Quantum batteries

Abstract

The huge interest is arising to various devices, operating on quantum scale, e.g., quantum batteries [1] and quantum heat engines or refrigerators [2]. Our interest is focused on operating the quantum batteries, physical systems able to exchange the energy and entropy with their surroundings, to store it in some appropriate form and finally release the energy and entropy to the surroundings again. We study the process of charging a quantum battery in our case represented by certain number of copies of non interacting two-level systems. For the sake of feasibility of its experimental realization, we focus on definite model of the battery: qubit is represented by a single photon that could propagate trough two fibers, [3] . We compare the effect of charging for different input system states, namely for quantum superposition state and incoherent thermal state. We also explore the additivity of the battery, i.e. with increase in the number of resources (qubits) we should be able to get more output (energy gain).

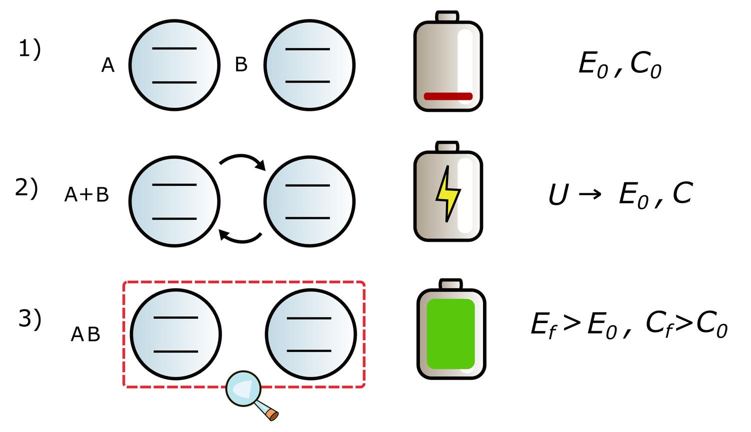

Figure 1: Protocol of charging a battery, represented by a pair of qubits A and B. In the first stage we have two independent copies of qubits with an initial energy. An interaction (coherence) between them is created (stage 2). The third and final step is a detection, which in case of success, increases the energy of the qubits, therefore, charging our battery. The coherence is also increased for some range of parameters.

Battery source

The source of our battery is represented by N two-level systems (TLS), namely, qubits with energy gap $E = Const.$
The preferred energy basis of the non interacting TLS is given as $\{|0\rangle_i, |1\rangle_i\}$, where $i = 1, \ldots, N$ is the number of TLS.
The Hamiltonians defining the basis state of each TLS read

$$\hat{H}_i = E|1\rangle_i\langle 1|_i, \quad i = 1, \ldots, N,$$

where $|1\rangle_i = |0\rangle_{i_0}|1\rangle_{i_E}$ denotes that photon is in the excited state of the i-th TLS.
Initial state of the battery

$$|\Psi\rangle = |\psi\rangle_1 \otimes |\psi\rangle_2 \ldots \otimes |\psi\rangle_N, \tag{1}$$

Charging procedure

Charging procedure, schematically shown in Figure 1, consists of

1. mixing lower energy states with BS
   - unitary operation
   - energy conserving
   - changes the coherence of system
2. projective measurement (detection of "no photon" in the lower states)
   - increases average energy of qubits
   - conditional, however with sufficient probability of success
   - increases the coherence of system

The final state

$$|\tau\rangle \propto {}_{2_0}\langle 0|_{3_0}\langle 0| \ldots {}_{N_0}\langle 0|\Psi'\rangle. \tag{2}$$

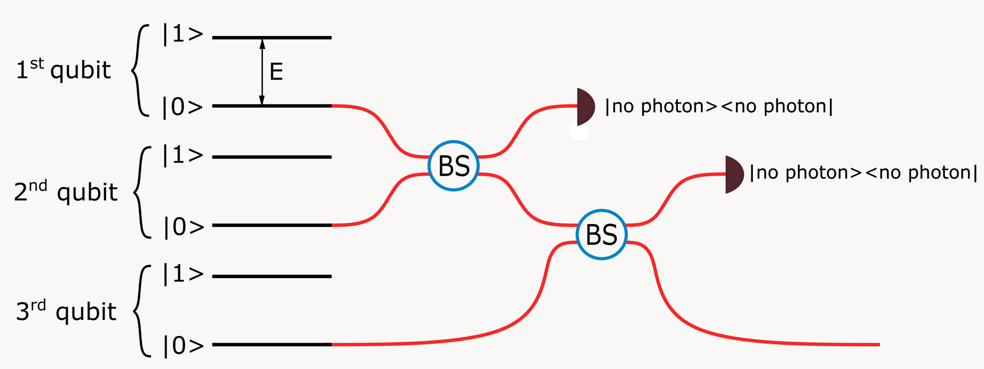

Figure 2: Schematic representation of the charging process for a battery consisting of 3 qubits, inspired by the experimental setup in [3]. The lower energy states are mixed with the help of beam-splitters. One of the outputs of BS is used for the detection, the other is sent as the input for the further BS. The projective measurement(detection) is successful when no photon is detected.

Input states

To compare the effect, this procedure has been done for different TLS's input states:

- pure states

$$|\psi\rangle = \sqrt{1-p}|0\rangle + \sqrt{p}|1\rangle, p \in [0, 1] \tag{3}$$

- mixed thermal states

$$\hat{\rho}_{therm} = (1-p)|0\rangle\langle 0| + p|1\rangle\langle 1|, p \in [0, \frac{1}{2}].$$

Measure of coherence

Measure of coherence was first introduced in [4] as

$$C_{\text{rel.ent.}} = S(\hat{\rho}_{\text{diag}}) - S(\hat{\rho})$$

where S is the von Neumann entropy and $\hat{\rho}_{\text{diag}}$ denotes the state obtained from $\hat{\rho}$ by deleting all off-diagonal elements.

Results

Remarks

- Incoherent quantum states exhibit the same results in the gain of energy as pure states, however, they were incoherent initially and didn't gain any coherence during the process of charging.
- The main advantage of the charging protocol is its additive feature, the energy gain increases with the increase in the number of qubits N, constituting the battery; the coherence increases with N less pronounced.

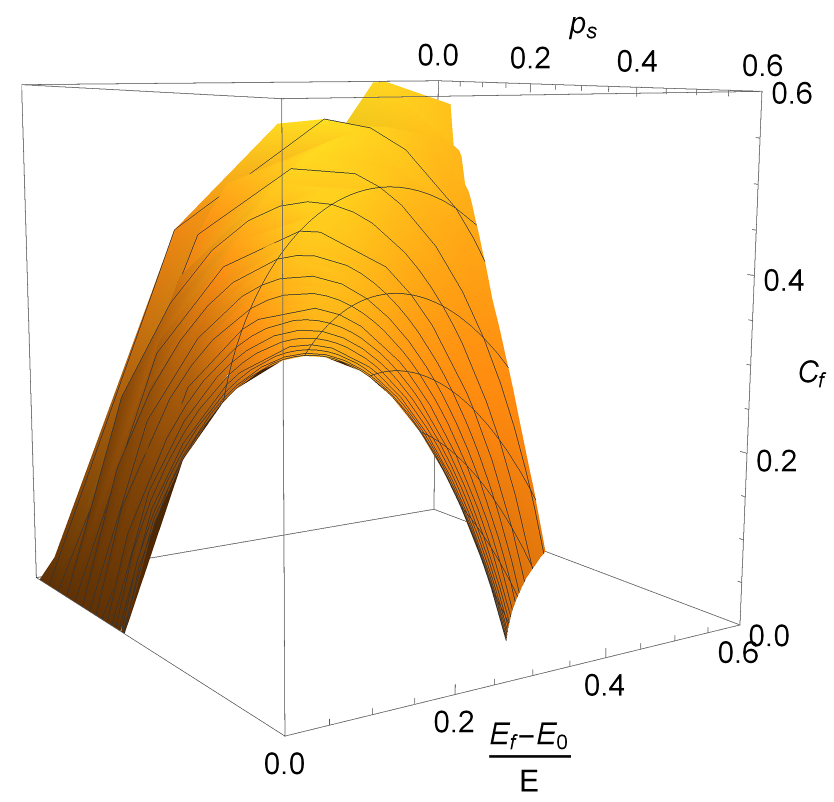

Figure 3: Parametric 3D plot of the relative entropy of coherence difference $(C_f - C_0)$ on z-axis, the normalized average energy difference $(E_f - E_0)/E$ on y-axis and probability of success p_s on x-axis for the battery of $N = 2$ qubits. The plot is parametrized by the probability of excitation of a qubit p in a pure state. It reflects the main results: increase in the energy and coherence of the battery after the charging process with sufficient and non-negligible probability of success.

Charging with a pair of TLS

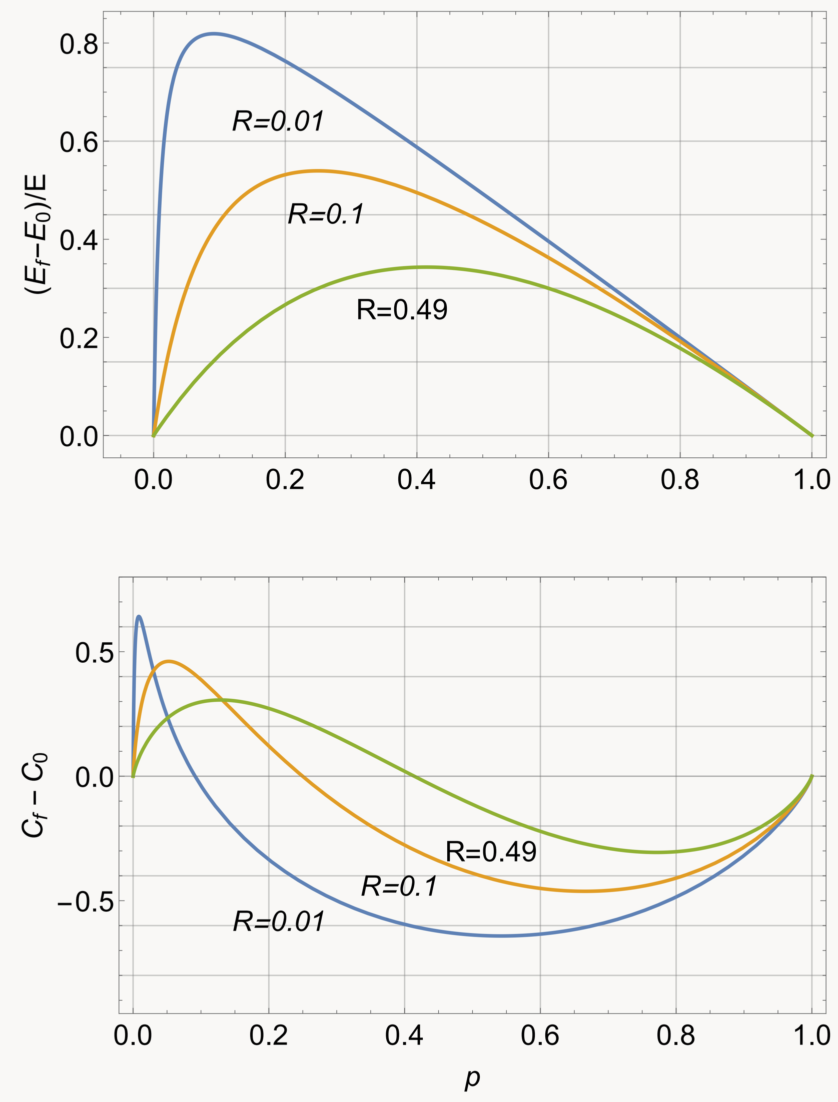

Figure 4: The plot of the normalized average energy difference $(E_f - E_0)/E$ and the relative entropy of coherence difference $(C_f - C_0)$ for the final state $|\tau\rangle$, Eq. (2), and the initial state $|\Psi\rangle$, Eq. (1), respectively, of the battery of $N = 2$ qubits. The difference is plotted versus the higher level excitation probability p, Eq. (3). The curves are parametrized by the parameter R, the beam splitter reflectance. The conditional increase of the final energy over the initial one, $(E_f - E_0)/E > 0$, is achieved for all parameters with maximum in the region of small p. With higher reflectance R the gained energy is lower. At the same time for any reflectance the final entropy of coherence can be conditionally increased above the initial one $C_f - C_0 > 0$in a region of the small excitation probabilities p.

Charging with $N = 3$ of TLS

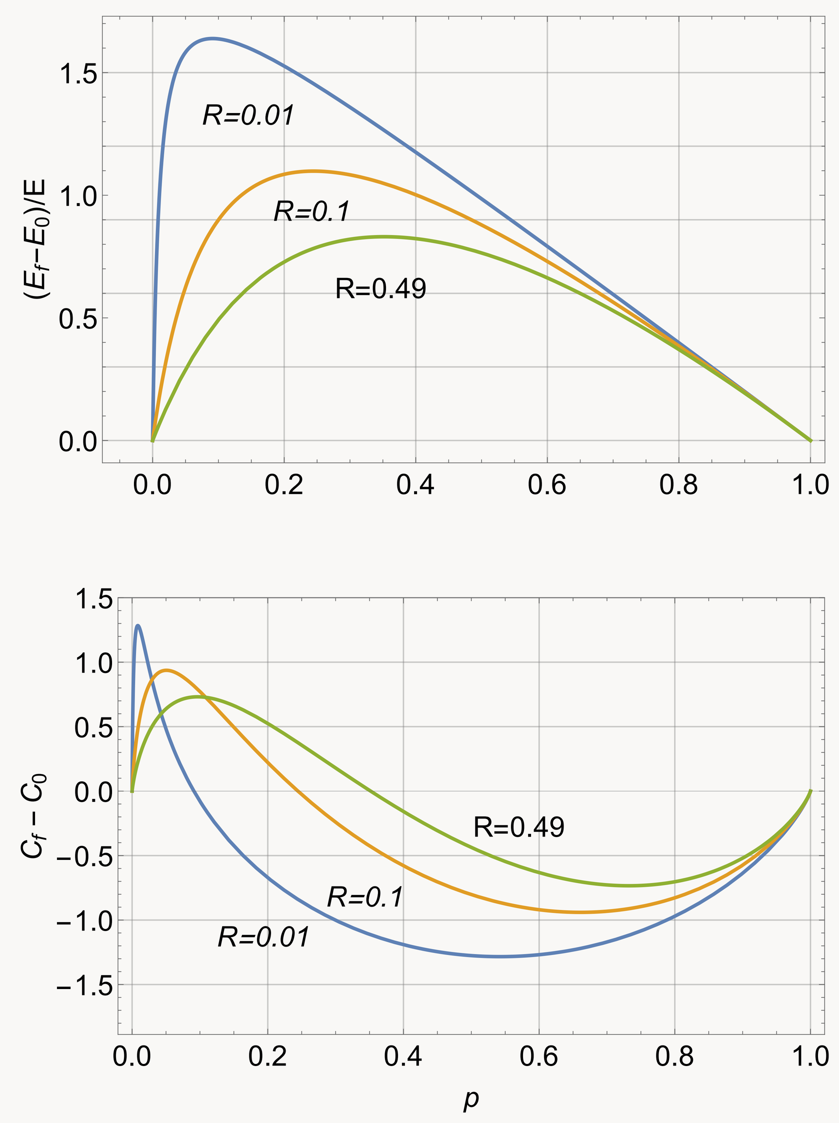

Figure 5: The plot of the normalized average energy difference $(E_f - E_0)/E$ and the relative entropy of coherence difference $(C_f - C_0)$ for the final state $|\tau\rangle$, Eq. (2), and the initial state $|\Psi\rangle$, Eq. (1), of the battery of $N = 3$ qubits. The difference is plotted versus the higher level excitation probability p, Eq. (3). The curves are parametrized by the parameter R, the beam splitter reflectance. The conditional increase of the final energy over the initial one, $(E_f - E_0)/E > 0$, is achieved for all parameters and is sufficiently higher than in the system of $N = 2$ qubits, Fig. 4. Gained energy gradually decreases with increasing parameter R. The relative entropy of coherence maxima values vary with parameter R .

Charging with $N = 4$ of TLS

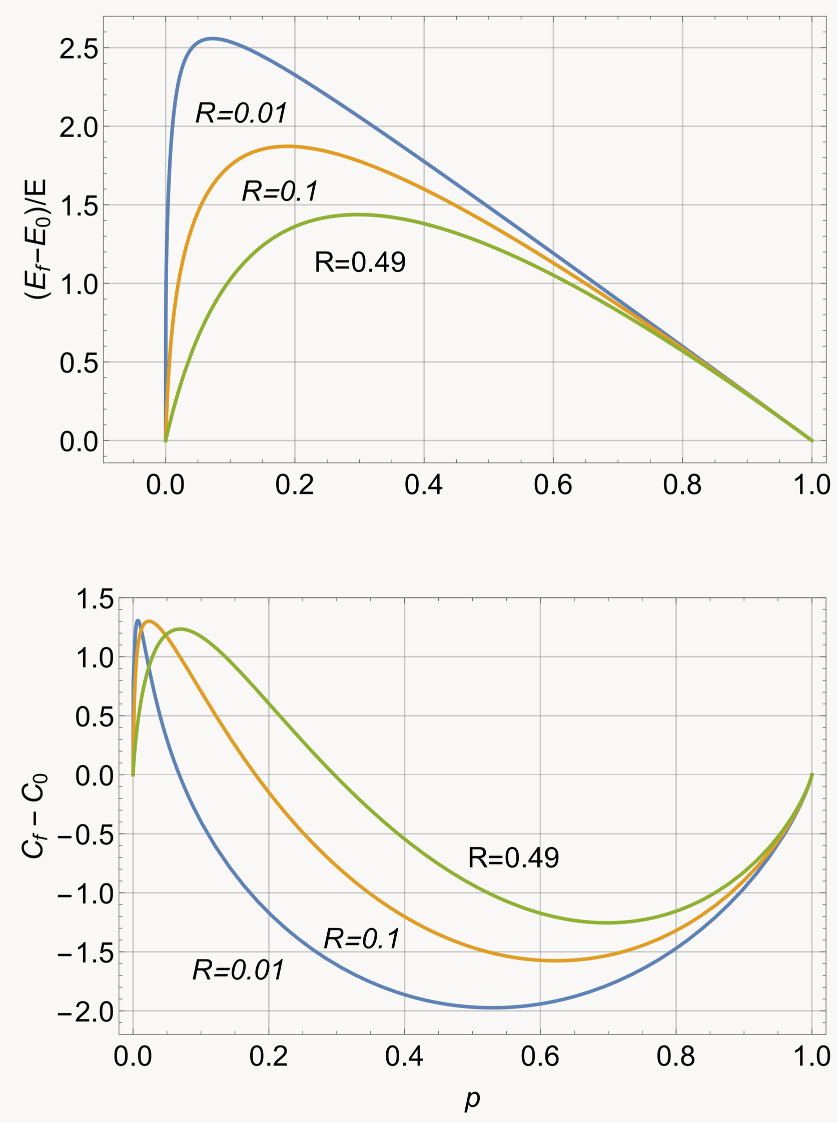

Figure 6: The plot of the normalized average energy difference $(E_f - E_0)/E$ and the relative entropy of coherence difference $(C_f - C_0)$ for the final state $|\tau\rangle$, Eq. (2), and the initial state $|\Psi\rangle$, Eq. (1), of the battery of $N = 4$ qubits. The difference is plotted versus the higher level excitation probability p, Eq. (3). The curves are parametrized by the parameter R, the beam splitter reflectance. The amount of gained energy varies with reflectance R and is significantly bigger than for systems of $N = 2$ qubits, Fig. 4, and $N = 3$ qubits, Fig. 5. The maxima of the coherence $C_f - C_0$ is gradually decreasing with the growth of R.

References

[1] Felix C. Binder et al. Quantacell: Powerful charging of quantum batteries. New J. Phys. 17, 075015(2015).
[2] James Millen and André Xuereb. The rise of the quantum machines. Phys. Rev. Lett. 29(1) 23(2016).
[3] Lucie Bartůšková et al. Optical implementation of the encoding of two qubits to a single qutrit. Phys. Rev. A 74, 022325(2006).
[4] T. Baumgratz, M. Cramer, and M.B. Plenio. Quantifying coherence. Phys. Rev. Lett. 113, 140401(2014).

Conclusions

- We have presented a charging scheme, consisting of application of BS and projective measurement, capable of simultaneous increase of energy and relative entropy of coherence of a system, composed of 2 photons distributed over four fibers. Such settings represent a pair of noninteracting two-level systems (TLS) each having a certain energy splitting E. The results for such charging scheme are shown in Fig. 3.
- There is a possibility to make projective measurement on one photon in the ground state, it gives increase in the energy of our battery, however with increasing the number of qubits N the energy gain doesn't increase as well. Therefore it is not suitable for our purpose.
- Incoherent and coherent quantum states charge our battery for the same amount of energy, however, the former stay incoherent during the process, while the latter gain coherence within small p.
- The charging procedure was generalized to the case of a battery, consisting of N copies of qubits with energy gap E. The positive effect is preserved for higher number of qubits N, namely, we have increase in the energy gain with higher number of two-level systems.

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