Multiparameter quantum metrology of incoherent point sources

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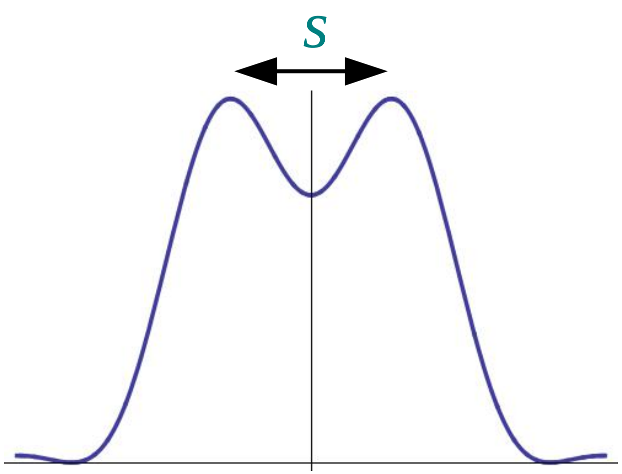
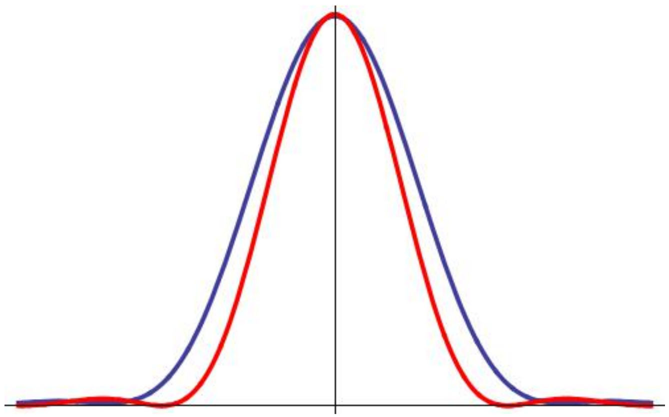
two-point resolution

- linear invariant system
- PSF – point spread function
- two mutually incoherent point sources



standard resolution limit

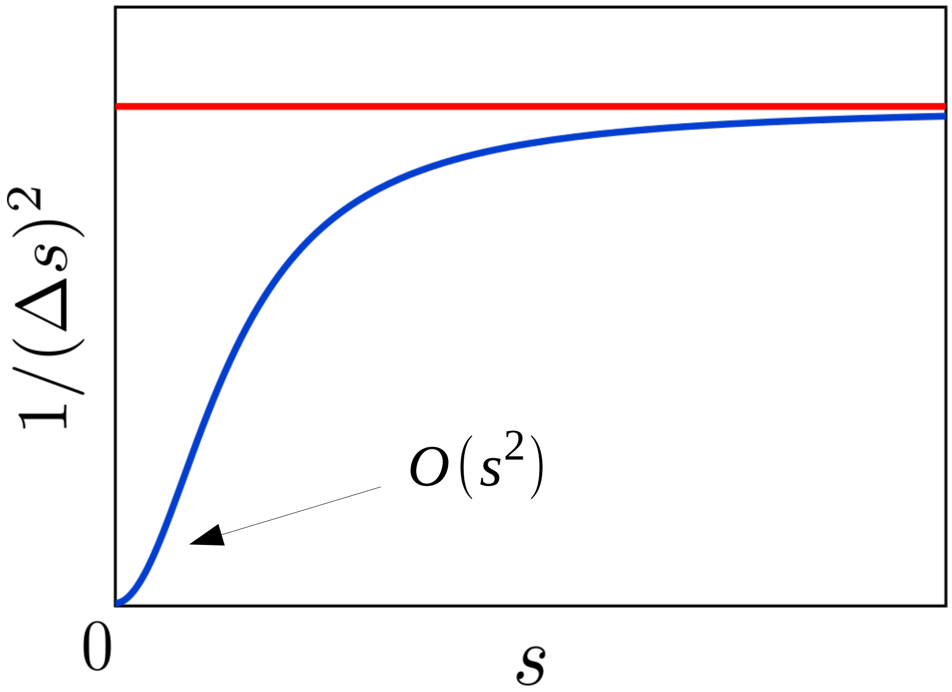


super-resolution

- detection noise sets the ultimate limits

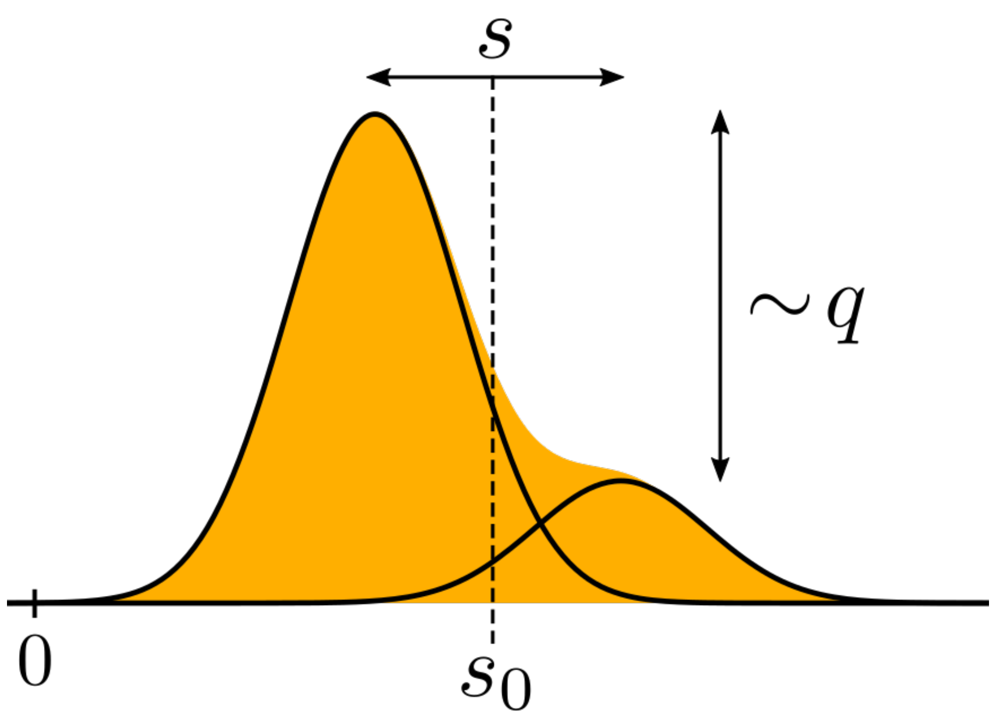
efficient unbiased estimators

direct CCD detection vs quantum CRLB for equally bright sources



- turns resolution into localization (M. Tsang and others)
- super-resolution is cheap (in theory ...)

multiparameter case



real impulse response $\psi(x) = \langle x|\psi\rangle = \psi(x)^*$

signal $\rho(s_0, s, q) = q|\psi_+\rangle\langle\psi_+| + (1-q)|\psi_-\rangle\langle\psi_-|$

signal components $|\psi_\pm\rangle = e^{-i(s_0 \pm s/2)P}|\psi\rangle$

QFI

rank 2 signal $\longrightarrow$ eigenstates expressed in terms of $|\psi_\pm\rangle$

QFI

$$Q = \begin{pmatrix} \cdots & 4(q-1/2)p^2 & \cdots \\ 4(q-1/2)p^2 & p^2 & 0 \\ \cdots & 0 & \dfrac{1-w^2}{q(1-q)} \end{pmatrix}$$

overlap $\quad w = \langle \psi_\pm | \psi_\mp \rangle$

PSF width $\quad p^2 = \langle \psi | P^2 | \psi \rangle$

- separation, centroid and intensity should be estimated simultaneously

bounds on precisions

precisions

$$H_s = 1/Q_{ss}^{-1} > 1/(\Delta s)^2$$

quantum limits for unbalanced sources

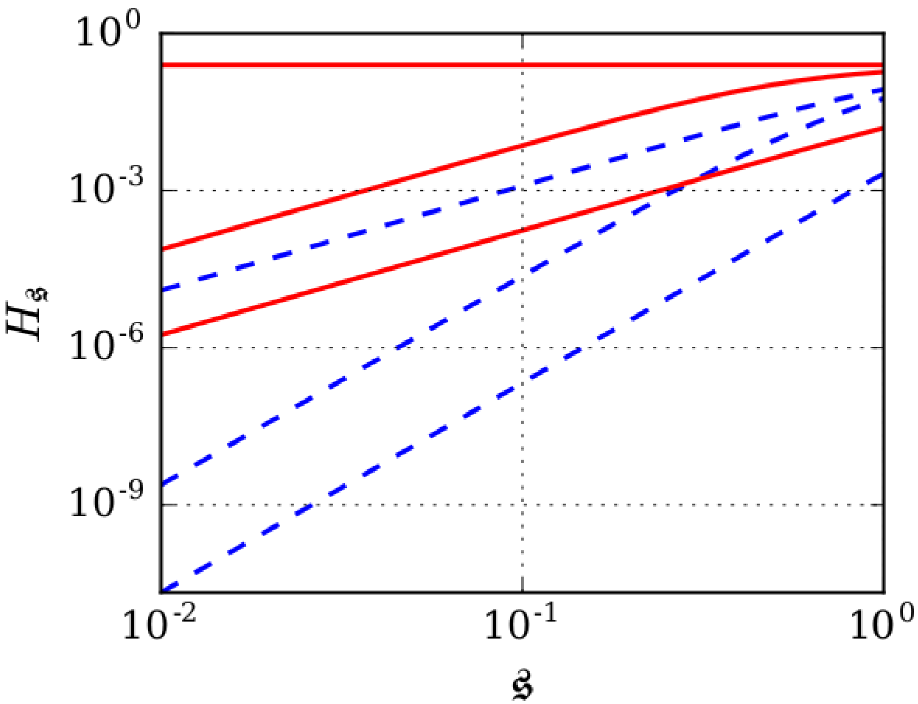
$$\begin{aligned} H_{s_0} &\propto s^2 \\ H_s &\propto s^2 \qquad , \quad s \ll 1 \\ H_q &\propto s^4 \end{aligned}$$

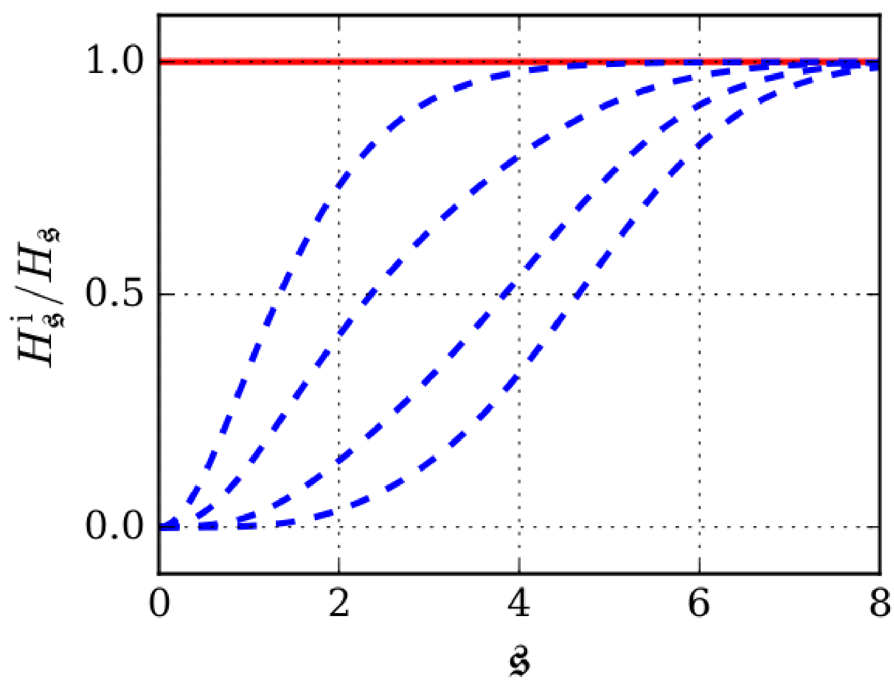
signal PSF enters through

$$\mathrm{Var}(P^2) = \langle \psi | P^4 | \psi \rangle - \langle \psi | P^2 | \psi \rangle^2$$

example: Gauss PSF

Gauss PSF $\sigma = 1$





- quantum improvement is always $\propto 1/s^2$
- improvement gets larger for more unbalanced signals

optimal measurements

super-resolution regime $s \ll 1$

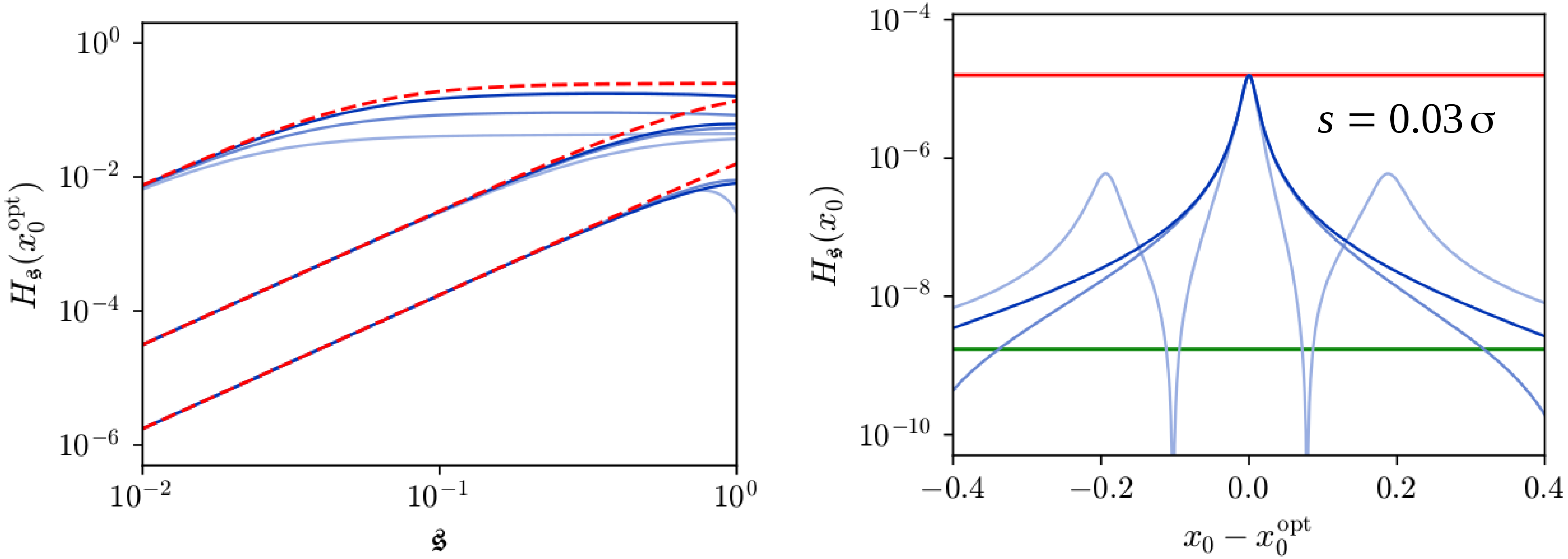
3 parameters $\longrightarrow$ 4-channel minimal POVM

PSF independent formulation – convenient basis

- PSF derivatives $\langle x|\psi_n\rangle = \frac{\partial^n}{\partial x^n}\psi(x-x_0)$
- orthonormalization $|\psi_n\rangle \to |\phi_n\rangle$
- measurement states $|\pi_j\rangle$, $j=0,1,2$, opt. conditions $\langle \pi_{0,1}|\phi_0\rangle = 0$
- displacement optimization $x_0 \to x_0^{\mathrm{opt}}$
- family of measurements saturating the QCRLB for all the parameters !

example: Gauss PSF

one particular family of optimal measurements applied to Gauss PSF



- performance at moderate separations and robustness varies among optimal sets
- adaptive strategies

conclusions

- QFI was derived for multiparameter two-point resolution with arbitrary real PSFs
- Rayleigh curse reappears in multiparameter scenarios
- still quantum measurements bring about significant improvements over the direct CCD imaging
- optimal 4-channel measurements were discussed attaining the QCRLB in the super-resolution regime