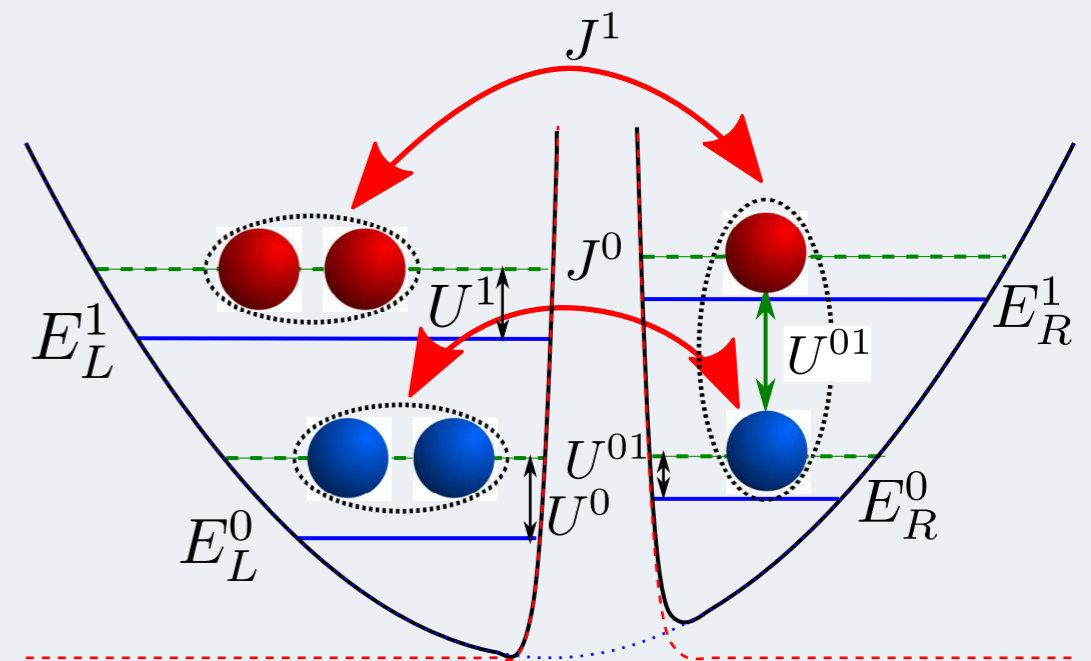
**All of these papers use infinitely sized baths**

# Papers exploring engines with finite baths

Fialko & Hallwood: PRL **108**, 085303 2012

*“First demonstration of a heat engine with finite heat baths”*

...however there is no discussion on thermodynamic quantities!



Uzdin et al.: arXiv:1610.02671

Small number of spins acting as a bath for an engine spin —  
artificially induced dephasing

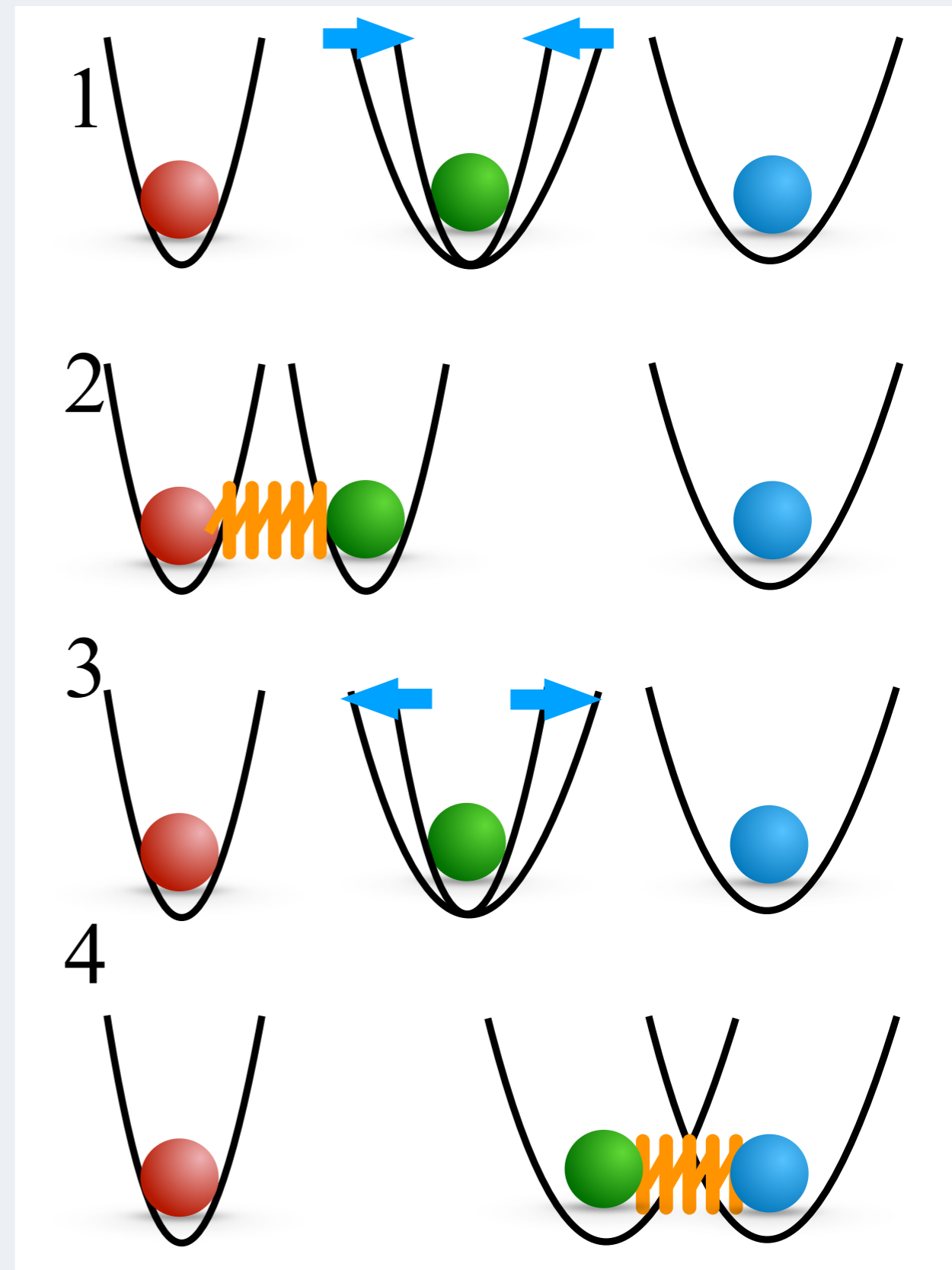
Aspects of finite bath engines not discussed; solely  
aiming to simulate Markovian dynamics

# Our setup

Three oscillators: hot bath, working medium, cold bath

## Otto cycle:

1. *Compression*
2. *Heating*
3. *Expansion*
4. *Cooling*



## Necessary but boring details

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$$H_i = m\omega_i^2 x_i^2 + p_i^2/m$$

$$H_{ij} = \alpha_{ij} \left( a_i a_j^\dagger + a_i^\dagger a_j \right) \quad (\text{Heating, cooling strokes})$$

If two oscillators are resonant...

$$\omega_i = \omega_j = \omega \rightarrow [H_i + H_j, H_{ij}] = 0$$

*(compression/expansion strokes)*

$$\omega_2(\omega_i, \omega_f) = \sqrt{\omega_i^2 + (\omega_f^2 - \omega_i^2)t/\tau_R}$$

Parameter Count:

~~$\omega_1$~~   ~~$\omega_2$~~   $\omega_3$

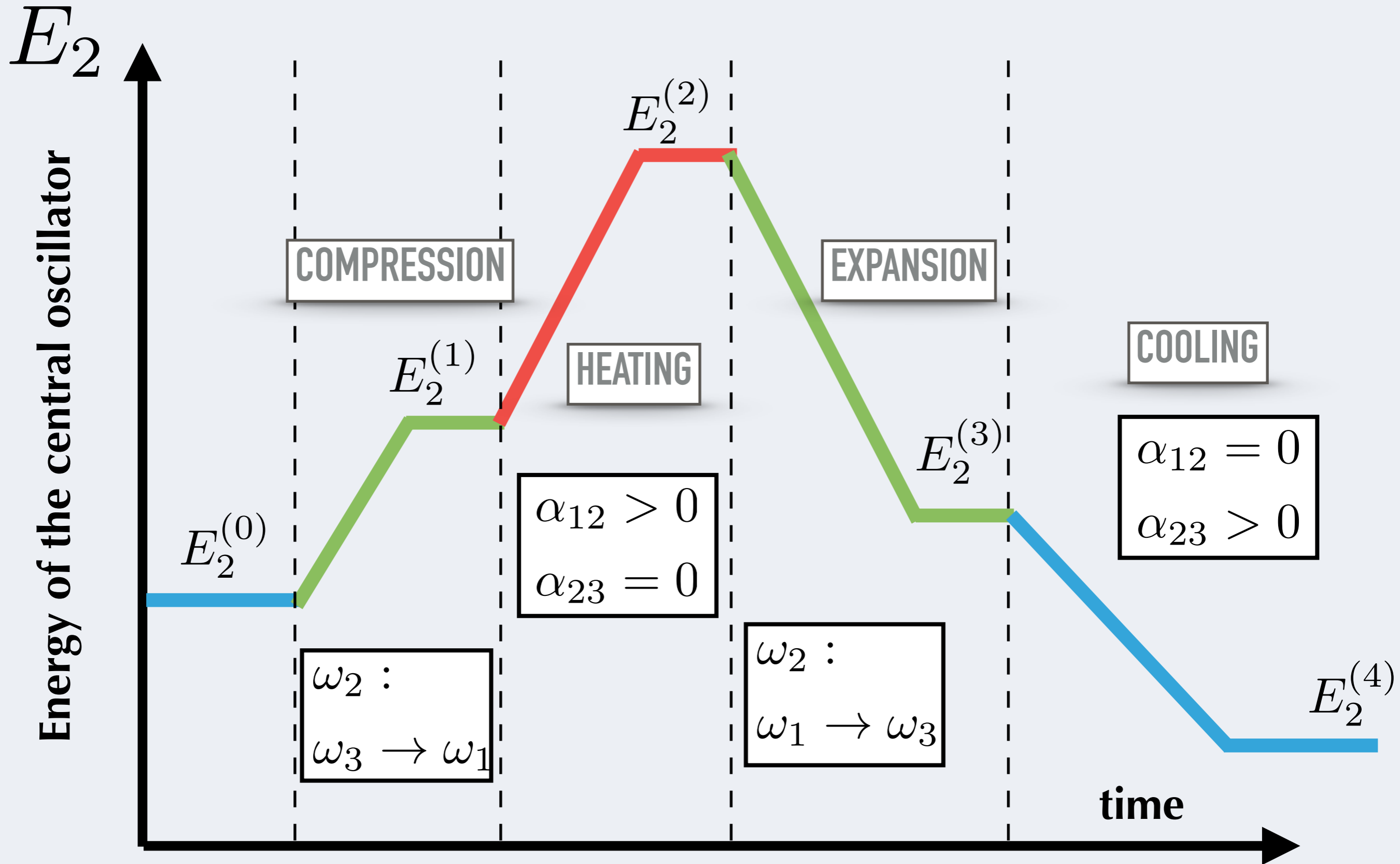
$\alpha_{12}$   $\alpha_{23}$

$\tau_H$   $\tau_C$

$\tau_R$



# The Otto cycle in oscillators



## More details

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Ordered operator basis  $R = \{x_1, x_2, x_3, p_1, p_2, p_3\}$

Covariance matrix  $\sigma_{ij} = \frac{1}{2} \langle R_i R_j + R_j R_i \rangle - \langle R_i \rangle \langle R_j \rangle$

**Lyapunov Equation**  $\dot{\sigma}(t) = A\sigma(t) + \sigma(t)A$

$$A = \Omega H$$

*Covariance matrix is the matrix of your unknowns  
- if you know it, you know everything!*



How are we measuring work?

$$\mathcal{W} = \text{Tr} [H \{ \sigma[n\tau] - \sigma[(n-1)\tau] \}]$$

Work defined on a per cycle basis as the energy of the system decreases every cycle

The baths are 'finite'...no thermalisation!  
As a result every cycle is different!

***Thermal states***

$$\bigotimes_{i=1}^3 \frac{\exp(-\beta_i H_i)}{Z_i}$$

***v***

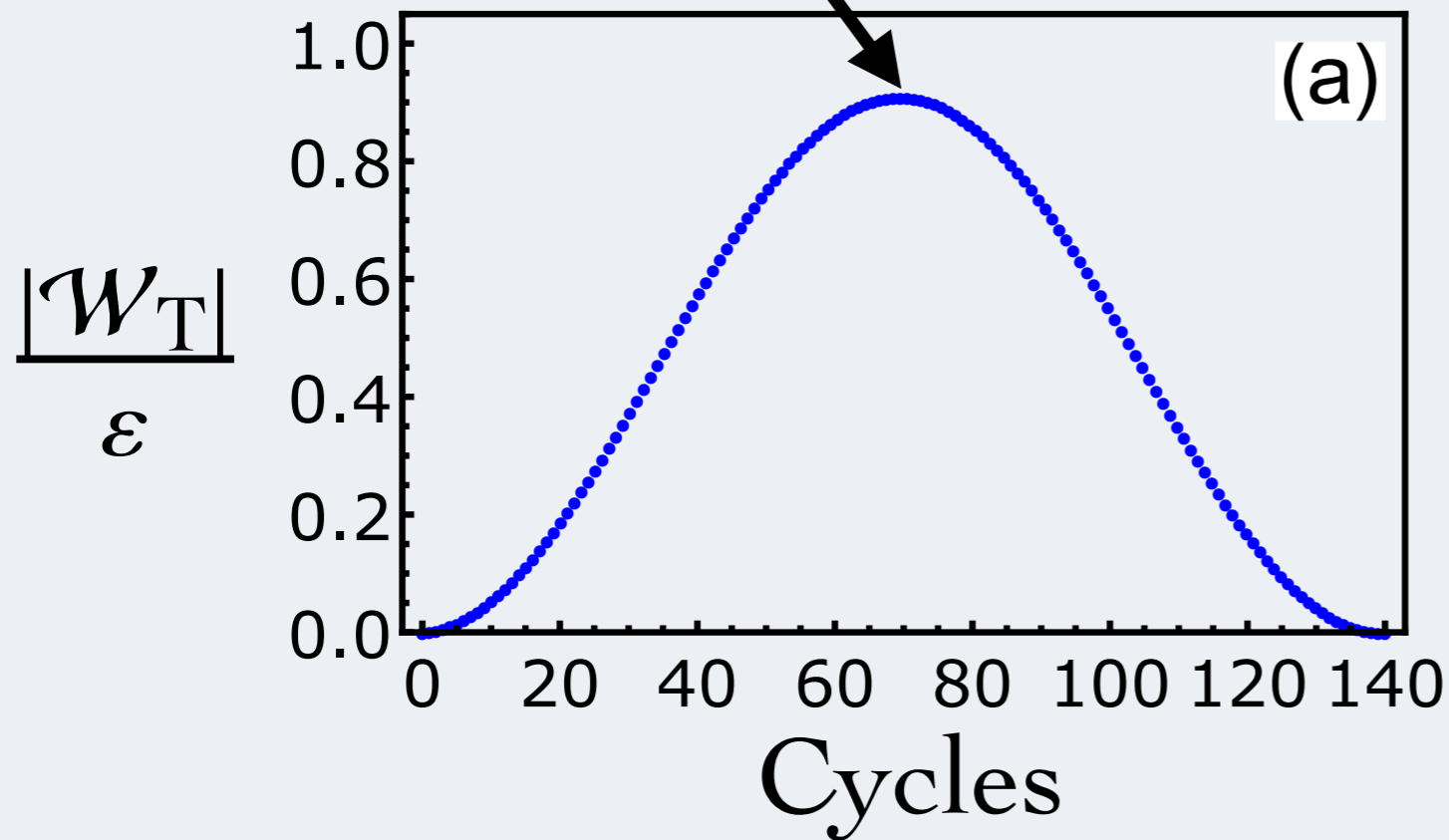
***Squeezed states***

$$\bigotimes_{i=1}^3 S(r_i) |0\rangle \langle 0| S^\dagger(r_i)$$

Engine stops here

$$\omega_3 = 0.1\omega_1$$

$$\beta_{2,3} \rightarrow \infty$$

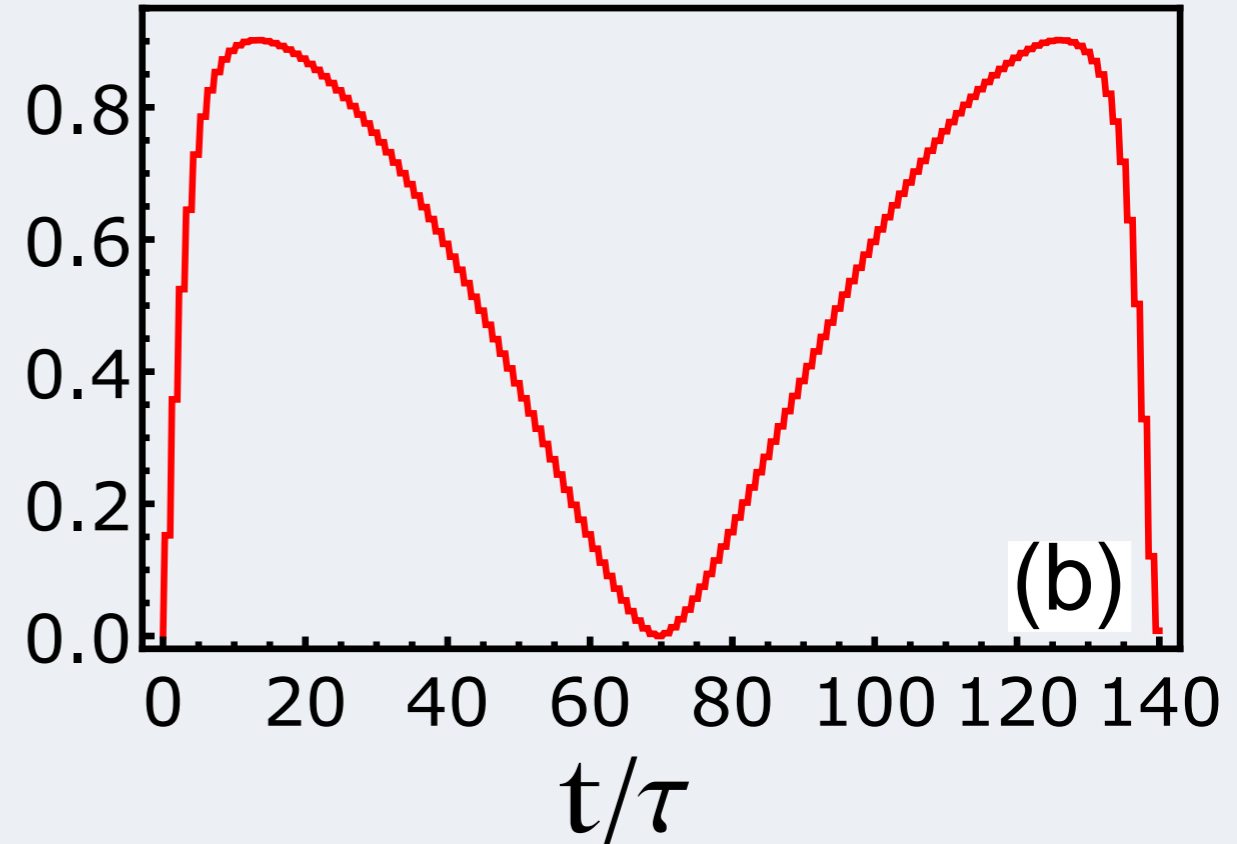


Ergotropy is the maximum extractable work

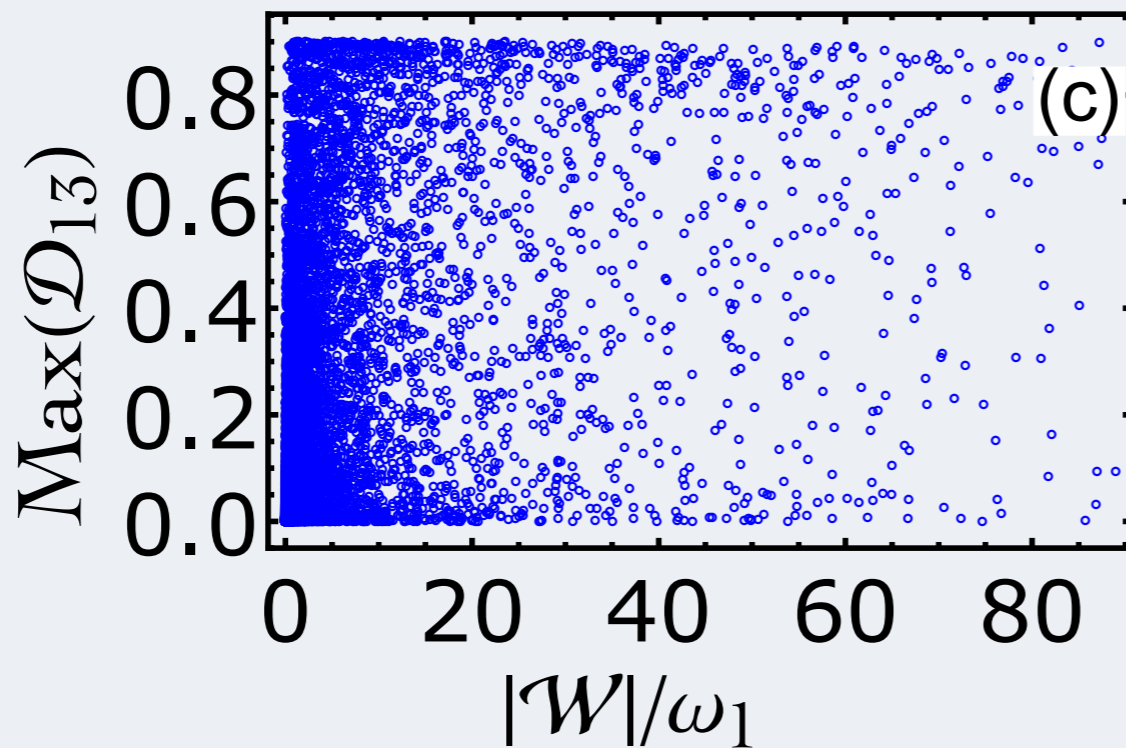
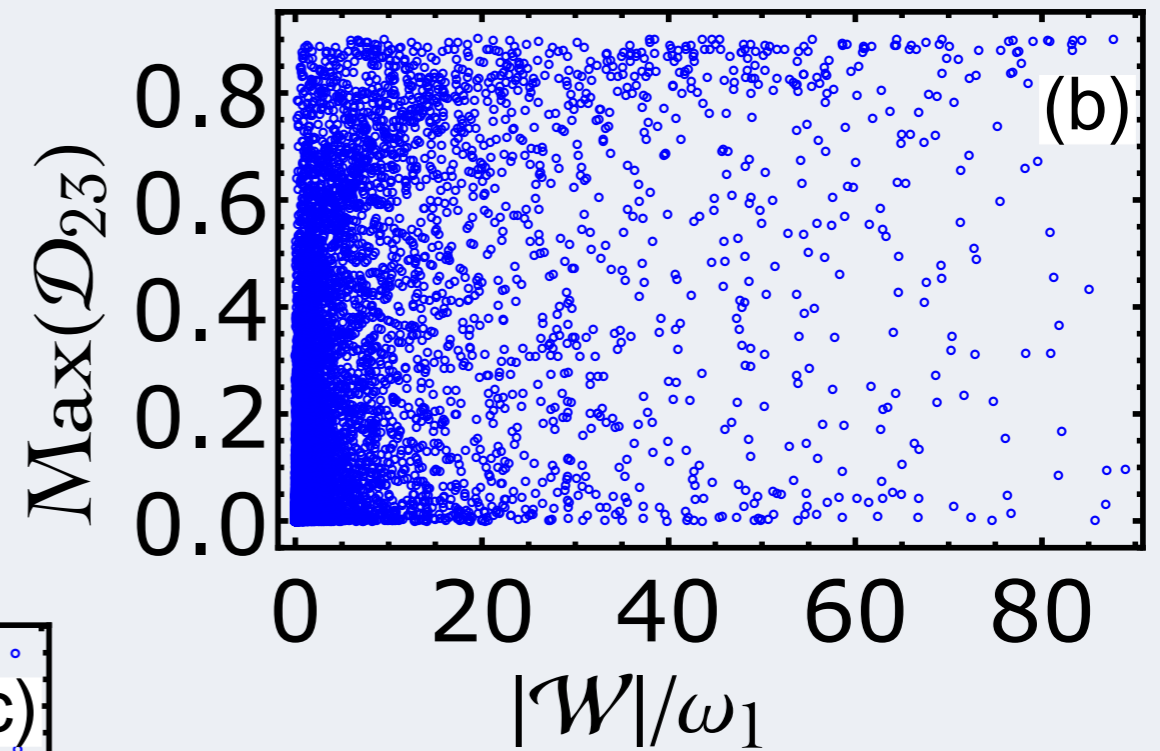
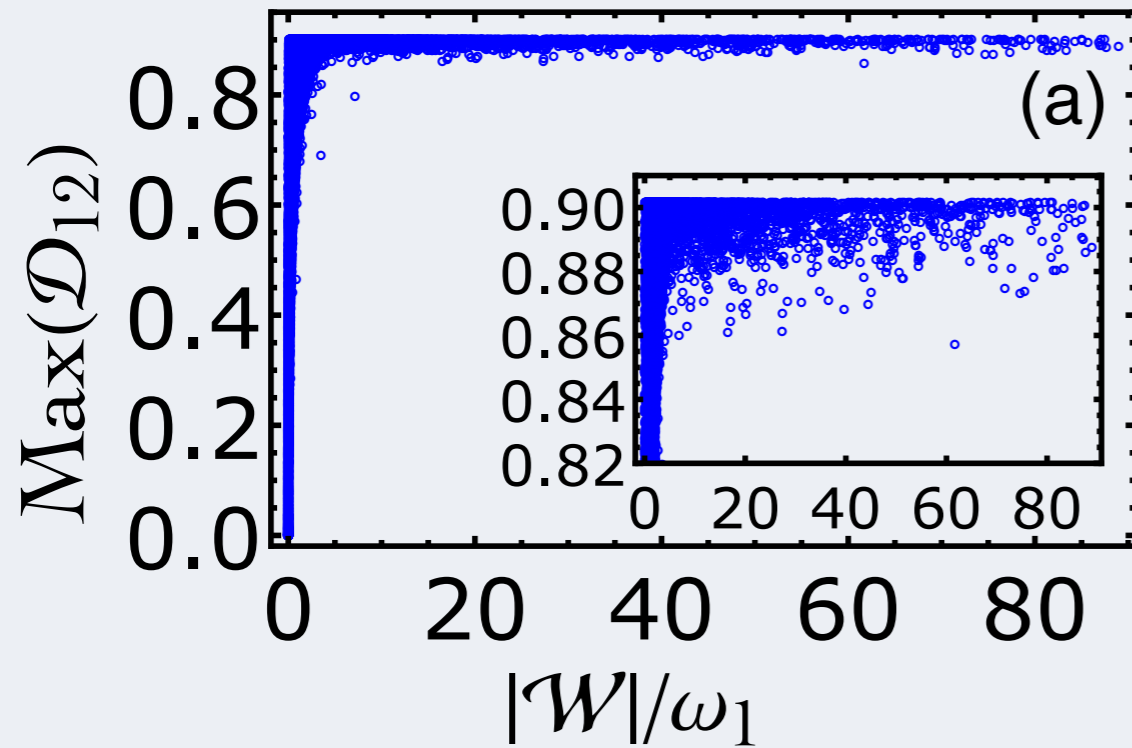
*No entanglement for thermal states!*

*Only non-zero discord is between oscillators 1 and 2*

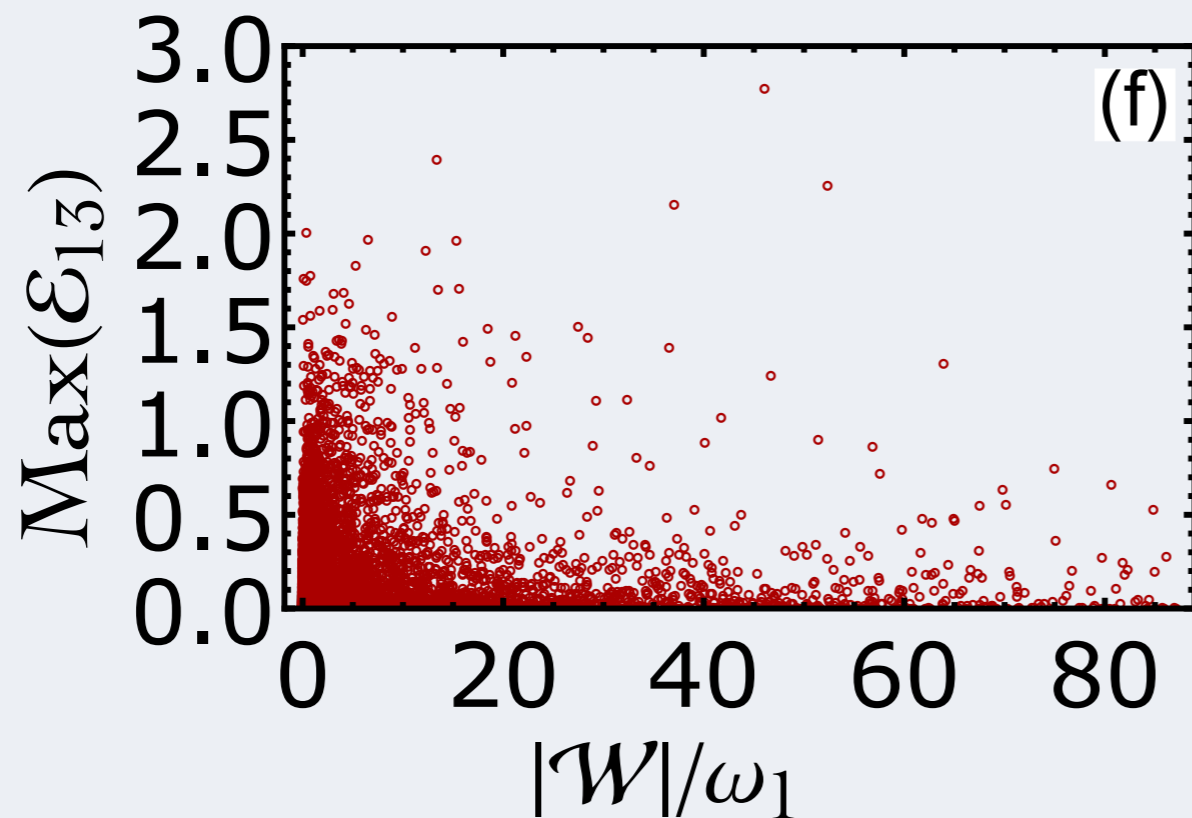
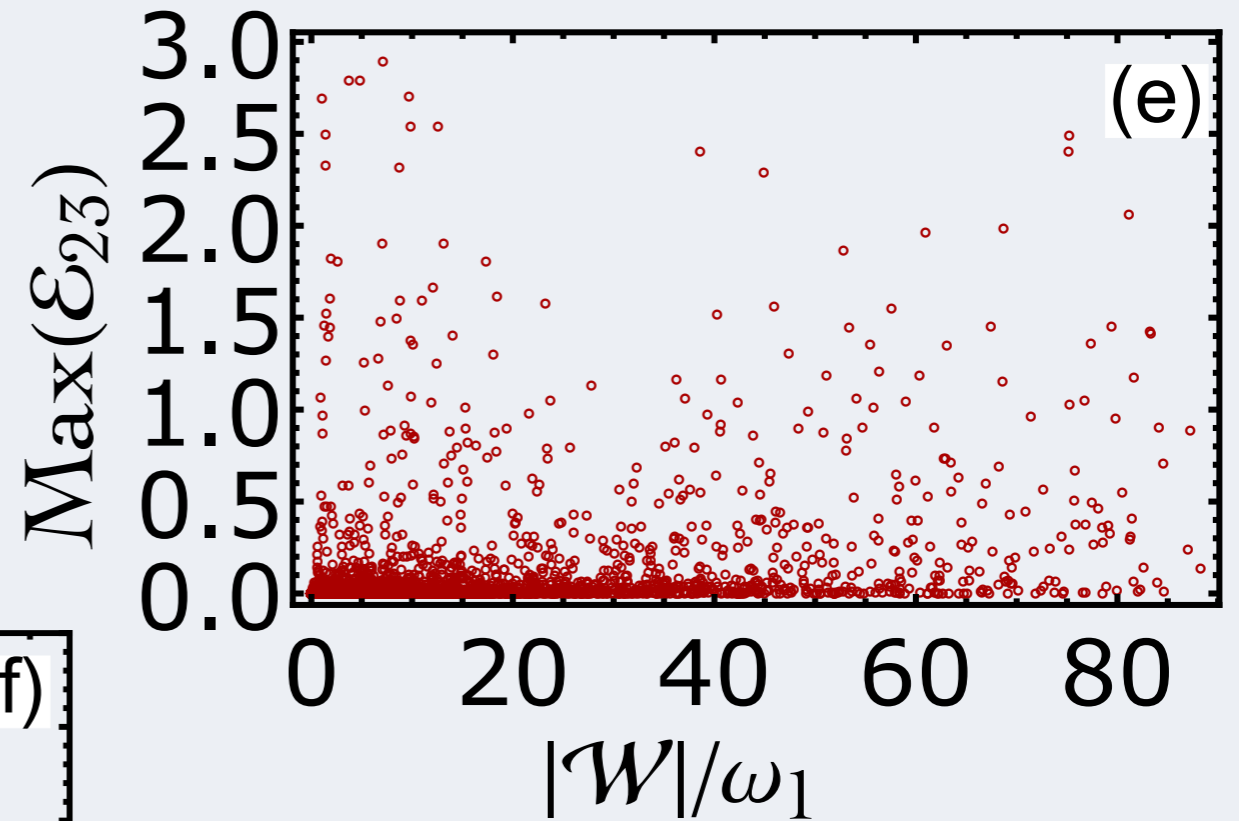
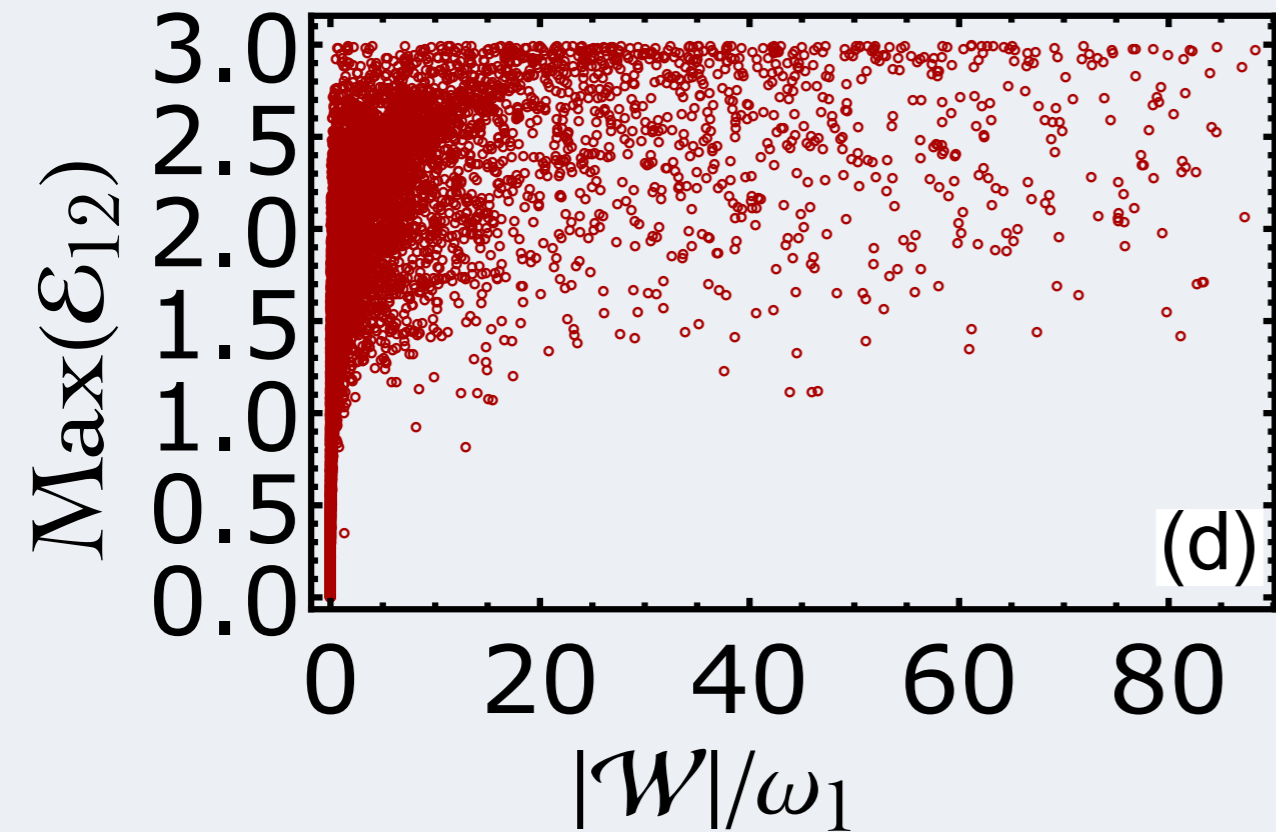
$$\mathcal{D}_{12}$$



# Thermal states – randomised variables



# Squeezed states — random variables



## *Talking points & Future work*

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Natural extension: bigger baths!

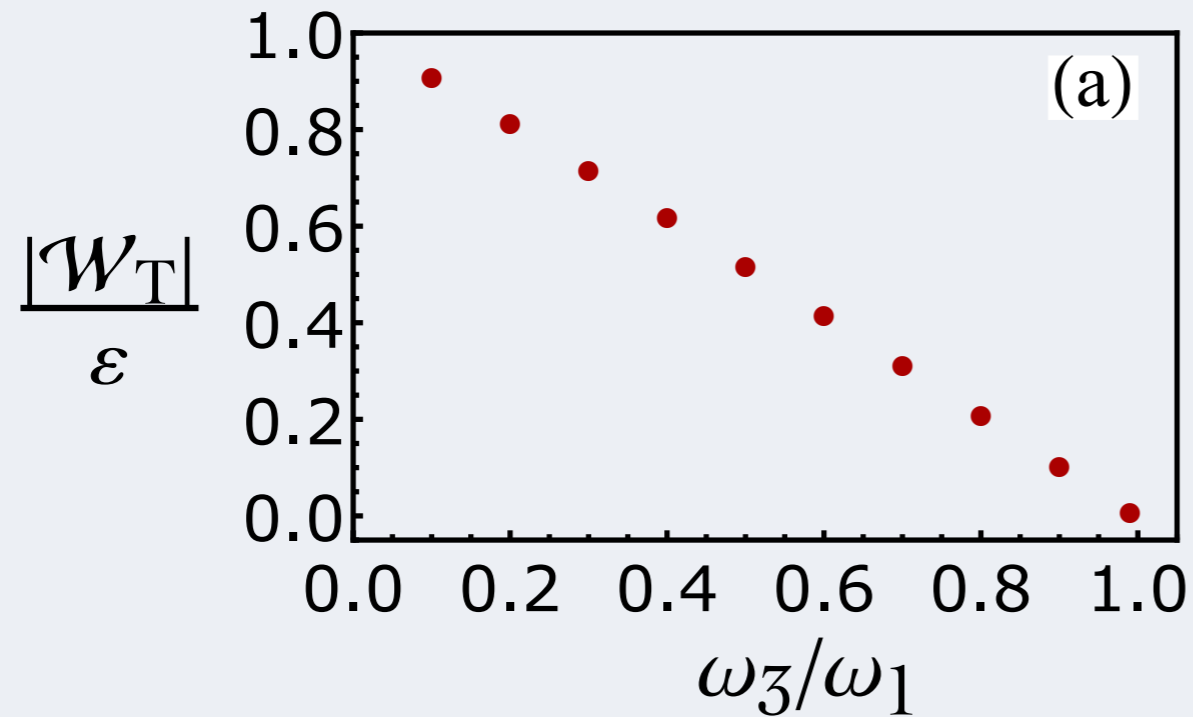
Pozas-Kerstjens et al. NJP **20** 043034

Large but finite baths allow for emulation of infinite baths

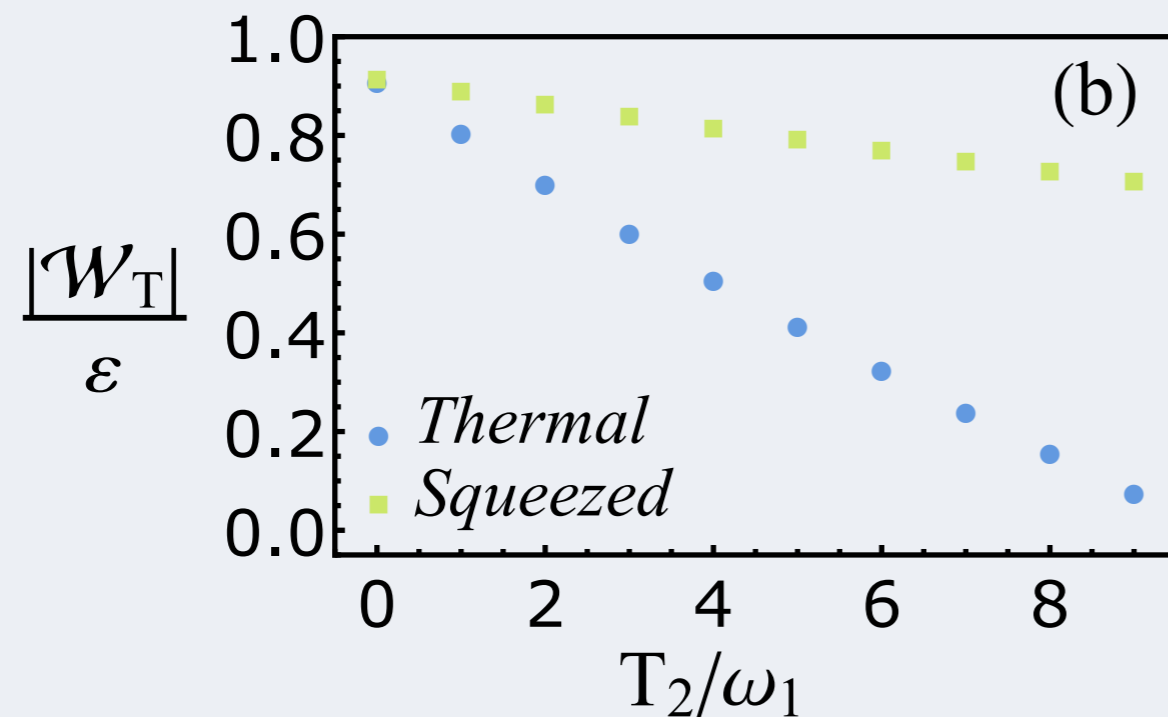
### ***To mention:***

Third oscillator kind of useless — not necessary for work extraction!!!

# Other results



Optimised, maximum work extraction





# *Two Italians and a Frenchman walk into a bar*

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