Continuous Variable Quantum Cryptography

Towards High Speed Quantum Cryptography

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OP Vzdělávání pro konkurenceschopnost
INVESTICE DO ROZVOJE VZDĚLÁVÁNÍ

Palacký University, Olomouc, 2011

Conditions for Perfect Secrecy

Alice sends a secret message to **Bob**

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Alice sends a secret message to **Bob** through a channel observed by **Eve**.

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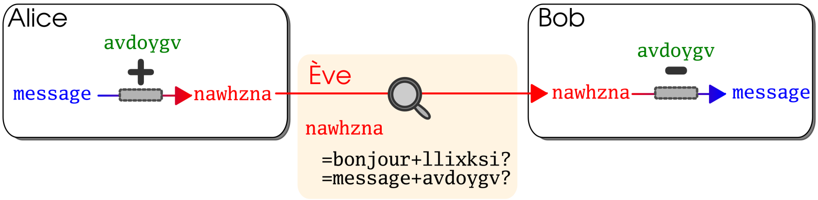

She encrypts the message with a secret key

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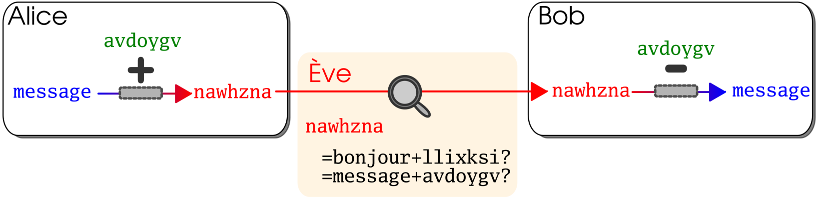

She encrypts the message with a secret key as long as the message.

Quantum Key Distribution

Alice sends quantum objects to Bob

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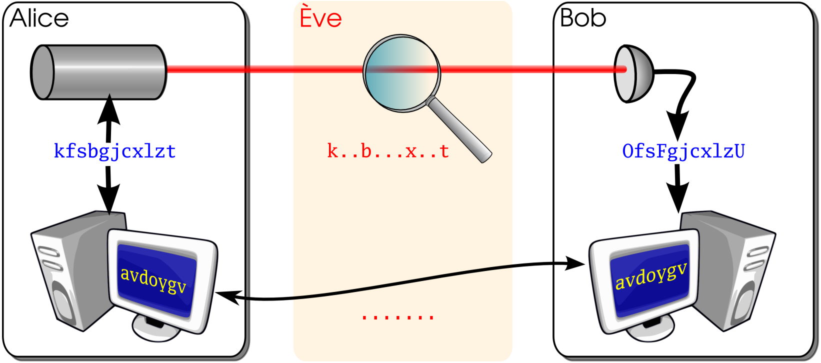

Eve's Measurenents

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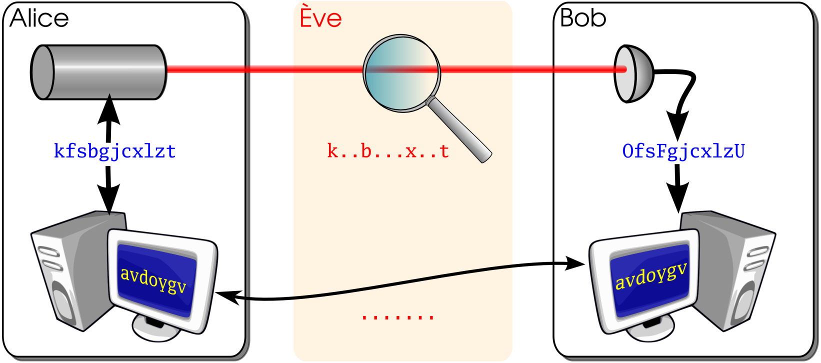

Eve's Measurenents ⇒ measurable perturbations

⇒ secret key generation

Unconditionnally Secure Systems . . .

Single Photon QKD

- Long Range ($\sim$ 100 km)
- Low rate (kbit/s)

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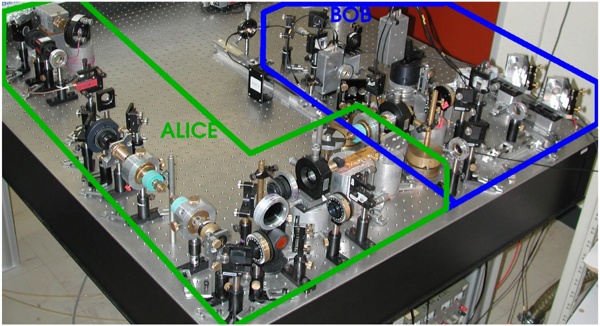
  - 1 CD / year = 180 bits/s
  - 1 iPod (160 GB)/ year = 40 kbit/s
- But the data has to stay here

. . . and Continuous Variable

- Medium Range :∼ 25 km
- Medium Rate :∼ a few kbit/s



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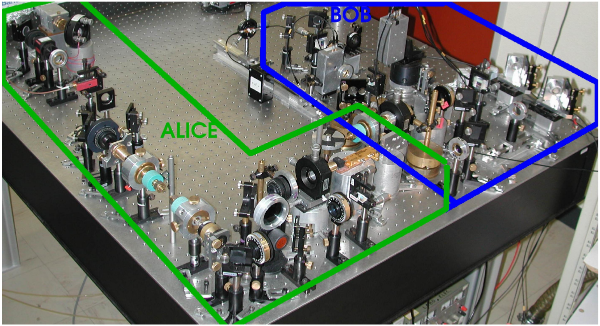
- Medium Range :∼ 25 km
- Medium Rate :∼ a few kbit/s
- Much less mature



. . . and Continuous Variable

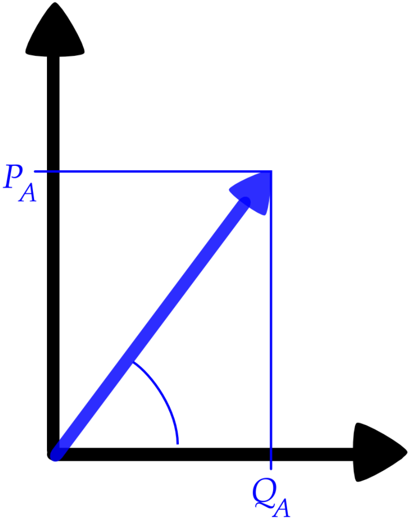
- Medium Range :∼ 25 km ; 80 km soon ?
- Medium Rate :∼ a few kbit/s ; Mbits/s soon ?
- Much less mature ⇒ Much room for improvements

Field quadratures

Classical field

Electromagnetic field described by Q_A and P_A

$E(t) = Q_A \cos \omega t + P_A \sin \omega t$



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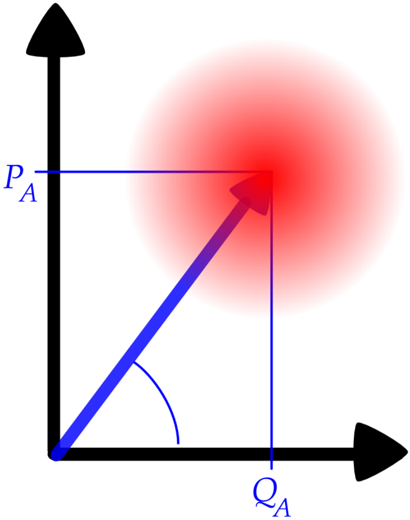
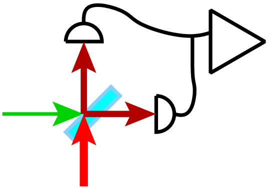
Quantum description

Q and P do not commute:
$$[Q, P] \propto i\hbar.$$
Add a "quantum noise":
$$Q = Q_A + B_Q \text{ et } P = P_A + B_P$$
Heisenberg $\Longrightarrow \Delta B_Q \Delta B_P \geq 1$



Homodyne Detection : Theory



Photocurrents:

$$i_\pm \propto \overline{(E_{\text{osc.}}(t) \pm E_{\text{signal}}(t))^2}$$
$$\propto \overline{E_{\text{osc.}}(t)^2 \pm 2E_{\text{osc.}}(t)E_{\text{signal}}(t)}$$

after substraction:

$$\delta i \propto \overline{E_{\text{osc.}}(t)E_{\text{signal}}(t)}$$
$$\propto \boxed{E_{\text{osc.}}(Q_{\text{signal}} \cos\varphi + P_{\text{signal}} \sin\varphi)}$$

XXth century CVQKD

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Problem : discrete bits $\neq$ continuous variable

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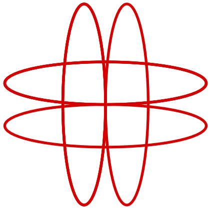
Adapting BB84?

Mark Hillery, "Quantum Cryptography with Squeezed States",
`arXiv:quant-ph/9909006`/PRA **61** 022309



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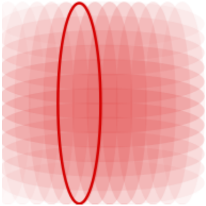
Problem : discrete bits $\neq$ continuous variable

Natural modulation + information theory!
Nicolas J. Cerf, Marc Lévy, Gilles Van Assche : "Quantum distribution of Gaussian keys using squeezed states", `arXiv:quant-ph/0008058`/PRL **63** 052311



Where are the bits ?

Quite frequent discussion with discrete quantum cryptographers :

DQC : How do you encode a 0 or a 1 in CVQKD?

Me : I don't care, C. E. Shannon tells me "$\forall \varepsilon > 0, \exists$ code of rate $I - \varepsilon$."

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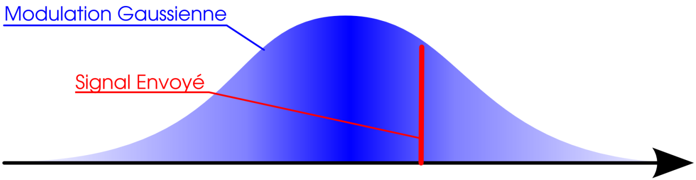
Computation of the ideal code performance is easy !

They're hidden

Availaible informationin a continuous signal



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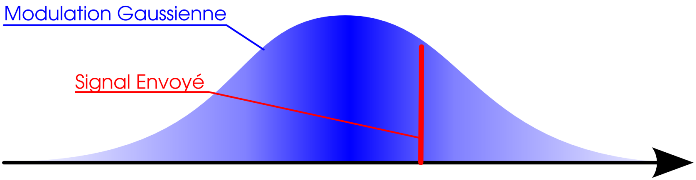

Differential entropy

$$H(X) = -\sum \mathcal{P}(x)\,dx \log \mathcal{P}(x)\,dx$$

$$\simeq \underbrace{\int dx\, \mathcal{P}(x) \log \mathcal{P}(x)}_{\mathcal{H}(X)} - \underbrace{\log dx}_{\text{constante}}$$

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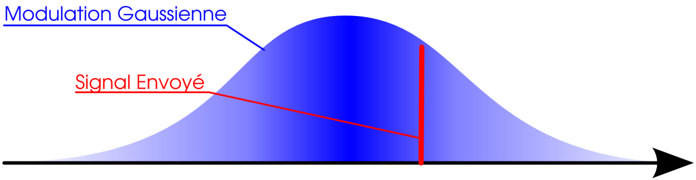
with noise ?



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$$\mathcal{H}(X) = \log \Delta X + \text{constante}$$

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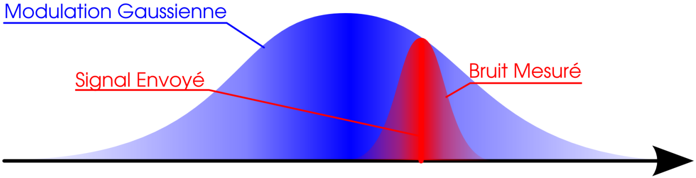

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Mutual information

$$\begin{aligned} I(X:Y) &= H(Y) - H(Y|X) \\ &= \mathcal{H}(Y) - \mathcal{H}(Y|X) \\ &= \frac{1}{2} \log \frac{\Delta Y^2}{\Delta Y^2|X} \end{aligned}$$

The spy's power

Heisenberg :

$$\Delta B_{\text{Eve}} \Delta B_{\text{Bob}} \geq 1$$

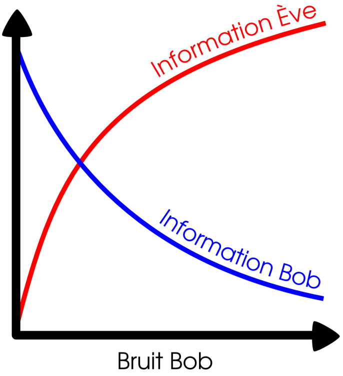
The spy's power

Heisenberg :

$$\Delta B_{\text{Eve}} \Delta B_{\text{Bob}} \geq 1$$

$\Rightarrow \Delta B_{\text{Bob}}$ gives

- I_{Eve}
- I_{Bob}



Quantum Key Distribution Protocols

Channel Evauation (noise measure)

Alice&Bob evaluate I_{Eve}

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Privacy Amplification

Alice&Bob share $I_{\text{Bob}} - I_{\text{Eve}}$ identical bits.

Ève knows ~ 0.

Theoretical Progresses in the last 10 years

We went from a protocol

- using squeezed states,
- insecure beyond 50% losses (15 km),
- proved secure against Gaussian individual attack

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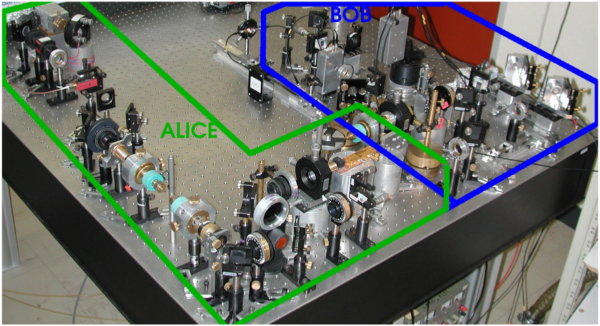
- and experimentally working

1st generation demonstrator

F. Grosshans *et. al.*, *Nature* (2003) & Brevet US

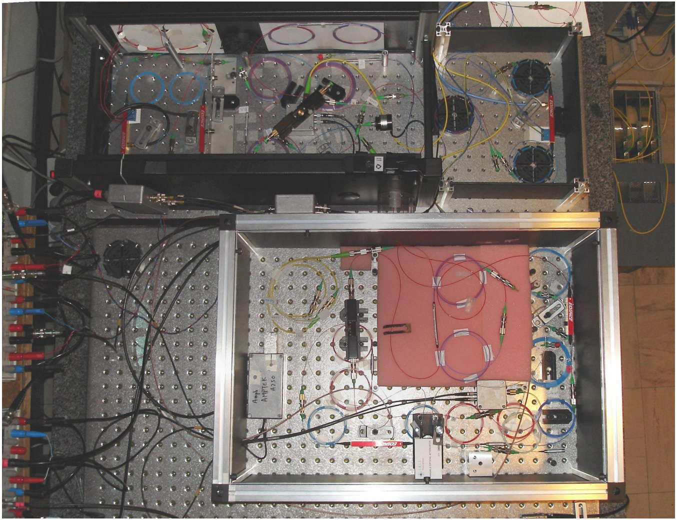


m

Key rate

- 75 kbit/s 3.1 dB (51%) losses
- 1.7 Mbit/s without losses

Integrated system



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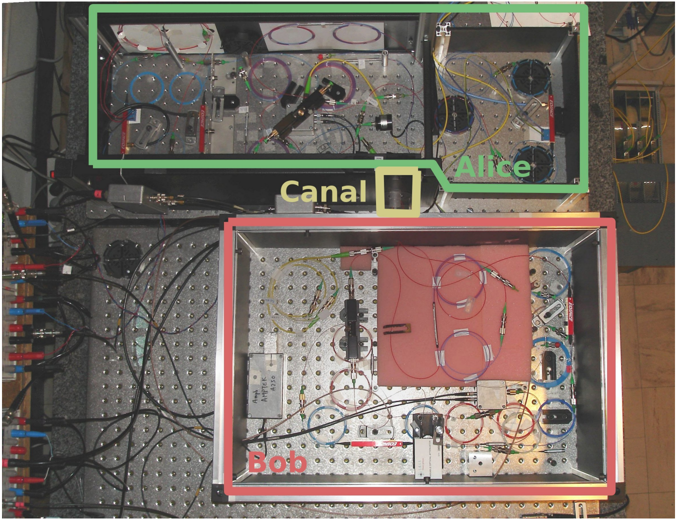

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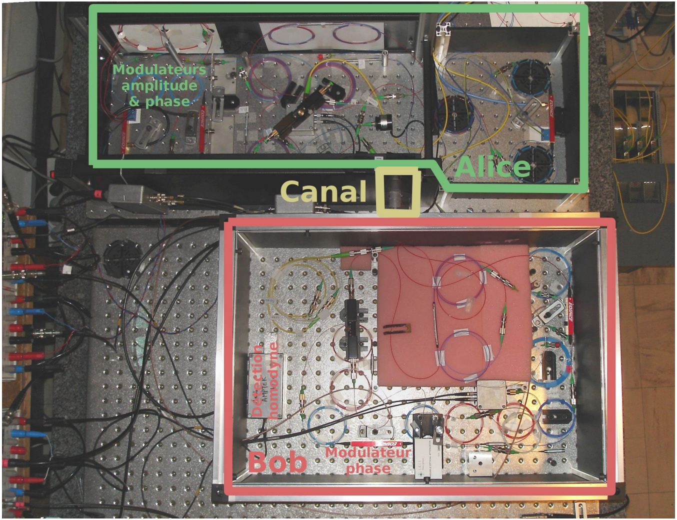
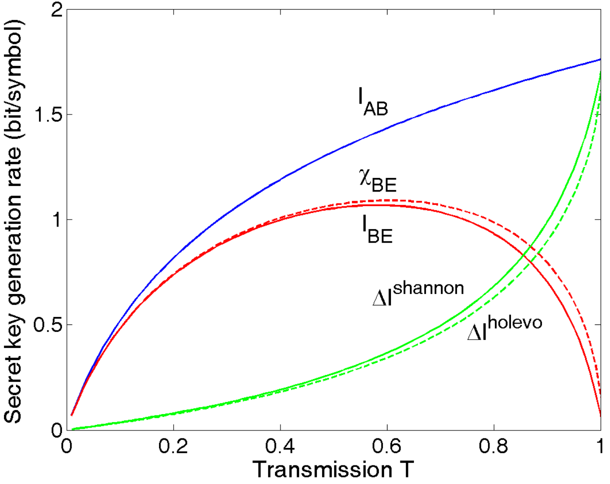
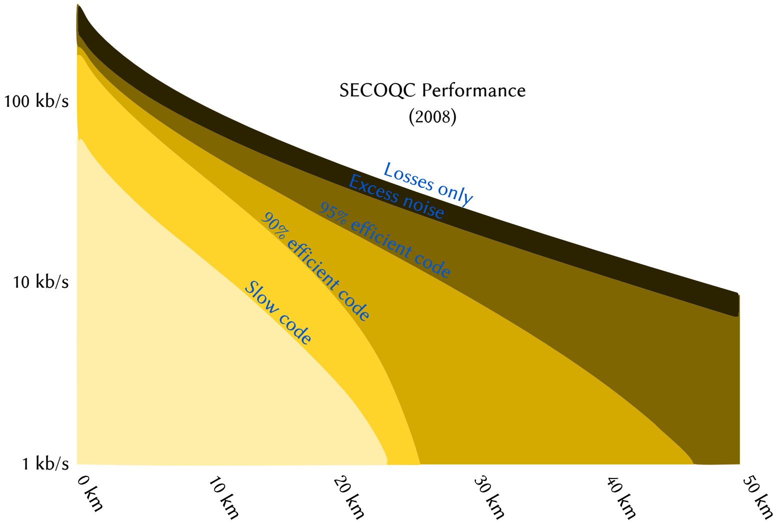


Key-Rates

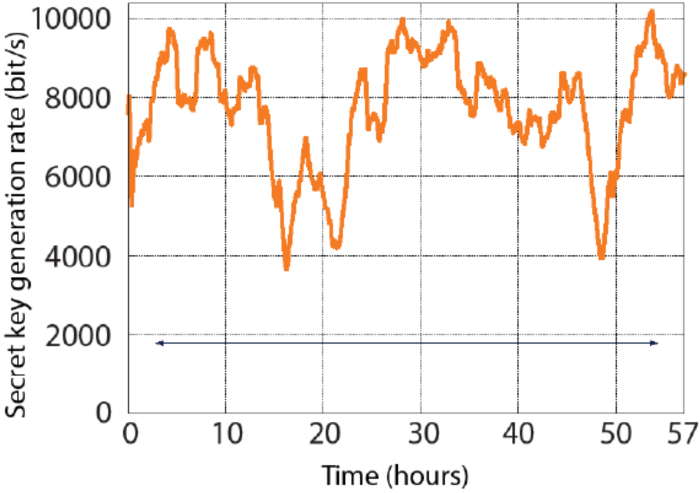


Secret key generation rate (bit/symbol)
Transmission T
I_{AB}
χ_{BE}
I_{BE}
$\Delta I^{shannon}$
ΔI^{holevo}
0
0.5
1
1.5
2
0
0.2
0.4
0.6
0.8
1

Key-Rates

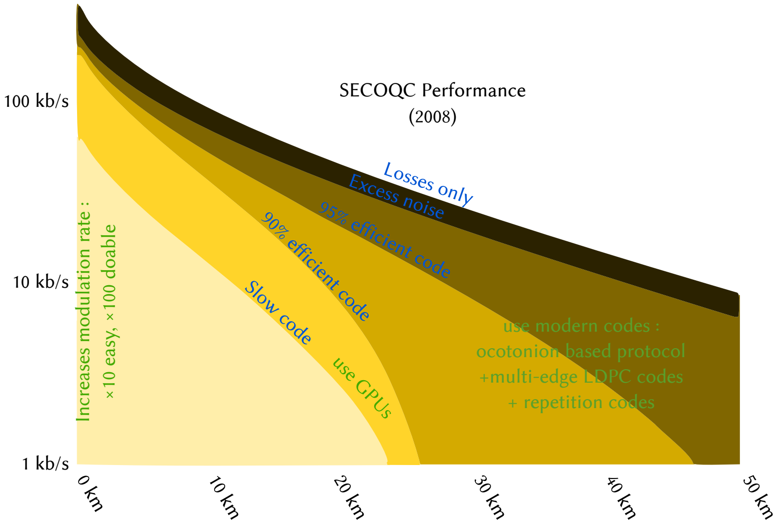

SECOQC Performance
(2008)
100 kb/s
10 kb/s
1 kb/s
0 km
10 km
20 km
30 km
40 km
50 km
Losses only
Excess noise
95% efficient code
90% efficient code
Slow code

Key-Rates



10000
8000
6000
4000
2000
0
Secret key generation rate (bit/s)
0
10
20
30
40
50
57
Time (hours)

Key-Rates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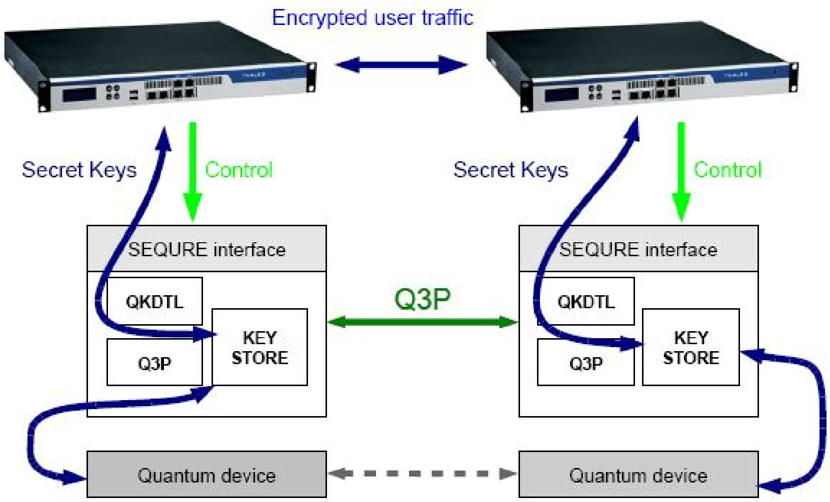
30 km
40 km
50 km
Losses only
Excess noise
95% efficient code
90% efficient code
Slow code
use GPUs
Increases modulation rate :
×10 easy, ×100 doable
use modern codes :
ocotonion based protocol
+multi-edge LDPC codes
+ repetition codes

Integration with classical cryptography



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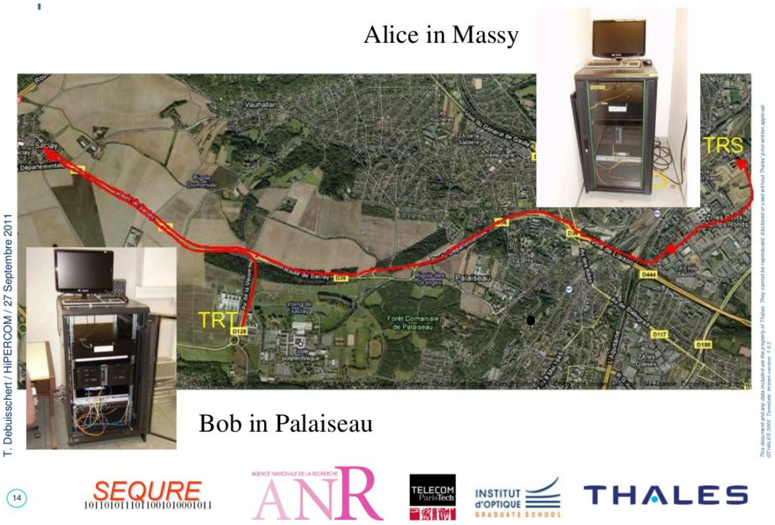

Open Problems

- Finite size effects
- Link with post-selection based protocols (`.de`, `.au`)
- Side-channels and quantum hacking
- Other cryptographic applications