# The optimal strategy for photonic quantum tomography

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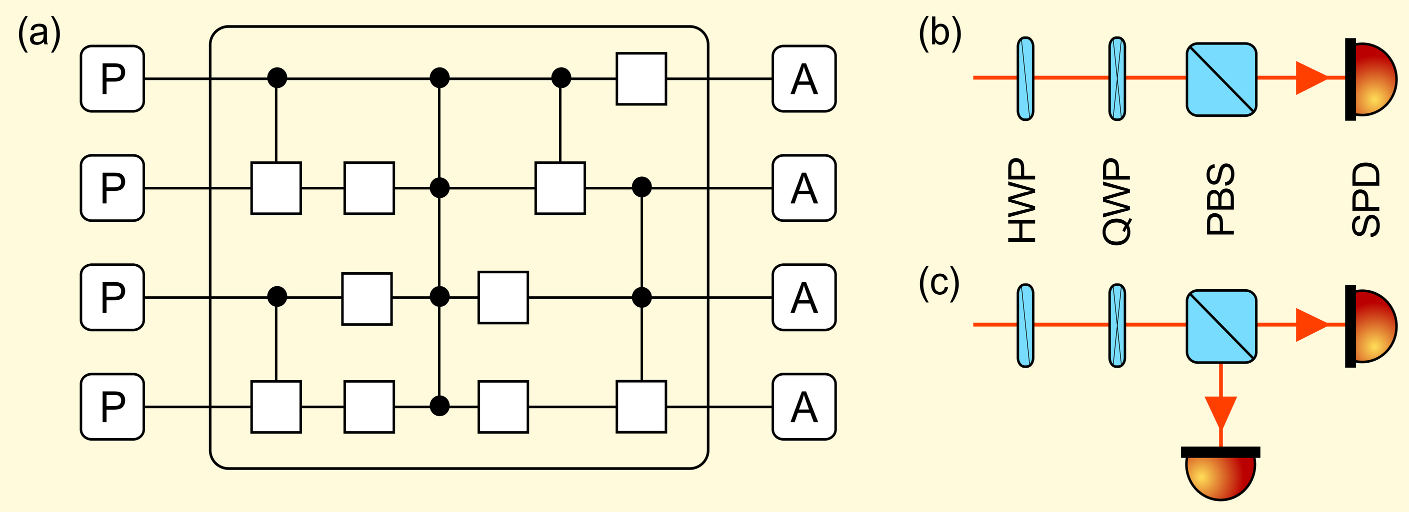
## Abstract

We report quantum tomography speedup by properly arranging the individual component measurements. For a quantum system of up to five polarization encoded optical qubits, we obtain the optimal order of tomographical polarization projections, minimizing the total time spent on reorienting wave plates between projection measurements. An increase in the speedup is seen with each added qubit, with a speedup factor reaching 2 already for three-qubit state tomography. The speedup has been verified experimentally for quantum state tomography and also for full quantum process characterization, without any changes needed to be made to the optical setup hardware-wise. The same technique can be used to speed up an input-output characterization of various optical transformations and scattering processes.

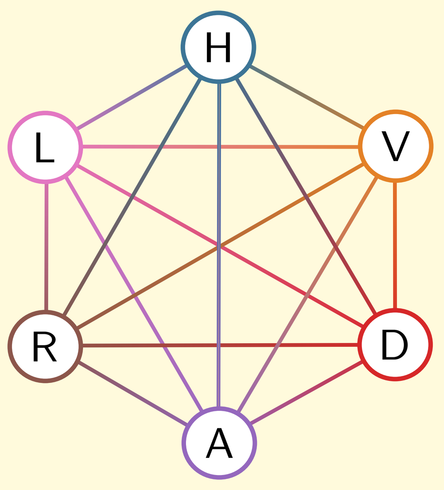
## Quantum state preparation and analysis schematic




(a) Characterization of a quantum circuit consists of probing (P) the input qubits and analysing (A) the output qubits. (b) Both probing and analysis of polarization-encoded qubits rely on a sequence of wave plates (half-wave, HWP; quarter-wave, QWP), a polarization beam splitter (PBS), and a single-photon detector (SPD, analysis only). The wave plates are rotated to perform the projections to particular polarization states. (c) When both outputs of the PBS are detected by SPDs, two orthogonal projections are measured at the same time.

## The traveling salesman problem (TSP) in tomography: one-qubit example

For one-qubit tomography, six polarization projection measurements are needed. The six measurements and all the possible transitions between them can be visualized as a graph:




| | $\alpha_{\mathrm{HWP}}$ | $\alpha_{\mathrm{QWP}}$ |
|---|---|---|
| H | 0 | 0 |
| V | 45 | 0 |
| D | 22.5 | 0 |
| A | -22.5 | 0 |
| R | 0 | 45 |
| L | 0 | -45 |

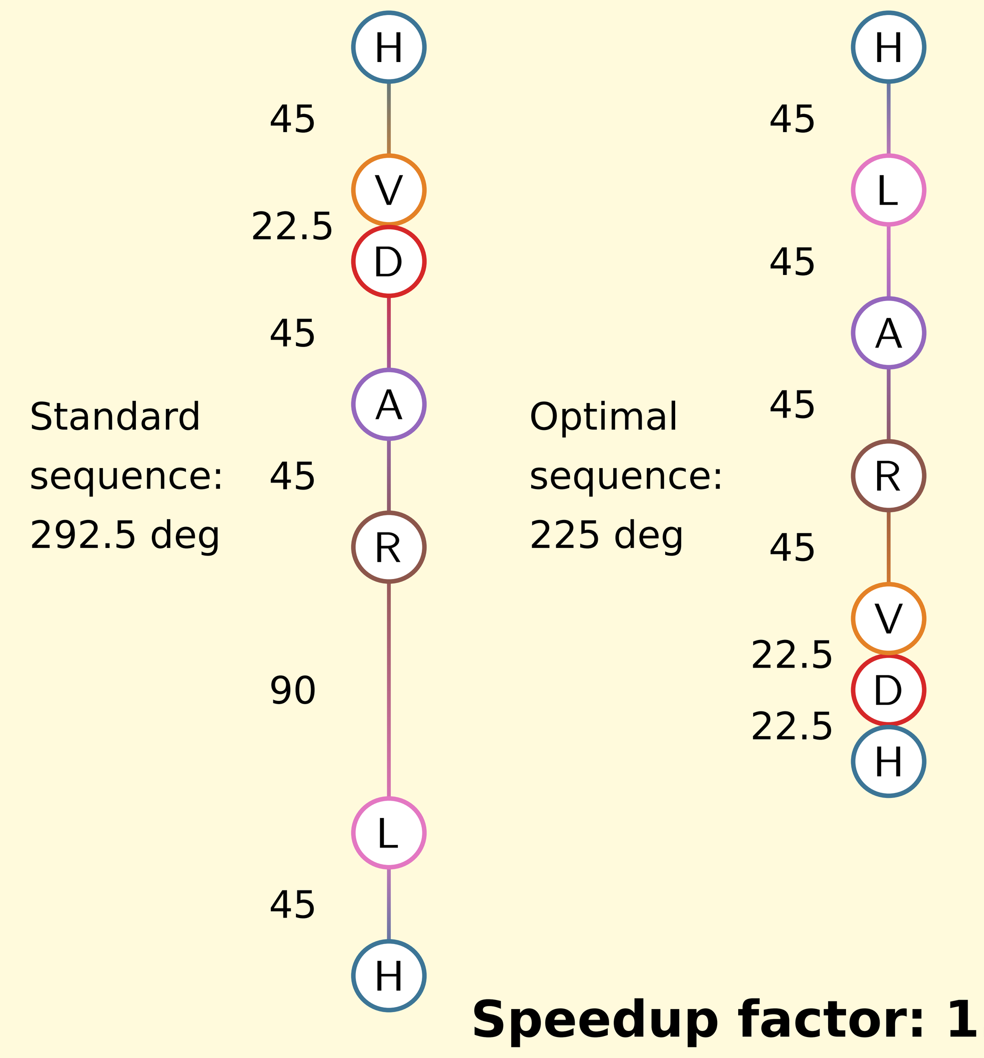
(a)

| | H | V | D | A | R | L |
|---|---|---|---|---|---|---|
| H | 0 | 45 | 22.5 | 22.5 | 45 | 45 |
| V | 45 | 0 | 22.5 | 67.5 | 45 | 45 |
| D | 22.5 | 22.5 | 0 | 45 | 45 | 45 |
| A | 22.5 | 67.5 | 45 | 0 | 45 | 45 |
| R | 45 | 45 | 45 | 45 | 0 | 90 |
| L | 45 | 45 | 45 | 45 | 90 | 0 |

(b)

(a) Wave plate angles for the six polarization projections. The TSP can be formally specified in the form of adjacency matrix (b). Its elements indicate for each transition the absolute value of the largest angle that any of the wave plates has to be rotated by. This matrix is given as input to a TSP solver.

The solver finds the order of polarization projection measurements that minimizes the total angle traveled by the waveplates during tomography, which leads to a reduction of the temporal duration of the procedure.




**Speedup factor: 1.3**

## Adding qubits - increasing speedup

For the tomography of a multi-qubit system it is not enough to perform the one-qubit procedure for each individual qubit. It is necessary to measure the projection onto every combination of the single-qubit polarization states. This leads to a $p^n$ scaling of the number of projection measurements, $p$ being the number of measured polarization projections for a single-qubit case, and $n$ being the number of qubits in the quantum system.

Below: the speedup factor - the amount by which the total duration of a procedure is reduced using the TSP optimization - for analysis with either one SPD (blue squares) or two SPDs (orange circles). Full symbols represent calculated speedup, empty symbols correspond to speedup achieved in an experimental setting. The speedup factor for the single-SPD scheme reaches a value of 2 for three-qubit systems already. For greater systems, duration reduction in the orders of tens of minutes, or even hours, can be achieved with TSP-optimized procedures.

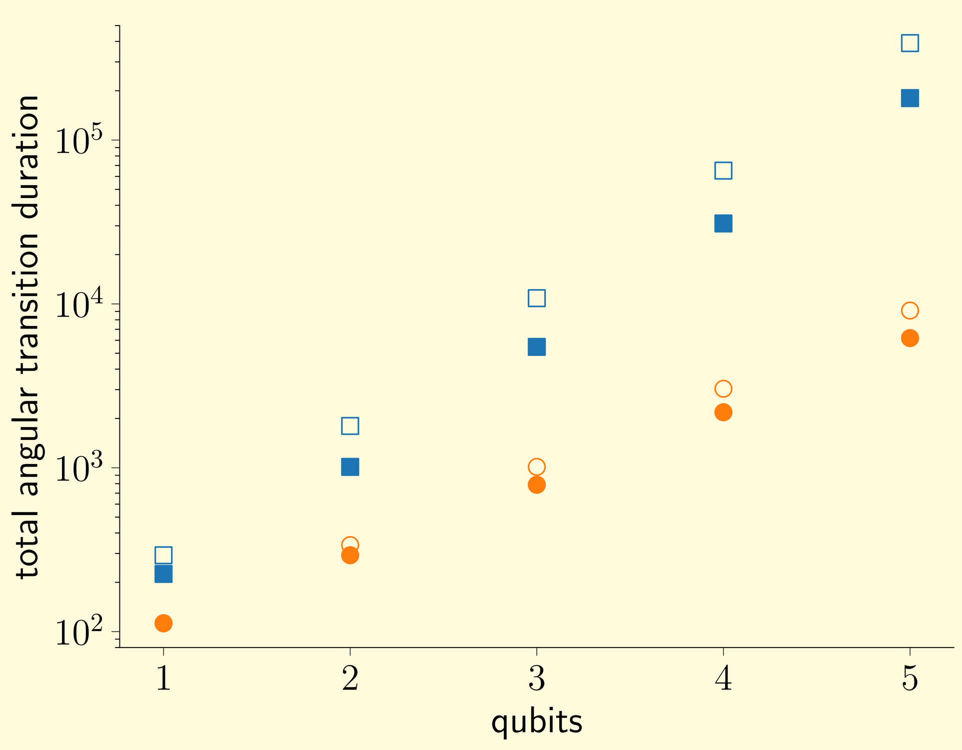

Characterization of a 3-qubit control-control-phase gate (**measurement sped up from original 23 hours to 11 hours**)

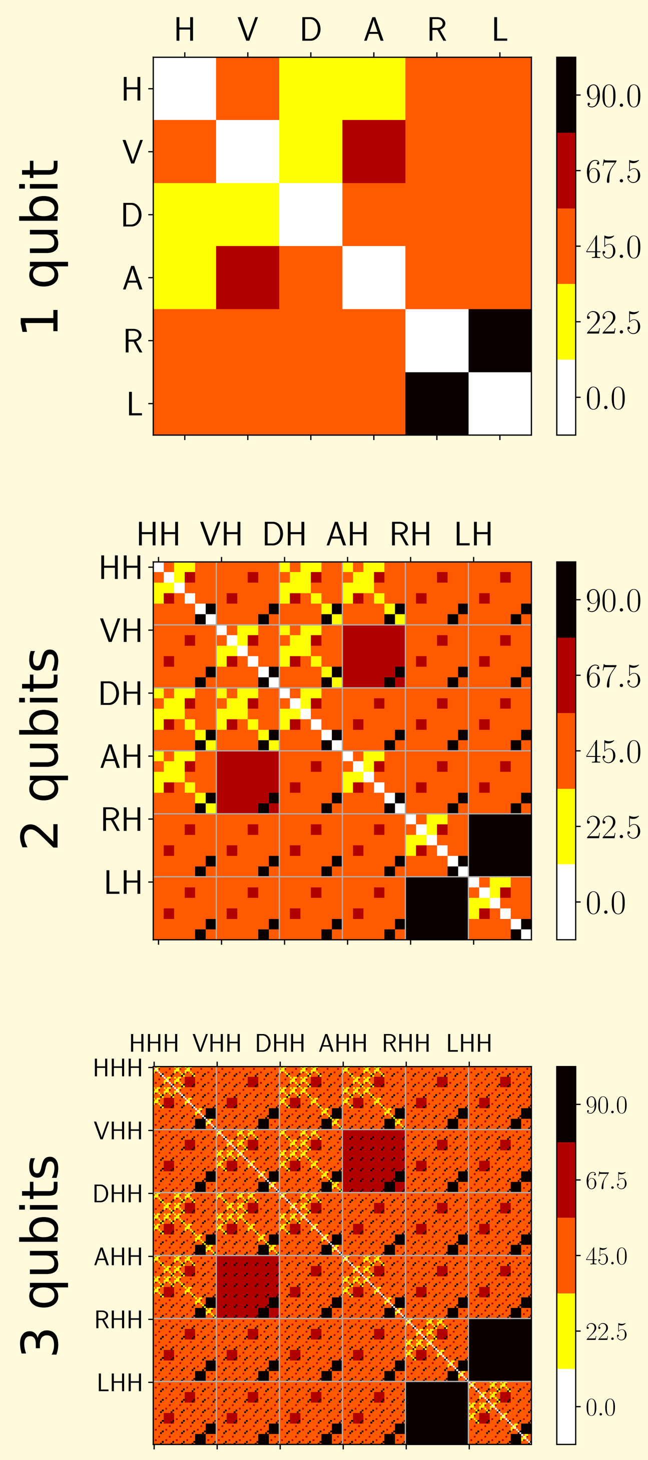
The TSP-based optimization of quantum state, or process, characterization was performed for quantum systems of up to five qubits. Both analysis configurations, with either one or two SPDs,were considered.

The TSPs were formulated and solved analogously to the single-qubit example above. On the right, TSP adjacency matrices are shown for 1-qubit, 2-qubit, and 3-qubit problems (single-SPD configuration).

While the TSP-optimized procedures are faster than the conventionally ordered ones, the scaling itself is not affected. Nevertheless, it is important to notice that despite this fact, a significant amount of time can be saved. For systems of larger sizes, this can be hours.

Below: the $p^n$ scaling of characterization of quantum states and processes. The analysis scheme with one SPD is represented by the blue squares, the two-SPD scheme is shown using orange circles. Empty symbols represent the conventionally ordered procedures, while full symbols represent the TSP-optimized ones.




Above: the adjacency matrices for TSP problems of this kind share an interesting property. As we increase the number of qubits, we can see that self-similar pattern forms in the sequence of matrices.

## Acknowledgements

This work was supported by the Czech Science Foundation (project 17-26143S). MJ acknowledges the support of HYPER-U-P-S project funded from the MEYS and European Union's Horizon 2020 (2014-2020) research and innovation framework programme under grant agreement No 731473 (project No 8C18002). RH and RS also acknowledge the support by the Palacký University (project IGA-PrF-2018-010).